

Thermomechanical interactions in transversely isotropic magneto thermoelastic solid with two temperatures and without energy dissipation

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(Received April 25, 2019, Revised August 27, 2019, Accepted September 3, 2019)

Abstract. The purpose of this research paper is to depict the thermomechanical interactions in transversely isotropic magneto thermoelastic solid with two temperatures and without energy dissipation in generalized LS theories of thermoelasticity. The Laplace and Fourier transform techniques have been used to find the solution of the problem. The displacement components, stress components, and conductive temperature distribution with the horizontal distance are computed in the transformed domain and further calculated in the physical domain numerically. The effect of two temperature and relaxation time are depicted graphically on the resulting quantities.

Keywords: transversely isotropic thermoelastic; magneto generalized thermoelastic solid; Laplace and Fourier transform; mechanical and thermal sources

1. Introduction

It is well known that all the rotating large bodies have angular velocity, as well as magnetism, therefore, the thermoelastic interactions in a rotating medium under magnetic field is of importance. The study of interaction between mechanical and thermal fields in the anisotropic media particularly transversely isotropic is one of the most extensive and productive areas of continuum dynamics. Therefore, in an unbounded rotating elastic medium with angular velocity, with two temperature, rotation and relaxation time and without energy dissipation in generalized thermoelasticity has been studied in this research.

Marin (1997) had proved the Cesaro means of strain and kinetic energies of dipolar bodies with finite energy. Marin (1998) investigated and solved the initial-boundary value problem without recourse either to an energy conservation law or to any boundedness assumptions on the thermoelastic coefficients in thermoelastic bodies with voids. Ailawalia *et al.* (2010) had studied a rotating generalized thermoelastic medium in presence of two temperatures beneath hydrostatic stress and gravity with different kinds of sources using integral transforms. Singh and Yadav (2012) solved the transversely isotropic rotating magnetothermoelastic medium by cubic velocity equation of three plane waves without anisotropy, rotation, and thermal and magnetic effects. Banik and Kanoria (2012) studied the thermoelastic interaction in an isotropic infinite elastic body with a spherical cavity for the TPL (Three-

Phase-Lag) heat equation with two-temperature generalized thermoelasticity theory and has shown variations between two models: the two-temperature GN theory in presence of energy dissipation and two-temperature TPL model and has shown the effects of ramp parameters and two-temperature.

Mahmoud (2012) considered the impact of rotation, relaxation times, magnetic field, gravity field and initial stress on Rayleigh waves and attenuation coefficient in an elastic half-space of granular medium and obtained the analytical solution of Rayleigh waves velocity by using Lamé's potential techniques. Abd-alla and Alshaikh (2015) discussed the influence of magnetic field and rotation on plane waves in transversely isotropic thermoelastic medium under the GL theory in presence of two relaxation times to show the presence of three quasi plane waves in the medium. Marin *et al.* (2013) modelled a micro stretch thermoelastic body with two temperatures and eliminated divergences among the classical elasticity and research.

Sharma *et al.* (2015) investigated the 2-D deception in a transversely isotropic homogeneous thermoelastic solids in presence of two temperatures in GN-II theory with an inclined load (linear combination of normal load and tangential load). Kumar *et al.* (2016a, b) investigated the impact of Hall current in a transversely isotropic magnetothermoelastic in presence and absence of energy dissipation due to the normal force. Kumar *et al.* (2016c) studied the conflicts caused by thermomechanical sources in a transversely isotropic rotating homogeneous thermoelastic medium with magnetic effect as well as two temperature and applied to the thermoelasticity Green-Naghdi theories with and without energy dissipation using thermomechanical sources. Lata *et al.* (2016) studied two temperature and rotation aspect for GN-II and GN-III theory of thermoelasticity in a homogeneous transversely isotropic magnetothermoelastic medium for the case of the plane wave propagation and reflection. Ezzat *et al.* (2017b)

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proposed a mathematical model of electro-thermoelasticity for heat conduction with memory-dependent derivative. Kumar *et al.* (2017) analyzed the Rayleigh waves in a transversely isotropic homogeneous magnetothermoelastic medium in presence of two temperature, with Hall current and rotation. Marin *et al.* (2017) studied the GN-thermoelastic theory for a dipolar body using mixed initial BVP and proved a result of Hölder's-type stability. Lata (2018a) studied the impact of energy dissipation on plane waves in sandwiched layered thermoelastic medium of uniform thickness, with two temperature, rotation, and Hall current in the context of GN Type-II and Type-III theory of thermoelasticity. Ezzat and El-Bary (2017) had applied the magneto-thermoelasticity model to a one-dimensional thermal shock problem of functionally graded half-space based on memory-dependent derivative. Despite of this several researchers worked on different theory of thermoelasticity as Marin (1999, 1996, 1998a, 2009, 2010), Mahmoud *et al.* (2011, 2015), Atmane *et al.* (2015), Meradjah *et al.* (2015), Bousahla *et al.* (2016), Menasria *et al.* (2017), Ezzat *et al.* (2016), Marin and Öchsner (2017), Ezzat *et al.* (2012, 2015, 2017a, b), Ezzat and El-Bary (2017), Lata (2018b), Othman and Marin (2017), Chauthale and Khobragade (2017), Lata and Kaur (2018, 2019a, b, c, d), Kumar *et al.* (2018), Kaur and Lata (2019a, b), Marin and Craciun (2017), Marin *et al.* (2017).

In spite of these, not much work has been carried out in thermomechanical interactions in transversely isotropic magneto thermoelastic solid with two temperature, rotation and with relaxation time and without energy dissipation in generalized thermoelasticity. Keeping these considerations in mind, analytic expressions for the displacements, stresses, and temperature distribution in two-dimensional homogeneous, transversely isotropic magneto-thermoelastic solids with two temperatures and without energy dissipation has been obtained.

2. Basic equations

Consider a transversely isotropic thermoelastic medium, with the constitutive relation (Kumar *et al.* 2016a) given by

$$t_{ij} = C_{ijkl}e_{kl} - \beta_{ij}T. \quad (1)$$

Equation of motion as described by Schoenberg and Censor (1973) for a transversely isotropic thermoelastic medium rotating uniformly with an angular velocity $\Omega = \Omega \mathbf{n}$, where \mathbf{n} is a unit vector representing the direction of the axis of rotation and taking into account Lorentz force

$$t_{ij,j} + F_i = \rho \{ \ddot{u}_i + (\Omega \times (\Omega \times \mathbf{u}))_i + (2\Omega \times \dot{\mathbf{u}})_i \}, \quad (2)$$

where $\Omega = \Omega \mathbf{n}$, \mathbf{n} is a unit vector representing the direction of the axis of rotation, The term $\Omega \times (\Omega \times \mathbf{u})$ is the additional centripetal acceleration due to the time-varying motion only, and the term $2\Omega \times \dot{\mathbf{u}}$ is the Coriolis acceleration. All other terms are as usual

$$F_i = \mu_0 (\dot{\mathbf{j}} \times \vec{H}_0)_i.$$

The heat conduction equation without energy dissipation using Lord-Shulman (1967) model is

$$K_{ij}\varphi_{,ij} + \rho(Q + \tau_0\dot{Q}) = \beta_{ij}T_0(\dot{e}_{ij} + \tau_0\ddot{e}_{ij}) + \rho C_E(\dot{T} + \tau_0\ddot{T}), \quad (3)$$

where

$$\beta_{ij} = C_{ijkl}\alpha_{ij}, \quad (4)$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad i, j = 1, 2, 3. \quad (5)$$

$$T = \varphi - a_{ij}\varphi_{,ij}$$

$$\beta_{ij} = \beta_i\delta_{ij}, \quad K_{ij} = K_i\delta_{ij}, \quad i \text{ is not summed.}$$

Here C_{ijkl} ($C_{ijkl} = C_{klij} = C_{jikl} = C_{ijlk}$) are elastic parameters.

3. Formulation and solution of the problem

We consider a homogeneous transversely isotropic magnetothermoelastic medium, permeated by an initial magnetic field $\vec{H}_0 = (0, H_0, 0)$ acting along y -axis. The rectangular Cartesian co-ordinate system (x, y, z) having origin on the surface ($z = 0$) with z -axis pointing vertically into the medium is introduced. The surface of the half-space is subjected to an inclined load acting at $z = 0$.

In addition, we consider that

$$\Omega = (0, \Omega, 0).$$

From the generalized Ohm's law

$$J_2 = 0.$$

The density components J_1 and J_3 are given as

$$J_1 = -\varepsilon_0\mu_0 H_0 \frac{\partial^2 w}{\partial t^2}, \quad (6)$$

$$J_3 = \varepsilon_0\mu_0 H_0 \frac{\partial^2 u}{\partial t^2}. \quad (7)$$

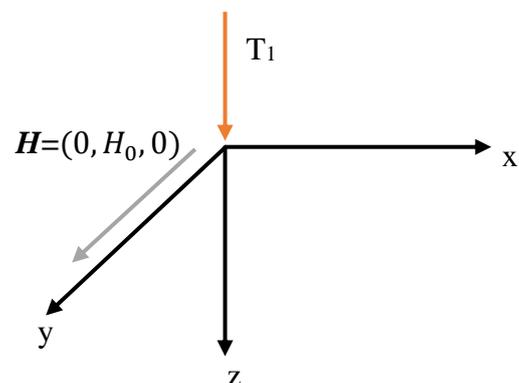


Fig. 1 Geometry of the problem

In addition, the equations of displacement vector $(\vec{u}, \vec{v}, \vec{w})$ and conductive temperature φ for transversely isotropic thermoelastic solid in presence of two temperature and without energy dissipation, using the proper transformation on Eqs. (1)-(2) following Slaughter (2002) are as under

$$\begin{aligned} \vec{u} &= u(x, z, t), \vec{v} = 0, \vec{w} \\ &= w(x, z, t) \text{ and } \varphi = \varphi(x, z, t). \end{aligned} \tag{8}$$

Eqs. (1)-(3) with the aid of (8), yield

$$\begin{aligned} C_{11} \frac{\partial^2 u}{\partial x^2} + C_{13} \frac{\partial^2 w}{\partial x \partial z} + C_{44} \left(\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial x \partial z} \right) \\ - \beta_1 \frac{\partial}{\partial x} \left\{ \varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial x^2} + a_3 \frac{\partial^2 \varphi}{\partial z^2} \right) \right\} - \mu_0 J_3 H_0 \tag{9} \\ = \rho \left(\frac{\partial^2 u}{\partial t^2} - \Omega^2 u + 2\Omega \frac{\partial w}{\partial t} \right), \end{aligned}$$

$$\begin{aligned} (C_{13} + C_{44}) \frac{\partial^2 u}{\partial x \partial z} + C_{44} \frac{\partial^2 w}{\partial x^2} + C_{33} \frac{\partial^2 w}{\partial z^2} \\ - \beta_3 \frac{\partial}{\partial z} \left\{ \varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial x^2} + a_3 \frac{\partial^2 \varphi}{\partial z^2} \right) \right\} - \mu_0 J_1 H_0 \tag{10} \\ = \rho \left(\frac{\partial^2 w}{\partial t^2} - \Omega^2 w - 2\Omega \frac{\partial u}{\partial t} \right), \end{aligned}$$

$$\begin{aligned} K_1 \frac{\partial^2 \varphi}{\partial x^2} + K_3 \frac{\partial^2 \varphi}{\partial z^2} + \rho(Q + \tau_0 \dot{Q}) \\ = \rho C_E (\dot{T} + \tau_0 \ddot{T}) + T_0 \frac{\partial}{\partial t} \left\{ \begin{aligned} &\beta_1 \left(1 + \tau_0 \frac{\partial}{\partial t} \right) \frac{\partial u}{\partial x} \\ &+ \beta_3 \left(1 + \tau_0 \frac{\partial}{\partial t} \right) \frac{\partial w}{\partial z} \end{aligned} \right\}, \tag{11} \end{aligned}$$

and

$$t_{11} = C_{11} e_{11} + C_{13} e_{13} - \beta_1 T, \tag{12}$$

$$t_{33} = C_{13} e_{11} + C_{33} e_{33} - \beta_3 T, \tag{13}$$

$$t_{13} = 2C_{44} e_{13}, \tag{14}$$

where

$$\begin{aligned} T &= \varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial x^2} + a_3 \frac{\partial^2 \varphi}{\partial z^2} \right), \\ \beta_1 &= (C_{11} + C_{12}) \alpha_1 + C_{13} \alpha_3, \\ \beta_3 &= 2C_{13} \alpha_1 + C_{33} \alpha_3, \end{aligned}$$

We consider that the medium is initially at rest. Therefore, the preliminary and symmetry conditions are given by

$$\begin{aligned} u(x, z, 0) = 0 = \dot{u}(x, z, 0), \\ w(x, z, 0) = 0 = \dot{w}(x, z, 0), \\ \varphi(x, z, 0) = 0 = \dot{\varphi}(x, z, 0) \text{ for } z \geq 0, -\infty < x < \infty, \\ u(x, z, t) = w(x, z, t) = \varphi(x, z, t) = 0 \\ \text{for } t > 0 \text{ when } z \rightarrow \infty. \end{aligned}$$

To simplify the solution, mention below dimensionless quantities are used

$$\begin{aligned} x' &= \frac{x}{L}, \quad u' = \frac{\rho c_1^2}{L \beta_1 T_0} u, \quad t' = \frac{c_1}{L} t, \\ w' &= \frac{\rho c_1^2}{L \beta_1 T_0} w, \quad T' = \frac{T}{T_0}, \quad t'_{11} = \frac{t_{11}}{\beta_1 T_0}, \\ t'_{33} &= \frac{t_{33}}{\beta_1 T_0}, \quad t'_{31} = \frac{t_{31}}{\beta_1 T_0}, \quad \varphi' = \frac{\varphi}{T_0}, \tag{15} \\ a'_1 &= \frac{a_1}{L^2}, \quad z' = \frac{z}{L}, \quad a'_3 = \frac{a_3}{L^2}, \\ h' &= \frac{h}{H_0}, \quad \Omega' = \frac{\Omega}{C_1}. \end{aligned}$$

Making use of (15) in Eqs. (9)-(11), after suppressing the primes, yield

$$\begin{aligned} \frac{\partial^2 u}{\partial x^2} + \delta_4 \frac{\partial^2 w}{\partial x \partial z} + \delta_2 \left(\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial x \partial z} \right) \\ - \frac{\partial}{\partial x} \left\{ \varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial x^2} + a_3 \frac{\partial^2 \varphi}{\partial z^2} \right) \right\} \tag{16} \\ = \left(\frac{\varepsilon_0 \mu_0^2 H_0^2}{\rho} + 1 \right) \frac{\partial^2 u}{\partial t^2} - \Omega^2 u + 2\Omega \frac{\partial w}{\partial t}, \end{aligned}$$

$$\begin{aligned} \delta_1 \frac{\partial^2 u}{\partial x \partial z} + \delta_2 \frac{\partial^2 w}{\partial x^2} + \delta_3 \frac{\partial^2 w}{\partial z^2} \\ - \frac{\beta_3}{\beta_1} \frac{\partial}{\partial z} \left\{ \varphi - \left(a_1 \frac{\partial^2 \varphi}{\partial x^2} + a_3 \frac{\partial^2 \varphi}{\partial z^2} \right) \right\} \tag{17} \\ = \left(\frac{\varepsilon_0 \mu_0^2 H_0^2}{\rho} + 1 \right) \frac{\partial^2 w}{\partial t^2} - \Omega^2 w + 2\Omega \frac{\partial u}{\partial t}, \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 \varphi}{\partial x^2} + \frac{K_3}{K_1} \frac{\partial^2 \varphi}{\partial z^2} + \rho \left(1 + \tau_0 \frac{c_1}{L} \frac{\partial}{\partial t} \right) Q \\ = \delta_5 \frac{\partial}{\partial t} \left(1 + \tau_0 \frac{c_1}{L} \frac{\partial}{\partial t} \right) \left[\varphi - a_1 \frac{\partial^2 \varphi}{\partial x^2} - a_3 \frac{\partial^2 \varphi}{\partial z^2} \right] \tag{18} \\ + \delta_6 \frac{\partial}{\partial t} \left(1 + \tau_0 \frac{c_1}{L} \frac{\partial}{\partial t} \right) \left[\beta_1 \frac{\partial u}{\partial x} + \beta_3 \frac{\partial w}{\partial z} \right]. \end{aligned}$$

Where

$$\begin{aligned} \delta_1 &= \frac{C_{13} + C_{44}}{c_{11}}, \quad \delta_2 = \frac{C_{44}}{c_{11}}, \quad \delta_3 = \frac{C_{33}}{c_{11}}, \\ \delta_4 &= \frac{C_{13}}{c_{11}}, \quad \delta_5 = \frac{\rho C_E C_1 L}{K_1}, \quad \delta_6 = \frac{T_0 \beta_1 L}{\rho C_1 K_1} \end{aligned}$$

Apply Laplace and Fourier transforms defined by

$$\tilde{f}(x, z, s) = \int_0^\infty f(x, z, t) e^{-st} dt \tag{19}$$

$$\hat{f}(\xi, z, s) = \int_{-\infty}^\infty \tilde{f}(x, z, s) e^{i\xi x} dx \tag{20}$$

on Eqs. (16)-(18), we obtain a system of equations

$$\begin{aligned} [-\xi^2 + \delta_2 D^2 - \delta_7 s^2 + \Omega^2] \hat{u}(\xi, z, s) \\ + [\delta_4 D i \xi + \delta_2 D i \xi - 2\Omega s] \hat{w}(\xi, z, s) \\ + (-i\xi) [1 + a_1 \xi^2 - a_3 D^2] \hat{\varphi}(\xi, z, s) = 0, \tag{21} \end{aligned}$$

$$\begin{aligned} & [\delta_1 D i \xi + 2 \Omega s] \hat{u}(\xi, z, s) \\ & + [-\delta_2 \xi^2 + \delta_3 D^2 - \delta_7 s^2 + \Omega^2] \hat{w}(\xi, z, s) \\ & - \frac{\beta_3}{\beta_1} D [1 + a_1 \xi^2 - a_3 D^2] \hat{\phi}(\xi, z, s) = 0, \end{aligned} \quad (22)$$

$$\begin{aligned} & [\delta_6 s \delta_8 \beta_1 i \xi] \hat{u}(\xi, z, s) \\ & + [\delta_6 s \delta_8 \beta_3 D] \hat{w}(\xi, z, s) \\ & + \left[\xi^2 - \frac{K_3}{K_1} D^2 + \delta_5 \delta_8 s (1 + a_1 \xi^2 - a_3 D^2) \right] \hat{\phi}(\xi, z, s) \\ & = \rho \delta_8 \hat{Q}(\xi, z, s), \end{aligned} \quad (23)$$

where

$$\delta_7 = \frac{\varepsilon_0 \mu_0^2 H_0^2}{\rho} + 1, \quad \delta_8 = 1 + \tau_0 \frac{C_1}{L} s.$$

By taking $\hat{Q}(\xi, z, s) = 0$, we obtain homogeneous system of Eqs. (21)-(23) whose nontrivial solution exists when the determinant value of coefficient matrix for $(\hat{u}, \hat{w}, \hat{\phi})$ is zero.

$$(AD^6 + BD^4 + CD^2 + E)(\hat{u}, \hat{w}, \hat{\phi}) = 0, \quad (24)$$

where

$$\begin{aligned} A &= \delta_2 \delta_3 \vartheta_7 - \vartheta_5 \delta_2 \frac{\beta_3}{\beta_1} a_3, \\ B &= \Lambda_3 \vartheta_1 \vartheta_7 - a_3 \vartheta_1 \vartheta_5 \frac{\beta_3}{\beta_1} + \delta_2 \delta_3 \vartheta_6 + \delta_2 \vartheta_7 \vartheta_3 \\ &\quad - \vartheta_5 \vartheta_9 \delta_2 - \vartheta_8 \delta_1 i \xi \vartheta_7 + \vartheta_8 \vartheta_4 \frac{\beta_3}{\beta_1} a_3 \\ &\quad - a_3 \xi^2 \vartheta_5 \delta_1 - a_3 \delta_3 \vartheta_4 i \xi, \\ C &= \delta_3 \vartheta_1 \vartheta_6 + \vartheta_1 \vartheta_3 \vartheta_7 - \vartheta_1 \vartheta_5 \vartheta_9 + \Lambda_2 \vartheta_6 \vartheta_3 \\ &\quad + \vartheta_4 \vartheta_8 \vartheta_9 - \vartheta_8 \delta_1 i \xi \vartheta_6 + 4 \Omega^2 s^2 \vartheta_7 \\ &\quad + \vartheta_2 \delta_1 i \xi \vartheta_5 - \vartheta_2 \vartheta_4 \delta_3 - a_3 \vartheta_4 i \xi \vartheta_3, \\ E &= \vartheta_3 \vartheta_1 \vartheta_6 + 4 \Omega^2 s^2 \vartheta_6 - \vartheta_2 \vartheta_4 \vartheta_3, \\ \vartheta_1 &= \xi^2 - \delta_7 s^2 + \Omega^2, \\ \vartheta_2 &= -i \xi (1 + a_1 \xi^2), \\ \vartheta_3 &= -\delta_2 \xi^2 - \delta_5 s^2 + \Omega^2, \\ \vartheta_4 &= \delta_6 \delta_8 s \beta_1 i \xi, \\ \vartheta_5 &= \delta_6 \delta_8 s \beta_3, \\ \vartheta_6 &= \xi^2 + \delta_5 \delta_8 s (1 + a_1 \xi^2), \\ \vartheta_7 &= -\frac{K_3}{K_1} - a_3 \delta_5 \delta_8 s, \\ \vartheta_8 &= \delta_1 i \xi, \\ \vartheta_9 &= -(1 + a_1 \xi^2) \frac{\beta_3}{\beta_1}. \end{aligned}$$

The roots of the Eq. (24) are $\pm \lambda_i$, ($i = 1, 2, 3$), the result of the Eq. (24) sustaining the radiation condition that $\hat{u}, \hat{v}, \hat{w}$ can be written as

$$\bar{u}(\xi, z, s) = \sum_{i=1}^3 A_i e^{-\lambda_i z}, \quad (25)$$

$$\bar{w}(\xi, z, s) = \sum_{i=1}^3 d_i A_i e^{-\lambda_i z}, \quad (26)$$

$$\bar{\phi}(\xi, z, s) = \sum_{i=1}^3 l_i A_i e^{-\lambda_i z}, \quad (27)$$

where A_i , $i = 1, 2, 3$ being undetermined constants and d_i and l_i are given by

$$\begin{aligned} d_i &= \frac{\delta_2 \vartheta_7 \lambda_i^4 + (\vartheta_7 \vartheta_1 - a_3 \vartheta_4 i \xi + \delta_2 \vartheta_6) \lambda_i^2 + \vartheta_1 \vartheta_6 - \vartheta_4 \vartheta_2}{\left(\delta_3 \vartheta_7 - \frac{\beta_3}{\beta_1} a_3 \vartheta_5 \right) \lambda_i^4 + (\delta_3 \vartheta_6 + \vartheta_3 \vartheta_7 - \vartheta_5 \vartheta_9) \lambda_i^2 + \vartheta_3 \vartheta_6}, \\ l_i &= \frac{\delta_2 \delta_3 \lambda_i^4 + (\delta_2 \vartheta_3 + \vartheta_1 \delta_3 - \delta_1 \vartheta_8 i \xi) \lambda_i^2 + 4 \Omega^2 s^2 + \vartheta_3 \vartheta_1}{\left(\delta_3 \vartheta_7 - \frac{\beta_3}{\beta_1} a_3 \vartheta_5 \right) \lambda_i^4 + (\delta_3 \vartheta_6 + \vartheta_3 \vartheta_7 - \vartheta_5 \vartheta_9) \lambda_i^2 + \vartheta_3 \vartheta_6}. \end{aligned}$$

4. Boundary conditions

Thermal source and normal force are applied on the half-space ($z = 0$) surface.

The applicable boundary conditions are

$$t_{33}(x, z, t) = -F_1 \psi_1(x) \delta(t), \quad (28)$$

$$t_{31} = 0, \quad (29)$$

$$\frac{\partial \varphi}{\partial z}(x, z, t) = F_2 \psi_2(x) \delta(t), \quad (30)$$

where $\delta(t)$ is the Dirac's delta function, F_1 is the magnitude of the force applied, F_2 is the thermal source applied on the boundary, $\psi_1(x)$ specifies the source distribution function on the x-axis, $\psi_2(x)$ specifies the source distribution function on z-axis.

Applying the Laplace and Fourier transform defined by (19) and (20) on the boundary conditions (28)-(30), using (12)-(14) and with the help of Eqs. (25)-(27), we obtain the components of displacement, normal stress, tangential stress, and conductive temperature as

$$\hat{u} = \frac{F_1 \hat{\psi}_1(\xi)}{\Lambda} \left[\sum_{i=1}^3 \Lambda_{1i} e^{-\lambda_i z} \right] + \frac{F_2 \hat{\psi}_2(\xi)}{\Lambda} \left[\sum_{i=1}^3 \Lambda_{2i} e^{-\lambda_i z} \right], \quad (31)$$

$$\begin{aligned} \hat{w} &= \frac{F_1 \hat{\psi}_1(\xi)}{\Lambda} \left[\sum_{i=1}^3 d_i \Lambda_{1i} e^{-\lambda_i z} \right] \\ &+ \frac{F_2 \hat{\psi}_2(\xi)}{\Lambda} \left[\sum_{i=1}^3 d_i \Lambda_{2i} e^{-\lambda_i z} \right], \end{aligned} \quad (32)$$

$$\begin{aligned} \hat{\phi} &= \frac{F_1 \hat{\psi}_1(\xi)}{\Lambda} \left[\sum_{i=1}^3 l_i \Lambda_{1i} e^{-\lambda_i z} \right] \\ &+ \frac{F_2 \hat{\psi}_2(\xi)}{\Lambda} \left[\sum_{i=1}^3 l_i \Lambda_{2i} e^{-\lambda_i z} \right], \end{aligned} \quad (33)$$

$$\begin{aligned} \hat{t}_{11} &= \frac{F_1 \hat{\psi}_1(\xi)}{\Lambda} \left[\sum_{i=1}^3 S_i \Lambda_{1i} e^{-\lambda_i z} \right] \\ &+ \frac{F_2 \hat{\psi}_2(\xi)}{\Lambda} \left[\sum_{i=1}^3 S_i \Lambda_{2i} e^{-\lambda_i z} \right], \end{aligned} \quad (34)$$

$$\hat{t}_{13} = \frac{F_1 \hat{\psi}_1(\xi)}{\Lambda} \left[\sum_{i=1}^3 N_i \Lambda_{1i} e^{-\lambda_i z} \right] \quad (35)$$

$$+ \frac{F_2 \hat{\psi}_2(\xi)}{\Lambda} \left[\sum_{i=1}^3 N_i \Lambda_{2i} e^{-\lambda_i z} \right], \tag{35}$$

$$\begin{aligned} \hat{t}_{33} = & \frac{F_1 \hat{\psi}_1(\xi)}{\Lambda} \left[\sum_{i=1}^3 M_i \Lambda_{1i} e^{-\lambda_i z} \right] \\ & + \frac{F_2 \hat{\psi}_2(\xi)}{\Lambda} \left[\sum_{i=1}^3 M_i \Lambda_{2i} e^{-\lambda_i z} \right], \end{aligned} \tag{36}$$

where

$$\begin{aligned} \Lambda_{11} &= -N_2 R_3 + R_2 N_3, \\ \Lambda_{12} &= N_1 R_3 - R_1 N_3, \\ \Lambda_{13} &= -N_1 R_2 + R_1 N_2, \\ \Lambda_{21} &= M_2 N_3 - N_2 M_3, \\ \Lambda_{22} &= -M_1 N_3 + N_1 M_3, \\ \Lambda_{23} &= M_1 N_2 - N_1 M_2, \\ \Lambda &= -M_1 \Lambda_{11} - M_2 \Lambda_{12} - M_3 \Lambda_{13} \\ N_j &= -\delta_2 \lambda_j + i \xi d_j, \\ M_j &= i \xi - \delta_3 d_j \lambda_j - \frac{\beta_3}{\beta_1} l_j [(1 + a_1 \xi^2) - a_3 \lambda_j^2], \\ R_j &= -\lambda_j l_j [(1 + a_1 \xi^2) - a_3 \lambda_j^2], \\ S_j &= -i \xi - \delta_4 d_j \lambda_j - l_j [(1 + a_1 \xi^2) - a_3 \lambda_j^2]. \end{aligned}$$

5. Special cases

a. Mechanical force on a half-space surface

By taking $F_2 = 0$ in Eqs. (31)-(36), we obtain the components of displacement, normal stress, tangential stress and conductive temperature due to mechanical force.

b. Thermal source on the half-space surface

By considering $F_1 = 0$ in equations (31)-(36), we obtain the components of displacement, normal stress, tangential stress and conductive temperature due to a thermal source.

5.1 Green's function

The solution due to a concentrated normal force applied on the half-space is obtained by setting

$$\psi_1(x) = \delta(x), \psi_2(x) = \delta(x). \tag{37}$$

By applying (20) on (37), we obtain

$$\hat{\psi}_1(\xi) = 1, \hat{\psi}_2(\xi) = 1 \tag{38}$$

By using (38) in (31)-(36), we obtained the displacement components, stress components, and conductive temperature.

5.2 Influence function

The solution due to a uniformly distributed force applied on the half-space is obtained by setting

$$\psi_1(x), \psi_2(x) = \begin{cases} 1 & \text{if } |x| \leq m \\ 0 & \text{if } |x| > m \end{cases} \tag{39}$$

The Fourier transforms of $\psi_1(x)$ and $\psi_2(x)$ w. r. t. the pair (x, ξ) for uniform strip load of dimensionless width $2m$ applied at $x = z = 0$ in the dimensionless form after stifling the primes becomes

$$\hat{\psi}_1(\xi) = \hat{\psi}_2(\xi) = \left\{ \frac{2 \sin(\xi m)}{\xi} \right\}, \xi \neq 0 \tag{40}$$

By using (40) in (31)-(36), we obtained the displacement components, stress components, and conductive temperature.

5.3 Linearly distributed function

The solution due to a linearly distributed force applied on the half-space is obtained by setting

$$\{\psi_1(x), \psi_2(x)\} = \begin{cases} 1 - \frac{|x|}{m} & \text{if } |x| \leq m \\ 0 & \text{if } |x| > m \end{cases} \tag{41}$$

Here $2m$ is the width of the strip load, using (15) and applying the transform defined by (20) on (41), we get

$$\hat{\psi}_1(\xi) = \hat{\psi}_2(\xi) = \left\{ \frac{2[1 - \cos(\xi m)]}{\xi^2 m} \right\}, \xi \neq 0 \tag{42}$$

By using (42) in (31)-(36), we obtained the displacement components, stress components, and conductive temperature.

6. Inversion of the transformation

For obtaining the result in the physical domain, invert the transforms in Eqs. (31)-(36) using

$$\begin{aligned} \tilde{f}(x, z, s) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\xi x} \hat{f}(\xi, z, s) d\xi \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} |\cos(\xi x) f_e - i \sin(\xi x) f_o| d\xi, \end{aligned} \tag{43}$$

where f_o is odd and f_e is the even parts of $\hat{f}(\xi, z, s)$ respectively. Following Honig and Hirdes (1984), the Laplace transform function $\tilde{f}(x, z, s)$ can be inverted to $f(x, z, t)$ by

$$f(x, z, t) = \frac{1}{2\pi i} \int_{e^{-i\infty}}^{e^{+i\infty}} \tilde{f}(x, z, s) e^{-st} ds. \tag{44}$$

Two methods are used to reduce the total error. First, the ‘‘Korrektur’’ method is used to reduce the discretization error. Next, the -algorithm is used to reduce the truncation error and hence to accelerate convergence. The details of these methods can be found in Honig and Hirdes (1984) and integrate these equations as described in Press (1986) by using Romberg’s integration and trapezoidal rule followed by extrapolation.

7. Numerical results and discussion

To demonstrate the theoretical results and effect of rotation, relaxation time and two temperature, the physical data for cobalt material, which is transversely isotropic, is taken from Dhaliwal and Singh (1980) is given as

$$\begin{aligned} c_{11} &= 3.07 \times 10^{11} Nm^{-2}, \\ c_{33} &= 3.581 \times 10^{11} Nm^{-2}, \\ c_{13} &= 1.027 \times 10^{10} Nm^{-2}, \\ c_{44} &= 1.510 \times 10^{11} Nm^{-2}, \\ \beta_1 &= 7.04 \times 10^6 Nm^{-2} deg^{-1}, \\ \beta_3 &= 6.90 \times 10^6 Nm^{-2} deg^{-1}, \\ \rho &= 8.836 \times 10^3 Kg m^{-3}, \\ C_E &= 4.27 \times 10^2 jKg^{-1} deg^{-1}, \\ K_1 &= 0.690 \times 10^2 Wm^{-1} K deg^{-1}, \\ K_3 &= 0.690 \times 10^2 Wm^{-1} K^{-1}, \\ T_0 &= 298 K, H_0 = 1 Jm^{-1} nb^{-1}, \\ \varepsilon_0 &= 8.838 \times 10^{-12} Fm^{-1}, \\ L &= 1, \\ \tau_0 &= 1. \end{aligned}$$

The numerical calculations have been obtained by developing a FORTRAN program using the above values for cobalt material. With the above values, a comparison of the dimensionless form of the field variables displacement components (u and w), Conductive temperature φ and stress components (t_{11} , t_{13} and t_{33}) for a transversely isotropic plate with two temperature, relaxation time and rotation have been studied for concentrated force, linearly distributed force and uniformly distributed force with mechanical and thermal sources are demonstrated graphically as:

- (1) The black line with square symbol relates to two temperature and relaxation time i.e., $a_1 = 0.0$, $a_3 = 0.0$ for $\tau_0 = 0.0$ at angular velocity $\Omega = 0.5$,
- (2) The blue line with circle symbol relates to two temperature and relaxation time i.e., $a_1 = 0.0$, $a_3 = 0.0$ for $\tau_0 = 0.5$ at angular velocity $\Omega = 0.5$,
- (3) The red line with triangle symbol relates to two temperature and relaxation time i.e., $a_1 = 0.02$, $a_3 = 0.04$ for $\tau_0 = 0.0$ at angular velocity $\Omega = 0.5$,
- (4) The green line with star symbol relates to two temperature and relaxation time i.e., $a_1 = 0.02$, $a_3 = 0.04$ for $\tau_0 = 0.5$ at angular velocity $\Omega = 0.5$.

Case I: Concentrated force and mechanical force

Figs. 2-7 shows the variations of the displacement components (u and w), Conductive temperature φ , stress components (t_{11} , t_{13} and t_{33}) for a concentrated and mechanical force with effects of rotation with and without two temperature in generalized thermoelasticity and LS theory without energy dissipation respectively. The displacement components (u and w), Conductive temperature φ and stress components (t_{11} , t_{13} and t_{33}) illustrate the same pattern but having different magnitudes for different value of frequency. These components vary

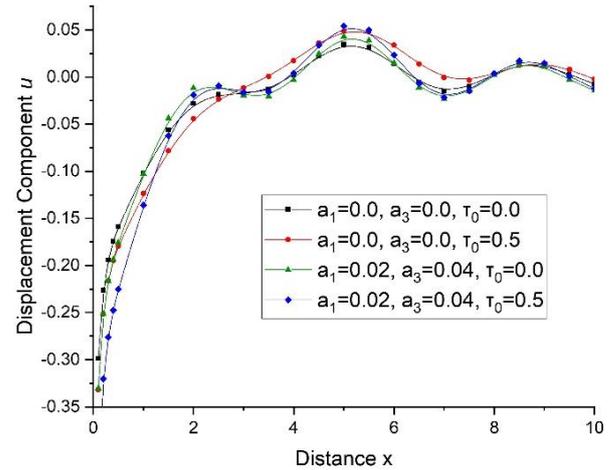


Fig. 2 Variations of displacement component u with distance x

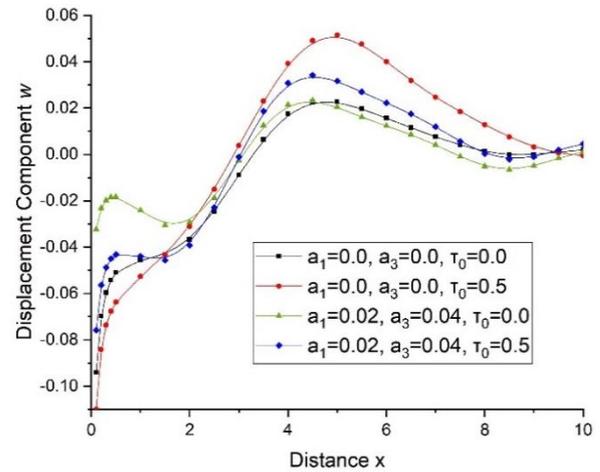


Fig. 3 Variations of displacement component w with distance x

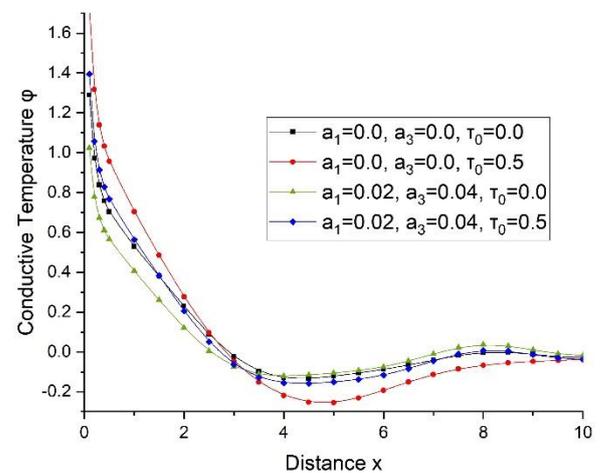


Fig. 4 Variations of conductive temperature φ with distance x

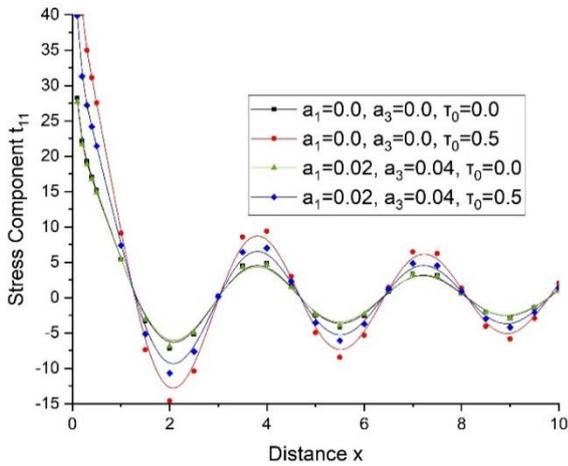


Fig. 5 Variations of stress component t_{11} with distance x

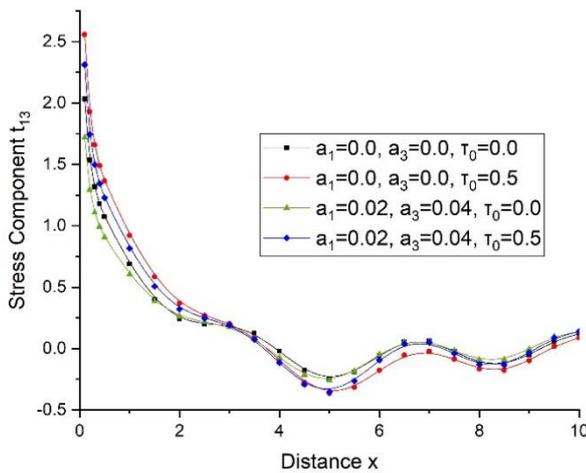


Fig. 6 Variations of the stress component t_{13} with distance x

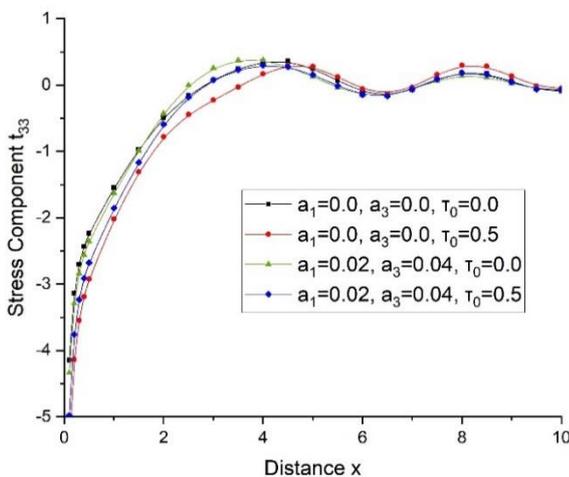


Fig. 7 Variations of the stress component t_{33} with distance x

during the initial range of distance near the loading surface of the mechanical source with and without two temperature and follow small oscillatory pattern for rest of the range of distance.

Case II: Uniformly distributed force and mechanical force

Figs. 8-13 illustrates the variations of the displacement components (u and w), Conductive temperature φ and stress components (t_{11} , t_{13} and t_{33}) with uniformly distributed force and mechanical force with joint effects of rotation with and without two temperature in generalized thermoelasticity and LS theory without energy dissipation respectively. The displacement components (u and w), Conductive temperature φ and stress components (t_{11} , t_{13} and t_{33}) illustrate the same pattern but having different magnitudes for different value of frequency. These components vary (increases or decreases) during the initial range of distance near the loading surface of the mechanical source with and without two temperature and follow small oscillatory pattern for rest of the range of distance.

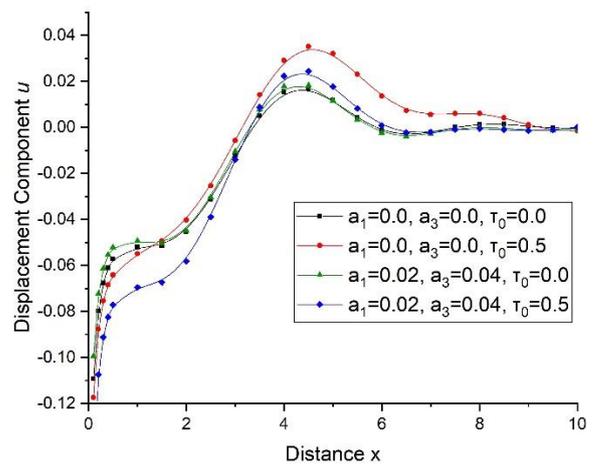


Fig. 8 Variations of displacement component u with distance x

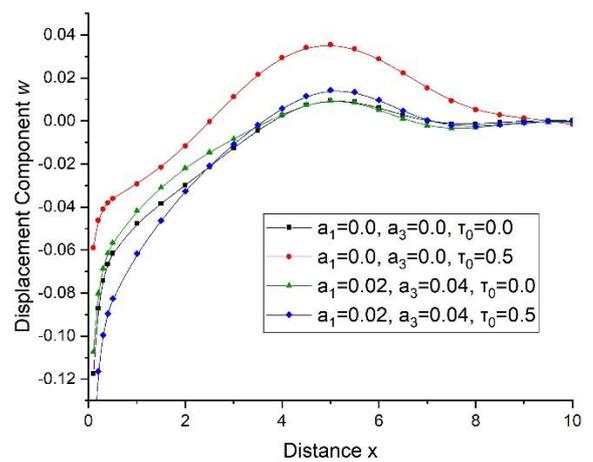


Fig. 9 Variations of displacement component w with distance x

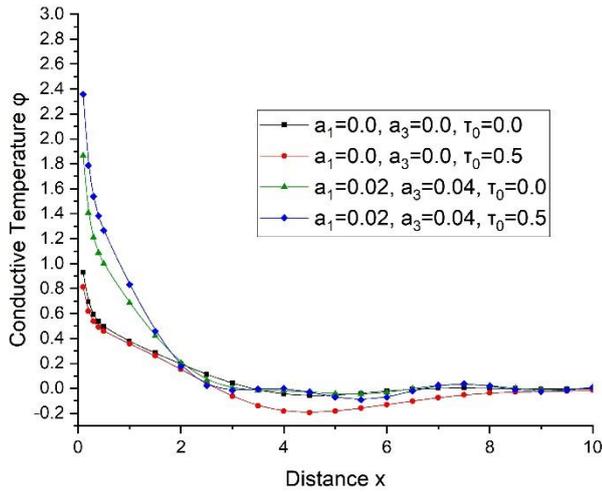


Fig. 10 Variations of conductive temperature ϕ with distance x

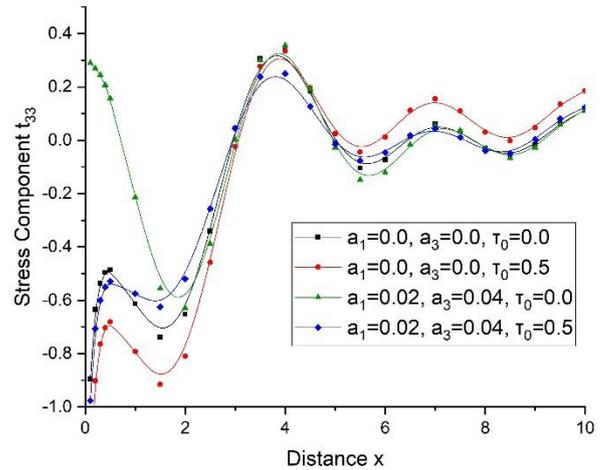


Fig. 13 Variations of the stress component t_{33} with distance x

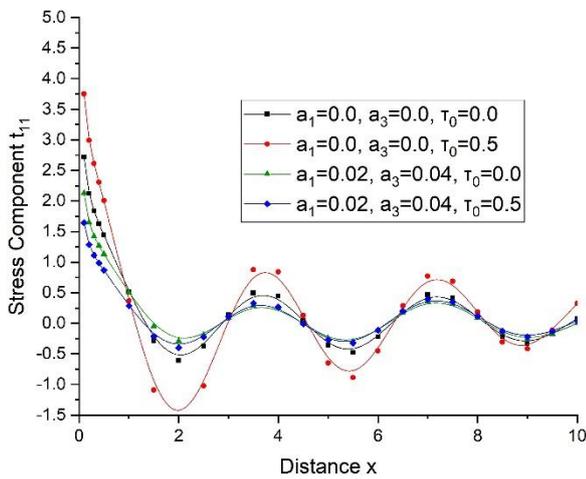


Fig. 11 Variations of stress component t_{11} with distance x

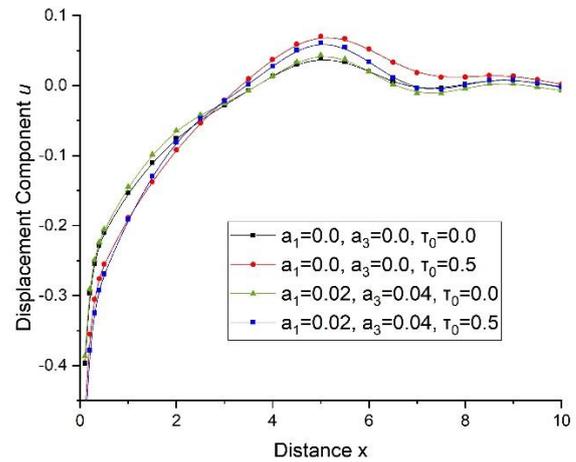


Fig. 14 Variations of displacement component u with distance x

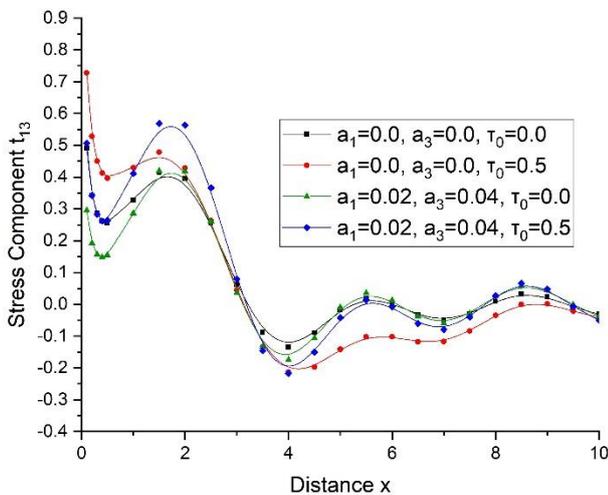


Fig. 12 Variations of the stress component t_{13} with distance x

Case III: Linearly Distributed force and mechanical force

Figs. 14-19 demonstrates the variations of the displacement components (u and w), Conductive temperature ϕ and stress components (t_{11} , t_{13} and t_{33}) with linearly distributed force and mechanical force with effects of rotation with and without two temperature in generalized thermoelasticity and LS theory without energy dissipation respectively. The displacement components (u and w), Conductive temperature ϕ and stress components (t_{11} , t_{13} and t_{33}) illustrate the same pattern but having different magnitudes for different value of frequency. These components vary during the initial range of distance near the loading surface of the mechanical source with and without two temperature and follow small oscillatory pattern for rest of the range of distance.

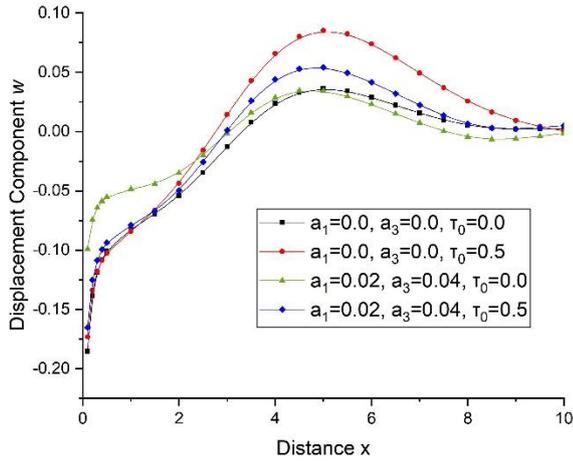


Fig. 15 Variations of displacement component w with distance x

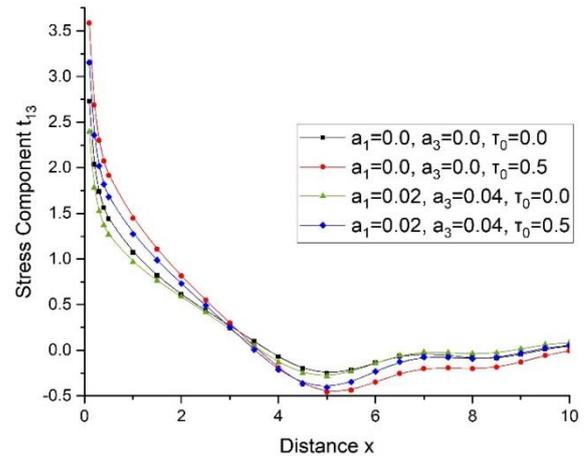


Fig. 18 Variations of the stress component t_{13} with distance x

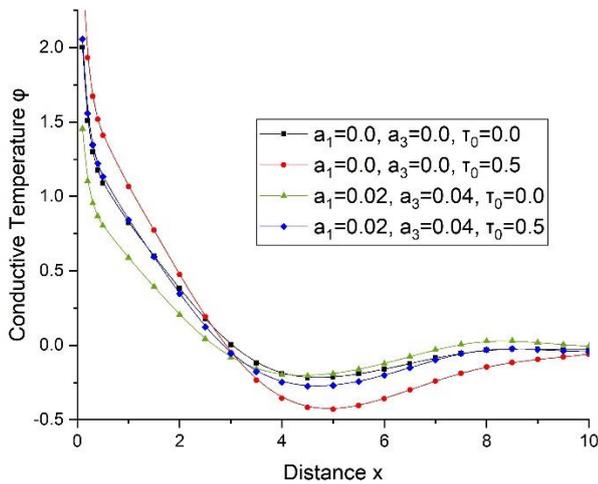


Fig. 16 Variations of conductive temperature ϕ with distance x

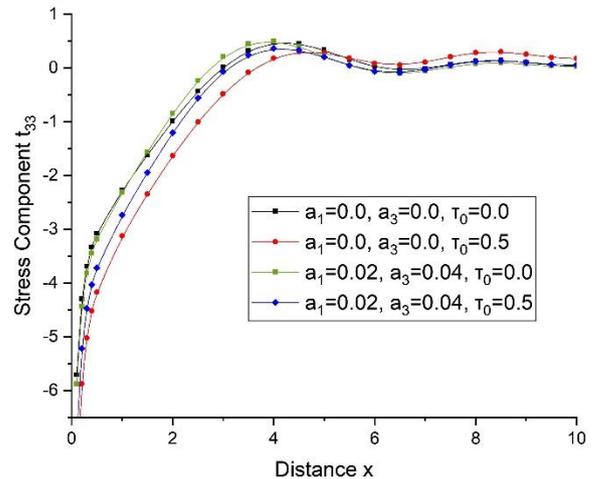


Fig. 19 Variations of the stress component t_{33} with distance x

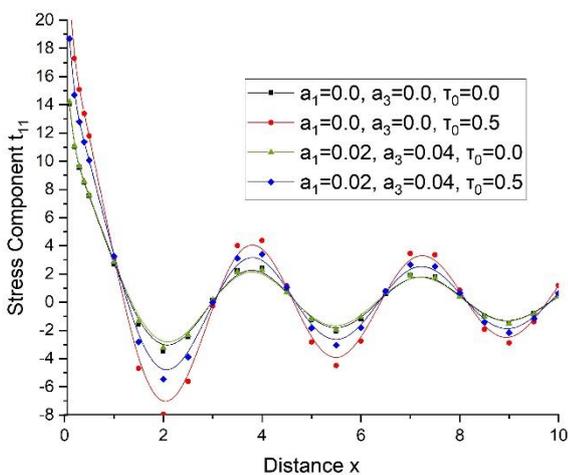


Fig. 17 Variations of stress component t_{11} with distance x

Case IV: Concentrated force and thermal source

Figs. 20-25 depicts the variations of the displacement components (u and w), Conductive temperature ϕ and stress components (t_{11} , t_{13} and t_{33}) with concentrated force and thermal force with effects of rotation with and without two temperature in generalized thermoelasticity and LS theory without energy dissipation respectively. The displacement components (u and w), Conductive temperature ϕ and stress components (t_{11} , t_{13} and t_{33}) illustrate the same pattern but having different magnitudes for different value of frequency. These components vary during the initial range of distance near the loading surface of the mechanical source with and without two temperature and follow small oscillatory pattern for rest of the range of distance.

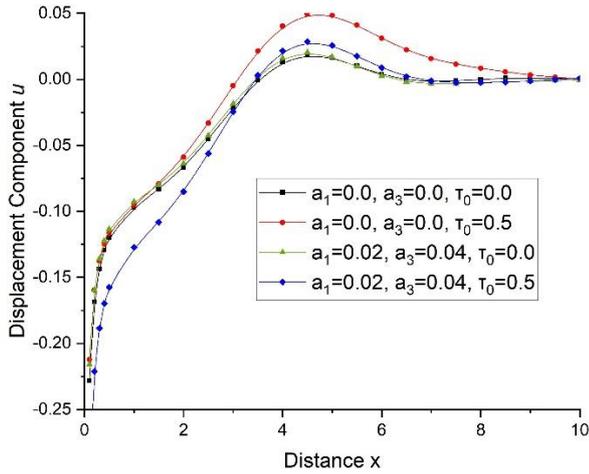


Fig. 20 Variations of displacement component u with distance x

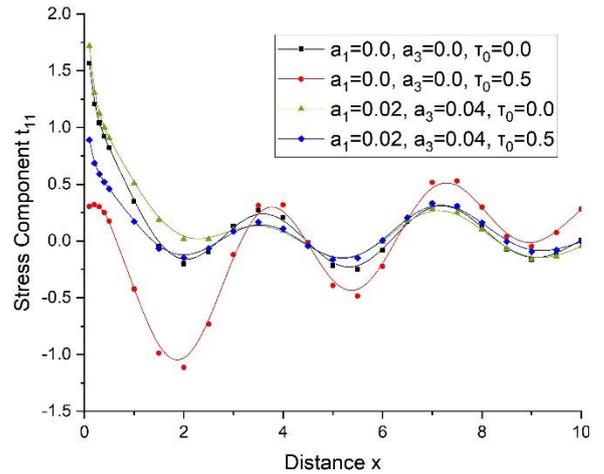


Fig. 23 Variations of stress component t_{11} with distance x

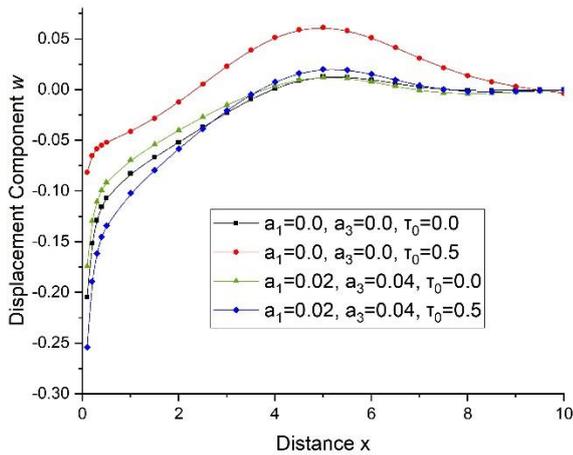


Fig. 21 Variations of displacement component w with distance x

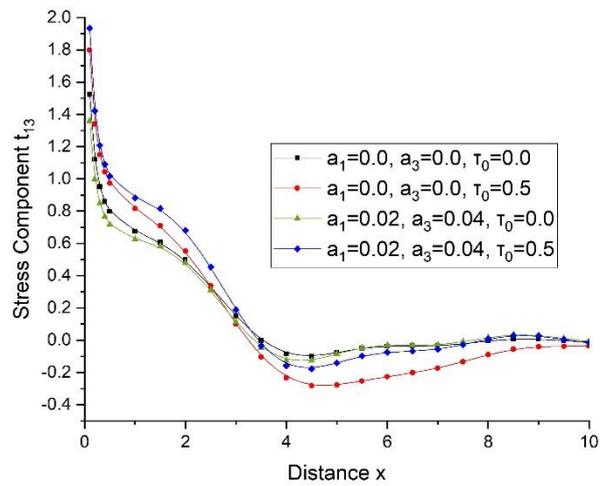


Fig. 24 Variations of the stress component t_{13} with distance x

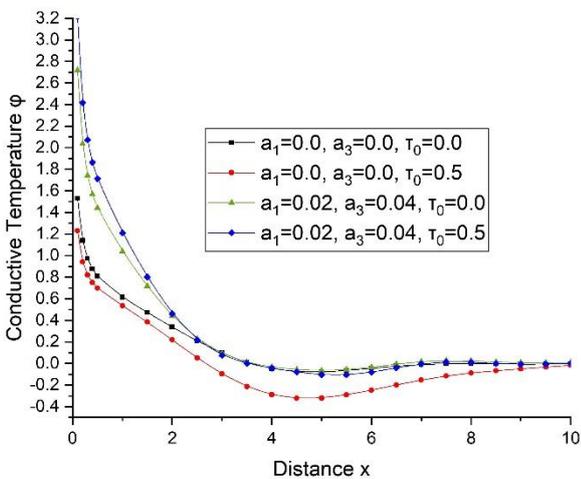


Fig. 22 Variations of conductive temperature ϕ with distance x

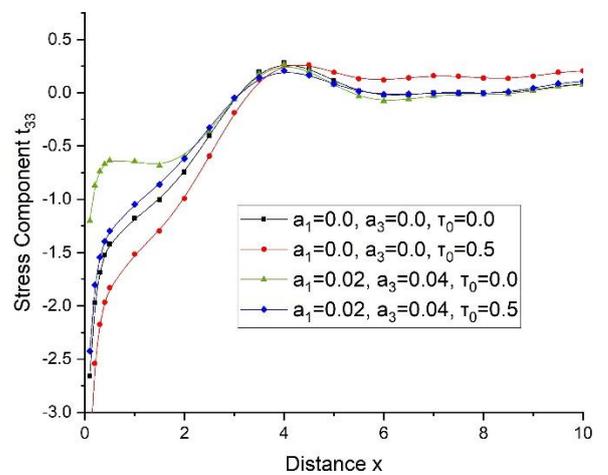


Fig. 25 Variations of the stress component t_{33} with distance x

Case V: Uniformly distributed force and thermal source

Figs. 26-31 displays the variations of the displacement components (u and w), Conductive temperature φ and stress components (t_{11} , t_{13} and t_{33}) with uniformly distributed force and thermal force with effects of rotation with and without two temperature in generalized thermoelasticity and LS theory without energy dissipation respectively. The displacement components (u and w), Conductive temperature φ and stress components (t_{11} , t_{13} and t_{33}) illustrate the same pattern but having different magnitudes for different value of frequency. These components varies (increases or decreases) during the initial range of distance near the loading surface of the mechanical source with and without two temperature and follow small oscillatory pattern for rest of the range of distance.

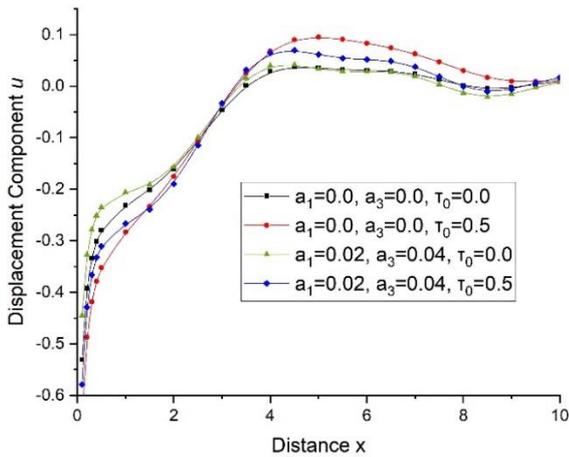


Fig. 26 Variations of displacement component u with distance x

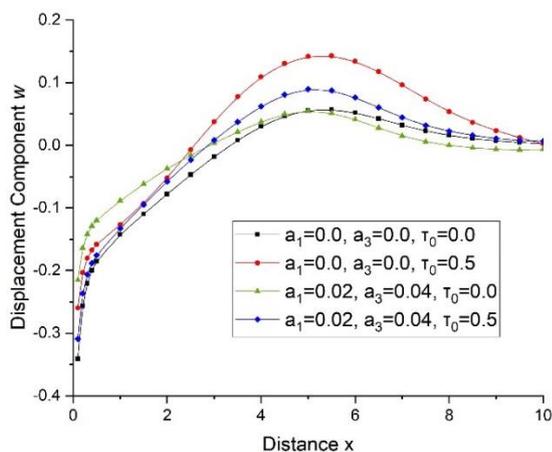


Fig. 27 Variations of displacement component w with distance x

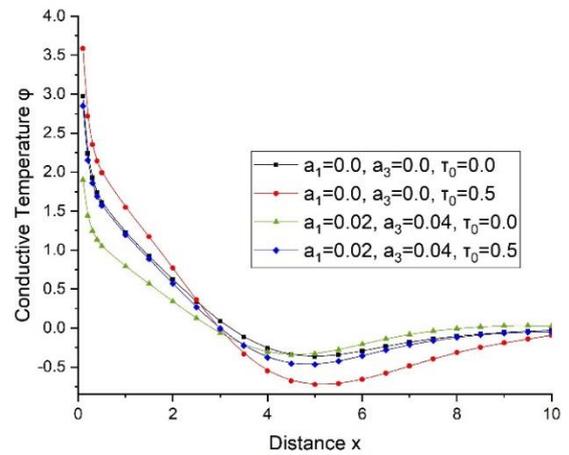


Fig. 28 Variations of conductive temperature φ with distance x

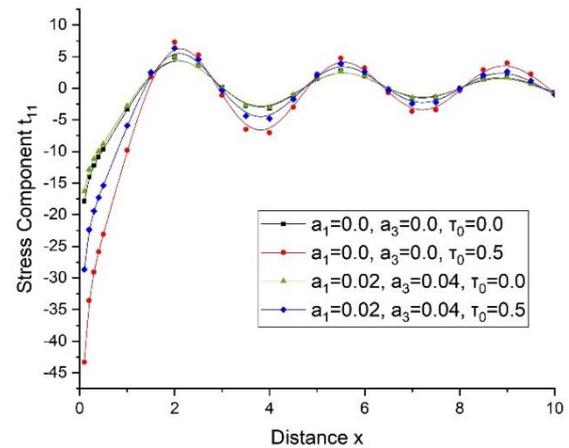


Fig. 29 Variations of stress component t_{11} with distance x

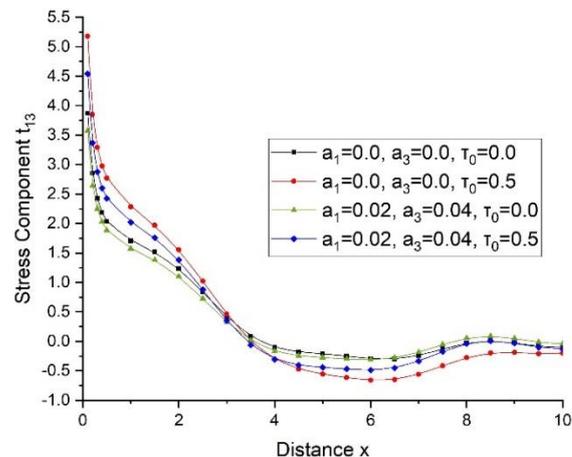


Fig. 30 Variations of the stress component t_{13} with distance x

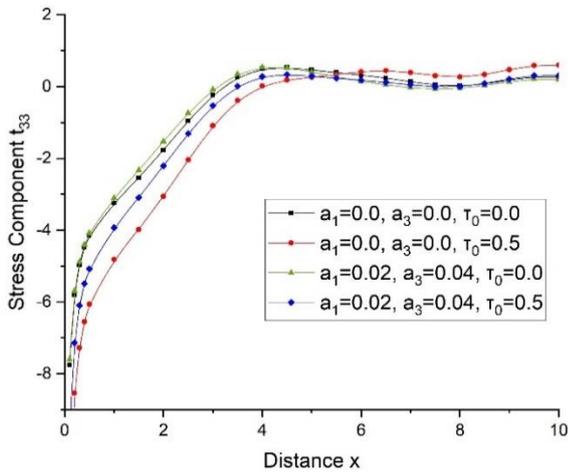


Fig. 31 Variations of the stress component t_{33} with distance x

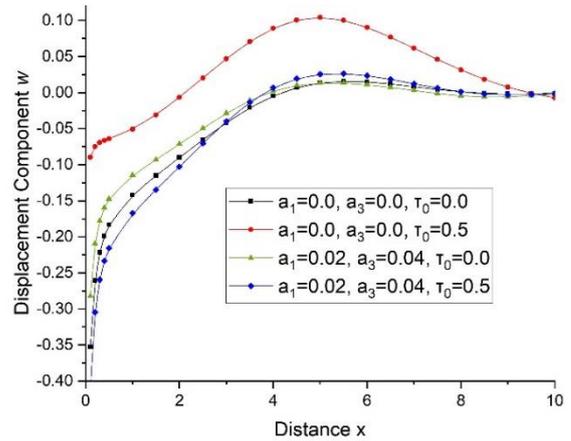


Fig. 33 Variations of displacement component w with distance x

Case VI: Linearly distributed force and thermal source

Figs. 32-37 shows the variations of the displacement components (u and w), Conductive temperature ϕ and stress components (t_{11} , t_{13} and t_{33}) with linearly distributed force and thermal force with effects of rotation with and without two temperature in generalized thermoelasticity and LS theory without energy dissipation respectively. The displacement components (u and w), Conductive temperature ϕ and stress components (t_{11} , t_{13} and t_{33}) illustrate the same pattern but having different magnitudes for different value of frequency. These components vary during the initial range of distance near the loading surface of the mechanical source with and without two temperature and follow small oscillatory pattern for rest of the range of distance.

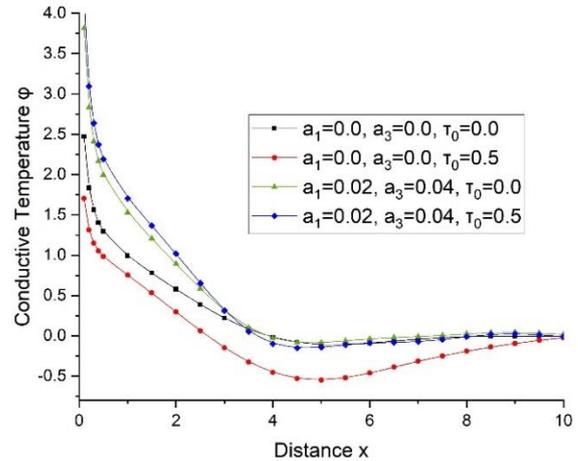


Fig. 34 Variations of conductive temperature ϕ with distance x

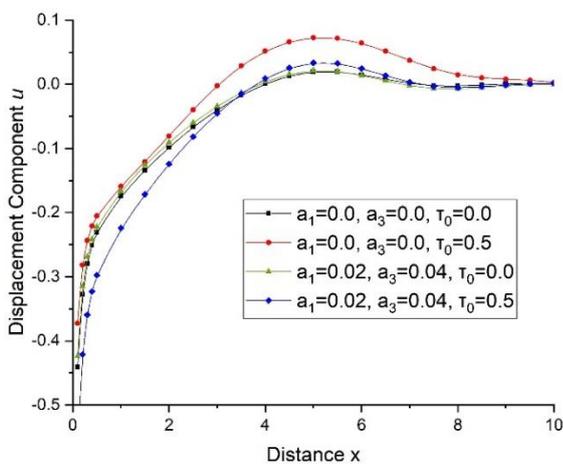


Fig. 32 Variations of displacement component u with distance x

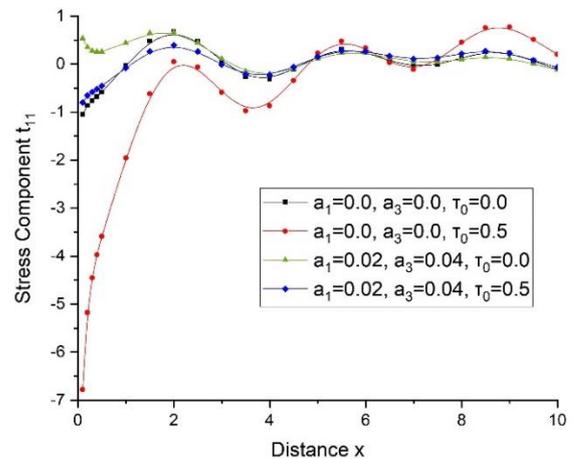


Fig. 35 Variations of stress component t_{11} with distance x

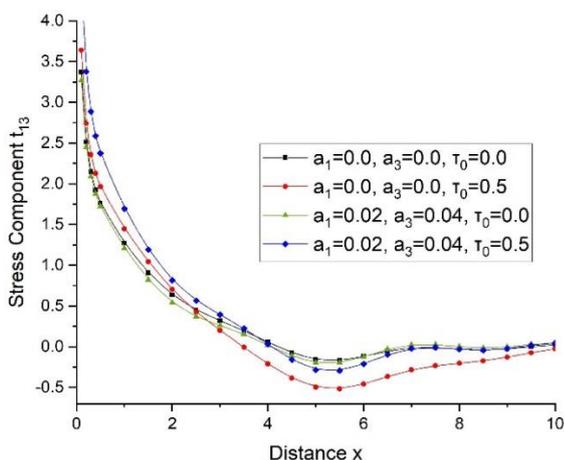


Fig. 36 Variations of the stress component t_{13} with distance x

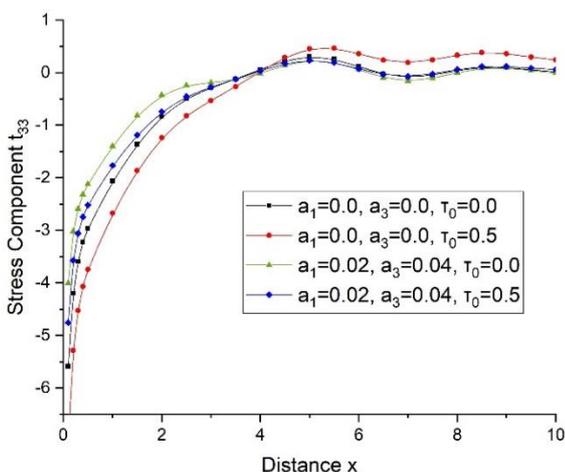


Fig. 36 Variations of the stress component t_{33} with distance x

9. Conclusions

From above investigation, the displacement components, stress components, and conductive temperature distribution with the horizontal distance are computed in the transformed domain and further calculated in the physical domain numerically. It is observed that thermal source and mechanical force is an important factor for the variation of physical quantities both near the source as well as far from the source. Moreover, the effect of relaxation time and two temperature plays a key part in the deformation of all the physical quantities. The physical quantities amplitude differ (i.e., either rise or fall) with and without two temperatures in generalized LS theories of thermoelasticity. In the presence of two temperature and τ_0 , the displacement components and stress components exhibit an oscillatory pattern w.r.t. x . The result gives the inspiration to study magneto-thermoelastic materials as an innovative domain of applicable thermoelastic solids. The shape of curves shows the impact of thermal and mechanical source with and without two temperatures in

generalized and LS theories of thermoelasticity on the body and fulfills the purpose of the study. The outcomes of this research are extremely helpful in the 2-D problem with dynamic response in transversely isotropic magneto-thermoelastic medium with rotation and two temperature which is beneficial in detection of deformation field such as geothermal engineering; advanced aircraft structure design, thermal power plants, composite engineering, geology, high-energy particle accelerators and in real life as in geophysics, auditory range, geomagnetism etc.

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Nomenclature

δ_{ij}	Kronecker delta,
C_{ijkl}	Elastic parameters,
β_{ij}	Thermal elastic coupling tensor,
T	Absolute temperature,
T_0	Reference temperature,
φ	conductive temperature,
t_{ij}	Stress tensors,
e_{ij}	Strain tensors,
u_i	Components of displacement,
ρ	Medium density,
C_E	Specific heat,
a_{ij}	Two temperature parameters,
α_{ij}	Linear thermal expansion coefficient,
K_{ij}	Materialistic constant,
K_{ij}^*	Thermal conductivity,
ω	Frequency
τ_0	Relaxation Time
Ω	Angular Velocity of the Solid
F_i	Components of Lorentz force
\vec{H}_0	Magnetic field intensity vector
\vec{j}	Current Density Vector
\vec{u}	Displacement Vector
μ_0	Magnetic permeability
ε_0	Electric permeability
$\delta(t)$	Dirac's delta function