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On the stability of isotropic and composite thick plates

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Abstract. This proposed project presents the bi-axial and uni-axial stability behavior of laminated composite plates based on an original three variable “refined” plate theory. The important “novelty” of this theory is that besides the inclusion of a cubic distribution of transverse shear deformations across the thickness of the structure, it treats only three variables such as conventional plate theory (CPT) instead five as in the well-known theory of “first shear deformation” (FSDT) and theory of “higher order shear deformation” (HSDT). A “shear correction coefficient” is therefore not employed in the current formulation. The computed results are compared with those of the CPT, FSDT and exact 3D elasticity theory. Good agreement is demonstrated and proved for the present results with those of “HSDT” and elasticity theory.

Keywords: stability analysis; isotropic; laminated composite plate

1. Introduction

With the novel trend of employing composites materials as laminated structures in the automotive, aerospace, and civil “*industries*”, the theoretical predictions of local stability (buckling) response of such studied structures has widely attracted the newly attention of many scientists. Buckling is often considered as one of the important mechanisms of failure in layered composite plates. A very large amount of mathematical and theoretical scientific studies have been considered and proposed on the stability of “*isotropic*”, “*transversely isotropic*”, “*orthotropic*” and stratified composite plates. According to reports by several researchers (Noor and Burton 1989, Reddy and Arciniega 2004, Ghugal and Shimpi 2002, Kreja 2011, Carrera 2003, Wanji and Zhen 2008, Demasi 2009a, b, c, d, e) many equivalent monolayer theories have been proposed and developed in recent decades to examine the stability response of “*isotropic*”, “*orthotropic*” and stratified composite plates. The simplest from these theories is the classical plate theory (CPT) based on the Kirchhoff considerations, which ignores normal transverse and “*shear stresses*”. Since the transverse “*shear deformation*” is omitted in this model, it gives acceptable and reasonable results only for thin structures such plates, but not for thick ones. Xiang and Wang (2002) obtained the critical stability load by considering the Levy procedure and the CPT-based state space method. Jafari and Eftekhari (2011) applied the “*Ritz-differential quadrature technique*” for dynamic and stability investigation of rectangular orthotropic plates via CPT. Shojaee *et al*. (2012) proposed an “*isogeometric finite element method*” based on non-uniform rational “*B-spline basic functions*” for free vibration and stability analysis of thin symmetric stratified composite plates via CPT.

In the aim to avoid the CPT limitation, the “*First Order Shear Deformation Theory*” (FSDT) which considers the transverse shear influence is strongly recommended. The Reissner (1945) and Mindlin (1951) theories are respectively known as FSDTs based on stress and displacement, and explain the transverse shear influence by the linear distribution of plane displacements across the plate thickness. However, these models do not meet the conditions without shear stress on the upper and lower surfaces of the structure, and must utilize the “*shear correction coefficient*” to compensate in general the error of considering a constant transverse shear stress variation (Alimirzaei *et al*. 2019). These coefficients depend on the type of material, and both boundary and loading conditions. By applying the method of differential quadrature element, Liu (2001) presented the stability investigation of discontinuous and rectangular plates via Mindlin's plate theory. Liew and Huang (2003) employed the mobile differential quadratics method to the bending and stability analyzes of symmetric plates (laminates), while Huang and Li (2004) utilized it to bending and stability analyzes of anti-symmetric laminated plates via the FSDT. Using a radial point interpolation technique, Liew and Chen (2004) studied the buckling response of rectangular Mindlin plates under partial edge loads in the plane. Liew *et al*. (2004) used a non-mesh method for dynamic and stability analyzes of “*shear-deformable plates*” by considering Mindlin and Reissner plate models. Shukla *et al*. (2005) investigated and discussed the buckling of stratified composite rectangular plates on the basis of FSDT and von Karman nonlinearity. Zhong and Gu (2007) proposed exact theoretical solutions for stability of symmetric cross-ply “*composite rectangularplates*” with simply supported edges subjected

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to a linear edge load based on the FSDT. Hashemi *et al*. (2008) applied Mindlin's plate theory to examine the buckling of plane-loaded isotropic plates with different types of boundary conditions. Akhavan *et al*. (2009) provided exact mathematical solutions for the stability analysis of rectangular Mindlin plates subjected to planar loadings uniformly and linearly distributed on two simply supported opposite edges resting on elastic foundations. Ferreira *et al*. (2011) performed the buckling and dynamic investigation of isotropic and stratified plates via radial basic functions. Gilat *et al*. (2001) investigated the bifurcation stability problem of stratified composite structures such as plates by considering the global local plate model. Azhari and Kassaei (2004) formulated the buckling analysis for thick and anisotropic plates under arbitrary loads. Using the "*generalized quadrature method*" and the "*Rayleigh-Ritz procedure*", Darvizeh *et al*. (2004) performed a comparative investigation on the stability of composite plates. Shufrin *et al*. (2008) presented an extended semi-analytical Kantorovich formulation for the study of stability of symmetrically stratified rectangular structures such as plates with different boundary conditions. Barton (2008) provided an "*approximate quadratic*" closed form relation for the critical buckling investigation of a combined bending and compressing plate. Liu and Pavlovi (2008) presented an analytical method to the elastic buckling of rectangular plates simply supported under arbitrary external forces. Kuo and Shiau (2009) examined the stability and dynamic response of composite stratified plates with "*variable fiber spacing*". Panda and Singh (2010a) investigated the nonlinear free vibration response of thermally post-buckled composite spherical shell panel. Fiedler *et al*. (2010) studied the stability of square multilayer stratified composite plates under unidirectional plane loads via generalized HSDT. By employing HSDT, Panda and Ramachandra (2010) studied the buckling forces of composite structures such as rectangular plates with the consideration of nine types of different "*boundary conditions*" and subjected to non-uniform plane forces. Using non-linear finite element method, Panda and Singh (2010b) studied the thermal post-buckling response of a laminated composite spherical shell panel embedded with shape memory alloy fibres. Nali *et al*. (2011) evaluated the refined models for the stability analysis of stratified plates. Ruocco and Fraldi (2012) provided an analytical method for the buckling investigation of rectangular plate under mixed types of boundary conditions. Kheirikhah *et al*. (2012) proposed a novel high order improved theory for the biaxial stability investigation of sandwich structures such as plates with a soft orthotropic nucleus. Wang *et al*. (2000) provided the exact expressions between buckling forces of the "*classical Kirchhoff*" plate model, "*Mindlin*" plate model and "*Reddy*" plate model for simply supported plates with plane forces. Wang and Reddy (1997) and Wang and Lee (1998) determined the buckling force expressions between both "*Reddy*" and "*Kirchhoff*" plate model for circular and polygonal plates. Shaikh and Ganeshan (2012) performed a buckling investigation of composite structures such as plates via the Ritz procedure and the FSDT. The elastic buckling of thin and rectangular plates with different boundary conditions is considered and discussed by Ruocco *et al*. (2011). Verma and Singh (2009) used the finite element formulation for the thermal buckling investigation of stratified composite plates. A semi-analytical formulation for flexure, global stability, and dynamic investigation of sandwich panels with square honeycomb cores is developed and discussed by Liu *et al*. (2010). Panda and Singh (2013a) employed a nonlinear finite element method for analyzing the thermal post-buckling vibration of laminated composite shell panel embedded with SMA fibre. Panda and Singh (2013b) analyzed the post-buckling response of laminated composite doubly curved panel embedded with SMA fibers subjected to thermal environment. Panda and Singh (2013c) discussed the thermal post-buckling behavior of laminated composite spherical shell panel using NFEM. Panda and Singh (2013d) presented a large amplitude free vibration analysis of thermally post-buckled composite doubly curved panel embedded with SMA fibers. Katariya and Panda (2014) analyzed the thermo-mechanical stability of composite cylindrical panels. Panda and Katariya (2015) studied stability and free vibration behaviour of laminated composite panels under thermo-mechanical loading. Katariya and Panda (2016) analyzed the thermal buckling and vibration behavior of laminated composite curved shell panel.

A type of refined shear deformation models employing exponential, trigonometric and hyperbolic functions to consider shear deformation influences exist in the scientific literature. Levy (1877) proposed for the first time a refined model for thick and isotropic plates via sinusoidal functions in terms of z axis in the kinematic. Theories employing trigonometric functions considering z coordinates in the kinematics of plate models are referred to as the "*trigonometric shear deformation*" models. Stein and Bains (1990), Touratier (1991), Soldatos (1992), Shimpi and Ghugal (2002), Arya *et al*. (2002) and Shimpi *et al*. (2003) also developed similar models and used them to stratified and isotropic plates. For the first time, Ferreira *et al*. (2005) utilized a shear trigonometric deformation theory to model symmetric composite structures discretized by a non-mesh method by considering global multiquadric radial basic functions. Xiang *et al*. (2009) have employed these models for the modeling of discrete stratified plates by a non-mesh method via an "*inverse multiquadric radial basic functions*". Sayyad and Ghugal (2013) evaluated the influence of concentration of stress because of the concentrated loading on stratified plates by employing the theory of trigonometric shear deformation with considering the influences of transverse shear and normal transverse deformations. Akavci (2007) proposed secant and tangent hyperbolic functions in terms of z axis in the kinematics of the theory for stability and dynamic investigation of symmetrical and anti-symmetrical composite plates resting on elastic foundations.

The goal of this work is to construct a new and simple HSDT for the stability of isotropic, transversely isotropic, orthotropic and cross laminated composite rectangular plates under uniaxial and biaxial plane compression. The proposed model contains fewer variables and motion equations than FSDT, but satisfies the equilibrium conditions on the upper and lower surfaces of the plate without the use of shear correction factors. Indeed, contrary to the theories mentioned above, the number of variables in the current theory is the same as in the CPT. The results determined for critical buckling forces are compared to other HSDTs and 3D elasticity solutions provided by Noor (1975) to prove the validity of the model.

2. Mathematical and theoretical formulations

A rectangular plate is considered here with sides a and$b$ and a non-variable thickness of h. The coordinates system (x; y; z) is used and the coordinate parameters are chosen such that $0 \le x \le a$, $0 \le y \le b$ and $-h/2 \le x \le h/2$. The structure is assumed to consist of an arbitrary number "*N*" of linearly elastic "orthotropic layers".

2.1 Displacement field

The original three variable "*refined*" plate theory proposed in this article is presented by the following displacement field

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\frac{\partial w_0}{\partial x} - \beta\, f(z)\frac{\partial^3 w_0}{\partial x^3} \\ v(x,y,z) &= v_0(x,y) - z\frac{\partial w_0}{\partial y} - \beta\, f(z)\frac{\partial^3 w_0}{\partial y^3} \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{1}$$

where u_0, v_0, and w_0 are three variables of displacement functions of mid-surface of the plate and β is a coefficient of the proposed displacement model.The displacement field in the plane uses the cubic function in terms of the thickness co-ordinate to introduce the transverse shear deformation influence and is given by

$$f(z) = \frac{5}{4}\left(z - \frac{4}{3h^2}z^3\right) \tag{2}$$

The relations of linear deformations deduced from the kinematics provided by Eq. (1), valid for the thick and thin plates, are

$$\begin{aligned} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} &= \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} + \beta f(z)\begin{Bmatrix} \eta_x \\ \eta_y \\ \eta_{xy} \end{Bmatrix}, \\ \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} &= \beta g(z)\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix}, \end{aligned} \tag{3}$$

where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial x} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial^2 w_0}{\partial y^2} \\ -2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}, \tag{4}$$

$$\begin{Bmatrix} \eta_x \\ \eta_y \\ \eta_{xy} \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^4 w_0}{\partial x^4} \\ -\dfrac{\partial^4 w_0}{\partial y^4} \\ -\dfrac{\partial^2 (\nabla^2 w_0)}{\partial x \partial y} \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^3 w_0}{\partial y^3} \\ -\dfrac{\partial^3 w_0}{\partial x^3} \end{Bmatrix} \tag{4}$$

and

$$g(z) = f'(z), \qquad \nabla^2 w_0 = \frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{\partial y^2} \tag{5}$$

2.2 Constitutive relations

The stress-strain expressions for the k^{th} orthotropic ply of the stratified plate in the coordinate axes of the material are provided by Jones (1975)

$$\begin{pmatrix} \sigma_x \\ \sigma_x \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{pmatrix}^k = \begin{bmatrix} \overline{C}_{11} & \overline{C}_{12} & 0 & 0 & 0 \\ \overline{C}_{12} & \overline{C}_{22} & 0 & 0 & 0 \\ 0 & 0 & \overline{C}_{44} & 0 & 0 \\ 0 & 0 & 0 & \overline{C}_{55} & 0 \\ 0 & 0 & 0 & 0 & \overline{C}_{66} \end{bmatrix}^k \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{pmatrix} \tag{6}$$

The transformed mechanical material constants of each orthotropic ply are given as

$$\begin{pmatrix} \overline{C}_{11} \\ \overline{C}_{12} \\ \overline{C}_{22} \\ \overline{C}_{66} \end{pmatrix}^k = \begin{bmatrix} c^4 & 2c^2s^2 & s^4 & 4c^2s^2 \\ c^2s^2 & c^4+s^4 & c^2s^2 & -4c^2s^2 \\ s^4 & 2c^2s^2 & c^4 & 4c^2s^2 \\ c^2s^2 & -2c^2s^2 & c^2s^2 & (c^2-s^2)^2 \end{bmatrix} \begin{pmatrix} C_{11} \\ C_{12} \\ C_{22} \\ C_{66} \end{pmatrix} \tag{7a}$$

$$\begin{pmatrix} \overline{C}_{44} \\ \overline{C}_{55} \end{pmatrix}^k = \begin{bmatrix} c^2 & s^2 \\ s^2 & c^2 \end{bmatrix} \begin{pmatrix} C_{44} \\ C_{55} \end{pmatrix} \tag{7b}$$

where $c = \cos\theta_k$ and $s = \sin\theta_k$. θ_k is the generally defined angle between two axes such as the global x-axis and the local x-axis of each ply. C_{ij} are the engineering constants.

2.3 Governing equations

In this study, constant applied edge in-plane loads N_x^0, N_y^0 and N_{xy}^0 are considered. By using the principle of virtual work, the governing equations become

$$\begin{aligned} \delta\, u_0:&\quad \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \\ \delta\, v_0:&\quad \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \\ \delta\, w_0:&\quad \frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} \\ &\quad + \left(\frac{\partial^4 S_x}{\partial x^4} + \frac{\partial^4 S_{xy}}{\partial x^3 \partial y} + \frac{\partial^4 S_{xy}}{\partial y^3 \partial x} + \frac{\partial^4 S_y}{\partial y^4}\right) \\ &\quad - \left(\frac{\partial^3 Q_{xz}}{\partial x^3} + \frac{\partial^3 Q_{yz}}{\partial y^3}\right) + N_x^0\left(\frac{\partial^2 w_0}{\partial x^2}\right) \\ &\quad + 2N_{xy}^0\left(\frac{\partial^2 w_0}{\partial x \partial y}\right) + N_y^0\left(\frac{\partial^2 w_0}{\partial y^2}\right) = 0 \end{aligned} \tag{8}$$

where the stress resultants used in the above equations are defined as

$$(N_i, M_i, S_i) = \int_{-\frac{h}{2}}^{\frac{h}{2}} (1, z, \beta f)(\sigma_i) dz, \quad (i = x, y, xy),$$
$$Q_i = \int_{-h/2}^{h/2} (\tau_i)\, \beta g(z) dz, \qquad (i = xz, yz) \tag{9}$$

The governing expressions defined by Eq. (8) can be written in terms of displacements

$$\begin{aligned}
&A_{11}\frac{\partial^2 u_0}{\partial x^2} + A_{66}\frac{\partial^2 u_0}{\partial y^2} + (A_{12}+A_{66})\frac{\partial^2 v_0}{\partial x \partial y} \\
&-B_{11}\frac{\partial^3 w_0}{\partial x^3} - (B_{12}+2B_{66})\frac{\partial^3 w_0}{\partial x \partial y^2} \\
&-\beta\left(B_{11}^s\frac{\partial^5 w_0}{\partial x^5} + (B_{12}^s + B_{66}^s)\frac{\partial^5 w_0}{\partial x \partial y^4}\right. \\
&\left.+B_{66}^s\left(\frac{\partial^5 w_0}{\partial x^3 \partial y^2}\right)\right) = 0,
\end{aligned} \tag{10a}$$

$$\begin{aligned}
&A_{22}\frac{\partial^2 v_0}{\partial x^2} + A_{66}\frac{\partial^2 v_0}{\partial x^2} + (A_{12}+A_{66})\frac{\partial^2 u_0}{\partial x \partial y} \\
&-B_{22}\frac{\partial^3 w_0}{\partial y^3} - (B_{12}+2B_{66})\frac{\partial^3 w_0}{\partial x^2 \partial y} \\
&-\beta\left(B_{22}^s\frac{\partial^5 w_0}{\partial y^5} + (B_{12}^s + B_{66}^s)\frac{\partial^5 w_0}{\partial x^4 \partial y}\right. \\
&\left.+B_{66}^s\left(\frac{\partial^5 w_0}{\partial x^2 \partial y^3}\right)\right) = 0,
\end{aligned} \tag{10b}$$

$$\begin{aligned}
&B_{11}\frac{\partial^3 u_0}{\partial x^3} + (B_{12}+2B_{66})\left(\frac{\partial^3 u_0}{\partial x \partial y^2} + \frac{\partial^3 v_0}{\partial x^2 \partial y}\right) \\
&+B_{22}\frac{\partial^3 v_0}{\partial y^3} - D_{11}\frac{\partial^4 w_0}{\partial x^4} \\
&-2(D_{12}+2D_{66})\frac{\partial^4 w_0}{\partial x^2 \partial y^2} - D_{22}\frac{\partial^4 w_0}{\partial y^4} \\
&+\beta\left[B_{11}^s\frac{\partial^5 u_0}{\partial x^5} + (B_{12}^s + B_{66}^s)\left(\frac{\partial^5 u_0}{\partial x \partial y^4} + \frac{\partial^5 v_0}{\partial x^4 \partial y}\right)\right. \\
&+B_{22}^s\frac{\partial^5 v_0}{\partial y^5} + B_{66}^s\left(\frac{\partial^5 v_0}{\partial x^3 \partial y^2} + \frac{\partial^5 v_0}{\partial x^2 \partial y^3}\right) - 2D_{11}^s\frac{\partial^6 w_0}{\partial x^6} \\
&\left.-2(D_{12}^s + 2D_{66}^s)\left(\frac{\partial^6 w_0}{\partial x^2 \partial y^4} - \frac{\partial^6 w_0}{\partial x^4 \partial y^2}\right) - 2D_{22}^s\frac{\partial^6 w_0}{\partial y^6}\right] \\
&\beta^2\left[H_{11}^s\frac{\partial^8 w_0}{\partial x^8} + 2(H_{12}^s + H_{66}^s)\frac{\partial^8 w_0}{\partial x^4 \partial y^4}\right. \\
&+H_{22}^s\frac{\partial^8 w_0}{\partial y^8} + H_{66}^s\left(\frac{\partial^8 w_0}{\partial x^6 \partial y^2} + \frac{\partial^8 w_0}{\partial x^2 \partial y^6}\right) \\
&\left.+H_{22}^s\frac{\partial^8 w_0}{\partial y^8} - A_{44}^s\frac{\partial^6 w_0}{\partial x^6} - A_{55}^s\frac{\partial^6 w_0}{\partial y^6}\right] \\
&+N_x^0\left(\frac{\partial^2 w_0}{\partial x^2}\right) + 2N_{xy}^0\left(\frac{\partial^2 w_0}{\partial x \partial y}\right) + N_y^0\left(\frac{\partial^2 w_0}{\partial y^2}\right) = 0
\end{aligned} \tag{10c}$$

where the plate stiffnesses A_{ij}, B_{ij}, etc. are expressed as follows

$$\begin{aligned}
&\left(A_{ij}, B_{ij}, B_{ij}^s, D_{ij}, B_{ij}^s, H_{ij}^s\right) \\
&= \sum_{k=1}^{n}\int_{z_k}^{z_{k+1}} \overline{C}_{ij}^k \left(1, z, f(z), z^2, zf(z), f(z)^2\right) dz, \\
&\qquad (i, j = 1,2,6),
\end{aligned} \tag{11a}$$

$$A_{ij}^s = \sum_{k=1}^{n}\int_{z_k}^{z_{k+1}} \overline{C}_{ij}^k (g(z))^2\, dz, (i, j = 4,5) \tag{11b}$$

3. Analytical solutions

The Navier procedure is employed for simply supported rectangular plates. The solution used in Navier method to the Eq. (10) are

$$\begin{Bmatrix} u_0(x,y) \\ v_0(x,y) \\ w_0(x,y) \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\begin{Bmatrix} U_{mn}\, cos(\lambda\, x)\, sin(\mu\, y) \\ V_{mn}\, sin(\lambda\, x)\, cos(\mu\, y) \\ W_{mn}\, sin(\lambda\, x)\, sin(\mu\, y) \end{Bmatrix} \tag{12}$$

where U_{mn}, V_{mn} and W_{bmn} are arbitrary parameters to be determined, m and n are positive integers and $\lambda = m\pi/a$ and $\mu = m\pi/b$. The in-plane compressive loads are considered as follows

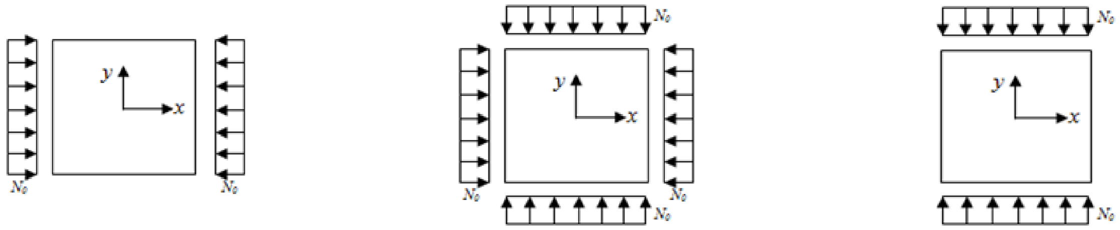
$$N_x^0 = -k_1 N_0, \quad N_y^0 = -k_2 N_0 \text{ and } N_{xy}^0 = 0 \tag{13}$$

By substituting Eqs. (12) and (13) into the Eq. (13), the following equation is found

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}\begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{14}$$

in which

$$\begin{aligned}
a_{11} &= -(A_{11}\lambda^2 + A_{66}\mu^2) \\
a_{12} &= -\left(\lambda\, \mu\, (A_{12}+A_{66})\right) \\
a_{13} &= B_{11}\lambda^3 + (B_{12}+2B_{66})\,\lambda\mu^2 \\
&\quad -\beta\left(B_{11}^s\lambda^5 + B_{12}^s\lambda\mu^4 + B_{66}^s(\lambda^3\mu^2 + \lambda\mu^4)\right) \\
a_{22} &= -(A_{66}\lambda^2 + A_{22}\mu^2) \\
a_{23} &= B_{22}\mu^3 + (B_{12}+2B_{66})\,\lambda^2\mu \\
&\quad -\beta\left(B_{22}^s\mu^5 + B_{12}^s\lambda^4\mu + B_{66}^s(\lambda^2\mu^3 + \lambda^4\mu)\right) \\
a_{33} &= -D_{11}\lambda^4 - 2(D_{12}+2D_{66})\lambda^2\mu^2 - D_{22}\mu^4 \\
&\quad -\beta[-2(D_{11}^s\lambda^6 + D_{22}^s\mu^6) \\
&\quad -2(\lambda^4\mu^2 + \lambda^2\mu^4)(D_{12}^s + 2D_{66}^s)] \\
&\quad -\beta^2[H_{11}^s\lambda^8 + H_{22}^s\mu^8 + 2\lambda^4\mu^4(H_{12}^s + H_{66}^s) \\
&\quad +(\lambda^6\mu^2 + \lambda^2\mu^6)H_{66}^s + A_{44}^s\mu^6 + 2A_{45}^s\lambda^3\mu^3 \\
&\quad +A_{55}^s\lambda^6] - N_x^0\lambda^2 - N_y^0\mu^2 + 2N_{xy}^0\lambda\mu
\end{aligned} \tag{15}$$



(a) (b) (c)

Fig. 1 Plate under in-plane compressive loads: (a) “uniaxial compression” along x-direction; (b) “biaxial compression”; and (c) “uniaxial compression” along y-direction

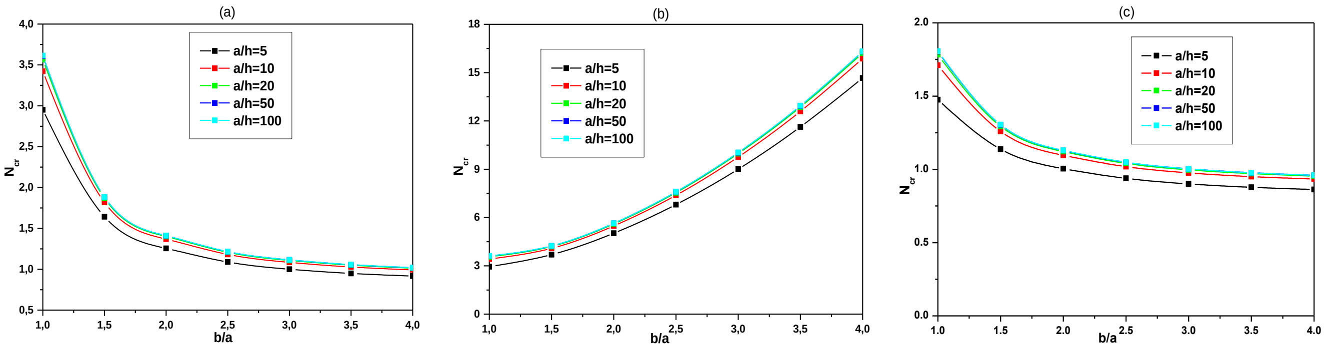


Fig. 2 Variations of non-dimensional critical stability force versus (b/a) ratio for rectangular isotropic plate using current model: (a) uni-axial compression along x-axis; (b) uni-axial compression along y-axis; and (c) biaxial compression

4. Numerical results and discussion

To test the validity of the proposed three-variable “refined” plate theory for evaluating the buckling loads of simply supported plates under the in-plane compression loads, the plate presented in Fig. 1 is examined. The model can be applied for all these types of structures: isotropic, transversely isotropic, orthotropic and symmetric cross-ply stratified composite plates. The influence of the aspect ratio of the structure like plate (a/b), the lateral ratio to the thickness (a/h) and modular ratio (E_1/E_2) on the stability characteristics of the structures is examined and discussed.

The critical stability loads computed by the proposed model are compared numerically with those of the Kirchhoff CPT, FSDT of Mindlin (1951), HSDT of Reddy (1984), RPT of Bourada *et al*. (2016), theory of Sayyad and Ghugal (2014) and 3D elasticity solutions provided by Noor (1975) wherever applicable, for prove the validity of the model. The following mechanical properties are utilized in this study.

“***Isotropic plate***”:

$$E = 210\ GPa, \quad \nu = 0.3, \quad G = \frac{E}{2(1+\nu)}$$

“***Transverse isotropic plate***”:

$$C_{11} = C_{22} = 20 \times 10^{10}; \quad C_{11} = 12 \times 10^{10};$$

$$C_{22} = C_{22} = 1 \times 10^{10}; \quad C_{12} = 4 \times 10^{10};$$

$$C_{44} = C_{55} = 3 \times 10^{10}; \quad C_{66} = \frac{(C_{11} - C_{12})}{2}$$

“***Orthotropic plate***”:

$$\frac{E_1}{E_2} = open, \quad \frac{E_2}{E_3} = 1.0, \quad \frac{G_{12}}{E_2} = \frac{G_{13}}{E_2} = 0.5$$

$$\frac{G_{23}}{E_2} = 0.2, \quad \nu_{12} = \nu_{13} = 0.25$$

“***Laminated composite plate***”:

$$\frac{E_1}{E_2} = open, \quad \frac{E_2}{E_3} = 1.0, \quad \frac{G_{12}}{E_2} = \frac{G_{13}}{E_2} = 0.6$$

$$\frac{G_{23}}{E_2} = 0.5, \quad \nu_{12} = \nu_{13} = 0.25$$

The “*nondimensional*” critical stability forces for isotropic, transversely isotropic, orthotropic and symmetric cross-ply stratified composite plates are provided in Tables 1-5 and their changes versus the “a/b ratio, a/h ratio, and the degree of orthotropy E_1/E_2 of the individual plies are shown in Figs. 2-6.

In Table 1, a comparison of the dimensionless critical stability force (N_{cr}) for isotropic and rectangular plates subjected to uni-axial compression along x-axis (1, 0), uni-axial compression along y-axis (0,1) and biaxial compression (1, 1) is demonstrated in Table 1. The numerical results are provided for different ratios of both (a/h) and (b/a) ratios. By examining Table 1, we found that the results computed by the current model are in well agreement with those of the “HSDT” and the “FSDT” for all the geometric ratios (a/h) and (b/a). The CPT over-predicts the critical stability forces when compared to

Table 1 Comparison of non-dimensional stability forces ($N_{Cr} = a^2 N_0 / E h^3$), for "*isotropic plates*" under uni-axial and biaxial compressions

(k_1, k_2)	a/h	Theory	Model	Non-dimensional critical stability force (N_{Cr}) (b/a) 1.0	1.5	2.0	2.5	3.0	3.5	4.0
(1,0)	5	Present	03-variable	2.9512	1.6431	1.2558	1.0883	1.0008	0.9494	0.9165
		Sayyad and Ghugal (2014)	TSDT	3.0266	1.6541	1.2599	1.0936	1.0078	0.9575	0.9255
		Reddy (1984)	HSDT	2.9512	1.6219	1.2379	1.0756	0.9917	0.9425	0.9112
		Mindlin (1951)	FSDT	2.9496	1.6215	1.2377	1.0754	0.9915	0.9424	0.9111
		Kirchhoff	CPT	3.6152	1.8857	1.4122	1.2161	1.1158	1.0574	1.0203
	10	Present	03-variable	3.4224	1.8182	1.3694	1.1813	1.0846	1.0281	0.9922
		Sayyad and Ghugal (2014)	TSDT	3.4541	1.8258	1.3738	1.1857	1.0891	1.0328	0.9971
		Reddy (1984)	HSDT	3.4224	1.8120	1.3641	1.1777	1.0819	1.0261	0.9906
		Mindlin (1951)	FSDT	3.4222	1.8119	1.3641	1.1776	1.0819	1.0261	0.9906
		Kirchhoff	CPT	3.6152	1.8857	1.4122	1.2161	1.1158	1.0574	1.0203
	20	Present	03-variable	3.5650	1.8684	1.4012	1.2073	1.1078	1.0499	1.0131
		Sayyad and Ghugal (2014)	TSDT	3.5821	1.8749	1.4058	1.2113	1.1117	1.0537	1.0169
		Reddy (1984)	HSDT	3.5649	1.8667	1.3999	1.2063	1.1071	1.0494	1.0127
		Mindlin (1951)	FSDT	3.5649	1.8667	1.3999	1.2063	1.1071	1.0494	1.0127
		Kirchhoff	CPT	3.6152	1.8857	1.4122	1.2161	1.1158	1.0574	1.0203
	50	Present	03-variable	3.6071	1.8829	1.4104	1.2147	1.1145	1.0562	1.0192
		Sayyad and Ghugal (2014)	TSDT	3.6213	1.8897	1.4154	1.2190	1.1185	1.0599	1.0228
		Reddy (1984)	HSDT	3.6071	1.8827	1.4100	1.2145	1.1143	1.0559	1.0189
		Mindlin (1951)	FSDT	3.6071	1.8827	1.4103	1.2146	1.1145	1.0561	1.0190
		Kirchhoff	CPT	3.6152	1.8857	1.4122	1.2161	1.1158	1.0574	1.0203
	100	Present	03-variable	3.6132	1.8850	1.4118	1.2158	1.1155	1.0571	1.0200
		Sayyad and Ghugal (2014)	TSDT	3.6256	1.8913	1.4165	1.2198	1.1192	1.0606	1.0234
		Reddy (1984)	HSDT	3.6130	1.8847	1.4116	1.2153	1.1152	1.0567	1.0195
		Mindlin (1951)	FSDT	3.6130	1.8851	1.4116	1.2157	1.1152	1.0571	1.0201
		Kirchhoff	CPT	3.6152	1.8857	1.4122	1.2161	1.1158	1.0574	1.0203
(0,1)	5	Present	03-variable	2.9512	3.6969	5.0234	6.8020	9.0075	11.6299	14.6643
		Sayyad and Ghugal (2014)	TSDT	3.0266	3.7216	5.0395	6.8352	9.0699	11.729	16.269
		Reddy (1984)	HSDT	2.9512	3.6494	4.9518	6.7228	8.9254	11.546	14.580
		Mindlin (1951)	FSDT	2.9496	3.6485	4.9508	6.7216	8.9239	11.545	14.578
		Kirchhoff	CPT	3.6152	4.2429	5.6488	7.6010	10.042	12.953	16.325
	10	Present	03-variable	3.4224	4.0911	5.4774	7.3833	9.7610	12.5938	15.8744
		Sayyad and Ghugal (2014)	TSDT	3.4541	4.1081	5.4951	7.4104	9.8021	12.652	15.953
		Reddy (1984)	HSDT	3.4224	4.0769	5.4566	7.3603	9.7373	12.570	15.850
		Mindlin (1951)	FSDT	3.4222	4.0768	5.4565	7.3603	9.7373	12.570	15.850
		Kirchhoff	CPT	3.6152	4.2429	5.6488	7.6010	10.042	12.953	16.325
	20	Present	03-variable	3.5650	4.2038	5.6049	7.5454	9.9704	12.8613	16.2099
		Sayyad and Ghugal (2014)	TSDT	3.5821	4.2185	5.6231	7.5707	10.005	12.907	16.269
		Reddy (1984)	HSDT	3.5649	4.2001	5.5994	7.5392	9.9642	12.855	16.204
		Mindlin (1951)	FSDT	5.5649	4.2002	5.5994	7.5394	9.9643	12.855	16.204
		Kirchhoff	CPT	3.6152	5.2429	5.6488	7.6010	10.042	12.953	16.325

Table 1 Continued

(k_1, k_2)	a/h	Theory	Model	Non-dimensional critical stability force (N_{Cr}) (b/a) 1.0	1.5	2.0	2.5	3.0	3.5	4.0
(0,1)	50	Present	03-variable	3.6071	4.2366	5.6417	7.5921	10.0308	12.9383	16.3065
		Sayyad and Ghugal (2014)	TSDT	3.6213	4.2519	5.6601	7.6188	10.066	12.984	16.365
		Reddy (1984)	HSDT	3.6068	4.2359	5.6402	7.5904	12.028	12.935	16.303
		Mindlin (1951)	FSDT	3.6071	4.2360	5.6412	7.5914	10.030	12.937	16.305
		Kirchhoff	CPT	3.6152	4.2429	5.6488	7.6010	10.042	12.953	16.325
	100	Present	03-variable	3.6132	4.2413	5.6470	7.5988	10.0394	12.9494	16.3204
		Sayyad and Ghugal (2014)	TSDT	3.6256	4.2555	5.6658	7.6240	10.073	12.992	16.375
		Reddy (1984)	HSDT	6.6130	4.2407	5.6466	7.5954	10.037	12.944	16.312
		Mindlin (1951)	FSDT	3.6130	4.2414	5.6465	7.5983	10.037	12.950	16.321
		Kirchhoff	CPT	3.6152	4.2429	5.6488	7.6010	10.042	12.953	16.325
(1,1)	5	Present	03-variable	1.4756	1.1375	1.0047	0.9382	0.9008	0.8773	0.8626
		Sayyad and Ghugal (2014)	TSDT	1.5133	1.1451	1.0079	0.9428	0.9070	0.8853	0.8711
		Reddy (1984)	HSDT	1.4756	1.1229	0.9904	0.9273	0.8925	0.8714	0.8577
		Mindlin (1951)	FSDT	1.4748	1.1226	0.9902	0.9271	0.8924	0.8713	0.8575
		Kirchhoff	CPT	1.8076	1.3055	1.1298	1.0484	1.0042	0.9776	0.9603
	10	Present	03-variable	1.7112	1.2588	1.0955	1.0184	0.9761	0.9508	0.9338
		Sayyad and Ghugal (2014)	TSDT	1.7271	1.2640	1.0990	1.0221	0.9802	0.9549	0.9384
		Reddy (1984)	HSDT	1.7112	1.2544	1.0913	1.0152	0.9737	0.9487	0.9324
		Mindlin (1951)	FSDT	1.7111	1.2544	1.0913	1.0152	0.9737	0.9487	0.9324
		Kirchhoff	CPT	1.8076	1.3055	1.1298	1.0442	1.0042	0.9776	0.9603
	20	Present	03-variable	1.7825	1.2935	1.1210	1.0407	0.9970	0.9707	0.9535
		Sayyad and Ghugal (2014)	TSDT	1.7910	1.2980	1.1246	1.0459	1.0005	0.9742	0.9570
		Reddy (1984)	HSDT	1.7825	1.2923	1.1199	1.0399	0.9964	0.9702	0.9532
		Mindlin (1951)	FSDT	1.7825	1.2924	1.1199	1.0399	0.9964	0.9702	0.9532
		Kirchhoff	CPT	1.8076	1.3055	1.1298	1.0484	1.0042	0.9776	0.9603
	50	Present	03-variable	1.8036	1.3036	1.1283	1.0472	1.0031	0.9765	0.9521
		Sayyad and Ghugal (2014)	TSDT	1.8106	1.3083	1.1323	1.0509	1.0066	0.9799	0.9626
		Reddy (1984)	HSDT	1.8035	1.3033	1.1280	1.0469	1.0028	0.9762	0.9590
		Mindlin (1951)	FSDT	1.8035	1.3034	1.1282	1.0471	1.0030	0.9764	0.9591
		Kirchhoff	CPT	1.8076	1.3055	1.1298	1.0484	1.0042	0.9776	0.9632
	100	Present	03-variable	1.8066	1.3050	1.1294	1.0481	1.0039	0.9773	0.9600
		Sayyad and Ghugal (2014)	TSDT	1.8126	1.3094	1.1332	1.0516	1.0073	0.9806	0.9632
		Reddy (1984)	HSDT	1.8065	1.3048	1.1293	1.0476	1.0037	0.9769	0.9595
		Mindlin (1951)	FSDT	1.8065	1.3050	1.1293	1.0480	1.0037	0.9774	0.9601
		Kirchhoff	CPT	1.8076	1.3055	1.1298	1.0484	1.0042	0.9776	0.9603

results given by the present original three variable "*refined*" plate theory and other "*refined theories*" because of neglecting of the influence of shear deformation in the CPT. It is remarked from the found results that when the structure is loaded by an uni-axial compression in the x-axis and bi-axial compression, the critical stability force increases with increasing a/h and decreases with the increase of b/a. When the isotropic structure is loaded by an uni-axial compression in they-axis, the critical stability force increases with increasing a/h and b/a.

The changes of the non-dimensional critical stability force (N_{cr}) versus the geometric ratio b/a for the isotropic plate via the current model are demonstrated in Fig. 2.

Table 2 presents the dimensionless critical stability forces (N_{cr}) provided by current proposed model, TSDT, HSDT, FSDT and CPT for "*transversely isotropic*" plates loaded by uni-axial and bi-axial compressions. It can be deduced from this example that the values of N_{cr}

Table 2 Comparison of non-dimensional stability forces ($N_{Cr} = a^2 N_0 / Q_{11} h^3$), for "*transversely isotropic plates*" under uni-axial and biaxial compressions

				Non-dimensional critical stability force (N_{Cr})						
				(*b*/*a*)						
(k_1, k_2)	*a*/*h*	Theory	Model	1.0	1.5	2.0	2.5	3.0	3.5	4.0
(1,0)	5	Present	03-variable	2.1600	1.2775	0.9967	0.8693	0.8013	0.7608	0.7348
		Sayyad and Ghugal (2014)	TSDT	2.1559	1.2455	0.9685	0.8490	0.7866	0.7498	0.7263
		Reddy (1984)	HSDT	2.1600	1.2449	0.9679	0.8486	0.7863	0.7496	0.7261
		Mindlin (1951)	FSDT	2.2868	1.3032	1.0086	0.8823	0.8164	0.7777	0.7530
		Kirchhoff	CPT	3.2899	1.7160	1.2851	1.1067	1.0154	0.9622	0.9285
	10	Present	03-variable	2.9078	1.5794	1.1977	1.0355	0.9514	0.9022	0.8708
		Sayyad and Ghugal (2014)	TSDT	2.8970	1.5585	1.1800	1.0215	0.9398	0.8921	0.8619
		Reddy (1984)	HSDT	2.9078	1.5672	1.1875	1.0293	0.9463	0.8983	0.8679
		Mindlin (1951)	FSDT	2.9648	1.5901	1.2027	1.0405	0.9571	0.9084	0.8774
		Kirchhoff	CPT	3.2899	1.7160	1.2851	0.1067	1.0154	0.9622	0.9285
	20	Present	03-variable	3.1851	1.6796	1.2620	1.0880	0.9986	0.9465	0.9134
		Sayyad and Ghugal (2014)	TSDT	3.1545	1.6589	1.2459	1.0744	0.9864	0.9352	0.9027
		Reddy (1984)	HSDT	3.1851	1.6762	1.2592	1.0860	0.9972	0.9454	0.9125
		Mindlin (1951)	FSDT	3.2021	1.6827	1.2635	1.0894	1.0002	0.9482	0.9152
		Kirchhoff	CPT	3.2899	1.7160	1.2851	1.1067	1.0154	0.9622	0.9285
	50	Present	03-variable	3.2726	1.7101	1.2814	1.1037	1.0127	0.9597	0.9260
		Sayyad and Ghugal (2014)	TSDT	3.2338	1.6890	1.2655	1.0900	1.0002	0.9479	0.9147
		Reddy (1984)	HSDT	3.2726	1.7094	1.2809	1.1033	1.0124	0.9595	0.9259
		Mindlin (1951)	FSDT	3.2755	1.7107	1.2817	1.1039	1.0130	0.9600	0.9264
		Kirchhoff	CPT	3.2899	1.7160	1.2851	1.1067	1.0154	0.9622	0.9285
	100	Present	03-variable	3.2855	1.7145	1.2842	1.1059	1.0147	0.9616	0.9279
		Sayyad and Ghugal (2014)	TSDT	3.2454	1.6934	1.2683	1.0923	1.0022	0.9498	0.9165
		Reddy (1984)	HSDT	3.2853	1.7140	1.2840	1.1057	1.0144	0.9615	0.9279
		Mindlin (1951)	FSDT	3.2864	1.7148	1.2846	1.1064	1.0150	0.9616	0.9281
		Kirchhoff	CPT	3.2899	1.7160	1.2851	1.1067	1.0154	0.9622	0.9285
(0,1)	5	Present	03-variable	2.1600	2.8744	3.9867	5.4328	7.2113	9.3198	11.7569
		Sayyad and Ghugal (2014)	TSDT	2.1559	2.8023	3.8739	5.3062	7.0791	9.1851	11.621
		Reddy (1984)	HSDT	2.1600	2.8010	3.8717	5.3036	7.0763	9.1822	11.617
		Mindlin (1951)	FSDT	2.2868	2.9321	4.0344	5.5141	7.3779	9.5272	12.048
		Kirchhoff	CPT	3.2899	3.8610	5.1404	6.9169	9.1385	11.787	14.855
	10	Present	03-variable	2.9078	3.5536	4.7910	6.4719	8.5630	11.0520	13.9335
		Sayyad and Ghugal (2014)	TSDT	2.8970	3.5066	4.7202	6.3841	8.4580	10.928	13.787
		Reddy (1984)	HSDT	2.9078	3.5263	4.7501	6.4269	8.5164	11.004	13.885
		Mindlin (1951)	FSDT	2.9648	3.5777	4.8107	6.5033	6.6138	11.127	14.038
		Kirchhoff	CPT	3.2899	3.8610	5.1404	6.9169	9.1385	11.787	14.855
	20	Present	03-variable	3.1851	3.7791	5.0482	6.7998	8.9873	11.5942	14.6137
		Sayyad and Ghugal (2014)	TSDT	3.1545	3.7325	4.9835	6.7148	8.8778	11.456	14.442
		Reddy (1984)	HSDT	3.1851	3.7714	5.0369	6.7875	8.9745	11.581	14.600
		Mindlin (1951)	FSDT	3.2021	3.7861	5.0539	6.8088	9.0016	11.615	14.642
		Kirchhoff	CPT	3.2899	3.8610	5.1404	6.9169	9.1385	11.787	14.855

Table 2 Continued

(k_1, k_2)	a/h	Theory	Model	Non-dimensional critical stability force (N_{Cr}) (b/a) 1.0	1.5	2.0	2.5	3.0	3.5	4.0
(0,1)	50	Present	03-variable	3.2726	3.8477	5.1254	6.8979	9.1140	11.7560	14.8165
		Sayyad and Ghugal (2014)	TSDT	3.2338	3.8003	5.0619	6.8128	9.0020	11.612	14.635
		Reddy (1984)	HSDT	3.2726	3.8462	5.1236	6.8958	9.1118	11.753	14.814
		Mindlin (1951)	FSDT	3.2755	3.8490	5.1267	6.8995	9.1169	11.759	14.822
		Kirchhoff	CPT	3.2899	3.8610	5.1404	6.9169	9.1385	11.787	14.855
	100	Present	03-variable	3.2855	3.8577	5.1367	6.9122	9.1324	11.7794	14.8460
		Sayyad and Ghugal (2014)	TSDT	3.2454	3.8102	5.0733	6.8270	9.0199	11.634	14.663
		Reddy (1984)	HSDT	3.2853	3.8566	5.1360	6.9108	9.1298	11.778	14.845
		Mindlin (1951)	FSDT	3.2864	3.8582	5.1385	6.9153	9.1347	11.779	14.849
		Kirchhoff	CPT	3.2899	3.8610	5.1404	6.9169	9.1385	11.787	14.855
(1,1)	5	Present	03-variable	1.0800	0.8844	0.7973	0.7494	0.7211	0.7034	0.6916
		Sayyad and Ghugal (2014)	TSDT	1.0779	0.8622	0.7748	0.7319	0.7079	0.6932	0.6836
		Reddy (1984)	HSDT	1.0800	0.8618	0.7743	0.7315	0.7076	0.6930	0.6834
		Mindlin (1951)	FSDT	1.1434	0.9022	0.8069	0.7606	0.7348	0.7190	0.7087
		Kirchhoff	CPT	1.6449	1.1880	1.0281	0.9541	0.9139	0.8896	0.8739
	10	Present	03-variable	1.4539	1.0934	0.9582	0.8927	0.8563	0.8341	0.8196
		Sayyad and Ghugal (2014)	TSDT	1.4485	1.0790	0.9440	0.8806	0.8458	0.8248	0.8111
		Reddy (1984)	HSDT	1.4539	1.0850	0.9500	0.8865	0.8516	0.8305	0.8168
		Mindlin (1951)	FSDT	1.4824	1.1008	0.9621	0.8970	0.8614	0.8398	0.8258
		Kirchhoff	CPT	1.6449	1.1880	1.0281	0.9541	0.9139	0.8896	0.8739
	20	Present	03-variable	1.5926	1.1628	1.0096	0.9379	0.8987	0.8750	0.8596
		Sayyad and Ghugal (2014)	TSDT	1.5772	1.1484	0.9967	0.9262	0.8878	0.8646	0.8496
		Reddy (1984)	HSDT	1.5926	1.1604	1.0074	0.9362	0.8974	0.8741	0.8589
		Mindlin (1951)	FSDT	1.6011	1.1650	1.0108	0.9391	0.9002	0.8766	0.8613
		Kirchhoff	CPT	1.6449	0.1880	1.0281	0.9541	0.9139	0.8896	0.8739
	50	Present	03-variable	1.6363	1.1839	1.0251	0.9514	0.9114	0.8872	0.8716
		Sayyad and Ghugal (2014)	TSDT	1.6169	1.1693	1.0124	0.9397	0.9002	0.8764	0.8609
		Reddy (1984)	HSDT	1.6363	1.1835	1.0247	0.9511	0.9112	0.8871	0.8714
		Mindlin (1951)	FSDT	1.6378	1.1843	1.0253	0.9517	0.9117	0.8875	0.8719
		Kirchhoff	CPT	1.6449	1.1880	1.0281	0.9541	0.9139	0.8896	0.8739
	100	Present	03-variable	1.6428	1.1870	1.0273	0.9534	0.9132	0.8890	0.8733
		Sayyad and Ghugal (2014)	TSDT	1.6227	1.1724	1.0147	0.9417	0.9020	0.8781	0.8626
		Reddy (1984)	HSDT	1.6427	1.1866	1.0272	0.9532	0.9130	0.8889	0.8733
		Mindlin (1951)	FSDT	1.6432	1.1871	1.0277	0.9538	0.9135	0.8890	0.8735
		Kirchhoff	CPT	1.6449	1.1880	1.0281	0.9541	0.9139	0.8896	0.9739

determined by the proposed "*three-variable*" plate theory, TSDT, HSDT and FSDT are in very good agreement for all geometric ratios (a/h and b/a), whereas CPT over-predicts the buckling forces.

Fig. 3 gives the changes of dimensionless critical stability force versus the b/a for "*transversely isotropic*" plate by employing the proposed formulation. It is observed that the tendency of the response of buckling in this case is the same as for the "*isotropic*" plates.

Table 3 presents a comparison of the dimensionless critical stability forces (N_{cr}) for square "*orthotropic plates*" loaded by uni-axial and bi-axial compression for different geometric and material ratios (a/h and E_1/E_2). For all ratios of (a/h) and (E_1/E_2), the dimensionless critical stability forces provided by the present model are in "*excellent agreement*" with those reported by RPT. It is remarked that the difference between the stability force of the present method and the "CPT" diminishes with

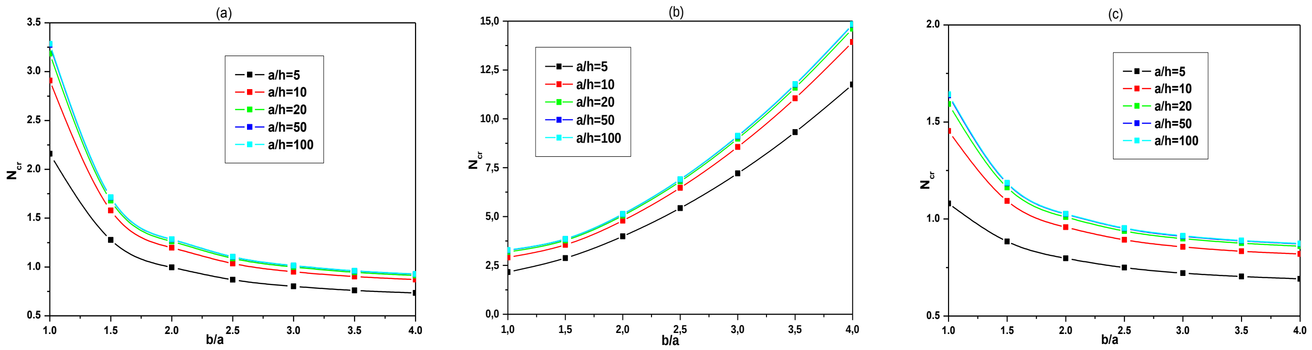


Fig. 3 Variations of non-dimensional critical stability force versus (b/a) ratio for "*transversely isotropic plate*" using current theory: (a) uni-axial compression along "*x-direction*"; (b) uni-axial compression along "*y-direction*"; and (c) biaxial compression

Table 3 Comparison of non-dimensional stability force ($N_{Cr} = a^2 N_0 / E_2 h^3$), for "*orthotropic square plates*" under uni-axial and biaxial compressions

				Non-dimensional critical stability force (N_{Cr})				
				Modular ratio (E_1/E_2)				
(k_1, k_2)	a/h	Theory	Model	3	10	20	30	40
(1,0)	5	Present	03-variable	3.9587	6.3478	8.3967	9.6821	10.578
		Sayyad and Ghugal (2014)	TSDT	4.0062	6.3018	7.9324	8.8418	9.4502
		Bourada *et al*. (2016)	RPT	-	6.3478	-	-	10.579
		Reddy (1984)	HSDT	3.9434	6.2072	7.8292	8.7422	9.3472
		Mindlin (1951)	FSDT	3.9386	6.1804	7.7450	8.5848	9.1084
		Kirchhoff	CPT	5.4248	11.163	19.383	27.606	35.830
	10	Present	03-variable	4.9837	9.3732	14.5633	18.7723	22.2581
		Sayyad and Ghugal (2014)	TSDT	4.9817	9.3189	14.064	17.661	20.490
		Bourada *et al*. (2016)	RPT	-	9.3732	-	-	22.258
		Reddy (1984)	HSDT	4.9568	9.2772	14.001	17.577	20.386
		Mindlin (1951)	FSDT	4.9562	9.2734	13.982	17.532	20.304
		Kirchhoff	CPT	5.4248	11.163	19.383	27.606	35.830
	20	Present	03-variable	5.3016	10.6534	17.8981	24.6897	31.0685
		Sayyad and Ghugal (2014)	TSDT	5.3098	10.635	17.687	24.140	30.067
		Bourada *et al*. (2016)	RPT	-	10.653	-	-	31.069
		Reddy (1984)	HSDT	5.2994	10.621	17.664	24.108	30.025
		Mindlin (1951)	FSDT	5.2994	10.620	17.662	24.102	30.014
		Kirchhoff	CPT	5.4248	11.163	19.383	27.606	35.830
	50	Present	03-variable	5.4047	11.0780	19.1292	27.0940	34.9717
		Sayyad and Ghugal (2014)	TSDT	5.4099	11.078	19.088	26.979	34.750
		Bourada *et al*. (2016)	RPT	-	11.078	-	-	34.972
		Reddy (1984)	HSDT	5.4040	11.072	19.085	26.976	34.748
		Mindlin (1951)	FSDT	5.4046	11.072	19.085	29.976	34.748
		Kirchhoff	CPT	5.4248	11.163	19.383	27.606	35.830
	100	Present	03-variable	5.4197	11.1415	19.3192	27.4767	35.5591
		Sayyad and Ghugal (2014)	TSDT	5.4194	11.140	19.310	27.450	35.562
		Bourada *et al*. (2016)	RPT	-	11.142	-	-	35.559
		Reddy (1984)	HSDT	5.4192	11.139	19.307	27.466	35.553
		Mindlin (1951)	FSDT	5.4206	11.142	19.309	27.448	35.554
		Kirchhoff	CPT	5.4248	11.163	19.383	27.606	35.830

Table 3 Continued

				Non-dimensional critical stability force (N_{Cr})				
				Modular ratio (E_1/E_2)				
(k_1, k_2)	a/h	Theory	Model	3	10	20	30	40
(1,1)	5	Present	03-variable	1.9793	3.1739	4.1984	4.8411	5.2892
		Sayyad and Ghugal (2014)	TSDT	2.0031	3.1509	3.9662	4.4209	4.7251
		Bourada *et al*. (2016)	RPT	-	3.1739	-	-	5.2895
		Reddy (1984)	HSDT	1.9717	3.1036	3.9146	4.3711	4.6736
		Mindlin (1951)	FSDT	1.9693	3.0902	3.8725	4.2924	4.5542
		Kirchhoff	CPT	2.7124	5.5814	9.6917	13.8034	17.9154
	10	Present	03-variable	2.4818	4.6866	7.2816	9.3862	11.1291
		Sayyad and Ghugal (2014)	TSDT	2.4909	4.6595	7.0322	8.8308	10.2454
		Bourada *et al*. (2016)	RPT	-	4.6866	-	-	11.1290
		Reddy (1984)	HSDT	2.4784	4.6486	7.0002	8.7885	10.1929
		Mindlin (1951)	FSDT	2.4781	4.6367	6.9910	8.7662	10.1522
		Kirchhoff	CPT	2.7124	5.5814	9.6917	13.8034	17.9154
	20	Present	03-variable	2.6508	5.3267	8.9490	12.3448	15.5343
		Sayyad and Ghugal (2014)	TSDT	2.6549	5.3178	8.8439	1.0703	15.0336
		Bourada *et al*. (2016)	RPT	-	5.2265	-	-	15.5345
		Reddy (1984)	HSDT	2.6497	5.3101	8.8320	12.0540	15.0127
		Mindlin (1951)	FSDT	2.6497	5.3100	8.8311	12.0513	15.0070
		Kirchhoff	CPT	2.7124	5.5814	9.6917	13.8034	17.7154
	50	Present	03-variable	2.7023	5.5390	9.5646	13.5470	17.4859
		Sayyad and Ghugal (2014)	TSDT	2.7059	5.5390	9.5463	13.4932	17.3801
		Bourada *et al*. (2016)	RPT	-	5.5390	-	-	17.4860
		Reddy (1984)	HSDT	2.7020	5.5360	9.5424	13.4884	17.3744
		Mindlin (1951)	FSDT	2.7023	5.5362	9.5425	13.4885	17.3745
		Kirchhoff	CPT	2.7124	5.5814	2.6917	13.8034	17.9154
	100	Present	03-variable	2.7099	5.5707	9.6596	13.7384	17.8060
		Sayyad and Ghugal (2014)	TSDT	2.7124	5.5723	9.6571	13.7269	17.7811
		Bourada *et al*. (2016)	RPT	-	5.5710	-	-	17.8060
		Reddy (1984)	HSDT	2.7096	5.5697	9.6533	13.7230	17.7767
		Mindlin (1951)	FSDT	2.7103	5.5710	9.6544	13.7241	17.7772
		Kirchhoff	CPT	2.7124	5.5814	9.6917	13.8034	17.7154

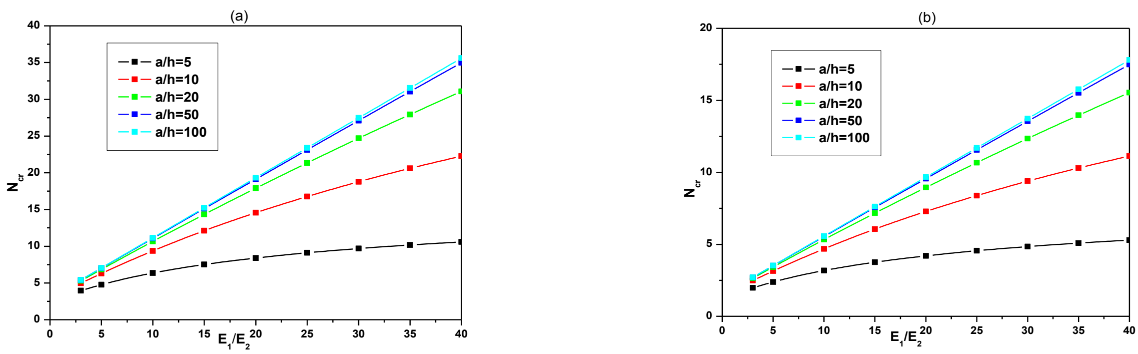


Fig. 4 Variations of non-dimensional critical stability force versus (E_1/E_2) ratio for square "*orthotropic plate*" using current theory: (a) uni-axial compression; and (b) biaxial compression

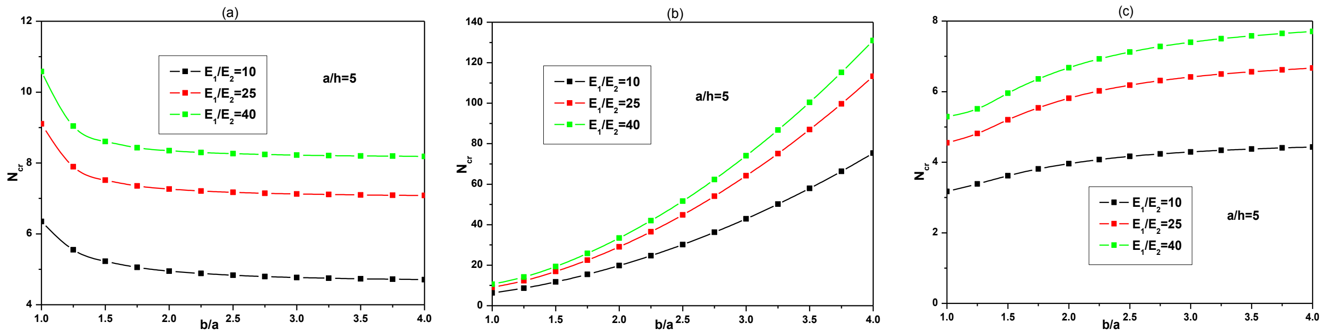


Fig. 5 Variations of non-dimensional critical buckling load versus (b/a) ratio for "*orthotropic plate*" using current theory: (a) uni-axial compression along "*x-direction*"; (b) uni-axial compression along "*y-direction*"; and (c) biaxial compression

Table 4 Comparison of non-dimensional critical stability forces ($N_{Cr} = a^2 N_0 / E_2 h^3$) of "*orthotropic plates*" (*a*/*h* = 5) under uni-axial and biaxial compressions

				Non-dimensional critical stability force (N_{Cr})						
				(*b*/*a*)						
(k_1, k_2)	E_1/E_2	Theory	Model	1.0	1.5	2.0	2.5	3.0	3.5	4.0
(1,0)	10	Present	03-variable	6.3478	5.2255	4.9502	4.8323	4.7700	4.7328	4.7088
		Sayyad and Ghugal (2014)	TSDT	6.301	5.287	4.994	4.868	4.803	4.765	4.741
		Reddy (1984)	HSDT	6.207	5.224	4.941	4.822	4.761	4.725	4.702
		Bourada et al. (2016)	RPT	6.347	-	5.011	-	-	-	-
		Mindlin (1951)	FSDT	6.180	5.202	4.920	4.802	4.741	4.705	4.683
		Kirchhoff	CPT	11.16	9.355	8.843	8.627	8.515	8.450	8.408
	25	Present	03-variable	9.1039	7.5142	7.2632	7.1722	7.1265	7.0997	7.0826
		Sayyad and Ghugal (2014)	TSDT	8.439	7.541	7.293	7.191	7.139	7.109	7.0905
		Reddy (1984)	HSDT	8.339	7.493	7.263	7.170	7.123	7.096	7.079
		Bourada et al. (2016)	RPT	9.103	-	7.541	-	-	-	-
		Mindlin (1951)	FSDT	8.219	7.380	7.153	7.061	7.015	6.988	6.971
		Kirchhoff	CPT	23.49	21.69	21.18	20.96	20.85	20.78	20.74
	40	Present	03-variable	10.5785	8.6039	8.3466	8.2632	8.2233	8.2003	8.1857
		Sayyad and Ghugal (2014)	TSDT	9.450	8.601	8.372	8.279	8.232	8.205	8.188
		Reddy (1984)	HSDT	9.347	8.554	8.345	8.262	8.221	8.198	8.183
		Bourada et al. (2016)	RPT	10.57	-	8.758	-	-	-	-
		Mindlin (1951)	FSDT	9.108	8.323	8.117	8.036	7.995	7.972	7.958
		Kirchhoff	CPT	35.83	34.02	33.52	33.30	33.18	33.12	33.08
(0,1)	10	Present	03-variable	6.3478	11.7574	19.8007	30.2021	42.9295	57.9763	75.3406
		Sayyad and Ghugal (2014)	TSDT	6.301	11.907	19.975	30.426	43.229	58.375	75.859
		Reddy (1984)	HSDT	6.207	11.755	19.765	30.139	42.849	57.885	75.242
		Bourada et al. (2016)	RPT	6.347	-	20.044	-	-	-	-
		Mindlin (1951)	FSDT	6.180	11.705	19.682	30.013	42.670	57.644	74.929
		Kirchhoff	CPT	11.16	21.048	35.371	53.918	76.638	103.51	134.53
	25	Present	03-variable	9.1039	16.9070	29.0527	44.8262	64.1383	86.9716	113.3214
		Sayyad and Ghugal (2014)	TSDT	8.439	16.968	29.171	44.943	64.253	87.089	113.44
		Reddy (1984)	HSDT	8.339	16.859	29.052	44.813	64.110	86.931	113.27
		Bourada et al. (2016)	RPT	9.103	-	30.164	-	-	-	-
		Mindlin (1951)	FSDT	8.219	16.606	28.611	44.131	63.132	85.604	111.54
		Kirchhoff	CPT	23.49	48.803	84.716	131.02	187.66	254.63	331.92

Table 4 Continued

				Non-dimensional critical stability force (N_{Cr})						
				(b/a)						
(k_1, k_2)	E_1/E_2	Theory	Model	1.0	1.5	2.0	2.5	3.0	3.5	4.0
(0,1)	4	Present	03-variable	10.5785	19.3587	33.3863	51.6453	74.0095	100.4540	130.9719
		Sayyad and Ghugal (2014)	TSDT	9.450	19.353	33.487	51.744	74.092	100.52	131.02
		Reddy (1984)	HSDT	9.347	19.246	33.382	51.642	73.955	100.42	130.93
		Bourada et al. (2016)	RPT	10.57	-	35.034	-	-	-	-
		Mindlin (1951)	FSDT	9.108	18.728	32.471	50.226	71.962	97.667	127.33
		Kirchhoff	CPT	35.83	76.560	134.06	208.12	298.69	405.76	529.31
(1,1)	10	Present	03-variable	3.1739	3.6177	3.9601	4.1658	4.2929	4.3756	4.4318
		Sayyad and Ghugal (2014)	TSDT	3.1606	3.6639	3.9952	4.1967	4.3230	4.4057	4.4623
		Reddy (1984)	HSDT	3.1036	3.6170	3.9530	4.1571	4.2849	4.3687	4.4260
		Bourada et al. (2016)	RPT	3.1739	-	4.0087	-	-	-	-
		Mindlin (1951)	FSDT	3.0902	3.6017	3.9364	4.1398	4.2671	4.3505	4.4076
		Kirchhoff	CPT	5.5814	6.4765	7.0743	7.4371	7.6638	7.8122	7.9137
	25	Present	03-variable	4.5519	5.2022	5.8105	6.1829	6.4138	6.5639	6.6660
		Sayyad and Ghugal (2014)	TSDT	7.2199	5.2210	5.8343	6.1991	6.4253	6.5728	6.6734
		Reddy (1984)	HSDT	4.1697	5.1874	5.8105	6.1811	6.4110	6.5609	66631
		Bourada et al. (2016)	RPT	4.1619	-	6.0327	-	-	-	-
		Mindlin (1951)	FSDT	4.1099	5.1096	5.7224	6.0870	6.3132	6.4607	6.5613
		Kirchhoff	CPT	11.747	15.016	16.943	18.072	18.767	19.217	19.524
	40	Present	03-variable	5.2892	5.9565	6.6773	7.1235	7.4009	7.5814	7.7042
		Sayyad and Ghugal (2014)	TSDT	4.7251	5.9549	6.6975	7.1372	7.4092	7.5863	7.7071
		Reddy (1984)	HSDT	4.6736	5.9221	6.6764	7.1231	7.3995	7.5796	7.7023
		Bourada et al. (2016)	RPT	5.2895	-	7.0069	-	-	-	-
		Mindlin (1951)	FSDT	4.5542	5.7626	6.4942	6.9278	7.1963	7.3711	7.4903
		Kirchhoff	CPT	17.915	23.557	26.813	28.707	29.870	30.623	31.136

increasing the geometric ratio (a/h) and increases with the increase in the "*degree of orthotropy*" (E_1/E_2).

The changes of the critical stability force versus the material ratio (E_1/E_2) using the present "*three-variable*" plate theory are demonstrated in Fig. 4.

The dimensionless critical stability forces of rectangular "*orthotropic plates*" for a/h = 5 and E_1/E_2 = 10, 25, 40 are tabulated in Table 4. It is remarked, that the critical stability force is reduced with increasing the geometric ratios (b/a) when the structure is loaded by uni-axial compression along the "*x-axis*" and increased with increasing the material ratios when the structure is loaded by both uni-axial and bi-axial compressions. The critical stability forces predicted using the current formulation, RPT, HSDT, TSDT and FSDT are in good agreement with "*each other*" for all geometric ratios b/a, whereas the CPT overestimated the values. The changes of dimensionless critical stability forces versus the geometric ratio (b/a) for "*orthotropic plates*" are given in Fig. 5.

Table 5 presents a comparison of the critical stability forces computed by the proposed formulation, TSDT, FSDT and CPT for a "*symmetric cross-ply*" stratified composite plates loaded by uni-axial and bi-axial compressions for different geometric ratios (a/h and b/a) with a given material ratio E_1/E_2 = 40. For all geometric ratios, the TSDT is in good agreement with the current theory. The stability forces provided by the CPT are clearly higher than the values calculated by the present theory. This is the result of the neglect of the "*transverse shear deformation*" influence in the CPT. The changes of dimensionless critical stability forces versus the geometric ratio (b/a) for "*symmetric cross-ply*" stratified composite plates are presented in Fig. 6. It is remarked from Table 5 and Fig. 6 that the critical stability forces diminish with increasing the geometric ratio of plate in the case of uni-axial compression in the "*x-axis*" and increases with increasing the plate geometric ratio when it is loaded by uni-axial compression in the "*y-direction*" and bi-axial compression.

5. Conclusions

In this current work, we study the stability behavior of "*isotropic*", "*transversely isotropic*", "*orthotropic*", "*laminated composite plates*", based on a new and a simplest and accurate 3-unknowns "*refined*" plate theory.

Table 5 Comparison of non-dimensional stability forces ($N_{Cr} = a^2 N_0 / E_2 h^3$, for (0/90/90/0) "*symmetric cross-ply laminated plate*" under uni-axial and biaxial compressions with $E_1/E_2 = 40$

				Non-dimensional critical stability force (N_{Cr})					
				(*b*/*a*)					
(k_1, k_2)	*a*/*h*	Theory	Model	1.5	2.0	2.5	3.0	3.5	4.0
(1,0)	5	Present	03-variable	9.4910	8.7771	8.5589	8.4659	8.4182	8.3906
		Sayyad and Ghugal (2014)	TSDT	9.404	8.780	8.561	8.463	8.412	8.382
		Mindlin (1951)	FSDT	9.320	8.673	8.453	8.357	8.308	8.279
		Kirchhoff	CPT	30.950	29.833	29.440	29.259	29.161	29.102
	10	Present	03-variable	19.5638	18.4506	18.1214	17.9833	17.9124	17.8711
		Sayyad and Ghugal (2014)	TSDT	19.390	18.500	18.194	18.057	17.984	17.941
		Mindlin (1951)	FSDT	19.286	18.398	18.096	17.962	17.891	17.849
		Kirchhoff	CPT	30.950	29.833	29.440	29.259	29.161	29.102
	20	Present	03-variable	26.9925	25.8220	25.4430	25.2760	25.1876	25.1350
		Sayyad and Ghugal (2014)	TSDT	26.890	25.850	25.488	25.322	25.334	25.181
		Mindlin (1951)	FSDT	26.839	25.802	25.442	25.278	25.190	25.137
		Kirchhoff	CPT	30.950	29.833	29.440	29.259	29.161	29.102
	50	Present	03-variable	30.2399	29.1083	28.7169	28.5386	28.4424	28.3846
		Sayyad and Ghugal (2014)	TSDT	30.220	29.116	28.728	28.550	28.454	28.396
		Mindlin (1951)	FSDT	30.208	29.105	28.717	28.540	28.443	28.386
		Kirchhoff	CPT	30.950	29.833	29.440	29.259	29.161	29.102
	100	Present	03-variable	30.7698	29.6486	29.2558	29.0756	28.9782	28.9195
		Sayyad and Ghugal (2014)	TSDT	30.766	29.652	29.260	29.080	28.983	28.924
		Mindlin (1951)	FSDT	30.763	29.648	29.256	29.077	28.979	28.921
		Kirchhoff	CPT	30.950	29.833	29.440	29.259	29.161	29.102
(0,1)	5	Present	03-variable	21.3547	35.1085	53.4934	76.1930	103.122	134.2489
		Sayyad and Ghugal (2014)	TSDT	21.159	35.122	53.507	76.171	103.05	134.12
		Mindlin (1951)	FSDT	20.970	34.693	52.832	75.216	101.77	132.47
		Kirchhoff	CPT	69.639	119.33	183.99	263.33	357.22	465.63
	10	Present	03-variable	44.0185	73.8025	113.2585	161.8498	219.4273	285.9371
		Sayyad and Ghugal (2014)	TSDT	43.629	74.002	113.71	162.51	220.31	287.06
		Mindlin (1951)	FSDT	43.395	73.595	113.10	161.66	219.17	285.59
		Kirchhoff	CPT	69.939	119.33	193.99	263.33	357.22	465.63
	20	Present	03-variable	60.7354	103.2880	159.0187	227.4840	308.5485	402.1605
		Sayyad and Ghugal (2014)	TSDT	60.503	103.40	159.29	227.90	309.12	402.89
		Mindlin (1951)	FSDT	60.389	103.21	159.04	227.50	308.58	402.20
		Kirchhoff	CPT	69.639	119.33	183.99	263.33	357.22	465.63
	50	Present	03-variable	68.0398	116.4333	179.4809	256.8470	348.4198	348.4198
		Sayyad and Ghugal (2014)	TSDT	67.995	116.46	179.55	256.95	348.56	454.34
		Mindlin (1951)	FSDT	67.968	116.42	179.48	256.86	348.43	454.17
		Kirchhoff	CPT	69.639	119.33	183.99	263.33	357.22	465.63
	100	Present	03-variable	69.2321	118.5945	182.8487	261.6805	354.9832	462.7124
		Sayyad and Ghugal (2014)	TSDT	69.225	118.61	182.87	261.72	355.04	462.79
		Mindlin (1951)	FSDT	69.217	118.59	182.85	261.69	354.99	462.73
		Kirchhoff	CPT	69.639	119.33	183.99	263.33	357.22	465.63

Table 5 Continued

				Non-dimensional critical stability force (N_{Cr})					
				(b/a)					
(k_1, k_2)	a/h	Theory	Model	1.5	2.0	2.5	3.0	3.5	4.0
(1,0)	5	Present	03-variable	6.5707	7.0217	7.3784	7.6193	7.7828	7.8970
		Sayyad and Ghugal (2014)	TSDT	6.5107	7.0244	7.3803	7.6171	7.7775	7.8896
		Mindlin (1951)	FSDT	6.4524	6.9387	7.2872	7.5216	7.6811	7.7928
		Kirchhoff	CPT	21.427	23.866	25.379	26.333	26.960	27.390
	10	Present	03-variable	13.5441	14.7605	15.6219	16.1850	16.5606	16.8198
		Sayyad and Ghugal (2014)	TSDT	13.424	14.800	15.684	16.251	16.627	16.8863
		Mindlin (1951)	FSDT	13.352	14.719	15.600	16.166	16.541	16.799
		Kirchhoff	CPT	21.427	23.866	25.379	26.333	26.960	27.390
	20	Present	03-variable	18.6878	20.6576	21.9336	22.7484	23.2867	23.6565
		Sayyad and Ghugal (2014)	TSDT	18.616	20.680	21.972	22.790	23.329	23.700
		Mindlin (1951)	FSDT	18.581	20.642	21.932	22.750	23.289	23.659
		Kirchhoff	CPT	21.427	23.866	25.379	26.333	26.960	27.390
	50	Present	03-variable	20.9353	23.2867	24.7560	25.6847	26.2958	26.7149
		Sayyad and Ghugal (2014)	TSDT	20.921	23.293	24.766	25.695	26.307	26.726
		Mindlin (1951)	FSDT	20.913	23.284	24.756	25.686	26.297	26.716
		Kirchhoff	CPT	21.427	23.866	25.379	26.333	26.960	27.390
	100	Present	03-variable	21.3022	23.7189	25.2205	26.1680	26.7912	27.2184
		Sayyad and Ghugal (2014)	TSDT	21.300	23.722	25.224	26.172	26.795	27.223
		Mindlin (1951)	FSDT	21.297	23.718	25.220	26.169	26.792	27219
		Kirchhoff	CPT	21.427	23.866	25.379	26.333	26.960	27.390

The equilibrium equations are deduced by considering the “principle of virtual work”. A “*Navier-type*” analytical solution is determined and the study demonstrates that the proposed 3-unknowns “*refined*” plate model is able to provide accurate numerical results than the FSDT and other HSDTs. An improvement of the present formulation will be considered in the future work to consider other type of materials (Panda and Singh 2009, 2011, Benferhat *et al*. 2016, Kar *et al*. 2016, 2017, Kolahchi *et al*. 2016, Kar and Panda 2016, 2017, Fadoun *et al*. 2017, Panda *et al*. 2017, Katariya *et al*. 2017a, b, 2018, Eltaher *et al*. 2018, Panjehpour *et al*. 2018, Bensattalah *et al*. 2018, Karami *et al*. 2018, Selmi and Bisharat 2018, Belkacem *et al*. 2018, Shahadat *et al*. 2018, Akbaş 2019, Hussain and Naeem 2019, Avcar 2019, Hadji and Zouatnia 2019, Mehar and Panda 2019, Fenjan *et al*. 2019, Abualnour *et al*. 2019, Batou *et al*. 2019, Salah *et al*. 2019, Mehar *et al*. 2019).

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