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Shape and size optimization of trusses with dynamic constraints using a metaheuristic algorithm

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Abstract. Metaheuristic algorithm is used to solve the weight minimization problem of truss structures considering shape, and sizing design variables. The cross-sectional areas of the line element in trusses are the design variables for size optimization and the changeable joint coordinates are the shape optimization used in this study. The design of plane and spatial truss structures are optimized by metaheuristic technique named Teaching-Learning-Based Optimization (TLBO). Finite element analyses of structures and optimization process are carried out by the computer program visually developed by the authors coded in MATLAB. The four benchmark problems (trusses 2D ten-bar, 3D thirty-seven-bar, 3D seventy-two-bar and 2D two-hundred-bar) taken from literature are optimized and the optimal solution compared the results given by previous studies.

Keywords: truss structures; shape and size optimization; TLBO algorithm; dynamic constraints

1. Introduction

Optimization of trusses taking into account the dynamic constraints is made by several researches. Lingyun *et al*. (2004) used genetic algorithm (GA) for the size and shape optimization of trusses. Sedaghati *et al*. (2002) presented a study on optimization of structure using integrated force method and sequential quadratic programming. Wang *et al*. (2004) used evolutionary optimization method for trusses with dynamic constraints. A particle swarm algorithm (PSO) is used by Gomes (2011) for the optimal solution of truss structure. A new algorithm named democratic particle swarm optimization (DPSO) suggested by Kaveh and Zolghadr (2014) for the truss structures. Kaveh and Zolghadr (2012) used a hybridized CSS-BBBC algorithm, Kaveh and Mahdavi (2015) developed two-dimensional colliding bodies algorithm, Grzywiński *et al*. (2019) studied on the optimization of middle-size of dome structures by using continuous design variables. They preferred to use Jaya algorithm in the optimization process. Another study is made by Tejani *et al*. (2016) for the design optimization of trusses using an optimization technique called adaptive symbiotic organisms. Optimal design of steel space frames using GA is made by Artar and Daloğlu (2015). Using TLBO (teaching-learning based algorithm), the optimization of the braced steel structures is presented by the Artar (2016a). Another study for the comparative study on truss structure is studied by Artar (2016b). Dede (2018) presented a study using the Jaya algorithm for the size optimization of the steel grillage structures.

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2. Optimization of truss structures

The general purpose in the optimization of the trusses is to minimize the total weight of structure. But, while minimizing the weight the constraint should not be violated. In this study, the first natural frequencies are used as a constraint. The objective function written in the terms of the weight of structure is given by Eq. (1).

$$W(\boldsymbol{A}) = \sum_{i=1}^{nm} A_i\, \rho_i\, L_i \quad i = 1,2,\ldots,nm \tag{1}$$

where W is the truss structure's weight, A_i is the cross-sectional for a truss member, ρ_i is the material density and L_i, the length of the truss member, and "nm" is the number of truss member.

$$\omega_j \geq \omega_j^* \qquad j = 1, 2, \ldots, n \tag{2}$$

$$\omega_k \leq \omega_k^* \qquad k = n+1, n+2, \ldots, m \tag{3}$$

Where "n" is the number of constraint which must be greater or equal to the given frequency limit and similar way, "m" is the number of constraint which must be smaller or equal to the given frequency limit. For the size optimization the Eq. (4) is used and for the shape optimization the Eq. (5) is used.

$$A_l^{low} \leq A_l \leq A_l^{up} \qquad l = 1, 2, \ldots, nm \tag{4}$$

$$x_m^{low} \leq x_m \leq x_m^{up} \qquad m = 1, 2, \ldots, nc \tag{5}$$

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where ω_j is the frequency values of the trusses, A_l^{low} and A_l^{up} are minimum and maximum value of the bounds for design variables, x_m^{low} and x_m^{up} are minimum and maximum value of the bounds for design coordinate variables and "nc" is the number of joint which it's coordinates are changeable for shape optimization.

The penalized objective function in terms of the constraint can be written as given in Eq. (6);

Penalty function is given as:

$$PF = W * (1 + \varphi * C) \tag{6}$$

where "φ" is a constant value which can be adapted according to the optimization problem, "ω*" is the allowable frequency, "C" summation of violations of the constraints. For example, in the case of "$\omega_k \leq \omega_k^*$" the calculation for the violation of the constraint is demonstrated by the help of the Eq. (7).

$$g_i = \frac{\omega_k}{\omega_k{}^*} - 1 \leq 0 \tag{7}$$

$$if \quad g_i < 0 \quad then \quad C_i = 0, \tag{8}$$

$$if \quad g_i > 0 \quad then \quad C_i = g_i \tag{9}$$

3. TLBO algorithm

As an efficient and effective metaheuristics algorithm, the Teaching-Learning-Based Optimization was firstly presented by Rao and his colleagues in Rao (2016) and Rao *et al*. (2011). Like the many optimization techniques, the TLBO use a randomly created initial population and then this technique uses a maximum iteration and a number of population sizes. There are two main part in this algorithm named as teacher phase and student phase. In the first part, that is teacher phase, the individual which is the candidate solution to become a best solution learn some useful knowledge from the teacher which is the best individual in the current generation.

For the teacher phase, updating the population is given in the following equation for one individual.

$$Pop_{i,new} = Pop_i + rand * (Pop_{teacher} - T_F \cdot Pop_{mean}) \tag{10}$$

where Pop_i is the "i-th" individual in the population, $Pop_{teacher}$ is the best solution in the current generation, Pop_{mean} is the mean solution, $rand$ is a random number between with zero and one. The T_F is calculated as 1 or 2. The detailed knowledge about this calculation can be found in the main paper Rao (2016).

After updating the population, there are two individuals named the old one and the new one in the student phase. In this phase, the individual called as a student compare his/her knowledge with the randomly selected any student. After this comparison, if a student has better objective functions he/she use own knowledge. Otherwise, the student uses the randomly selected student's knowledge. This comparison is formulated by using the following equations.

$$Pop_{i,new} = Pop_i + rand(Pop_j - Pop_i) \quad \text{if} \quad f(Pop_i) > f(Pop_j) \tag{11}$$

$$Pop_{new} = Pop_i + rand(Pop_i - Pop_j) \quad \text{if} \quad f(Pop_i) < f(Pop_j) \tag{12}$$

If $Pop_{i,new}$ is better, it is accepted for the next generation. By this way, this algorithm is carried out until the maximum iteration.

4. Numerical examples

To present the proposed algorithm named TLBO technique, four different 2D and 3D trusses are taken into account for the optimization process. The trusses: planar Ten-bar (with 10 Design Variables), Two-hundred-bar (29 DV), and spatial truss Seventy-two-bar (16 DV) are size optimization problems, whereas the planar truss Thirty-seven-bar (19 DV), is size and shape optimization.

4.1 Planar truss ten-bar

As a first structural example, ten-bar truss is optimized using the proposed algorithm. The configuration and added mass distribution are given in the Fig. 1. For ten-bar structure only size optimization is considered by using ten independent size design variables which are the cross-sectional area of the truss bar members. The additional mass is applied for the joints on 1, 2, 3 and 4. This additional mass is 454 kg. The density of material, Young modulus, additional mass, design variables and the constraint in the terms of the frequency are given in the Table 1. Ten-bar benchmark structure was studied before by many authors such as Segadhati *et al*. (2002), Wang *et al*. (2004), Lingyun *et al*. (2004) and Kaveh and Zolghadr (2014).

For ten-bar benchmark problem, the size of population and the maximum iteration number are taken as 50 and 200, respectively. But, after the generation 184 the convergence is achieved. The optimal solutions obtained by using proposed algorithm are compared with the best results given

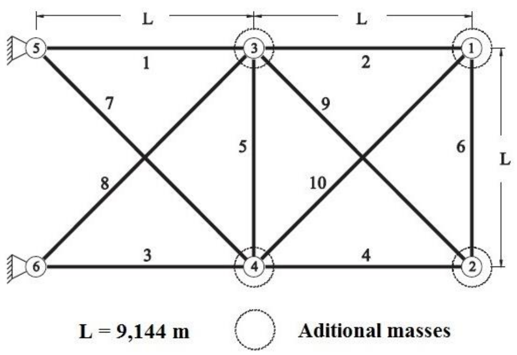

Fig. 1 Configuration of the ten-bar truss

Table 1 Structural constraints and material properties for ten-bar structure

Symbols	Definitions	Value	Unit
E	Young modulus	$6{,}89 \times 10^{10}$	N/m^2
ρ	Density of materials	2770	kg/m^3
M	Additional masses	454	kg
A	Cross-sectional area	$0{,}645 \le A \le 50$	cm^2
ω	Frequency constraints	$\omega_1 \ge 7$, $\omega_2 \ge 15$, $\omega_3 \ge 20$	Hz

Table 2 Natural frequencies of the optimum designs for ten-bar structure

Frequency number	Sedaghati *et al.* (2002)	Wang *et al.* (2004)	Lingyun *et al.* (2004)	Kaveh and Zolghdar (2014)	This study
1	7,000	7,011	7,008	7,000	7,0001
2	17,620	17,302	18,148	16,187	16,1741
3	20,000	20,001	20,000	20,000	20,0001
4	20,000	20,100	20,508	20,021	20,0093
5	28,200	30,869	27,797	28,470	28,3625

in previous studies in Table 2 for the frequency and Table 3 for the total weight of ten-bar structure.

As seen from these comparisons, it should be highlighted that the best results obtained with the proposed algorithm named TLBO studied in this study. The optimal solutions obtained by using proposed algorithm do not violate the constraints of the ten-bar structure.

4.2 Planar truss thirty-seven-bar

The next example is truss bridge (type Pratt). Initial configuration and added mass distribution are given in the Fig. 2(a). The lower chord elements have constant cross-section area equal 40 cm^2. The thirty-seven-bar structure is size and shape optimization. There are fourteen independent size design variables (DV) which are the cross-sectional area of the truss bar members, and five shape variables (SV) of the truss upper nodes (using symmetry of structure). The additional mass is applied only the free nodes of the lower chord. This additional mass is 10 kg. The density of material, Young modulus, additional mass, design variables and the constraint in the terms of the frequency are given in the Table 4. Thirty-seven-bar benchmark structure was

Table 3 Optimal solutions cross-section (cm^2) for ten-bar structure

Design variable	Sedaghati *et al.* (2002)	Wang *et al.* (2004)	Lingyun *et al.* (2004)	Kaveh and Zolghdar (2014)	This study
A1	38,245	32,456	42,234	35,944	34,9857
A2	9,916	16,577	18,555	15,530	14,0963
A3	38,619	32,456	38,851	35,285	35,0343
A4	18,232	16,577	11,222	15,385	4,9628
A5	4,419	2,115	4,783	0,648	0,6528
A6	4,419	4,467	4,451	4,583	4,5870
A7	20,097	22,810	21,049	23,610	22,8338
A8	24,097	22,810	20,949	23,599	24,9355
A9	13,890	17,490	10,257	13,135	13,0128
A10	11,452	17,490	14,342	12,357	11,9419
Weight (kg)	**537,01**	**553,80**	**542,75**	**532,39**	**524,729**

Table 4 Structural constraints and material properties for thirty-seven-bar truss

Symbols	Definitions	Value	Unit
E	Young modulus	$2{,}1 \times 10^{11}$	N/m^2
ρ	Density of materials	7800	kg/m^3
M	Additional masses	10	kg
A	Cross-sectional area	$1 \le A \le 10$	cm^2
Y	Node coordination	$0{,}1 \le Y \le 3{,}0$	m
ω	Frequency constraints	$\omega_1 \ge 20$, $\omega_2 \ge 40$, $\omega_3 \ge 60$	Hz

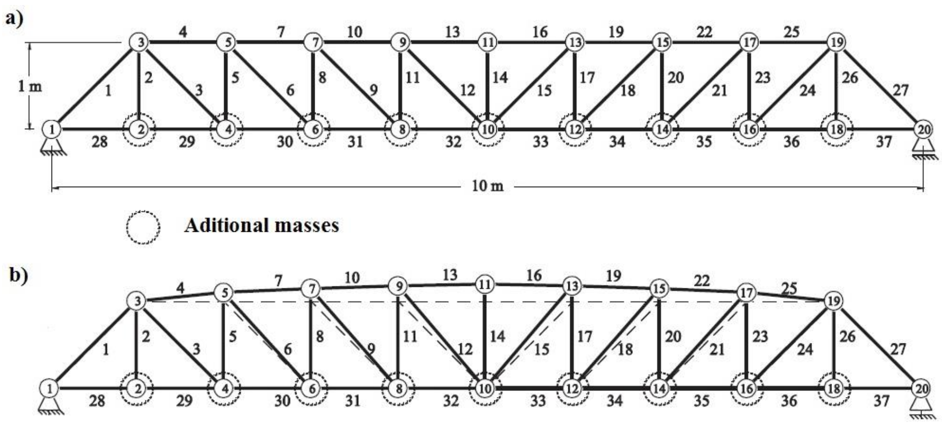

Fig. 2 The thirty-seven-bar truss: (a) initial shape; (b) optimized shape

Table 5 Natural frequencies of the optimum designs for thirty-seven-bar truss

Frequency number	Wang *et al.* (2004)	Lingyun *et al.* (2004)	Kaveh and Zolghdar (2012)	Farshchin *et al.* (2016)	This study
1	20,0850	20,0013	20,0194	20,0001	20,0001
2	42,0743	40,0305	40,0113	40,0005	40,0029
3	62,9383	60,0000	60,0082	60,0066	60,0164
4	74,4539	73,0444	76,9896	76,4395	75,9988
5	90,0576	89,8244	97,2222	95,9402	95,8489

Table 6 Optimal solutions cross-section (cm^2) and nodal position (m) for thirty-seven-bar truss

Design variable	Wang *et al.* (2004)	Lingyun *et al.* (2004)	Kaveh and Zolghdar (2012)	Farshchin *et al.* (2016)	This study
A1, A27	3,2508	2,8932	2,6208	2,9055	2,7473
A2, A26	1,2364	1,1201	1,0397	1,0012	1,0191
A3, A24	1,0000	1,0000	1,0464	1,0001	1,0157
A4, A25	2,5386	1,8655	2,7163	2,5598	2,6216
A5, A23	1,3714	1,5962	1,0252	1,2523	1,0917
A6, A21	1,3681	1,2642	1,5081	1,2141	1,2734
A7, A22	2,4290	1,8254	2,3750	2,3851	2,5043
A8, A20	1,6522	2,0009	1,4498	1,3881	1,4790
A9, A18	1,8257	1,9526	1,4499	1,5235	1,4542
A10, A19	2,3022	1,9705	2,5327	2,6065	2,4546
A11, A17	1,3103	1,8294	1,2358	1,1378	1,1798
A12, A15	1,4067	1,2358	1,3528	1,3078	1,3319
A13, A16	2,1896	1,4049	2,9144	2,6205	2,5279
A14	1,0000	1,0000	1,0085	1,0003	1,0000
Y3, Y19	1,2086	1,1998	0,9482	0,9639	1,0006
Y5, Y17	1,5788	1,6553	1,3439	1,3551	1,3699
Y7, Y15	1,6719	1,9652	1,5043	1,5338	1,5402
Y9, Y13	1,7703	2,0737	1,6350	1,6367	1,6702
Y11	1,8502	2,3050	1,7182	1,7052	1,7458
Weight (kg)	**366,50**	**368,84**	**360,40**	**359,88**	**359,94**

studied before by many authors such as Wang *et al*. (2004), Lingyun *et al*. (2004), Kaveh and Zolghadr (2012), and Farshchin *et al*. (2016). For thirty-seven-bar benchmark problem, the size of population and the maximum iteration number are taken as 30 and 250, respectively. But, after the generation 213 the convergence is achieved. The optimal solutions obtained by using proposed algorithm are compared with the best results given in previous studies in Table 5 for the frequency and Table 6 for the total weight of ten-bar structure. Fig. 2(b) shows the optimal shape thirty-seven-bar truss structure.

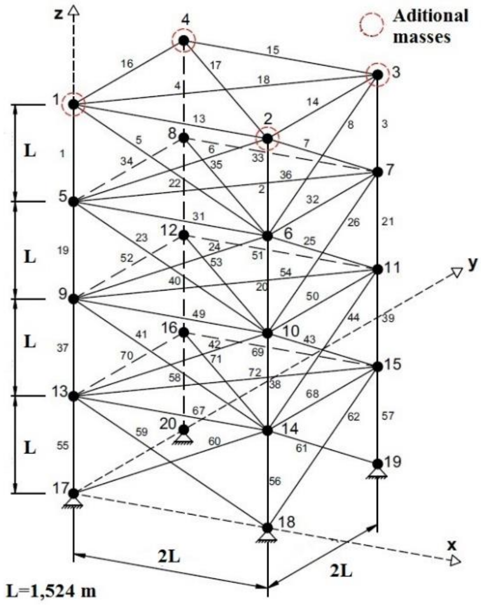

Fig. 3 Configuration of the seventy-two-bar truss

4.3 Spatial truss seventy-two-bar

The configuration and added mass distribution for seventy-two-bar truss are given in the Fig. 3. For seventy-two-bar structure only size optimization is considered by using ten independent size design variables which are the cross-sectional area of the truss bar members. The additional mass is applied for the joints on 1, 2, 3 and 4. This additional mass is 2270 kg. The density of material, Young modulus, additional mass, design variables and the constraint in the terms of the frequency are given in the Table 7. The seventy-two-bar benchmark structure was

Table 7 Structural constraints and material properties for seventy-two-bar truss

Symbols	Definitions	Value	Unit
E	Young modulus	$6,89 \times 10^{10}$	N/m^2
ρ	Density of materials	2770	kg/m^3
M	Additional masses	2270	kg
A	Cross-sectional area	$0,645 \leq A \leq 30$	cm^2
ω	Frequency constraints	$\omega_1 = 4$, $\omega_3 \geq 6$	Hz

Table 8 Natural frequencies of the optimum designs for seventy-two-bar truss

Frequency number	Sedaghati *et al.* (2002)	Gomes (2011)	Kaveh and Zolghdar (2012)	Farshchin *et al.* (2016)	This study
1	4,000	4,000	4,0000	4,0000	4,0000
2	4,000	4,000	4,0000	4,0000	4,0000
3	6,000	6,000	6,0040	6,0000	6,0000
4	6,247	6,219	6,2491	6,2515	6,2930
5	9,074	8,976	8,9726	9,0799	9,1174

Table 9 Optimal solutions cross-section (cm^2) for seventy-two-bar truss

Design variable	Sedaghati *et al.* (2002)	Gomes (2011)	Kaveh and Zolghdar (2012)	Farshchin *et al.* (2016)	This study
A1-A4	3,499	2,987	2,854	3,5491	3,5952
A5-A12	7,932	7,849	8,301	7,9676	7,8589
A13-A16	0,645	0,645	0,645	0,6450	0,6452
A17-A18	0,645	0,645	0,645	0,6450	0,6452
A19-A22	8,056	8,765	8,202	8,1532	8,4094
A23-A30	8,011	8,153	7,043	7,9667	7,8724
A31-A34	0,645	0,645	0,645	0,6450	0,6452
A34-A36	0,645	0,645	0,645	0,6450	0,6452
A37-A40	12,812	13,450	16,328	12,9272	13,3221
A41-A48	8,061	8,073	8,299	8,1226	7,9985
A49-A52	0,645	0,645	0,645	0,6452	0,6452
A53-A54	0,645	0,645	0,645	0,6450	0,6452
A55-A58	17,279	16,684	15,048	17,0524	16,0000
A59-A66	8,088	8,159	8,268	8,0618	8,0241
A67-A70	0,645	0,645	0,645	0,6450	0,6452
A71-A72	0,645	0,645	0,645	0,6450	0,6452
Weight (kg)	**327,605**	**328,823**	**327,507**	**327,568**	**324,457**

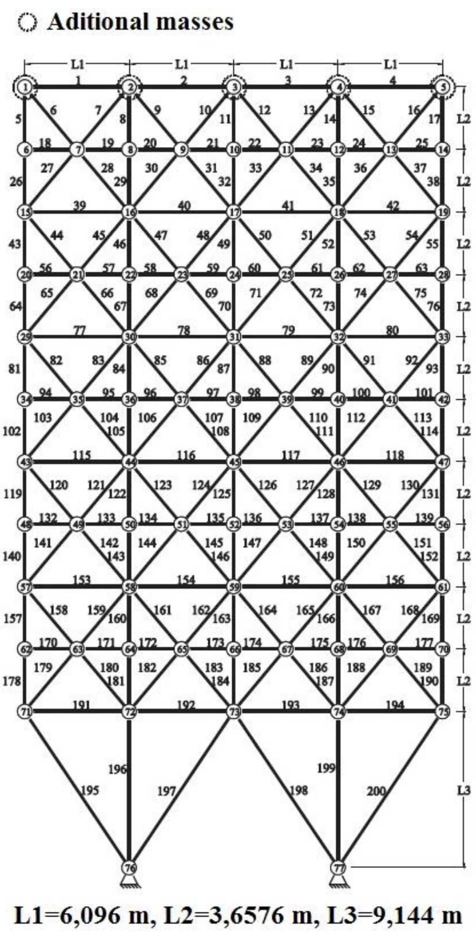

Fig. 4 Configuration of the two-hundred-bar truss

Table 10 Structural constraints and material properties for two-hundred -bar truss

Symbols	Definitions	Value	Unit
E	Young modulus	$2{,}1 \times 10^{11}$	N/m^2
ρ	Density of materials	7860	kg/m^3
M	Additional masses	100	kg
A	Cross-sectional area	$1 \leq A \leq 30$	cm^2
ω	Frequency constraints	$\omega_1 \geq 5$, $\omega_2 \geq 10$, $\omega_3 \geq 15$	Hz

studied before by many authors such as Segadhati *et al.* (2002), Gomes (2011), Kaveh and Zolghadr (2012), and Farshchin *et al.* (2016).

For seven-two-bar benchmark problem, the size of population and the maximum iteration number are taken as 40 and 2500, respectively. But, after the generation 1506 the convergence is achieved. The optimal solutions obtained by using proposed algorithm are compared with the best results given in previous studies in Table 8 for the frequency and Table 9.

4.4 Planar truss two-hundred-bar

The configuration and added mass distribution for two-hundred-bar truss are given in the Fig. 4. For two-hundred-bar structure only size optimization is considered by using ten independent size design variables which are the cross-sectional area of the truss bar members. The additional mass is applied for the joints on 1, 2, 3, 4, and 5. This additional mass is 100 kg. The density of material, Young modulus, additional mass, design variables and the constraint in the terms of the frequency are given in the Table 10. The two-hundred-bar benchmark structure was studied before by many authors such as Kaveh and Mohdavi (2015), Farshchin *et al.* (2016), and Tejani *et al.* (2016). For two-hundred-bar benchmark problem, the size of population and the maximum iteration number are taken as 50 and 500, respectively. But, after the generation 491 the convergence is achieved. The optimal solutions obtained by using proposed algorithm are compared with the best results given in previous studies in Table 11 for the frequency and Table 12.

Table 11 Natural frequencies of the optimum designs for two-hundred -bar truss

Frequency number	Kaveh and Mohdavi (2015)	Farshchin *et al.* (2016)	Tejani *et al.* (2016)	This study
1	5,0016	5,0000	5,0001	5,0000
2	13,3868	12,2305	12,1388	12,1782
3	15,1981	15,0259	15,1284	15,0409
4	17,0921	16,6805	16,7317	16,7347
5	21,2002	21,4089	21,1952	21,3380

Table 12 Optimal solutions cross-section (cm^2) for two-hundred-bar truss

DV groups	Elements	Kaveh and Mohdavi (2015)	Farshchin *et al*. (2016)	Tejani *et al*. (2016)	This study
DV1	1, 2, 3, 4	0,4460	0,3067	0,2822	0,2954
DV2	5, 8, 11, 14, 17	0,4556	0,4449	0,5014	0,4596
DV3	19, 20, 21, 22, 23, 24	0,1519	0,1000	0,1071	0,1002
DV4	18, 25, 56, 63, 94, 101, 132, 139, 170, 177	0,1000	0,1000	0,1002	0,1001
DV5	26, 29, 32, 35, 38	0,4723	0,5077	0,5277	0,5207
DV6	6, 7, 9, 10, 12, 13, 15, 16, 27, 28, 30, 31, 33, 34, 36, 37	0,7543	0,8241	0,8248	0,8166
DV7	39, 40, 41, 42	0,1024	0,1001	0,13	0,1028
DV8	43, 46, 49, 52, 55	1,4924	1,4367	1,4016	1,4192
DV9	57, 58, 59, 60, 61, 62	0,1000	0,1000	0,1000	0,1021
DV10	64, 67, 70, 73, 76	1,6060	1,5787	1,4657	1,6463
DV11	44, 45, 47, 48, 50, 51, 53, 54, 65, 66, 68, 69, 71, 72, 74, 75	1,2098	1,1586	1,1327	1,1445
DV12	77, 78, 79, 80	0,1061	0,1000	0,1196	0,1544
DV13	81, 84, 87, 90, 93	3,0909	2,9573	3,0262	3,0223
DV14	95,96, 97, 98, 99, 100	0,7916	0,1000	0,2527	0,1045
DV15	102, 105, 108, 111, 114	3,6095	3,2569	3,3267	3,2818
DV16	82, 83, 85, 86, 88, 89, 91, 92, 103, 104, 106, 107, 109, 110, 112, 113	1,4999	1,5733	1,5963	1,5958
DV17	115, 116, 117, 118	0,1000	0,2674	0,2417	0,2544
DV18	119, 122, 125, 128, 131	5,2951	5,0867	4,8557	5,2039
DV19	133, 134, 135, 136, 137, 138	0,1000	0,1004	0,1001	0,1001
DV20	140, 143, 146, 149, 152	4,5288	5,4551	5,4975	5,4880
DV21	120,121,123,124, 126, 127, 129, 130, 141,142, 144, 145, 147,148, 150, 151	2,2178	2,0998	2,0829	2,1282
DV22	153, 154, 155, 156	0,7571	0,7156	0,8522	0,6509
DV23	157, 160, 163, 166, 169	7,7999	7,6425	7,5480	7,5512
DV24	171, 172, 173, 174, 175, 176	0,3506	0,1049	0,1279	0,1993
DV25	178, 181, 184, 187, 190	7,8943	7,9352	7,6278	7,9193
DV26	158, 159, 161, 162, 164, 165, 167, 168, 179, 180, 182, 183, 185, 186, 188, 189	2,8097	2,8261	3,0233	2,8504
DV27	191, 192, 193, 194	10,4220	10,4388	10,3024	10,6251
DV28	195, 197, 198, 200	21,2576	21,2125	21,4034	21,2733
DV29	196, 199	11,9061	10,8347	10,481	11,1346
	Weight (kg)	**2189,080**	**2156,639**	**2164,884**	**2176,038**

5. Conclusions

The TLBO algorithm is implemented in this paper for the size and shape optimization of planar, and spatial truss structures. This optimization algorithm consists of two main phases, i.e., "Teacher phase" and "Student phase". Like other nature-inspired algorithms. TLBO is also a population-based method using a population of solutions to proceed to the global solution. The design results are compared with the previous. To optimize the truss structures a new and efficient algorithm called TLBO is coded in the Matlab.

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