Steel and Composite Structures, *Vol. 33, No. 1 (2019) 123-131*
DOI: https://doi.org/10.12989/scs.2019.33.1.123

Effect of nonlocal parameter on nonlocal thermoelastic solid due to inclined load

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(*Received May 23, 2019, Revised September 5, 2019, Accepted September 29, 2019*)

Abstract. The present investigation is concerned with two dimensional deformation in a homogeneous nonlocal thermoelastic solid with two temperature. The nonlocal thermoelastic solid is subjected to inclined load. Laplace and Fourier transforms are used to solve the problem. The bounding surface is subjected to concentrated and distributed sources. The analytical expressions of displacement, stress components, temperature change are obtained in the transformed domain. Numerical inversion technique has been applied to obtain the results in the physical domain. Numerical simulated results are depicted graphically to show the effect of angle of inclination and nonlocal parameter on the components of displacements, stresses and conductive temperature. Some special cases are also deduced from the present investigation.

Keywords: thermoelasticity; nonlocality; nonlocal theory of thermoelasticity; Eringen model of nonlocal theories; two temperature

1. Introduction

The theory of elasticity is concerned with the study of elastic properties of a material having the property that once the deformation forces are removed, the material recovers back its original shape and size. The deformations arising due to both mechanical and thermal causes were the reasons for the development of the subject of thermoelasticity. The nonlocal theory of thermoelasticity considers that the various physical quantities defined at a point are not just a function of the values of independent constitutive variables at that point only but a function of their values over the whole body. So the nonlocal stress forces can be termed as remote action forces. The nonlocal theory can be termed as a generalization of the classical field theory in two respects: (i) the energy balance law is valid for the whole body, and (ii) the state of the body at a material point is considered to be attracted by all points of the body. Nonlocal effects are dominant in nature. If the effects of strains at points other than the reference point are neglected, classical theory is recovered.

In the theory of thermoelasticity with two temperatures, the heat conduction depends upon the variations on two distinct temperatures termed as the conductive temperature and the thermodynamic temperature. In case of time-independent problems, the difference of these temperatures is proportionally equivalent to the heat supplied to the body but the conductive temperature and the thermodynamic temperature are the same if the body is not supplied any heat. In case of time dependent situations, regardless of the heat supply, the two temperatures are generally different.

The theory of nonlocal thermoelasticity was developed due to combined contribution of many researchers. Kroner (1967) developed a continuum theory for long range cohesive forces in elastic materials. He explained how the range effects can be important in materials having Vander Waals interactions as local theory gave a zero force. Edelen and Law (1971) discussed a theory of nonlocal interactions and agreed to the concept of nonlocality as suggested by Kroner. Edelen *et al*. (1971) discussed the consequences of global postulate of energy balance and obtained the constitutive equations for the nonlinear theory. They called this nonlinear theory of nonlocal elasticity as protoelasticity. The nonlocal elasticity theory was developed by Eringen and Edelen (1972) by making use of the global balance laws and the second law of thermodynamics. They proved that the stress field at a particular point is affected due to the strain at all the other point of the body also. Wang and Dhaliwal (1993) established a reciprocity relation and addressed certain issues addressing nonlocal thermo-elasticity. They extended the concept of nonlocality further to other fields. It was Artan (1996), who proved the superiority of the nonlocal theory by comparing the results of local and nonlocal elasticity theories to differentiate between the stress distributions of the local and the nonlocal theories. Marin (1996) derived the generalized solutions in elasticity and discussed their contributions on uniqueness in thermoelastodynamics.

Polizzotto (2001) assumed an attenuation function and used it to further refine the Eringen model of nonlocal elasticity theory. The attenuation function assumed by him was supposed to be dependent on the geodetical distance

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ISSN: 1229-9367 (Print), 1598-6233 (Online)

and not on the Eucledian distance between material particles. Eringen (2002) developed nonlocal continuum field theories for prevalent nonlocal intermolecular attractions in material bodies. He presented a unified approach to the development of the basic field equations for nonlocal continuum field theories. Sharma and Ganti (2003) described the size dependent elastic stress state of inclusions in nonlocal media. Paola *et al.* (2010) presented a mechanical based approach to three dimensional nonlocal elasticity theory and proved the high dependence of the results on the size. The model proposed by them yielded that the equilibrium is not attained just by contact forces at adjacent elements but also by long range forces due to non-adjacent elements. Simsek (2011) conducted a detailed parametric study for studying the influences of the nonlocal parameter of an embedded single walled carbon nanotube.

Khurana and Tomar (2013) studied the propagation of plane longitudinal waves through an isotropic nonlocal micropolar elastic medium and showed that four dispersive waves and two sets of coupled transverse waves may propagate. Belkorissat *et al.* (2015) presented a new nonlocal hyperbolic refined plate model for free vibration properties of functionally graded plates. Salehipour *et al.* (2015) gave a modified nonlocal elasticity theory stating that the strain tensor at all the neighbouring points contribute to the imaginary nonlocal strain tensor at a particular point. They used a nonlocal strain tensor for obtaining the nonlocal stress tensor. This assumed strain tensor was very similar to the strain tensor as used by Eringen in his nonlocal theory. Then, Vasiliev and Lurie (2016) developed a new nonlocal generalized theory. Using a variational approach, they developed a new variant of nonlocal elasticity theory for generalized stresses by introducing high gradient equilibrium equations. Marin and Nicaise (2016) derived existence and stability results for thermoelastic dipolar bodies with double porosity.

Khetir *et al.* (2017) proposed a new nonlocal trigonometric shear deformation theory. Khurana and Tomar (2017) studied the propagation of Rayleigh surface waves and explored the conditions for their existence. Bellifa *et al.* (2017) developed a nonlocal zeroth-order shear deformation theory. Singh *et al.* (2017) studied the propagation of plane harmonic waves and derived the governing relations in nonlocal elastic solid with voids. Marin *et al.* (2017) discussed various results and problems for elastic dipolar bodies. Othman and Marin (2017) studied the effect of thermal loading due to laser pulse on thermoelastic porous medium under G-N theory. Kaur *et al.* (2018) derived dispersion relation and investigated the propagation of Rayleigh type surface wave in nonlocal elastic solid.

Bachher and Sarkar (2018) postulated a new nonlocal theory of thermoelasticity, which is based on Eringen's nonlocal elasticity theory for thermoelastic materials with voids. A material is needed to be classified by its fractional and elastic nonlocality parameter according to this theory. Arefi (2018) studied nonlocal free vibration analysis of a doubly piezoelectric nanoshell. He employed nonlocal elasticity theory to derive governing equations. Karami *et al.* (2018) developed a three dimensional elasticity theory in conjunction with nonlocal strain gradient theory. Lata (2018a, b) studied the plane waves in a layered medium of two semi-infinite nonlocal solids with anisotropic thermoelasic medium. She also depicted the nonlocal parameter effects graphically. Benahmed *et al.* (2019) presented an efficient higher order nonlocal beam theory for the critical buckling of functionally graded nanobeams with porosities. Soleimani *et al.* (2019) investigated the effects of inevitable out of plane defects on the postbuckling behavior of single layered graphene sheets under in-plane loadings based on nonlocal first order shear deformation theory.

Chen and Gurtin (1968) developed a theory of heat conduction. They suggested that in case of bodies being deformable the said theory is dependent on two temperatures. Two distinct temperatures are known as: the thermodynamic temperature and the conductive temperature. Chen *et al.* (1969) suggested that the heat supplied is directly proportional to the difference between the thermodynamic temperature and the conductive temperature. A generalized two temperature theory was developed by Youssef (2005). He obtained the uniqueness theorem for equations of two temperature generalized thermoelasticity. Youssef and Al-Lehaibi (2007), after investigating various problems, gave an indication that the two temperature generalized thermoelasticity is more realistic in describing the state of an elastic body as compared to one temperature. Said and Othman (2016) applied a general model of equations of the two-temperature theory of generalized thermoelasticity to study the wave propagation in a fibre reinforced magneto-thermoelastic medium. Kumar *et al.* (2016a, b) studied the disturbances in a homogeneous transversely iosotropic thermoelastic rotating medium with two temperatures, in the presence of Hall currents and magnetic field due to thermomechanical sources. Sharma *et al.* (2015) carried the investigation regarding the two dimensional deformation in a transversely isotropic medium with two temperatures. The disturbances due to inclined load were studied along with graphical representations of the effects of two temperatures.

2. Basic equations

Following Youssef (2005) and Eringen (2002), the equations of motion and the constitutive relations in a homogeneous non local thermoelastic solid with two temperatures are given by

$$(\lambda + 2\mu)\nabla(\nabla.\boldsymbol{u}) - \mu\,(\nabla\times\nabla\times\boldsymbol{u}) - \beta\nabla\theta = (1-\epsilon^2\nabla^2)\rho\frac{\partial^2\boldsymbol{u}}{\partial t^2}, \tag{1}$$

$$K^*\nabla^2\varphi = \rho\, C^*\frac{\partial\theta}{\partial t} + \beta\theta_0\frac{\partial}{\partial t}(\nabla.u), \tag{2}$$

where

$$\theta = (1 - a\nabla^2)\,\varphi, \tag{3}$$

$$t_{ij} = \lambda u_{k,k}\delta_{ij} + \mu\left(u_{i,j} + u_{j,i}\right) - \beta\theta\delta_{ij}. \tag{4}$$

where λ, μ are material constants, ϵ is the nonlocal parameter, ρ is the mass density, $u = (u_1, 0, u_3)$ is the displacement vector, $\boldsymbol{\varphi}$ is the conductive temperature, a is two temperature parameter, θ is absolute temperature and θ_0 is reference temperature, K^* is the coefficient of the thermal conductivity, C^* the specific heat at constant strain, $\beta = (3\lambda + 2\mu)\alpha$, where α is coefficient of liner thermal expansion, e_{ij} are components of strain tensor, e_{kk} is the dilatation, δ_{ij} is the Kronecker delta, t_{ij} are the components of stress tensor.

3. Formulation of the problem

We consider a homogeneous non local isotropic thermoelastic solid in an initially undeformed state at temperature θ_0. For two dimensional problem, we take

$$\boldsymbol{u} = (u_1, 0, u_3). \tag{5}$$

Using Eq. (5) in Eqs. (1)-(2), yields

$$(\lambda+\mu)\frac{\partial e}{\partial x_1} + \mu\,\nabla^2 u_1 - \beta\frac{\partial\theta}{\partial x_1} = (1-\epsilon^2\nabla^2)\,\rho\frac{\partial^2 u_1}{\partial t^2}, \tag{6}$$

$$(\lambda+\mu)\frac{\partial e}{\partial x_3} + \mu\,\nabla^2 u_3 - \beta\frac{\partial\theta}{\partial x_3} = (1-\epsilon^2\nabla^2)\,\rho\frac{\partial^2 u_3}{\partial t^2}, \tag{7}$$

$$K^*\nabla^2\varphi = \rho\, C^*\frac{\partial\theta}{\partial t} + \beta\theta_0\frac{\partial e}{\partial t}. \tag{8}$$

where

$$e = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}, \qquad \nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}.$$

we define the following dimensionless quantities

$$\begin{aligned}
&(x_1', x_3') = \frac{\omega_1}{c_2}(x_1, x_3), (u_1', u_3') = \frac{\omega_1}{c_2}(u_1, u_3),\\
&t_{ij}' = \frac{t_{ij}}{\beta T_0}, t' = \omega_1 t, a' = \frac{\omega_1^2}{c_2^2}a, \mathrm{K}_n' = \frac{c_2}{\lambda\omega_1}K_n,\\
&F_1{}' = \frac{F_1}{\beta T_0} \text{ and } F_2{}' = \frac{F_2}{\beta T_0}.
\end{aligned} \tag{9}$$

where

$$c_2^2 = \frac{\mu}{\rho} \quad \text{and} \quad \omega_1 = \frac{\rho\, C^* c_2^2}{K^*}$$

Upon introducing the quantities defined by Eq. (9) in equations Eqs. (6)-(8), and suppressing the primes, yields

$$\begin{aligned}
&\left(\frac{\lambda+2\mu}{\mu}\right)\frac{\partial^2 u_1}{\partial x_1{}^2} + \left(\frac{\lambda+\mu}{\mu}\right)\frac{\partial^2 u_3}{\partial x_1\partial x_3} + \frac{\partial^2 u_1}{\partial x_3{}^2} - \beta\frac{\theta_0}{\mu}\frac{\partial\theta}{\partial x_1}\\
&= (1-\epsilon^2\nabla^2)\frac{\partial^2 u_1}{\partial t^2},
\end{aligned} \tag{10}$$

$$\begin{aligned}
&\left(\frac{\lambda+2\mu}{\mu}\right)\frac{\partial^2 u_3}{\partial x_3{}^2} + \left(\frac{\lambda+\mu}{\mu}\right)\frac{\partial^2 u_1}{\partial x_1\partial x_3} + \frac{\partial^2 u_3}{\partial x_1{}^2} - \beta\frac{\theta_0}{\mu}\frac{\partial\theta}{\partial x_3}\\
&= (1-\epsilon^2\nabla^2)\frac{\partial^2 u_3}{\partial t^2},
\end{aligned} \tag{11}$$

Also

$$\begin{aligned}
&K^*\nabla^2\varphi = \rho\, C^*\frac{\partial\theta}{\partial t} + \beta\theta_0\frac{\partial e}{\partial t}.\\
&\Rightarrow \nabla^2\varphi - \frac{\rho\, C^* c_2{}^2}{K^*\omega_1}\frac{\partial}{\partial t}(1-a\nabla^2)\,\varphi\\
&= \frac{\beta c_2{}^2}{K^*\omega_1{}^2}\frac{\partial}{\partial t}\left(\frac{\partial \mathrm{u}_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}\right)
\end{aligned} \tag{12}$$

Introducing potential functions defined by

$$u_1 = \frac{\partial \mathrm{q}}{\partial x_1} - \frac{\partial\psi}{\partial x_3}, \qquad u_3 = \frac{\partial \mathrm{q}}{\partial x_3} + \frac{\partial\psi}{\partial x_1}. \tag{13}$$

Using Eq. (13) in Eqs. (10)-(12), where $\mathrm{q}(\mathrm{x}_1, \mathrm{x}_3, \mathrm{t})$, and $\psi(\mathrm{x}_1, \mathrm{x}_3, \mathrm{t})$, are scalar potential functions, we obtain

$$\begin{aligned}
&\left(\frac{\lambda+2\mu}{\mu}\right)\left(\frac{\partial^3 q}{\partial x_1{}^3} + \frac{\partial^3 q}{\partial x_1\partial x_3{}^2}\right) - \frac{\partial^3\psi}{\partial x_3{}^3}\\
&-\frac{\partial^3\psi}{\partial x_1{}^2\partial x_3} - \beta\frac{\theta_0}{\mu}(1-a\nabla^2)\frac{\partial\varphi}{\partial x_1}\\
&= (1-\epsilon^2\nabla^2)\frac{\partial^2}{\partial t^2}\left(\frac{\partial \mathrm{q}}{\partial x_1} - \frac{\partial\psi}{\partial x_3}\right),
\end{aligned} \tag{14}$$

$$\begin{aligned}
&\left(\frac{\lambda+2\mu}{\mu}\right)\left(\frac{\partial^3 q}{\partial x_3{}^3} + \frac{\partial^3 q}{\partial x_1{}^2\partial x_3}\right) + \frac{\partial^3\psi}{\partial x_1{}^3}\\
&+\frac{\partial^3\psi}{\partial x_1\partial x_3{}^2} - \beta\frac{\theta_0}{\mu}(1-a\nabla^2)\frac{\partial\varphi}{\partial x_3}\\
&= (1-\epsilon^2\nabla^2)\frac{\partial^2}{\partial t^2}\left(\frac{\partial \mathrm{q}}{\partial x_3} + \frac{\partial\psi}{\partial x_1}\right),
\end{aligned} \tag{15}$$

$$\begin{aligned}
&\nabla^2\varphi - \frac{\rho\, C^* c_2{}^2}{K^*\omega_1}\frac{\partial}{\partial t}(1-a\nabla^2)\,\varphi\\
&= \frac{\beta c_2{}^2}{K^*\omega_1{}^2}\frac{\partial}{\partial t}\left(\frac{\partial^2 q}{\partial x_1{}^2} + \frac{\partial^2 q}{\partial x_3{}^2}\right).
\end{aligned} \tag{16}$$

Laplace & Fourier Transforms are defined by

$$\bar{f}(x_1, x_3, s) = \int_0^\infty f(x_1, x_3, t)\, e^{-st} dt, \tag{17}$$

$$\hat{f}(\boldsymbol{\xi}, x_3, s) = \int_{-\infty}^{\infty} \bar{f}(x_1, x_3, \boldsymbol{s})\, \boldsymbol{e}^{i\xi x_1}\boldsymbol{d}x_1. \tag{18}$$

Using Laplace and Fourier transforms defined by Eqs. (17)-(18), upon Eqs. (14)-(16), we obtain a system of equations

$$\begin{aligned}
&\left[(a_1+\epsilon^2 s^2)\frac{d^2}{dx_3^2} - (a_1\boldsymbol{\xi}^2 + s^2 + \epsilon^2\boldsymbol{\xi}^2)\right]\hat{q}\\
&-a_2\left[1 + a\boldsymbol{\xi}^2 - a\frac{d^2}{dx_3^2}\right]\hat{\varphi} = 0,
\end{aligned} \tag{19}$$

$$\begin{aligned}
&a_2\left[1 + a\boldsymbol{\xi}^2 - a\frac{d^2}{dx_3^2}\right]\hat{\varphi} = 0,\\
&\left[(1+\epsilon^2\mathrm{s}^2)\frac{d^2}{dx_3^2} - (\mathrm{s}^2 + \boldsymbol{\xi}^2 + \epsilon^2\mathrm{s}^2\boldsymbol{\xi}^2)\right]\hat{\psi} = 0.
\end{aligned} \tag{20}$$

$$\left\{a_4 s\left(-\boldsymbol{\xi}^2+\frac{d^2}{dx_3^2}\right)\right\}\hat{q} \\ -\left\{(1+aa_3 s)\frac{d^2}{dx_3^2}-(\boldsymbol{\xi}^2+a_3 s+aa_3 s\boldsymbol{\xi}^2)\right\}\hat{\varphi}=0. \tag{21}$$

From Eqs. (19) and (21), we yield a set of homogeneous equations which will have a nontrivial solution if determinant of coefficient $\hat{q}$ and $\hat{\varphi}$ vanishes so as to give a characteristic equation as

$$\left[\mathrm{P}\frac{d^4}{dx_3^4}+Q\frac{d^2}{dx_3^2}+R\right](\hat{q},\hat{\varphi})=0.$$

where

$$\begin{aligned} \mathrm{P}&=-[a_1+aa_1a_3s+\epsilon^2s^2 \\ &+aa_3\epsilon^2s^3+aa_2a_4s], \\ Q&=2a_1\boldsymbol{\xi}^2+a_2a_4s+2aa_2a_4s\boldsymbol{\xi}^2 \\ &+aa_3s\epsilon^2\boldsymbol{\xi}^2(1+s^2)+2aa_1a_3s\boldsymbol{\xi}^2 \\ &+a_1a_3s+s^2+aa_3s^3 \\ &+\epsilon^2\boldsymbol{\xi}^2(1+s^2), \\ R&=-\left[\begin{pmatrix} a_1+aa_3s\epsilon^2+\epsilon^2 \\ +aa_1a_3s+aa_2a_4s\end{pmatrix}\boldsymbol{\xi}^4\right. \\ &+\begin{pmatrix} a_1a_3s+s^2+aa_3s^3 \\ +a_3s\epsilon^2+a_2a_4s\end{pmatrix}\boldsymbol{\xi}^2 \\ &+a_3s^3. \end{aligned} \tag{22}$$

From Eq. (20)

$$\left[P'\frac{d^2}{dx_3^2}-Q'\right]\widehat{\Psi}=0.$$

where (23)

$$\begin{aligned} P'&=(1+\epsilon^2\mathrm{s}^2), \\ Q'&=(\mathrm{s}^2+\boldsymbol{\xi}^2+\epsilon^2\mathrm{s}^2\boldsymbol{\xi}^2). \end{aligned}$$

The roots are $\pm\lambda_i(i=1,2)$ and $\pm\lambda_i(i=3)$ making use of the radiation conditions that $\hat{q},\hat{\varphi}\to 0$ as $x_3\to\infty$, the solutions of Eqs. (22)-(23) may be written as

$$\hat{q}=A_1e^{-\lambda_1x_3}+A_2e^{-\lambda_2x_3}, \tag{24}$$

$$\hat{\varphi}=d_1A_1e^{-\lambda_1x_3}+d_2A_2e^{-\lambda_2x_3}, \tag{25}$$

$$\widehat{\Psi}=A_3e^{-\lambda_3x_3}. \tag{26}$$

where

$$d_i=\frac{(1+aa_3s)\lambda_i{}^2-(\boldsymbol{\xi}^2+a_3s+aa_3s\boldsymbol{\xi}^2)}{(a_1+\epsilon^2s^2)\lambda_i{}^2-(a_1\boldsymbol{\xi}^2+s^2+\epsilon^2\boldsymbol{\xi}^2)}, \\ i=1,2. \tag{27}$$

3.1 Boundary conditions

We consider a normal line load F_1 per unit length acting in the positive x_3 axis on the plane boundary $x_3=0$along the x_2 axis and a tangential load F_2, per unit length, acting at the origin in the positive x_1 axis. The boundary conditions are

(1) $t_{33}(x_1,x_3,t)=-F_1\psi_1(x)H(t)$,

(2) $t_{31}(x_1,x_3,t)=-F_2\psi_2(x)H(t)$,

(3) $\frac{\partial}{\partial x_3}\varphi(x_1,x_3,t)=0$.

where, F_1 and F_2 are the magnitudes of the forces applied, $\psi_1(x)$ and $\psi_2(x)$ specify the vertical and horizontal load distribution functions respectively along x_3 and x_1 axis, H(t) is the Heaviside unit step function.

Using the dimensionless quantities defined by Eq. (9) and substituting values of $\hat{q}$, $\hat{\varphi}$ and $\widehat{\Psi}$ from Eqs. (24)-(26), and solving, we obtain the components of displacement, normal stress, tangential stress and conductive temperature as

$$\hat{q}=\frac{F_1\widehat{\psi_1}(\xi)}{\mathrm{s}\Delta}\mu(\lambda_1d_1-\lambda_2d_2)(\lambda_3{}^2+\boldsymbol{\xi}^2) \\ +\frac{F_2\widehat{\psi_2}(\xi)}{\mathrm{s}\Delta}\iota 2\xi\lambda_3\mu(\lambda_1d_1-\lambda_2d_2), \tag{28}$$

$$\hat{\varphi}=\frac{F_1\widehat{\psi_1}(\xi)}{\mathrm{s}\Delta}(\lambda_1-\lambda_2)\mu d_1d_2(\lambda_3{}^2+\boldsymbol{\xi}^2) \\ +\frac{F_2\widehat{\psi_2}(\xi)}{\mathrm{s}\Delta}\iota 2\xi\lambda_3(\lambda_1-\lambda_2)\mu d_1d_2, \tag{29}$$

$$\widehat{\Psi}=\frac{F_1\widehat{\psi_1}(\xi)}{\mathrm{s}\Delta}\iota 2\mu\xi\lambda_1\lambda_2(d_1-d_2) \\ +\frac{F_2\widehat{\psi_2}(\xi)}{\mathrm{s}\Delta}\begin{bmatrix} \lambda\boldsymbol{\xi}^2(\lambda_2d_2-\lambda_1d_1) \\ +(\lambda+2\mu)\lambda_1\lambda_2(\lambda_2d_1-\lambda_1d_2) \\ +\beta a\frac{\theta_0\omega_1{}^2}{c_2{}^2}(\lambda_2-\lambda_1)(\boldsymbol{\xi}^2+\lambda_1\lambda_2)d_1d_2\end{bmatrix}. \tag{30}$$

where

$$\Delta=(\lambda_3{}^2+\boldsymbol{\xi}^2)\begin{bmatrix} \lambda\mu\boldsymbol{\xi}^2(\lambda_2d_2-\lambda_1d_1) \\ +\mu(\lambda+2\mu)\lambda_1\lambda_2(\lambda_2d_1-\lambda_1d_2) \\ -\beta\mu d_1d_2(\lambda_1-\lambda_2) \\ +\beta a\mu\frac{\theta_0\omega_1{}^2}{c_2{}^2}d_1d_2(\lambda_2-\lambda_1)(\boldsymbol{\xi}^2+\lambda_1\lambda_2)\end{bmatrix} \\ -4\mu^2\boldsymbol{\xi}^2\lambda_1\lambda_2\lambda_3(d_1-d_2). \tag{31}$$

Using Laplace and Fourier transform as defined by Eqs. (17)-(18) in Eq. (13), then using Eqs. (28) and (30) and Using Eq. (29) in Eq. (3), yields

$$\widehat{\widehat{u_1}}=\frac{F_1\widehat{\psi_1}(\xi)}{\mathrm{s}\Delta}\begin{bmatrix}\iota\xi\mu(\lambda_1d_1-\lambda_2d_2)(\lambda_3{}^2+\boldsymbol{\xi}^2) \\ +2\lambda_1\lambda_2(d_1-d_2)\end{bmatrix} \\ +\frac{F_2\widehat{\psi_2}(\xi)}{\mathrm{s}\Delta}\begin{bmatrix} \boldsymbol{\xi}^2(\lambda+2\mu\lambda_3)(\lambda_2d_2-\lambda_1d_1) \\ +(\lambda+2\mu)\lambda_1\lambda_2(\lambda_2d_1-\lambda_1d_2) \\ +\beta a\frac{\theta_0\omega_1{}^2}{c_2{}^2}(\lambda_2-\lambda_1)(\boldsymbol{\xi}^2+\lambda_1\lambda_2)d_1d_2\end{bmatrix}, \tag{32}$$

$$\widehat{\widehat{u_3}}=\frac{F_1\widehat{\psi_1}(\xi)}{\mathrm{s}\Delta}\mu\lambda_1\lambda_2(d_2-d_1)(\lambda_3{}^2+3\boldsymbol{\xi}^2) \tag{33}$$

$$+\frac{F_2\widehat{\overline{\psi_2}}(\xi)}{s\Delta}\begin{bmatrix} \iota 2\mu\xi\lambda_1\lambda_2\lambda_3(d_2-d_1) \\ +\iota\lambda\boldsymbol{\xi}^3(\lambda_2 d_2-\lambda_1 d_1) \\ +\iota\xi(\lambda+2\mu)\lambda_1\lambda_2(\lambda_2 d_1-\lambda_1 d_2) \\ +\iota\xi\beta a\dfrac{\theta_0{\omega_1}^2}{{c_2}^2}(\lambda_2-\lambda_1)(\boldsymbol{\xi}^2+\lambda_1\lambda_2)d_1 d_2 \end{bmatrix}, \tag{33}$$

$$\hat{\bar{\theta}}=\frac{F_1\widehat{\overline{\psi_1}}(\xi)}{s\Delta}({\lambda_3}^2+\boldsymbol{\xi}^2)\begin{bmatrix} (1+a\boldsymbol{\xi}^2)d_1 d_2-a\lambda_1\lambda_2(\lambda_2 d_1-\lambda_1 d_2)] \\ +\dfrac{F_2\widehat{\overline{\psi_2}}(\xi)}{s\Delta}[\iota 2\xi\lambda_3(\lambda_1-\lambda_2)\mu d_1 d_2(1+a\boldsymbol{\xi}^2+a\lambda_1\lambda_2) \end{bmatrix}, \tag{34}$$

$$\widehat{\overline{t_{33}}}=\frac{F_1\widehat{\overline{\psi_1}}(\xi)}{s\Delta}\left\{\begin{matrix}\mu({\lambda_3}^2+\boldsymbol{\xi}^2)\begin{bmatrix}\boldsymbol{\xi}^2(\lambda_2 d_2-\lambda_1 d_1) \\ +\lambda_1\lambda_2(\lambda+2\mu)(\lambda_2 d_1-\lambda_1 d_2) \\ +\beta(\lambda_2-\lambda_1)(1+a\boldsymbol{\xi}^2+a\lambda_1\lambda_2\lambda_3)d_1 d_2\end{bmatrix} \\ -2\xi\lambda_1\lambda_2(d_2-d_1)(\iota+\mu\xi\lambda\lambda_3+2\mu^2\xi\lambda_3)\end{matrix}\right\}$$
$$-\frac{F_2\widehat{\overline{\psi_2}}(\xi)}{s\Delta}\left\{\begin{matrix}\iota\boldsymbol{\xi}^3(\lambda+2\mu\lambda_3-\lambda^2\lambda_3-2\mu\lambda\lambda_3)(\lambda_2 d_2-\lambda_1 d_1) \\ +\iota\xi\lambda_1\lambda_2(\lambda+2\mu)(1-\lambda\lambda_3)(\lambda_2 d_1-\lambda_1 d_2) \\ +\iota\xi\beta a\dfrac{\theta_0{\omega_1}^2}{{c_2}^2}(\lambda_2-\lambda_1)(\boldsymbol{\xi}^2+\lambda_1\lambda_2)(1-2\mu\lambda_3-\lambda\lambda_3)d_1 d_2 \\ +\iota 2\mu\lambda_3\xi\beta(\lambda_2-\lambda_1)(1+a\boldsymbol{\xi}^2+a\lambda_1\lambda_2)d_1 d_2\end{matrix}\right\}, \tag{35}$$

$$\widehat{\overline{t_{31}}}=\widehat{\overline{t_{13}}}=\frac{F_1\widehat{\overline{\psi_1}}(\xi)}{s\Delta}\left[4\iota\xi\mu^2\lambda_1\lambda_2(d_2-d_1)({\lambda_3}^2+\boldsymbol{\xi}^2)\right]$$
$$-\frac{F_2\widehat{\overline{\psi_2}}(\xi)}{s\Delta}\mu\left\{\begin{matrix}({\lambda_3}^2+\boldsymbol{\xi}^2)\begin{bmatrix}\lambda\boldsymbol{\xi}^2(\lambda_2 d_2-\lambda_1 d_1) \\ +\lambda_1\lambda_2(\lambda+2\mu)(\lambda_2 d_1-\lambda_1 d_2) \\ +\beta a\dfrac{\theta_0{\omega_1}^2}{{c_2}^2}(\lambda_2-\lambda_1)(\boldsymbol{\xi}^2+\lambda_1\lambda_2)d_1 d_2\end{bmatrix} \\ +4\mu\lambda_1\lambda_2\lambda_3\xi^2(d_2-d_1)\end{matrix}\right\}. \tag{36}$$

Concentrated force: The solution due to concentrated normal force on the half space is obtained by setting

$$\psi_1(x)=\delta(x), \qquad \psi_2(x)=\delta(x)$$

where, $\delta(x)$ is dirac delta functions. Applying Laplace and Fourier transform, we obtain

$$\widehat{\overline{\psi_1}}(\xi)=1, \qquad \widehat{\overline{\psi_2}}(\xi)=1. \tag{37}$$

Using Eq. (37) in Eq. (29), Eq. (33) and Eqs. (35)-(36), we obtain the components of displacement, stress and conductive temperature.

Uniformly distributed force: The solution due to uniformly distributed force applied in the half space is obtained by setting

$$\{\psi_1(x),\psi_2(x)\}=\begin{cases}1 \ if \ \ |x|\le m \\ 0 \ if \ \ |x|>m\end{cases}$$

The Laplace and Fourier transforms of $\psi_1(x)$ and $\psi_2(x)$ with respect to the pair (x, ξ) for the case of a uniform strip load of non-dimensional width 2 m applied at origin of coordinate system $x_1=x_3=0$ is given by

$$\{\widehat{\overline{\psi_1}}(\xi), \qquad \widehat{\overline{\psi_2}}(\xi)\}=\left[{2\sin(\xi m)}/{\xi}\right]\xi\neq 0. \tag{38}$$

Using Eq. (38) in Eq. (29), Eq. (33) and Eqs. (35)-(36), we obtain the components of displacement, stress and conductive temperature.

4. Particular cases

- If a = 0, then from Eqs. (28)-(36), we obtain the corresponding expressions for displacements, stresses and conductive temperature for nonlocal isotropic solid without two temperature.
- If ϵ = 0, then from Eqs. (28)-(36), we obtain the corresponding expressions for displacements, stresses and conductive temperature for isotropic solid without nonlocal effects and with two temperature.

5. Inversion of the transformation

To obtain the solution of the problem in physical domain, we must invert the transforms in Eqs. (28)-(36). Here the displacement components, normal and tangential stresses and conductive temperature are functions ofx_3, the parameters of Laplace and Fourier transforms s and ξ respectively and hence are of the form $f(\xi,x_3,s)$. To obtain the function $f(x_1,x_3,t)$ in the physical domain, we first invert the Fourier transform using

$$\begin{aligned}\bar{f}(x_1,x_3,s)&=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-i\xi x_1}f(\xi,x_3,s)d\xi \\ &=\frac{1}{2\pi}\int_{-\infty}^{\infty}|\cos(\xi x_1)f_e-i\sin(\xi x_1)f_0|\,d\xi.\end{aligned} \tag{39}$$

where,f_e and $f_0$are respectively the odd and even parts of $f(\xi,x_3,s)$. Thus the Eq. (39) gives the Laplace transform $\bar{f}(x_1,x_3,s)$ of the function$f(x_1,x_3,t)$. Following Honig and Hirdes (1984), the Laplace transform function $\bar{f}(x_1,x_3,s)$ can be inverted to $f(x_1,x_3,t)$. The last step is to calculate the integral in Eq. (39). The method for evaluating this integral is described in Press *et al.* (1986). It involves the use of Romberg's integration with adequate step size. The results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero are also used.

6. Numerical results and discussion

Magnesium material is chosen for the purpose of numerical calculation which is isotropic and according to Dhaliwal and Singh (1980), the physical data for it is given as

$$\begin{aligned}&\lambda=9.4\times 10^{10}Nm^{-2},\mu=3.278\times 10^{10}Nm^{-2}, \\ &K^*=1.7\times 10^2 Wm^{-1}K^{-1},\rho=1.74\times\ 10^3 Kgm^{-3}, \\ &T_0=298\ K,C^*=10.4\times 10^2 JKg^{-1}deg^{-1}, \\ &\omega_1=3.58,a=0.05.\end{aligned}$$

A comparison of values of normal displacement u_3, normal force stress t_{33} , tangential stress t_{31} and conductive temperature φ for a transversely isotropic thermoelasic solid with distance x has been made for local and nonlocal parameter $\epsilon=2.5$ and is presented graphically at $\theta=45^0$ and $\theta=90^0$ in Figs. 1-8.

(1) The blue coloured solid line with circles as symbols and small dashed purple line with triangles as symbols respectively corresponds to $\epsilon = 0$ at $\theta = 45^0$ and $\theta = 90^0$.

(2) The solid black line and small dashed red line respectively corresponds to $\epsilon = 2.5$ at $\theta = 45^0$ and $\theta = 90^0$.

6.1 Concentrated force

From Fig. 1, it is clear that the variation in values of normal stress t_{33} is higher due to effects of nonlocality for both the angles. In Fig. 2, near the loading surface, the value of normal displacement u_3 is higher for $\theta = 90^0$ due to nonlocal effects as compared to all the other values but it is more oscillatory and for the other values it is lessoscilla-tory. Also it is evident that as the displacement increases the value of normal displacement at the other values is becoming almost stationary. From Fig.3, for the values of tangential stress t_{31}, variation is more when nonlocal effect is zero at both angles and is clearly more prominent for $\theta = 90^0$.The values of conductive temperature φ increase initially for both the angles with more dominance for $\theta = 45^0$ and then showing oscillatory behavior with comparatively less variations (Fig. 4). But for $5 \le x \le 12$, the behavior is opposite. Also the effects of local and nonlocal parameters are clearly visible at both the angles.

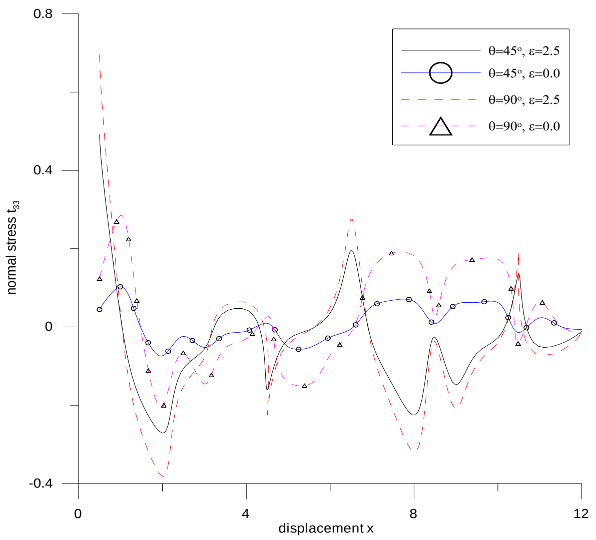

Fig. 1 Variation of normal stress t_{33} with displacement x (concentrated force)

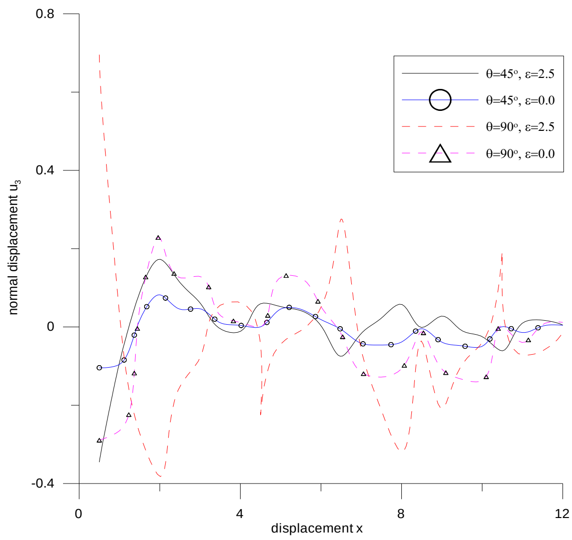

Fig. 2 Variation of normal displacement u_3 with displacement x (concentrated force)

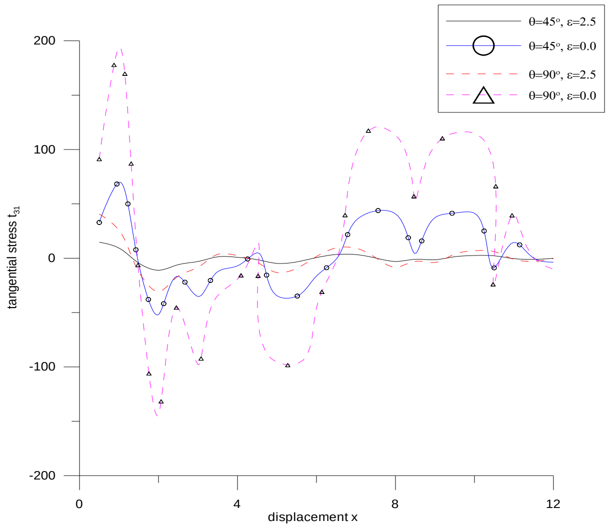

Fig. 3 Variation of tangential stress t_{31} with displacement x (concentrated force)

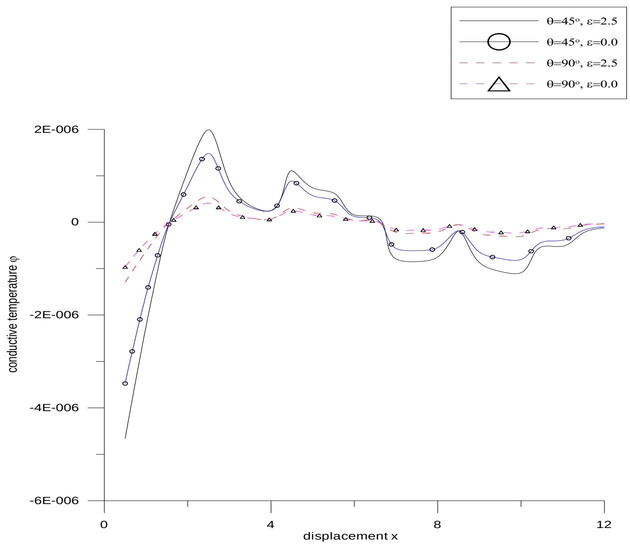

Fig. 4 Variation of conductive temperature φ with displacement x (concentrated force)

6.2 Uniformly distributed force

Fig. 5 depicts that the variation in the values of normal stress t_{33} is comparatively less due to the effects of nonlocality for both angles but more prominent at $\theta = 45^0$

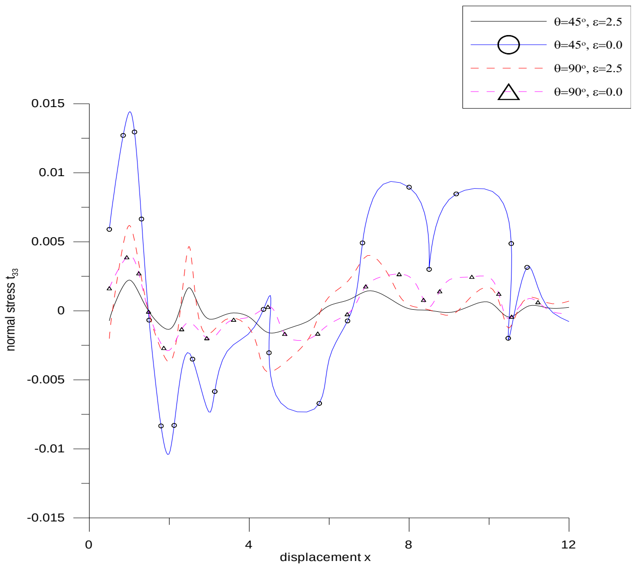

Fig. 5 Variation of normal stress t_{33} with displacement x (uniformly distributed force)

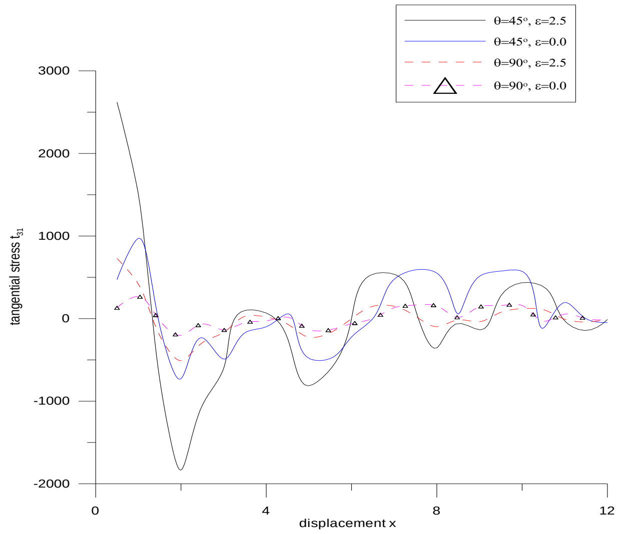

Fig. 7 Variation of tangential stress t_{31} with displacement x (uniformly distributed force)

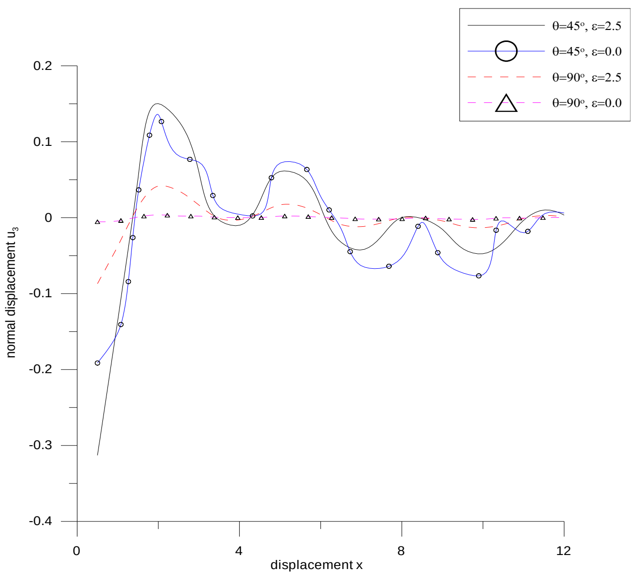

Fig. 6 Variation of normal displacement u_3 with displacement x (uniformly distributed force)

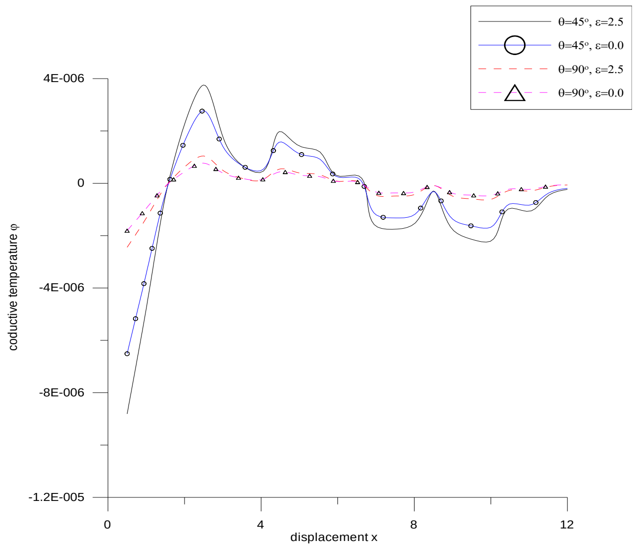

Fig. 8 Variation of conductive temperature φ with displacement x (uniformly distributed force)

as compared to $\theta = 90^0$. For the values of normal displacement u_3 (Fig. 6), there is comparatively lesser variation due to the effects of nonlocality for both angles but it is clearly more visible for $\theta = 45^0$. For the values of tangential stress t_{31}, variation is more for $\theta = 45^0$ at initial stages i.e., for $x < 5$ and is showing comparatively lesser variation for $\theta = 90^0$ (Fig. 7). The values of conductive temperature φ (Fig. 8), is showing lesser variations comparatively at $\theta = 90^0$ but the variations are more visible at $\theta = 45^0$ due to nonlocal effects.

7. Conclusions

In the present discussion the numerical results have been depicted graphically showing the effects of nonlocal parameter on the components of displacements, stresses and conductive temperature. From above investigation it is observed that there is a significant impact on normal displacement, normal stress, tangential stress and conductive temperature due to effects of nonlocality. The variation of the components is dependent upon the nonlocal parameters as well as the variations in the absolute temperature. The amplitude of all the physical quantities (discussed above) either increase or decrease with nonlocal parameters as well as the angle of inclined load. In presence and absence of nonlocal parameters the stress and displacement components follow an oscillatory path with respect to *x*. The inclined load plays a significant role in the distribution of all the physical quantities. The results of this paper give an inspiration to study nonlocal parameter effects at a higher level. These results will be very useful for the researchers working in the field of material science, geophysics, acoustics etc.

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