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Nonlocal vibration of DWCNTs based on Flügge shell model using wave propagation approach

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Abstract. In this article, free vibration attributes of double-walled carbon nanotubes based on nonlocal elastic shell model have been investigated. For this purpose, a nonlocal Flügge shell model is established to observe the small scale effect. The wave propagation is employed to frame the governing equations as eigenvalue system. The influence of nonlocal parameter subjected to different end supports has been overtly examined. A suitable choice of material properties and nonlocal parameter been focused to analyze the vibration characteristics. The new set of inner and outer tubes radii investigated in detail against aspect ratio and length. The dominance of boundary conditions via nonlocal parameter is shown graphically. The results generated furnish the evidence regarding applicability of nonlocal shell model and also verified by earlier published literature.

Keywords: free vibration; nonlocal parameter; double-walled CNTs; Flügge shell model; wave propagation approach

1. Introduction

The rapid development of nano science and nano technology is phenomenal as echoed with an increase of its application in scientific research. Carbon nanotubes (CNTs) is such discovery by Iijima (1991), that may be used in a variety of fields like material reinforcement, aerospace, medicine, defense and microelectronic devices (Sosa *et al*. 2014, Soldano 2015, Fakhrabadi *et al*. 2015, Mouffoki *et al*. 2017, Bouadi *et al*. 2018). Owing the striking mechanical properties through the cylindrical mechanism CNT hold purposeful role in conveying fluid and gas. With a vast area of potential innovation, however CNTs demands more understanding to investigate its mechanical properties. Free vibration analysis of CNTs have been influential aspect in dynamical science for the last one decade. Vibration characteristics are investigated using thin shell theory by Yakobson *et al*. (1996), beam theory by Wang *et al*. (2006) and nonlocal beam theory (Zermi *et al*. 2015, Youcef *et al*. 2018). An eminent study found in based upon ring theory by Vodenitcharova and Zhang (2003) whereas theories of continuum models developed by Li and Chou (2003) in literature. Well known two main classes of models used to analyze the theoretical aspects of CNTs have been atomic model and other is continuum model. The classical molecular dynamics (MD) has shown to exceed those of other techniques such as tight-binding molecular dynamics and ab initio method included in class of atomic modeling (Iijima *et al*. 1996, Yakobson *et al*. 1997, Hernandez *et al*. 1998, Sanchez *et al*. 1999, Qian *et al*. 2002).

The main reason continuum mechanics (Yoon *et al*. 2003, Fu *et al*. 2006, Kuang *et al*. 2009, Ansari *et al*. 2011) turned noticeable tool is its computational capability to generate results of large range system in nanometer range. The nonlocal elasticity introduced by Eringen (1983, 2002) becomes a turning point as small scale effect was inculcated in to fundamental equations as simply material parameter. Therefore, scientific community now propose to apply nonlocal continuum models to investigate nano-structured materials (Sudak 2003, Wang *et al*. 2006, Pradhan and Phadikar 2009, Ansari *et al*. 2010, Hao *et al*. 2010, Amara *et al*. 2010, Shen and Zhang 2010). The first ever work presented on use of nonlocal elasticity was by Peddieson *et al*. (2003). Prominent computational competence and accuracy makes nonlocal models an attractive choice for further advancements in field. Donnell (1996) and Flügge (1962) have been two substantial shell theories practiced extensively in study of static and dynamic characteristics of CNTs. Flügge shell theory takes promising place to generate remarkably accurate developments to examine the CNTs. In another paper, Natuski *et al*. (2006) carried out the vibration analysis of nested CNTs in elastic matrix. Flügge shell theory again had been engaged to establish administrative shell equations while proposed method was wave propagation. Natuski and Qing *et al*. (2007) investigated single and double-walled CNTs filled with fluids by adopting wave propagation approach. Flügge shell theory was proposed to form governing equations of motion for CNTs. Rouhi and Ansari (2012) executed the axial buckling of double-walled CNT subject to various layer-wise conditions by using Rayleigh-Ritz based upon

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nonlocal Flügge shell theory. Their study showed that the number of different layer-wise boundary conditions dominates the choice of values for nonlocal parameter. Usuki and Yogo (2009) formed beam equations again based on Flugge shell theory, they concluded that if nonlocality and refined model are ignored then the generalized Beam theory and Flügge theory produce alike results. Further Wang and Zhang (2007) examined the bending and torsional stiffness of single-walled CNT applying the Flügge shell equations. They presented three-dimensional model of single-walled CNT in their work with effect of thickness. Ansari and Rouhi (2013) summarized the effect of small scale, geometrical parameter and layer-wise end conditions of double-walled CNT by adopting Flügge shell model (FSM). They depicted that the continuum model considering the nonlocal effect compels the short double-walled CNT more flexible.

In recent studies double-walled carbon nanotubes (DWCNTs) have been intensively attracted as that of single-walled CNT due to its effectively applicable thermal, mechanical and electronic features. Hu *et al*. (2008) reported a study on the transverse and torsion waves based on nonlocal shell model for single-walled and double-walled CNTs. Xu *et al*. (2008) modeled the nested tubes of double-walled CNT as separate elastic beam. Their work revealed that double-walled CNT had no change for a particular invariable frequency subject to distinct edge conditions. Using nonlocal Timoshenko beam theory, Ke *et al*. (2009) investigated free nonlinear vibrations of double-walled CNT and applied differential quadrature technique to derive frequency equations. Khosrozadeh and Hajabasi (2012) carried out vibration analysis of double-walled CNT subject to nonlinear van der Waals forces. The length of the tube with surrounding elastic medium was found with nonlocal parameters. Rouhi and Ansari (2013) adapted new numerical approach with nonlocal Donnell shell theory to inquire the small-scale effect on double walled-CNT depending on boundary conditions. Moreover, Benguidiab *et al*. (2014) explored the mechanical buckling features of zigzag double-walled CNT. A comprehensive research presented by Salvatore Brischetto (2015) to analyze the vibration characteristic of double-walled CNT by considering shell continuum model. The findings of article were evolved around effects of van der Waals interaction in terms of frequency ratio. . Further Rouhi *et al*. (2015) investigated the vibration analysis of the multi-walled CNT by developing nonlocal FSM and presented the frequency spectrum against layer wise boundary conditions.

Arani *et al*. (2016) used the nonlinear buckling of SWCNTs resting on elastic foundation. The mixture rule was employed for buckling analysis of embded CNTs with Euler and Timoshenko beam model. The influence of geometrical parameter and elastic foundation with different boundary conditions was investigated. Ehyaei and Daman (2017) investigated the vibration characteristics of SWCNTs and DWCNTs using initial perfection and continuum mechanics approach. The general equation of motion was obtained by Hamiltonian principle and energy equivalent model. The numerical frequencies of DWCNTs and SWCNTs were determined by Navier method and finite element method. Bilouei *et al*. (2016) and Zamanian *et al*. (2017) studied the buckling behavior of concrete columns with nanofiber reinforced polymer and SiO_2 nano-particles. By using the strain-displacements, Hamilton's principles and Mori- Tanka approach, the governing equation was derived. Numerical results were presented with the variation of elastic foundations. Madini *et al*. (2016) investigated the vibration of embedded FG-CNT-reinforced piezoelectric cylindrical shells using differential quadrature method (DQM). The mixture rule of four different types of distribution was used in the thickness direction. Kolahchi and Reza (2017) and Kolahchi *et al*. (2017c, d) studied the bending and buckling of viscoelastic and non-viscoelastic sandwich nanocomposits using DQM, zigzag theory and Grey Wolf algorithm. Numerical results for volume fraction, and piezoelectric layers for the role of actuator and sensor. Avcar (2019) presented the vibration of FG beam and effect of rotary inertia of beam by the process of manufacturer. The thickness was controlled by the rule of mixture with volume fraction law. The governing equation was derived by classical theory with power law. The frequencies for span to depth ratio with varying volume fraction index were examined in detail. Semmah *et al*. (2019) investigated the buckling analysis of zigzag single walled boron nitride based on Winkler foundation. The governing equation was taken into account with the shear deformation theory. Effect of different nonlocal parameter was investigated with closed form solution. Recently Hussain and Naeem (2019a, b, c, d) performed the vibration of SWCNTs based on wave propagation approach and Galerkin's method. Many material researchers used various methods for new results of nanocomposits (Akbaş 2015, Farahani and Barati 2015, Moradi-Dastjerdi 2016, Hussain and Naeem 2017, Hussain *et al*. 2017, Nikkar *et al*. 2017, Zarei *et al*. 2017, Kumar 2018, Hajmohammad *et al*. 2018a, Amnieh *et al*. 2018, Hajmohammad *et al*. 2018a, Fakhar *et al*. 2018, Hussain and Naeem 2018b, Hosseini *et al*. 2018, Jassas *et al*. 2019, Fatahi-Vajari 2019).

Vibration analysis of armchair DWCNTs are rarely done in recent past. A limited number of researchers performed analysis first time to investigate the vibration of DWCNTs (Wang *et al*. 2006, Natuski *et al*. 2007, Kuang *et al*. 2009, Shen and Zhang 2010, Ansari and Rouhi 2012, Ansari and Arash 2013). So far as reviewed from the literature, vibration response of armchair double-walled CNT using wave propagation approach based on nonlocal Flügge shell model (FSM) has not been investigated/assumed. Many material researchers calculated the frequency of CNTs using different techniques, for example, structural mechanics approach (Li and Chou 2003, Tahouneh 2017, Moradiand Payganeh 2017, Shafiei and Setoodeh 2017), non-local theory of elasticity (Kolahchi *et al*. 2019), differential quadrature method (Azmi *et al*. 2019), shear deformation theory (Arefi *et al*. 2018, Lei and Zhang, 2018), nonlocal continuum models (Sudak, 2003, Wang *et al*. 2006, Pradhan and Phadikar 2009, Ansari *et al*. 2010, Hao *et al*. 2010, Amara *et al*. 2010, Shen and Zhang 2010, She *et al.* 2019, Hussain *et al*. 2019, Asghar *et al*. 2019)), stress and strain theory (Karami *et al*. 2018), *quasi-3D beam* (Tlidji *et al*. 2019*)*, shell theory (Yakobson *et al*. 1996), Mori-Tanak

(MT) homogenization technique (Selmi and Bisharat (2018), beam theory (Wang *et al*. 2006), Flügge's shell model (Hussain *et al*. 2019b), atomic modeling (Iijima *et al*. 1996, Yakobson *et al*. 1997, Hernandez *et al*. 1998, Sanchez *et al*. 1999, Qian *et al*. 2002), Rayleigh-Ritz (Ansari and Rouhi 2012), Galerkin method (Do *et al*. 2019), isotropic truncated conical shell (Sofiyev *et al*. 2009) and axially loaded double beam system (Xiaobin *et al*. 2014, Sharma *et al*. 2019)). Moreover, the existing novel theoretical model contributes inventive computational outputs for the vibration of CNTs as compare to prior models presented (Iijima *et al*. 1996, Qian *et al*. 2002, Peddison *et al*. 2003, Sudak 2003, Natuski *et al*. 2006, Shen and Zhang 2010, Ansari and Rouhi, 2012).

The foremost intension of this paper to investigate vibrations characteristics of armchair double-walled CNT by means of nonlocal elasticity shell model. The nonlocal shell model is established by inferring the nonlocal elasticity equations into Flügge shell theory, which is our particular motivation. The suggested method to investigate the solution of fundamental eigen relations is wave propagation, which is a well-known and efficient technique to develop the fundamental frequency equations. It is keenly seen from the literature, no evidence is found concerning current model where such problem has been studied so it gave impetus to conduct present work. The specific influence of four different end supports based on nonlocal FSM such as clamped-clamped (FSM-CC), clamped-simply supported (FSM-CS), simply supported-simply supported (FSM-SS) and clamped-free (FSM-CF) with assorted values of nonlocal parameter and distinguish inner tube radii is examined in detail.

2. Formation of nonlocal Flügge shell equations

Eringen (1983, 2002) acquainted the nonlocal elasticity theory as the stress on a given reference point is a function of strain field at each point in the body. This is how simply scale effect is treated as material parameter in fundamental equations of problem. On the other hand, because of unique dependence of stress state on strain state, classical elasticity cannot be useful for the scale effect. According to nonlocal elasticity theory, the stress at a reference point $x$is taken as a function of strain field at all other points $x^{/}$ of the body. The basic expression in terms of the nonlocal stress tensor σ is written as follows

$$\sigma(x)=\int_V \lambda(\left|x-x^{/}\right|,\mu)t(x^{/})dV(x^{/}).\forall x\varepsilon V \tag{1}$$

where $\lambda(\left|x-x'\right|,\mu)$ stands for nonlocal modulus or attenuation function whose arguments are the Euclidean distance and t for macroscopic stress tensor. In $\mu=e_0a/l$ as a is the internal characteristic length (e.g., length of C-C bond, lattice parameter, granular bond), l an external characteristic length (e.g., crack length, wave length) and $e_o a$ be pertinent material parameter. The equivalent of the Eq. (1), in two-dimensional nonlocal elasticity theory can be written in differential form as

$$(1-(e_o a)^2\nabla^2)\sigma=t \tag{2}$$

The term e_0a describes the characteristic length known as nonlocal parameter. For stress tensor, the generalized Hook's law is used as

$$t=S:\epsilon \tag{3}$$

Here S reads as fourth order elasticity tensor and ":" as double dot product.Thus, the relationship between stress and strain is expressed as

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{\theta\theta}\\ \sigma_{x\theta}\end{Bmatrix}-(e_o a)^2\nabla^2\begin{Bmatrix}\sigma_{xx}\\ \sigma_{\theta\theta}\\ \sigma_{x\theta}\end{Bmatrix}=\begin{pmatrix}\frac{E}{1-\nu^2} & \frac{\nu E}{1-\nu^2} & 0\\ \frac{\nu E}{1-\nu^2} & \frac{E}{1-\nu^2} & 0\\ 0 & 0 & \frac{E}{2(1-\nu^2)}\end{pmatrix}\begin{Bmatrix}\sigma_{xx}\\ \sigma_{\theta\theta}\\ \sigma_{x\theta}\end{Bmatrix} \tag{4}$$

here E symbolizes Young modulus of the material and ν known as Poisson ratio, X and θ are longitudinal and angular circumferential coordinates. Whereas $\sigma_{xx},\sigma_{\theta\theta}$ and $\sigma_{x\theta}$ are normal and shear stress terms, $\varepsilon_{xx},\varepsilon_{\theta\theta}$ and $\varepsilon_{x\theta}$ present the normal and shear strains. DWCNTs comprised of two embedded tubes in which each tube is regarded as autonomous cylindrical shell assumes radius $\boldsymbol{R}$, length $\boldsymbol{L}$ and thickness h shown in Fig. 1.

The displacement components u_x , u_y and u_z in three directions x,θ and z, according to classical shell theory are as

$$u_x(x,\theta,z,t)=u(x,\theta,t)-z\frac{\partial w}{\partial x}(x,\theta,t) \tag{5a}$$

$$u_y(x,\theta,z,t)=v(x,\theta,t)-z\frac{\partial w}{\partial \theta}(x,\theta,t) \tag{5b}$$

$$u_z(x,\theta,z,t)=w(x,\theta,t) \tag{5c}$$

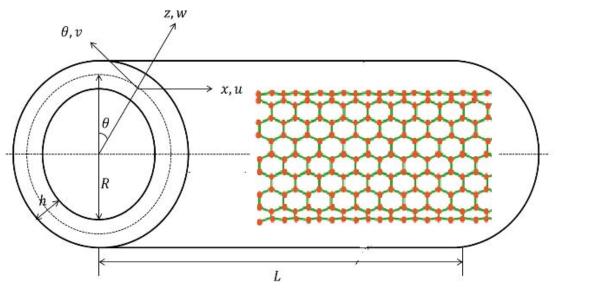


Fig. 1 A geometrical diagram of double-walled CNT

Where u, v and z signify surface displacements. The relations of middle surface strains and middle surface curvatures are symbolized as

$$\varepsilon_{xx}=\frac{\partial u}{\partial x},\varepsilon_{\theta\theta}=\frac{1}{R}\frac{\partial v}{\partial\theta}+\frac{w}{R},\gamma_{x\theta}=\frac{\partial v}{\partial x}+\frac{1}{R}\frac{\partial u}{\partial\theta},$$
$$k_{xx}=-\frac{\partial^2 w}{\partial x^2},k_{\theta\theta}=-\frac{1}{R^2}\left(\frac{\partial^2 w}{\partial\theta^2}-\frac{\partial v}{\partial\theta}\right),k_{x\theta}=-\frac{2}{R}\left(\frac{\partial^2 w}{\partial\theta\partial x}-\frac{\partial v}{\partial x}\right) \tag{6}$$

The kinematics expressions are written as

$$\begin{aligned}\varepsilon^{o}{}_{\theta\theta}&=\varepsilon_{\theta\theta}+zk_{\theta\theta}\\ \varepsilon^{o}{}_{xx}&=\varepsilon_{xx}+zk_{xx}\\ \gamma^{o}{}_{x\theta}&=\gamma_{x\theta}+zk_{x\theta}\end{aligned} \tag{7}$$

The stress and moment resultants are established using the stress components in Eq. (4) and formulated in terms of kinematic relation in Flügge shell theory (Benguediab *et al.* 2014).

$$\begin{Bmatrix}N_{xx}\\N_{\theta\theta}\\N_{x\theta}\\M_{xx}\\M_{\theta\theta}\\M_{x\theta}\end{Bmatrix}-(e_oa)^2\begin{Bmatrix}N_{xx}\\N_{\theta\theta}\\N_{x\theta}\\M_{xx}\\M_{\theta\theta}\\M_{x\theta}\end{Bmatrix}=\begin{pmatrix}\frac{Eh}{1-\nu^2}\frac{\partial}{\partial x} & \frac{1}{R}\frac{\nu Eh}{1-\nu^2}\frac{\partial}{\partial\theta} & \frac{1}{R}\frac{\nu Eh}{1-\nu^2}\\ \frac{\nu Eh}{1-\nu^2}\frac{\partial}{\partial x} & \frac{1}{R}\frac{Eh}{1-\nu^2}\frac{\partial}{\partial\theta} & \frac{1}{R}\frac{Eh}{1-\nu^2}\\ \frac{1}{R}\frac{Eh}{2(1+\nu)}\frac{\partial}{\partial\theta} & \frac{Eh}{2(1+\nu)}\frac{\partial}{\partial x} & 0\\ 0 & D\frac{\nu}{R^2}\frac{\partial}{\partial\theta} & -D\left(\frac{\partial^2}{\partial x^2}+\frac{\nu}{R^2}\frac{\partial^2}{\partial^2\theta}\right)\\ 0 & D\frac{1}{R^2}\frac{\partial}{\partial\theta} & -D\left(\nu\frac{\partial^2}{\partial x^2}+\frac{1}{R^2}\frac{\partial^2}{\partial^2\theta}\right)\\ 0 & \frac{D}{R}(1-\nu)\frac{\partial}{\partial x} & -\frac{D}{R}(1-\nu)\frac{\partial^2}{\partial\theta\partial x}\end{pmatrix}\begin{Bmatrix}u\\v\\w\end{Bmatrix} \tag{8}$$

Here D stands for effective bending rigidity. The governing equations established on Flügge shell theory are written as (Ansari and Arash 2013).

$$\begin{aligned}&\frac{\partial N_{xx}}{\partial x}+\frac{1}{R}\frac{\partial N_{x\theta}}{\partial\theta}=\rho h\frac{\partial^2 u}{\partial^2 t}\\ &\frac{1}{R}\frac{\partial N_{\theta\theta}}{\partial\theta}+\frac{\partial N_{x\theta}}{\partial x}+\frac{1}{R^2}\frac{\partial M_{\theta\theta}}{\partial\theta}+\frac{1}{R}\frac{\partial M_{x\theta}}{\partial x}=\rho h\frac{\partial^2 v}{\partial^2 t}\\ &\frac{\partial^2 M_{xx}}{\partial x^2}+\frac{1}{R^2}\frac{\partial^2 M_{\theta\theta}}{\partial^2\theta}+\frac{2}{R}\frac{\partial^2 M_{x\theta}}{\partial\theta\partial x}-\frac{N_{\theta\theta}}{R}+p=\rho h\frac{\partial^2 w}{\partial^2 t}\end{aligned} \tag{9}$$

Where p denotes the exerted pressure on i tube through van der Waals (vdW) interaction forces. The proposed vdW model accounts the effects of interlayer interactions between the tubes of double-walled CNT.

$$p=w_i\sum_{j=1}^{2}c_{ij}-\sum_{j=1}^{2}c_{ij}w_j \quad (i=1,2) \tag{10}$$

c_{ij} is vdW coefficient, depicting the pressure increment contributing from ith to jth tube.

$$c_{ij}=\left[\frac{1001\pi\varepsilon\sigma^{12}}{3a^4}E_{ij}{}^{13}-\frac{1120\pi\varepsilon\sigma^6}{9a^4}E_{ij}{}^{7}\right]R_j \tag{11}$$

Here C-C bond length is given by $a=1.42\dot{A}$,depth of potential by ε, σ as parameter concluded by equilibrium distance, R_j as radius of j^{th} tube and $E_{ij}{}^{m}$ be as elliptic integral which is given as

$$E_{ij}{}^{m}=(R_j+R_i)^{-m}\int_0^{\pi/2}\frac{d\theta}{(1-K_{ij}\cos^2\theta)^{m/2}} \tag{12}$$

being m as integer and coefficient K_{ij} is defined by

$$K_{ij}=\frac{4R_jR_i}{(R_j+R_i)^2} \tag{13}$$

By incorporating Eq. (8) into Eq. (9), developed the set of partial differential equations written in terms of three field variables $u^i, v^i, w^i\,(i=1,2)$ for the ith tube of double-walled CNT.

$$g_{11}{}^{(1)}u^1+g_{12}{}^{(1)}v^1+g_{13}{}^{(1)}w^1=\rho h\left(\ddot{u}^{(1)}-(e_oa)^2(\ddot{u}^{(1)}_{xx}+\frac{1}{R_1^2}\ddot{u}^{(1)}_{xx}\right) \tag{14a}$$

$$g_{21}{}^{(1)}u^1+g_{22}{}^{(1)}v^1+g_{23}{}^{(1)}w^1=\rho h\left(\ddot{v}^{(1)}-(e_oa)^2(\ddot{v}^{(1)}_{xx}+\frac{1}{R_1^2}\ddot{v}^{(1)}_{xx})\right) \tag{14b}$$

$$\begin{aligned}&g_{31}{}^{(1)}u^1+g_{32}{}^{(1)}v^1+g_{33}{}^{(1)}w^1=\rho h\ddot{w}^{(1)}+w^{(1)}\sum_{\substack{j=1\\j\neq i}}^{2}c_{1j}-\sum_{\substack{j=1\\j\neq i}}^{2}c_{1j}w^{(j)}-\\ &(e_oa)^2\left[\rho h(\ddot{w}^{(1)}_{xx}+\frac{1}{R_1^2}\ddot{w}^{(1)}_{\theta\theta})+(\ddot{w}^{(1)}_{xx}+\frac{1}{R_1^2}\ddot{w}^{(1)}_{\theta\theta})\sum_{\substack{j=1\\j\neq i}}^{2}c_{1j}-\sum_{\substack{j=1\\j\neq i}}^{2}c_{1j}(\ddot{w}^{(j)}_{xx}+\frac{1}{R_1^2}\ddot{w}^{(j)}_{\theta\theta})\right]\end{aligned} \tag{14c}$$

$$\begin{aligned}&g_{11}{}^{(2)}u^2+g_{12}{}^{(2)}v^2+g_{13}{}^{(2)}w^2=\\ &\rho h\left(\ddot{u}^{(2)}-(e_oa)^2(\ddot{u}^{(2)}_{xx}+\frac{1}{R_2^2}\ddot{u}_{\theta\theta}{}^{(2)})\right)\end{aligned} \tag{14d}$$

$$\begin{aligned}&g_{21}{}^{(2)}u^2+g_{22}{}^{(2)}v^2+g_{23}{}^{(2)}w^2=\\ &\rho h\left(\ddot{v}^{(2)}-(e_oa)^2(\ddot{v}^{(2)}_{xx}+\frac{1}{R_2^2}\ddot{v}^{(2)}_{xx})\right)\end{aligned} \tag{14e}$$

$$\begin{aligned}&g_{31}{}^{(2)}u^2+g_{32}{}^{(2)}v^2+g_{33}{}^{(2)}w^2=\\ &\rho h\ddot{w}^{(2)}+w^{(2)}\sum_{\substack{j=1\\j\neq 2}}^{2}c_{2j}-\sum_{\substack{j=1\\j\neq 2}}^{2}c_{2j}w^{(j)}-\\ &(e_oa)^2\left[\begin{aligned}&\rho h(\ddot{w}^{(2)}_{xx}+\frac{1}{R_2^2}\ddot{w}^{(2)}_{\theta\theta})+(\ddot{w}^{(2)}_{xx}+\\ &\frac{1}{R_2^2}\ddot{w}^{(2)}_{\theta\theta})\sum_{\substack{j=1\\j\neq 2}}^{2}c_{2j}-\sum_{\substack{j=1\\j\neq 2}}^{2}c_{2j}(\ddot{w}^{(j)}_{xx}+\frac{1}{R_2^2}\ddot{w}^{(j)}_{\theta\theta})\end{aligned}\right]\end{aligned} \tag{14f}$$

where $g_{pq} = (p, q = 1, 2, 3)$ are the partial operators can be seen in Appendix-I.

3. Solution using the wave propagation approach

Over the past several years, various theories of vibration of tube/shell structures of various configurations and boundary conditions have been extensively studied (Iijima *et al*. 1996, Natuski *et al*. 2006, Shen and Zhang 2010, Ansari and Rouhi 2012, Hussain *et al.* 2018a). The Wave propagation approach is one of the widely and effectively used numerical technique by researchers to study the free vibrations of plates, shells and single-walled CNTs problems (Hussain and Naeem 2018a, Hussain *et al*. 2018b, Hussain *et al*. 2018c). The three modal displacement functions of the shell for ith tube can be regarded as

$$u^{(i)}(x,\theta,t) = a_m \cos(n\theta) e^{(i\omega t - ik_m x)} \tag{15a}$$

$$v^{(i)}(x,\theta,t) = b_m \sin(n\theta) e^{(i\omega t - ik_m x)} \tag{15b}$$

$$w^{(i)}(x,\theta,t) = c_m \cos(n\theta) e^{(i\omega t - ik_m x)} \tag{15c}$$

In which a_m, b_m, c_m define the displacement amplitude in x, θ and z directions respectively. The angular frequency is denoted by ω, circumferential wave number by n and k_m regarded as axial wave number associated with end conditions imposed on DWCNTs. Substituting the functions and derivatives into the field equations, hence obtained a set of simultaneous as follows

$$\begin{aligned} &G_{11}{}^{(i)} a_m^i + G_{12}{}^{(i)} b_m^i + G_{13}{}^{(i)} c_m^i = \\ &-\omega^2 \left(1 - (e_o a)^2 \nabla^2\right) \rho h a_m^i \end{aligned} \tag{16a}$$

$$\begin{aligned} &G_{21}{}^{(i)} a_m^i + G_{22}{}^{(i)} b_m^i + G_{23}{}^{(i)} c_m^i = \\ &-\omega^2 \left(1 - (e_o a) \nabla^2\right) \rho h b_m^i \end{aligned} \tag{16b}$$

$$\begin{aligned} &G_{31}{}^{(i)} a_m^i + G_{32}{}^{(i)} b_m^i + G_{33}{}^{(i)} c_m^i + \left(1 - (e_o a)^2 \nabla^2\right) \left[\sum_{\substack{j=1 \\ j \neq i}}^{2} c_{ij} c_m^i - \sum_{\substack{j=1 \\ j \neq i}}^{2} c_{ij} c_m^i \right] = \\ &-\omega^2 \left(1 - (e_o a)^2 \nabla^2\right) \rho h c_m^i \end{aligned} \tag{16c}$$

Where $i = (1, 2)$ and the algebraic operators $G_{pq}^{(i)}$ are derived using Appendix-II with $p, q = (1, 2, 3)$. The frequency vibration of double-walled CNT is exhibited based on nonlocal FSM subject to four end supports clamped -clamped (FSM-CC), clamped-simply supported (FSM-CS), simply supported-simply supported (FSM-SS) and clamped-free (FSM-CF).

4. Results and discussion

In this portion of writing, the significance of boundary conditions on the vibration behavior of DWCNTs is investigated employing wave propagation approach. The versatility and accuracy of proposed method is observed by numerous studies (Natuski *et al*. 2006, Natuski *et al*. 2007) to determine natural frequencies in shell and CNTs. This study specifically scrutinizes the small scale effect in the vibration analysis of double-walled CNT. The numerical values of Young modulus, Poisson's ratio, thickness and density are $E = 1TPa, \nu = 0.3, h = 0.34nm$ and $\rho = 2.3g/cm^3$ reported (Ansari and Arash 2013). Moreover, distinguished values of inner tube radius together with nonlocal parameter signifies the present non-local shell-based model to analyze frequency spectra. CNT is well known structure in shapes of i) armchair ii) chiral and iii) zigzag, here the vibration analysis is carried out of armchair CNT subjected to four conditions FSM-CC,FSM-CS,FSM-SS and FSM-CF. For the convergence rate of CNT, the non-dimensional frequency parameters enumerated in the current work, i.e., using FSM, are happened to be in a good consistency along with the so-called exact results furnished by Loy *et al*. (Loy *et al*. 1999), those were established by working out with the deformation theory provided in Table 1. The frequencies are described for non-dimensional frequency parameters as: $\xi = \omega R\sqrt{(1-\nu^2)\rho / E}$ as shown in Table 1 and positive coherence is achieved.

The percentage difference is negligible as n = 1, 3, 4 are 0.006%, 0.01%, 0.002% and at n = 2 by 0.0061% and present FSM result are lower than equivalent results executed by Loy *et al*. (1999). The frequency parameters for circumferential wave numbers n = 5, 6 are same with the outcomes of Loy *et al*. (1999). A non-dimensional frequency parameter ξ is defined for a CNT as: $\xi = \omega R\sqrt{(1-\nu^2)\rho / E}$. The obtained results are cross-compared with external data and provide agreement between modeling, computation and experimental outcomes as shown in Tables 1 and 2. Fig. 2 plots the fundamental frequency versus **L/d** for FSM-CC end condition for different modes of vibration. It should be mentioned for both cases, the values of *L/d* varies from 4.67 ~ 35.34. It is found that from Fig. 3, that frequencies of first (1, 1)

Table 1 Comparison of FSM double-walled CNT frequencies with Ref. (Loy *et al*. 1999). (*L/R* = 20, *h/R* = 0.02)

Method	*n*					
	1	2	3	4	5	6
Loy *et al.* (1999)	0.016102	0.009382	0.022105	0.042095	0.06801	0.09973
FSM	0.016101	0.009378	0.022103	0.042094	0.04209	0.09973

Table 2 FSM frequencies of clamped double-walled CNTs (h/R = 0.05, L/R = 2.5)

m	V	N	Heydarpour *et al*. (2014)	Present
0	0.12	7	0.6240	0.6228
		9	0.6240	0.6234
		11	0.6240	0.6239
	0.17	7	0.8157	0.8143
		9	0.8157	0.8152
		11	0.8157	0.8155
	0.28	7	0.8553	0.8541
		9	0.8553	0.8547
		11	0.8553	0.8550

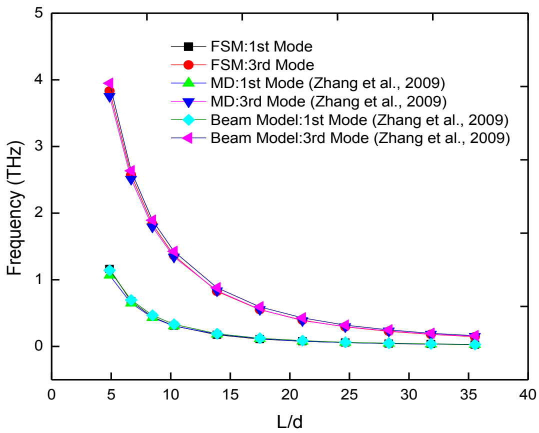
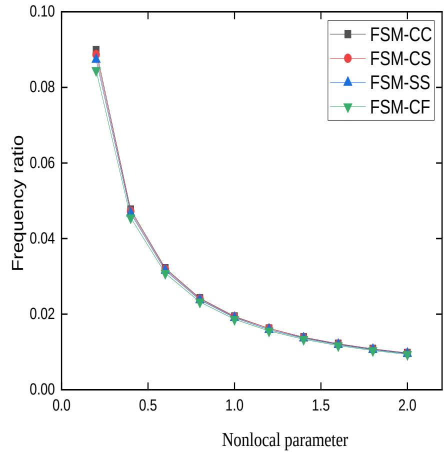
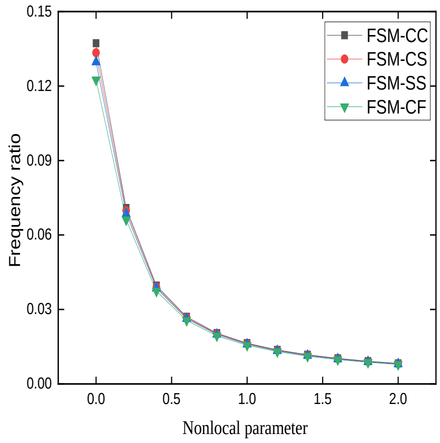


Fig. 2 Frequency comparison of FSM-CC double-walled CNTs for 1st and 3rd mode against L/d with FSM and MD simulations (Zhang *et al*. 2009)

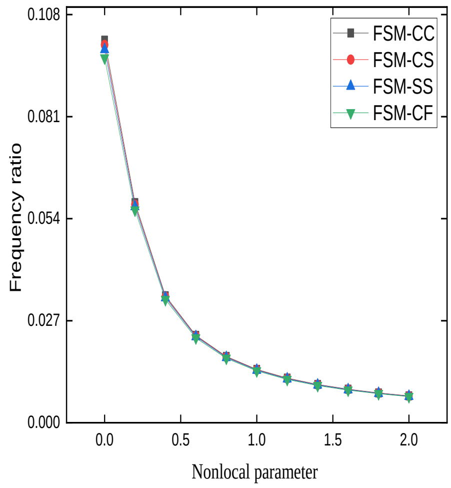
and third (3, 1) vibration modes decrease and reaches the constant values on increasing of ***L/d***. The influence of ***L/d*** on the frequency of present model has been discussed and checked with MD simulation as shown in Fig. 3 for FSM-CC end condition. The obtained results are well agreed with the reported results of MD simulation (Zhang *et al*., 2009). Particularly, the frequencies (THz) of double-walled CNTs correspond to L/d = 6.71 are 0.671, 1.565, 2.552, 3.523 for present model and 0.681, 1.535, 2.536, 3.588, as given by Duan *et al*. (2007), respectively. The vibrations of FSM-CC double-walled CNTs have been investigated both by simulations techniques (Li and Chou 2003, Li and Chou 2004, Zhang *et al*. 2009) and experimentally (Yakobson *et al*. 1996, Hsu *et al*. 2008). It is seen that the frequencies have a notable effect on the vibration of double-walled CNTs with shorter length-to-diameter ratio.



(a)

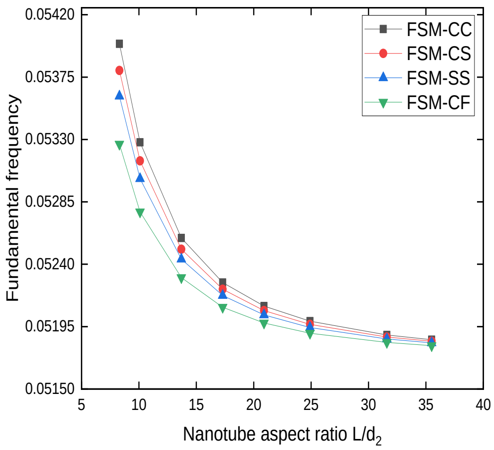


(b)



(c)

Fig. 3. Frequency with respect to nonlocal parameter $e_o a$ for aspect ratio $L/R_1 = 5, 10, 20$.



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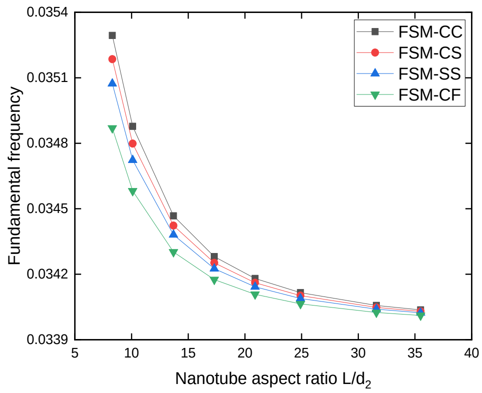

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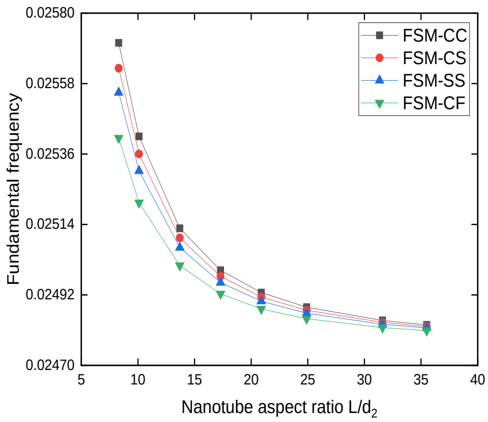


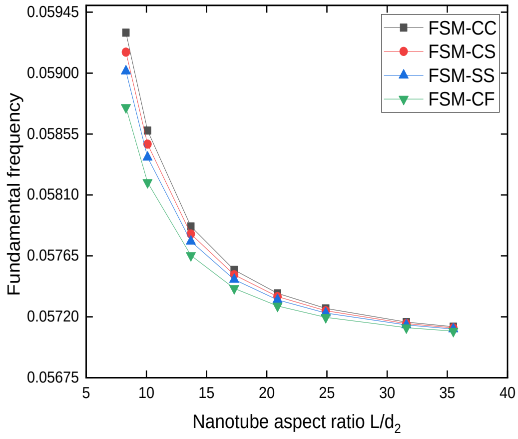
(c)

Fig. 4 Influence of distinct boundary condition against numerous values of $e_o a$ = 0.2, 0.35, 0.5 of armchair (5, 5) double-walled CNTs with $R_1 = 0.35nm$

Fig. 3 exhibits the variation of fundamental eigen frequencies against values of nonlocal parameter that changes within a limit from 0 to 2. Three distinct aspect ratio, $L/R_1 = 5, 10, 20$ are discussed subject to four boundary conditions FSM-CC, FSM-CS, FSM-SS and FSM-CF. The radius of inner tube is considered here as $R_1 = 0.35nm$ with all above mentioned numerical estimates of physical parameters incorporating also with vdW interaction between two tubes of double–walled CNT. The graph in figure shows that with a decrease in values of nonlocal parameter, frequency corresponding to each boundary condition tends to decrease. For lower values of $e_o a$ there is slight variation in frequencies of FSM-CC, FSM-CS, FSM-SS and FSM-CF respectively at the same time for lower aspect ratio the observation remains alike. Two main findings depicted by graph are, calculated frequencies coincide for all boundary condition and continue to decline with a rise in aspect ratio. The rooted nonlocal elasticity model also produces more significant results for minimal radius of tubes.

The graphs in Fig. 4 compares the fundamental frequencies of armchair (5, 5) with three different values of nonlocal parameter $e_o a = 0.2, 0.35$ and 0.5 versus length to diameter (L/d_2) lies in range of 8 nm to 36 nm. The all other numerical estimates are same as quoted above. The curves in three graphs shows the validity of small-scale effect as the frequencies decreases with an increase of nonlocal parameter. Also, it is observed that as length to radius expands so the fundamental frequencies for all end conditions coincide. The FSM-CC attains highest fundamental frequency chased by FSM-CS after that FSM-SS and at last FSM-CF comes. The graphs in Fig. 5 included the fundamental frequencies of armchair (7, 7) and (9, 9) showing diversity with the $e_o a = 0.2, 0.35$ and 0.5. The all depicted frequencies in graphs is according to length to diameter ratio.

It is noticed that there is uniform increase in frequencies of arm chair corresponding to all four conditions FSM-CC, FSM-CS, FSM-SS and FSM-CF. Corresponding to $e_o a = 0.2$, the clamped-clamped (FSM-CC) condition of armchair (7, 7) and (9, 9) obtained frequencies 0.054, 0.0595 and 0.0619 respectively. It is obviously seen there is an increasing trend and which remains unchanged for all boundary conditions as well as other two values of nonlocal parameter possess the identical behavior. Moreover, the more accretion in the nonlocal parameter, the lower the fundamental frequencies are observed. In Fig. 6, the inner tube radius is taken as $R_1 = 1.5nm$ with other estimates remained same. The graph 6 represented the frequency 0.01245 against the first length to diameter ratio for FSM-CC of armchair (5, 5), whereas in Fig. 4(a) it was espied as 0.05399.It shows a descent in fundamental frequencies with an ascent in the inner tube radius. Similarly, in Figs. 5(b) and 5(c) the patter recognized the fact. One of the observations, that in long carbon nanotubes the difference among end conditions diminishes which



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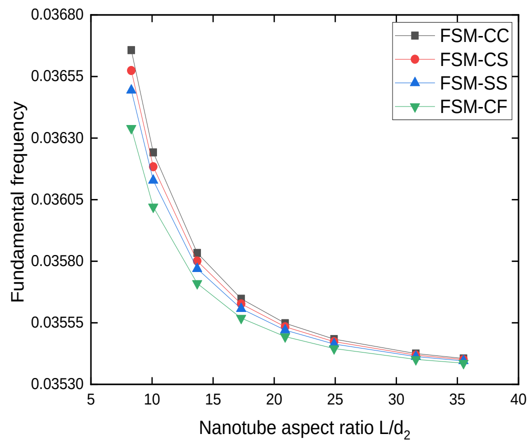

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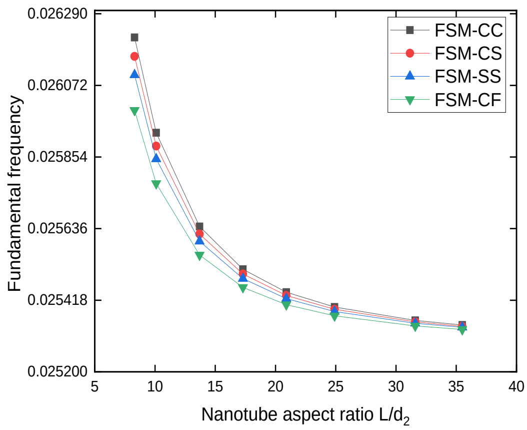

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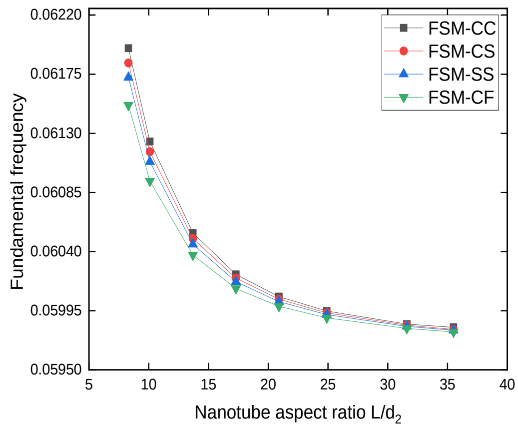

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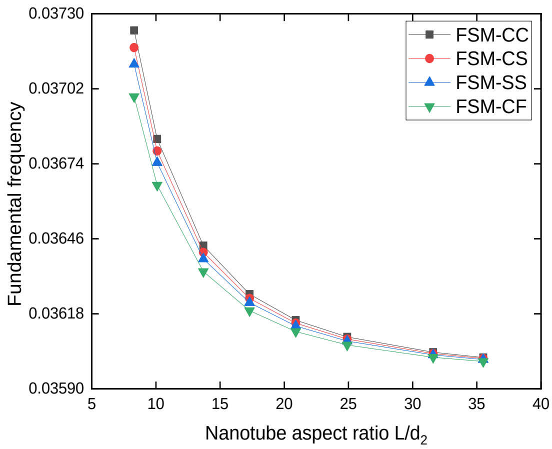

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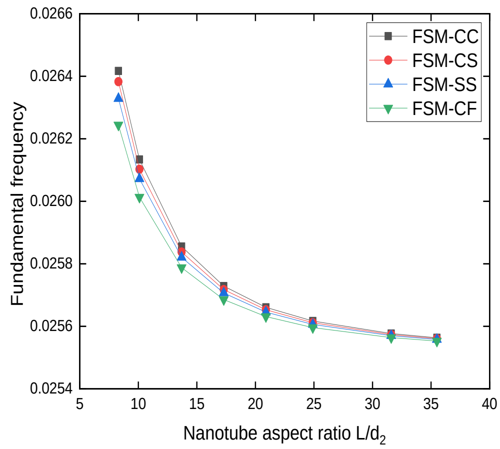
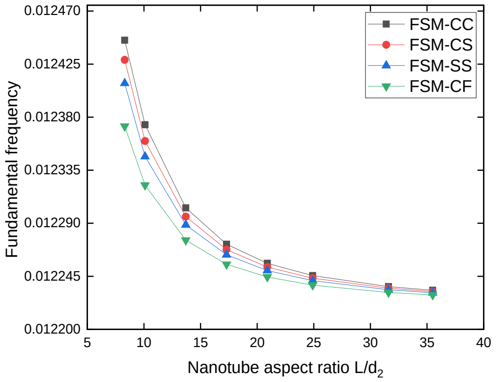
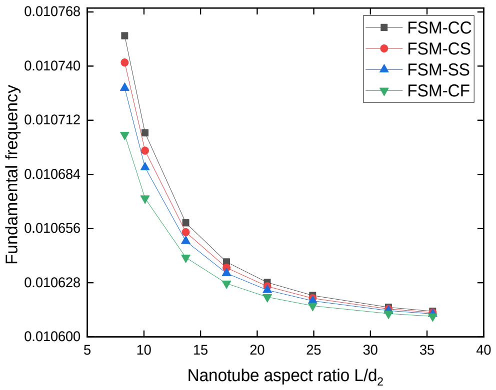


(f)

Fig. 5 Influence of distinct boundary condition against numerous values of $e_o a$ = 0.2, 0.35, 0.5 of armchair (7, 7) (a)-(c) and armchair (9, 9) (d)-(f) double-walled CNTs with $R_1 = 0.35nm$



(a)



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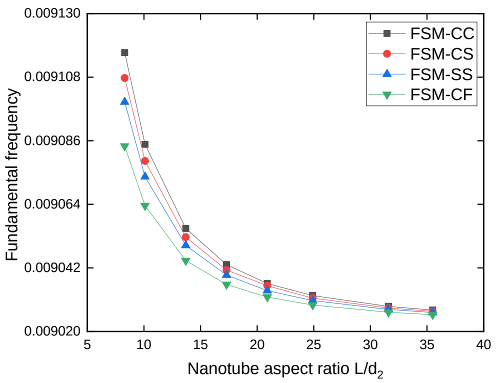
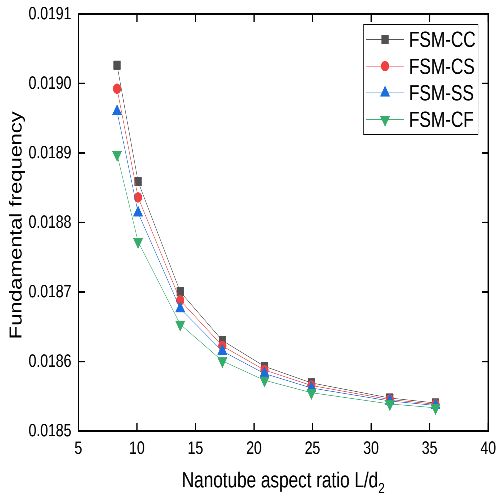


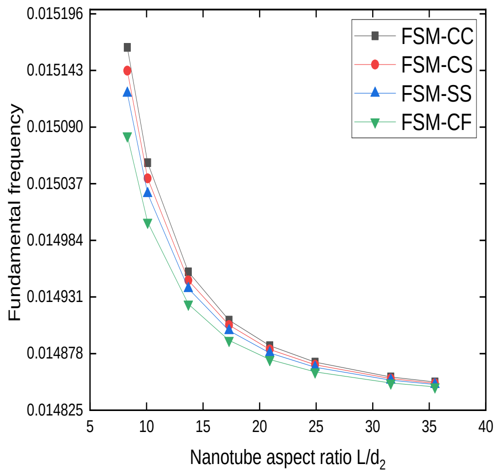
(c)

Fig. 6 Influence of distinct boundary condition against numerous values of $e_o a$ = 0.2, 0.32, 0.5 of armchair (5, 5) double-walled CNTs with $R_1 = 1.5nm$

shows the accuracy of the purposed non-local elastic model along with wave propagation technique. Figs. 7 and 8 illustrates the influence of boundary conditions for armchair (7, 7) and (9, 9) respectively considering the $R_1 = 1.5nm$ .The decline of the curves opposite of length to diameter ratio affirms the nonlocal effect. Corresponding to armchair (5, 5), (7, 7) and (9, 9), there is seen drop in the frequencies as inflates the nonlocal parameter value.



(a)



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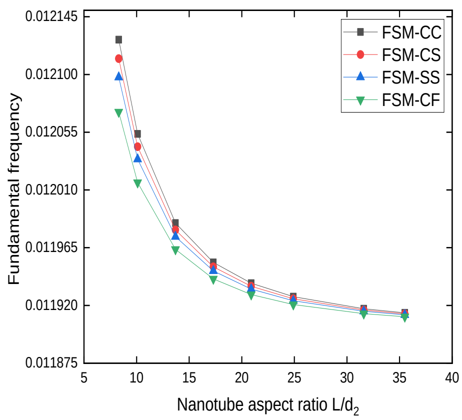
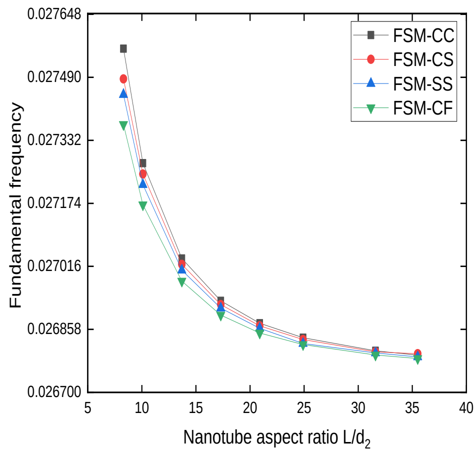
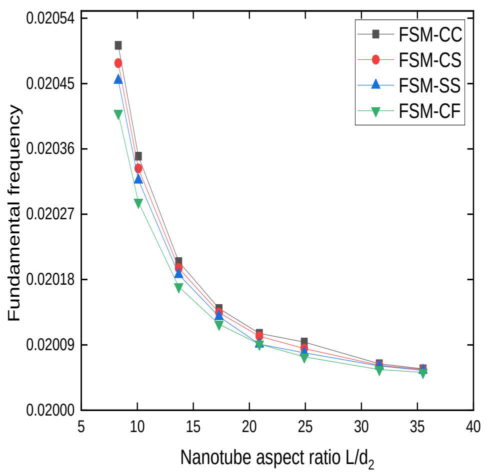


(c)

Fig. 7 Influence of distinct boundary conditions against numerous values of $e_o a$ = 0.2, 0.35, 0.5 of armchair (7, 7) double-walled CNTs with $R_1 = 1.5nm$



(a)



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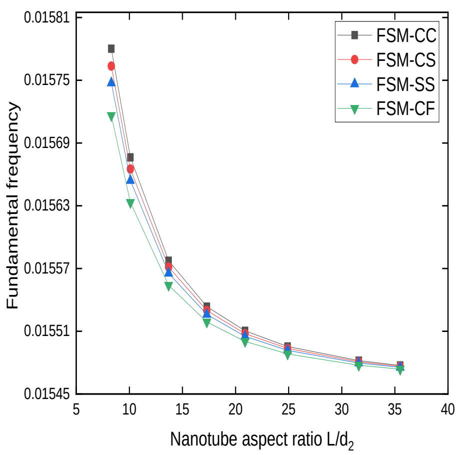

(c)

Fig. 8 Influence of distinct boundary conditions against numerous values of $e_o a$ = 0.2, 0.35, 0.5 of armchair (9, 9) double-walled CNTs with $R_1 = 1.5nm$

However, as enlarges the indices of armchair, the curves indicated escalation in frequencies. The expanded values of length to diameter ratio exhibits the reality that nonlocal effect becomes negligible on boundary conditions. On the other hand, the frequency curve showed the difference in contrast of the boundary conditions becomes infinitesimal with an increase in inner tube radius. The gap presented in four boundary conditions is obvious in start of the curves as FSM-CF secures the lowest frequency in comparison of FSM-SS, FSM-CS and FSM-CC.

5. Conclusions

The Flügge shell theory based on nonlocal elasticity investigates the vibration characteristics of double-walled CNT. Theoretical formation of the nonlocal model involves the van der Waals interactions between the tubes and impact of small-scale effect subjected to four boundary supports. The wave propagation approach is exercised to determine eigen frequencies for armchair CNT. The fundamental frequencies scrutinized with assorted length to diameter ratios. The analysis done with the findings

* The raised in value of nonlocal parameter reduces the corresponding fundamental frequency estimates.
* Due to small scale effect fundamental frequency ratio decreases as length to diameter ratio increases.
* Small scale effect becomes negligible on all end supports for the higher values of aspect ratio.
* With the smaller inner tube radius double-walled CNT behaves more sensitive towards nonlocal parameter.

The present study can be appropriate to employ for analyzing the vibrations in double-walled CNTs with Galerkin and finite element methods.

Declaration of Conflicting Interests

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Appendix-I

$$g_{11}^{(i)} = \frac{Eh}{1-\nu^2}\frac{\partial^2}{\partial x^2} + \frac{1}{R_i^2}\frac{Eh}{2(1+\nu)}\frac{\partial^2}{\partial\theta^2}$$

$$g_{12}^{(i)} = \frac{1}{R_i}\left(\frac{Eh\nu}{1-\nu^2} + \frac{Eh}{2(1+\nu)}\right)\frac{\partial^2}{\partial\theta\partial x}$$

$$g_{13}^{(i)} = \frac{1}{R_i}\frac{Eh\nu}{1-\nu^2}\frac{\partial}{\partial x},$$

$$g_{21}^{(i)} = g_{12}^{(i)}$$

$$g_{22}^{(i)} = \left(\frac{Eh}{2(1+\nu)} + \frac{D(1-\nu)}{R_i^2}\right)\frac{\partial^2}{\partial x^2} + \frac{1}{R_i^2}\left(\frac{Eh}{1-\nu^2} + \frac{D}{R_i^2}\right)\frac{\partial^2}{\partial\theta^2}$$

$$g_{23}^{(i)} = -\frac{D}{R_i^2}\frac{\partial^3}{\partial x^2\partial\theta} - \frac{\nu D}{R_i^4}\frac{\partial^3}{\partial\theta^3} + \frac{1}{R_i^2}\frac{Eh}{1-\nu^2}\frac{\partial}{\partial\theta}$$

$$g_{31}^{(i)} = -g_{13}^{(i)}$$

$$g_{32}^{(i)} = \frac{D}{R_i^2}(2-\nu)\frac{\partial^3}{\partial x^2\partial\theta} + \frac{D}{R_i^4}\frac{\partial^3}{\partial\theta^3} - \frac{1}{R_i^2}\frac{Eh}{1-\nu^2}\frac{\partial}{\partial\theta}$$

$$g_{33}^{(i)} = -D\frac{\partial^4}{\partial x^4} - \frac{2D}{R_i^2}\frac{\partial^4}{\partial x^2\partial\theta^2} - \frac{D}{R_i^4}\frac{\partial^4}{\partial\theta^4} - \frac{1}{R_i^2}\frac{Eh}{1-\nu^2}$$

Appendix-II

$$G_{11}^{(i)} = \frac{Eh}{1-\nu^2}(-k_m^{\ 2}) + \frac{1}{R_i^2}\frac{Eh}{2(1+\nu)}(-n^2)$$

$$G_{12}^{(i)} = \frac{1}{R_i}\left(\frac{Eh\nu}{1-\nu^2} + \frac{Eh}{2(1+\nu)}\right)(-nik_m)$$

$$G_{13}^{(i)} = \frac{1}{R_i}\frac{Eh\nu}{1-\nu^2}(-ik_m)$$

$$G_{21}^{(i)} = -G_{12}^{(i)}$$

$$G_{22}^{(i)} = \left(\frac{Eh}{2(1+\nu)} + \frac{D(1-\nu)}{R_i^2}\right)(-k_m^{\ 2}) + \frac{1}{R_i^2}\left(\frac{Eh}{1-\nu^2} + \frac{D}{R_i^2}\right)(-n^2)$$

$$G_{23}^{(i)} = -\frac{D}{R_i^2}(nk_m^{\ 2}) - \frac{\nu D}{R_i^4}n^3 + \frac{1}{R_i^2}\frac{Eh}{1-\nu^2}(-n)$$

$$G_{31}^{(i)} = -G_{13}^{(i)}$$

$$G_{32}^{(i)} = \frac{D}{R_i^2}(2-\nu)(-nk_m^{\ 2}) - \frac{D}{R_i^4}n^3 - \frac{1}{R_i^2}\frac{Eh}{1-\nu^2}n$$

$$G_{33}^{(i)} = -Dk_m^{\ 4} - \frac{2D}{R_i^2}n^2k_m^{\ 2} - \frac{D}{R_i^4}n^4 - \frac{1}{R_i^2}\frac{Eh}{1-\nu^2}$$