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Wave dispersion properties in imperfect sigmoid plates using various HSDTs

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Abstract. In this paper, wave propagations in sigmoid functionally graded (S-FG) plates are studied using new Higher Shear Deformation Theory (HSDT) based on two-dimensional (2D) elasticity theory. The current higher order theory has only four unknowns, which mean that few numbers of unknowns, compared with first shear deformations and others higher shear deformations theories and without needing shear corrector. The material properties of sigmoid functionally graded are assumed to vary through thickness according sigmoid model. The S-FG plates are supposed to be imperfect, which means that they have a porous distribution (even and uneven) through the thickness of these plates. The governing equations of S-FG plates are derived employed Hamilton's principle. Using technique of Navier, differential equations of S-FG in terms displacements are solved. Extensive results are presented to check the efficient of present methods to predict wave dispersion and velocity wave in S-FG plates.

Keywords: wave propagation; S-FG; porosity; higher shear deformations theories

1. Introduction

A new material called functionally graded materials, which results from the demand for improved structural efficiency in the aerospace field, after it is used to others fields as mechanical and civil engineering (Shen and Wang 2012, Avcar 2015, Kar and Panda 2015a, b, Ait Atmane *et al*. 2016, Bellifa *et al*. 2017a, Fahsi *et al*. 2017, Aldousari 2017, Karami *et al*. 2018a, Selmi and Bisharat 2018, Faleh *et al*. 2018, Hussain and Naeem 2019). FGMs are manufactured in such a way that the properties of the materials vary evenly and continuously in the thickness, from the surface of a ceramic exposed to a high temperature to that of a metal to the other surface (Attia *et al*. 2015, Kar and Panda 2016a, b, Beldjelili *et al*. 2016, Kar *et al*. 2017, Nebab *et al*. 2018, Mahapatra *et al*. 2017, Younsi *et al*. 2018, Attia *et al*. 2018, Yousfi *et al*. 2018, Zarga *et al*. 2019, Bennai *et al*. 2019, Boussoula *et al*. 2019).

The functionally graded materials and structures are subject to study by many researchers. Chi and Chung (2006) are investigated a simply supported, functionally graded material (FGM) plate of medium thickness subjected to transverse loading. Chung and Chang (2008) are studied a rectangular functionally graded material (FGM) plate with medium thickness subjected to linear temperature change through the thickness of plates in z-direction. Ait Atmane *et al*. (2010) are presented a new shear deformation theory free vibration analysis of functionally graded plates resting on elastic foundations. Fekrar *et al*. (2012) are analyzed buckling analysis of functionally graded plates using a refined plate theory. Menaa *et al*. (2012) are studied analytical Solutions for Static Shear Correction Factor of Functionally Graded Beams. Kettaf *et al*. (2013) are presented a new hyperbolic shear displacement model to study Thermal buckling of functionally graded sandwich plates. Tounsi and co-works have been studied static, buckling, vibrations and thermo-mechanical functionally graded structures of functionally graded plates and beams (Meziane *et al*. 2014, Ait Atmane *et al*. 2015, 2016, 2017, Bennai *et al*. 2015, Bousahla *et al*. 2016, Abdelaziz *et al*. 2017, El-Haina et al. 2017, Menasria *et al*. 2017, Hellal *et al*. 2019, Meksi *et al*. 2019, Bourada *et al*. 2019, Mahmoudi *et al*. 2019). Dynamic analysis of S-FGM on elastic foundations using a four-variable refined plate theory was investigated by (Han *et al*. 2015). Jung and Han (2015) is examined bending, vibration and buckling analysis of sigmoid functionally graded materials micro-scale plates via the modified couple stress theory. Lee *et al*. (2015) is presented a refined higher order shear and normal deformation theory for exponential, power-law and sigmoid functionally graded material plates on elastic foundation. Fazzolari (2016) is investigated on free vibration of functionally graded plates with temperature-dependent materials in a thermal environment. Hamed *et al*. (2016) was investigated no vibration characteristics of both nonlinear symmetric power and sigmoid functionally graded nonlocal nano-beams. Han *et al*. (2016) are investigated vibration characteristics of longitudinally moving sigmoid functionally graded material (S-FGM) plates containing porosities. Benferhat *et al*. (2016) analyzed the bending response of FG plate with porosities. Gupta and Talha (2018) are studied static and stability

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behaviors of geometrically imperfect functionally graded material (FGM) plate with a microstructural defect resting on Pasternak elastic foundation. Thang and Lee (2018) are investigated the positive influence of the stiffeners on the vibration of functionally graded plates. Wang and Zu (2018) are presented a study on nonlinear vibration of inhomogeneous functional plates composed of sigmoid graded metal-ceramic materials.

Scholars have been investigated the study of wave propagations in functionally graded structures. Chakraborty and Gopalakrishnan (2004) are purposed a new higher-order spectral element for wave propagation analysis of a functionally graded material (FGM) beam in the presence of thermal and mechanical loading. Chen *et al*. (2007) are studied the dispersion behavior of waves in an elastic plate with material properties varying along the thickness direction. Kudela *et al*. (2007) are presented numerical simulation of the propagation of transverse elastic waves corresponding to the A0 mode of Lamb waves in a composite plate. Cao *et al*. (2009) are studied the propagation of Love waves in a layered structure consisting of two different homogenous piezoelectric materials, an upper layer and a substrate. Yu *et al*. (2010) are presented guided thermo-elastic waves in functionally graded plates with two relaxation times. Besseghier *et al*. (2011) are studied the thermal effect on wave propagation in double-walled carbon nanotubes embedded in a polymer matrix via nonlocal elasticity. Idesman (2014) is purposed an accurate numerical solution for wave propagation in inhomogeneous materials under impact loading. Wave propagation in functionally graded nano-composites reinforced with carbon nanotubes is investigated, based on shear deformation theory (Ghorbanpour Arani *et al*. 2016, Janghorban and Nami 2016). Arefi and Zenkour (2017) are studied wave propagation analysis of a nanobeam made of functionally graded magneto-electro-elastic materials with rectangular cross section rest on visco-pasternak foundation. Ebrahimi and Dabbagh (2017) are presented a nonlocal strain gradient theory to capture size effects in wave propagation analysis of compositionally graded smart nano-plates. Fadodun *et al*. (2017) presented fractional wave propagation in radially vibrating non-classical cylinder. Kolahchi *et al*. (2017) examined wave propagation of embedded viscoelastic FG-CNT-reinforced sandwich plates integrated with sensor and actuator based on refined zigzag theory. Recently, Aminipour *et al*. (2018) are investigated on the wave propagation in doubly-curved shell made of Functionally Graded Anisotropic materials via higher order shear deformation theory. Later, other researches are dedicated to study wave propagations in functionally graded materials structures as plates , nano-plates and beams in (Benadouda *et al*. 2017, Ayache *et al*. 2018, Fourn *et al*. 2018, Karami *et al*. 2018a-e, 2019a, b, Bouanati *et al*. 2019, Azizi *et al*. 2019, Bennai *et al*. 2019, Nebab *et al*. 2019). The first study on imperfect sigmoid beams is done by Avcar (2019).

In this work, wave propagation in porous sigmoid plates is investigated for the first time. By introduction undefined integral, the shear deformation theory has only four unknowns. The S-FGM plates are supposed to be porous with distribution of porosity through thickness of plates. The study indicates effect of some parameters such as index porosity, power-law index and side-to-thickness.

2. Problem formulation

2.1 Sigmoid functionally graded materials and plates

Consider a plate have two dimensions, length a and width b, with other one dimension, thickness h is negligible compared with others two dimensions. This plate is made by sigmoid functionally graded materials, which their materials proprieties change through thickness namely in z direction, as shown in Fig. 1. The S-FG plate is discussed to the (x, y, z) set of directions. We supposed in part of our study the functionally graded materials to be an imperfect (porous) due to effect even, uneven, logarithmic-uneven and exponential-uneven porosities in its material properties distributions. The volume fractions of the constituents are given according to the structure of the S-FG plates. In this article, the S-FG plates are investigated:

(i) S-FG perfect plates:

The volume fraction of the ceramic phase is defined according to the following power law (Bourada *et al*. 2018)

$$\begin{aligned} V_c^1 &= 1 - \frac{1}{2}\left(\frac{(h/2) - z}{h/2}\right)^p \\ V_c^2 &= \frac{1}{2}\left(\frac{(h/2) + z}{h/2}\right)^p \end{aligned} \tag{1}$$

Where, in the case of S-FG isotropic plates, h is thickness and p is the volume fraction index indicating the material variation through-the-plate-thickness direction.

2.2 Voigt's modified rule ofmixture for Porous S-FG Plates

The modified rule of the mixture taking into account microstructural defect, as proposed by Gupta and Talha (2018), the modules of young $E(z)$ and materials density ρ are given by the following law of mixtures:

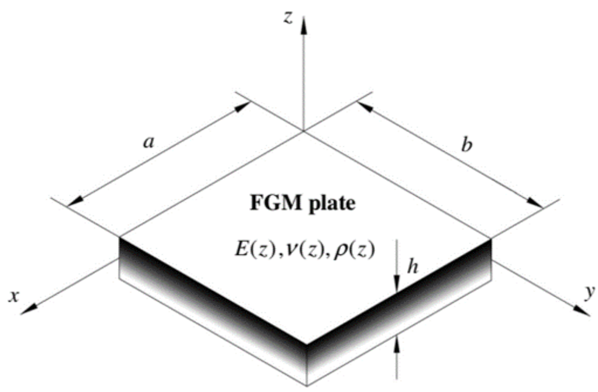


Fig. 1 Typical S-FG plates with Cartesian coordinates

(i) Type-I, porosities evenly distributed through cross section

$$E(z) = E_c V_c^1(z) + \left(1 - V_c^1(z)\right)E_m - \frac{\alpha}{2}(E_c + E_m) \quad \text{for} \quad 0 \le z \le \frac{h}{2}$$
$$E(z) = E_c V_c^2(z) + (1 - V_c^2)E_m V_c^2 - \frac{\alpha}{2}(E_c + E_m) \quad \text{for} \quad -h/2 \le z \le h/2 \tag{2a}$$

$$\rho(z) = \rho_c V_c^1(z) + \left(1 - V_c^1(z)\right)\rho_m - \frac{\alpha}{2}(\rho_c + \rho_m) \quad \text{for} \quad 0 \le z \le \frac{h}{2}$$
$$\rho(z) = \rho_c V_c^2(z) + (1 - V_c^2)\rho_m V_c^2 - \frac{\alpha}{2}(\rho_c + \rho_m) \quad \text{for} \quad -h/2 \le z \le h/2 \tag{2b}$$

(ii) Type-II, porosities unevenly distributed through cross-section and mainly concentrated in the central area of the plates

$$E(z) = E_c V_c^1(z) + \left(1 - V_c^1(z)\right)E_m - \Omega\left(1 - \frac{2|z|}{h}\right)(E_c + E_m) \quad \text{for} \quad 0 \le z \le \frac{h}{2}$$
$$E(z) = E_c V_c^2(z) + (1 - V_c^2)E_m V_c^2 - \Omega\left(1 - \frac{2|z|}{h}\right)(E_c + E_m) \quad \text{for} \quad -h/2 \le z \le h/2 \tag{3a}$$

$$\rho(z) = \rho_c V_c^1(z) + \left(1 - V_c^1(z)\right)\rho_m - \Omega\left(1 - \frac{2|z|}{h}\right)(\rho_c + \rho_m) \quad \text{for} \quad 0 \le z \le \frac{h}{2}$$
$$\rho(z) = \rho_c V_c^2(z) + (1 - V_c^2)\rho_m V_c^2 - \Omega\left(1 - \frac{2|z|}{h}\right)(\rho_c + \rho_m) \quad \text{for} \quad -h/2 \le z \le h/2 \tag{3b}$$

Table 1 Factor of the distribution of porosity Ω

Sources	Distribution kind	Ω
Wattanasakulpong and Ungbhakorn (2014)	Linear	$\frac{\alpha}{2}$
Gupta and Talha (2017)	Logarithmic	$log\left(1 + \frac{\alpha}{2}\right)$
Ayache *et al*. (2018)	Exponentially	$1 - e^{-\frac{\alpha}{2}}$

In which, subscripts c and m represent the ceramic and metal, respectively. In addition, p is power-law index that defines the material variation characterization through the thickness of the plate. The Poisson's coefficient is considered constant. (<< 1) is the porosity coefficient. The effect of porosities is proposed as two different models. In Eq. (2), the first model has even distribution of porosity in cross-section of plates, as given Ref (Fazzolari 2018). In Eq. (3), the second model of porosity has uneven distributions concentered in the middle of plates with three models, (linear, logarithmic and exponential), as given in Refs (Gupta and Talha 2017, 2018, Ayache *et al*. 2018) as shown in Table 1.

2.3 Kinematics and strains

The present displacement field is proposed to reduce the numbers of the unknown from five to only four unknown, based on higher shear deformation theories for plates, as following

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) - z\frac{\partial w_0}{\partial x} + P_1 A' f(z)\frac{\partial \theta}{\partial x} \\ v(x,y,z,t) &= v_0(x,y,t) - z\frac{\partial w_0}{\partial y} + P_1 B' f(z)\frac{\partial \theta}{\partial y} \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{4}$$

Where, u_0 and v_0 are the mid-plane displacement of the plate in the x and y directions, w_0 and θ are the bending and shear components of transverse displacement, respectively.

$f(z)$ is a shape function determining the distribution of the transverse shear are chosen to satisfy the stress-free boundary conditions on the top and bottom surfaces of the plate, thus a shear correction factor is not required. The different shapes function for $f(z)$ are presented in this current study, as expressed in Table 2.

Agreeing to the displacement field of the current four unknown shear deformation theories, non-zero strains can be expressed as follows

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + f(z)\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g(z)\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} \tag{5}$$

Table 2 Various shear deformation theories (HSDTs)

Models	$f(z)$
Third plate deformation theory (TSDT)	$1 - \frac{4z^3}{3h^2}$
Sinusoidal plate deformation theory (SSDT)	$\frac{z\left[\pi + 2\cos\left(\frac{\pi z}{h}\right)\right]}{2 + \pi}$
Exponential plate deformation theory(ESDT)	$ze^{-2\left(\frac{z}{h}\right)^2}$
Hyperbolic plate deformation theory (HSDT)	$h\sinh\frac{z}{h} + z\cosh\frac{1}{2}$

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} 1 = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial x} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial^2 w_0}{\partial y^2} \\ -2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix},$$

$$\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} P_1 A' \dfrac{\partial^2 \theta}{\partial^2 x} \\ P_2 B' \dfrac{\partial^2 \theta}{\partial^2 y} \\ (P_1 A' + P_2 B') \dfrac{\partial^2 \theta}{\partial x \partial y} \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} P_2 B' \dfrac{\partial \theta}{\partial y} \\ P_1 A' \dfrac{\partial \theta}{\partial x} \end{Bmatrix} \tag{6}$$

$$g(z) = \frac{df(z)}{dz} \tag{7}$$

Where the coefficients A' and B' are expressed according to the type of solution used, in this case via dispersions relations. Therefore, A', B' and P_1, P_2 are written as follows

$$A' = -\frac{1}{{k_1}^2}, \quad B' = -\frac{1}{{k_2}^2}; \quad P_1 = k_1^2; \quad P_2 = k_2^2 \tag{8}$$

The general relations between stress-strain for a linear isotropic sigmoid functionally graded plate are described as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix}^{(n)} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{66} & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & C_{55} \end{bmatrix}^{(n)} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}^{(n)} \tag{9}$$

where (σ_x, σ_y, τ_{xy}, τ_{yz}, τ_{xz}) and (ε_x, ε_y, γ_{xy}, γ_{yz}, γ_{xz}) are the stress and strain components, respectively. Using the material properties defined in Eqs. (2) and (3), stiffness Coefficients, C_{ij}, can be given as

$$C_{11} = C_{22} = \frac{E(z)}{1-\nu^2}, \quad C_{12} = \frac{\nu\, E(z)}{1-\nu^2}, \\ C_{44} = C_{55} = C_{66} = \frac{E(z)}{2(1+\nu)} \tag{10}$$

2.4 Equations of motion

Hamilton's principle is used to guide the equations that govern and can be expressed by the following relationship (Bounouara *et al.* 2016, Berghouti *et al.* 2019, Chaabane *et al.* 2019)

$$0 = \int_0^t (\delta U - \delta K) dt \tag{11}$$

Where δU is the variation of strain energy; and δK is the variation of kinetic energy, respectively.

The variation of strain energy of the plate is given by (Mahi *et al.* 2015)

$$\begin{aligned} \delta\, U = \int_V \big[& \sigma_x \delta\, \varepsilon_x + \sigma_y \delta\, \varepsilon_y + \tau_{xy} \delta\, \gamma_{xy} \\ & + \tau_{yz} \delta\, \gamma_{yz} + \tau_{xz} \delta\, \gamma_{xz} \big] dV \\ = \int_A \big[& N_x \delta\, \varepsilon_x^0 + N_y \delta\, \varepsilon_y^0 + N_{xy} \delta\, \gamma_{xy}^0 \\ & + M_x \delta\, k_x^b + M_y \delta\, k_y^b + M_{xy} \delta\, k_{xy}^b \\ & + S_x \delta\, k_x^s + S_y \delta\, k_y^s + S_{xy} \delta\, k_{xy}^s \\ & + T_{yz} \delta\, \gamma_{yz}^0 + T_{xz} \delta\, \gamma_{xz}^0 \big]\, dA = 0 \end{aligned} \tag{12}$$

Where A is the top surface and the stress resultants N, M, and S are defined by

$$(N_i, M_i, S_i) = \sum_{n=1}^{2} \int_{h_n}^{h_{n+1}} (1, z, f) \sigma_i dz, \quad (i = x, y, xy) \tag{13a}$$

$$(T_{xz}, T_{yz}) = \sum_{n=1}^{2} \int_{h_n}^{h_{n+1}} g(\tau_{xz}, \tau_{yz}) dz \tag{13b}$$

Where, h_{n+1} and h_n are the top and bottom z-coordinates of the nth layer.

The variation of kinetic energy of the plate can be expressed as

$$\delta\, K = \int_V [\dot{u} \delta\, \dot{u} + \dot{v} \delta\, \dot{v} + \dot{w} \delta\, \dot{w}]\, \rho(z)\, dV \tag{14a}$$

$$\begin{aligned} = \int_A \Bigg\{ & I_0 [\dot{u}_0 \delta \dot{u}_0 + \dot{v}_0 \delta \dot{v}_0 + \dot{w}_0 \delta \dot{w}_0] \\ & - I_1 \left(\dot{u}_0 \frac{\partial \delta \dot{w}_0}{\partial x} + \frac{\partial \dot{w}_0}{\partial x} \delta\, \dot{u}_0 + \dot{v}_0 \frac{\partial \delta \dot{w}_0}{\partial y} + \frac{\partial \dot{w}_0}{\partial y} \delta\, \dot{v}_0 \right) \\ & + J_1 \left((k_1 A') \left(\dot{u}_0 \frac{\partial \delta \dot{\theta}}{\partial x} + \frac{\partial \dot{\theta}}{\partial x} \delta\, \dot{u}_0 \right) + (k_2 B') \left(\dot{v}_0 \frac{\partial \delta \dot{\theta}}{\partial y} + \frac{\partial \dot{\theta}}{\partial y} \delta\, \dot{v}_0 \right) \right) \\ & + I_2 \left(\frac{\partial \dot{w}_0}{\partial x} \frac{\partial \delta\, \dot{w}_0}{\partial x} + \frac{\partial \dot{w}_0}{\partial y} \frac{\partial \delta\, \dot{w}_0}{\partial y} \right) \\ & + K_2 \left((k_1 A')^2 \left(\frac{\partial \dot{\theta}}{\partial x} \frac{\partial \delta\, \dot{\theta}}{\partial x} \right) + (k_2 B')^2 \left(\frac{\partial \dot{\theta}}{\partial y} \frac{\partial \delta\, \dot{\theta}}{\partial y} \right) \right) \\ & - J_2 \left(\begin{aligned} & (k_1 A') \left(\frac{\partial \dot{w}_0}{\partial x} \frac{\partial \delta\, \dot{\theta}}{\partial x} + \frac{\partial \dot{\theta}}{\partial x} \frac{\partial \delta\, \dot{w}_0}{\partial x} \right) \\ & + (k_2 B') \left(\frac{\partial \dot{w}_0}{\partial y} \frac{\partial \delta\, \dot{\theta}}{\partial y} + \frac{\partial \dot{\theta}}{\partial y} \frac{\partial \delta\, \dot{w}_0}{\partial y} \right) \end{aligned} \right) \Bigg\} dA \end{aligned} \tag{14b}$$

Where the point-exponent convention indicates the differentiation with respect to the time variable; $\rho(z)$ is the mass density given by Eq. (1); and (I_i, J_i, K_i) are mass inertias expressed by

$$(I_0, I_1, I_2) = \sum_{n=1}^{2} \int_{h_n}^{h_{n+1}} (1, z, z^2) \rho(z) dz \\ (J_1, J_2, L_2) = \sum_{n=1}^{2} \int_{h_n}^{h_{n+1}} (f, z\, f, f^2) \rho(z) dz \tag{15}$$

By substituting Eqs. (12) and (14) into Eq. (11), the following can be derived

$$
\begin{aligned}
&\delta\, u_0\colon\ \frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=I_0\ddot{u}_0-I_1\frac{\partial \ddot{w}_0}{\partial x}+P_1A^{\prime J_1\frac{\partial\ddot{\theta}}{\partial x}}\\
&\delta\, v_0\colon\ \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_{y}}{\partial y}=I_0\ddot{v}_0-I_1\frac{\partial \ddot{w}_0}{\partial y}+P_2B^{\prime J_1\frac{\partial\ddot{\theta}}{\partial y}}\\
&\delta\, w_0\colon\ \frac{\partial^2 M_x}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}+\frac{\partial^2 M_y}{\partial y^2}\\
&\quad=I_0\ddot{w}_0+I_1\left(\frac{\partial \ddot{u}_0}{\partial x}+\frac{\partial \ddot{v}_0}{\partial y}\right)-I_2\nabla^2\ddot{w}_0\\
&\qquad+J_2\left(P_1A'\frac{\partial^2\ddot{\theta}}{\partial x^2}+P_2B'\frac{\partial^2\ddot{\theta}}{\partial y^2}\right)\\
&\delta\, \theta\colon\ -P_1A^{\prime\frac{\partial^2 S_x}{\partial x^2}}-P_2B^{\prime\frac{\partial^2 S_y}{\partial y^2}}\\
&\qquad-(P_1A'+P_2B')\frac{\partial^2 S_{xy}}{\partial x\partial y}+P_1A^{\prime\frac{\partial T_{xz}}{\partial x}}+P_2B^{\prime\frac{\partial T_{yz}}{\partial y}}\\
&\quad=-J_1\left(P_1A'\frac{\partial \ddot{u}_0}{\partial x}+P_2B'\frac{\partial \ddot{v}_0}{\partial y}\right)\\
&\qquad-L_2\left((P_1A')^2\frac{\partial^2\ddot{\theta}}{\partial x^2}+(P_2B')^2\frac{\partial^2\ddot{\theta}}{\partial y^2}\right)\\
&\qquad+J_2\left(P_1A'\frac{\partial^2\ddot{w}_0}{\partial x^2}+P_2B'\frac{\partial^2\ddot{w}_0}{\partial y^2}\right)
\end{aligned}
\tag{16}
$$

Substituting Eq. (5) in Eq. (9) and subsequent results in Eq. (16), the constraint results can be represented in terms of displacement fields (u_0, v_0, w_0 and θ) as

$$
\begin{Bmatrix} N\\ M\\ S\end{Bmatrix}=\begin{bmatrix} A & B & B^s\\ B & D & D^s\\ B^s & D^s & H^s\end{bmatrix}\begin{Bmatrix}\varepsilon\\ k^b\\ k^s\end{Bmatrix},\qquad T=A^s\gamma,
\tag{17}
$$

In which

$$
N=\{N_x,N_y,N_{xy}\}^t,\qquad M=\{M_x,M_y,M_{xy}\}^t\qquad S=\{S_x,S_y,S_{xy}\}^t
\tag{18a}
$$

$$
\varepsilon=\{\varepsilon_x^0,\varepsilon_y^0,\gamma_{xy}^0\}^t,\qquad k^b=\{k_x^b,k_y^b,k_{xy}^b\}^t,\qquad k^s=\{k_x^s,k_y^s,k_{xy}^s\}^t
\tag{18b}
$$

$$
A=\begin{bmatrix}A_{11}&A_{12}&0\\A_{12}&A_{22}&0\\0&0&A_{66}\end{bmatrix},\quad B=\begin{bmatrix}B_{11}&B_{12}&0\\B_{12}&B_{22}&0\\0&0&B_{66}\end{bmatrix},\quad D=\begin{bmatrix}D_{11}&D_{12}&0\\D_{12}&D_{22}&0\\0&0&D_{66}\end{bmatrix}
\tag{18c}
$$

$$
B^s=\begin{bmatrix}B_{11}^s&B_{12}^s&0\\B_{12}^s&B_{22}^s&0\\0&0&B_{66}^s\end{bmatrix},\quad D^s=\begin{bmatrix}D_{11}^s&D_{12}^s&0\\D_{12}^s&D_{22}^s&0\\0&0&D_{66}^s\end{bmatrix},\quad H^s=\begin{bmatrix}H_{11}^s&H_{12}^s&0\\H_{12}^s&H_{22}^s&0\\0&0&H_{66}^s\end{bmatrix}
\tag{18d}
$$

$$
T=\{T_{xz},T_{yz}\}^t,\qquad \gamma=\{\gamma_{xz}^0,\gamma_{yz}^0\}^t,\qquad A^s=\begin{bmatrix}A_{44}^s&0\\0&A_{55}^s\end{bmatrix}
\tag{18e}
$$

In which, stiffness components are given as

$$
\begin{Bmatrix}A_{11} & B_{11} & D_{11} & B_{11}^s & D_{11}^s & H_{11}^s\\ A_{12} & B_{12} & D_{12} & B_{12}^s & D_{12}^s & H_{12}^s\\ A_{66} & B_{66} & D_{66} & B_{66}^s & D_{66}^s & H_{66}^s\end{Bmatrix}=\int_{-h/2}^{h/2}C_{11}\left(1,z,z^2,f(z),z\,f(z),f^2(z)\right)\begin{Bmatrix}1\\ \nu\\ \frac{1-\nu}{2}\end{Bmatrix}dz
\tag{19a}
$$

$$
\begin{aligned}
&(A_{22},B_{22},D_{22},B_{22}^s,D_{22}^s,H_{22}^s)\\
&=(A_{11},B_{11},D_{11},B_{11}^s,D_{11}^s,H_{11}^s)\\
&C_{11}^{(n)}=\frac{E(z)}{1-\nu^2},
\end{aligned}
\tag{19b}
$$

$$
A_{44}^s=A_{55}^s=\sum_{n=1}^{3}\int_{h_n}^{h_{n+1}}\frac{E(z)}{2(1+\nu)}\left[g(z)\right]^2dz\,,
\tag{19c}
$$

Substituting Eqs. (17)-(19) in Eq. (16), the equations of motion for simply supported S-FG plates via the current method can be rewritten in terms of displacements (u_0, v_0, w_0 and θ) as follows

$$
\begin{aligned}
&A_{11}\frac{\partial^2 u_0}{\partial^2 x}+A_{66}\frac{\partial^2 u_0}{\partial y^2}+(A_{12}+A_{66})\frac{\partial^2 v_0}{\partial x\,\partial y}-B_{11}\frac{\partial^3 w_0}{\partial x^3}\\
&-(B_{12}+2B_{66})\frac{\partial^3 w_0}{\partial x\,\partial y^2}+B_{11}^sP_1A'\frac{\partial^3\theta}{\partial x^3}\\
&+(B_{12}^sP_2B'+B_{66}^s(P_1A'+P_2B'))\frac{\partial^3\theta}{\partial x\,\partial y^2}=I_0\ddot{u}_0-I_1\frac{\partial\ddot{w}}{\partial x}+J_1P_1A'\frac{\partial\ddot{\theta}}{\partial x},\\[2ex]
&A_{22}\frac{\partial^2 v_0}{\partial y^2}+A_{66}\frac{\partial^2 v_0}{\partial x^2}+\left(A_{12}+A_{66}\right)\frac{\partial^2 u_0}{\partial x\,\partial y}-B_{22}\frac{\partial^3 w_0}{\partial y^3}-\left(B_{12}+2B_{66}\right)\frac{\partial^3 w_0}{\partial x^2\partial y}\\
&+\left(B_{66}^s\left(P_1A'+P_2B'\right)+B_{12}^sP_1A'\right)\frac{\partial^3\theta}{\partial x^2\partial y}\\
&+B_{22}^sP_2B'\frac{\partial^3\theta}{\partial y^3}=I_0\ddot{v}_0-I_1\frac{\partial\ddot{w}_0}{\partial y}+J_1B'P_2\frac{\partial\ddot{\theta}}{\partial y},\\[2ex]
&B_{11}\frac{\partial u_0}{\partial x^3}+(B_{12}+2B_{66})\frac{\partial^3 u_0}{\partial x\partial y^2}\\
&+(B_{12}+2B_{66})\frac{\partial^3 v_0}{\partial x^2\partial y}+B_{22}\frac{\partial^3 v_0}{\partial y^3}-D_{11}\frac{\partial^4 w_0}{\partial x^4}\\
&-2(D_{12}+2D_{66})\frac{\partial^4 w_0}{\partial x^2\partial y^2}-D_{22}\frac{\partial^4 w_0}{\partial y^4}+D_{11}^sP_1A'\frac{\partial^4\theta}{\partial x^4}\\
&+\left((D_{12}^s+2D_{66}^s)(P_1A'+P_2B')\right)\frac{\partial^4\theta}{\partial x^2\partial y^2}\\
&+D_{22}^sP_2B'\frac{\partial^4\theta}{\partial y^4}\\
&=I_0\ddot{w}_0+I_1\left(\frac{\partial\ddot{u}_0}{\partial x}+\frac{\partial\ddot{v}_0}{\partial y}\right)-I_2\left(\frac{\partial^2\ddot{w}_0}{\partial x^2}+\frac{\partial^2\ddot{w}_0}{\partial y^2}\right)\\
&+J_2\left(P_1A'\frac{\partial^2\ddot{\theta}}{\partial x^2}+P_2B'\frac{\partial^2\ddot{\theta}}{\partial y^2}\right),\\
&P_1A'B_{11}^s\frac{\partial^3 u_0}{\partial x^3}+\left(B_{12}^sP_2B'+B_{66}^s(P_1A'+P_2B')\right)\frac{\partial^3 u_0}{\partial x\partial y^3}
\end{aligned}
\tag{20}
$$

$$
\begin{aligned}
&+\left(B_{12}^s P_1 A' + B_{66}^s (P_1 A' + P_2 B')\right)\frac{\partial^3 v_0}{\partial x^2 \partial y} \\
&+B_{22}^s P_2 B'\frac{\partial^3 v_0}{\partial y^3} - D_{11}^s P_1 A'\frac{\partial^4 w_0}{\partial x^4} \\
&-\left((D_{12}^s + 2D_{66}^s)(P_1 A' + P_2 B')\right)\frac{\partial^4 w_0}{\partial x^2 \partial y^2} \\
&-D_{22}^s P_2 B'\frac{\partial^4 w_0}{\partial y^4} + H_{11}^s (P_1 A')^2\frac{\partial^4 \theta}{\partial x^4} + H_{22}^s (P_2 B')^2\frac{\partial^4 \theta}{\partial y^4} \\
&+(2H_{12}^s P_1 P_2 A'B' + (P_1 A' + P_2 B')^2 H_{66}^s)\frac{\partial^4 \theta}{\partial x^2 \partial y^2} \\
&-A_{44}^s (P_1 A')^2\frac{\partial^2 \theta}{\partial x^2} - A_{55}^s (P_2 B')^2\frac{\partial^2 \theta}{\partial y^2} \\
&= J_1\left(P_1 A'\frac{\partial \ddot{u}_0}{\partial x} + P_2 B'\frac{\partial \ddot{v}_0}{\partial y}\right) \\
&-J_2\left(P_1 A'\frac{\partial^2 \ddot{w}_0}{\partial x^2} + P_2 B'\frac{\partial^2 \ddot{w}_0}{\partial y^2}\right) \\
&+L_2\left((P_1 A')^2\frac{\partial^2 \ddot{\theta}}{\partial x^2} + (P_2 B')^2\frac{\partial^2 \ddot{\theta}}{\partial y^2}\right)
\end{aligned}
\tag{20}
$$

2.5 Dispersion relations

Solutions of the equation of motion for wave propagation in simply supported plates are expressed by the following dispersion relationships

$$
\begin{Bmatrix} u_0(x,y,t) \\ v_0(x,y,t) \\ w_0(x,y,t) \\ \theta_0(x,y,t) \end{Bmatrix} = \begin{Bmatrix} U\, exp[i(\kappa_1 x + \kappa_2 y - \omega\, t)] \\ V\, exp[i(\kappa_1 x + \kappa_2 y - \omega\, t)] \\ W\, exp[i(\kappa_1 x + \kappa_2 y - \omega\, t)] \\ X\, exp[i(\kappa_1 x + \kappa_2 y - \omega\, t)] \end{Bmatrix}
\tag{21}
$$

where U; V; W and X are the coefficients of the wave amplitude, κ_1 and κ_2 are the wave numbers of wave propagation along x-axis and y-axis directions respectively, ω is the frequency, $\sqrt{i} = -1$ the imaginary unit.

Substituting Eq. (21) into Eq. (20), the following problem is obtained

$$
\left(\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{12} & a_{22} & a_{23} & a_{24} \\ a_{13} & a_{23} & a_{33} & a_{34} \\ a_{14} & a_{24} & a_{34} & a_{44} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & 0 & m_{13} & m_{14} \\ 0 & m_{22} & m_{23} & m_{24} \\ m_{13} & m_{23} & m_{33} & m_{34} \\ m_{14} & m_{24} & m_{34} & m_{44} \end{bmatrix}\right) \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ \theta_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}
\tag{22}
$$

where

$$
\begin{aligned}
&a_{11} = -\left(A_{11}{k_1}^2 + A_{66}{k_2}^2\right), \\
&a_{12} = -k_1 k_2 (A_{12} + A_{66}), \\
&a_{13} = i\left(B_{11}{k_1}^3 + (B_{12} + 2B_{66})k_1{k_2}^2\right), \\
&a_{14} = -i\left((k_1 k_2^2 (B_{12}^s P_2 B' + (P_1 A' + P_2 B')B_{66}^s)\right. \\
&\qquad \left. + P_1 A'^{B_{11}^s {k_1}^3}\right), \\
&a_{21} = -(A_{12} + A_{66})k_1 k_2 \\
&a_{22} = -\left(A_{66}{k_1}^2 + A_{22}{k_2}^2\right), \\
&a_{23} = i\left(B_{22}{k_2}^3 + (B_{12} + 2B_{66}){k_1}^2 k_2\right) \\
&\boldsymbol{a_{24} = \left(-i{k_1}^2 k_2 P_1 A'^{B_{12}^s} - i k_1^2 k_2 B_{66}^s (P_1 A' + P_2 B')\right.} \\
&\qquad \boldsymbol{\left. - i B_{22}^s k_2^3 P_2 B'\right)} \\
&\boldsymbol{a_{31} = -i k_1^3 B_{11} - i(B_{12} + 2B_{66})k_1 k_2^2;} \\
&\boldsymbol{a_{32} = -i k_2^3 B_{22} - i(B_{12} + 2B_{66})k_1^2 k_2} \\
&\boldsymbol{a_{33} = -(D_{11}{k_1}^4 + 2(D_{12} + 2D_{66}){k_1}^2{k_2}^2} \\
&\qquad \boldsymbol{+ D_{22}{k_2}^4),} \\
&a_{34} = P_1 A'^{D_{11}^s {k_1}^4} \\
&\qquad + \left((D_{12}^s + 2D_{66}^s)(P_1 A' + P_2 B')\right){k_1}^2{k_2}^2 \\
&\qquad + P_2 B'^{D_{22}^s {k_2}^4} \\
&\boldsymbol{a_{41} = i k_1 k_2^2 (B_{12}^s P_2 B' + (P_1 A' + P_2 B')B_{66}^s)} \\
&\qquad \boldsymbol{+ i B_{11}^s P_1 A'^{k_1^3}} \\
&\boldsymbol{a_{42} = i P_2 B'^{B_{22}^s k_2^3} + i(B_{12}^s P_1 A'} \\
&\qquad \boldsymbol{+ (P_1 A' + P_2 B')B_{66}^s)k_1^2 k_2^2} \\
&\boldsymbol{a_{43} = a_{34}} \\
&a_{44} = -(P_1 A')^2 H_{11}^s {k_1}^4 - (2P_1 P_2 A'B'H_{12}^s \\
&\qquad + (P_1 A' + P_2 B')^2 H_{66}^s){k_1}^2{k_2}^2 - (P_2 B')^2 H_{22}^s {k_2}^4 \\
&\qquad - (P_2 B')^2 A_{55}^s {k_2}^2 - (P_1 A')^2 A_{44}^s {k_1}^2 \\
&m_{11} = -I_0, \\
&m_{13} = i k_1 I_1, \\
&m_{14} = -i J_1 P_1 A'^{k_1}, \\
&m_{22} = -I_0, \\
&m_{23} = i k_2 I_1, \\
&m_{24} = -i P_2 B'^{k_2 J_1}, \\
&m_{31} = -i k_1 I_1, \\
&m_{32} = -i k_2 I_1, \\
&m_{33} = -I_0 - I_2\left({k_1}^2 + {k_2}^2\right), \\
&m_{41} = i P_1 A'^{k_1 J_1}, \\
&m_{34} = J_2\left(P_1 A'^{k_1^2} + P_2 B'^{k_2^2}\right), \\
&m_{33} = -I_0 - I_2\left({k_1}^2 + {k_2}^2\right), \\
&m_{42} = i P_2 B' k_2 J_1, \\
&m_{43} = J_2 (P_1 A' k_1^2 + P_2 B' k_2^2), \\
&m_{44} = -L_2\left((P_1 A')^2{k_1}^2 + (P_2 B')^2{k_2}^2\right)
\end{aligned}
\tag{23}
$$

The dispersion relations of wave propagation in the sigmoid functionally graded plate are presented by

$$
|\, [K] - \omega^2 [M] \,| = 0 \tag{24}
$$

The roots of Eq. (24) can be presented as

$$
\begin{aligned}
&\omega_1 = W_1(\kappa), \quad \omega_2 = W_2(\kappa), \quad \omega_3 = W_3(\kappa) \\
&\text{and} \qquad\qquad\quad \omega_4 = W_4(\kappa)
\end{aligned}
\tag{25}
$$

Its roots correspond to the wave modes M_1, M_2, M_3 and M_4 respectively. The wave modes M_1 and M_4 correspond to the flexural wave, the wave mode M_2 and M_3 corresponds to the extensional wave. The phase velocity of wave propagation in the FG plate can be obtained by

$$
C_i = \frac{W_i(\kappa)}{\kappa}, \qquad (i = 1, 2, 3, 4) \tag{26}
$$

3. Numerical results

In the results section, the wave dispersion analysis in simply supported imperfect S-FG plates is performed using various higher order shear theories. The frequencies and the

Table 3 Material properties of the S-FG plates

Material	Properties		
	$E(GPa)$	ν	$\rho(Kg/m^3)$
Ceramic (Si_3N_4)	348.43	0.3	2370
Metal (SUS304)	201.04	0.3	8166

phases of the waves are obtained according to the thickness, the porosities, the index of power and the number of waves in Table 3. The material properties of the S-FG plates are taken to calculate the current results for ceramics and metals and come from the Refs (Boukhari *et al*. 2016, Fourn *et al*. 2018), as follows

3.1 Sigoimd functionally graded perfect plates

In Fig. 2, the dispersion wave curve for S-FG plates is plotted utilizing various higher shear deformation theories in function wave number with different values of power index (p = 0, 0.5, 1 and 2). It can be seen from Fig. 2, that all curves of dispersion predicted via three shear deformation theories proposed are identical, given the same results, to each other; in function the power index and wave modes (M_0, M_1, M_2, and M_2). The frequencies of wave modes in sigmoid functionally graded plates increase with the decrease of the power law index p for all modes. The maximum wave propagations frequencies are for the homogeneous plate with (p = 0).

In Fig. 3, the curve of phase velocity are given for sigmoid functionally graded plates versus wave number with different power index values using higher order shear plates theories. We observe that the phase velocity of wave propagation in sigmoid-FG increases with the decrease of power index values (p = 0, 0.5, 1, and 2). In addition, the phase velocity of the waves modes M2 for all values power index and M3 for (p = 0) of the plate is constant. As well as the frequency, for the homogeneous plate (p = 0), the phase velocity takes the maximum among those of all other compositions.

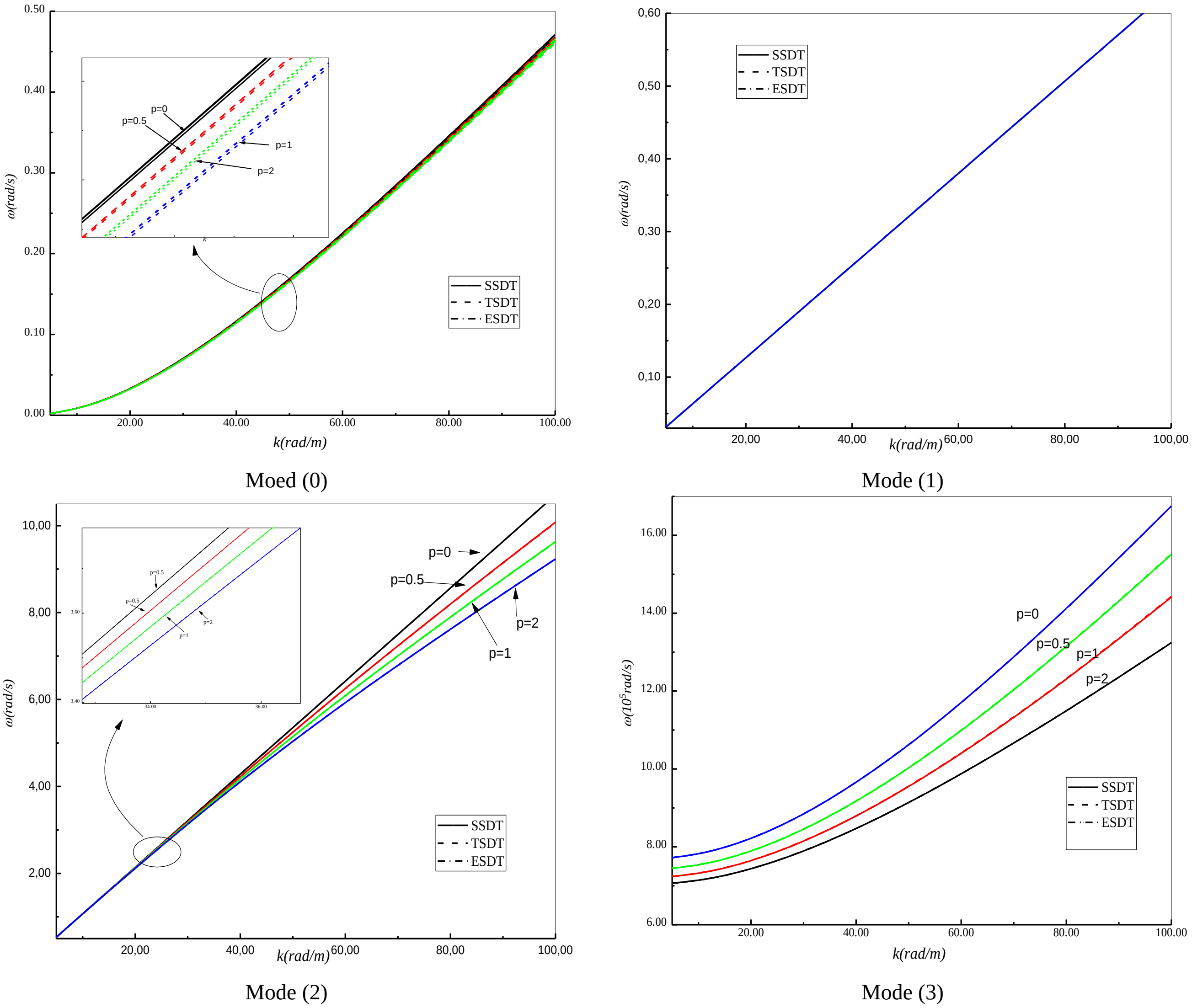
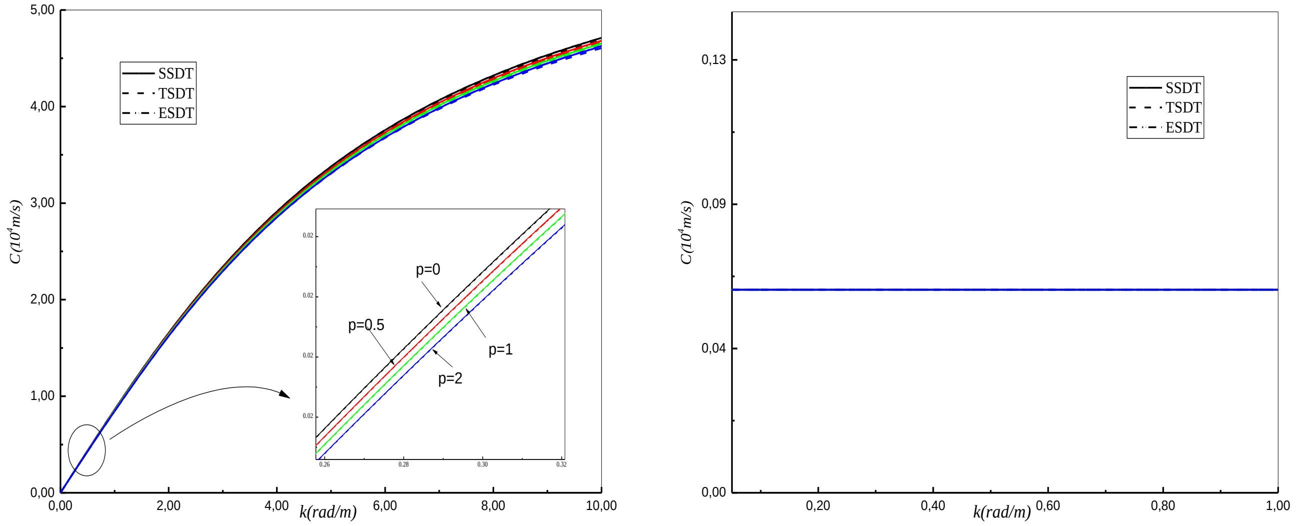
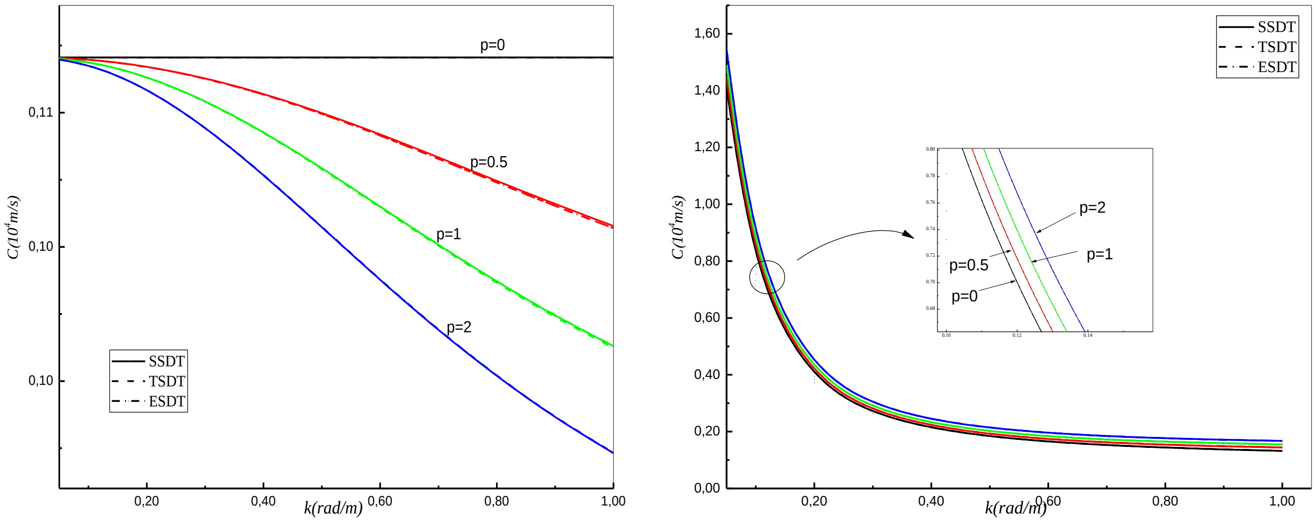


Fig. 2 The natural frequency curves in S-FG plate in terms of wave number (h = 0.02, p = 2)

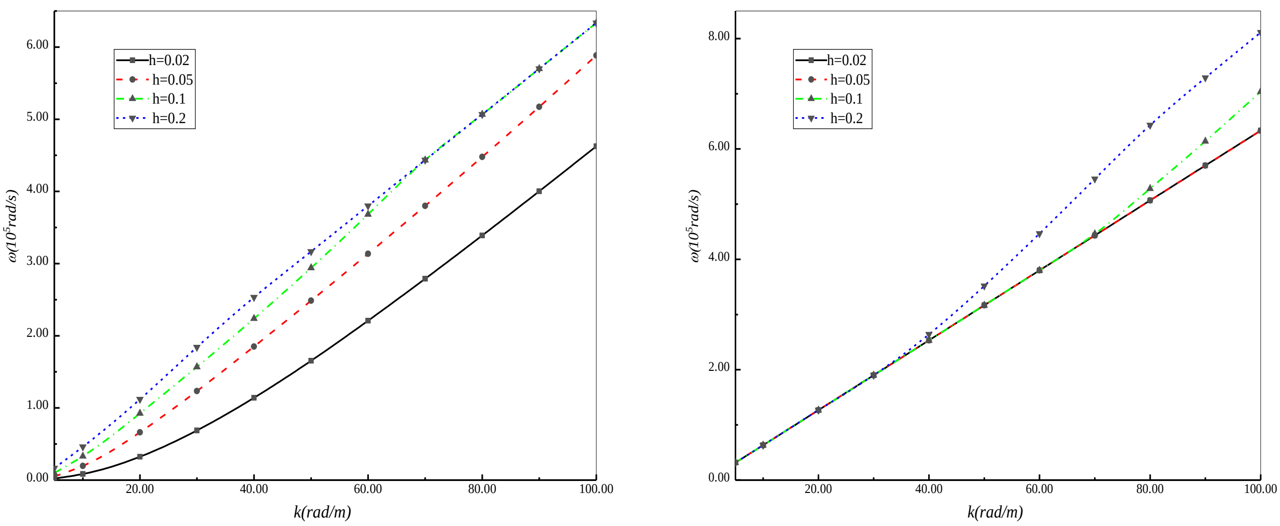


Moed (0) Mode (1)



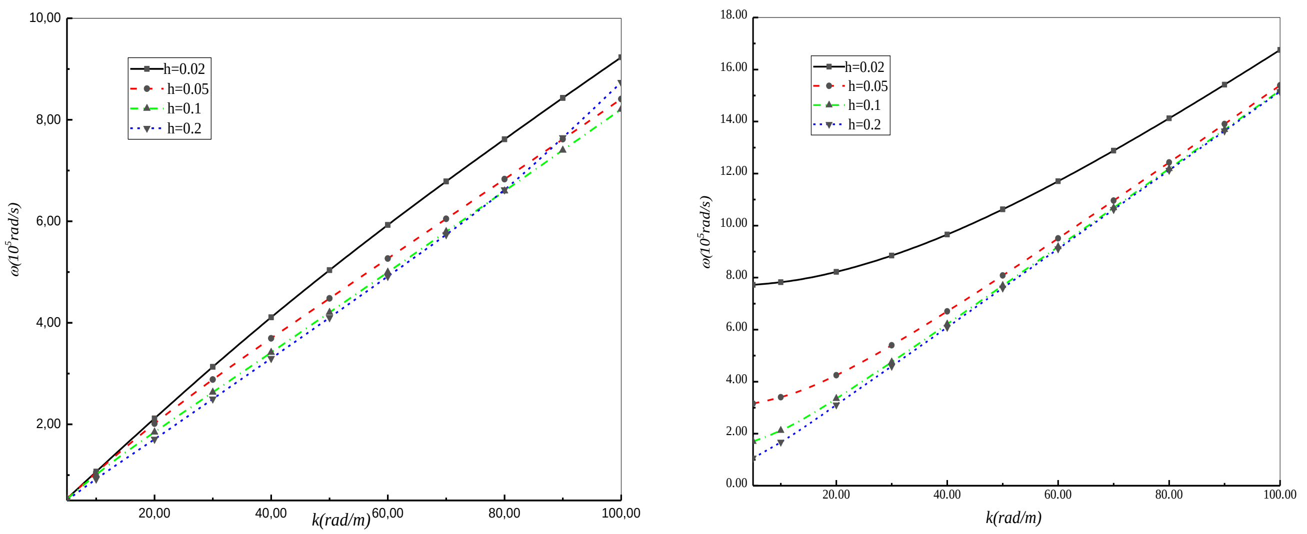
Mode (2) Mode (3)

Fig. 3 The phase velocities curves in S-FG plate in terms of wave number (h = 0.02, p = 2)



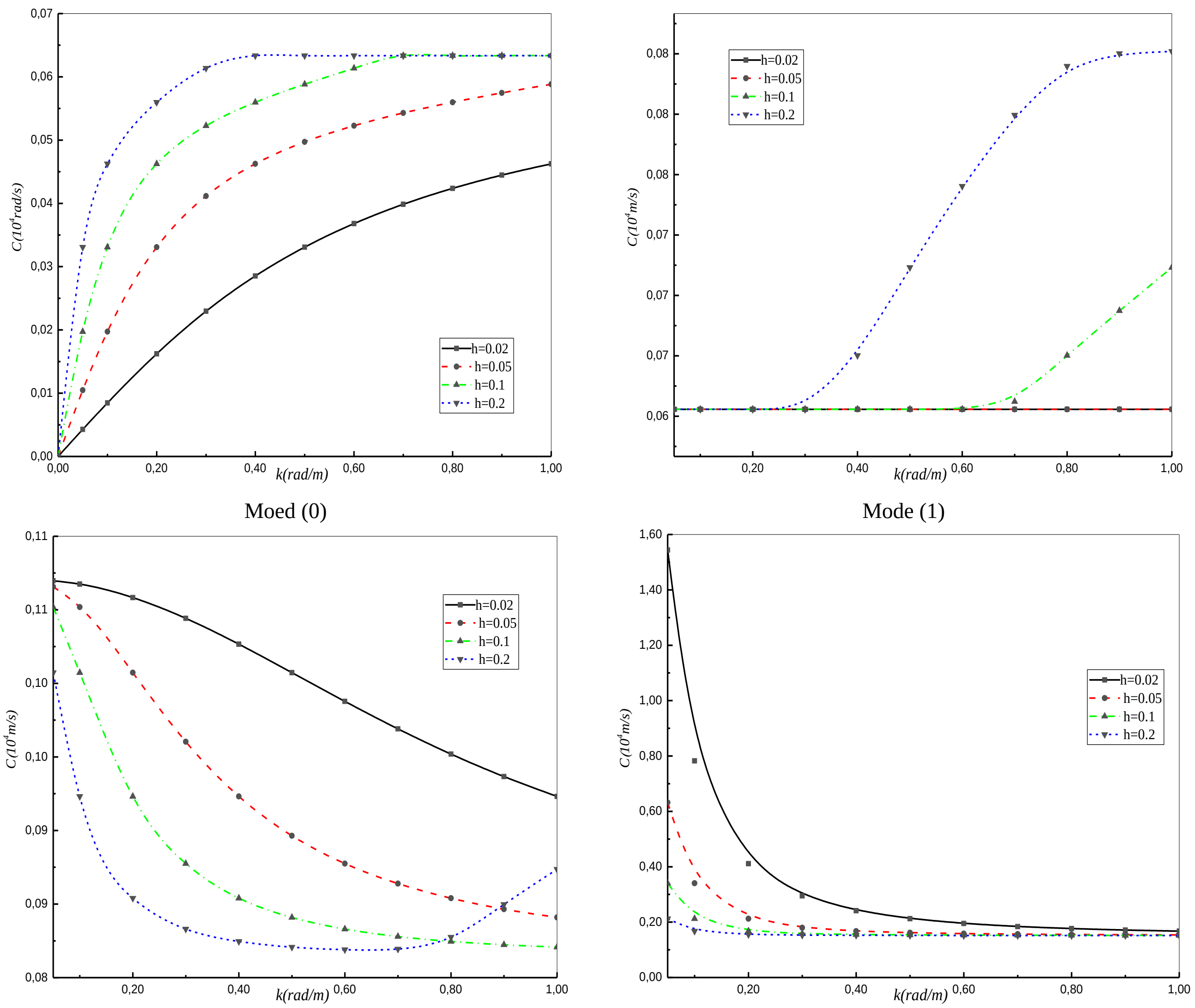
Moed (0) Mode (1)

Fig. 4 Effect of side-to-thickness ratio on dispersion curves in S-FG plate (k = 10 and p = 2)



Mode (2) Mode (3)

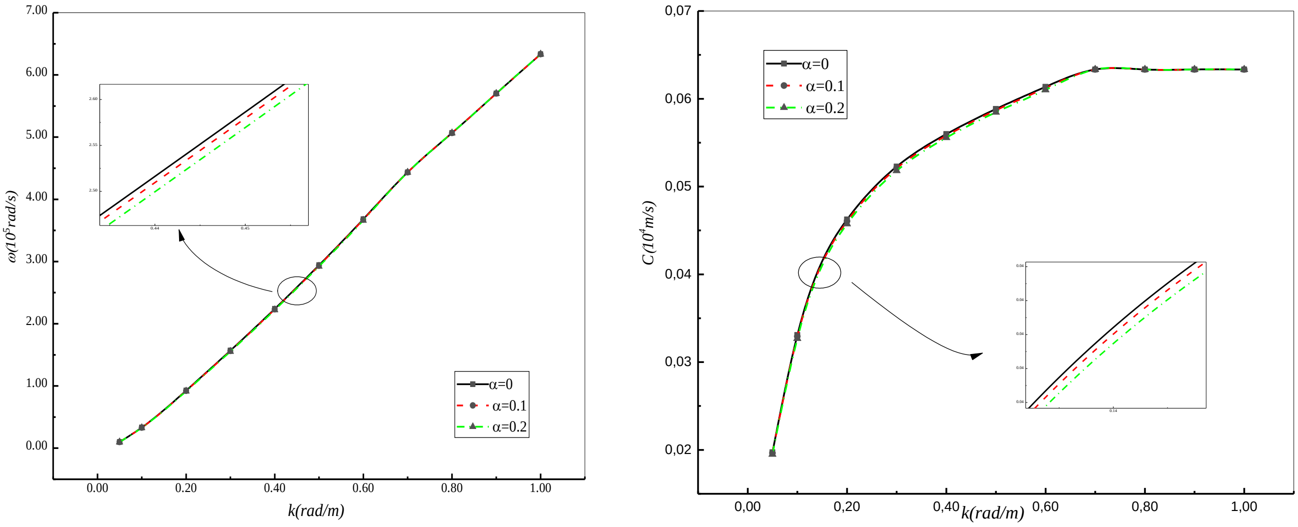
Fig. 4 Continued



Moed (0) Mode (1)

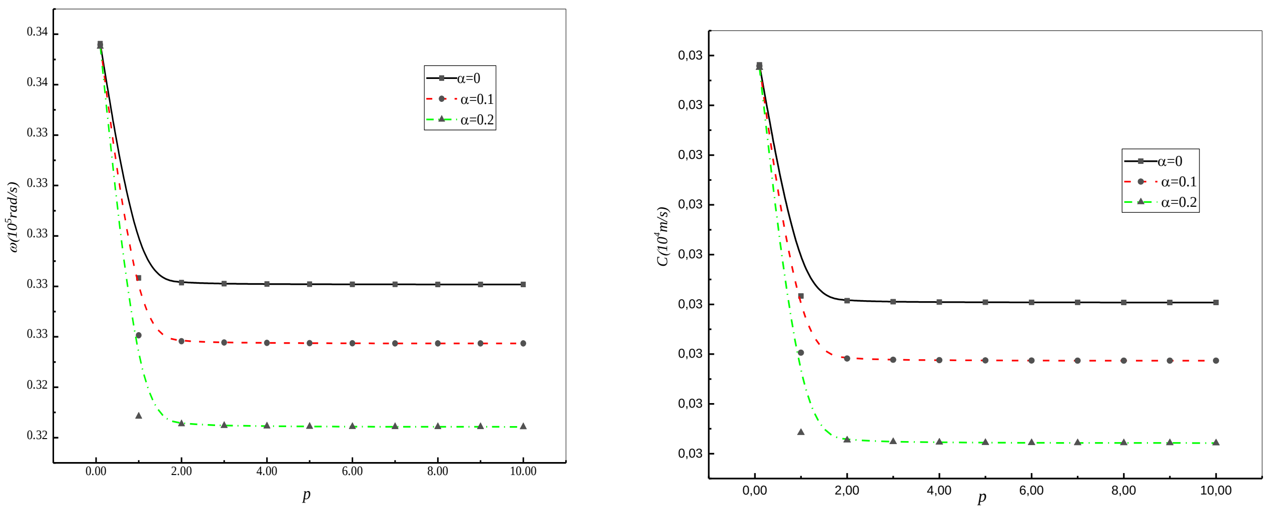
Mode (2) Mode (3)

Fig. 5 Effect of side-to-thickness ratio on velocities phases curves in S-FG plate (k = 10 and p = 2)



(a) Fundamental frequency (b) Phase velocity

Fig. 6 Fundamental frequency and phase velocity in S-FG plates with even porosity (p = 1, h = 0.1)



(a) Fundamental frequency (b) Phase velocity

Fig. 7 Fundamental frequency and phase velocity in S-FG plates with even porosity (k = 10, h = 0.1)

In Fig. 4, curves dispersion of the wave propagation in S-FG plates are presented, with power index p = 2, in terms side-to-thickness ratio. It is clearly the curves dispersion for all modes of the wave propagation are increased with a decrease of side-to-thickness ratio.

The velocity phase of wave propagation in S-FG plates are presented with p = 2 with a different side-to-thickness ratio in terms wave number, from Fig. 5. It can be seen that the phase of the velocity of the wave propagation in mode M_1 and M_2 decrease with increasing wave number.

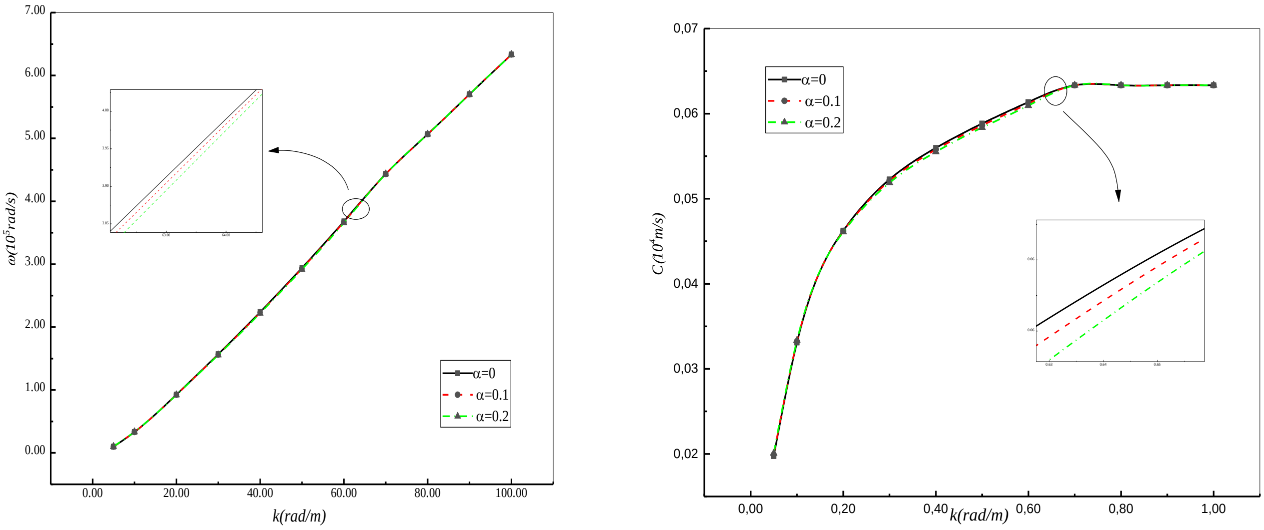
It can be concluded that the present method predicted the curve dispersion and phase velocity of wave propagation in S-FG with accurate and efficient, using various shear deformation theories.

3.2 Sigoimd functionally graded imperfect plates

3.2.1 Uneven porosity distribution

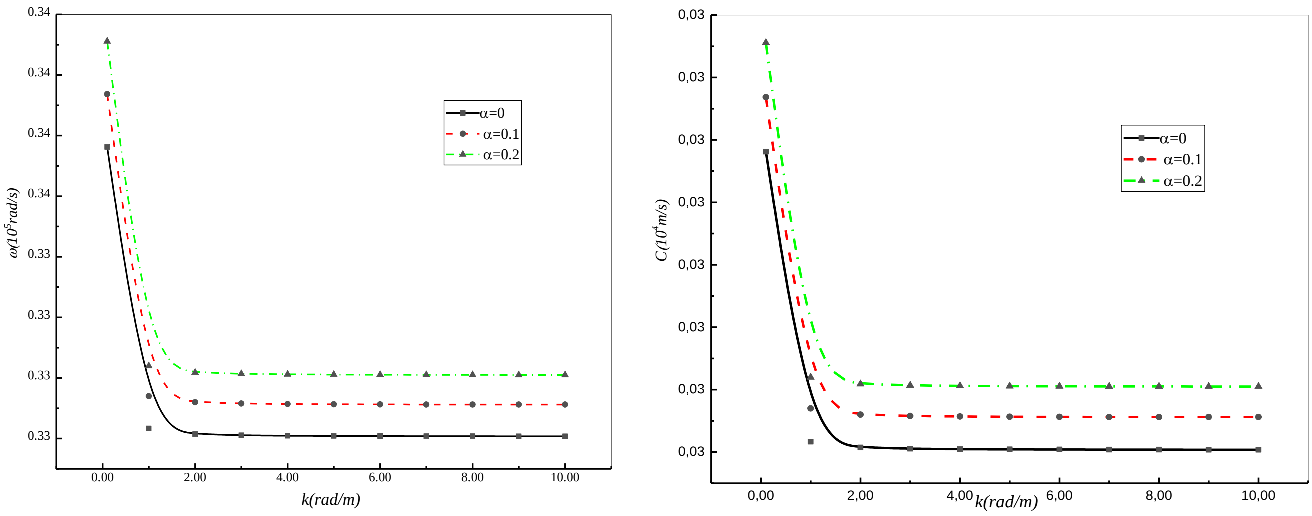
In Fig. 6 shows the effect of even distribution on undamental frequency and phase of the velocity of wave propagation in S-FG plates in function of wave number. It can be observed that fundamental frequency and phase of the velocity of S-FG plates are increased with increased wave number. The even porosity has decreased the frequency and phase velocity.

In Fig. 7, frequency and phase velocity of wave propagation in S-FG plates in terms of power index with fixed wave number (k = 10). It can be seen that the even distribution of porosity decreased the fundamental frequency and phase of the velocity of S-FG plates.



(a) Fundamental frequency (b) Phase velocity

Fig. 8 Effect uneven porosity of linear distribution on fundamental frequency and phase velocity in S-FG plates in terms wave number (p = 10, h = 0.1)



(a) Fundamental frequency (b) Phase velocity

Fig. 9 Effect uneven porosity of linear distribution on fundamental frequency and phase velocity in S-FG plates in terms power index (k = 10, h = 0.1)

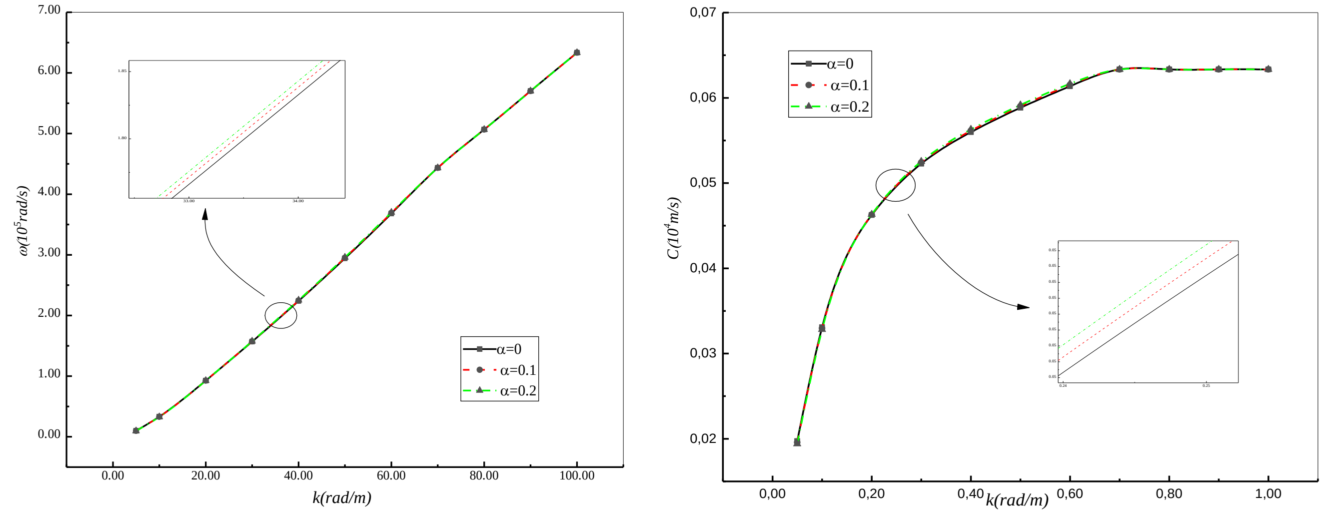
3.2.2 Even porosity distribution

Figs. 8, 10 and 12 reported the present results of the effect of the uneven distribution, with three shapes are linear, logarithmic and exponentially, on fundamental frequency and phase of the velocity of wave propagation in S-FG plates in function of wave number. It can be seen that fundamental frequency and phase of the velocity of S-FG plates are increased with increasing wave number. In Figs. 8 and 12, the uneven porosity has decreased the frequency and phase velocity of S-FG plate but it increases them in Fig. 10.

Frequency and phase velocity of wave propagation in S-FG plates in terms of power index with uneven distributions of porosities are presented in Figs. 9, 11 and 13. It can be seen that uneven linear distribution is increased the frequency and phase velocity but logarithmic and exponentially distribution is decreased them.

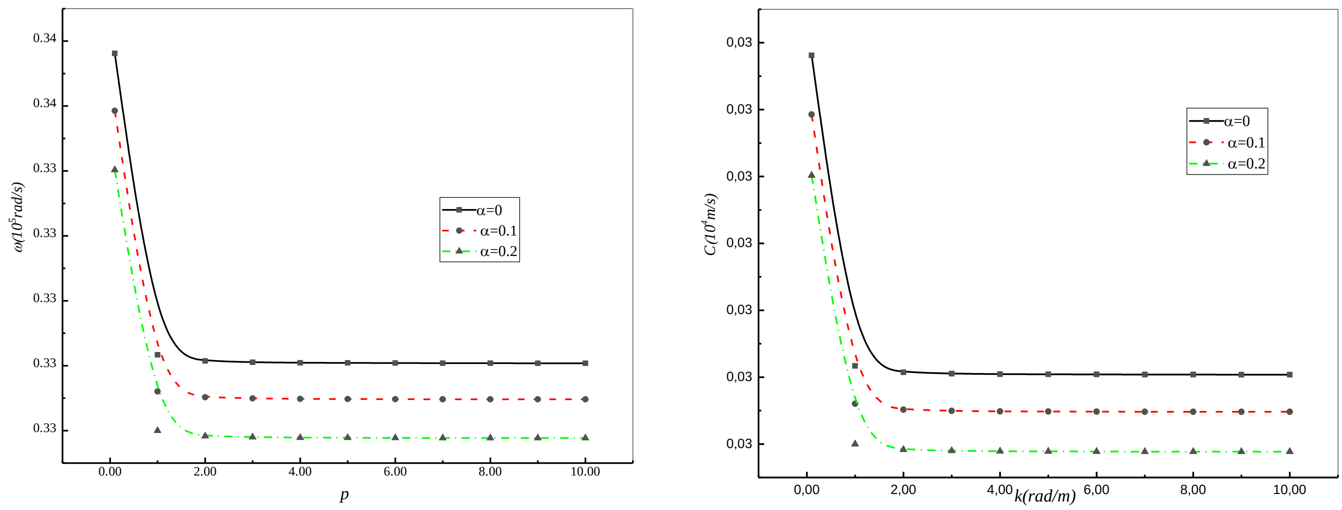
4. Conclusions

In this research, wave propagation in sigmoid functionally graded materials plates with effect porosity is achieved with success using new higher shear deformation theories (HSDT). The HSDT has only four variables which means the time of calculating and using three function shape of desaturation shear deformation (three order, sinusoidal and exponentially). The material properties of



(a) Fundamental frequency (b) Phase velocity

Fig. 10 Effect uneven porosity of logarithmic distribution on fundamental frequency and phase velocity in S-FG plates in terms wave number (p = 10, h = 0.1)

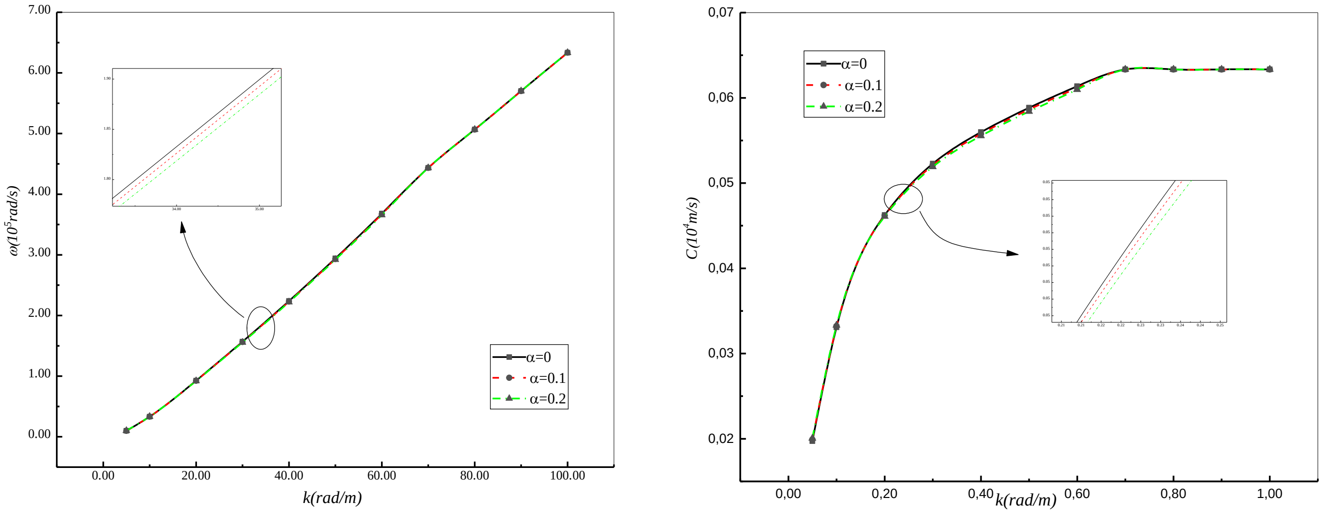


(a) Fundamental frequency (b) Phase velocity

Fig. 11 Effect uneven porosity of logarithmic distribution on fundamental frequency and phase velocity in S-FG plates in terms wave number (p = 10, h = 0.1)

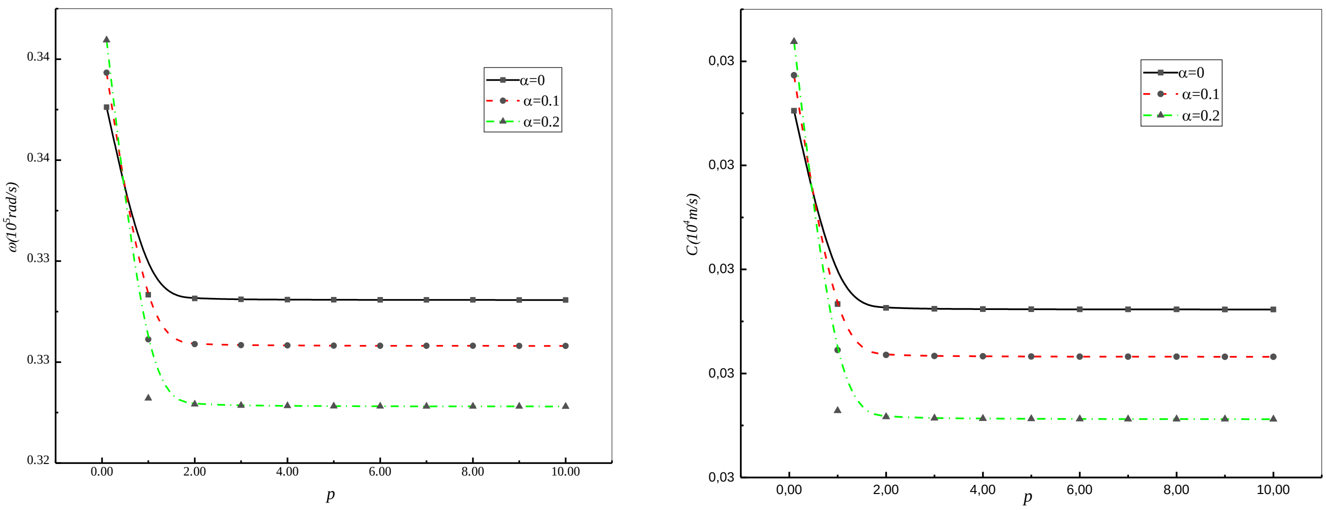
S FG plates are changed cross section smoothly by sigmoidal low. The desaturation of porosity was considered to have even distribution or uneven. The uneven porosity distributions have three kinds (linear, logarithmic and exponentially).

We employed the principle Hamilton to derive the equations motion for sigmoid functionally graded imperfect plates. A comprehensive study was presented in section results and validation. Finally, it can be seen the effectiveness of the current method to predict wave dispersion in imperfect and perfect S-FG plates. An improvement of the present formulation will be considered in the future work to consider other type of materials (Al-Basyouni *et al*. 2015, Mahapatra *et al*. 2016a, b, Bellifa *et al*. 2017b, Yeghnem *et al*. 2017 Karami *et al*. 2017, Daouadji 2017, Panjehpour *et al*. 2018, Bensaid *et al*. 2018, Hirwani and Panda 2018, Ayat *et al*. 2018, Behera and Kumari 2018, Bakhadda *et al*. 2018, Youcef *et al*. 2018, Shahadat *et al*. 2018, Cherif *et al*. 2018, Karami *et al*. 2018b, 2019c, Zine *et al*. 2018, Hirwani *et al*. 2018a, b, Bisen *et al*. 2018, Katariya *et al*. 2019, Bensattalah *et al*. 2019, Medani *et al*. 2019, Hussain *et al*. 2019, Benmansour *et al*. 2019, Draiche *et al*. 2019, Draoui *et al*. 2019, Hirwani and Panda 2019) and also to consider the stretching effect (Draiche *et al*. 2016, Chikh *et al*. 2017, Karami *et al*. 2018c, d,Abualnour *et al*. 2018, Bouhadra *et al*. 2018, Benchohra *et al*. 2018, Boulefrakh *et al*. 2019, Tlidji *et al*. 2019, Semmah *et al*. 2019, Boutaleb *et al*. 2019, Bendaho *et al*. 2019, Addou *et al*. 2019, Tounsi *et al*. 2019, Zaoui *et al*. 2019, Khiloun *et al*. 2019, Boukhlif *et al*. 2019).



(a) Fundamental frequency (b) Phase velocity

Fig. 12 Effect uneven porosity of exponentially distribution on fundamental frequency and phase velocity in S-FG plates in terms wave number (p = 10, h = 0.1)



(a) Fundamental frequency (b) Phase velocity

Fig. 13 Effect uneven porosity of exponentially distribution on fundamental frequency and phase velocity in S-FG plates in terms wave number (k = 10, h = 0.1)

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