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Effect of thermal conductivity on isotropic modified couple stress thermoelastic medium with two temperatures

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Abstract. The objective is to study the deformation in a homogeneous isotropic modified couple stress thermoelastic medium with mass diffusion and with two temperatures due to a thermal source and mechanical force. Laplace and Fourier transform techniques are applied to obtain the solutions of the governing equations. The displacements, stress components, conductive temperature, mass concentration and couple stress are obtained in the transformed domain. Numerical inversion technique has been used to obtain the solutions in the physical domain. Isothermal boundary and insulated boundaryconditions are used to investigate the problem. Some special cases of interest are also deduced.

Keywords: isotropic solid; two temperatures; Laplace and Fourier transforms; mass concentration; modified couple stress; thermoelastic; mass diffusion

1. Introduction

Couple stress theory is an extension to continuum theory that includes the effects of couple stresses, in addition to the classical direct and shear forces per unit area. This is a higher order continuum theory capable of predicting the size dependency at micro and nanoscale. First mathematical model to examine the materials with couple stresses was presented by Cosserat and Cosserat (1909).This theory could not establish the constitutive relationships. Mindlin and Tierstein (1962) and Koiter (1964) developed initial version of couple stress theory, based on the Cosserat continuum theory (1909).It involves four material constants for isotropic elastic materials where two of them are separate material length scale parameters which are very difficult to determine. So, modified couple stress theory (M-CST) with one length scale parameter was presented by yang *et al.* (2002) using the balance law for moments of couple besides the balance laws for forces and moment of forces, with symmetric couple-stress tensor. This theory suffers from some inconsistencies. So, Hadjesfandiari *et al.* (2011) gave consistent couple stress theory (C-CST) with the skew-symmetric couple-stresses, which resolves all the discrepancies of modified couple stress theory. Modified couple stress theory was not applicable to anisotropic materials. Now a day, new modified couple stress theory (NM-CST) for anisotropic materials containing three length scale parameters has been used presented by Chen *et al.* (2014). Ke *et al*. (2011)investigated the size effect on dynamic stability of functionally graded microbeams based on a modified couple stress theory. Chen *et al.* (2011) presented a new modified couple stress model for bending analysis of composite laminated beams with first order shear deformation. Asghari (2012) studied the geometrically nonlinear micro-plate formulation based on the modified couple stress theory. Simsek *et al*. (2013) investigated the bending and vibration of functionally graded microbeams using a new higher order beam theory and the modified couple stress theory. Fang *et al.*(2013) examined the problem of thermoelastic damping in the axisymmetric vibration of circular microplate resonators using two dimensional couple stress heat conduction model. Ansari *et al.* (2014) studied the free vibration behavior of post-buckled functionally graded (FG) Mindlin rectangular microplates based on the modified couple stress theory (MCST). Ansari *et al.* (2014) introduced an exact solution for the vibration analysis of piezoelectric microbeams for both Euler-Bernoulli and Timoshenko beam models using modified couple stress theory. It was shown that when the length of microbeams is decreased, effects of piezoelectricity and size effects are more prominent. Shaat *et al.* (2014) studied the bending analysis of nano-sized Kirchhoff plates using modified couple-stress theory and surface elasticity theory of Gurtin and Murdochto consider the surface energy effects. Mehralian and TediBeni (2017) studied the buckling conduct of piezoelectrical cylindrical nanoshell subjected to conditions of uniform temperature, electrical and mechanical interactions on Winkler-Pasternak medium on the basis of modified couple stress theory and shell model. They illustrated the influence of various parameters e.g. thickness ratio, length ratio, material length scale parameter and Winkler and Pasternak foundation stiffness parameter on the critical buckling load. Free vibration frequencies of nonuniform microbeams were examined by Khaniki *et al.* (2017) on the basis of modified couple stress theory. Nonuniformity was introduced by

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exponentially varying width among the microbeam, thickness remaining constant. By analytically solving the problem for various boundary conditions, effect of nonuniformity and small scale effects were observed on varying the frequency terms. Anaxisymmetric problem of thick circular plate in modified couple stress theory of thermoelastic diffusion using Laplace and Hankel transforms technique have been investigated by Kumar and Devi (2016). Kumar and Devi (2015) studied the influence of Hall current and rotation in thermoelastic diffusive media caused by ramp type loading on the basis of modified couple stress theory using Laplace and Fourier Transform techniques. Atanasov *et al.* (2017) examined the thermal effect on the free vibration and buckling of the Euler-Bernoulli double microbeam system based on the modified couple stress theory using Bernoulli–Fourier method. Togun *et al.* (2017) presented the linear free vibration of a simply-supported nanobeam by using modified couple stress theory and Hamilton's principle. Also, they studied the effects of the length scale parameter and the Poisson's ratio on natural frequency. Malikan investigated the buckling of a thick sandwich plate under the biaxial non-uniform compression using the modified couple stress theory with various boundary conditions. Abbas *et al.* (2009,2014) studied different problems under two-temperature generalized thermoelastic theory by finite element method.Alimirzaei*et al.* (2019) presented the nonlinear static, buckling and vibration analysis of viscoelastic micro-composite beam reinforced by various distributions of boron nitride nanotube (BNNT) with initial geometrical imperfection by modified strain gradient theory (MSGT) using finite element method (FEM). Boukhlif *et al.* (2019) presented a dynamic investigation of functionally graded (FG) plates resting on elastic foundation using a simple quasi-3D higher shear deformation theory. Boulefrakh *et al.* (2019) employed a quasi 3D hyperbolic shear deformation model for bending and dynamic behavior of functionally graded (FG) plates resting on visco-Pasternak foundations. Bourada *et al.* (2019) presented the free vibration analysis of simply supported perfect and imperfect (porous) FG beams using a high order trigonometric deformation theory assuming that the material properties of the porous beam vary across the thickness. Boutaleb *et al.* (2019) studied the dynamic analysis of the functionally graded rectangular nanoplates. Chaabane *et al.* (2019) studied bending and free vibration responses of functionally graded beams resting on elastic foundation analytically. Karami *et al.* (2019) investigated the buckling behavior of functionally graded (FG) nanoplates made of anisotropic material (beryllium crystal as a hexagonal material). Karami *et al.* (2019) dealt with the size-dependent wave propagation analysis of functionally graded (FG) anisotropic nanoplates based on a nonlocal strain gradient refined plate model. Medani *et al.* (2019) studied the static and dynamic behavior of Functionally Graded Carbon Nanotubes (FG-CNT)-reinforced porous sandwich (PMPV) polymer plate. Zarga *et al.* (2019) employed a simple quasi-3D shear deformation theory for thermo-mechanical bending analysis of functionally graded material (FGM) sandwich plates. Unlike the other high order shear deformation theories (HSDTs), they considered a new kinematic which includes undetermined integral variables. Othman and Marin (2017) studied the effect of thermal loading due to laser pulse on thermoelastic porous medium under G-N theory. Marin *et al.* (2017) studied the Saint-Venant's problem in the context of the theory of porous dipolar bodies. Despite of this several researchers worked on different theory of thermoelasticity as Hassan *et al.* (2018), Marin and Nicaise (2016), Marin and Craciun (2017), Sharma *et al.* (2016), Marin *et al.* (2017), Jahangir *et al.* (2015,2018,2019) and Lata and Kaur (2019,2019a), Ezzat*et al.*(2016), Sharma *et al*. (2015), Kumar *et al.* (2016), Lata (2018a, b).

In the present investigation, our objective is to study the deformation in homogeneous isotropic modified couple stress thermoelastic medium with two temperatures using isothermal and insulated boundary conditions, with and without energy dissipationin the presence of mass diffusion sources. The medium is employed to the thermal and mechanical sources. Laplace and Fourier transform technique is applied to obtain the solutions of the governing equations. The displacement components, stress components, conductive temperature and couple stress are obtained in the transformed domain and are presented graphically for different values of displacement. The effect of K^* on the resulting quantities is depicted graphically. Analysis is done without using the potential functions. In the absence of these equation of motion simplifies to the classical equations.

2. Basic equations

Following Kumar and Devi (2017) and Youssef (2013),the field equations for isotropic modified couple stress thermoelastic medium with mass diffusion, with and without energy dissipation in the absence of body forces, body couples are given by

(a) Constitutive relationships

$$t_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij} - \frac{1}{2}e_{kij}m_{lk,l} - \beta_1 T\delta_{ij} - \beta_2 C\delta_{ij}, \quad (1)$$

$$m_{ij} = 2\alpha\chi_{ij} \quad (2)$$

$$\chi_{ij} = \frac{1}{2}\left(\omega_{i,j} + \omega_{j,i}\right) \quad (3)$$

$$\omega_i = \frac{1}{2}e_{ijk}u_{k,j}. \quad (4)$$

(b) Equation of motion

$$\left(\lambda + \mu + \frac{\alpha}{4}\Delta\right)\nabla(\nabla.\vec{u}) + \left(\mu - \frac{\alpha}{4}\Delta\right)\nabla^2\vec{u} - \beta_1\nabla T - \beta_2\nabla C = \rho\ddot{\vec{u}} \quad (5)$$

(c) Equation of heat conduction

$$K\nabla^2\varphi + K^*\nabla^2\dot{\varphi} = \rho C^*\ddot{T} + \beta_1 T_0\nabla.\ddot{\vec{u}}, \quad (6)$$

$$T = (1 - \alpha_1 \nabla^2)\varphi \tag{7}$$

(d) Equation of mass diffusion

$$D\beta_2 \nabla^2(\nabla.\vec{u}) + Da\nabla^2 T - Db\nabla^2 C + \dot{C} + \tau\ddot{C} = 0 \tag{8}$$

Here $u = (u, v, w)$ is the components of displacement vector, σ_{ij} are the components of stress tensor, ε_{ij} are the components of strain tensor,e_{ijk} is alternate tensor, $m_{ij}$are the components of couple-stress, α_1 is the two temperature parameter,β_{ij} is thermal tensor,T is thethermodynamical temperature, φ is the conductive temperature, K^* is the coefficient of thermal conductivity,$\chi_{ij}$is curvature,$\omega_i$is the rotational vector, ρ is the density, K_{ij} isthe thermal conductivity,C^* is the specific heat at constant strain, $T_0$is the reference temperature assumed to be such that$T/_{T_0} \ll 1$,G_i are the elasticity constants and , $\beta_1 = (3\lambda + 2\mu)\alpha_t$,$\beta_3 = (3\lambda + 2\mu)\alpha_t$. Here α_t and α_v are the coefficients of linear thermal expansion and diffusion expansion respectively, $C$is the mass concentration, $\alpha$is the couple stress parameter, $b$is the coefficient describing the measure of mass diffusion effects ,$a$is the coefficient describing the measure of thermoelastic diffusion, $D$is the thermoelastic diffusion constant, Δ $\left(= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)$ is the Laplacian operator ,∇ is del operator, $\delta_{ij}$is Kronecker's delta.

3. Formulation and solution of the problem

We consider a two dimensional homogeneous isotropic modified couple stress thermoelastic medium initially at uniform temperature T_0 occupying the region of a half space $z \geq 0$ .A rectangular coordinate system (x, y, z) having origin on the surface $z = 0$ has been taken. All the field quantities depend on (x, z, t).The half surface is subjected to isothermal and insulated boundary conditions.

The initial and regularity conditions are given by

$$u(x,z,0) = 0 = \dot{u}(x,z,0)$$

$$v(x,z,0) = 0 = \dot{v}(x,z,0)$$

$$\varphi(x,z,0) = 0 = \dot{\varphi}(x,z,0) for \quad x_3 \geq 0, -\infty < x < \infty$$

$$u(x,z,0) = v(x,z,0) = \varphi(x,z,0) = 0 \;\; for \quad t > 0 \;\; when \quad z \to \infty.$$

(a) Equations of motion in $u - w$ plane are

$$(\lambda + \mu)\frac{\partial e}{\partial x} + \mu\nabla^2 u + \frac{\alpha}{4}\nabla^2\left(\frac{\partial e}{\partial x} - \nabla^2 u\right) - \beta_1\frac{\partial T}{\partial x} - \beta_2\frac{\partial C}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{9}$$

$$(\lambda + \mu)\frac{\partial e}{\partial z} + \mu\nabla^2 w + \frac{\alpha}{4}\nabla^2\left(\frac{\partial e}{\partial z} - \nabla^2 w\right) - \beta_1\frac{\partial T}{\partial z} - \beta_2\frac{\partial C}{\partial z} = \rho\frac{\partial^2 w}{\partial t^2}, \tag{10}$$

(b) Equation of heat conduction is

$$K\nabla^2\varphi + K^*\nabla^2\dot{\varphi} = \rho C^*(1 - \alpha_1\nabla^2)\ddot{\varphi} + \beta_1 T_0\frac{\partial^2 e}{\partial t^2}, \tag{11}$$

(c) Equation of mass diffusion is

$$\beta_2\nabla^2 \mathrm{e} + a\nabla^2 T - b\nabla^2 C + \frac{1}{D}\left(\frac{\partial C}{\partial t} + \tau\frac{\partial^2 C}{\partial t^2}\right) = 0 \tag{12}$$

And components of stress and couple stress

$$t_{33} = \left(\frac{\partial w}{\partial z} + \frac{\partial u}{\partial x}\right) + 2\mu\frac{\partial w}{\partial x} - \beta_1(1 - \nabla^2\alpha_1)\varphi - \beta_2 C \tag{13}$$

$$t_{31} = \mu\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) - \frac{\alpha}{4}\nabla^2\left(-\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) \tag{14}$$

$$m_{32} = \frac{\alpha}{2}\left(\frac{\partial^2 u}{\partial z^2} - \frac{\partial^2 w}{\partial x\partial z}\right) \tag{15}$$

where $e = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}$.

To facilitate the solution, the following dimensionless quantities are introduced

$$x' = \frac{\omega^*}{c_1}x, \quad z' = \frac{\omega^*}{c_1}z, \quad u' = \frac{\omega^*}{c_1}u, \quad w' = \frac{\omega^*}{c_1}w, \quad t' = \omega^* t, \quad t_{ij}' = \frac{t_{ij}}{\beta_{1T_0}}, m_{ij}' = \frac{m_{ij}}{c_1\beta_{1T_0}}, T' = \frac{\beta_1 T}{\rho c_1^2}, \varphi' = \frac{\beta_1\varphi}{\rho c_1^2}, \quad c' = \frac{\beta_2 C}{\rho c_1^2}, c_1^2 = \frac{\lambda + 2\mu}{\rho}, \quad \omega^{*2} = \frac{\lambda}{\mu t^2 + \rho\alpha}, \quad \alpha_1' = \frac{\omega^*}{c_1}\alpha_1 \tag{16}$$

Using the dimensionless quantities defined by (16) in the Eqs. (9)-(15) and after suppressing the primes yield

$$a_8\frac{\partial e}{\partial x} + a_1\nabla^2 u + \frac{\alpha}{4}a_2\nabla^2\left(\frac{\partial e}{\partial x} - \nabla^2 u\right) - \frac{\partial T}{\partial x} - \frac{\partial C}{\partial x} = \frac{\partial^2 u}{\partial t^2} \tag{17}$$

$$a_8\frac{\partial e}{\partial z} + a_1\nabla^2 w + \frac{\alpha}{4}a_2\nabla^2\left(\frac{\partial e}{\partial z} - \nabla^2 w\right) - \frac{\partial T}{\partial z} - \frac{\partial C}{\partial z} = \frac{\partial^2 w}{\partial t^2}, \tag{18}$$

$$\nabla^2\varphi + \frac{K^*}{K}\nabla^2\dot{\varphi} = a_3\left(1 - \alpha_1\frac{\omega^*}{c_1}\nabla^2\right)\ddot{\varphi} + a_4\frac{\partial^2 e}{\partial t^2}, \tag{19}$$

$$\nabla^2 e + a_5\frac{\rho c_1^2}{\beta_1}\omega^*\nabla^2 T - a_6\nabla^2 C + a_7\left(\frac{\partial C}{\partial t} + \tau\frac{\partial^2 C}{\partial t^2}\right) = 0 \tag{20}$$

$$t_{33} = \frac{\lambda}{\beta_1 T_0}\left(\frac{\partial w}{\partial z} + \frac{\partial u}{\partial x}\right) + \frac{2\mu}{\beta_1 T_0} + 2\mu\frac{\partial w}{\partial x} - \frac{\rho c_1^2}{\beta_1 T_0}\left(\left(1 - \alpha_1\frac{\omega^*}{c_1}\nabla^2\right)\varphi - C\right), \tag{21}$$

$$t_{31}=\frac{\mu}{\beta_1 T_0}\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)-\frac{\alpha}{4\beta_1 T_0}\left(\frac{\omega^*}{c_1}\right)^2\nabla^2\left(-\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right) \quad (22)$$

$$m_{32}=\frac{\alpha\omega^*}{2\beta_1 T_0 c_1^2}\left(\frac{\partial^2 u}{\partial z^2}-\frac{\partial^2 w}{\partial x\partial z}\right). \quad (23)$$

where

$$a_1=\frac{\mu}{\rho c_1^2},\quad a_2=\frac{\omega^{*2}}{\rho c_1^4},\quad a_3=\frac{\rho c_1^2 C^*}{K\omega^*},\quad a_4=\frac{\beta_1^2 T_0}{\rho K\omega^*},\quad a_5=\frac{a\rho c_1^2}{\beta_1\beta_2},\quad a_6=\frac{b\rho c_1^2}{{\beta_2}^2},\quad a_7=\frac{\rho c_1^4}{{\beta_2}^2 D\omega^*},\quad a_8=\frac{(\lambda+\mu)}{\rho c_1^2}$$

Applying Laplace and Fourier transformation defined by

$$(x_1,x_3,s)=\int_0^\infty f(x_1,x_3,t)e^{-st}dt \quad (24)$$

$$\hat{f}(\xi,z,s)=\int_{-\infty}^\infty \overline{f}(x,z,s)e^{i\xi x}dx \quad (25)$$

on the set of Eqs. (17)-(20), we obtain system of four homogeneous equations. These resulting equations have non trivial solution if the determinant of the coefficients of $(\hat{u},\hat{w},\hat{\varphi},\hat{C})$ vanishes, which yields the following characteristic equation

$$P\frac{d^{10}}{dz^{10}}+Q\frac{d^8}{dz^8}+R\frac{6}{dz^6}+S\frac{d^4}{dz^4}+T\frac{d^2}{dz^2}+U)(\hat{u},\hat{w},\hat{\varphi},\hat{C})=0 \quad (26)$$

where

$$P=\gamma p_{18}+\gamma^2 a_6 p_8,$$

$$Q=-\gamma p_{19}+p_{13}p_{18}+(a_8-\gamma\xi^2)\gamma a_6 p_8+\gamma p_{26}-a_6 p_2\gamma\xi^2 a_4 s^2+p_5\gamma\xi^2,$$

$$R=p_{18}p_{21}+p_{13}p_{19}+(a_8-\gamma\xi^2)p_{26}+\gamma(p_{27}-p_{20})+\xi^2 a_4 s^2\left(\gamma p_{23}-a_6 p_2\left(a_1+\frac{\alpha}{2}a_2\xi^2\right)\right)+\gamma\xi^2(p_6-2\xi^2)$$

$$S=p_{19}p_{21}+p_{13}p_{20}-\gamma p_{15}p_{19}+(a_8-\gamma\xi^2)p_{27}+\gamma p_8-\xi^2 a_4 s^2(a_6 p_2(s^2+p_3\xi^2)+\gamma p_{22}+p_{23}\left(a_1+\frac{\alpha}{2}a_2\xi^2\right)+\gamma\xi^2(p_7-2\xi^2 p_6)+(s^2+\gamma\xi^4)$$

$$T=p_{20}p_{21}+p_{13}p_{15}+(a_8-\gamma\xi^2)(p_{28}-p_{22})-\xi^2 a_4 s^2(\left(a_1+\frac{\alpha}{2}a_2\xi^2\right)+p_{23}(s^2+p_{13}\xi^2)+\xi^2(-2\xi^2\gamma p_7+(s^2+\gamma\xi^4)p_6$$

$$U=p_9 p_{21}p_{15}-\xi^2 a_4 s^2 p_{22}(s^2+p_{13}\xi^2)+p_7\xi^2(s^2+p_{13}\xi^2)$$

The roots of the Eq. (24) are $\pm\lambda_i$ (i = 1, 2, 3, 4, 5), using the radiation condition that $\hat{u},\hat{w},\hat{\varphi},\widehat{C}\rightarrow 0$ as $x_3\rightarrow\infty$ the solution of Eq. (24) may be written as

$$\hat{u}=\sum_{i=1}^5 A_i e^{-\lambda_i z} \quad (27)$$

$$\hat{w}=\sum_{i=1}^5 Q_i A_i e^{-\lambda_i z}, \quad (28)$$

$$\hat{\varphi}=\sum_{i=1}^5 R_i A_i e^{-\lambda_i z}, \quad (29)$$

$$\hat{C}=\sum_{i=1}^5 S_i A_i e^{-\lambda_i z} \quad (30)$$

where

$$Q_i=\frac{p_8 a_6\lambda_i^8-(a_1 a_6 p_8-\gamma\, p_{36})\lambda_i^6+(-Ea_6 p_8+a_1\, p_{36}-\gamma IK+\xi^2 a_4 s^2 p_2(a_6-a_5)-p_8\xi^4)\lambda_i^4+\left(E\, p_{36}+a_1 IK-a_4 s^2\xi^2(p_{14}a_6+p_2K-a_5L)-\xi^2(-\xi^2 p_8+I)\right)\lambda_i^2+EIK}{(a_4 s^2 p_2(a_5+a_6)-p_8)\lambda_i^6+\left(\left(-Ga_6 p_8+\, p_{29}\, p_{36}-a_4 s^2(a_5L+p_2K+p_{14}a_6+p_8\xi^2)\right)\right)\lambda_i^4+(G\, p_{36}+\, p_{29}IK+a_4 s^2(p_{14}K+p_{14}a_5\xi^2)+I\xi^2)\lambda_i^2+GIK}$$

$$R_i=\frac{-\gamma(p_{30}+a_6 p_{25})\lambda_i^8+\left(a_1 p_{32}-p_{25}+\gamma(p_{30}-F\xi^2 a_6+\xi^2 p_{31})\right)\lambda_i^6+\left(Ep_{32}-a_1 p_{30}-\gamma GK+\xi^2(F(p_{31}-1)+p_{29})\right)\lambda_i^4+((a_1GK-Ep_{30}+\xi^2(KF^2+G-p_{29}\xi^2))\lambda_i^2+EGK-\xi^4}{(a_4 s^2 p_2(a_5+a_6)-p_8)\lambda_i^6+\left(\left(-Ga_6 p_8+\, p_{29}\, p_{36}-a_4 s^2(a_5L+p_2K+p_{14}a_6+p_8\xi^2)\right)\right)\lambda_i^4+(G\, p_{36}+\, p_{29}IK+a_4 s^2(p_{14}K+p_{14}a_5\xi^2)+I\xi^2)\lambda_i^2+GIK}$$

$$S_i=\frac{\left(-\gamma p_{34}+\gamma(FI+s^2 a_4 p_{14})\right)\lambda_i^8+(a_1 p_{34}+\gamma(-p_{33}+Fp_8+p_{35}+s^2 a_4\xi^2 p_2))\lambda_i^6+\left(Ep_{34}+a_1 p_{33}-\gamma GI+s^2 a_4 p_{14}(1+\gamma\xi^2)+Fp_{35}-s^2 a_4\xi^2 p_2(F-\, p_{29})\right)\lambda_i^4+(Ep_{33}+(a_1G+\xi^2F^2)I-s^2 a_4\xi^2 p_{14}\, p_{29}+p_2(G+F))\lambda_i^2+EGI}{(a_4 s^2 p_2(a_5+a_6)-p_8)\lambda_i^6+\left(\left(-Ga_6 p_8+\, p_{29}\, p_{36}-a_4 s^2(a_5L+p_2K+p_{14}a_6+p_8\xi^2)\right)\right)\lambda_i^4+(G\, p_{36}+\, p_{29}IK+a_4 s^2(p_{14}K+p_{14}a_5\xi^2)+I\xi^2)\lambda_i^2+GIK}$$

$$\gamma=\frac{\alpha}{4}a,\quad p_1=a_7(s+\tau s^2),\quad p_2=\alpha_1\frac{\omega^*}{c_1},p_3=1+\frac{K^*}{K}s,p_4=p_3+p_2 s^2,$$

$$p_5=s^2 a_4 a_5 p_2-p_4,p_6=-s^2 a_4 a_5(1+2p_2\xi^2)+2p_4\xi^2-a_3 s^2,\quad p_7=s^2 a_4 a_5(\xi^2+p_2\xi^4)-p_4\xi^4+a_3 s^2\xi^2,$$

$$p_8=p_3-p_2 s^2,p_9=p_1+a_6\xi^2,\quad p_{10}=-p_8\xi^2-a_3 s^2,$$

$$p_{11}=-s^2 a_4 a_5 p_2+p_8,$$

$$p_{12}=s^2 a_4 a_5(1+p_2\xi^2)-a_3 s^2+p_2 s^2\xi^2,$$

$$
\begin{aligned}
p_{13} &= a_1+\gamma\xi^2, p_{14}=1+p_2\xi^2 \;, p_{15}\\
&= s^2\xi^2-p_{13}\xi^2, p_{16}\\
&= p_6p_8-p_{10}a_6 \;,\quad p_{17}\\
&= a_6p_4-p_2p_9 \;, p_{18}\\
&= -a_6p_8(a_8+p_{13})+\; a_4a_6p_2s^2\\
&+p_{11} \;, p_{19}\\
&= a_{16}(a_8+p_{13})-\; p_8a_6p_{15}\\
&-p_{11}\xi^2+p_{18} \;, p_{20}\\
&= p_9(a_8+p_{13})+p_{16}p_{15}+a_4p_{14}s^2\\
&-p_{18}\xi^2, p_{21}= B+a_1\xi^2, p_{21}\\
&= B+a_1\xi^2, p_{22}\\
&= p_1+p_2a_6\xi^4\\
&+(a_6+p_1p_2)\xi^2,\; p_{23}\\
&= p_1p_2+a_6+2a_6p_2\xi^2,\\
&p_{24}= -p_8\xi^4-a_3s^2\xi^2
\end{aligned}
$$

$$
\begin{aligned}
p_{13} &= a_1+\gamma\xi^2, p_{14}=1+p_2\xi^2 \;, p_{15}=s^2\xi^2-p_{13}\xi^2, p_{16}\\
&= p_6p_8-p_{10}a_6 \;,\quad p_{17}\\
&= a_6p_4-p_2p_9 \;, p_{18}\\
&= -a_6p_8(a_8+p_{13})+\; a_4a_6p_2s^2\\
&+p_{11} \;, p_{19}\\
&= a_{16}(a_8+p_{13})-\; p_8a_6p_{15}-p_{11}\xi^2\\
&+p_{18} \;, p_{20}\\
&= p_9(a_8+p_{13})+p_{16}p_{15}+a_4p_{14}s^2\\
&-p_{18}\xi^2, p_{21}= B+a_1\xi^2, p_{21}\\
&= B+a_1\xi^2, p_{22}\\
&= p_1+p_2a_6\xi^4+(a_6+p_1p_2)\xi^2,\; p_{23}\\
&= p_1p_2+a_6+2a_6p_2\xi^2,\\
&p_{24}= -p_8\xi^4-a_3s^2\xi^2
\end{aligned}
$$

$$
\begin{aligned}
p_{24} &= -p_8\xi^4-a_3s^2\xi^2, \qquad p_{25}=\gamma\xi^2,\\
&p_{26}\\
&= (a_8-p_{25})p_8a_6\\
&+\gamma(p_{10}a_6+p_8p_9)+a_3s^2p_2a_6\\
&-a_8,\\
&p_{27}\\
&= (a_8-p_{25})(p_{10}a_6+p_8p_9)\\
&+\gamma p_{10}p_9-a_4s^2p_{23}\\
&+(-a_4a_5+a_{3)})s^2+2\xi^2p_8\\
p_{28} &= (a_8-p_{25})p_{10}p_9+a_4s^2(p_{22}+a_4a_5\xi^2),\\
&p_{29}=(a_8+p_{13}),\; p_{30}\\
&= Ga_6+\; p_{29}K-\xi^2,\\
&p_{31}=-Fa_6+\gamma K+1,\; p_{32}\\
&= -\; p_{29}a_6+1,\; p_{33}\\
&= Gp_8+(a_8+p_{13})\; I+a_4s^2p_{14},\\
&p_{34}=(a_8+p_{13})p_8-a_4s^2p_2,\; p_{35}\\
&= Fp_8+\gamma I-a_4s^2\; p_2,\; p_{36}\\
&= \left(-Ia_6+p_8K\right), E= -(\xi^2+s^2), F\\
&= a_8-\frac{\alpha}{4}\xi^2a_2 \;, G\\
&= -\left(s^2+\xi^2a_1+\frac{\alpha}{4}\xi^4a_2\right), I\\
&= (-\;p_3+p_2s^2)\xi^2-a_3s^2, K\\
&= p_1+a_6\xi^2, L= p_{14}+p_2\xi^2
\end{aligned}
$$

4. Boundary condition

The boundary conditions at $z=0$ are given by

$$t_{33}(x,z,t)=-F_1\psi_1(x)\delta(t) \tag{31}$$

Where $\delta(t)$ is the Dirac delta function, F_1 is the magnitude of the force applied, $\psi_1(x)$ specify the source distribution function along x axis.

$$t_{31}(x,z,t)=0 \tag{32}$$

$$m_{32}(x,z,t)=0 \tag{33}$$

Mass concentration boundary condition: we consider the boundary plane $z=0$ is iso-concentrated surface, so

$$C=0 \tag{34}$$

Thermal boundary condition: Thermal boundary condition is taken as

$$h_1\frac{\partial\phi}{\partial z}(x,z,t)+h_2\phi(x,z,t)=0 \tag{35}$$

where $h_1\to 0$ corresponds to isothermal boundaries and $h_2\to 0$ corresponds to insulated boundary.

Substituting the values of $\hat{u},\hat{w},\hat{\phi},\hat{C}$ from Eqs. (27)-(30) in the boundary conditions(31)-(34)and with the aid of (17)-(25), we obtain the components of displacement, normal stress, tangential stress, tangential couple stress and conductive temperature and mass concentration as

$$u=\frac{-F_1\widehat{\psi_1}(\xi)}{\Delta}\sum_{i=1}^{5}B_{1i}\,e^{-\lambda_i z} \tag{35}$$

$$w=\frac{-F_1\widehat{\psi_1}(\xi)}{\Delta}\sum_{i=1}^{5}Q_iB_{1i}\,e^{-\lambda_i z} \tag{36}$$

$$\varphi=\frac{-F_1\widehat{\psi_1}(\xi)}{\Delta}\sum_{i=1}^{5}R_iB_{1i}\,e^{-\lambda_i z} \tag{37}$$

$$C=\frac{-F_1\widehat{\psi_1}(\xi)}{\Delta}\sum_{i=1}^{5}S_iB_{1i}\,e^{-\lambda_i z} \tag{38}$$

$$
\begin{aligned}
t_{31}=\frac{-F_1\widehat{\psi_1}(\xi)}{\beta_1T_0\Delta}\sum_{i=1}^{5}(\mu(\mathrm{i}\xi Q_i+\lambda_i)\\
-\frac{\alpha}{4}\left(\frac{\omega^*}{c_1}\right)^2(-\xi^2+\lambda_i^2)(\lambda_i\\
+\mathrm{i}\xi Q_i))B_{1i}e^{-\lambda_i z},
\end{aligned} \tag{39}
$$

$$
\begin{aligned}
t_{33}=\frac{-F_1\widehat{\psi_1}(\xi)}{\beta_1T_0\Delta}\sum_{i=1}^{5}(\lambda(\mathrm{i}\xi-\lambda_iQ_i)-2\mu Q_i\lambda_i\\
-\rho c_1^2\left(1-\alpha_1\frac{\omega^*}{c_1}(-\xi^2+\lambda_i^2)\right)R_i\\
-S_i)B_{1i}e^{-\lambda_i z}
\end{aligned} \tag{40}
$$

$$m_{32}=\frac{-\alpha F_1\widehat{\psi_1}(\xi)\omega^*}{2\beta_1T_0\Delta c_1^2}\sum_{i=1}^{5}(\lambda_i^2+\mathrm{i}\xi Q_i\lambda_i)B_{1i}e^{-\lambda_i z} \tag{41}$$

Where

$$B_{11} = \Delta_1/A_{11},$$
$$B_{21} = -\Delta_2/A_{12},$$
$$B_{31} = \Delta_3/A_{13},$$
$$B_{41} = -\Delta_4/A_{14},$$
$$B_{51} = \Delta_5/A_{15},$$

$$A_{1i} = \lambda\frac{\mathrm{i}\xi - \lambda_i Q_i}{\beta_1 T_0} - \frac{2\mu Q_i\lambda_i}{\beta_1 T_0} - \frac{\rho c_1^2}{\beta_1 T_0}\left(\left(1 - \alpha_1\frac{\omega^*}{c_1}(-\xi^2 + \lambda_i^2)\right)R_i - S_i\right),$$

$$A_{2i} = \frac{\mu}{\beta_1 T_0}(\mathrm{i}\xi Q_i + \lambda_i) - \frac{\alpha}{4\beta_1 T_0}\left(\frac{\omega^*}{c_1}\right)^2(-\xi^2 + \lambda_i^2)(\lambda_i + \mathrm{i}\xi Q_i),$$

$$A_{3i} = \frac{\alpha\omega^*}{2\beta_1 T_0 c_1^2}(\lambda_i^2 + \mathrm{i}\xi Q_i\lambda_i),$$
$$A_{4i} = S_i$$
$$A_{5i} = (-h_1\lambda_i + h_2)R_i,$$
$$\Delta = \Delta_1 - \Delta_2 + \Delta_3 - \Delta_4 + \Delta_5,$$
$$\Delta_1 = A_{11}(A_{22}(A_{33}D_1 - A_{34}D_2 + A_{35}D_3) - A_{23}(A_{32}D_1 - A_{34}D_4 + A_{35}D_5) + A_{24}(A_{32}D_2 - A_{33}D_4 + A_{35}D_6) - A_{25}(A_{32}D_3 - A_{33}D_5 + A_{34}D_6),$$
$$\Delta_2 = A_{12}(A_{11}(A_{33}D_1 - A_{34}D_2 + A_{35}D_3) - A_{13}(A_{31}D_1 - A_{34}D_7 + A_{35}D_8) + A_{14}(A_{31}D_2 - A_{33}D_7 + A_{35}D_9) - A_{15}(A_{31}D_3 - A_{33}D_8 + A_{34}D_9),$$
$$\Delta_3 = A_{13}(A_{11}(A_{22}D_{10} - A_{24}D_{11} + A_{35}D_8) - A_{13}(A_{31}D_1 - A_{34}D_7 + A_{35}D_8) + A_{14}(A_{21}D_{11} - A_{22}D_{14} + A_{25}D_{13}) - A_{15}(A_{21}D_{12} - A_{22}D_{15} + A_{24}D_{13}),$$
$$\Delta_4 = A_{14}(A_{21}(A_{32}D_2 - A_{33}D_4 + A_{35}D_6) - A_{22}(A_{31}D_2 - A_{33}D_7 + A_{35}D_9) - A_{23}(A_{31}D_4 - A_{32}D_7 + A_{35}D_{16}) - A_{25}(A_{31}D_6 - A_{32}D_9 + A_{33}D_{16}),$$
$$\Delta_5 = A_{15}(A_{21}(A_{32}D_3 - A_{33}D_5 + A_{34}D_6) - A_{22}(A_{31}D_3 - A_{33}D_8 + A_{34}D_9) - A_{23}(A_{31}D_5 - A_{32}D_8 + A_{34}D_{16}) - A_{24}(A_{31}D_6 - A_{32}D_9 + A_{33}D_{16}),$$
$$D_1 = A_{44}A_{55} - A_{54}A_{45}, D_2 = A_{43}A_{55} - A_{53}A_{45}, D_3 = A_{43}A_{54} - A_{53}A_{44}, D_4 = A_{42}A_{55} - A_{52}A_{45}, D_5 = A_{42}A_{54} - A_{52}A_{44}\ , D_6 = A_{42}A_{53} - A_{52}A_{43}, D_7 = A_{41}A_{55} - A_{51}A_{45}, D_8 = A_{41}A_{54} - A_{51}A_{44}, D_9 = A_{41}A_{53} - A_{51}A_{43}, D_{10} = A_{34}A_{45} - A_{44}A_{35}, D_{11} = A_{32}A_{45} - A_{42}A_{35}, D_{12} = A_{32}A_{44} - A_{42}A_{34}, D_{13} = A_{31}A_{42} - A_{41}A_{32}, D_{14} = A_{31}A_{45} - A_{41}A_{35}, D_{15} = A_{31}A_{42} - A_{41}A_{32}, D_{16} = A_{41}A_{52} - A_{51}A_{42}$$

5. Inversion of the transformations

To obtain the solution of the problem in physical domain, we must invert the transforms in Eqs. (35)-(41). Here the displacement components, normal and tangential stresses, conductive temperature and couple stress are functions of z, the parameters of Laplace and Fourier transforms s and ξ respectively and hence are of the form $f\ (\xi\ , z, s)$. To obtain the function $f(x, z, t)$ in the physical domain, we first invert the Fourier transform using

$$\bar{f}(x,z,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-\mathrm{i}\xi x}\hat{f}(\xi\ ,z\ ,s)\,d\xi = \frac{1}{2\pi}\int_{-\infty}^{\infty} |cos\ (\xi\ x)f_e - \ isin(\xi\ x)f_0|d\xi\ . \tag{50}$$

where $f_e$and f_0 are respectively the odd and even parts of$\hat{f}(\xi\ ,z\ ,s)$. Thus the expression (24) gives the Laplace transform $\bar{f}(\xi\ ,z,s)$ of the function$f(x,z,t)$. Following Honig and Hirdes (1984), the Laplace transform function$\bar{f}(\xi\ ,z,s)$ can be inverted to $f(x,z,t)$.

The last step is to calculate the integral in Eq. (50). The method for evaluating this integral is described in Press *et al.*(1986). It involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

6. Results and disscussions

For numerical computations, following Sherief and Saleh (2005), we take the isotropic material (thermoelastic diffusion solid) as $\lambda\ = 7.76 \times 10^{10} Kgm^{-1}s^{-1}$, $\mu = 3.86 \times 10^{10} Kgm^{-1}s^{-1}$, $T_0 = 293K, C^* = 3831 \times 10^3 JKg^{-1}K^{-1}, \alpha_t = 1.78 \times 10^{-5}K^{-1}$,$\alpha_c = 1.98 \times 10^{-4}m^3Kg^{-1}, a = 1.02\ \times 10^4K^{-1}m^2s^{-1}, b = 9 \times 10^5 m^5s^{-2}Kg^{-1}, \tau = 0.5s,\ \rho = 8.954 \times 10^3 Kgm^{-3}, K = 1, D = .85 \times 10^{-8}Kgsm^3, \alpha\lambda\ = 7.76 \times 10^{10}Kgm^{-1}s^{-1}$, $\mu = 3.86 \times 10^{10}Kg = .05Kgms^{-2}, K^* =\ 0.3831 \times 10^3 W\ m^{-1}, \alpha_1 = .02$,and F_1 is the force of constant magnitude of 1N.

Components of displacement, stress, conductive temperatureand couple stress are computed numerically. Software GNU Octave has been used to determine and compare the values of normal stress, tangential stress, couple stress, conductive temperature and components of displacement for homogeneous isotropic thermoelastic medium with distance x for two different values of K^* graphically.

(a) In Figs. 1-7, solid line with centre symbol $(-\ o -)$ corresponds to $K^* =\ 0$ and solid line with centre symbol $(-\ \Delta\ -)$ corresponds to$K^* =\ 0.3831$ for isothermal boundaries.

In Figs. 8-14, solid line with centre symbol $(-\ o -)$ corresponds to $K^* =\ 0$ and solid line with centre symbol $(-\ \Delta\ -)$ corresponds to $K^* =\ 0.3831$ for insulated boundaries.

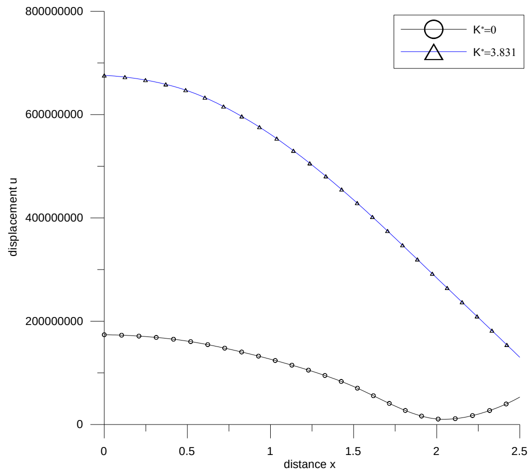

Fig. 1 Variation of distance u with the distance x(isothermal boundary)

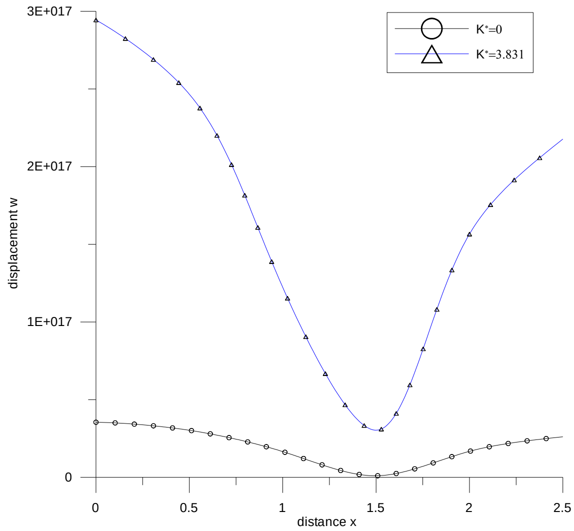

Fig. 2 Variation of distance w withthe distance x(isothermal boundary)

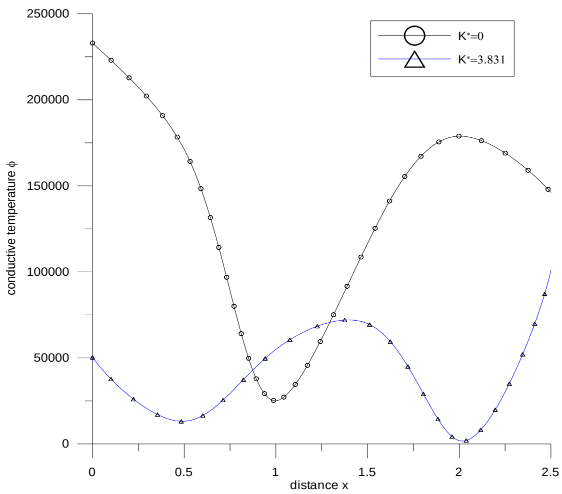

Fig. 3 Variation of conductive temperature φ with the distance x(isothermal boundary)

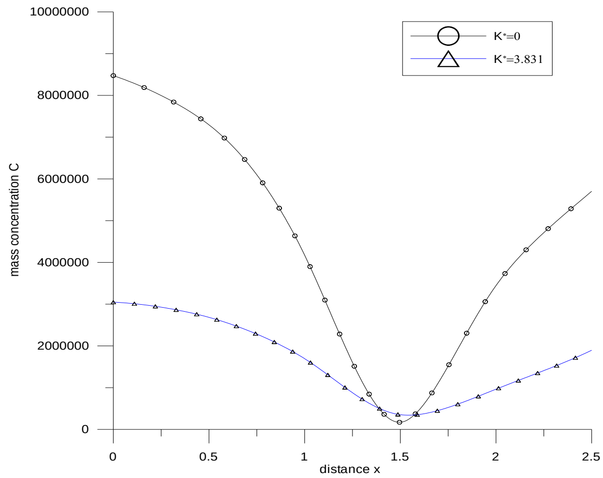

Fig. 4 Variation of mass concentration C with the distance x(isothermal boundary)

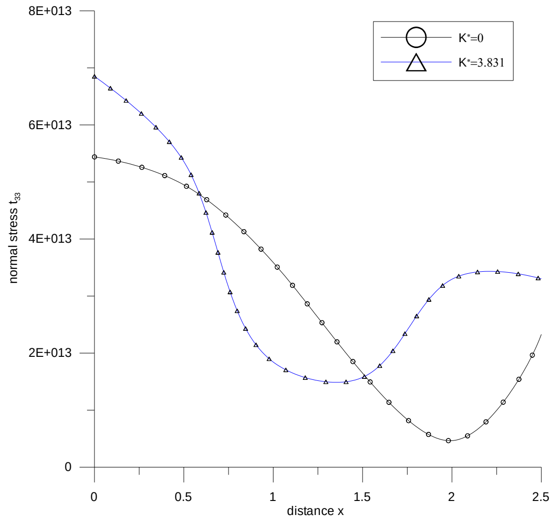

Fig. 5 Variation of normal stress t_{33} with the distance x(isothermal boundary)

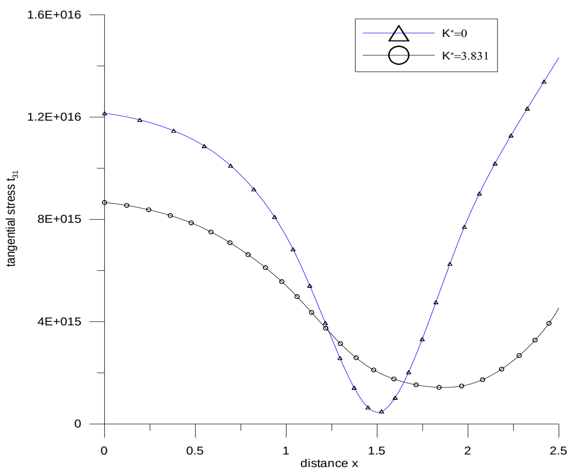

Fig. 6 Variation of tangential stresst_{31} with the distance x(isothermal boundary)

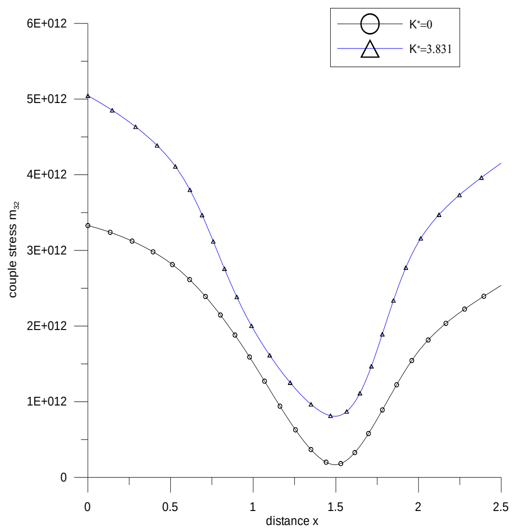

Fig. 7 Variation of couple stress m_{32} with the distance x(isothermal boundary)

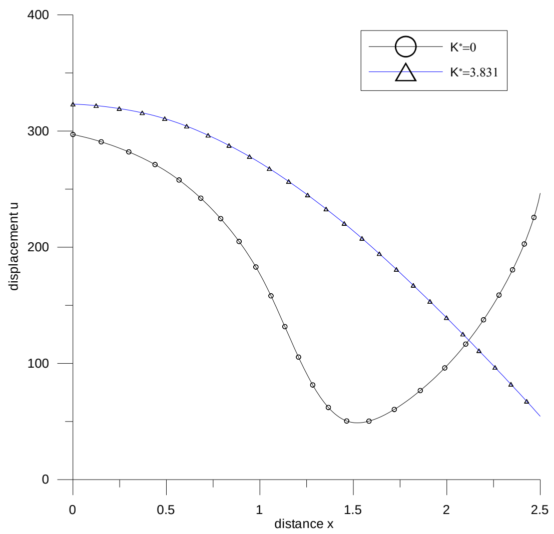

Fig. 8 Variation of distance u with the distance x(insulated boundary)

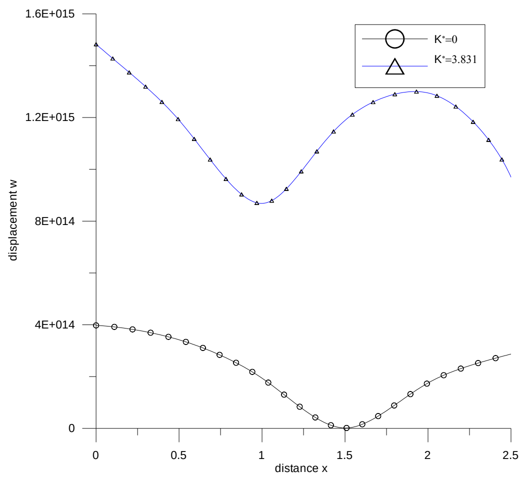

Fig. 9 Variation of distance w with the distance

x(insulated boundary)

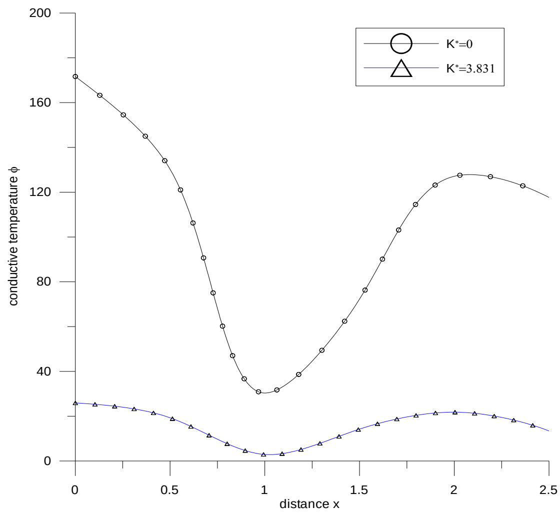

Fig. 10 Variation of conductive temperature φ with the distance x (insulated boundary)

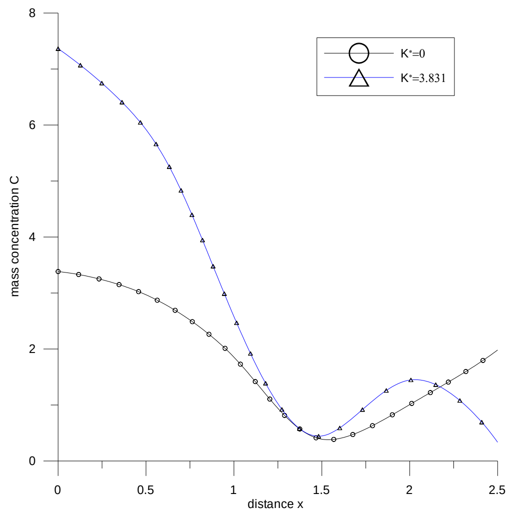

Fig. 11 Variation of mass concentration C with the distance x (insulated boundary)

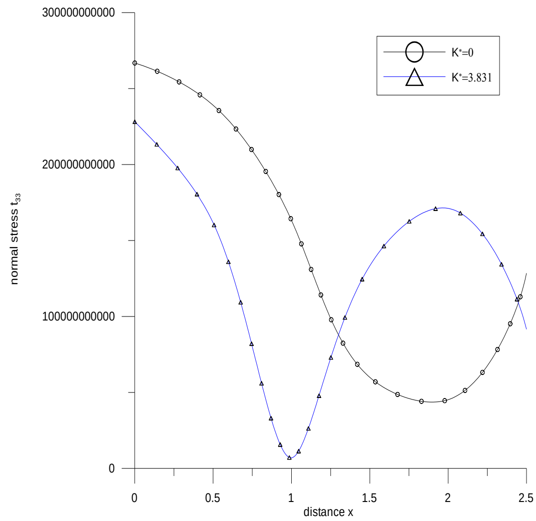

Fig. 12 Variation of normal stress t_{33} with the distance x(insulated boundary)

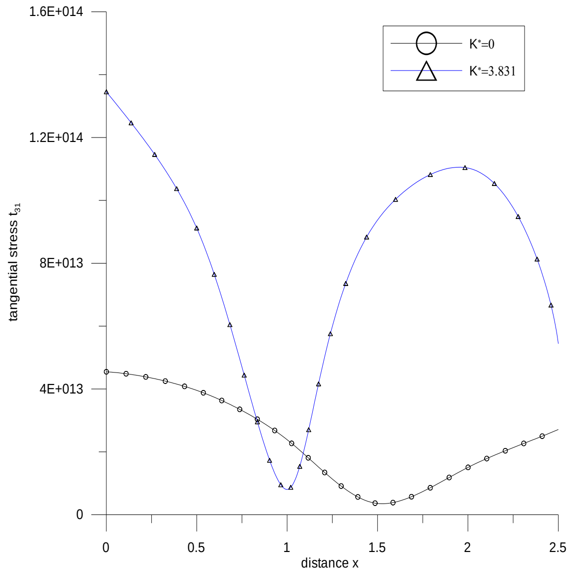

Fig. 13 Variation of tangential stress t_{31} with the distance x (insulated boundary)

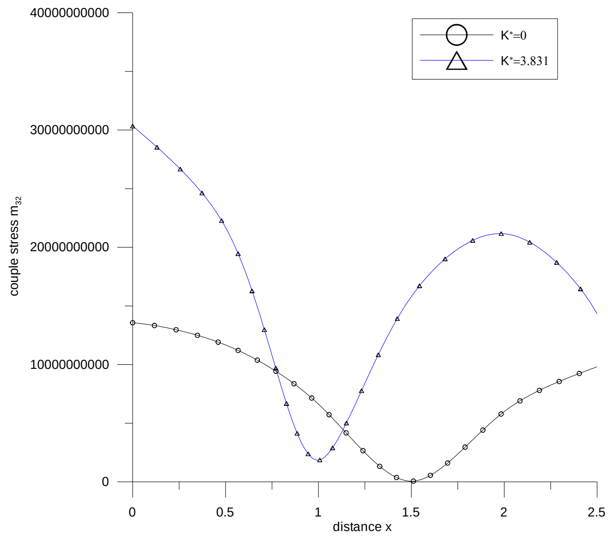

Fig. 14 Variation of couple stressm_{32} with the distance x(insulated boundary)

Figs. 1-7 shows the variation of displacement component u, displacement component w, conductive temperature φ, normal stress t_{33}, tangential stress t_{31}, couple stress $m_{32}$with the distance$x$ resp. for two different cases $K^* = 0.3831$ and $K^* = 0$ for isothermal boundaries and Figs. 8-14 corresponds to insulated boundaries. Displacement has oscillatory effect on all the physical components mentioned above.

(a) Isothermal boundary

In Fig. 1 displacement$u$ decreases as x increases for $K^* = 0$ and decreases for$0 \leq x \leq 2$and the increases very slightly corresponding to $K^* = .3831$. In Fig. 2 displacement w varies oscillatory with small amplitude for $K^* = 0$ and varies oscillatory with large amplitude for $K^* = .3831$. The value of w is less for $K^* = .3831$ than $K^* = 0$ in the range $1.25 \leq x \leq 1.75$. In Fig. 3, corresponding to variation of conductive temperature both the curves are oscillatory with opposite behavior. In Fig. 4 value of mass concentration decreases monotonically for $0 \leq x \leq 1.5$and increases monotonically for the rest of the range. Amplitude are greater for curve corresponding to $K^* = 0$ than $K^* = .3831$. in Fig.3 and Fig.4 In Fig. 5curve depicting the variation of t_{33},corresponding to $K^* = 0$ decreases regularly $0 \leq x \leq 2$and increases for rest .Curve corresponding to $K^* = .3831$ is oscillatory with blunt amplitude. In Fig. 6variation of t_{31} is similar to that of displacement$w$ except that curve corresponding to $K^* = 0$ has large but thick amplitude. In Fig.7 value of couple stress decreases for$0 \leq x \leq 1.5$and increases in the rest. Amplitudes of the curves are large.

(b) Insulated boundary

In Fig. 8 value of displacement u decreases monotonically for $0 \leq x \leq 1.5$ and increases smoothly in the rest of the range corresponding to $K^* = 0$ and decreases smoothly from $u = 325$ to $u = 70$ appx.corresponding to$K^* = .3831$.In Fig. 9displacement$w$ shows oscillatory conduct with the distance x. Values of w for $K^* = .3831$ are greater than that of $K^* = 0$ i.e. curve corresponding to $K^* = 0.3831$ is at higher position. In Fig. 10curves corresponding to $K^* = 0$ and $K^* = .3831$. In Fig. 11 Value of Ccorresponding to $K^* = 0.3831$ decreases for $0 \leq x \leq 1.5$ and follow descending oscillatory behavior in rest of distance axes, and corresponding to $K^* = 0$decreases for $0 \leq x \leq 1.5$ and increases in rest. In Fig. 12 normal stress curves corresponding to $K^* = 0.3831$ and $K^* = 0$ are oscillatory crossing each other at $x = 1.3$ and $x = 2.3$. Value of normal stress is higher near the loading surface.In Fig. 13 variation of the curves are similar to variation of the curve for the variation of couple stress in Fig. 14 except the magnitude/value of normal stress and couple stress. Curves decreases monotonically for $0 \leq x \leq 1.5$ and increases monotonically for the rest of the range for both the cases of K^*. Amplitude are smaller for curves corresponding to $K^* = 0$than $K^* = .3831$in both the Figs.

7. Conclusions

From the graphs, it is clear that there is a significant impact of thermal conductivity on the deformation of various components of stresses, displacement, conductive temperature, couple stress with two temperature. The effect of thermal conductivity in isotropic modified couple stress thermoelastic with two temperature and insulated and isothermal boundaries has an imperative impact in the investigation of the deformation of the body. As distance x varied from the point of application of the source, the components of normal stress, tangential stress, couple stress and conductive temperature pursue an oscillatory pattern. It is seen that as the disturbances travel through different constituents of the medium, the variations of displacement components, normal stress t_{33}, tangential stress t_{31} and conductive temperature φ, it suffers changes, result in an varying/ non- uniform pattern of curves. The trend of curves exhibits the effect of K^*on the medium and satisfies the required condition of the research. The results of this problem exceptionally valuable in the two dimensional problem of homogeneous isotropic thermoelastic solid with two temperature which has various geophysical and industrialised applications and helpful for designers of new materials.

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