Steel and Composite Structures, *Vol. 33, No. 1 (2019) 81-92*
DOI: https://doi.org/10.12989/scs.2019.33.1.081

Bending analysis of anti-symmetric cross-ply laminated plates under nonlinear thermal and mechanical loadings

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(*Received January 10, 2019, Revised September 25, 2019, Accepted September 29, 2019*)

Abstract. The present paper addresses a refined plate theoryin order to describe the response of anti-symmetric cross-ply laminated plates subjected to a uniformlydistributed nonlinear thermo-mechanical loading. In the present theory, the undetermined integral terms are used and the variables number is reduced to four instead of five or more in other higher-order theories. The boundary conditions on the top and the bottom surfaces of the plate are satisfied; hence the use of the transverse shear correction factors isavoided. The principle of virtual work is used to obtain governing equations and boundary conditions. Navier solution for simply supported plates is used to derive analytical solutions. For the validation of the present theory, numerical results for displacements and stressesare compared with those of classical, first-order, higher-order and trigonometricshear theories reported in the literature.

Keywords: anti-symmetric laminated plates; nonlinear thermo-mechanical loading; displacements; stresses

1. Introduction

Composite materials are widely used in structures subjected to severe thermal environment owing to their excellent mechanical and thermal properties such as high specific strength, high stiffness, corrosion resistance, light damping, temperature resistance and low thermal coefficient of expansion. In order to describe the correct thermo-mechanical behavior of laminated plate there is a necessity for the deployment of new refined theories.

Using the classical plate theory, thermal stresses in isotropic plate are given by Boley and Weiner (1960), whereas thermal stresses analysis of laminated plates under thermal loading is presented by Jones (1999), Reddy (1997), Wu and Tauchert (1980). This theory, however, gives inaccurate results for the laminated plates. This inaccuracy is due to the neglect of transverse stresses in the laminates. Reddy (1997) used the first order theory (FSDT) to analyze thermal stresses in laminated plates, this theory includes the transverse shear deformation in the governing equations, but it gives a constant transverses shear stresses through the thickness. To satisfy the boundary conditions on the top and the bottom surface of the plate, the FSDT uses a shear correction factor K, this factor depends on lamina properties and laminations scheme. However, the higher-order shear deformation theories (HSDT) don’t require the employment of the shear correction factors. Khdeir and Reddy (1991) developed an exact analytical solution of refined plate theories, stresses and deflections of laminated plate subjected to a single sinusoidal thermal loading are presented. The global-local higher theory has been simply derived by Zhen and Chen (2006) in order to obtain an efficient higher-order theory and finite element for laminated plates under sinusoidal thermal loading. Shinde *et al*. (2013) used the hyperbolic shear deformation theory to investigate the thermal bending of isotropic plates under uniformly distributed thermal loading. Thermal flexural analysis of cross-ply laminated plates subjected to a nonlinear sinusoidal thermal loading using trigonometric shear deformation theory has been presented by Ghugal and Kulkarni (2013a). Various plate theories have been used by Sayyad *et al*. (2014) to carry out a thermo-elastic analysis of cross-ply laminated plates under linear sinusoidal thermal loading. Thermal displacements and stresses of laminated plates subjected to a sinusoidally distributed linear thermal loading using a four-variable plate theory have been presented by Sayyad *et al*. (2015). In another article Sayyad *et al*. (2016) presented a thermal stress analysis of cross-ply laminated plate subjected to linear thermal load using an exponential shear deformation theory. Gandhe *et al*. (2018) have recently presented three variables trigonometric shear deformation theory to analyze flexural behavior of isotropic plates subjected to a single sinusoidal thermal loading.

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ISSN: 1229-9367 (Print), 1598-6233 (Online)

Many first and higher-order theories have been developed or extended to study the behaviour of laminated plates under thermo-mechanical loading. To start with Reddy and Hsu (1980) who suggested a finite element formulation of governing equations of laminated plates subjected to mechanical/thermal loading. (Fares and Zenkour 1999, and Fares *et al*. 2000) presented a mixed variational formula for the analysis of generally layered composite structures subjected to sinusoidal thermo-mechanical single loading. Han *et al*. (2017) proposed an enhanced first order shear deformation theory including the transverse normal strain effect for the analysis of the thermo-mechanical response of laminated composite and sandwich plates. By the use of a unified shear deformation plate theory, Zenkour (2004) investigated the static thermo-elastic response of symmetric and anti-symmetric cross-ply laminated plates under non-uniform sinusoidal mechanical and/or thermal loading. An equivalent single layer shear deformation theory has been presented by Ghugal and Kulkarni (2012, 2013b, c) using a trigonometric shear deformation theory in order to analyze displacements and stresses of cross ply laminated plates under uniformly distributed linear and non-linear thermo-mechanical loading. Chattibi *et al*. (2015) developed a four variable sinusoidal to investigate the thermo-mechanical bending response of anti-symmetric cross-ply composite plates. Based on the layer-wise displacement field of Reddy, Cetkovic (2015) proposed a mathematical model using small deflexion linear-elasticity theory to analyze the thermo-mechanical bending of laminated composites and sandwich plates subjected to a uniform or a single sinusoidally distributed gradient temperature along with sinusoidal mechanical loadings. Zen and Xiaohui (2016) proposed a new modal to analyze the thermo-mechanical behavior of multilayered composite plates under thermo-mechanical combined loading based on Reddy-type higher order theory. An analytical model of laminated composite plates based on an inverse hyperbolic shear deformation theory (IHSDT) has been proposed by Joshan *et al*. (2017), the thermo-mechanical response of cross-ply and angle-ply laminated composite plates has been investigated.

Several investigations delved on the study of the thermal or thermo-mechanical behaviour of functionally graded plates; various refined theories have been presented by (Zenkour and Alghamdi 2008, Bourada *et al*. 2012, Saidi *et al*. 2013, Kar *et al*. 2015, Mahapatra and Panda 2016, Mahapatra *et al*. 2016a, b, c, 2017a, b, Kolahchi and Moniri Bidgoli 2016, Singh *et al*. 2016, Hirwani and Panda 2018, Hirwani *et al*. 2017, 2018aa, b, c, d, e, Dutta *et al*. 2017, Kolahchi *et al*. 2017a, Sahoo *et al*. 2017a, b, Bachiri *et al*. 2018, Mehar *et al*. 2017, 2018, Bisen *et al*. 2018, Katariya *et al*. 2018a, b, Dash *et al*. 2019, Mehar *et al*. 2019, Sharma *et al*. 2019).

Based on the above-mentioned references, it is noticed that most studiesinvestigated the response of laminated plate under single sinusoidally distributed linear thermal and/or mechanical loadings. The present research accordingly attempts to provide a refined higher-order theory for the analysis of the response of laminated plates under combined uniformly distributed nonlinear thermo-mechanical loading. In this theory, the unknown number is reduced to four instead of five or more as suggested in the other theories. The obtained results are discussed and compared with those of classical, first-order, trigonometric and higher-order shear theories.

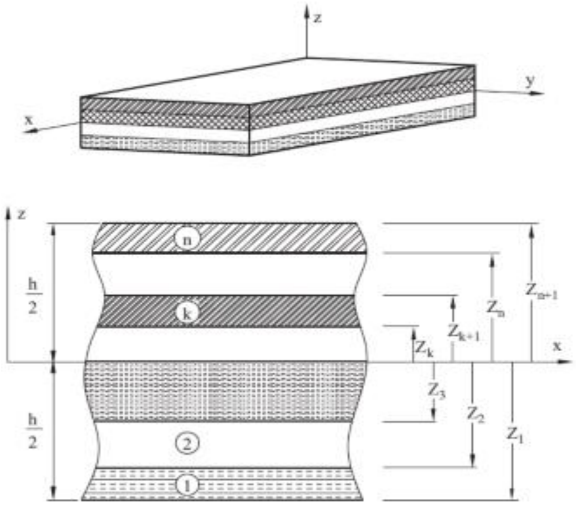

Fig. 1 Plaque geometry and coordinate system

2. Theoretical formulation

Consider a rectangular cross-ply laminated plate total thickness *h* composed of *n* orthotropic layers (see Fig. 1), which are perfectly bonded together. The material of each layer is assumed to posses on plane of elastic symmetry parallel to x-y plane. The upper surface of the plate is subjected to a mechanical load q (x,y) and thermal load T(x,y,z).

2.1 Kinematics

The displacement field of the conventional HSDT at a point in the laminated plate is expressed as

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\frac{\partial w_0}{\partial x} + f(z)\varphi_x(x,y) \\ v(x,y,z) &= v_0(x,y) - z\frac{\partial w_0}{\partial y} + f(z)\varphi_y(x,y) \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{1}$$

u_0, v_0, w_0, φ_x and φ_y are the five unknown displacements of a point on the mid-plane of the plate, supposing that $\varphi_x = \int \theta(x,y)dx$ and $\varphi_y = \int \theta(x,y)dy$, the displacement field mentioned above can be written in a simple form as

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\frac{\partial w_0}{\partial x} + k_1 f(z)\int \theta(x,y)dx \\ v(x,y,z) &= v_0(x,y) - z\frac{\partial w_0}{\partial y} + k_2 f(z)\int \theta(x,y)dy \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{2}$$

The integrals terms defined in the above equations shall be resolved by using Navier type method and the displacement field can be written as follows

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\frac{\partial w_0}{\partial x} + k_1 A_1 f(z)\frac{\partial \theta}{\partial x} \\ v(x,y,z) &= v_0(x,y) - z\frac{\partial w_0}{\partial y} + k_2 B_1 f(z)\frac{\partial \theta}{\partial y} \\ w(x,y,z) &= w_0(x,y) \end{aligned} \quad (3)$$

Where

$$k_1 = \mu^2, k_2 = \lambda^2, A_1 = -\frac{1}{\mu^2}, B_1 = -\frac{1}{\lambda^2} \quad (4a)$$

And

$$\mu = \frac{m\pi}{a}, \qquad \lambda = \frac{n\pi}{b} \quad (4b)$$

In the present article the shape function $f(z)$is given as follows

$$f(z) = \frac{h}{\pi} sin\frac{\pi z}{h} \quad (5)$$

The normal and shear strains associated with the displacement field (3) are as follows

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + f(z)\begin{Bmatrix} k_x^s \\ k_y^b \\ k_{xy}^s \end{Bmatrix}; \quad (6a)$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g(z)\begin{Bmatrix} \gamma_{yz}^s \\ \gamma_{xz}^s \end{Bmatrix} \quad (6b)$$

Where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial y} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}; \quad \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial^2 w_0}{\partial y^2} \\ -2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}; \quad (6c)$$

$$\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} k_1 A_1 \dfrac{\partial^2 \theta}{\partial x^2} \\ k_2 B_1 \dfrac{\partial^2 \theta}{\partial y^2} \\ (k_1 A_1 + k_2 B_1)\dfrac{\partial^2 \theta}{\partial x \partial y} \end{Bmatrix}; \quad \begin{Bmatrix} \gamma_{yz}^s \\ \gamma_{xz}^s \end{Bmatrix} = \begin{Bmatrix} k_2 B_1 \dfrac{\partial \theta}{\partial y} \\ k_1 A_1 \dfrac{\partial \theta}{\partial x} \end{Bmatrix} \quad (6d)$$

And

$$g(z) = \frac{df(z)}{dz} \quad (6e)$$

2.2 Constitutive equations

The stress-strain relationships, accounting for transverse shear deformation and thermal effects for a layer can be expressed as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_X - \alpha_x T \\ \varepsilon_y - \alpha_y T \\ \gamma_{xy} - \alpha_{xy} T \end{Bmatrix}; \quad (7a)$$

$$\begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{44} & Q_{45} \\ Q_{45} & Q_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \quad (7b)$$

Where Q_{ij} are the plane stress-reduced stiffnesses that are expressed as follows

$$\begin{aligned} Q_{11} &= \frac{E_1}{1 - \upsilon_{12}\upsilon_{21}}; \quad & Q_{12} &= \frac{\upsilon_{12} E_2}{1 - \upsilon_{12}\upsilon_{21}}; \\ Q_{22} &= \frac{E_2}{1 - \upsilon_{12}\upsilon_{21}}; \quad & Q_{66} &= G_{12}; \\ Q_{44} &= G_{23}; & Q_{55} &= G_{13} \end{aligned} \quad (8)$$

And E_i are Young's moduli, ν_{ij} are Poisson's rations, G_{ij} are shear moduli, α_x and α_y are the coefficients of linear thermal expansion in x and y directions respectively, and $T = T(x, y, z)$ is the temperature distribution.

The constitutive equations of each lamina are transformed to the plate coordinates (x, y, z) and the stress-strain relationships in the plate coordinate system for thek^{th} layer is expressed as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_{(k)} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_{(k)} \begin{Bmatrix} \varepsilon_X - \alpha_x T \\ \varepsilon_y - \alpha_y T \\ \gamma_{xy} - \alpha_{xy} T \end{Bmatrix}_{(k)}, \quad (9a)$$

$$\begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix}_{(k)} = \begin{bmatrix} \overline{Q}_{44} & \overline{Q}_{45} \\ \overline{Q}_{45} & \overline{Q}_{55} \end{bmatrix}_{(k)} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}_{(k)} \quad (9b)$$

Where $\overline{Q}_{ij}$ are the transformed elastic coefficient given by Reddy (1997).

2.3 Governing equations

The principle of virtual work is used in order to determine the governing equations as follows

$$\begin{aligned} &\int_{-\frac{h}{2}}^{\frac{h}{2}}\int_0^b\int_0^a (\sigma_x\,\delta\varepsilon_x + \sigma_y\delta\varepsilon_y + \tau_{yz}\delta\gamma_{yz} + \tau_{xz}\delta\gamma_{xz} \\ &+\tau_{xy}\delta\gamma_{xy})dxdydz - \int_0^b\int_0^a q\delta w dxdy = 0 \end{aligned} \quad (10)$$

By substituting Eqs. (6)-(9) into Eq. (10) and integrating through the thickness, Eq. (10) can be expressed as

$$\int_0^b\int_0^a (N_x\frac{\partial\delta u_0}{\partial x}-M_x^b\frac{\partial^2\delta w_0}{\partial x^2}+k_1A_1M_x^s\frac{\partial^2\delta\theta}{\partial x^2}$$
$$+N_y\frac{\partial\delta v_0}{\partial y}-M_y^b\frac{\partial^2\delta w_0}{\partial y^2}+k_2B_1M_y^s\frac{\partial^2\delta\theta}{\partial y^2}$$
$$+N_{xy}\left(\frac{\partial\delta u_0}{\partial y}+\frac{\partial\delta v_0}{\partial x}\right)-2M_{xy}^b\frac{\partial^2\delta w_0}{\partial x\partial y} \quad (11)$$
$$+(k_1A_1+k_2B_1)M_{xy}^s\frac{\partial^2\delta\theta}{\partial x\partial y}+k_1A_1S_{xz}^s\frac{\partial\delta\theta}{\partial x}$$
$$+k_2B_1S_{yz}^s\frac{\partial\delta\theta}{\partial y}-q\delta w_0)dxdy=0$$

The resulting stresses and moments are obtained by integrating Eq. (9) over the thickness, and are expressed as

$$\begin{Bmatrix}N\\M^b\\M^s\end{Bmatrix}=\begin{bmatrix}A & B & B^s\\B & D & D^s\\B^s & D^s & H^s\end{bmatrix}\begin{Bmatrix}\varepsilon\\k^b\\k^s\end{Bmatrix}+\begin{Bmatrix}N^T\\{M^b}^T\\{M^s}^T\end{Bmatrix}, S=A^s\gamma \quad (12)$$

Where

$$N=\{N_x,N_y,N_{xy}\}^t,\quad M^b=\{M_x^b,M_y^b,M_{xy}^b\}^t,\quad M^s=\{M_x^s,M_y^s,M_{xy}^s\}^t \quad (13a)$$

$$N^T=\{N_x^T,N_y^T,N_{xy}^T\}^t,\quad {M^b}^T=\{{M_x^b}^T,{M_y^b}^T,{M_{xy}^b}^T\}^t,\quad {M^s}^T=\{{M_x^s}^T,{M_y^s}^T,{M_{xy}^s}^T\}^t \quad (13b)$$

$$\varepsilon=\{\varepsilon_x^0,\varepsilon_y^0,\gamma_{xy}^0\}^t,\quad k^b=\{k_x^b,k_y^b,k_{xy}^b\}^t,\quad k^s=\{k_x^s,k_y^s,k_{xy}^s\}^t \quad (13c)$$

$$A=\begin{bmatrix}A_{11}&A_{12}&A_{16}\\A_{12}&A_{22}&A_{26}\\A_{16}&A_{26}&A_{66}\end{bmatrix},\quad B=\begin{bmatrix}B_{11}&B_{12}&B_{16}\\B_{12}&B_{22}&B_{26}\\B_{16}&B_{26}&B_{66}\end{bmatrix},\quad D=\begin{bmatrix}D_{11}&D_{12}&D_{16}\\D_{12}&D_{22}&D_{26}\\D_{16}&D_{26}&D_{66}\end{bmatrix} \quad (13d)$$

$$B^s=\begin{bmatrix}B_{11}^s&B_{12}^s&B_{16}^s\\B_{12}^s&B_{22}^s&B_{26}^s\\B_{16}^s&B_{26}^s&B_{66}^s\end{bmatrix},\quad D^s=\begin{bmatrix}D_{11}^s&D_{12}^s&D_{16}^s\\D_{12}^s&D_{22}^s&D_{26}^s\\D_{16}^s&D_{26}^s&D_{66}^s\end{bmatrix},\quad H^s=\begin{bmatrix}H_{11}^s&H_{12}^s&H_{16}^s\\H_{12}^s&H_{22}^s&H_{26}^s\\H_{16}^s&H_{26}^s&H_{66}^s\end{bmatrix} \quad (13e)$$

$$S=\{S_{yz}^s,S_{xz}^s\}^t,\quad \gamma=\{\gamma_{yz},\gamma_{xz}\}^t,\quad A^s=\begin{bmatrix}A_{44}^s&A_{45}^s\\A_{45}^s&A_{55}^s\end{bmatrix} \quad (13f)$$

Where the stiffness components are defined as

$$(A_{ij},B_{ij},D_{ij})=\sum_{k=1}^n\int_{z_k}^{z_{k+1}}\overline{Q}_{ij}^{(k)}(1,z,z^2)dz,\quad (i,j=1,2,6) \quad (14a)$$

$$(B_{ij}^s,D_{ij}^s,H_{ij}^s)=\sum_{k=1}^n\int_{z_k}^{z_{k+1}}\overline{Q}_{ij}^{(k)}(f(z),zf(z),f^2(z))dz,\quad (i,j=1,2,6), \quad (14b)$$

$$A_{ij}^s=\sum_{k=1}^n\int_{z_k}^{z_{k+1}}\overline{Q}_{ij}^{(k)}g^2(z)dz,\quad (i,j=4,5). \quad (14c)$$

In the present article, the thermal loading across the thickness is supposed to be

$$T(x,y,z)=T_1(x,y)+\frac{z}{h}T_2(x,y)+\frac{f(z)}{h}T_3(x,y) \quad (15)$$

With the integration by parts of Eq. (11) alongside collecting the coefficient of δu_0, δv_0, δw_0, $\delta\theta$ we can obtain the following governing equations

$$\delta u_0: \frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=0$$
$$\delta v_0: \frac{\partial N_y}{\partial y}+\frac{\partial N_{xy}}{\partial x}=0$$
$$\delta w_0: \frac{\partial^2 M_x^b}{\partial x^2}+\frac{\partial^2 M_y^b}{\partial y^2}+2\frac{\partial^2 M_{xy}^b}{\partial x\partial y}+q=0 \quad (16)$$
$$\delta\theta: k_1A_1\frac{\partial S_{xz}^s}{\partial x}+k_2B_1\frac{\partial S_{yz}^s}{\partial y}-k_1A_1\frac{\partial^2 M_x^s}{\partial x^2}$$
$$-k_2B_1\frac{\partial^2 M_y^s}{\partial y^2}-(k_1A_1+k_2B_1)\frac{\partial^2 M_{xy}^s}{\partial x\partial y}=0$$

By substituting Eq. (12) into Eq. (16), the governing equations can be written in terms of displacements (u_0, v_0, w_0, θ) as follows

$$A_{11}\frac{\partial^2 u_0}{\partial x^2}+2A_{16}\frac{\partial^2 u_0}{\partial x\partial y}+A_{66}\frac{\partial^2 u_0}{\partial y^2}$$
$$+A_{16}\frac{\partial^2 v_0}{\partial x^2}+A_{26}\frac{\partial^2 v_0}{\partial y^2}+(A_{12}+A_{66})\frac{\partial^2 v_0}{\partial x\partial y}$$
$$-B_{11}\frac{\partial^3 w_0}{\partial x^3}-B_{26}\frac{\partial^3 w_0}{\partial y^3}-3B_{16}\frac{\partial^3 w_0}{\partial x^2\partial y}$$
$$-(B_{12}+2B_{66})\frac{\partial^3 w_0}{\partial x\partial y^2}+k_1A_1B_{11}^s\frac{\partial^3\theta}{\partial x^3} \quad (17a)$$
$$+k_2B_1B_{26}^s\frac{\partial^3\theta}{\partial y^3}+k_2B_1B_{12}^s\frac{\partial^3\theta}{\partial x\partial y^2}$$
$$+(2k_1A_1+k_2B_1)B_{16}^s\frac{\partial^3\theta}{\partial x^2\partial y}$$
$$+(k_1A_1+k_2B_1)B_{66}^s\frac{\partial^3\theta}{\partial x\partial y^2}-\frac{\partial N_x^T}{\partial x}-\frac{\partial N_{xy}^T}{\partial y}=0$$

$$A_{12}\frac{\partial^2 u_0}{\partial x^2}+A_{26}\frac{\partial^2 u_0}{\partial y^2}+(A_{12}+A_{66})\frac{\partial^2 u_0}{\partial x\partial y}$$
$$+A_{22}\frac{\partial^2 v_0}{\partial y^2}+2A_{26}\frac{\partial^2 v_0}{\partial x\partial y}+A_{66}\frac{\partial^2 v_0}{\partial x^2} \quad (17b)$$
$$-B_{16}\frac{\partial^3 w_0}{\partial x^3}-B_{22}\frac{\partial^3 w_0}{\partial y^3}-B_{12}\frac{\partial^3 w_0}{\partial x^2\partial y}-3B_{26}\frac{\partial^3 w_0}{\partial x\partial y^2}$$

$$-2B_{66}\frac{\partial^3 w_0}{\partial x^2\partial y}+k_1A_1B_{16}^s\frac{\partial^3\theta}{\partial x^3}+k_2B_1B_{22}^s\frac{\partial^3\theta}{\partial y^3}$$
$$+k_1A_1B_{12}^s\frac{\partial^3\theta}{\partial x^2\partial y}+(k_1A_1+2k_2B_1)B_{26}^s\frac{\partial^3\theta}{\partial x\partial y^2} \quad (17b)$$
$$+(k_1A_1+k_2B_1)B_{66}^s\frac{\partial^3\theta}{\partial x^2\partial y}-\frac{\partial N_y^T}{\partial y}-\frac{\partial N_{xy}^T}{\partial x}=0$$

$$B_{11}\frac{\partial^3 u_0}{\partial x^3}+B_{26}\frac{\partial^3 u_0}{\partial y^3}+3B_{16}\frac{\partial^3 u_0}{\partial x^2\partial y}+B_{12}\frac{\partial^3 u_0}{\partial x\partial y^2}$$
$$+2B_{66}\frac{\partial^3 u_0}{\partial x\partial y^2}+B_{16}\frac{\partial^3 v_0}{\partial x^3}+B_{22}\frac{\partial^3 v_0}{\partial y^3}+B_{12}\frac{\partial^3 v_0}{\partial x^2\partial y}$$
$$+3B_{26}\frac{\partial^3 v_0}{\partial x\partial y^2}+2B_{66}\frac{\partial^3 v_0}{\partial x^2\partial y}-D_{11}\frac{\partial^4 w_0}{\partial x^4}-D_{22}\frac{\partial^4 w_0}{\partial y^4}$$
$$-2D_{12}\frac{\partial^4 w_0}{\partial x^2\partial y^2}-4D_{16}\frac{\partial^4 w_0}{\partial x^3\partial y}-4D_{26}\frac{\partial^4 w_0}{\partial x\partial y^3}$$
$$-4D_{66}\frac{\partial^4 w_0}{\partial x^2\partial y^2}+k_1A_1D_{11}^s\frac{\partial^4\theta}{\partial x^4}+k_2B_1D_{22}^s\frac{\partial^4\theta}{\partial y^4} \quad (17c)$$
$$+k_2B_1D_{12}^s\frac{\partial^4\theta}{\partial x^2\partial y^2}+(3k_1A_1+k_2B_1)D_{16}^s\frac{\partial^4\theta}{\partial x^3\partial y}$$
$$+(k_1A_1+3k_2B_1)D_{26}^s\frac{\partial^4\theta}{\partial x\partial y^3}+k_1A_1D_{12}^s\frac{\partial^4\theta}{\partial x^2\partial y^2}$$
$$+2(k_1A_1+k_2B_1)D_{66}^s\frac{\partial^4\theta}{\partial x^2\partial y^2}-\frac{\partial^2 M_x^{b^T}}{\partial x^2}$$
$$-\frac{\partial^2 M_y^{b^T}}{\partial y^2}-2\frac{\partial^2 M_{xy}^{b^T}}{\partial x\partial y}+q=0$$

$$-k_1A_1B_{11}^s\frac{\partial^3 u_0}{\partial x^3}-k_2B_1B_{26}^s\frac{\partial^3 u_0}{\partial y^3}-k_2B_1B_{12}^s\frac{\partial^3 u_0}{\partial x\partial y^2}$$
$$-(2k_1A_1+k_2B_1)B_{16}^s\frac{\partial^3 u_0}{\partial x^2\partial y}$$
$$-(k_1A_1+k_2B_1)B_{66}^s\frac{\partial^3 u_0}{\partial x\partial y^2}-k_1A_1B_{16}^s\frac{\partial^3 v_0}{\partial x^3}$$
$$-k_2B_1B_{22}^s\frac{\partial^3 v_0}{\partial y^3}-k_1A_1B_{12}^s\frac{\partial^3 v_0}{\partial x^2\partial y}$$
$$-(k_1A_1+2k_2B_1)B_{26}^s\frac{\partial^3 v_0}{\partial x\partial y^2}$$
$$-(k_1A_1+k_2B_1)B_{66}^s\frac{\partial^3 v_0}{\partial x^2\partial y} \quad (17d)$$
$$+k_1A_1D_{11}^s\frac{\partial^4 w_0}{\partial x^4}+k_2B_1D_{22}^s\frac{\partial^4 w_0}{\partial y^4}$$
$$+k_1A_1D_{12}^s\frac{\partial^4 w_0}{\partial x^2\partial y^2}+k_2B_1D_{12}^s\frac{\partial^4 w_0}{\partial x^2\partial y^2}$$
$$+(3k_1A_1+k_2B_1)D_{16}^s\frac{\partial^4 w_0}{\partial x^3\partial y}$$
$$+(k_1A_1+3k_2B_1)D_{26}^s\frac{\partial^4 w_0}{\partial x\partial y^3}$$
$$+2(k_1A_1+k_2B_1)D_{66}^s\frac{\partial^4 w_0}{\partial x^2\partial y^2}$$

$$-(k_1A_1)^2H_{11}^s\frac{\partial^4\theta}{\partial x^4}-(k_2B_1)^2H_{22}^s\frac{\partial^4\theta}{\partial y^4} \quad (17e)$$
$$+(k_1A_1)^2A_{55}^s\frac{\partial^2\theta}{\partial x^2}+(k_2B_1)^2A_{44}^s\frac{\partial^2\theta}{\partial y^2}$$
$$+2k_1A_1k_2B_1A_{45}^s\frac{\partial^2\theta}{\partial x\partial y}-2k_1A_1k_2B_1H_{12}^s\frac{\partial^4\theta}{\partial x^2\partial y^2}$$
$$-2k_1A_1(k_1A_1+k_2B_1)H_{16}^s\frac{\partial^4\theta}{\partial x^3\partial y}$$
$$-2k_2B_1(k_1A_1+k_2B_1)H_{26}^s\frac{\partial^4\theta}{\partial x\partial y^3} \quad (17e)$$
$$-(k_1A_1+k_2B_1)^2H_{66}^s\frac{\partial^4\theta}{\partial x^2\partial y^2}+k_1A_1\frac{\partial^2 M_x^{s^T}}{\partial x^2}$$
$$+k_2B_1\frac{\partial^2 M_y^{s^T}}{\partial y^2}+(k_1A_1+k_2B_1)\frac{\partial^2 M_{xy}^{s^T}}{\partial x\partial y}=0$$

3. Analytical solutions for anti-symmetric cross-ply laminated plates

By using the Navier approach, the closed form solution of Eqs. (17) is determined for simply-supported rectangular plates.

For anti-symmetric cross-ply laminates, the following stiffnesses are equal to zero

$$\begin{aligned}&A_{16}=A_{26}=D_{16}=D_{26}=D_{16}^s=D_{26}^s\\&=H_{16}^s=H_{26}^s=0\\&B_{12}=B_{16}=B_{26}=B_{66}=B_{12}^s=B_{16}^s\\&=B_{26}^s=B_{66}^s=0\end{aligned} \quad (18)$$

And for anti-symmetric plates, the thermal expansion coefficient equals zero, $\alpha_{xy}=0$.

The boundary condition for simply-supported edges could be expressed as

$$v_0=w_0=\frac{\partial\theta}{\partial y}=N_x=M_x^b=M_x^s=0 \quad at \quad x=0,a \quad (19a)$$

$$u_0=w_0=\frac{\partial\theta}{\partial x}=N_y=M_y^b=M_y^s=0 \quad at \quad x=0,b \quad (19b)$$

We assume that the thermal and transverse mechanical loadings are expanded in double Fourier series as

$$\begin{Bmatrix}q\\T_1\\T_2\\T_3\end{Bmatrix}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\begin{Bmatrix}q_{mn}\\T_{1mn}\\T_{2mn}\\T_{3mn}\end{Bmatrix}\sin\mu x\sin\lambda y \quad (20)$$

Where the coefficients T_{1mn}, T_{2mn}, T_{3mn} and q_{mn} are expressed as follows

$$\begin{Bmatrix}q_{mn}\\T_{1mn}\\T_{2mn}\\T_{3mn}\end{Bmatrix}=\frac{4}{ab}\int_0^a\int_0^b\begin{Bmatrix}q_{mn}\\T_1\\T_2\\T_3\end{Bmatrix}\sin\mu x\sin\lambda y\,dxdy \quad (21)$$

The coefficients T_{1mn}, T_{2mn}, T_{3mn} and q_{mn} can be evaluated by integrating Eq. (21) as:

T_{1mn}, T_{2mn}, $T_{3mn}=T_0$ for $m = n = 1$, and for a single sinusoidal thermal loading, and T_{1mn}, T_{2mn}, $T_{3mn}=\frac{16T_0}{\pi^2 mn}$ for m, n odd, in case of uniformly distributed thermal loading, where T_0 represents the intensity of thermal loading.

$q_{mn} = q_0$ for $m = n = 1$, for a single sinusoidal mechanical loading, and $q_{mn} = \frac{16q_0}{\pi^2 mn}$ for m, n odd, for uniformly distributed mechanical loading, where q_0 represents the intensity of mechanical loading.

The solution form for (u_0, v_0, w_0, θ) to solve the problem is adopted as follows

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn}\cos(\mu x)\sin(\lambda y) \\ V_{mn}\sin(\mu x)\cos(\lambda y) \\ W_{mn}\sin(\mu x)\cos(\lambda y) \\ \theta_{mn}\sin(\mu x)\cos(\lambda y) \end{Bmatrix} \tag{22}$$

Where $U_{mn}, V_{mn}, W_{mn}, \theta_{mn}$ are arbitrary parameters to be determined, substituting Eqs. (20)-(22) into governing Eq. (17), we obtain the following operator equation

$$[K]\{\delta\} = \{F\} \tag{23}$$

Where $\{\delta\} = \{U_{mn}, V_{mn}, W_{mn}, \theta_{mn}\}$ and [K] is the symmetric matrix given by

$$[K] = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{12} & K_{22} & K_{23} & K_{24} \\ K_{13} & K_{23} & K_{33} & K_{34} \\ K_{14} & K_{24} & K_{34} & K_{44} \end{bmatrix} \tag{24}$$

In which

$$\begin{aligned}
K_{11} &= -A_{11}\lambda^2 - A_{66}\mu^2,\ \ K_{12} = -(A_{12} + A_{66})\lambda\mu \\
K_{13} &= B_{11}\lambda^3,\ \ K_{14} = -B_{11}^s\lambda^3 k_1 A_1 \\
K_{22} &= -A_{22}\mu^2 - A_{66}\lambda^2,\ \ K_{23} = B_{22}\mu^3, \\
K_{24} &= -B_{22}^s k_2 B_1 \mu^3 \\
K_{33} &= -D_{11}\lambda^4 - 2(D_{12} + 2D_{66})\lambda^2\mu^2 - D_{22}\mu^4 \\
K_{34} &= D_{11}^s k_1 A_1 \lambda^4 + D_{22}^s k_2 B_1 \mu^4 \\
&\quad + D_{12}^s \lambda^2\mu^2(k_1 A_1 + k_2 B_1) \\
&\quad + 2D_{66}^s \lambda^2\mu^2(k_1 A_1 + k_2 B_1) \\
K_{44} &= -H_{11}^s (k_1 A_1)^2\lambda^4 - 2H_{12}^s k_1 A_1 k_2 B_1 \lambda^2\mu^2 \\
&\quad - H_{22}^s (k_2 B_1)^2\mu^4 - H_{66}^s (k_1 A_1 + k_2 B_1)\lambda^2\mu^2 \\
&\quad - S_{55}^s k_1 A_1 \lambda^2 - S_{44}^s k_2 B_1 \mu^2
\end{aligned} \tag{25}$$

And $\{F\} = \{F_1, F_2, F_3, F_4\}$ is the generalized force given by

$$\begin{aligned}
F_1 &= \lambda\begin{bmatrix} (L_{11} + L_{21})T_{1mn} + (P_{11} + P_{21})T_{2mn} \\ +(R_{11} + R_{21})T_{3mn} \end{bmatrix} \\
F_2 &= \mu\begin{bmatrix} (L_{12} + L_{22})T_{1mn} + (P_{11} + P_{22})T_{2mn} \\ +(R_{11} + R_{22})T_{3mn} \end{bmatrix} \\
F_3 &= -\lambda^2\begin{bmatrix} (S_{11} + S_{21})T_{1mn} \\ +(F_{11} + F_{21})T_{2mn} \\ +(U_{11} + U_{21})T_{3mn} \end{bmatrix} \\
&\quad -\mu^2\begin{bmatrix} (S_{11} + S_{22})T_{1mn} \\ +(F_{11} + F_{22})T_{2mn} \\ +(U_{11} + U_{22})T_{3mn} \end{bmatrix} - q_{mn} \\
F_4 &= -k_1 A_1 \lambda^2\begin{bmatrix} (V_{11} + V_{21})T_{1mn} \\ +(W_{11} + W_{21})T_{2mn} \\ +(X_{11} + X_{21})T_{3mn} \end{bmatrix} \\
&\quad -k_2 B_1 \mu^2\begin{bmatrix} (V_{11} + V_{22})T_{1mn} \\ +(W_{11} + W_{22})T_{2mn} \\ +(X_{11} + X_{22})T_{3mn} \end{bmatrix}
\end{aligned} \tag{26}$$

Where

$$(L_{ij}, P_{ij}, R_{ij}) = \sum_{k=1}^{n}\int_{z_k}^{z_{k+1}} \alpha_i^{(k)}\overline{Q}_{ij}^{(k)}\,(1, \frac{z}{h}, \frac{f(z)}{h}),\quad (i,j = 1,2) \tag{27a}$$

$$(S_{ij}, T_{ij}, U_{ij}) = \sum_{k=1}^{n}\int_{z_k}^{z_{k+1}} \alpha_i^{(k)}\overline{Q}_{ij}^{(k)}\,(z, \frac{z^2}{h}, \frac{f(z)z}{h}),\quad (i,j = 1,2) \tag{27b}$$

$$(V_{ij}, W_{ij}, X_{ij}) = \sum_{k=1}^{n}\int_{z_k}^{z_{k+1}} \alpha_i^{(k)}\overline{Q}_{ij}^{(k)} f(z)\,(1, \frac{z}{h}, \frac{f(z)}{h}),\quad (i,j = 1,2) \tag{27c}$$

4. Numerical results and discussion

To verify the accuracy of the present theory, simply-supported two layers (0°/90°) and four layers (0°/90°/ 0°/90°) anti-symmetric laminated plates under uniformly distributed nonlinear thermo-mechanical loadings are to be considered. In all cases, the lamina properties are assumed to be

$$\frac{E_1}{E_2} = 25,\ \ G_{12} = 0{,}5E_2,\ \ G_{13} = G_{12},\ \ G_{23} = 0{,}2E_2,\ \ \mu_{12} = 0{,}25, \frac{\alpha_1}{\alpha_2} = 3$$

4.1 Two layers (0°/90°) anti-symmetric plat

Dimensionless displacements ($\bar{u}$, $\bar{v}$, $\bar{w}$) and stresses ($\bar{\sigma}_x$,$\bar{\sigma}_y$,$\bar{\tau}_{xy}$,$\bar{\tau}_{xz}$,$\bar{\tau}_{yz}$) utilized for two layers (0°/90°) anti-symmetric plate expressed as

$$\begin{aligned}
\bar{u} &= u(0, \frac{b}{2}, -\frac{h}{2})\frac{1}{(q_0 h s^3/E_2) + (\alpha_1 T_1 a^2)}, \\
\bar{v} &= v(\frac{a}{2}, 0, -\frac{h}{2})\frac{1}{(q_0 h s^3/E_2) + (\alpha_1 T_1 a^2)}, \\
\bar{w} &= w(\frac{a}{2}, \frac{b}{2}, 0)\frac{100}{(q_0 a^4/E_2 h^3) + (\alpha_1 T_1 a^2/10h)}, \\
\bar{\sigma}_x &= \sigma_x(\frac{a}{2}, \frac{b}{2}, -\frac{h}{2})\frac{1}{(q_0 a^2/h^2) + (E_2\alpha_1 T_1 a^2)}, \\
\bar{\sigma}_y &= \sigma_y(\frac{a}{2}, \frac{b}{2}, +\frac{h}{2})\frac{1}{(q_0 a^2/h^2) + (E_2\alpha_1 T_1 a^2)}, \\
\bar{\tau}_{xy} &= \tau_{xy}(0, 0, -\frac{h}{2})\frac{1}{(q_0 a^2/h^2) + (E_2\alpha_1 T_1 a^2)}, \\
\bar{\tau}_{xz} &= \tau_{xz}(0, \frac{b}{2}, 0)\frac{1}{(q_0 a/h) + (E_2\alpha_1 T_1 a^2)}, \\
\bar{\tau}_{yz} &= \tau_{yz}(\frac{a}{2}, 0, 0)\frac{1}{(q_0 a/h) + (E_2\alpha_1 T_1 a^2)}.
\end{aligned}$$

Numerical results for two layers (0°/90°) anti-symmetric plates predicted in this work are discussed and compared with those of the classical, first-order, higher-order and trigonometric theories obtained by Ghugal and Kulkarni (2013b).

Table 1 Normalized displacements and in-plan stresses for square two layers (0°/90°) anti-symmetric laminated plate subjected to uniformly distributed nonlinear thermo-mechanical loading for aspect ratios 4 and 10

a/h	Theory	$\bar{u}$	$\bar{v}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$
4	Present	0.0481	0.0788	4.8625	-2.7199	2.7199	0.3183
	TSDT*	0.0470	0.0755	4.8397	-2.6430	2.6430	0.4662
	HSDT	0.0468	0.0756	4.8700	-2.6303	2.6303	0.4695
	FSDT*	0.0424	0.0736	5.0904	-2.4166	2.4166	0.4960
	CPT*	0.0442	0.0776	3.5430	-2.5043	2.5043	0.3112
10	Present	0.0204	0.0471	2.2105	-1.3506	1.3506	0.1327
	TSDT*	0.0203	0.0468	2.2084	-1.3428	1.3428	0.1505
	HSDT	0.0202	0.0468	2.2125	-1.3408	1.3408	0.1509
	FSDT*	0.0194	0.0465	2.2433	-1.3073	1.3073	0.1525
	CPT*	0.0196	0.0468	1.9926	-1.3165	1.3165	0.1300

*Ghugal and Kulkarni (2013)

Table 2 Normalized transverse shear stresses for square two layer (0°/90°) anti-symmetric laminated plates subjected to uniformly distributed nonlinear thermo-mechanical loading for aspect ratios 4 and 10 (T1 = 0)

a/h	Theory	$\bar{\tau}_{xz}$	$\bar{\tau}_{yz}$
4	Present	0.1682	0.1682
	TSDT	0.1925	0.1925
	HSDT	0.1917	0.1917
	FSDT	0.1996	0.1996
10	Present	0.2283	0.2283
	TSDT	0.2493	0.2493
	HSDT	0.2402	0.2402
	FSDT	0.2171	0.2171

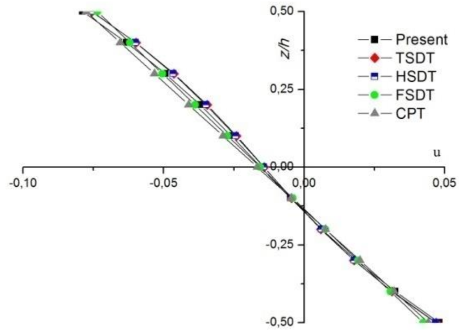

Fig. 2 Normalized in-plan displacement ($\bar{u}$) through the thickness for a two-layer laminated plate for aspect ratio 4

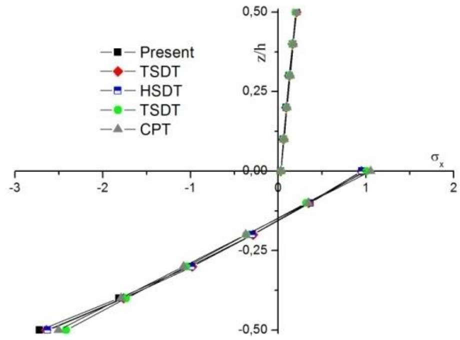

Fig. 3 Normalized in-plan normal stress ($\bar{\sigma}_x$) through the thickness of a two-layer plate for aspect ratio 4

The results of the in-plan displacements ($\bar{u}$, $\bar{v}$), the transversenormal displacement ($\bar{w}$), the in-plan normal stresses ($\bar{\sigma}_x$, $\bar{\sigma}_y$) and the in-plan shear stress ($\bar{\tau}_{xy}$) of two layers (0°/90°) anti-symmetric laminated plate subjected to combined uniformly distributed thermo-mechanical loading for aspect ratios 4 and 10 are reported in Table 1, whereas the transverse shear stresses ($\bar{\tau}_{xz}$, $\bar{\tau}_{yz}$) are shown in Table 2.

The examination of Table 1 reveals that the in-plan displacements ($\bar{u}$, $\bar{v}$) obtained using the present theory for two anti-symmetric layers plates are in good agreementwith those provided by the TSDT, HSDT, FSDT, and CPT for both aspect ratios 4 and 10. The transverse normal displacements ($\bar{w}$) predicted by the present theory are well converged with those given by the TSDT, HSDT, and FSDT whereas the results provided by the CPT are under predicted. Fig. 2 displays the variation of in-plan displacement ($\bar{u}$) through the thickness for a two anti-symmetric layers plate for aspect ratio 4. The results of the in-plan normal stresses ($\bar{\sigma}_x$, $\bar{\sigma}_y$) obtained using the present theory are comparable with those given by the TSDT, HSDT, FSDT, and CPT for aspect ratios 4 and 10, and thethrough-the-thickness variation of the in-plan normal stress ($\bar{\sigma}_x$) as shown in Fig. 3, it is observed that the results obtained by the use of the present theory converged very well along the width with those ofthe TSDT, HSDT, FSDT and CPT theories.The values of the in-plan stresses $\bar{\tau}_{xy}$ are found lower than those of the TSDT, HSDT,and FSDT, and are comparable with those provided by the CPT for thick and thin plates.

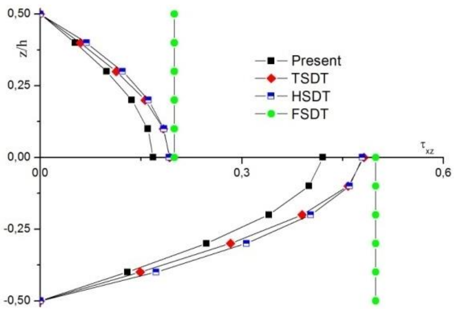

Fig. 4 Normalized transverse shear stress $(\bar{\tau}_{xz})$ through the thickness of a two-layer plate for aspect ratio 4

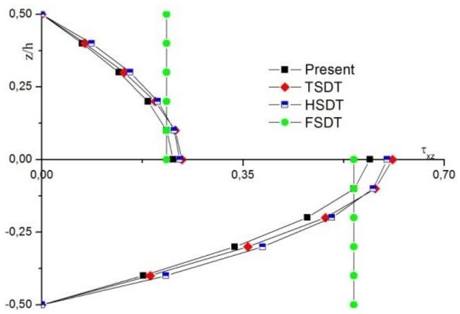

Fig. 5 Normalized transverse shear stress $(\bar{\tau}_{xz})$ through the thickness of a two-layer plate for aspect ratio 10

The results reported in Table 2 show that transverse shear stresses values estimated by the present theory are slightly lower than those given by the TSDT and HSDT for aspect ratios 4 and 10. The FSDT predicts the highest value for aspect ratio 4 while it gives the lowest values for aspect ratio 4, and these values are constant through the thickness. The variation of the transverse shear stresses $(\bar{\tau}_{xz})$ through the thickness for a two anti-symmetric layers plate for aspect ratios 4 and 10 is shown in Figs. 4 and 5 respectively. It is noticed that the stress continuity is not imposed in the present theories.

4.2 Fourlayers (0°/90°/0°/90°) anti-symmetric plate

Dimensionless displacements $(\bar{u}, \bar{v}, \bar{w})$ and stresses $(\bar{\sigma}_x, \bar{\sigma}_y, \bar{\tau}_{xy}, \bar{\tau}_{xz}, \bar{\tau}_{yz})$ utilized for four layers (0°/90°/0°/90°) anti-symmetric plate

$$\bar{u} = u\left(0,\frac{b}{2},-\frac{h}{2}\right)\frac{1}{\left(\frac{q_0 h s^3}{E_2}\right)+(\alpha_1 T_1 a^2)},$$
$$\bar{v} = v\left(\frac{a}{2},0,-\frac{h}{2}\right)\frac{1}{\left(\frac{q_0 h s^3}{E_2}\right)+(\alpha_1 T_1 a^2)},$$
$$\bar{w} = w(\frac{a}{2},\frac{b}{2},0)\frac{100}{(q_0 a^4/E_2 h^3)+(\alpha_1 T_1 a^2/10h)},$$
$$\bar{\sigma}_x = \sigma_x(\frac{a}{2},\frac{b}{2},-\frac{h}{2})\frac{1}{(q_0 a^2/h^2)+(E_2\alpha_1 T_1 a^2)},$$
$$\bar{\sigma}_y = \sigma_y(\frac{a}{2},\frac{b}{2},+\frac{h}{2})\frac{1}{(q_0 a^2/h^2)+(E_2\alpha_1 T_1 a^2)},$$
$$\bar{\tau}_{xy} = \tau_{xy}(0,0,-\frac{h}{2})\frac{1}{(q_0 a^2/h^2)+(E_2\alpha_1 T_1 a^2)},$$
$$\bar{\tau}_{xz} = \tau_{xz}(0,\frac{b}{2},-\frac{h}{4})\frac{1}{(q_0 a/h)+(E_2\alpha_1 T_1 a^2)},$$
$$\bar{\tau}_{yz} = \tau_{yz}(\frac{a}{2},0,-\frac{h}{4})\frac{1}{(q_0 a/h)+(E_2\alpha_1 T_1 a^2)}.$$

It is to be noticed that the results for four layers (0°/90°/0°/90°) anti-symmetric laminated plates under uniformly distributed nonlinear thermo-mechanical loadingsof the classical, first-order, higher-order, and trigonometric theories used in the discussions and comparisons with those predicted by the present theory are not available in the literature but generated using the aforementioned theories.

Table 3 reveals that the in-plan displacements $(\bar{u}, \bar{v})$ obtained using the present theory for a four-layersanti-symmetric plate are found to agree well with theTSDT, HSDT, and FSDT whereas the CPT underpredicts the in-plan displacements for aspect ratios 4 and 10. The transverse normal displacements $\bar{w}$ for a four anti-symmetric layers plate are in good agreement with the

Table 3 Normalized displacements and in-plan stresses for square four layers (0°/90°/0°/90°) antisymmetric laminated plates subjected to uniformly distributed nonlinear thermo-mechanical loading for aspect ratio 4 and 10 (T1 = 0)

a/h	Theory	$\bar{u}$	$\bar{v}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$
4	Present	0.0445	0.0567	4.1670	-2.4421	2.4421	0.2732
	TSDT*	0.0423	0.0534	4.1327	-2.2788	2.2788	0.4531
	HSDT	0.0425	0.0533	4.1536	-2.2853	2.2853	0.4604
	FSDT*	0.0405	0.0488	4.0321	-2.2469	2.2469	0.5107
	CPT*	0.0206	0.0342	1.7870	-1.5600	1.5600	0.1552
10	Present	0.0159	0.0232	1.3542	-1.0223	1.0223	0.0846
	TSDT*	0.0157	0.0228	1.3501	-1.0033	1.0033	0.1071
	HSDT	0.0157	0.0228	1.3528	-1.0050	1.0050	0.1079
	FSDT*	0.0154	0.0221	1.3308	-1.0030	1.0030	0.1105
	CPT*	0.0128	0.0192	0.9665	-0.8749	0.8749	0.0629

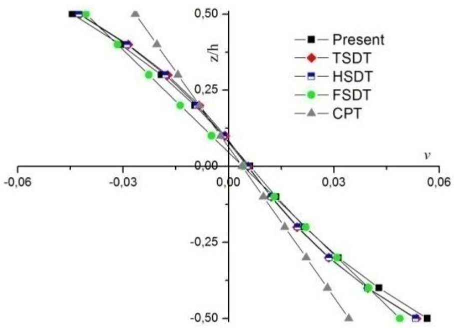

Fig. 6 Normalized in-plan displacement $(\bar{v})$ through the thickness for an anti-symmetric four-layer laminated plate for aspect ratio 4

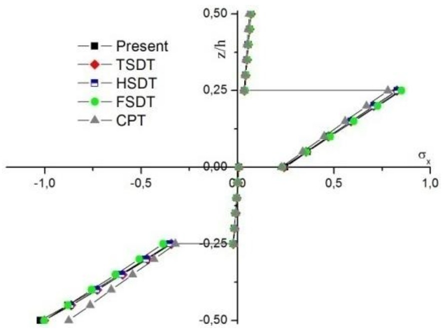

Fig. 7 Normalized in-plan normal stress $(\bar{\sigma}_x)$ through the thickness of an anti-symmetric four-layer laminated plate for aspect ratio 10

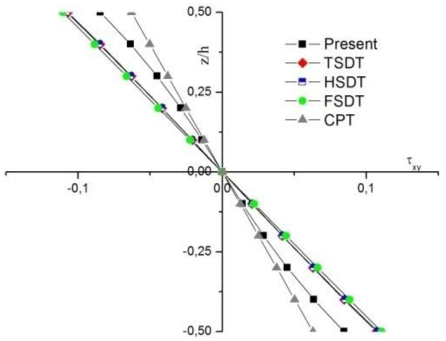

Fig. 8 Normalized in plane shear stress $(\bar{\tau}_{xy})$ through the thickness of an anti-symmetric four-layer laminated plate for aspect ratio 10

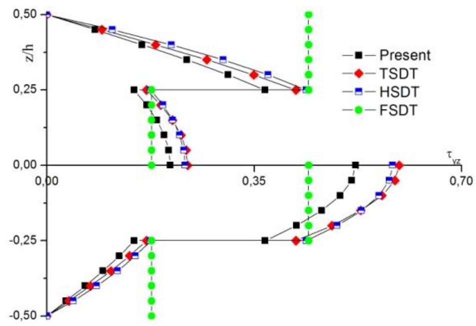

Fig. 9 Normalized transverse shear stress $(\bar{\tau}_{yz})$ through the thickness of an anti-symmetric four-layer laminated plate for aspect ratio 4

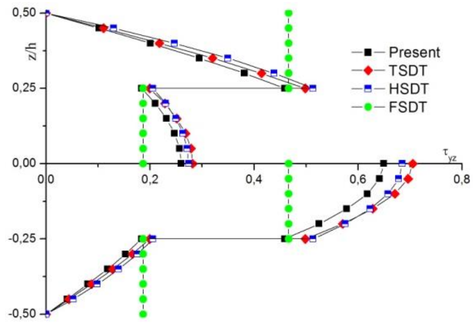

Fig. 10 Normalized transverse shear stress $(\bar{\tau}_{yz})$ through the thickness of an anti-symmetric four-layer laminated plate for aspect ratio 10

TSDT, HSDT, and FSDT whereas the results provided by the CPT are underestimated. Fig. 6 displays the distribution of the in-plan displacement $(\bar{v})$ through the thickness for an anti-symmetric four-layer laminated plate for aspect ratio 4, the results displayed in this figure show that the in-plan displacements given by the present theory, TSDT, HSDT, and FSDT are more or less identical along the thickness whereas those provided by the CPT are underpredicted. The results of the in-plan normal stresses $(\bar{\sigma}_x, \bar{\sigma}_y)$ obtained by the present theory are comparable withtheTSDT, HSDT, and FSDT whereas the CPT underestimates the same for both aspect ratios 4 and 10. The variation of normalized in-plan normal stress $(\bar{\sigma}_x)$ through the thickness of a four anti-symmetric layers laminated plate for aspect ratio 10 is displayed in Fig. 7, from whichit is noticed that the results obtained by the use of the present theory are closeto those given by the TSDT, HSDT, and FSDT whereas the results provided by the CPT are underestimated. The in-plan stresses $\bar{\tau}_{xy}$ show intermediate values between those of the TSDT, HSDT, FSDT and those of the CPT for thick and thin plates, the distribution of these stresses for aspect ratio 10 is displayed in Fig. 8.

From Table 4, it is observed that transverse shear stresses values given by the present theory converged accurately enough with the other theories; the TSDT, HSDT, and FSDT. Figs. 9 and10 displayed normalized transverse shear stress $(\bar{\tau}_{yz})$ through the thickness of an anti-symmetric four-layer laminated plate for aspect ratios 4 and 10 respectively. It is noticed from the results of the mentioned figures that the present theory converges well with the other higher-order theories whereas the FSDT gives a constant value of transverse shear stress trough the thickness.

5. Conclusions

The present article delves on the presentation of a refined four-variable theory for the analysis of the response of simply supported two layers (0°/90°) and four layers (0°/90°/0°/90°) anti-symmetric laminated composite plates subjected to a combined uniformly distributed nonlinear thermo-mechanical loading. The concluding remarks are as follows:

- The present theory does not require the use of a shear correction factor.
- The number of variables and the governing equations are reduced to four instead of five or more in the other theories.
- The results provided using the present theory are found to converge well with the trigonometric shear theory along with the higher-order shear one.

An improvement of the present formulation will be considered in the future work to consider other type of materials (Arani and Kolahchi 2016, Kolahchi *et al*. 2016a, b, 2017b Bilouei *et al*. 2016, Madani *et al*. 2016, Zamanian *et al*. 2017, Kolahchi and Cheraghbak 2017, Kolahchi 2017, Hajmohammad *et al*. 2017, 2018a, b, c, Fakhar and Kolahchi 2018, Amnieh *et al*. 2018, Golabchi *et al*. 2018, Hosseini and Kolahchi 2018, Alimirzaei *et al*. 2019).

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