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Influence of stiffened hangers on the structural behavior of all-steel tied-arch bridges

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Abstract. In tied-arch bridges, the way the arch and the deck are connected may become crucial. The deck is usually suspended from hangers made out of steel pinned cables capable of resisting axial forces only. However, a proper structural response may be ensured by fixing and stiffening the hangers in order to resist, additionally, shear forces and bending moments. Thus, this paper studies the effect of different pinned and stiffened hanger arrangements on the structural behavior of the tied-arch bridges, with the intention of providing designers with useful tools at the early steps of design. Longitudinally and transversally stiffened hangers (and the effect of hinges at the hangers and their locations) are studied separately because the in-plane and the out-of-plane behavior of the bridge are uncoupled due to its symmetry. As a major conclusion, regarding the in-plane behavior, hangers composed of cables (either with vertical, Nielsen-Löhse or network arrangements) are recommended due to its low cost and ease of erection. Alternatively, longitudinally stiffened hangers, fixed at both ends, can be used. Regarding the out-of-plane behavior, and in addition to three-dimensional arrangements of cables, of limited effectiveness, transversally stiffened hangers fixed at both ends are the most efficient arrangement. A configuration almost as efficient and, additionally, cheaper and easier to build can be achieved by locating a hinge at the end corresponding to the most flexible structural element (normally the arch). Its efficiency is further improved if the cross-section tapers from the fixed end to the pinned end.

Keywords: arch bridge; stiffened hanger; hinge; cable arrangement; preliminary design; conceptual design

1. Introduction

In arch bridges, the connection between the arch and the deck may become crucial. This link defines, for example, how internal forces are distributed between the arch and the deck, or how effective the bridge is in order to reduce the deflections of the deck under service loads. When the deck is suspended from the arch, hangers are usually made out of steel cables (see EC-3 2006b, Pfeifer 2017), because they are relatively cheap and easy to build (See Fig. 1). In practical terms, cables are assumed to have negligible flexural stiffness, and, subsequently, it is assumed they can resist only axial tension loads.

The in-plane cable arrangement may be modified, for example, to reduce the bending moments at the bridge. On this purpose, vertical hangers can be substituted by inclined hangers, as it happens in the so-called Nielsen-Löhse type, pioneered by bridges such as the Castelmoron Bridge, over the Lot River, France (Fig. 2). It should be mentioned that in this typology, and due to their inclination, the cables are subjected to high stress amplitudes, to such an extent that they may become slack for a combination of loads and according to the slope angle, which may lead to structural damage in the cables due to fatigue.

Fig. 1 Example of pinned hangers. Puente del Cañuelo, Spain. Photo: Juan José Jorquera-Lucerga

Fig. 2 Castelmoron Bridge, over the Lot River, France. Photo: Mossott, Wikimedia commons

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Fig. 3 Fehmarndsund Bridge.
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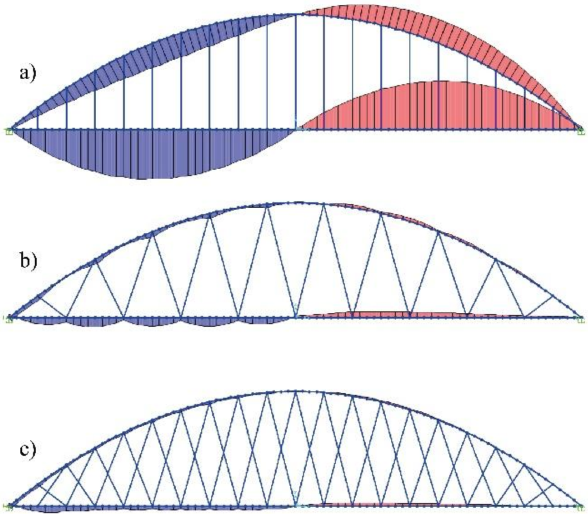


Fig. 4 Comparison of bending moment for different hanger arrangements, drawn at the same scale, under an UDL upon half the deck: (a) Vertical hangers; (b) Nielsen-Löhse hangers; (c) Network hangers

A further reduction of bending moments, both at the arch and at the deck, can be obtained with the so-called network bridge, an evolution of the Nielsen-Löhse type, where some of the hangers cross each other at least twice, and which leads to very slender arches. The network cable arrangement was proposed for the first time by P. Tveit in 1959 (see Tveit 2018). Perhaps the most well-known example of is the Fehrmandsund bridge shown in Fig. 3. The relative efficiency of the three cable arrangements (vertical, Nielsen-Löhse and Network) described so far are compared in Fig. 4 for a uniformly distributed load (UDL) acting upon half the deck.

However, in some cases, the designer decides that, in order to ensure a correct global structural behavior, the hangers must resist, in addition to axial forces, shear forces and bending moments. Consequently, the hangers must have bending resistant cross-sections, such as H or hollow box-sections, i.e., the hangers must be stiffened.

When H-sections are used, the orientation of the hangers determines the bending direction where the effect of the flexural stiffness is more relevant, such in La Devesa footbridge (Fig. 5) or the Merchants Bridge in Manchester (Fig. 6). The orientation of the H-shaped hangers is defined to provide either longitudinal stiffness, in order to support in-plane bending, or transversal stiffness, in order to support torsional forces at the deck or out-of-plane behavior at the arch. When hollow box-sections are used for the hangers, they are usually fully fixed at both ends, which provides simultaneously longitudinal in-plane and transversal out-of-plane stiffness, as it happens (See Fig. 7) in La Alameda bridge, in Valencia (Spain).

Fig. 5 Example of stiffened hangers. La Devesa Footbridge, Ripoll. Photo: Nicolas Janberg, structurae.net

Fig. 6 Example of stiffened hangers. Merchants Bridge, Manchester. Photo: Clem Rutter, Wikimedia commons

Fig. 7 Example of fixed hangers. La Alameda Bridge, Valencia. Photo: Juan José Jorquera-Lucerga

It is noteworthy that, both at the Devesa and the Merchants Bridge, although the H-sections of the hangers are transversally oriented, the hangers are not fully fixed to the arch in the transversal direction. It will be shown how, in these and in many other cases, a hinge with its top pinned is almost as efficient as a fully fixed hanger while it is much easier to build. However, as far as the authors know, the effect of the position of the hinge along the hanger on the structural response of the bridge has not been studied deeply except for the study carried out by García-Guerrero and Jorquera-Lucerga (2018), focused on the in-plane behavior of the arches and for hinges located only either at the top or the bottom of the hangers.

The way the arch and the deck are connected is also relevant when checking the arch sensitivity to buckling. Generally, stiffened hangers are more efficient than cables to prevent the arch from buckling, and, obviously, the stiffness of the hangers becomes of major importance, as it is shown, for example, in Palkowski (2012), Hu *et al*. (2015) or De Backer *et al*. (2014). Therefore, stiffening the hangers becomes one typological option available for the designer at the initial steps of the design, simply because the critical load of the arch may increase just by stiffening the hangers, i.e., without modifying the cross-section of the arch. This seems to be the case of the Destructor Bridge (Fig. 8) in Bath (UK), where the transversally stiffened hangers are simple flat steel plates of the same width as the arch, a decision made by the designer to prevent the very slender arch (with a depth of only 200 mm) from buckling.

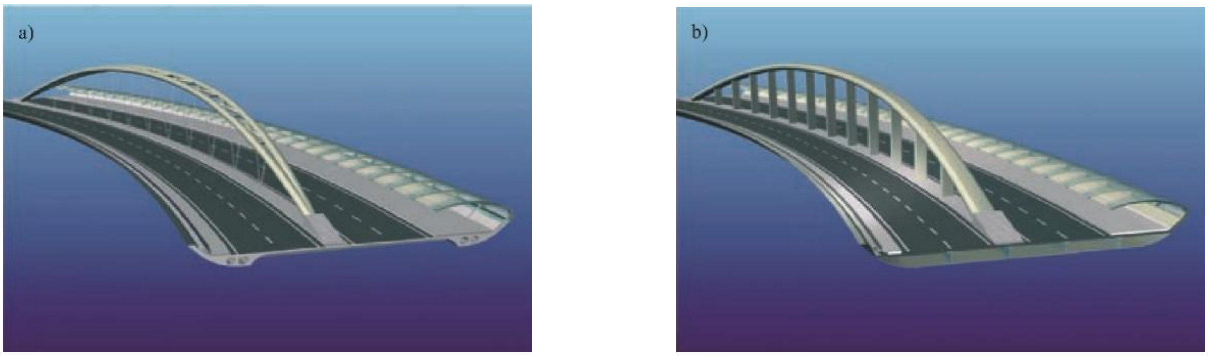
Pinned and stiffened hangers are seldom combined in the same bridge, and it is a decision usually based more on aesthetical purposes than on structural considerations. A known example is the Ondarroa Bridge, at northern Spain (see Fig. 9), designed by Calatrava (see Calatrava 2017, Tzonis 2007, or Jodidio 1998). This bridge has two separate decks: a curved deck for pedestrian traffic and a straight one for vehicles. The former is supported by stiffened hangers, whereas the latter is supported by cables.

Pinned hangers can also be used in three-dimensional arrangements. The Galindo Bridge (Fig. 10), in Bilbao, in Northern Spain (see Manterola-Armisén *et al*. (2011)), is an upper arch bridge whose plan follows the curved deck alignment, resulting in a warped geometry, i.e., not contained within a plane. The deck is linked to the arch by means of two sets of cables. The first one is composed of

Fig. 8 Destructor Bridge, Bath (UK). Photo: happypontist.com

Fig. 9 Ondarroa Bridge. Photo: Zarateman, Wikimedia commons

Fig. 10 Galindo Bridge, Spain. Foto: J. M. García-Guerrero



(a) Trussed arch (b) Transversally stiffened hangers

Fig. 11 Galindo Bridge Non-built solutions vertical hangers that have the same role as in planar arches, transferring vertical loads form the deck to the arch. The second one is composed of active pseudo-horizontal cables anchored both to the arch and to cantilever elements protruding from the inner edge of the deck. These cables introduce active forces in the arch to achieve an antifunicular configuration contained within the arch geometry (see Jorquera-Lucerga 2007).

It is interesting to mention that some alternatives, eventually non-built, were considered for the Galindo Bridge. Fig. 11 shows two of them (see Manterola-Armisén *et al*. 2011). In the first one (Fig. 11(a)) the horizontal unbalanced loads that appear at the arch due to its curved plan are resisted by increasing the transversal stiffness of the arch, transforming its cross-section into a horizontal truss, and by doubling the hangers, a decision that gives them some transversal inclination, and, subsequently, some capacity to resist radial loads. For our research, the second non-built option, shown in Fig. 11(b), is more relevant: the complex spatial arrangement of cables of the built solution is substituted by a set of transversally (i.e., radially) stiffened hangers. Although perhaps the large horizontal forces that appear at the Galindo Bridge could not have resisted only by the stiffened hangers, this alternative has inspired our research: Stiff hangers and hangers composed by more than one cable could be interchangeable.

1.1 Objectives and paper structure

The aim of this paper is to expand the structural possibilities of both fixed and pinned hangers to provide designers with more resources at the initial steps of the design of an arch bridge. For fixed hangers, the effect of the location of hinges both in the transversal and longitudinal directions is analyzed. Regarding the pinned hangers, different combinations of pinned hangers are studied with the aim of finding structural configurations that offer an intermediate point between the structural efficiency of the stiffened hangers and the constructive ease of the pinned ones.

The paper begins by defining, in Section 2, the so-called reference model, which is a common bridge configuration (a tied-arch bridge) on which all the bridges shown in this paper are based. This Section also shows how the in-plane and the out-of-plane behavior of the arch are uncoupled, a fact that allows them to be studied separately. Thus, Section 3 studies the effect of longitudinally stiffened hangers on the longitudinal behavior (i.e., contained within the plane of the arch) of the bridge. In order to illustrate the effect of the hinges at the hangers, a longitudinal hinge has been added to all the hangers of the bridge, and their locations have been modified, ranging from the top to the bottom ends of the hangers.

In Section 4, the effect of transversally stiffened hangers on the out-of-plane response of the bridge is studied. The effects of transversal hinges and their location is analyzed in a study similar to that carried out in Section 3. Section 4 also studies the effect of the transversal stiffness of the arch and the deck on the efficiency of the arch-hanger structural system to resist transversal loads. Then, hinged and fixed hangers are compared. According to the results, it is shown how hangers with a hinge at one of their ends are as practically as efficient as totally fixed hangers. Besides, Section 4 describes how the sensitivity to out-of-plane buckling of the arch can be reduced by stiffening transversally the hangers. Section 4 finishes by comparing tapered hangers to constant cross-section hangers.

In Section 5, a study has been carried out in order to find a configuration of cables whose effect on the structural response of the structure can be regarded as equivalent to a stiff hanger, in an attempt to combine structural efficiency of stiff hangers halfway between effectiveness and cost can be ac and ease of execution of cables. It seems clear that the efficiency of the totally fixed hanger cannot be achieved using the configurations of cables studied in this paper. However, solutions halfway between effectiveness and cost can be achieved by combining the most effective in-plane and out-of-plane configurations. The paper finishes with the conclusions section.

2. The reference bridge and general considerations

2.1 The reference bridge

All the bridges shown in this paper are based on a given configuration, the so-called reference bridge, shown in Fig.

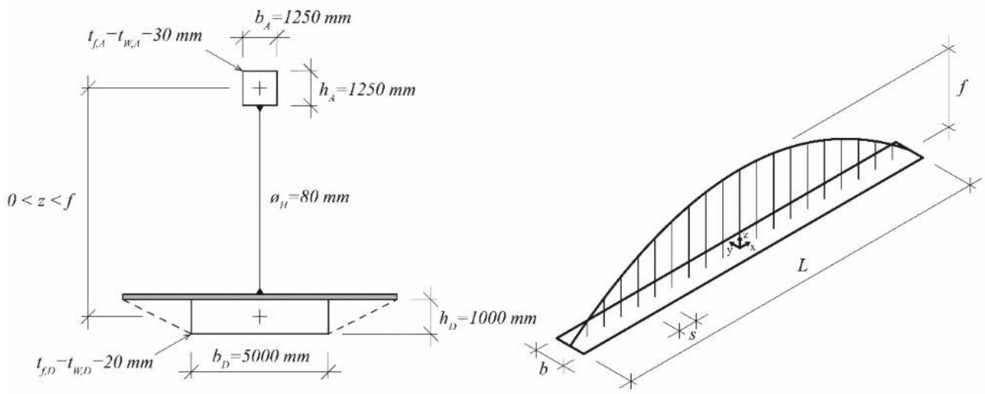


Fig. 12 Geometrical definition of the reference arch bridge (see Table 1)

Table 1 Reference bridge: cross-sections of the structural elements

Element	Cross-section	Dimensions (mm)	Young's modulus E (N/mm^2)
ARCH	Square box hollow section (SHS)	1250 × 1250, $t_{f,A} = t_{w,A} = 30$	2.1×10^5
HANGERS	Circular solid section	ø 80	1.6×10^5
DECK	Rectangular hollow section (RHS)	5000 × 1000, $t_{f,D} = t_{w,D} = 20$	2.1×10^5

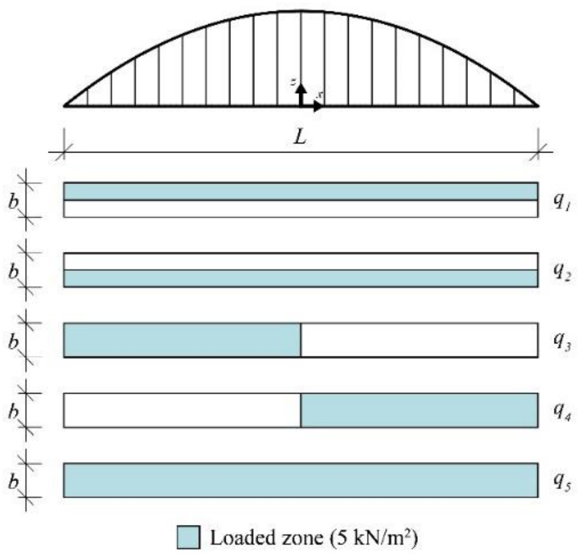


Fig. 13 Load cases considered, EC-1 (2003)

12. This reference bridge is composed of a straight deck supported by a vertical planar arch. The springing points of the arch are tied by a tensioned deck, as corresponds to the typology known as "tied-arch" bridge. The arch and the deck are linked by a set of vertical hangers attached to the deck centerline. In this reference configuration, the hangers are composed of cables and are, therefore, pinned at both ends.

The reference model is a 100 m span (*L*, Fig. 12) bridge. The rise of the arch (*f*) is 20.0 m. The loaded width of the deck, *b*, is 8.0 m. The deck spacing (*s*) between successive anchorages of hangers is 5.0 m. These dimensions have been inspired by real arch bridges and are relatively common (see for example Leonhardt 1982, Lebet and Hirt 2013 or Salonga and Gauvreau 2014).

The cross-sections of the structural elements of the bridge are shown in Table 1.

Regarding the objective of this research, it is not necessary to consider all the possible live load cases than may appear, and only pedestrian loads distributions have been considered, since they illustrate accurately enough the effect on the structural behavior when the hangers are stiffened. Thus, five live loads distributions (q_1 to q_5), shown in Fig. 13, have been considered, where the shadowed area is loaded with a vertical uniformly distributed load q_i of 5 kN/m^2 acting downwards, which corresponds to the pedestrian load model LM-4 defined in EC-1 (2003) or in Spanish IAP-11 (2011). In this paper, all the internal forces and deflections diagrams have been obtained from FEM models analyzed in SAP2000 (see Computers and Structures, Inc. 2013) and postprocessed with Matlab (see Matlab 2014).

2.2 Curtain effect

In a tied arch-bridge similar to the reference bridge defined in 2.1, the most adverse load case corresponds to an asymmetrical load distribution, where half-span is loaded with *q* and the other half with -*q*. This load case is equivalent to the combination q_3-q_4, according to Fig. 13. The acting loads are shared by the arch and the deck, according to their relative flexural stiffness EI_A and EI_D (see, for example Menn 1989).

$$M_A = M \cdot \frac{EI_A}{EI_A + EI_D} \tag{1}$$

$$M_D = M \cdot \frac{EI_D}{EI_A + EI_D} \quad (2$$

where *M* is the bending moment of a simply supported beam under q_3-q_4. However, the accuracy of Eqs. (1)-(2) depends on the hangers' axial stiffness, as shown, for example, in Jorquera-Lucerga (2007) or Siegrist (1997). This happens because, under live loads, the deck behaves as a continuous girder on spring supports. Therefore, when the vertical stiffness of the supports (i.e., the axial stiffness of the hangers) is low, the deflections at both ends of a given hanger are very different, being that at the deck end higher than at the arch end. As a result, the bending moments at the deck increase and the loads are not efficiently transferred from the deck to the arch.

The ideal situations happen when the vertical deflections at both ends are equal, i.e., the deflection of the deck is equal to that of the arch, something that may only happen when the stiffness of the vertical hanger can be assumed as infinite. This ideal situation is the so-called "curtain effect", as defined by Siegrist (1997), in which there is a negligible extension of the hangers. Although it cannot be achieved with real hangers, in practical terms it can be reproduced by assigning the hangers enough axial stiffness, i.e., enough area.

A sensitivity study for the reference model defined in Section 2.1 has been carried out to obtain the minimum hanger diameter from which this "curtain effect" happens, i.e., where the ratio between the arch and the deck deflection δ_A/δ_D can be considered close enough to one. The results show values for $(\delta_A/\delta_D)_{x=\pm L/4}$, at quarter span, under q_3, and $(\delta_A/\delta_D)_{x=0}$, at midspan, under q_5. For the former, $(\delta_A/\delta_D)_{x=\pm L/4}$ tends to be close to 1.00 for values of the hanger diameter around 80 mm and larger (Fig. 14(a)). For the latter, the ratio $(\delta_A/\delta_D)_{x=0}$ is near 1.00 for hanger diameter larger than 250 mm, although the absolute value of the deflections are much smaller (Fig. 14(b)). Therefore, in this paper, unless otherwise specified, 80 mm has been adopted for the diameters of pinned solid hangers. For this value, the error in the Eqs. (1)-(2) for the reference bridge is smaller than 2%, which is accurate enough for the aim of this research. For the stiffened hangers used in this paper, the axial stiffness needed is achieved by far due to the high values of their areas.

2.3 Uncoupled in and out-of-plane structural systems

The cross-section of the arch is assumed to be doubly symmetric, a configuration very common in arch bridges. This means that the behavior of the arch can be resolved into two uncoupled structural systems: Firstly, the in-plane behavior, that correspond to an arch subjected to in-plane bending moments and compressive forces, and secondly,the out-of-plane behavior, in which the arch behaves as a curved girder, where out-of-plane bending and torsional moments are coupled (see Jorquera-Lucerga 2007 or Li *et al*. 2017). The in-plane and out-plane responses are due respectively, to in-plane loads and out-plane loads or torsional moments.

Just to mention some consequences of the fact that the behavior is uncoupled, for example, a hinge may be located, at a given location of a stiff hanger, allowing a rotation depending of its orientation (i.e., contained within a given plane) that does not affect its behavior in the perpendicular plane. Similarly, the stiffness of the hangers can be tailored to provide higher stiffness within a chosen plane, as it happens in H-sections. An important consequence from the point of view of this research is that the analysis of the effect of hangers on in-plane and out-of-plane behavior can be analyzed separately, as it has been done, respectively, in Sections 3 and 4.

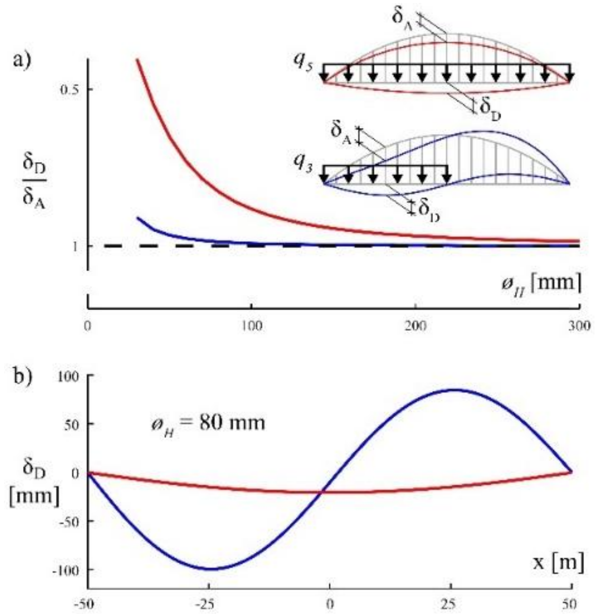


Fig. 14 (a) Ratio δ_A/δ_D vs. hanger diameter, ϕ_H; (b) Deck deflections δ_D under q_3 and q_5 loads for ϕ_H = 80 mm

3. In–plane behavior: pinned vs. longitudinally stiffened hangers

Stiffened hangers can be used to reduce internal forces and deflections at the whole bridge. To illustrate the efficiency of stiffened hangers compared to cables, Fig. 15 compares the bending moments with cables to the in-plane bending moments when the hangers have square hollow-box sections, (SHB, 400×400×20 mm and 800×800×20 mm), fixed at both ends, under the load cases q_3 and q_5 (see Fig. 13), for the reference model.

It can be seen how, for both load cases considered, the stiffer the hangers the smaller the bending moments are. The highest bending moments are obtained at quarterspan ($x = \pm L/4$) for a load acting half the span, i.e., q_3, which becomes the most adverse load case, as it is usual for arch

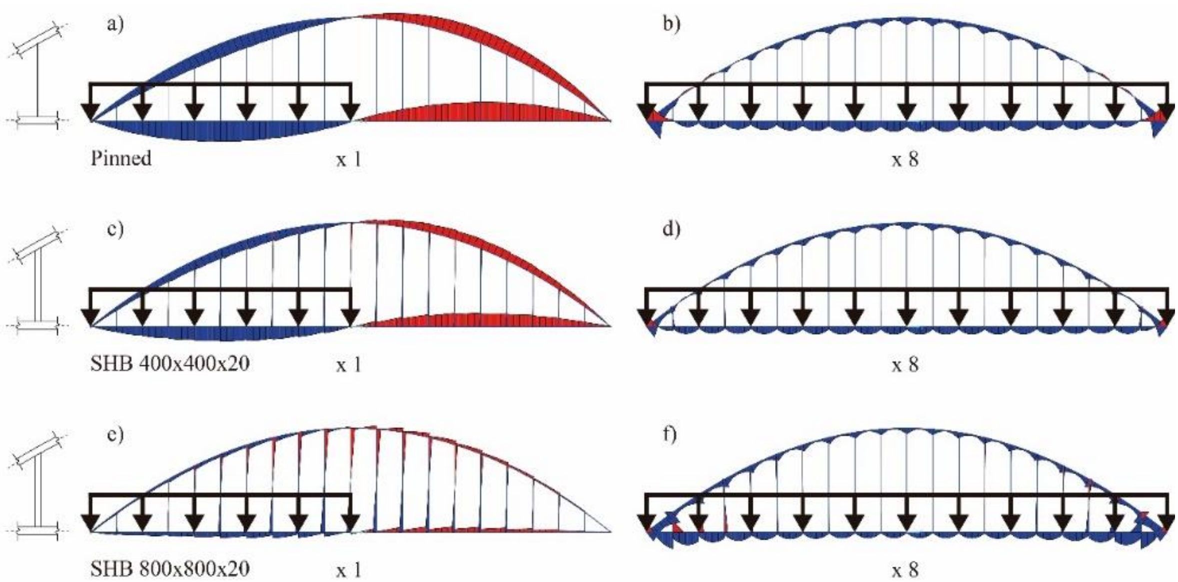


Fig. 15 Bending moment diagrams for the reference model (a, b), fixed SHB 400×400×20 mm stiffened hangers (c, d), and fixed SHB 800×800×20 mm stiffened hangers (e, f), for q_3 (a, c, e) and the whole deck, q_5 (b, d, f)

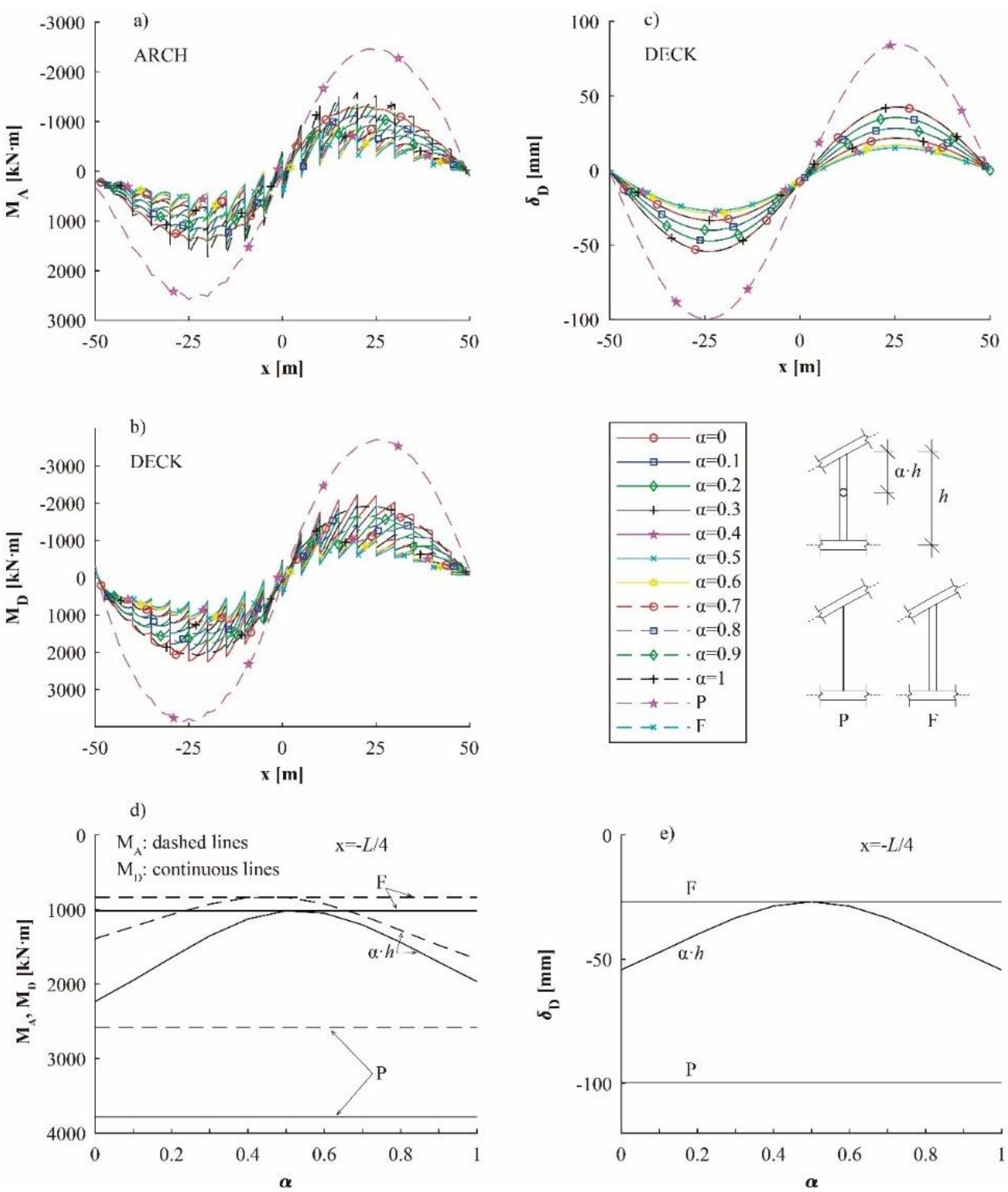


Fig. 16 (a) Arch and (b) deck bending moment diagrams vs. stiffness of hangers and hinge location; (c) Vertical deflection of the deck; (d) Bending moments detailed for quarter span; (e) Deck deflections detailed for quarter span

bridges (see Menn 1989, Manterola-Armisén 2006 or Karnowsky 2012).

3.1 Study of the hinge location

To study the effect of the location of the hinge at the hanger, a longitudinal hinge has been added to all the hangers in a reference model (see Section 2.1) with stiffened hangers (SHB 800×800×20 mm). The relative location of the hinge is at $\alpha \cdot h$, where h corresponds to the length of each hanger, and α ranges from 0 at the top end (at the arch) to 1 at the bottom end (at the deck). In this study, it has been considered that the relative position of the hinge, i.e., $\alpha \cdot h$, is the same for all the hangers, although, obviously, in a real design, pinned and stiffened hangers, with or without hinges, can be freely distributed.

Fig. 16 shows the bending moments distribution at the arch (Fig. 16(a)) and the deck (16(b)) for the different locations of the hinges. In addition, Fig. 16 also shows the values for the reference model defined in Section 2.1 both with pinned (ϕ_H = 80 mm, indicated by a P) and stiffened hangers without hinges (SHB 800×800×20 mm, indicated by a F). In Fig. 16(d) the values of the bending moments at the arch, M_A, and the deck, M_D, are shown at the quarterspan (x = -L/4), where the maximum bending moments appear. It is noteworthy to mention that the maximum bending moments always correspond to pinned hangers, whereas the minimum values correspond to fixed hangers. It is very interesting to confirm that, regarding bending moment reduction (Fig. 16(d)), when the hangers have their hinges located at α = 0.5 they are as effective as fully fixed hangers. This happens both for the arch and the

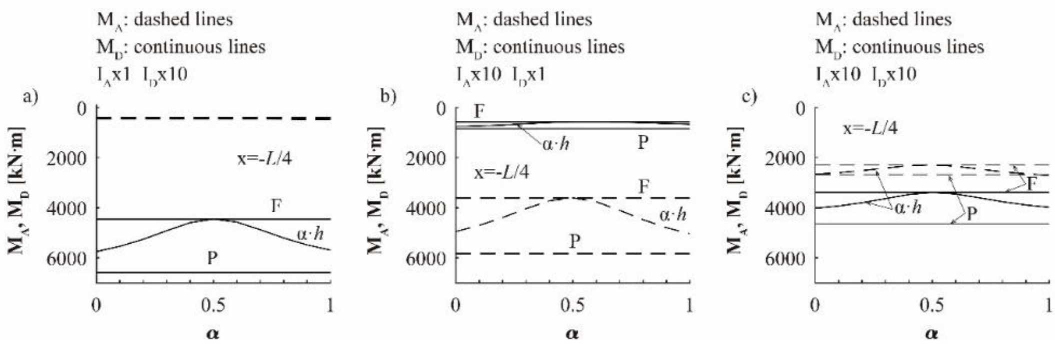


Fig. 17 Bending moment vs stiffness. In-plane stiffness of arch and deck factored, respectively, by k_A and k_D. (a) k_A = 1, k_D = 10; (b) k_A = 10, k_D = 1; (c) k_A = 10, k_D = 10

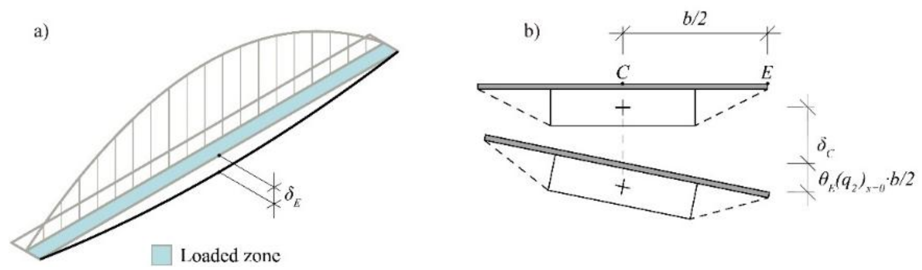


Fig. 18 (a) Deflected shape of the bridge under q_2. (b) deflected cross-section of the deck

deck. Moreover, the relative positions of the hinges (approximately α = 0.5) where the maximum efficiency of the hangers is achieved does not seem to depend on neither the relative stiffness of the cross-sections of the deck and the arch nor their absolute values (Fig. 17).

Similarly, Figs. 16(c) and (e) show the deflections at the deck for the different locations of the hinges, and for the two reference bridges, with cables and fixed hangers. As it happens for the results already described, pinned hangers always lead to larger deflections, whereas values of α close to 0.5 are as effective as fixed hangers in terms of deflection reduction.

4. Out-of-plane behavior: transversally stiffened hangers

In this section, the effect of stiffening hangers on the out-of-plane response of the bridge is studied. The effects of transversal hinges and their location is analyzed in a study similar to that carried out in Section 3. However, in this section, the response of the structure will be analyzed under q_1 or q_2 (See Fig. 13), i.e., uniform loads distributed longitudinally over the whole span and over half its width. Thus, torsional moments appear at the deck, as it is shown in Fig. 18, and the vertical deflection is different between the centerline of the cross section and the edge (points C and E, Fig. 18(b)) of the cross-section. The vertical deflection δ_E at the edge of the deck can be estimated as

$$\delta_E = \delta_C + \theta_C \cdot \frac{b}{2} \quad (3$$

where θ_c is the twist due to torsion at the center of the deck.

4.1 Effect of the location of the hinges

To study the effect of the location of the hinges at the hangers, a transversal hinge has been added to all the hangers of a reference model (see Section 2.1) with stiffened hangers (SHB 800×800×20 mm). The relative location of the hinge is $\alpha \cdot h$, where h corresponds to the length of each hanger, and α ranges from 0 at the top end (at the arch) to 1 at the bottom end (at the deck). In this study, it has been considered that the relative position of the hinge, $\alpha \cdot h$, is the same for all the hangers, although, obviously, in a real design, pinned and stiffened hangers, with or without hinges, could be freely distributed. In the study, out-of-plane bending moments at the arch and vertical axis moments at the deck (see Fig. 19) under q_2 have been analyzed, as well as torsional moments both for the arch and the deck (See Fig. 20). For the same load, vertical deflections have also been obtained at the edge of the deck, as well as transversal deflections at the crown of the arch (See Fig. 21).

As it is shown in Fig. 21(c), the transversal deflection at the crown decreases with α, whereas the vertical deflection at the edge of the deck increases. This fact is coherent with the torsion diagram shown in Fig. 20(b): as α increases, the torsional moment at the deck decreases, because it is partially supported by the arch-hanger structural system, reducing the twist along the deck and the deflections of its edge accordingly. It is noteworthy that the minimum deflection is obtained for α = 0 (hinges at the arch), and this value remains practically unchanged for fixed hangers, and the effect of fixing or not the hangers to the arch is practically negligible.

Some additional studies have been carried out in order

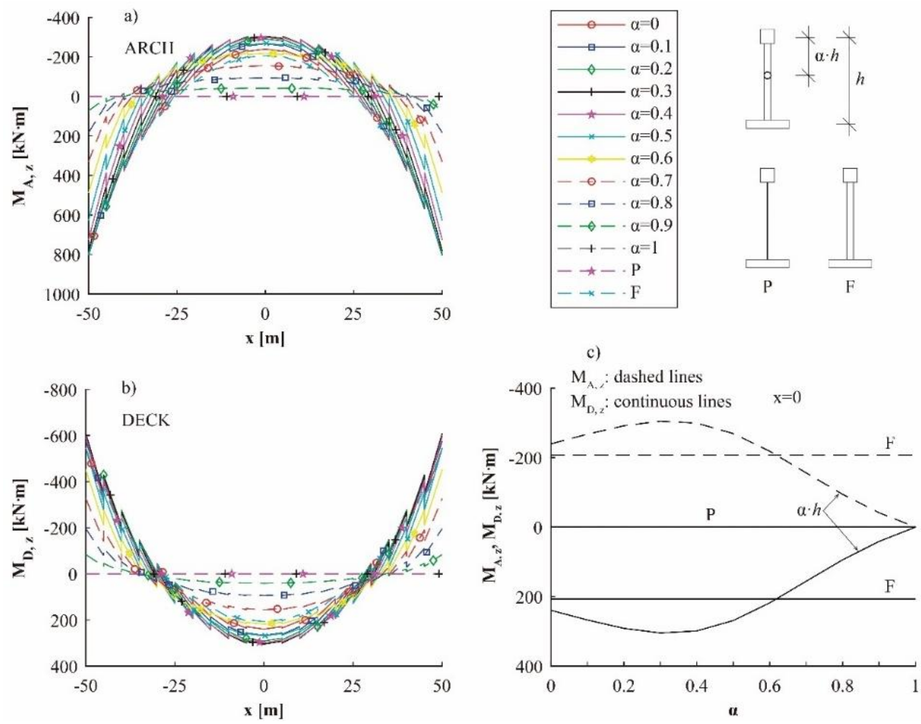


Fig. 19 Transversal arch (a) and deck (b) bending moment diagrams under q_2 vs. hinge location; (c) Detailed results at midspan

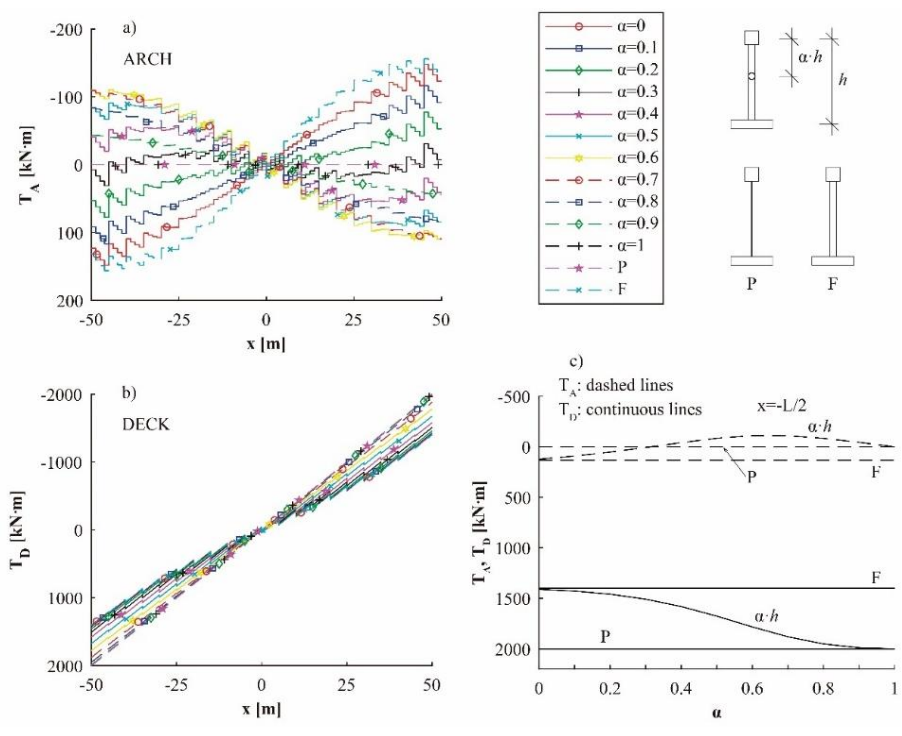


Fig. 20 Torsion under q_2 at the arch (a) and the deck (b) vs. hinge location; (c) Detailed results at springings and abutments

to understand completely the results shown in Figs. 19-21. Firstly, in Section 4.2 it is studied how the external loads are supported by the bridge, i.e., which structural systems of the deck, the arch and the hangers are involved. Then, in Section 4.3 it is shown how the external loads are partially resisted by the structural system composed by the arch and the stiffened hangers, and how its contribution is mainly governed, besides the location of the hinges, by the transversal stiffness of the hangers and the out-of-plane stiffness of the arch. Section 4.4 shows how, when realistic values are assigned to the out-of-plane stiffness of the arch, the fixed hangers are always more efficient than the hangers with a hinge, because they support a higher fraction of external torsional moments and, in addition, they induce smaller out-of-plane forces at the arch.

However, if the stiff hanger is hinged at an end, the solution retains much of its effectiveness. Section 4.5 describes how the sensitivity to out-of-plane buckling of the arch can be reduced by stiffening transversally the hangers and Section 4.6 compares tapered hangers to constant cross-section hangers.

4.2 Structural systems

The effect of the loads q_1 or q_2 can be resolved into a uniformly distributed vertical load $p = q \cdot b/2$ and torsional moment $m_t = q \cdot b^2/8$ acting at the centerline of the deck. The way p and m_t are supported jointly by the arch and the deck is explained in Fig. 22. First of all, regarding the vertical load, it can be assumed that each hanger supports approximately the tributary load P that corresponds to it, $P = p \cdot s$, where s is the longitudinal spacing of the hangers (see Fig. 12).

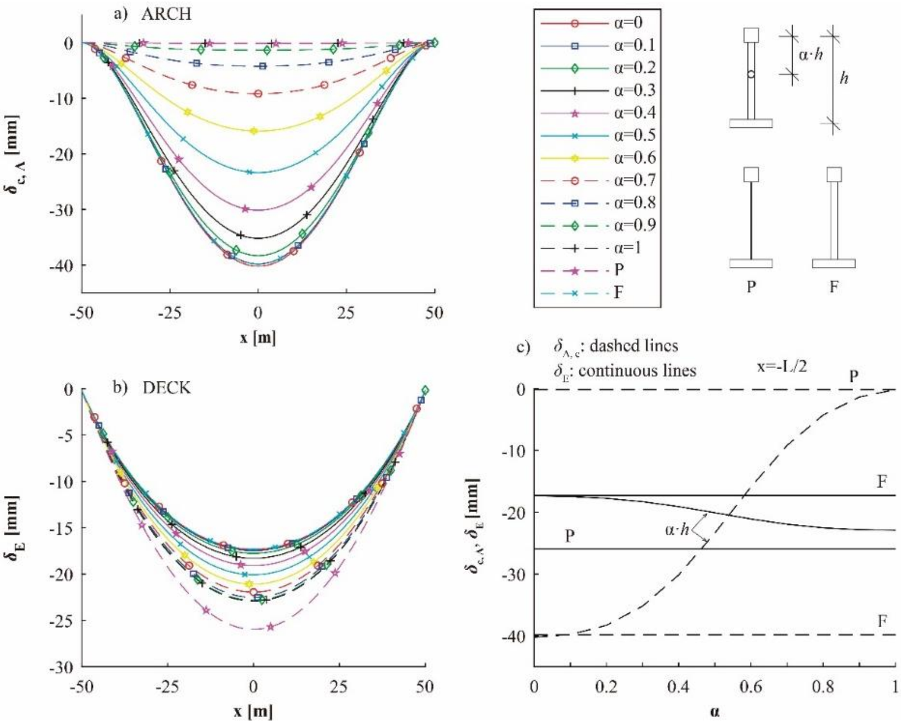


Fig. 21 (a) Transversal deflection at the crown under q_2; (b) Vertical deflection at the edge of the deck under q_2; (c) Detailed results at midspan

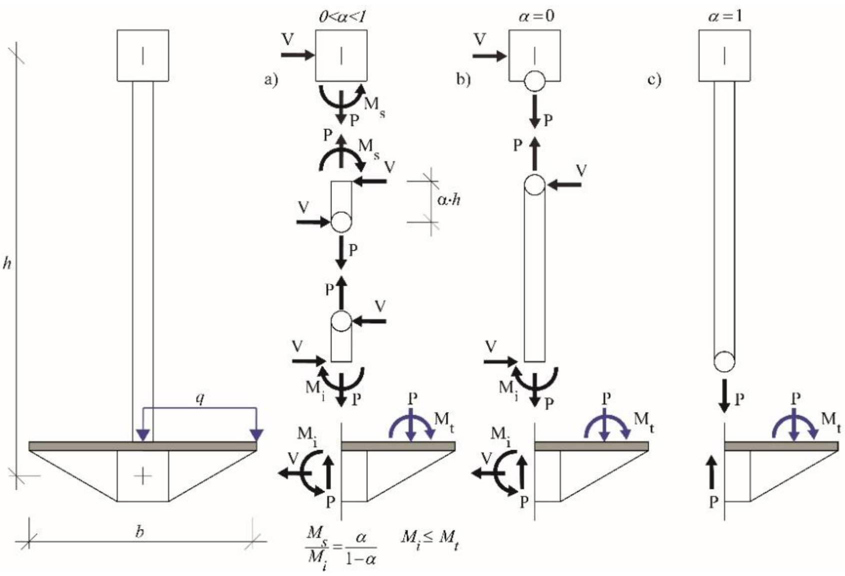


Fig. 22 Load transfer for asymmetrically loaded deck for the structural system composed of an arch and a hanger with a hinge

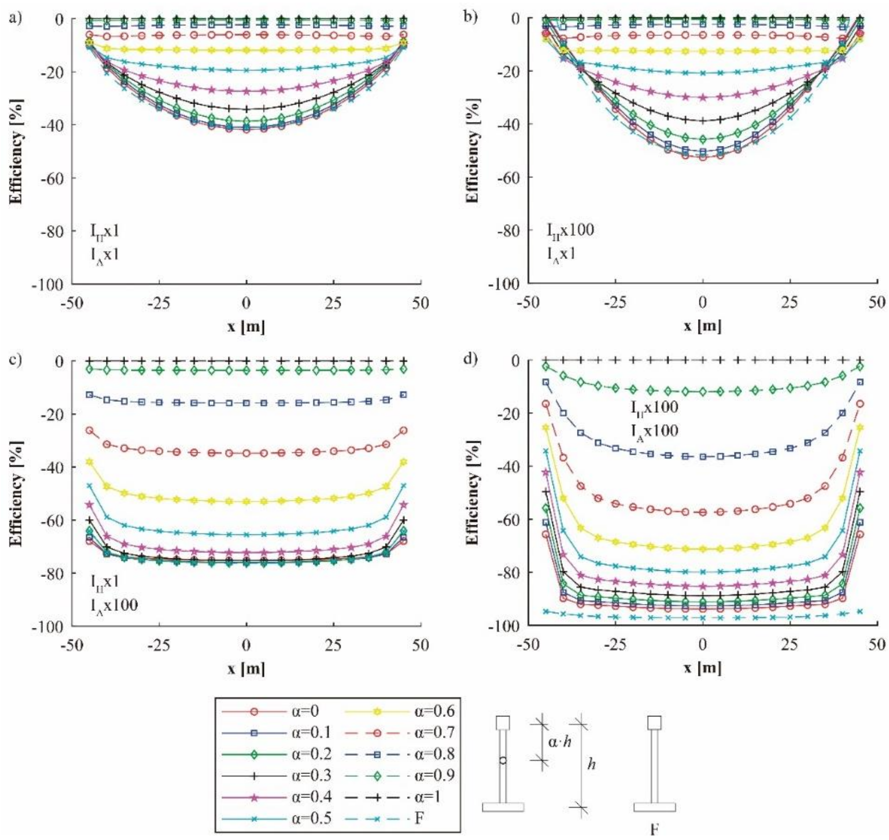


Fig. 23 Efficiency $e_H = M_i/M_t$ vs stiffness. Transversal stiffness of hangers factored by k_H, and transversal and torsional stiffness of arch factored by k_A. (a) $k_H = k_A = 1$; (b) $k_H = 100$, $k_A = 1$; (c) $k_H = 1$, $k_A = 100$; (d) $k_H = k_A = 100$

When the arch is linked to the deck by means either of cables or by hangers pinned at the bottom (Fig. 22(c)), the whole distributed torsional moment m_t due to the eccentric load is totally supported by the cross-section of the deck and entirely transferred to the abutments (See Fig. 20(b)). However, when the arch is linked to the deck by transversally stiffened hangers (Fig. 22(a)), a fraction of the tributary torsional moment, $M_t = m_t s$, corresponding to a hanger, is supported as a concentrated transversal bending moment at the bottom of the hanger, M_i. Of course, always $M_t \geq M_i$. M_s is the concentrated bending moment that appears at the top of the hanger and is transferred to the arch as a concentrated external moment.

Since the shear force V is the same at both sides of the hinge, the ratio M_s/M_i can be expressed, for hangers with a hinge, as

$$\frac{M_s}{M_i} = \frac{\alpha}{1-\alpha} \tag{4}$$

The horizontal shear force V acts at both ends of the hanger, and its value depends on the location of the hinge (Fig. 22(a)), although its value does depend on it. It causes vertical axis bending moments at the deck (Fig. 19(b)), which is seldom the most adverse load case, since the width of the deck is usually determined by functional requirements and it is typically high. Similarly, at the arch, V is a concentrated load acting out-of-plane, which makes the arch behave as a curved girder, where out-of-plane bending (Fig. 19(a)) and torsional moments (Fig. 20(a)) are coupled (see Jorquera-Lucerga 2007 and 2013).

4.3 Effect of transversal stiffness of the hanger and the arch

The transversal efficiency of stiffened hangers may be estimated, for example, by comparing, for the tributary length s of deck for each hanger, the transversal bending moment supported at the bottom of the hanger, M_i (See Fig. 22) with respect to the torsion produced by the eccentricity of the load q_2 acting on the deck, M_t. For the cases shown in Section 4.1, this efficiency of the hangers can be expressed by the coefficient $e_H \equiv M_i/M_t$. Fig. 23(a) shows how e_H is higher for the totally fixed hangers, whereas for the hangers with a hinge, the most effective location is $\alpha = 0$. The efficiency increases towards the midspan and reaches maximum values around 40%.

The transversal stiffness of the hangers is a parameter of major importance to define its efficiency. However, it has an upper bound: if the transverse stiffness of the hanger is factored by 100 (an unrealistic but very illustrative value), the efficiency of the hangers reaches maximum values around 55% (Fig. 23(b)). If the torsional and transversal stiffness of the arch if factored by 100 while the stiffness of the hangers remains unchanged (Fig. 23(c)), the efficiency reaches values around 75%. It is necessary to factor, simultaneously, the out-of-plane stiffness of the arch (transversal inertia and torsional stiffness, which jointly

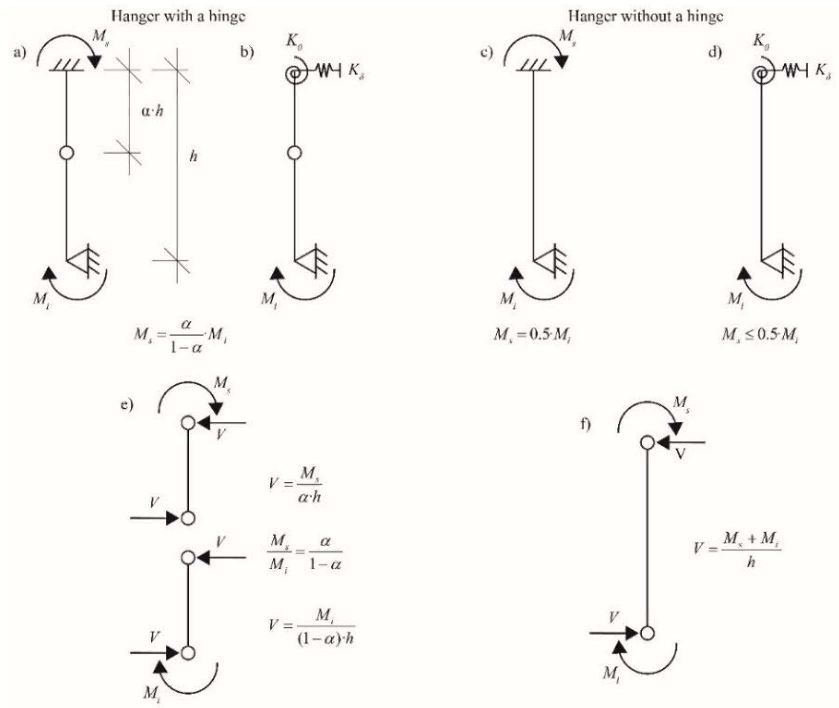


Fig. 24 Structural systems in hinged and non-hinged hangers

define its behavior as a curved beam) and the transversal stiffness of the hangers by very high factors (by 100 in Fig. 23(d)) to achieve values of efficiency close to 100%. In short, the efficiency of the hanger to transform part of the torsional moments of the deck into transversal bending moments at the bottom of the hangers M_i is defined by the combined out-of-plane arch-hanger stiffness. It is noteworthy that the highest efficiency is achieved, in all the cases (see Fig. 23), for the fixed hangers. The most efficient configurations, for hangers that have one hinge, appear for low values of α (i.e., hangers closer to the arch).

4.4 Hinged versus fixed hangers

Sections 4.1 to 4.3 have just shown how transversally stiffened hangers can be chosen by the designer to reduce the torsional moments at the deck and, subsequently, the deflections at the edge. However, if possible, the efficiency of the arch-hanger system should not be achieved at the expense of inducing high loads at the arch. Thus, it is preferable, to support a given moment M_i, to reduce M_s at a minimum (Fig. 22). With that objective, the configurations of Fig. 24 will be studied.

At first glance, it could be thought that the behavior of the transversally stiff hangers is properly described by Figs. 24(a) and (c), i.e., that the assumption that the twist of the arch is totally restrained can be considered valid, which means that the arch is a fixed support for the top end of the hanger. Let us assume that the criterion to make a choice among different hangers is given by the lowest ratio M_s/M_i. For the case (a) M_i is defined by Eq. (4), i.e., by the ratio $M_s = \alpha/(1-\alpha) \cdot M_i$, whereas, for the case (c) it is always $M_s = 0.5 \cdot M_i$. Thus, the hanger with a hinge would always more effective than a totally fixed hanger for $M_s < 0.5 \cdot M_i$, i.e., for $0 < \alpha < 1/3$.

However, this assumption is not valid, because the arch behaves as a spring support for the hanger, whose behavior is defined by Figs. 24(b) and (d). Let us assume that, the connection between the arch and the hanger can be modelled, in a simplified way, as a translational spring plus a rotational spring, which partially allow both the out-of-plane movement and the twist of the arch, and accordingly, of the top end of the hanger. The stiffness of these springs is given, respectively, by k_δ and k_θ (Figs. 24(b) and (d)). Therefore, for the fixed hanger, M_s is strongly reduced. However, for the hanger with a hinge (case b), M_s is still defined by Eq. (4), regardless the value of α. This fact is

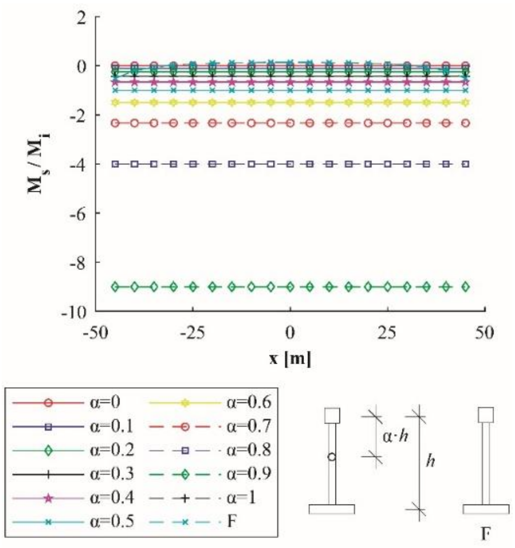


Fig. 25 M_s/M_i for q_2. Reference model

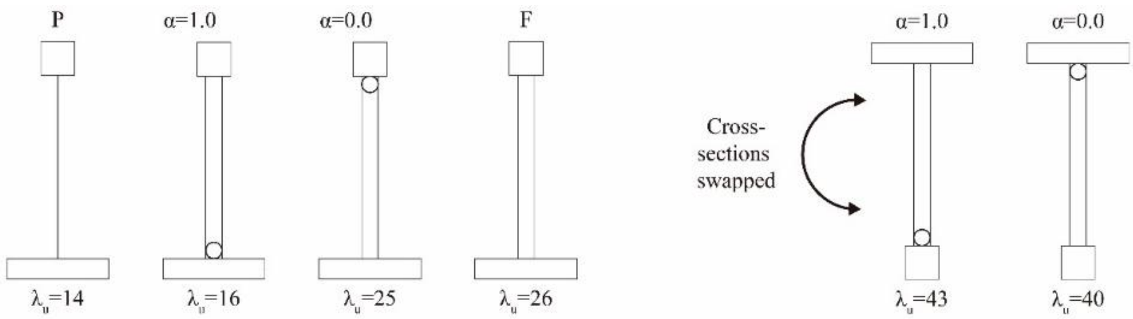


Fig. 26 Reference bridge: Coefficient λ_u for different hangers configuration, for load case B = 1.35·G+W+ $\lambda_u \cdot q_5$

crucial and can be seen in Fig. 25, which shows the ratio M_s/M_i for the reference bridge. It is noteworthy that the lines in Fig. 25 are horizontal, for hangers with a hinge, which means that M_s depends on the location of the hinge within the hanger (See Fig. 24(e)) and not on the location of the hanger in the bridge. As it can be seen, M_s/M_i falls well below 1% for fixed hangers, and these values are very similar to the values obtained for hangers with α = 0, i.e., with a hinge at the top end.

Therefore, as a general conclusion, it can be drawn that the hangers with transversal hinges are not as effective as totally fixed hangers, except for not realistic configurations in which the out-of-plane stiffness of the arch can be assumed as practically infinite. Additionally, hangers with a hinge at their top (α = 0) are as practically as efficient as totally fixed hangers, because the moment M_s tends to vanish due to the rotational flexibility of the arch.

4.5 Out-of-plane buckling

Similarly, the sensitivity to out-of-plane buckling of the arch can be reduced by stiffening the hangers. For the reference bridge, it has been obtained the bucking critical load for the simplified load combination B = 1.35·G + W + $\lambda \cdot q_5$, where G corresponds to dead loads, W to a transversal wind of 2 kN/m^2, and the coefficient λ has been gradually increased until $\lambda = \lambda_u$, which corresponds to collapse (see Fig. 26). It can be seen, and it is intuitive, how fixed hangers are the most efficient when preventing the arch from buckling.

If results for α = 0 and α = 1 are compared, λ_u is higher for α = 1, and it is clear that fixing the hanger to the deck is more efficient that fixing it to the arch. However, it does not mean that fixing the hanger to the deck is always the more effective way of increasing λ_u: if the cross-sections of the arch and the deck are swapped, the coefficients are λ_u = 43 and 40 for α = 1.0 and α = 0.0 respectively. These coefficients are much higher because the cross-section of the arch is the former cross-section of the deck, which is much stiffer. But the important fact, in this case, is that λ_u for α = 1.0 is higher than for α = 0 when the arch is stiffer than the deck.

Fig. 27 Pinned end achieved by means of a tapered hanger with a very low-stiffness cross-section at an end. Merchants bridge, Manchester. Photo: Paul Harrop, geograph.org.uk

4.6 Tapered hangers versus constant cross-section hangers

As it has been shown, hangers with a hinge at an end are practically as effective as totally fixed hangers. If hangers do not need bending capacity at one end, the arch-hanger connection is easier to build, as happens, for example, in the Merchants bridge and in La Devesa footbridge.

In this section, a constant cross-section (RHS 500×400×20) hanger with a pin at its end is compared to a tapered hanger, which varies from a RHS 800×400×20 to a RHS 200×400×20. These distributions of material along the hangers has been defined so that the amount of steel is equal for both hangers. Since the smallest cross-section has a very low transversal stiffness, in practical terms, it behaves as it was pinned at that end (Fig. 27).

Fig. 28(a) shows how the smallest value of the deflection at the edge of the deck corresponds to the tapered hanger, although all the values are very similar because of the high torsional stiffness of the deck. Fig. 28(b) shows the same results for the torsional stiffness of the deck factored by 0.2 (an unrealistic but very illustrative value), a fact that enhances the difference among the different types of hangers. Again, the most effective is the tapered hanger.

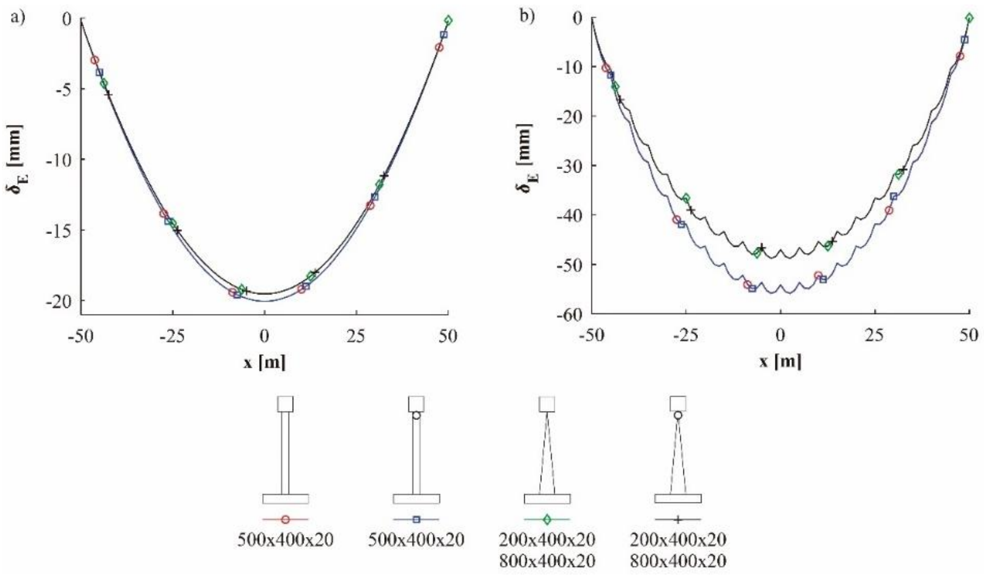


Fig. 28 Deflection at the edge of the deck under q_2: (a) Reference bridge; (b) Torsional stiffness of the deck factored by 0.2

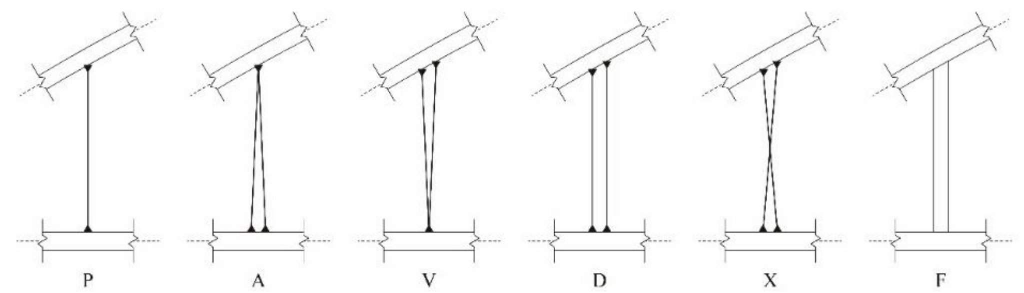


Fig. 29 Configurations of hangers within the plane of the arch

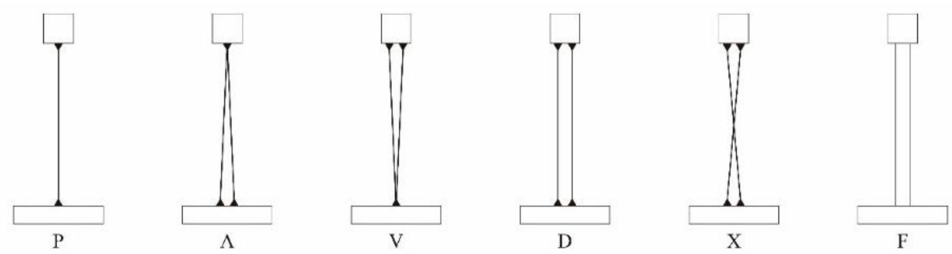


Fig. 30 Configurations of hangers within a plane perpendicular to the arch

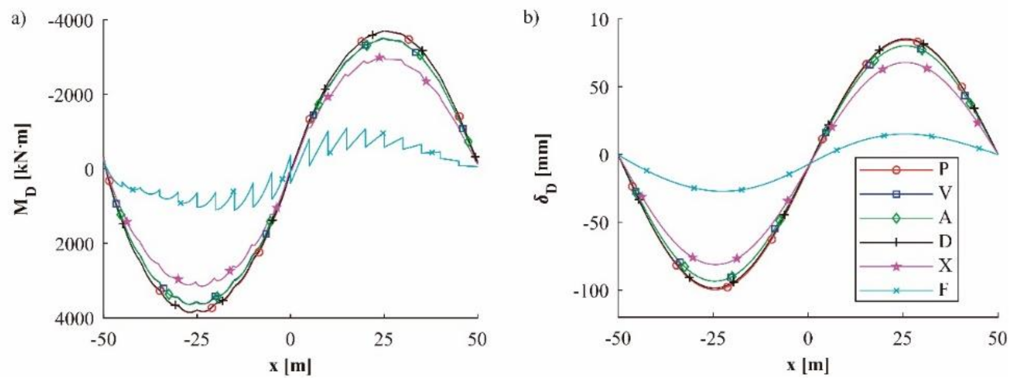


Fig. 31 Deck under q_3. (a) bending moments; (b) deflections at the centerline

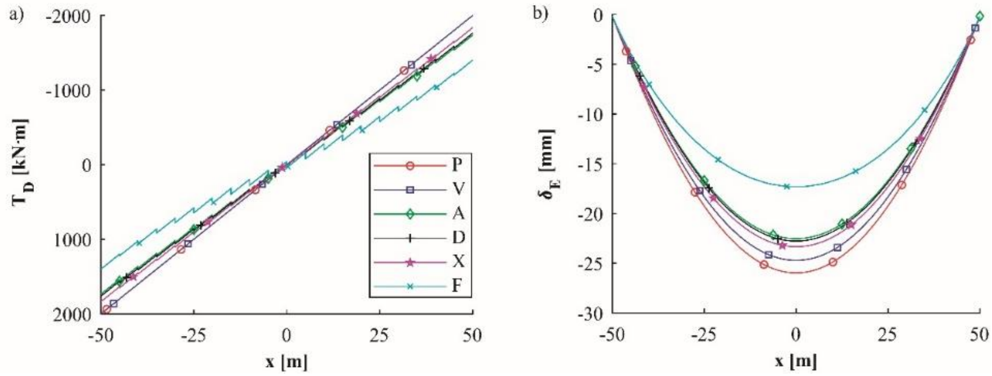


Fig. 32 Deck under q_2. (a) torsional moments; (b) deflections at the edge of the deck

5. Cable arrangement with bending capacity

Linking the arch and the deck by means of cables lead to overall cost savings in the bridge, since cables are also easier to build that stiff hangers. However, stiff hangers are generally more advantageous from the structural point of view, when reducing bending moments and deflections, as it has been shown in the previous sections.

Therefore, in this section, a study has been carried out in order to find a configuration of cables whose effect on the structural response of the structure can be regarded as equivalent to a stiff hanger, in an attempt to combine structural efficiency of stiff hangers and ease of execution of cables. Since the in-plane and the out-of-plane response of the bridge are uncoupled, they have been considered separately, as it was done for the stiff hangers.

Fig. 29 shows the studied configurations of hangers within the plane of the arch: A, V and X-shaped, as well as Double parallel vertical hangers (D). All of them have been compared to a cable (P) and a totally fixed hanger (F). The stiff hanger with a hinge at the arch (α = 0) has not been considered since its effect is very similar. The separation between the cables is 800 mm, which is equal to the size of the hangers.

Similar configurations have been studied for the out-of-plane behavior of the bridge (Fig. 30). They are obtained by rotating 90° the longitudinal configurations about a vertical axis.

Fig. 31 shows bending moments and vertical deflections at the deck under q_3, for the different in-plane configurations shown in Fig. 29. The most effective in-plane configuration, by far, is the totally fixed hanger. The X-shaped configuration is the best from the hangers composed only of cables.

Fig. 32 shows, again, how the totally fixed hanger is the most effective regarding transversal behavior. For the hangers composed only cables, the A-shaped is the most effective.

Therefore, it seems clear that the efficiency of the totally fixed hanger cannot be achieved using the configurations of cables defined in Figs. 29 and 30. However, solutions halfway between effectiveness and cost can be achieved by combining the most effective in-plane and out-of-plane configurations, as shown in Fig. 33.

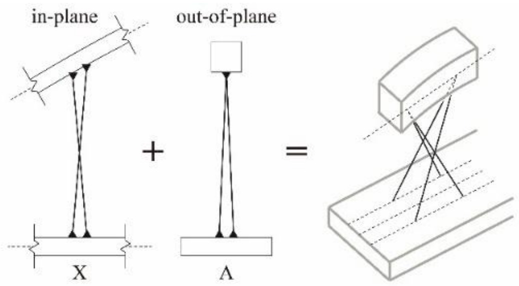


Fig. 33 Spatial cable arrangements obtained by combining most efficient planar configurations

6. Conclusions

In arch bridges, the way the arch and the deck are connected may become crucial. Both bending moments and deflections may be significantly reduced, provided the typology of the arch-deck connection is properly selected. Thus, the hanger arrangement becomes an additional tool available for designers, mainly at the early stages of design.

Cross section. In this paper it has been shown that the most effective type of hanger is, by far, the totally fixed hanger with bending and shear capacity, with cross-sections such as RHS or H-sections. This is the most effective hanger, from the structural point of view, both for in and out-of-plane behavior. The exception could be the network cable arrangement, beyond the scope of this paper, which is extremely efficient in order to reduce longitudinal bending moments and deflections.

Stiffness. Since, due to the longitudinal symmetry of the structure, the in-plane and out-of-plane responses of the structure are uncoupled, the effect of hangers can be considered independently for the plane of the arch and the perpendicular. For example, the orientation of the H-shaped hangers can be defined to provide either longitudinal stiffness, in order to support in-plane bending, or, alternatively, transversal stiffness, in order to support torsional forces at the deck or out-of-plane behavior at the arch.

The efficiency of the hanger not only depends on the stiffness of the hanger, but also on the combined stiffness of the arch or the deck. For example, the reduction of the deflection of the edge of the deck is governed by the transversal arch-hanger stiffness.

Location of the hinges. The studies carried out show that, in practical terms, the ends of the hanger are the only locations for the hinges that do not lead to a significant loss of efficiency.

Hinged hangers. If the stiff hanger is hinged at an end, the solution retains much of its effectiveness compared to the fully fixed hanger. The loss of effectiveness is smaller if the hanger is fixed to the stiffest element, either the arch or the deck. Since the connection at the hinged end does not need bending and shear capacity, the hinged hanger is easier to build and lead to overall cost savings.

Tapered hangers. The material of the hinged hanger may be distributed in such a way that is stiffer at the fixed end. This distribution increases the efficiency of the hanger, at the expense of using non-constant cross-sections.

Combination of cables. The efficiency of the totally fixed hanger (or similar solutions) cannot be totally achieved using the configurations of cables studied in the paper. However, solutions halfway between effectiveness and cost can be achieved by combining the X-shaped arrangement in the longitudinal plane and the A-shaped in the perpendicular plane.

Recommendations for the design. In general terms, regarding the in-plane behavior, the use of hangers composed of cables (either with vertical, Nielsen-Löhse or network arrangements) is recommended whenever possible, as a way of reducing in-plane forces and deflections, due to its comparatively low cost and ease of erection. Alternatively, longitudinally stiffened hangers, fixed at both ends, can be used with the same purpose.

Regarding out-of-plane behavior, three-dimensional arrangements of cables (A-shaped, for example) can be used, although their efficiency are limited. When transversally stiffened hangers are to be used, the most efficient arrangement is composed of hangers fixed at both ends. A configuration almost as efficient as this one, and, additionally, cheaper and easier to build, is achieved by locating a hinge at the end corresponding to the most flexible structural element (normally the arch). The efficiency of this configuration is further improved if the section tapers from the fixed end to the pinned end.

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