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Free vibration analysis of angle-ply laminated composite and soft core sandwich plates

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Abstract. In this work, a simple four-variable trigonometric shear deformation model with undetermined integral terms to consider the influences of transverse shear deformation is applied for the dynamic analysis of anti-symmetric laminated composite and soft core sandwich plates. Unlike the existing higher order theories, the current one contains only four unknowns. The equations of motion are obtained using the principle of virtual work. The analytical solution is determined by solving the eigenvalue problem. The influences of geometric ratio, modular ratio and fibre angle are critically evaluated for different problems of laminated composite and sandwich plates. The eigenfrequencies obtained using the current theory are verified by comparing the results with those of other theories and with the exact elasticity solution, if any.

Keywords: shear deformation; antisymmetric; laminated; sandwich; natural frequencies

1. Introduction

Laminated composite materials are widely employed in aerospace, civil, marine and other fields. Because of their high "*specific modulus*", "*high specific strength*" and adaptability to a specific application, laminated composites provide definite advantages over classical materials such as metal. The individual ply consists of "*high modulus*" and "*high strength*" fibers in a metallic, ceramic, or polymeric matrix material. With the continued development of the "*high-tech industry*", the demand for advanced materials has led to the development of alternative products to traditional "*engineering materials*" such as aluminum, steel, wood, concrete, etc. (Panjehpour *et al*. 2018). Therefore, a novel methodology for studying the behaviour of such materials is always desirable. Among the recent sophisticated mathematical models for studying bending, dynamic, buckling, etc., several classical theories have been developed to study laminated composite plates.

The classical laminated plate theory (CLPT), which does not consider the effects of transverse shear, guarantees reasonable results for thin plates (Fadoun *et al*. 2017). However, it underestimates the deflections and overestimates the frequencies as well as the buckling loads for moderately thick plates. Many "*shear deformation theories*" that take into account transverse shear influences have been proposed to solve this problem.

As a result, an improvement of the FSDT and HSDT (high shear deformation theory) have been developed. The FSDT is based on Reissner (1945) and Mindlin (1951) and takes into account the transverse shear effects assuming a linear variation the displacements across the thickness. Since FSDTs violate the equilibrium conditions on the upper and lower faces of the plate, shear correction factors are needed to correct the unrealistic variation of the shear strain/stress across the thickness. Many studies have been carried out using FSDT for the free mechanical analysis of structures (Yan *et al*. 1966, Whitney 1969, Bert and Chen 1978, Reddy 1979, Noor and Burton 1989, Kant and Swaminathan 2001a, b, Naserian-Nik and Tahani 2010, Eltaher *et al*. 2014, Akbaş 2016, 2018, Avcar 2019, Draiche *et al*. 2019). Higher order shear deformation theories (HSDTs) are developed to avoid the problems encountered in CPT and FSDT and to provide better modelling of the static and dynamic behaviour of laminated composite plates. Among the different theories of higher order plates, Reddy's theory of third order shear deformation (TSDT) (Reddy 1984) is the most widely known and used by many researchers in their work. Carrera (1999) investigated the influence of transverse shear and normal deformations on dynamic of multilayered plates. Ashour (2003) examined the buckling and vibration of symmetric laminated composite plates with edges elastically restrained. Ghugal and Pawar (2011) employed hyperbolic shear deformation model of Soldatos (1992) for the dynamic investigation of orthotropic plates. Karama *et al*. (2009) proposed exponential shear deformation model for the bending, buckling and dynamic response of "*laminated composite*

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plates". Liu and Zhao (2007) examined the influence of soft honeycomb core on dynamic of sandwich panel via lower order and higher order shear deformation models. Matsunaga (2001) analyzed the stability and dynamic of angle-ply laminated composite plates. Rao and Desai (2004) and Rao *et al*. (2004) proposed analytical solutions for the dynamic study of "*laminated composite and sandwich plates*". Kant and Swaminathan (2001a, b) performed a free-vibration analysis of cross-ply laminated composite and sandwich plates using the finite element method and "*higher order shear and normal deformation theory*". Aagaah *et al*. (2006) proposed a theory of third-order shear deformation for dynamic analysis of "*laminated composite plates*" by considering different boundary conditions. Chalak *et al*. (2013) presented free vibration analysis of "*laminated soft core sandwich plates*". Afsharmanesh *et al*. (2014) studied the buckling and vibration of laminated composite circular plate on Winkler-type foundation. Draiche *et al*. (2016) developed a refined theory with stretching effect for the flexure analysis of laminated composite plates. Chikh *et al*. (2017) proposed a simple HSDT for thermal buckling analysis of cross-ply laminated plates. Baltacioglu and Civalek (2018) presented a numerical approaches for vibration response of annular and circular composite plates. Javed *et al*. (2018) investigated the free vibration of cross-ply laminated plates based on higher-order shear deformation theory. Other HSDTs can be consulted in literature review such as (Benferhat *et al*. 2016, Kar and Panda 2016, Kolahchi 2017, Selmi and Bisharat 2018, Belkacem *et al*. 2018, Sahouane *et al*. 2019, Karami and arami 2019).

In this work, a simple HSDT is applied for the dynamic analysis of antisymmetric "*laminated composite*" and soft core "*sandwich plates*". Unlike the existing HSDTs, the current theory has only four unknown variables. Consequently, the current theory is a simple computational model in the class of equivalent monolayer theories. The displacement field of the current theory is much richer than other HSDTs because of the use of trigonometric functions in terms of *z*-coordinate thickness for calculating out-of-plane shear deformations. The eigenfrequencies of various cross-ply and angle-ply laminated composite and sandwich plates are computed and compared to the existing literature taking into account the influences of the geometric ratio (a/h), the "*modulus ratio*" (E_1/E_2) and the "*angle of the fiber*" (θ).

2. Mathematical formulation of present theory

A rectangular plate with length, width and uniform thickness equal to *a*, *b* and *h* respectively is shown in Fig. 1. The plate is composed of "*N*" number of orthotropic layers perfectly bonded together. Each layer of plate is made up of linearly elastic orthotropic materials. Rectangular Cartesian coordinates (x, y, z) are used to describe infinitesimal deformations of a plate occupying the region $[0, a]$ $\times[0, b]$ $\times[-h/2, h/2]$ in the unstressed reference configuration. The *z*-direction is taken positive in downward direction.

In this work, further simplifying supposition are made to the conventional HSDT so that the number of unknowns is reduced. The displacement field of the conventional HSDT is given by

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) - z\frac{\partial w_0}{\partial x} + f(z)\phi_x(x,y,t) \\ v(x,y,z,t) &= v_0(x,y,t) - z\frac{\partial w_0}{\partial y} + f(z)\phi_y(x,y,t) \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{1}$$

Where u_0, v_0, w_0, ϕ_x and ϕ_y are five unknown displacements of the mid-plane of the plate, $f(z)$ denotes shape function representing the variation of the transverse shear strains and stresses within the thickness. By considering that

$$\phi_x = \int \theta\,(x,y,t)dx \quad \text{and} \quad \phi_y = \int \theta\,(x,y,t)dy,$$

The displacement field of the present model can be expressed in a simpler form as follows

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) - z\frac{\partial w_0}{\partial x} + k_1 f(z)\int \theta\,(x,y,t)dx \\ v(x,y,z,t) &= v_0(x,y,t) - z\frac{\partial w_0}{\partial y} + k_2 f(z)\int \theta\,(x,y,t)dy \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{2}$$

The integrals defined in the above equations must be solved by the Navier method and the displacement field can be rewritten as follows

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) - z\frac{\partial w_0}{\partial x} + k_1 A' f(z)\frac{\partial \theta}{\partial x} \\ v(x,y,z,t) &= v_0(x,y,t) - z\frac{\partial w_0}{\partial y} + k_2 B' f(z)\frac{\partial \theta}{\partial y} \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \tag{3}$$

where the coefficients A', B', k_1 and k_2 are expressed as follows

$$\begin{aligned} &A' = -\frac{1}{\alpha^2}, \quad B' = -\frac{1}{\beta^2}, \quad k_1 = \alpha^2, \\ &k_2 = \beta^2, \quad \alpha = \frac{m\pi}{a}, \quad \beta = \frac{n\pi}{b} \end{aligned} \tag{4}$$

Clearly, the displacement field in Eq. (3) considers only four unknowns u_0, v_0, w_0 and θ. Where the shape function $f(z)$ is given as

$$f(z) = \frac{h}{\pi} sin\left(\frac{\pi z}{h}\right) \tag{5}$$

The shear function is presented in this theory to satisfy zero stresses on the top and bottom surfaces of the plate. The shear function is obtained as follows

$$g(z) = \frac{df(z)}{dz} \quad \text{where} \quad g(z)(z = \pm h/2) = 0 \tag{6}$$

The nonzero strains associated with the displacement field in Eq. (3) are

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + f(z) \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix}, \qquad \text{(7a)}$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g(z) \begin{Bmatrix} \gamma_{yz}^s \\ \gamma_{xz}^s \end{Bmatrix},$$

Where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial x} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix},$$

$$\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial^2 w_0}{\partial y^2} \\ -2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}, \qquad \text{(7b)}$$

$$\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} k_1 \theta \\ k_2 \theta \\ k_1 \dfrac{\partial}{\partial y} \int \theta dx + k_2 \dfrac{\partial}{\partial x} \int \theta dy \end{Bmatrix},$$

$$\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} k_2 \int \theta dy \\ k_1 \int \theta dx \end{Bmatrix}$$

For orthotropic laminated plate, the constitutive relations for each layer can be expressed as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}^k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}^k \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}^k, \qquad \text{(8)}$$

$$\begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix}^k = \begin{bmatrix} \bar{Q}_{55} & \bar{Q}_{45} \\ \bar{Q}_{45} & \bar{Q}_{44} \end{bmatrix}^k \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix}^k$$

where (σ_x, σ_y, τ_{xy}, τ_{yz}, τ_{xz}) and (ε_x, ε_y, γ_{xy}, γ_{yz}, γ_{xz}) are the stress and strain components, respectively. Where $\bar{Q}_{ij}$ are the transformed material constants and are given as

$$\begin{aligned}
\overline{Q}_{11}^k &= Q_{11}c^4 + 2(Q_{12} + 2Q_{66})s^2c^2 + Q_{22}s^4 \\
\overline{Q}_{12}^k &= (Q_{11} + Q_{22} - 4Q_{66})s^2c^2 + Q_{12}(s^4 + c^4) \\
\overline{Q}_{16}^k &= (Q_{11} - Q_{12} - 2Q_{66})sc^3 + (Q_{12} - Q_{22} + 2Q_{66})s^3c \\
\overline{Q}_{22}^k &= Q_{11}s^4 + 2(Q_{12} + 2Q_{66})s^2c^2 + Q_{22}c^4 \\
\overline{Q}_{26}^k &= (Q_{11} - Q_{12} - 2Q_{66})s^3c + (Q_{12} - Q_{22} + 2Q_{66})sc^3 \qquad \text{(9)} \\
\overline{Q}_{66}^k &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})s^2c^2 + Q_{66}(s^4 + c^4) \\
\overline{Q}_{44}^k &= Q_{44}c^2 + Q_{55}s^2, \\
\overline{Q}_{45}^k &= (Q_{55} - Q_{44})sc, \\
\overline{Q}_{55}^k &= Q_{44}s^2 + Q_{55}c^2
\end{aligned}$$

where $c = cos\,\theta^k$, $s = sin\,\theta^k$ and Q_{ij} are the stiffness coefficients as given below

$$\begin{aligned}
Q_{11} &= \frac{E_1}{1 - \nu_{12}\nu_{21}}, & Q_{22} &= \frac{E_2}{1 - \nu_{12}\nu_{21}}, \\
Q_{12} &= \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}}, & Q_{66} &= G_{12}, \qquad \text{(10)} \\
Q_{44} &= G_{23}, & Q_{55} &= G_{13}
\end{aligned}$$

where E_i, G_{ij} and ν_{ij} are the Young's moduli, shear moduli and Poisson's ratio, respectively

3. Equations of motion

In order to derive the equations of motion, the principle of virtual work is applied

$$\int_0^t (\delta\, U + \delta\, V - \delta\, K) dt = 0 \qquad \text{(11)}$$

where $\delta\, U$ is the virtual strain energy; $\delta\, V$ is the virtual work done by external loads; and $\delta\, K$ is the virtual kinetic energy. The virtual strain energy of the plate is computed by

$$\begin{aligned}
\delta U = \sum_{k=1}^{N} \int_{h_k}^{h_{k+1}} \int_A \big(&\sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \tau_{xy} \delta\gamma_{xy} \\
&+ \tau_{yz} \delta\gamma_{yz} + \tau_{xz} \delta\gamma_{xz} \big)\, dAdz
\end{aligned} \qquad \text{(12)}$$

by substituting Eqs. (7) and (8) in Eq. (12) and by integration through the thickness of the plate, the virtual strain energy can be put in the form

$$\begin{aligned}
\delta U = \int_A \Big\{ & N_x \frac{\partial \delta u_0}{\partial x} + N_y \frac{\partial \delta v_0}{\partial y} + N_{xy} \left(\frac{\partial \delta u_0}{\partial y} + \frac{\partial \delta v_0}{\partial x} \right) \\
& - M_x^b \frac{\partial^2 \delta w_0}{\partial x^2} - M_y^b \frac{\partial^2 \delta w_0}{\partial y^2} - 2M_{xy}^b \frac{\partial^2 \delta w_0}{\partial x \partial y} \\
& + k_1 A' M_x^s \frac{\partial^2 \delta\theta}{\partial x^2} + k_2 B' M_y^s \frac{\partial^2 \delta\theta}{\partial y^2} \\
& + (k_1 A' + k_2 B') M_{xy}^s \frac{\partial^2 \delta\theta}{\partial x \partial y} + k_1 A' S_{xz}^s \frac{\partial \delta\theta}{\partial x} \\
& + k_2 B' S_{yz}^s \frac{\partial \delta\theta}{\partial y} \Big\} dA
\end{aligned} \qquad \text{(13)}$$

where A is the top surface and the stress resultants N, M, and S are defined by

$$\begin{Bmatrix} N_x, & N_y, & N_{xy} \\ M_x^b, & M_y^b, & M_{xy}^b \\ M_x^s, & M_y^s, & M_{xy}^s \end{Bmatrix} = \sum_{k=1}^{N} \int_{h_k}^{h_{k+1}} (\sigma_x, \sigma_y, \tau_{xy}) \begin{Bmatrix} 1 \\ z \\ f(z) \end{Bmatrix} dz, \qquad \text{(14)}$$

$$(S_{xz}^s, S_{yz}^s) = \sum_{k=1}^{N} \int_{h_k}^{h_{k+1}} (\tau_{xz}, \tau_{yz})\, g(z)\, dz$$

using Eq. (8) in Eq. (14), the resultants of the forces of the plate can be expressed in terms of deformation

$$\begin{Bmatrix} N \\ M^b \\ M^s \end{Bmatrix} = \begin{bmatrix} A & B & B^s \\ B & D & D^s \\ B^s & D^s & H^s \end{bmatrix} \begin{Bmatrix} \varepsilon \\ k^b \\ k^s \end{Bmatrix}, \quad \mathrm{S} = A^s \gamma \qquad \text{(15a)}$$

Where

$$
\begin{aligned}
&N=\{N_x,\quad N_y,\quad N_{xy}\}^{Tr},\\
&M^b=\{M_x^b,\quad M_y^b,\quad M_{xy}^b\}^{Tr},\\
&M^s=\{M_x^s,\quad M_y^s,\quad M_{xy}^s\}^{Tr},\\
&\varepsilon=\{\varepsilon_x^0,\quad \varepsilon_y^0,\quad \varepsilon_{xy}^0\}^{Tr},\\
&k^b=\{k_x^b,\quad k_y^b,\quad k_{xy}^b\}^{Tr},\\
&k^s=\{k_x^s,\quad k_y^s,\quad k_{xy}^s\}^{Tr},\\
&S=\{S_{yz}^s,\quad S_{xz}^s\}^{Tr},\\
&S=\{\gamma_{yz},\quad \gamma_{xz}\}^{Tr}
\end{aligned}
\tag{15b}
$$

and the stiffness components are given as follows

$$
\begin{aligned}
&\left(A_{ij},B_{ij},D_{ij},B_{ij}^s,D_{ij}^s,H_{ij}^s\right)\\
&=\sum_{k=1}^{N}\int_{h_k}^{h_{k+1}}\bar{Q}_{ij}^{(k)}\left(1,z,z^2,f(z),z\,f(z),f^2(z)\right)d\,z,\\
&\qquad\qquad i,j=1,2,6
\end{aligned}
\tag{16a}
$$

$$
A_{ij}^s=\sum_{k=1}^{N}\int_{h_k}^{h_{k+1}}\overline{Q}_{ij}^{(k)}\,g^2(z)dz,\qquad i,j=4,5 \tag{16b}
$$

The virtual work due to transverse loads can be written as

$$
\delta\,V=-\int_A q\delta\,w\;dA \tag{17}
$$

And the virtual kinetic energy of the plate can be expressed as

$$
\begin{aligned}
\delta K&=\sum_{k=1}^{N}\int_{h_k}^{h_{k+1}}\int_A\rho(z)(\dot{u}\delta\,\dot{u}+\dot{v}\delta\,\dot{v}+\dot{w}\delta\,\dot{w})dAdz\\
&=\sum_{k=1}^{N}\int_A\{I_0(\dot{u}_0\delta\dot{u}_0+\dot{v}_0\delta\dot{v}_0+\dot{w}_0\delta\dot{w}_0)\\
&-I_1\left(\dot{u}_0\frac{\partial\delta\dot{w}_0}{\partial x}+\dot{v}_0\frac{\partial\delta\dot{w}_0}{\partial y}+\frac{\partial\dot{w}_0}{\partial x}\delta\dot{u}_0+\frac{\partial\dot{w}_0}{\partial y}\delta\dot{v}_0\right)\\
&+I_2\left(\frac{\partial\dot{w}_0}{\partial x}\frac{\partial\delta\dot{w}_0}{\partial x}+\frac{\partial\dot{w}_0}{\partial y}\frac{\partial\delta\dot{w}_0}{\partial y}\right)\\
&+J_1\left[+k_2B'\left(\dot{v}_0\frac{\partial\delta\dot{\theta}}{\partial y}+\frac{\partial\dot{\theta}}{\partial y}\delta\dot{v}_0\right)\right]\\
&-J_2\left[k_1A'\left(\frac{\partial\dot{w}_0}{\partial x}\frac{\partial\delta\dot{\theta}}{\partial x}+\frac{\partial\dot{\theta}}{\partial x}\frac{\partial\delta\dot{w}_0}{\partial x}\right)\right.\\
&\left.+k_2B'\left(\frac{\partial\dot{w}_0}{\partial y}\frac{\partial\delta\dot{\theta}}{\partial y}+\frac{\partial\dot{\theta}}{\partial y}\frac{\partial\delta\dot{w}_0}{\partial y}\right)\right]\\
&+K_2\left((k_1A')^2\frac{\partial\dot{\theta}}{\partial x}\frac{\partial\delta\dot{\theta}}{\partial x}+(k_2B')^2\frac{\partial\dot{\theta}}{\partial y}\frac{\partial\delta\dot{\theta}}{\partial y}\right)\Bigg\}dA
\end{aligned}
\tag{18}
$$

Where dot-superscript convention indicates the differentiation with respect to the time variable(t); $\rho(z)$ is the mass density and (I_i,J_i,K_i) are mass inertias expressed by

$$
\begin{aligned}
&(I_0,I_1,I_2,J_1,J_2,K_2)\\
&=\sum_{k=1}^{N}\int_{z_k}^{z_{k+1}}\rho^{(k)}(1,z,z^2,f(z),z\,f(z),f^2(z))d\,z
\end{aligned}
\tag{19}
$$

substituting Eqs. (13), (17), and (18) into Eq. (11), integrating by parts with respect to *x*, *y*, and *t*, and setting the coefficients of δu_0, δv_0, δw_0 and $\delta\theta$ to zero, individually. The following equations of motion are obtained

$$
\begin{aligned}
&\delta\,u_0:\quad \frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=I_0\ddot{u}_0-I_1\frac{\partial\ddot{w}_0}{\partial x}+k_1A^{\prime J_1\frac{\partial\ddot{\theta}}{\partial x}}\\
&\delta\,v_0:\quad \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=I_0\ddot{v}_0-I_1\frac{\partial\ddot{w}_0}{\partial y}+k_2B^{\prime J_1\frac{\partial\ddot{\theta}}{\partial y}}\\
&\delta\,w_0:\quad \frac{\partial^2M_x^b}{\partial x^2}+2\frac{\partial^2M_{xy}^b}{\partial x\partial y}+\frac{\partial^2M_y^b}{\partial y^2}+q\\
&\qquad=I_0\ddot{w}_0+I_1\left(\frac{\partial\ddot{u}_0}{\partial x}+\frac{\partial\ddot{v}_0}{\partial y}\right)-I_2\left(\frac{\partial^2\ddot{w}_0}{\partial x^2}+\frac{\partial^2\ddot{w}_0}{\partial y^2}\right)\\
&\qquad+J_2\left(k_1A'\frac{\partial^2\ddot{\theta}}{\partial x^2}+k_2B'\frac{\partial^2\ddot{\theta}}{\partial y^2}\right)\\
&\delta\,\theta:\quad -k_1M_x^s-k_2M_y^s-(k_1A'+k_2B')\frac{\partial^2M_{xy}^s}{\partial x\partial y}\\
&\qquad+k_1A^{\prime\frac{\partial S_{xz}^s}{\partial x}}+k_2B^{\prime\frac{\partial S_{yz}^s}{\partial y}}\\
&\qquad=-J_1\left(k_1A'\frac{\partial\ddot{u}_0}{\partial x}+k_2B'\frac{\partial\ddot{v}_0}{\partial y}\right)\\
&\qquad+J_2\left(k_1A'\frac{\partial^2\ddot{w}_0}{\partial x^2}+k_2B'\frac{\partial^2\ddot{w}_0}{\partial y^2}\right)\\
&\qquad-K_2\left((k_1A')^2\frac{\partial^2\ddot{\theta}}{\partial x^2}+(k_2B')^2\frac{\partial^2\ddot{\theta}}{\partial y^2}\right)
\end{aligned}
\tag{20}
$$

Substituting Eq. (15) into Eq. (20), the equations of motion can be expressed in terms of displacements δu_0, δv_0, δw_0 and $\delta\theta$

$$
\begin{aligned}
\delta u_0:\ &A_{11}\frac{\partial^2u_0}{\partial x^2}+2A_{16}\frac{\partial^2u_0}{\partial x\partial y}+A_{66}\frac{\partial^2u_0}{\partial y^2}\\
&+A_{16}\frac{\partial^2v_0}{\partial x^2}+A_{26}\frac{\partial^2v_0}{\partial y^2}+(A_{12}+A_{66})\frac{\partial^2v_0}{\partial x\partial y}\\
&-B_{11}\frac{\partial^3w_0}{\partial x^3}-B_{26}\frac{\partial^3w_0}{\partial y^3}-3B_{16}\frac{\partial^2w_0}{\partial x^2\partial y}\\
&-(B_{12}+2B_{66})\frac{\partial^2w_0}{\partial x\partial y^2}+(k_1B_{11}^s+k_2B_{12}^s)\frac{\partial\theta}{\partial x}\\
&+(k_1B_{16}^s+k_2B_{26}^s)\frac{\partial\theta}{\partial y}\\
&+(k_1A'+k_2B')B_{16}^s\frac{\partial^3\theta}{\partial x^2\partial y}\\
&+(k_1A'+k_2B')B_{66}^s\frac{\partial^3\theta}{\partial x\partial y^2}\\
&=I_0\ddot{u}_0-I_1\frac{\partial\ddot{w}_0}{\partial x}+k_1A'J_1\frac{\partial\ddot{\theta}_0}{\partial x}
\end{aligned}
\tag{21a}
$$

$$
\begin{aligned}
\delta v_0:\ &A_{16}\frac{\partial^2u_0}{\partial x^2}+A_{26}\frac{\partial^2u_0}{\partial y^2}+(A_{12}+A_{66})\frac{\partial^2u_0}{\partial x\partial y}\\
&+A_{22}\frac{\partial^2v_0}{\partial y^2}+A_{66}\frac{\partial^2v_0}{\partial x^2}+2A_{26}\frac{\partial^2v_0}{\partial x\partial y}-B_{16}\frac{\partial^3w_0}{\partial x^3}\\
&-B_{22}\frac{\partial^3w_0}{\partial y^3}-(B_{12}+2B_{66})\frac{\partial^3w_0}{\partial x^2\partial y}\\
&-3B_{26}\frac{\partial^2w_0}{\partial x\partial y^2}+(k_1B_{16}^s+k_2B_{26}^s)\frac{\partial\theta}{\partial x}
\end{aligned}
\tag{21b}
$$

$$
\begin{aligned}
&+(k_1B_{12}^s+k_2B_{22}^s)\frac{\partial\theta}{\partial y}\\
&+(k_1A'+k_2B')B_{66}^s\frac{\partial^3\theta}{\partial x^2\partial y}\\
&+(k_1A'+k_2B')B_{26}^s\frac{\partial^3\theta}{\partial x\partial y^2}\\
&=I_0\ddot{v}_0-I_1\frac{\partial\ddot{w}_0}{\partial y}+k_2B'J_1\frac{\partial\ddot{\theta}_0}{\partial y}
\end{aligned}
\tag{21b}
$$

$$
\begin{aligned}
\delta w_0:\ &B_{11}\frac{\partial^3u_0}{\partial x^3}+(B_{12}+2B_{66})\frac{\partial^2u_0}{\partial x\partial y^2}+B_{26}\frac{\partial^3u_0}{\partial y^3}\\
&+3B_{16}\frac{\partial^3u_0}{\partial x^2\partial y}+B_{16}\frac{\partial^3v_0}{\partial x^3}+B_{22}\frac{\partial^3v_0}{\partial y^3}\\
&+3B_{26}\frac{\partial^3v_0}{\partial x\partial y^2}+(B_{12}+2B_{66})\frac{\partial^3v_0}{\partial x^2\partial y}\\
&-D_{11}\frac{\partial^4w_0}{\partial x^4}-D_{22}\frac{\partial^4w_0}{\partial y^4}\\
&-2(D_{12}+2D_{66})\frac{\partial^4w_0}{\partial x^2\partial y^2}-4D_{26}\frac{\partial^4w_0}{\partial x\partial y^3}\\
&-4D_{16}\frac{\partial^4w_0}{\partial x^3\partial y}+2(k_1D_{12}^s+k_2D_{26}^s)\frac{\partial^2\theta}{\partial x\partial y}\\
&+(k_1A'+k_2B')D_{16}^s\frac{\partial^4\theta}{\partial x^3\partial y}\\
&+(k_1D_{11}^s+k_2D_{12}^s)\frac{\partial^2\theta}{\partial x^2}\\
&+(k_1D_{12}^s+k_2D_{22}^s)\frac{\partial^2\theta}{\partial y^2}\\
&+(k_1A'+k_2B')D_{26}^s\frac{\partial^4\theta}{\partial x\partial y^3}\\
&+2(k_1A'+k_2B')D_{66}^s\frac{\partial^4\theta}{\partial x^2\partial y^2}+q\\
&=I_0\ddot{w}_0+I_1\left(\frac{\partial\ddot{u}_0}{\partial x}+\frac{\partial\ddot{v}_0}{\partial y}\right)-I_2\left(\frac{\partial^2\ddot{w}_0}{\partial x^2}+\frac{\partial^2\ddot{w}_0}{\partial y^2}\right)\\
&+J_2\left(k_1A'\frac{\partial^2\ddot{\theta}}{\partial x^2}+k_2B'\frac{\partial^2\ddot{\theta}}{\partial y^2}\right)
\end{aligned}
\tag{21c}
$$

$$
\begin{aligned}
\delta\theta:\ &-(k_1B_{11}^s+k_2B_{12}^s)\frac{\partial u_0}{\partial x}-(k_1B_{16}^s+k_2B_{26}^s)\frac{\partial u_0}{\partial y}\\
&-(k_1A'+k_2B')B_{16}^s\frac{\partial^3u_0}{\partial x^2\partial y}\\
&-(k_1A'+k_2B')B_{66}^s\frac{\partial^3u_0}{\partial x\partial y^2}\\
&-(k_1B_{16}^s+k_2B_{26}^s)\frac{\partial v_0}{\partial x}-(k_1B_{12}^s+k_2B_{22}^s)\frac{\partial v_0}{\partial y}\\
&-(k_1A'+k_2B')B_{66}^s\frac{\partial^3v_0}{\partial x^2\partial y}\\
&-(k_1A'+k_2B')B_{26}^s\frac{\partial^3v_0}{\partial x\partial y^2}\\
&+(k_1D_{11}^s+k_2D_{12}^s)\frac{\partial^2w_0}{\partial x^2}\\
&+(k_1D_{12}^s+k_2D_{22}^s)\frac{\partial^2w_0}{\partial y^2}\\
&-2(k_1D_{16}^s+k_2D_{26}^s)\frac{\partial^2w_0}{\partial x\partial y}
\end{aligned}
\tag{21d}
$$

$$
\begin{aligned}
&+(k_1A'+k_2B')D_{26}^s\frac{\partial^4w_0}{\partial x\partial y^3}\\
&+(k_1A'+k_2B')D_{16}^s\frac{\partial^4w_0}{\partial x^3\partial y}\\
&+2(k_1A'+k_2B')D_{66}^s\frac{\partial^4w_0}{\partial x^2\partial y^2}\\
&-(k_1^2H_{11}^s-k_2^2H_{22}^s-2k_1k_2H_{12}^s)\theta\\
&-(k_1^2A'H_{16}^s+k_2^2B'H_{26}^s)\frac{\partial^2\theta}{\partial x\partial y}\\
&-k_1k_2(B'H_{16}^s+A'H_{26}^s)\frac{\partial^2\theta}{\partial x\partial y}\\
&-(k_1^2A'+k_1k_2B')H_{16}^s\frac{\partial^2\theta}{\partial x\partial y}\\
&-(k_2^2B'+k_1k_2A')H_{26}^s\frac{\partial^2\theta}{\partial x\partial y}\\
&-\left(k_1^2A'^2+k_2^2B'^2+2k_1k_2A'B'\right)H_{66}^s\frac{\partial^4\theta}{\partial x^2\partial y^2}\\
&+k_1^2A'^2A_{55}^s\frac{\partial^2\theta}{\partial x^2}+k_2^2B'^2A_{44}^s\frac{\partial^2\theta}{\partial y^2}\\
&+2k_1k_2A'B'^{'A_{45}^s\frac{\partial^2\theta}{\partial x\partial y}}\\
&=-J_1\left(k_1A'\frac{\partial\ddot{u}_0}{\partial x}+k_2B'\frac{\partial\ddot{v}_0}{\partial y}\right)\\
&+J_2\left(k_1A'\frac{\partial^2\ddot{w}_0}{\partial x^2}+k_2B'\frac{\partial^2\ddot{w}_0}{\partial y^2}\right)\\
&-K_2\left((k_1A')^2\frac{\partial^2\ddot{\theta}}{\partial x^2}+(k_2B')^2\frac{\partial^2\ddot{\theta}}{\partial y^2}\right)
\end{aligned}
\tag{21d}
$$

4. Analytical solutions for anti-symmetric laminated composite plates

The Navier method is employed to obtain the closed-form solutions of the partial differential equations in Eq. (21) for simply supported anti-symmetric laminated composite plates. Two different types are considered in this study, cross-ply $[0°/90°]_n$ and angle-ply $[\theta°/-\theta°]_n$. For the first type, the following stiffness components are identically zero

$$
\begin{aligned}
&A_{16}=A_{26}=B_{12}=B_{16}=B_{26}=B_{66}\\
&=B_{12}^s=B_{16}^s=B_{26}^s=B_{66}^s=D_{16}=D_{26}\\
&=D_{16}^s=D_{26}^s=H_{16}^s=H_{26}^s=A_{45}^s=0
\end{aligned}
\tag{22}
$$

Based on Navier method, the following expansions of generalized displacements are taken to automatically satisfy the simply supported boundary conditions of the laminated composite plates (for the vibration problems, the transverse load is set to be zero).

$$
\begin{Bmatrix}u_0\\v_0\\w_0\\\theta\end{Bmatrix}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\begin{Bmatrix}U_{mn}e^{i\omega t}\cos(\alpha x)\sin(\beta y)\\V_{mn}e^{i\omega t}\sin(\alpha x)\cos(\beta y)\\W_{mn}e^{i\omega t}\sin(\alpha x)\sin(\beta y)\\\Phi_{mn}e^{i\omega t}\sin(\alpha x)\sin(\beta y)\end{Bmatrix}
\tag{23}
$$

For the second type "angle-ply", the following stiffness components are identically zero

$$A_{16} = A_{26} = B_{11} = B_{12} = B_{22} = B_{66} = B_{11}^s = B_{12}^s = B_{22}^s = B_{66}^s = D_{16} = D_{26} = D_{16}^s = D_{26}^s = H_{16}^s = H_{26}^s = A_{45}^s = 0 \tag{24}$$

And the displacement variables which automatically satisfy the boundary conditions can be expressed in the following forms

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn} e^{i\omega t} \sin(\alpha x)\cos(\beta y) \\ V_{mn} e^{i\omega t} \cos(\alpha x)\sin(\beta y) \\ W_{mn} e^{i\omega t} \sin(\alpha x)\sin(\beta y) \\ \Phi_{mn} e^{i\omega t} \sin(\alpha x)\sin(\beta y) \end{Bmatrix} \tag{25}$$

Where $\alpha = m\pi/a$ and$\beta = n\pi/b$, ω is the frequency of free vibration of the plate, $\sqrt{i} = -1$ the imaginary unit. Substituting Eq. (23) into Eq. (21) and collecting the displacements and acceleration for any values of m and$n$, the following problem is obtained.

$$\left(\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix} - \omega^2 \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{12} & M_{22} & M_{23} & M_{24} \\ M_{13} & M_{23} & M_{33} & M_{34} \\ M_{14} & M_{24} & M_{34} & M_{44} \end{bmatrix} \right) \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ \Phi_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{26}$$

where $[S_{ij}]$ and $[M_{ij}]$ are elements of stiffness matrix and mass matrix, respectively. And can be defined for anti-symmetric cross-ply and angle-ply laminates as follows.

- ***For anti-symmetric cross-ply laminated plates***

$$\begin{aligned} &S_{11} = \alpha^2 A_{11} + \beta^2 A_{66}, \quad S_{12} = \alpha\beta(A_{12} + A_{66}), \\ &S_{13} = -\alpha^3 B_{11}, \quad S_{14} = -k_1 \alpha B_{11}^s, \\ &S_{22} = \alpha^2 A_{66} + \beta^2 A_{22}, \quad S_{23} = -\beta^3 B_{22}, \\ &S_{24} = -k_2 \beta B_{22}^s, \\ &S_{33} = \alpha^4 D_{11} + \beta^4 D_{22} + 2\alpha^2\beta^2(D_{12} + 2D_{66}), \\ &S_{34} = k_1 \alpha^2 D_{11}^s + (k_2\alpha^2 + k_1\beta^2) D_{12}^s \\ &\quad + k_2 \beta^2 D_{22}^s - 2(k_1 A' + k_2 B')\alpha^2\beta^2 D_{66}^s, \\ &S_{44} = k_1^2 H_{11}^s + k_2^2 H_{22}^s + 2k_1 k_2 H_{12}^s \\ &\quad + \left(k_1^2 A'^2 + k_2^2 B'^2 + 2k_1 k_2 A' B'\right)\alpha^2\beta^2 H_{66}^s \\ &\quad + k_2^2 B'^2 \beta^2 A_{44}^s + k_1^2 A'^2 \alpha^2 A_{55}^s \end{aligned} \tag{27}$$

- ***For anti-symmetric angle-ply laminated plates***

$$\begin{aligned} &S_{11} = \alpha^2 A_{11} + \beta^2 A_{66}, \\ &S_{12} = \alpha\beta(A_{12} + A_{66}), \\ &S_{13} = -3\alpha^2\beta B_{16} - \beta^3 B_{26}, \\ &S_{14} = -k_1 \beta B_{16}^s - k_2 \beta B_{26}^s + (k_1 A' + k_2 B')\alpha^2\beta B_{16}^s, \\ &S_{22} = \alpha^2 A_{66} + \beta^2 A_{22}, \\ &S_{23} = -3\alpha\beta^2 B_{26} - \alpha^3 B_{16}, \\ &S_{24} = -k_1 \alpha B_{16}^s - k_2 \alpha B_{26}^s + (k_1 A' + k_2 B')\alpha\beta^2 B_{26}^s, \\ &S_{33} = \alpha^4 D_{11} + \beta^4 D_{22} + 2\alpha^2\beta^2(D_{12} + 2D_{66}), \\ &S_{34} = k_1 \alpha^2 D_{11}^s + (k_2\alpha^2 + k_1\beta^2) D_{12}^s + k_2\beta^2 D_{22}^s \\ &\quad -2(k_1 A' + k_2 B')\alpha^2\beta^2 D_{66}^s, \\ &S_{44} = k_1^2 H_{11}^s + k_2^2 H_{22}^s + 2k_1 k_2 H_{12}^s \\ &\quad + \left(k_1^2 A'^2 + k_2^2 B'^2 + 2k_1 k_2 A' B'\right)\alpha^2\beta^2 H_{66}^s \\ &\quad + k_2^2 B'^2 \beta^2 A_{44}^s + k_1^2 A'^2 \alpha^2 A_{55}^s \end{aligned} \tag{28}$$

Elements of mass matrix $[M]$ for both cross-ply and angle-ply laminated plates

$$\begin{aligned} &M_{11} = I_0, \quad M_{12} = 0, \quad M_{13} = 0, \quad M_{14} = 0, \\ &M_{22} = I_0, \quad M_{23} = 0, \quad M_{24} = 0, \\ &M_{33} = I_0 + I_2(\alpha^2 + \beta^2), \\ &M_{34} = -(k_1 A' \alpha^2 + k_2 B' \beta^2) J_2, \\ &M_{44} = \left(k_1^2 A'^2 \alpha^2 + k_2^2 B'^2 \beta^2\right) K_2 \end{aligned} \tag{29}$$

5. Numerical results and discussions

Various numerical examples are solved to ensure the accuracy of the proposed mathematical model of this theory for the prediction of dynamic response of multi-layered antisymmetric laminated composite and sandwich plates, the closed form solution are obtained using the Navier solution for free vibration analysis of simply supported cross-ply and angle-ply laminated composite plates on all edges. The validity of the present theory is demonstrated by comparison with previously published results. For this purpose, suitable sets of material properties will be used in the numerical studies as follows

***Laminated composite*:**

Material 1:

$E_1/E_2 = open$, $G_{12}/E_2 = G_{13}/E_2 = 0.6$,
$G_{23}/E_2 = 0.5$, $\nu_{12} = 0.25$

Material 2:

$E_1/E_2 = 40$, $G_{12}/E_2 = G_{13}/E_2 = 0.6$,
$G_{23}/E_2 = 0.5$, $\nu_{12} = 0.25$

Material 3:

$E_1 = 276GPa$, $E_2 = E_3 = 6.9GPa$,
$G_{12} = G_{13} = 4.14GPa$, $G_{23} = 3.45GPa$,
$\nu_{12} = 0.25$, $\rho = 1578kg/m^3$

Material 4:

$E_1/E_2 = 15$, $G_{12}/E_2 = G_{13}/E_2 = 0.5$,
$G_{23}/E_2 = 0.35$, $\nu_{12} = 0.3$

***Sandwich*:**

Material 5:

$E = 73GPa$, $\nu_{12} = 0.3$,
$\rho = \dfrac{2800kg}{m^3}$, (Aluminum alloy for face sheets)

Material 6:

$E = 180GPa$, $\nu_{12} = 0.37$,
$\rho = 50kg/m^3$, (PVC material for foam core)

Material 7:

$E_1 = 132.38GPa$, $E_2 = E_3 = 10.756GPa$,
$G_{12} = G_{13} = 5.6537GPa$, $G_{23} = 3.603GPa$,
$\nu_{12} = 0.24$, $\rho = 1600kg/m^3$

Material 8:

$E_1 = 131.0GPa,$ $E_2 = E_3 = 10.34GPa,$
$G_{12} = G_{13} = 6.895GPa,$ $G_{23} = 6.205GPa,$
$\nu_{12} = 0.22,$ $\rho = 1627kg/m^3$

Material 9:

$E_1 = E_2 = E_3 = 00.689GPa,$
$G_{12} = G_{13} = G_{23} = 00.345GPa,$
$\nu_{12} = 0,$ $\rho = 97kg/m^3$

Table 1 Non-dimensional natural frequencies $(\bar{\omega})$ of multilayered $(0/90)_n$ anti-symmetric cross-ply laminated composite square plates ($a/h = 5$, material 1)

N	Lay-ups	Theory	E_1/E_2				
			3	10	20	30	40
2	$(0/90)_1$	Present	6.2188	6.9964	7.8379	8.5316	9.1236
		Sayyad and Ghugal (2017)	6.2190	6.9967	7.8385	8.5320	9.1246
		Thai and Kim (RPT1) (2010)	6.2169	6.9887	7.8210	8.5050	9.0871
		Thai and Kim (RPT2) (2010)	6.2167	6.9836	7.8011	8.4646	9.0227
		Sayyad and Ghugal (2015)	6.2417	7.0150	7.8537	8.5452	9.1357
		Reddy (1984)	6.2169	6.9887	7.8210	8.5050	9.0871
		Mindlin (1951)	6.2085	6.9392	7.7060	8.3211	8.8383
		CPT [(a)]	6.7705	7.7420	8.8555	9.8337	10.721
		Exact (1990)	6.2578	6.9845	7.6745	8.1763	8.5625
4	$(0/90)_2$	Present	6.5012	8.1929	9.6205	10.5268	11.1628
		Sayyad and Ghugal (2017)	6.5012	8.1929	9.6205	10.5268	11.1628
		Thai and Kim (RPT1) (2010)	6.5008	8.1954	9.6265	10.5348	11.1716
		Thai and Kim (RPT2) (2010)	6.5008	8.1949	9.6252	10.5334	11.1705
		Sayyad and Ghugal (2015)	6.5255	8.2177	9.6437	10.5477	11.1815
		Reddy (1984)	6.5008	8.1954	9.6265	10.5348	11.1716
		Mindlin (1951)	6.5043	8.2246	9.6885	10.6198	11.2708
		CPT [(a)]	7.1690	9.7192	12.476	14.7250	16.6725
		Exact (1990)	6.5455	8.1445	9.4055	10.1650	10.6789
6	$(0/90)_3$	Present	6.5567	8.4066	9.9210	10.8603	11.5102
		Sayyad and Ghugal (2017)	6.5567	8.4065	9.9210	10.8603	11.5100
		Thai and Kim (RPT1) (2010)	6.5558	8.4052	9.9181	10.8547	11.5012
		Thai and Kim (RPT2) (2010)	6.5558	8.4052	9.9181	10.8547	11.5009
		Sayyad and Ghugal (2015)	6.5815	8.4305	9.9407	10.855	11.5025
		Reddy (1984)	6.5558	8.4052	9.9181	10.8547	11.5012
		Mindlin (1951)	6.5569	8.4183	9.9427	10.8828	11.5264
		CPT [(a)]	7.2415	10.053	13.058	15.4907	17.5897
		Exact (1990)	6.6100	8.4143	9.8398	10.6958	11.2728
10	$(0/90)_5$	Present	6.5854	8.5156	10.0740	11.0309	11.6893
		Sayyad and Ghugal (2017)	6.5854	8.5156	10.0740	11.0309	11.6893
		Thai and Kim (RPT1) (2010)	6.5842	8.5126	10.0674	11.0197	11.6730
		Thai and Kim (RPT2) (2010)	6.5842	8.5126	10.0671	11.0186	11.6705
		Sayyad and Ghugal (2015)	6.6100	8.5397	10.0957	11.0500	11.6855
		Reddy (1984)	6.5842	8.5126	10.0614	11.0197	11.6730
		Mindlin (1951)	6.5837	8.5132	10.0638	11.0058	11.6444
		CPT [(a)]	7.2415	10.053	13.0585	15.4907	17.5897
		Exact (1990)	6.6458	8.5625	10.0843	11.0027	11.6245

Table 2 Non-dimensional natural frequencies $(\bar{\omega})$ of multilayered $(0/90)_n$ anti-symmetric cross-ply laminated composite square plates (material 2)

N	Lay-ups	Theory	a/h				
			5	10	20	50	100
2	$(0/90)_1$	Present	9.1236	10.5811	11.1089	11.2757	11.3003
		Sayyad and Ghugal (2017)	9.1246	10.5815	11.1090	11.2757	11.3003
		Thai and Kim (RPT1) (2010)	–	10.5680	11.1052	11.2751	11.3002
		Thai and Kim (RPT2) (2010)	–	10.5480	11.0997	11.2742	11.2999
		Sayyad and Ghugal (2015)	–	10.5930	11.1320	11.3000	11.3000
		Reddy (1984)	9.0871	10.5680	11.1052	11.2751	11.3002
		Mindlin (1951)	8.8383	10.4731	11.0779	11.2705	11.2990
		CPT [(a)]	10.721	11.1537	11.2693	11.3023	11.3070
4	$(0/90)_2$	Present	11.1628	14.8376	16.5700	17.1843	17.2782
		Sayyad and Ghugal (2017)	11.1628	14.8376	16.5700	17.1842	17.2782
		Thai and Kim (RPT1) (2010)	–	14.8463	16.5733	17.1849	17.2784
		Thai and Kim (RPT2) (2010)	–	14.8433	16.5719	17.1847	17.2783
		Sayyad and Ghugal (2015)	–	14.8570	16.6080	17.2250	17.3000
		Reddy (1984)	11.1716	14.8463	16.5733	17.1849	17.2784
		Mindlin (1951)	11.2708	14.9214	16.6008	17.1899	17.2796
		CPT [(a)]	16.6725	17.1448	17.2682	17.3032	17.3082
6	$(0/90)_3$	Present	11.5102	15.4633	17.3768	18.0643	18.1698
		Sayyad and Ghugal (2017)	11.5100	15.4633	17.3768	18.0642	18.1698
		Thai and Kim (RPT1) (2010)	–	15.4632	17.3772	18.0644	18.1698
		Thai and Kim (RPT2) (2010)	–	15.4627	17.3769	18.0643	18.1698
		Sayyad and Ghugal (2015)	–	15.4830	17.4160	18.1250	18.2000
		Reddy (1984)	11.5012	15.4632	17.3772	18.0644	18.1698
		Mindlin (1951)	11.5264	15.5010	17.3926	18.0673	18.1706
		CPT [(a)]	17.5897	18.0461	18.1652	18.1990	18.2038
10	$(0/90)_5$	Present	11.6893	15.7739	17.7751	18.4985	18.6097
		Sayyad and Ghugal (2017)	11.6893	15.7739	17.7751	18.4985	18.6097
		Thai and Kim (RPT1) (2010)	–	15.7700	17.7743	18.4984	18.6097
		Thai and Kim (RPT2) (2010)	–	15.7700	17.7743	18.4984	18.6097
		Sayyad and Ghugal (2015)	–	15.7930	17.8160	18.5500	18.6000
		Reddy (1984)	11.6730	15.7700	17.7743	18.4984	18.6097
		Mindlin (1951)	11.6444	15.7790	17.7800	18.4995	18.6100
		CPT [(a)]	17.5897	18.0461	18.1652	18.1990	18.2038

The following non-dimensional form is used while presenting numerical result of natural frequencies.

$$\bar{\omega} = \omega(b^2/h)\sqrt{(\rho/E_2)} \tag{30}$$

5.1 Free vibration analysis of anti-symmetric laminated composite plates

Example 1*: *Free vibration analysis of cross-ply $(0°/90°)_n$ laminated composite plates

In this example, free vibration analysis of anti-symmetric cross-ply laminated square plates is investigated using Eq. (26) in the absence of external load. In Table 1, the non-dimensional natural frequencies of multilayered $(0/90)_n$ laminated composite plates by using different theories are shown for various numbers of layers, varied from 2 to 10. The modulus ratio E_1/E_2 is varied from 3 to 40. All the layers have the same thickness and made up of Material 1. The present results are compared with those presented by Mindlin (1951), Reddy (1984), Thai and Kim (RPT2) (2010), Sayyad and Ghugal (2015, 2017) and the exact elasticity solution given by Noor and Burton (1990). It is observed that the present approach can provide accurate results in comparison with the three-dimensional elasticity solutions given by Noor and Burton (1990) and the previous studies based on the higher-order shear

Table 3 Non-dimensional natural frequencies $(\bar{\omega})$ of anti-symmetric $(45°/-45°)$ angle-ply laminated composite square plates (material 1)

E_1/E_2	Theory	E_1/E_2				
		4	10	20	50	100
3	Present	6.0900	7.0747	7.2706	7.3293	7.3378
	Sayyad and Ghugal (2017)	6.0902	7.0747	7.2706	7.3293	7.3382
	Thai and Kim (RPT1) (2010)	6.0861	7.0739	7.2705	7.3293	7.3378
	Thai and Kim (RPT2) (2010)	6.0852	7.0738	7.2704	7.3293	7.3378
	Kant and Manjunatha (1988)	6.1223	7.1056	7.3001	7.3583	7.3666
	Pandya and Kant (1988)	6.0803	7.0728	7.2702	7.3295	7.3383
	Reddy (1984)	6.0861	7.0739	7.2705	7.3293	7.3378
	Mindlin (1951)	6.0665	7.0700	7.2694	7.3291	7.3378
	CPT [(a)]	6.9251	7.2699	7.3228	7.3378	7.3400
10	Present	7.3670	8.9709	9.3279	9.4379	9.4541
	Sayyad and Ghugal (2017)	7.3676	8.9711	9.3279	9.4379	9.4541
	Thai and Kim (RPT1) (2010)	7.3470	8.9660	9.3266	9.4377	9.4540
	Thai and Kim (RPT2) (2010)	7.3259	8.9621	9.3255	9.4376	9.4540
	Kant and Manjunatha (1988)	7.2647	8.9893	9.3265	9.4377	9.5123
	Pandya and Kant (1988)	7.2159	8.9328	9.3174	9.4363	9.4540
	Reddy (1984)	7.3470	8.966	9.3266	9.4377	9.4540
	Mindlin (1951)	7.2169	8.9324	9.3173	9.4362	9.4537
	CPT [(a)]	8.7950	9.3444	9.4304	9.4548	9.4583
20	Present	8.4595	10.728	11.281	11.456	11.482
	Sayyad and Ghugal (2017)	8.4606	10.728	11.281	11.456	11.482
	Thai and Kim (RPT1) (2010)	8.4152	10.715	11.277	11.455	11.481
	Thai and Kim (RPT2) (2010)	8.3396	10.698	11.272	11.454	11.481
	Kant and Manjunatha (1988)	8.049	10.641	11.298	11.507	11.539
	Pandya and Kant (1988)	8.0074	10.588	11.240	11.449	11.480
	Reddy (1984)	8.4152	10.715	11.277	11.455	11.482
	Mindlin (1951)	8.1185	10.627	11.252	11.451	11.481
	CPT [(a)]	10.631	11.341	11.453	11.484	11.489
30	Present	9.2434	12.118	12.872	13.116	13.153
	Sayyad and Ghugal (2017)	9.2448	12.119	12.872	13.116	13.153
	Thai and Kim (RPT1) (2010)	9.1752	12.097	12.866	13.115	13.152
	Thai and Kim (RPT2) (2010)	9.0341	12.062	12.856	13.113	13.152
	Kant and Manjunatha (1988)	8.5212	11.893	12.842	13.157	13.204
	Pandya and Kant (1988)	8.4847	11.844	12.789	13.102	13.149
	Reddy (1984)	9.1752	12.097	12.866	13.115	13.152
	Mindlin (1951)	8.7213	11.946	12.821	13.108	13.151
	CPT [(a)]	12.159	12.989	13.120	13.158	13.163
40	Present	9.2434	12.118	12.872	13.116	13.153
	Sayyad and Ghugal (2017)	9.2448	12.119	12.872	13.116	13.153
	Thai and Kim (RPT1) (2010)	9.1752	12.097	12.866	13.115	13.152
	Thai and Kim (RPT2) (2010)	9.0341	12.062	12.856	13.113	13.152
	Kant and Manjunatha (1988)	8.5212	11.893	12.842	13.157	13.204
	Pandya and Kant (1988)	8.4847	11.844	12.789	13.102	13.149
	Reddy (1984)	9.1752	12.097	12.866	13.115	13.152
	Mindlin (1951)	8.7213	11.946	12.821	13.108	13.151
	CPT [(a)]	12.159	12.989	13.120	13.158	13.163

[(a)] Results taken from reference Sayyad and Ghugal (2017) deformation theories.

Table 2 demonstrates the comparison of non-dimensional natural frequencies of multilayered antisymmetric cross-ply laminated composite plates for various side-to-thickness ratio a/h, ranging from 5 to 100 (corresponding to from thick to thin plates), and the plate consists of material 2. The obtained results are compared with the solution reported by Sayyad and Ghugal (2015, 2017), Thai and Kim (2010) and Reddy (1984). This comparison demonstrates clearly that the present results are in good agreement with them. Whereas the classical plate theory (CPT) overestimates the natural frequency as compared to the results of other theories due to neglect of transverse shear strains and provided acceptable results only for thin laminated plates.

Example 2: *Free vibration analysis of angle-ply laminated* (*45°/-45°*) *composite plates*

In the next example, anti-symmetric angle-ply laminated (45°/-45°) square plates with two layers are considered. The modulus ratio E_1/E_2 is varied from 3 to 40. All layers are of equal thickness and made up of Material 1. The side-to-thickness ratio a/h varied from 4 to 100. Numerical results of non-dimensional natural frequencies ($\bar{\omega}$) are listed in Table 3. It is observed that the present method can provide accurate results in comparison with those generated by Sayyad and Ghugal (2017), Thai and Kim (2010), Kant and Manjunatha (1988), Pandya and Kant (1988), and Reddy (1984). It can be seen again that the computed results are in very good agreement with those calculated by other shear deformation theories for different values of thickness ratio ranging from thick to thin laminated composite plates.

The non-dimensional natural frequencies of anti-symmetric angle-ply laminated square plates with two layers made up of material 2 for both fiber orientation angle ($\theta = 15°$) and ($\theta = 30°$) are given in Table 4 and compared to other theories cited previously in the literature. It is evident from the obtained results that the present computations are in good concordance with the analytical results reported by Sayyad and Ghugal (2017) and Senthilnathan *et al*. (1988). Moreover, it can be noticed that the increase of the thickness ratio has a significant effect on the increase of the natural frequencies.

Further the comparison of natural frequencies for simply supported anti-symmetric angle-ply laminated square plates with two layers made up of material 1 are illustrated in Table 5 for different values of side-to-thickness ratio ($a/h = 5, 10, 20, 50, 100$) and for various values of both modulus ratio ($E_1/E_2 = 3, 10, 20, 30, 40$) and fiber orientation angle ($\theta = 15°, 30°, 60°$). From the examination of Table 5, it can be seen that the increase of modulus ratio leads to an increase of non-dimensional natural frequencies and this is due to the increase of the stiffness of the anti-symmetric angle-ply laminated composite plates.

Example 3: *Free vibration analysis of angle-ply laminated* (*45°/-45°*)$_2$ *composite plates*

This example is performed for free vibration analysis of (45°/-45°)$_2$ anti-symmetric angle-ply laminated plate to investigate the accuracy and applicability of the present theory. The thickness of the four layers was taken as

Table 4 Non-dimensional natural frequencies ($\bar{\omega}$) of anti-symmetric ($\theta/-\theta$) angle-ply laminated composite square plates (material 2)

θ	Theory	a/h				
		4	10	20	50	100
15°	Present	9.4639	13.1956	14.3222	14.7059	14.7638
	Sayyad and Ghugal (2017)	9.5421	13.4284	14.6293	15.0420	15.1044
	Senthilnathan *et al*. (1988)	9.4119	13.1793	14.3173	14.7050	14.6745
	Kant and Manjunatha (1988)	8.5142	12.7600	14.2324	14.7629	14.8445
	Pandya and Kant (1988)	8.4789	12.6928	14.1507	14.6754	14.7563
	Reddy (1984)	8.8117	12.8126	14.1881	14.6819	14.7577
	Mindlin (1951)	8.4662	12.6802	14.1457	14.6745	14.7557
	Present	9.6358	12.9540	13.8744	14.1783	14.2238
	Sayyad and Ghugal (2017)	9.6610	13.0383	13.9852	14.2992	14.3461
30°	Senthilnathan *et al*. (1988)	9.5564	12.9283	13.8667	14.1770	14.2235
	Kant and Manjunatha (1988)	8.6739	12.5935	13.8010	14.2137	14.2763
	Pandya and Kant (1988)	8.6393	12.5442	13.5452	14.1562	14.2184
	Reddy (1984)	9.4455	12.8730	13.8487	14.1738	14.2225
	Mindlin (1951)	8.9169	12.6807	13.7896	14.1637	14.2198
	Present	9.4639	13.1956	14.3222	14.7059	14.7638
	Sayyad and Ghugal (2017)	9.5421	13.4284	14.6293	15.0420	15.1044
	Senthilnathan *et al*. (1988)	9.4119	13.1793	14.3173	14.7050	14.6745
	Kant and Manjunatha (1988)	8.5142	12.7600	14.2324	14.7629	14.8445

Table 5 Non-dimensional natural frequencies $(\bar{\omega})$ of anti-symmetric $(\theta/-\theta)$ angle-ply laminated composite square plates (material 1)

θ	a/h	Theory	E_1/E_2				
			3	10	20	30	40
15°	4	Present	6.1752	7.5093	8.4440	9.0274	9.4639
		Sayyad and Ghugal (2017)	6.5130	7.7721	8.6007	9.1337	9.5421
	10	Present	7.1890	9.3233	11.0766	12.2692	13.1956
		Sayyad and Ghugal (2017)	7.7624	9.8333	11.4492	12.5576	13.4284
	20	Present	7.3904	9.7347	11.7544	13.1827	14.3222
		Sayyad and Ghugal (2017)	8.0191	10.3180	12.2060	13.5489	14.6293
	50	Present	7.4506	9.8622	11.9725	13.4859	14.7059
		Sayyad and Ghugal (2017)	8.0965	10.4697	12.4520	13.8810	15.0420
	100	Present	7.4594	9.8809	12.0049	13.5313	14.7638
		Sayyad and Ghugal (2017)	8.1078	10.4920	12.4886	13.9309	15.1044
30°	4	Present	6.1083	7.3253	8.3400	9.0670	9.6358
		Sayyad and Ghugal (2017)	6.2027	7.4276	8.3994	9.1516	9.6610
	10	Present	7.0987	8.9264	10.5771	11.8668	12.9540
		Sayyad and Ghugal (2017)	7.2635	9.1279	10.7242	12.0686	13.0383
	20	Present	7.2956	9.2816	11.1209	12.5975	13.8744
		Sayyad and Ghugal (2017)	7.4772	9.5113	11.2978	12.8437	13.9852
	50	Present	7.3545	9.3909	11.2929	12.8339	14.1783
		Sayyad and Ghugal (2017)	7.5413	9.6299	11.4800	13.0958	14.2992
	100	Present	7.3631	9.4070	11.3183	12.8690	14.2238
		Sayyad and Ghugal (2017)	7.5506	9.6472	11.5069	13.1334	14.3461
60°	4	Present	6.1083	7.3253	8.3400	9.0670	9.6358
		Sayyad and Ghugal (2017)	6.2027	7.4276	8.3994	9.1516	9.6610
	10	Present	7.0987	8.9264	10.5771	11.8668	12.9540
		Sayyad and Ghugal (2017)	7.2635	9.1279	10.7242	12.0686	13.0383
	20	Present	7.2956	9.2816	11.1209	12.5975	13.8744
		Sayyad and Ghugal (2017)	7.4772	9.5113	11.2978	12.8437	13.9852
	50	Present	7.3545	9.3909	11.2929	12.8339	14.1783
		Sayyad and Ghugal (2017)	7.5413	9.6299	11.4800	13.0958	14.2992
	100	Present	7.3631	9.4070	11.3183	12.8690	14.2238
		Sayyad and Ghugal (2017)	7.5506	9.6472	11.5069	13.1334	14.3461

0.25h/*0.25h*/*0.25h*/*0.25h*, and the plate is made of material 3. The variation of natural frequencies for first six modes with respect to side-to-thickness ratio (*a*/*h*) is presented in Table 6. In order to assure the accuracy of the present theory, the numerical results obtained for this example are compared with the results predicted by Sayyad and Ghugal (2017) using a simple four-variable trigonometric shear deformation theory, the analytical solutions reported by Matsunaga (2001) using the method of power series expansion of displacement components and a global higher-order plate theory, the quadrilateral element results achieved by Kulkarni and Kapuria (2008) based on the third-order zigzag theory, the finite element solutions presented by Chalak *et al*. (2013) and the solutions given by Chakrabarti and Sheikh (2004) based on the refined higher-order shear deformation plate theory. It should be clearly pointed out that the present theory gives more accurate results in predicting the natural frequencies when compared to Sayyad and Ghugal (2017) and Kulkarni and Kapuria (2008).

Example 4*: *Free vibration analysis of angle-ply laminated (45°/-45°)$_5$ composite plates

This study discusses the free vibration analysis of (45°/-45°)$_5$ anti-symmetric angle-ply laminated square composite plates, with layers of the same thickness and made up of material 4. The natural frequencies computed using present theory and other shear deformation theories with the three-dimensional elasticity solutions given by Noor and Burton (1990) are listed in Table 7. It can be confirmed from the Table 7 that, the results of the proposed theory agree well with the results of Sayyad and Ghugal (2017), Reddy (1984), Thai and Kim (2010) and FSDT of Mindlin (1951). By comparing the results to those obtained by CPT, it can

Table 6 Non-dimensional natural frequencies $(\bar{\omega})$ of four-layer $(45°/-45°)_2$ anti-symmetric angle-ply laminated composite square plates (material 3)

a/h	Theory	Modes of vibration					
		1	2	3	4	5	6
5	Present	12.5295	21.7713	21.7713	29.4406	32.4781	32.4781
	Sayyad and Ghugal (2017)	12.5295	21.7713	21.7713	29.4406	32.4780	32.4780
	Kulkarni and Kapuria (2008)	12.5293	21.4012	21.4012	29.3154	32.0688	32.0688
	Chakrabarti and Sheikh (2004)	11.8130	18.7780	18.9260	23.9570	25.3290	25.4380
	Chalak *et al*. (2013)	11.9131	20.2298	20.2298	27.2263	30.1399	30.3087
10	Present	18.3062	35.0905	35.0905	50.1181	54.8631	54.8631
	Sayyad and Ghugal (2017)	18.3062	35.0905	35.0905	50.1181	54.8630	54.8630
	Matsunaga (2001)	17.5885	32.6571	32.6571	46.6888	50.6147	50.6147
	Kulkarni and Kapuria (2008)	18.3144	34.5392	34.5392	50.0729	53.8869	53.8869
	Chakrabarti and Sheikh (2004)	17.9340	33.3000	33.4640	47.2370	50.5840	50.6340
	Chalak *et al*. (2013)	17.6921	32.8839	32.8839	47.2914	51.5889	52.0283

Table 7 Non-dimensional natural frequencies $(\bar{\omega})$ of ten-layer $(45°/-45°)_5$ anti-symmetric angle-ply laminated composite square plates (material 4)

Theory	a/h		
	5	10	100
Present	10.1672	13.6111	15.9482
Sayyad and Ghugal (2017)	10.1672	13.6111	15.9482
Reddy (1984)	10.1537	13.6078	15.9482
Thai and Kim (RPT1) (2010)	10.1537	13.6078	15.9482
Thai and Kim (RPT2) (2010)	10.1516	13.6078	15.9482
Mindlin (1951)	10.1288	13.6140	15.9484
CPT [(a)]	15.4661	15.8460	15.9775
Exact (1990)	9.9825	13.5100	15.9500

be shown that the effect of shear deformation is to decrease the natural frequencies

5.2 Free vibration analysis of sandwich plates

Example 1**: *Free vibration analysis of symmetric sandwich plates* (*0°/core/0°*)**

In this section, efficiency of proposed theory is proved for the free vibration response of simply supported symmetric square and rectangular sandwich plates with thin face sheets and thick core for different values of side-to-thickness ratio. The thickness of each face sheet is *0.15h* and made up of isotropic aluminum alloy (material 5) whereas thickness of central core is *0.7h* and made up of PVC foam (material 6).The non-dimensional frequencies of first three vibration modes obtained by present theory are presented in Table 8 and are compared with those obtained by exact elasticity solution given by Brischetto (2014) and the four-variable trigonometric shear deformation theory developed by Sayyad and Ghugal (2017). Examination of Table 8 also reveals that, the present theory gives excellent results for the frequencies of second and third modes for square and rectangular sandwich plates. However, there is a considerable difference with 3D-elasticity solution for non-dimensional frequencies of first mode.

Table 8 Non-dimensional natural frequencies $(\bar{\omega})$ of three-layer $(0°/Core/0°)$ symmetric rectangular sandwich plates (material 5 and 6)

b/a	Theory	Modes	a/h			
			5	10	50	100
1	Present	I	4.4221	6.5923	8.5408	8.6349
		II	13.535	27.070	135.35	270.70
		III	22.885	45.771	228.85	457.71
	Sayyad and Ghugal (2017)	I	4.4220	6.5923	8.5408	8.6384
		II	13.535	27.070	135.35	270.70
		III	22.885	45.771	228.85	457.70
	Exact 3D (2014)	I	1.4786	2.4879	7.0764	8.1693
		II	6.8059	27.045	135.35	270.70
		III	13.473	28.081	228.77	457.67
3	Present	I	2.9876	4.0581	4.7757	4.8051
		II	10.088	20.177	100.88	201.77
		III	17.058	34.116	170.57	341.16
	Sayyad and Ghugal (2017)	I	2.9876	4.0581	4.7756	4.8050
		II	10.088	20.176	100.88	201.77
		III	17.057	34.115	170.56	341.14
	Exact 3D (2014)	I	1.0092	1.7567	4.2583	4.6553
		II	6.9197	20.167	100.88	201.77
		III	10.066	24.201	170.55	341.14

Example 2**: *Free vibration analysis of symmetric cross-ply sandwich plates* (*0°/90°/core/90°/0°*)**

In second example, present theory is applied for the free vibration analysis of simply supported (0°/90°/core/90°/0°) symmetric sandwich plates. The sandwich plate is

Table 9 Non-dimensional natural frequencies ($\bar{\omega}$) of five-layer ($0°/90°/Core/90°/0°$) symmetric rectangular sandwich plates (material 6 and 7)

b/a	Theory	Modes	a/h			
			5	10	50	100
1	Present	I	5.7104	9.6654	15.702	16.129
		II	38.581	77.161	385.80	771.61
		III	43.074	86.148	430.74	861.48
	Sayyad and Ghugal (2017)	I	5.7107	9.6657	15.702	16.129
		II	38.580	77.161	385.80	771.60
		III	43.074	86.148	430.73	861.48
	Exact 3D (2014)	I	3.2639	5.9275	14.440	15.754
		II	17.398	71.631	385.73	771.58
		III	37.351	76.817	430.56	861.39
3	Present	I	4.1627	7.0263	11.169	11.449
		II	17.122	34.245	171.22	342.45
		III	39.554	79.108	395.54	791.08
	Sayyad and Ghugal (2017)	I	4.1626	7.0265	11.169	11.448
		II	17.122	34.245	171.22	342.45
		III	39.554	79.108	395.54	791.08
	Exact 3D (2014)	I	2.4968	4.5385	10.421	11.231
		II	19.965	34.219	171.22	342.45
		III	17.876	48.469	395.40	791.01

consisting of two face sheets at the top and bottom surfaces of the plate and made up of graphite-epoxy orthotropic composite material (material 7) whereas the flexible core at the center made up of PVC foam (material 6). The thickness of each face sheet is *0.075h* and thickness of soft core is *0.7h*. The comparison of non-dimensional frequencies for first three vibration modes of free vibration of square and rectangular sandwich plates is reported in Table 9 for four values of the thickness ratio (a/h = 5, 10, 50, 100). It is evident from the obtained results that the present computations are in an excellent agreement with those of exact elasticity solution and the trigonometric shear deformation theory given by Sayyad and Ghugal (2017).

Example 3: *Free vibration analysis of anti-symmetric cross-ply sandwich plates (0°/90°/core/0°/90°)*

The third example is carried out for simply supported anti-symmetric cross-ply square sandwich plate (0°/90°/core/0°/90°) with side-to-thickness ratio varied from 2 to 100. The thickness of the core to thickness of the face sheets is adopted($t_c/t_f = 10$). The face sheets of the plate are made of an orthotropic composite material 8 whereas the soft core is made of material 9. The non-dimensional natural frequencies obtained by the present solution are compared with those predicted by available results in Table 10 whereas non-dimensional natural frequencies for first six modes of vibration are mentioned in Table 11. The present results are compared with those provided by other existing theories such as the one proposed by Sayyad and Ghugal (2015, 2017), Reddy (1984), Rao *et al*. (2004), Kant and Manjunatha (1988), Pandya and Kant (1988), Senthilnathan *et al*. (1988) and Mindlin (1951). It can be seen that the results of present study again agree well with those reported by Sayyad and Ghugal (2017) using a four-variable trigonometric shear deformation theory and to those reported by Reddy (1984) based on HSDT. On the other hand, Table 12 presents comparison of non-dimensional natural frequencies for antisymmetric rectangular sandwich plates.

Example 4: *Free vibration analysis of anti-symmetric angle-ply sandwich plates (θ/-θ/core/θ/-θ)*

In last example, anti-symmetric (*θ/-θ/core/θ/-θ*) angle-ply square sandwich plates are considered for the calculation of non-dimensional natural frequencies with respect to the several values of both thickness ratio ($a/h = 10, 20, 50, 100$) and fiber orientation angle ($\theta = 15°, 30°, 45°$) whereas the thickness of the core to thickness of the face sheets is taken ($t_c/t_f = 4,\ 10$). The Face sheets of the plate are made up of orthotropic material 8 whereas the isotropic core is made of material 9. The obtained numerical results are presented in Table 13 and have been compared with previously published results obtained from other plate theories. Examination of

Table 10 Non-dimensional natural frequencies ($\bar{\omega}$) of five-layer ($0°/90°/Core/0°/90°$) anti-symmetric sandwich square plates (material 8 and 9,$t_c/t_f = 10$)

Theory	a/h					
	2	4	10	20	50	100
Present	0.8718	1.6694	4.0051	7.2849	12.3028	14.3519
Sayyad and Ghugal (2017)	0.8209	1.6439	3.9964	7.2820	12.3004	14.3474
Sayyad and Ghugal (2015)	0.8778	1.6767	4.1312	7.5829	13.3791	15.5978
Reddy (1984)	1.6252	3.1013	7.0473	11.2664	15.0323	15.9522
Rao *et al*. (2004)	0.7141	0.9363	1.8480	3.4791	7.7355	11.9400
Kant and Manjunatha (1988)	1.1941	2.1036	4.8594	8.5955	13.6899	15.5093
Pandya and Kant (1988)	1.1734	2.0913	4.8519	8.5838	13.6577	15.4647
Senthilnathan *et al*. (1988)	1.6252	3.1013	7.0473	11.2664	15.0323	15.9522
Mindlin (1951)	5.2017	9.0312	13.869	15.5295	16.1264	16.2175

Table 11 Non-dimensional natural frequencies ($\bar{\omega}$) of five-layer ($0°/90°/Core/0°/90°$) anti-symmetric sandwich square plates (material 8 and 9,$t_c/t_f = 10$)

a/h	Theory	Modes of vibration					
		1	2	3	4	5	6
10	Present	4.0051	6.5102	8.2788	9.3364	10.6617	12.6177
	Sayyad and Ghugal (2017)	3.9964	6.4622	8.1987	9.1760	10.4767	11.9465
	Sayyad and Ghugal (2015)	4.1312	6.7339	8.6150	9.6638	11.0885	13.1232
	Reddy (1984)	7.0473	11.9087	15.2897	17.3211	19.8121	23.5067
	Rao and Desai (2004)	4.9624	8.1934	10.5172	11.9867	13.7488	16.4514
	Kant and Manjunatha (1988)	4.8594	8.0187	10.2966	11.7381	13.4706	16.1320
	Pandya and Kant (1988)	4.8519	7.9965	10.2550	11.6809	13.3889	16.0039
	Senthilnathan *et al*. (1988)	7.0473	11.9624	15.2897	17.3698	19.8325	23.5067
	Mindlin (1951)	13.869	30.6432	41.5577	50.9389	58.3636	71.3722
100	Present	14.3519	35.5662	49.2113	64.7584	74.0710	92.2246
	Sayyad and Ghugal (2017)	14.3474	35.5583	49.2015	64.7474	74.0586	92.2090
	Sayyad and Ghugal (2015)	15.5978	38.3778	53.5165	69.8024	80.0727	100.3965
	Reddy (1984)	15.9521	42.2271	60.1272	83.9982	96.3132	124.2047
	Rao and Desai (2004)	15.5480	39.2652	73.4951	55.1512	84.2919	106.5897
	Kant and Manjunatha (1988)	15.5093	39.0293	54.7618	72.7572	83.4412	105.3781
	Pandya and Kant (1988)	15.4646	38.9232	54.6330	72.5925	83.2699	105.1807
	Senthilnathan *et al*. (1988)	15.9521	42.3708	60.1272	84.4215	96.7259	124.2047
	Mindlin (1951)	16.2175	44.7072	64.5044	94.9097	108.9049	143.7969

Table 12 Non-dimensional natural frequencies ($\bar{\omega}$) of five-layer ($0°/90°/Core/0°/90°$) anti-symmetric rectangular sandwich plates (material 8 and 9,$t_c/t_f = 10, a/h = 10$)

Theory	a/b						
	0.5	1	1.5	2	2.5	3	5
Present	12.5275	4.0051	2.3075	1.6275	1.2638	1.0374	0.6184
Sayyad and Ghugal (2017)	12.5073	3.9964	2.2980	1.6155	1.2489	1.0195	0.5886
Rao *et al*. (ESL) (2004)	15.3407	4.9624	2.8797	2.0483	1.6057	1.3317	0.8342
Rao *et al*. (LW) (2004)	5.7328	1.8480	1.0884	0.8049	0.6626	0.5792	0.4493
Reddy (1984)	21.4500	7.0473	4.1587	3.6444	2.3324	1.9242	1.1541
Kant and Manjunatha (1988)	15.0316	4.8594	2.8188	2.4560	1.5719	1.3040	0.8187
Pandya and Kant (1988)	15.0128	4.8519	2.8130	2.4469	1.5660	1.2976	0.8102
Senthilnathan *et al*. (1988)	21.6668	7.0473	4.1725	3.6582	2.3413	1.9216	1.1550
Mindlin (1951)	39.4840	13.869	10.165	9.4910	6.5059	5.6588	3.6841

Table 13 also reveals that, the present computations are in an excellent agreement with the analytical solutions provided by Ghugal and Sayyad (2017); however, a significant difference is observed as compared to other shear deformation theories as Kant and Manjunatha (1988), Pandya and Kant (1988), Reddy (1984) and Senthilnathan *et al*. (1988). This is due to the different approaches used to predict the natural frequencies. Moreover, the first-order shear deformation theory of Mindlin (1951) overestimates the natural frequency values for all thickness ratios.

6. Conclusions

A simple four-variable trigonometric shear deformation model with undetermined integral terms is developed for the free vibration analysis of simply supported antisymmetric laminated composite and soft core sandwich plates. The most important feature of this theory is that it has only four-unknown variables and four governing equations derived from the principle of virtual work and does not require any shear correction factors. Various numerical examples are presented and compared with those provided by other existing theories to prove the validity of

Table 13 Non-dimensional natural frequencies ($\bar{\omega}$) of five-layer ($\theta/-\theta/Core/\theta/-\theta$) anti-symmetric angle-ply sandwich square plates (material 8 and 9)

		Theory	a/h			
			10	20	50	100
4	15°	Present	7.5923	12.1491	16.2539	17.2622
		Sayyad and Ghugal (2017)	7.5929	12.1489	16.2527	17.2608
		Kant and Manjunatha (1988)	8.8342	12.9787	16.2421	16.9744
		Pandya and Kant (1988)	8.8109	12.9633	16.2330	16.9666
		Reddy (1984)	10.585	14.3884	16.6537	17.0840
		Senthilnathan *et al*. (1988)	11.284	14.9062	16.7857	17.1196
		Mindlin (1951)	14.360	16.3410	17.0808	17.1965
	30°	Present	7.7412	12.8130	17.9614	19.3510
		Sayyad and Ghugal (2017)	7.7421	12.8128	17.9602	19.3495
		Kant and Manjunatha (1988)	9.5383	14.4318	18.2621	19.1154
		Pandya and Kant (1988)	9.5153	14.4130	18.2465	19.1005
		Reddy (1984)	11.631	16.0979	18.7384	19.2378
		Senthilnathan *et al*. (1988)	11.832	16.2517	18.7787	19.2487
		Mindlin (1951)	16.096	18.3818	19.2351	19.3685
	45°	Present	7.7982	13.0779	18.7132	20.3004
		Sayyad and Ghugal (2017)	7.7993	13.0778	18.7119	20.2988
		Kant and Manjunatha (1988)	9.8197	15.0371	19.1695	20.0845
		Pandya and Kant (1988)	9.7973	15.0173	19.1513	20.0667
		Reddy (1984)	12.051	16.8312	19.6858	20.2163
		Senthilnathan *et al*. (1988)	12.051	16.8312	19.6858	20.2163
		Mindlin (1951)	16.848	19.3022	20.2263	20.3573
10	30	Present	4.0530	7.5673	13.8217	16.9422
		Sayyad and Ghugal (2017)	4.0393	7.5623	13.8192	16.9372
		Kant and Manjunatha (1988)	5.0035	9.0294	15.5303	18.4008
		Pandya and Kant (1988)	4.9949	9.0227	15.5216	18.3900
		Reddy (1984)	7.3280	12.2477	17.6159	19.1603
		Senthilnathan *et al*. (1988)	7.4382	12.4504	17.7286	19.1974
		Mindlin (1951)	15.926	18.5408	19.5550	19.7222
	45	Present	4.0624	7.6287	14.2074	17.6676
		Sayyad and Ghugal (2017)	4.0471	7.6230	14.2049	17.6626
		Kant and Manjunatha (1988)	5.0653	9.2740	16.2062	19.3098
		Pandya and Kant (1988)	5.0566	9.2675	16.1965	19.2970
		Reddy (1984)	7.4895	12.6964	18.4604	20.1355
		Senthilnathan *et al*. (1988)	7.4895	12.6964	18.4604	20.1355
		Mindlin (1951)	16.654	19.4671	20.5661	20.7477

the proposed mathematical model. The effects of number of layers, modulus ratio, side-to-thickness ratio and fiber orientation angle are examined and discussed. It is observed from this entire investigation that the present theory with four unknowns predicts excellent results for natural frequencies as compared to those obtained using other refined shear deformation theories for all modes of vibrations. Finally, the present mathematical model is found to be appropriate and efficient in analyzing vibration problem of laminated composite and soft core sandwich plates. An improvement of the present formulation will be considered in the future work to consider other type of materials (Avcar 2015, 2016, Hadji *et al*. 2016, Mehar *et al*. 2016, Kar and Panda 2015a, b, 2017, Chandra Mouli *et al*. 2018, Belmahi *et al*. 2018, Bensattalah *et al*. 2018, Chemi *et al*. 2018, Kumar and Srinivas 2018, Faleh *et al*. 2018, Shahadat *et al*. 2018, Safa *et al*. 2019).

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