Steel and Composite Structures, *Vol. 32, No. 4 (2019) 455-466*
DOI: https://doi.org/10.12989/scs.2019.32.4.455

A response surface modelling approach for multi-objective optimization of composite plates

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(*Received May 25, 2018, Revised January 9, 2019, Accepted August 1, 2019*)

Abstract. Despite the rapid advancement in computing resources, many real-life design and optimization problems in structural engineering involve huge computation costs. To counter such challenges, approximate models are often used as surrogates for the highly accurate but time intensive finite element models. In this paper, surrogates for first-order shear deformation based finite element models are built using a polynomial regression approach. Using statistical techniques like Box-Cox transformation and ANOVA, the effectiveness of the surrogates is enhanced. The accuracy of the surrogate models is evaluated using statistical metrics like R^2, ${R^2}_{adj}$, ${R^2}_{pred}$ and ${Q^2}_{F3}$. By combining these surrogates with nature-inspired multi-criteria decision-making algorithms, namely multi-objective genetic algorithm (MOGA) and multi-objective particle swarm optimization (MOPSO), the optimal combination of various design variables to simultaneously maximize fundamental frequency and frequency separation is predicted. It is seen that the proposed approach is simple, effective and good at inexpensively producing a host of optimal solutions.

Keywords: FE-surrogate; metamodel; multi-objective genetic algorithm (MOGA); multi-objective particle swarm optimization (MOPSO); pareto front

1. Introduction

Favorable qualities like high strength-to-weight and high stiffness-to-weight ratios combined with low operational cost have led to increased use of composites in the structural engineering field. Often, these composite structures act as the load carrying members and thus are subjected to various static and dynamic loads. Consequently, it is desirable that any machinery installed on these structures are not in resonance with them. A straightforward way to ensure this is by allowing the machinery to operate well outside the range of the intrinsic frequency of the structure. The ability to fine-tune structures in order to maximize or minimize its natural frequencies would thus be a lucrative option for design engineers.

With the rapid advancement in computing power, there has been a competitive development of numerical tools and theories in the structural optimization field. A plethora of nature-inspired techniques has been developed in the last three decades to solve multimodal and computationally intensive optimization problems from heuristic approaches. While such approaches are known to escape the pit of local optima— a serious drawback of classical optimization techniques, many of these metaheuristics (Boussaid *et al*. 2013) are known to depend on several parameters to be set by the user *a priori*. The choice of these parameters considerably influences the success of the approach. For example, in case of Genetic Algorithm— a nature-inspired technique that has been around for almost four decades now, there is still considerable disagreement (Mills *et al*. 2015) amongst researchers regarding the various tuning parameter settings. In fact, De Jong (2007) suggested that any numerical experimentation based on evolutionary algorithms should first conduct a few preliminary experiments to determine the optimal parameter settings. Thus, an optimization problem may need to be solved several times using different combinations of these tuning parameters in order to gain enough confidence in the predicted output. Though this can be easily done for problems involving just a few parameters and small search space, it can be a tedious and time-consuming exercise in case of structural optimization problems involving finite element simulations. Finite element approaches are known to be accurate but computationally intensive.

A remarkable reduction in the total computational effort can be obtained by reducing the number of structural analyses. This can be done by developing globally robust approximation routines to simulate the relevant structural

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http://www.techno-press.org/?journal=scs&subpage=6
ISSN: 1229-9367 (Print), 1598-6233 (Online)

phenomena. Such an approximation routine or surrogate metamodel eliminates the need to run the computationally intensive FE models iteratively (Abu-Odeh and Jones 1998). By replacing the original FE model with a surrogate metamodel, the objective function can be evaluated at a fraction of the original cost (Pajunen and Heinonen 2014). Response surface methodology (RSM) (Box and Wilson 1992), artificial neural network (ANN) (Haykin 2001) and radial basis functions (RBF) (Hardy 1971) are the most popular and widely studied metamodeling techniques in structural engineering. In RSM, the basis functions for approximation are polynomials chosen *a priori*. RSM is widely used in structural optimization problems like helicopter rotor (Ganguli 2002), truss (Ju *et al*. 2013) (Fang and Tee 2017), stiffened plates (Heinonen and Pajunen 2011), marine structures (Pajunen and Heinonen 2014), laminated plates subject to stress and displacement constraints (Abu-Odeh and Jones 1998), lateral stability of arch bridge (Pan *et al*. 2011), FRP composite deck (Kim *et al*. 2009), composite shells (Dey *et al*. 2016) etc.

Existing literature reveals that RSM metamodels are useful in structural engineering. However, metamodels have been sparingly used in the optimization of frequency parameters, where there is scope for further exploration. One exciting avenue is to test its applicability in a multi-objective optimization scenario, along which there has been very little work. In tune with genetically modified organisms whose genetic material is altered to increase yield or quality, the present research work explores genetically optimized composite laminates, whose material, geometric or layup orientation has been altered with the help of the in-silico counterpart of genetic engineering, viz. multi-objective genetic algorithm (MOGA). The algorithm simultaneously maximizes the fundamental frequency and frequency separation between the first two modes. Additionally, another robust swarm intelligence based multi-objective optimization algorithm, viz. multi-objective particle swarm optimization (MOPSO), is used alongside to predict the Pareto optimal solutions for comparison with one another. To the best of the authors' knowledge, this is the first application of such multi-objective optimization using state-of-the-art nature-inspired heuristic algorithms with RSM metamodels.

The rest of the paper is organized as follows: Section 2 presents a brief overview of the overall design and optimization framework. The response surface model used in the research is discussed in detail to ensure reproducibility. A brief discussion of the two multi-objective optimization algorithms (MOGA and MOPSO) are also included. In Section 3, the FE model is validated with published results. The validity and efficacy of the RSM metamodels built to replace the FE model are then illustrated. Subsequently, in the final part of Section 3, these second-order polynomial metamodels are used as objective functions in the MOGA and MOPSO algorithm to yield the multi-objective optimal solutions. Section 4 lists the key

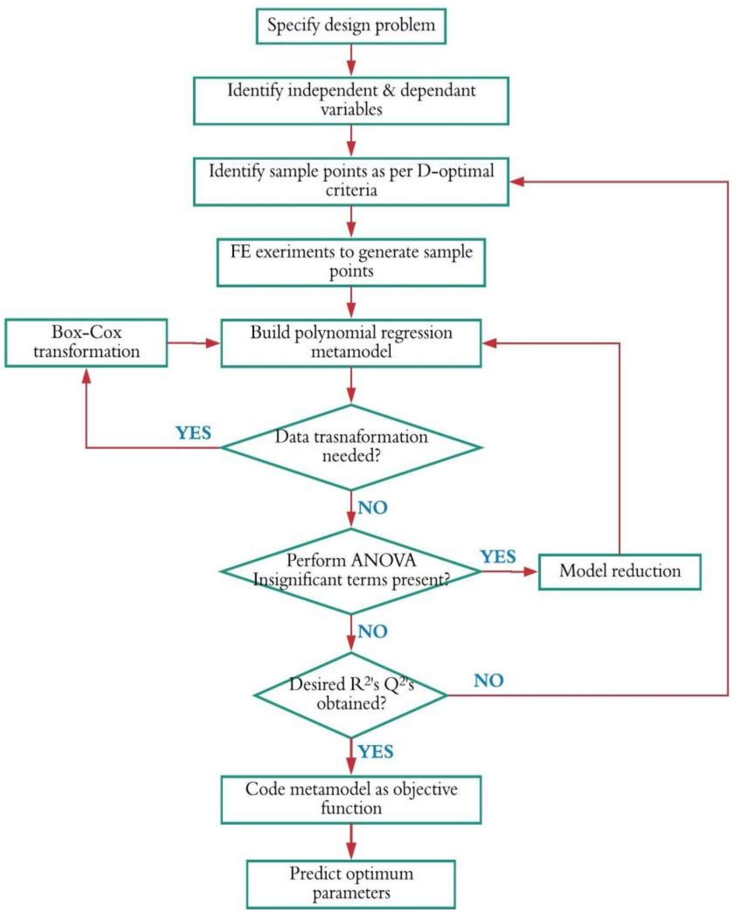

Fig. 1 Design and optimization framework used in the current study

findings and recommendations.

2. Design and optimization framework

A multi-objective multiparameter optimization procedure is developed by combining polynomial regression-based response surface models with robust metaheuristic algorithms, MOGA and MOPSO— the multi-objective procedures for Genetic Algorithm and Particle Swarm Optimization. The RSM metamodel is developed by using the highly accurate numerical data from a finite element program developed by the authors. Fig. 1 shows the design and optimization framework for the current problem. Based on the design problem, the input and output parameters are identified. While the independent input parameters for each problem are presented in Table 1, the output parameters are taken as the fundamental frequency (λ_1) and frequency separation between the first two modes (λ_{21}) in each problem. A design of experimentation scheme based on the RSM design (described in section 2.2) is selected and the numerical experiments are conducted using the finite element formulation reported in section 2.1. A polynomial regression based metamodel is developed based on these RSM sampling points using the FE data. With the help of Box-Cox plots, the need for any data transformation is identified and subsequently implemented. The model is tested for the desired level of accuracy and analysis of variance (ANOVA) test is performed to remove the non-significant terms from the metamodel. The metamodel is then used as the objective function for the multi-objective optimization using MOGA and MOPSO.

To depict the efficacy of the developed approach, four design and optimization test problems (referred to hereafter as TP-01, TP-02, TP-03 and TP-04) are used.

Test Problem 1: TP-01 is a 4-design variable optimization problem where the objective is to find an optimal combination of material constants (E_1/E_2 , G_{12}/E_2 , G_{23}/E_2 and ϑ_{12}) such that λ_1 and λ_{21} are simultaneously maximized. An all-side simply-supported, 8 layer symmetric ($[45/-45/45/-45]_s$), 30° skew composite plate is chosen as the structure of interest. The aspect ratio (b/a) is taken as 1, and the thickness-to-length ratio (h/a) is taken as 0.01.

Test Problem 2: The goal in TP-02 is simultaneous maximization of λ_1 and λ_{21} by using optimal geometric parameters (b/a , h/a , α and n) in an all-side clamped symmetric composite laminate. Since the number of plies (n) cannot be a fraction, additional constraints are imposed on it during the optimization phase, such that only integer values are considered. The material properties are chosen as $E_1/E_2 = 40$, $G_{12}/E_2 = 0.6$, $G_{23}/E_2 = 0.5$ and $\vartheta_{12} = 0.25$.

Test Problem 3: In TP-03 the 4 fiber angles (θ_1, θ_2, θ_3 and θ_4) of an 8-layer symmetric all-sides simply-supported composite plate are chosen as the design variables. The material properties are the same as those used in TP-02. The composite plate is of rectangular in shape with an aspect ratio of 2 (i.e., $b/a = 2$, $h/a = 0.1$, $\alpha = 0$, and $n = 8$.

Test Problem 4: TP-04 is developed as an 11-variable (E_1/E_2 , G_{12}/E_2 , G_{23}/E_2 , ϑ_{12}, b/a , h/a , α, $\theta_1, \theta_2, \theta_3$ and θ_4) design and optimization problem for an all-side simply-supported 8-layer composite skew plate.

The range of all the design variables used in the four test problems is reported in Table 1.

2.1 Finite element formulation

A nine-node isoparametric plate bending element is used in the current finite element analysis. The rotary inertia and shear deformation effect are considered in the finite element analysis by considering first order shear deformation theory. In literature, this finite element formulation has been used by the authors for free vibration analysis of various plate structures (Kalita and Haldar 2017) (Kalita *et al*. 2019a) (Kalita *et al*. 2016). In those past works, the FEM procedure has been shown to be in very close approximation to exact solutions and thus it is not presented here.

2.2 Response surface methodology

Response surface methodology (RSM) generates an approximate equation relating the independent (input) parameters to the dependent (output) parameters. The inherent statistical and mathematical analysis fits an equation of the following form

$$y = f(x_1, x_2, x_3, \dots, x_k) + \varepsilon \tag{1}$$

Table 1 Range of design variables

Variable type	Design variable	Symbol	Range
Material parameters	Orthotropy ratio	E_1/E_2	$[20, 60]$
	Major shear modulus to Young's modulus ratio	G_{12}/E_2	$[0.4, 0.8]$
	Minor shear modulus to Young's modulus ratio	G_{23}/E_2	$[0.3, 0.7]$
	Poisson's ratio	ϑ_{12}	$[0.19, 0.31]$
Geometric parameters	Height-to-width ratio	b/a	$[1, 5]$
	Thickness-to-width ratio	h/a	$[0.01, 0.1]$
	Skew angle	α	$[0, 60]$
	Number of plies	n	$[2, 10]$
Layup angles	Fiber angles	θ_1, θ_2, θ_3, θ_4	$[-90, 90]$

Here, f denotes the approximate response surface and ε is the normally distributed statistical error. Each x_i represents an independent parameter, while k is the maximum number of independent parameters. In RSM, a second-order polynomial form of the model is fitted as

$$y = \beta_0 + \sum_{i=1}^{n} \beta_i \, x_i + \sum_{i=1}^{n} \sum_{j>i}^{n} \beta_{ij} \, x_i x_j + \sum_{i=1}^{n} \beta_{ii} x_i^2 + \varepsilon \quad (2)$$

In this manuscript, all sampling data are modeled using D-optimal criteria. D-optimality is achieved if the determinant of $(X^t X^{-1})$ is minimal. Here, X denotes the design matrix as a set of value combinations of coded parameters, and X^t is the transpose of X (Mukhopadhyay *et al*. 2015).

Since a second-order polynomial may not be able to capture the high degree of nonlinearity in some systems, an appropriate data transformation technique is used. The Box-Cox transformation (Box 1964) represents a family of power transformations that incorporates and extends the traditional options to help researchers easily find the optimal normalizing transformation for each variable. A Box-Cox plot can reveal the needed transformation for the data. The minimum point of the curve generated by the natural log of the sum of squares of the residuals at various powers of transformation (Λ) represents the appropriate transformation parameter. Thus, wherever needed the training dataset is transformed using Box-Cox transformations calculated as

$$y' = \begin{cases} (y + C)^{\Lambda} & (\Lambda \neq 0), \\ \ln(y + C) & (\Lambda = 0); \end{cases} \quad (3)$$

Where C is a constant and Λ is the power of transformation.

The RSM model is then constructed using the D-optimal design identified, where FEM simulations are carried out at the required design points and fitted using the multiple regression technique. The difference between the FE design points (y_i) and the RSM model predicted points $(\hat{y}_i)$ is called residual.

$$\varepsilon_i = y_i - \hat{y}_i \quad (4)$$

The β_i estimates in Eq. (2) is selected such that the sum of squares of the residuals is minimized.

$$SS_R = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 \quad (5)$$

The statistically non-significant terms are screened and removed from the RSM model. This is done with analysis of variance (ANOVA), where the effect of each independent variable on the entire model variance is quantitatively evaluated. The accuracy of the model is evaluated using a goodness of fit criteria like R^2, $R^2{}_{adj}$ and $R^2{}_{pred}$. These can be calculated as

$$R^2 = 1 - \frac{SS_R}{SS_T}, \qquad \text{where } SS_T = \sum_{i=1}^{n} (y_i - \bar{y})^2 \quad (6)$$

$$R^2{}_{adj} = 1 - \frac{n-1}{n-k-1}(1 - R^2) \quad (7)$$

$$R^2{}_{pred} = 1 - \frac{SS_{R_{pred}}}{SS_T}, \quad \text{where } SS_{R_{pred}} = \sum_{i=1}^{n} (y_i - \hat{y}_{i/i})^2 \quad (8)$$

$\hat{y}_{i/i}$ is the observed $\hat{y}_i$ calculated by the model when the i^{th} sample is left out from the training set. This corresponds to the leave-one-out cross validation scheme.

Since R^2, $R^2{}_{adj}$ and $R^2{}_{pred}$ are based on use or re-use of the training dataset, an additional metric $Q^2{}_{F3}$ based on the use of independent test data is used. For calculation of $Q^2{}_{F3}$, 100 independent test sample points are randomly generated for each test problem and the corresponding $Q^2{}_{F3}$ metric proposed by Consonni *et al*. (2010) and suggested for structural metamodels in (Kalita *et al*. 2019b) is calculated.

$$Q^2{}_{F3} = 1 - \frac{\sum_{i=1}^{n_{test}} (\hat{y}_i - y_i)^2 / n_{test}}{\sum_{i=1}^{n_{train}} (y_i - \bar{y}_{train})^2 / n_{train}} \quad (9)$$

2.3 Multi-objective optimization

In most design and optimization problems, there are several objectives that often need to be tackled simultaneously. Such optimization problems are called multi-objective optimization. One way to solve them is by converting them into an equivalent single objective optimization using the weighted sum approach. By multiplying each normalized objective by a suitable weight and then combining them would lead to an equivalent single objective optimization (Abachizadeh and Tahani 2009) (Jakob and Blume 2014). However, such optimal solutions are dependent on the choice of weight. Another approach is to predict a host of non-dominated solutions, referred to as Pareto optimal solutions which together constitute a Pareto front. Each solution of the Pareto front is a chosen such that solution to one objective function can be improved only by worsening at least one of the other objective functions.

2.3.1 Multi-objective genetic algorithm

Genetic Algorithm (GA) is good at taking colossal search spaces and navigating them, looking for the best combinations of parameters and predicting optimal solutions. It works on Darwin’s principle of natural selection (Goldberg 2006). GA is superior to most conventional search techniques in three significant ways. It does not get trapped in local optima as it performs parallel search throughout the population of solutions. Secondly rather than optimizing the parameters themselves, GA works on chromosomes which constitute an encrypted form of a potential solution, effectively bringing about a faster convergence. Thirdly the algorithm uses a fitness score based on the objective function to predict a feasible solution, which invites better performing solutions to influence successive searches. The user typically chooses the best structure from the last generation as the optimal

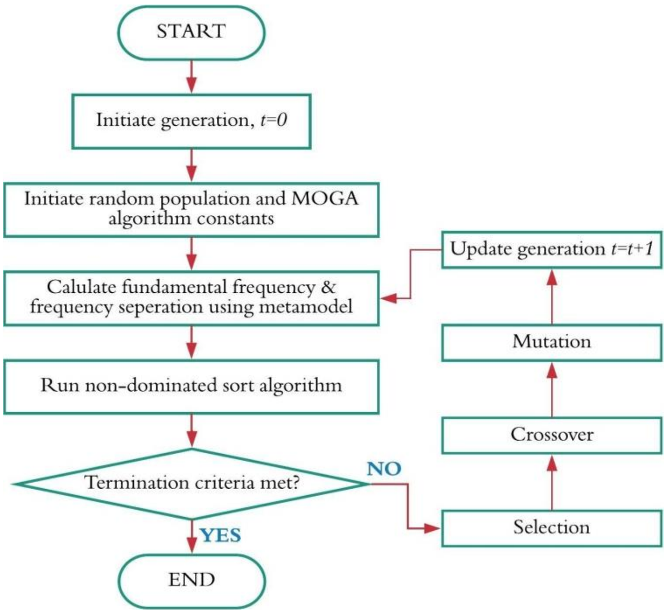

Fig. 2 Implementation scheme of MOGA

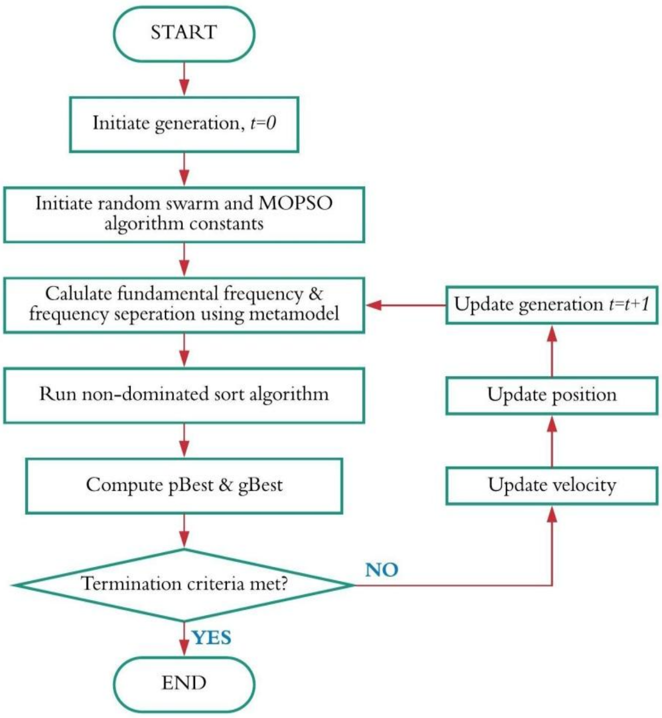

Fig. 3 Implementation scheme of MOPSO

solution if the algorithm is set to terminate after a certain tolerance level is reached. However, running the algorithm for a predetermined number of times is more common among researchers. In this case, the algorithm terminates when the total predetermined number of iterations is reached, and it reports back the best solution encountered among all the generations. The implementation scheme of the multi-objective genetic algorithm (MOGA) used in the current study is highlighted in Fig. 2.

2.3.2 Multi-objective particle swarm optimization

Particle Swarm Optimization (PSO) originally developed by Eberhart and Kennedy in 1995 (Eberhart and Kennedy 1995), is an algorithm that was first used to model the social behavior of birds and fish communicating among themselves in search for food. The standard PSO (SPSO) algorithm is relatively straightforward. It has since developed into a widely researched algorithm to optimize solutions to many kinds of problems. It works by first

Table 2 Performance of the RSM metamodels

Test problem→	TP-01		TP-02		TP-03		TP-04	
Metric ↓	λ_1	λ_{21}	λ_1	λ_{21}	λ_1	λ_{21}	λ_1	λ_{21}
R^2	1.0000	1.0000	0.9925	0.9938	0.9941	0.9499	0.9747	0.8886
$R^2{}_{adj}$	1.0000	1.0000	0.9862	0.9901	0.9905	0.9075	0.9607	0.8384
$R^2{}_{pred}$	1.0000	1.0000	0.9644	0.9808	0.9829	0.7758	0.9374	0.7491
Q^2_{F3}	1.0000	1.0000	0.9795	0.9165	0.8090	0.7793	0.8802	0.6124

assuming a swarm, S_t of n particles. These particles are analogous to individual birds or fish in the real world. These particles use the information available to them to either explore new solutions or move closer to already known solutions. By doing this over an extended period, the particles eventually find or get very close to the optimal solution to a problem. Every particle that makes up the swarm has access to some information. Firstly, they know the current value of their solution and their current position which is a solution to the problem the algorithm is trying to solve. Each particle tracks its personal best value it has attained (‘*pBest*’) and the position that was achieved. Each particle also has access to the global best solution value (‘*gBest*’) and the position at which this was discovered. Lastly, a particle is aware of its current velocity, i.e., how fast its position is changing (Kalita *et al*. 2017). Any k^{th} particle continuously updates its velocity and position as follows

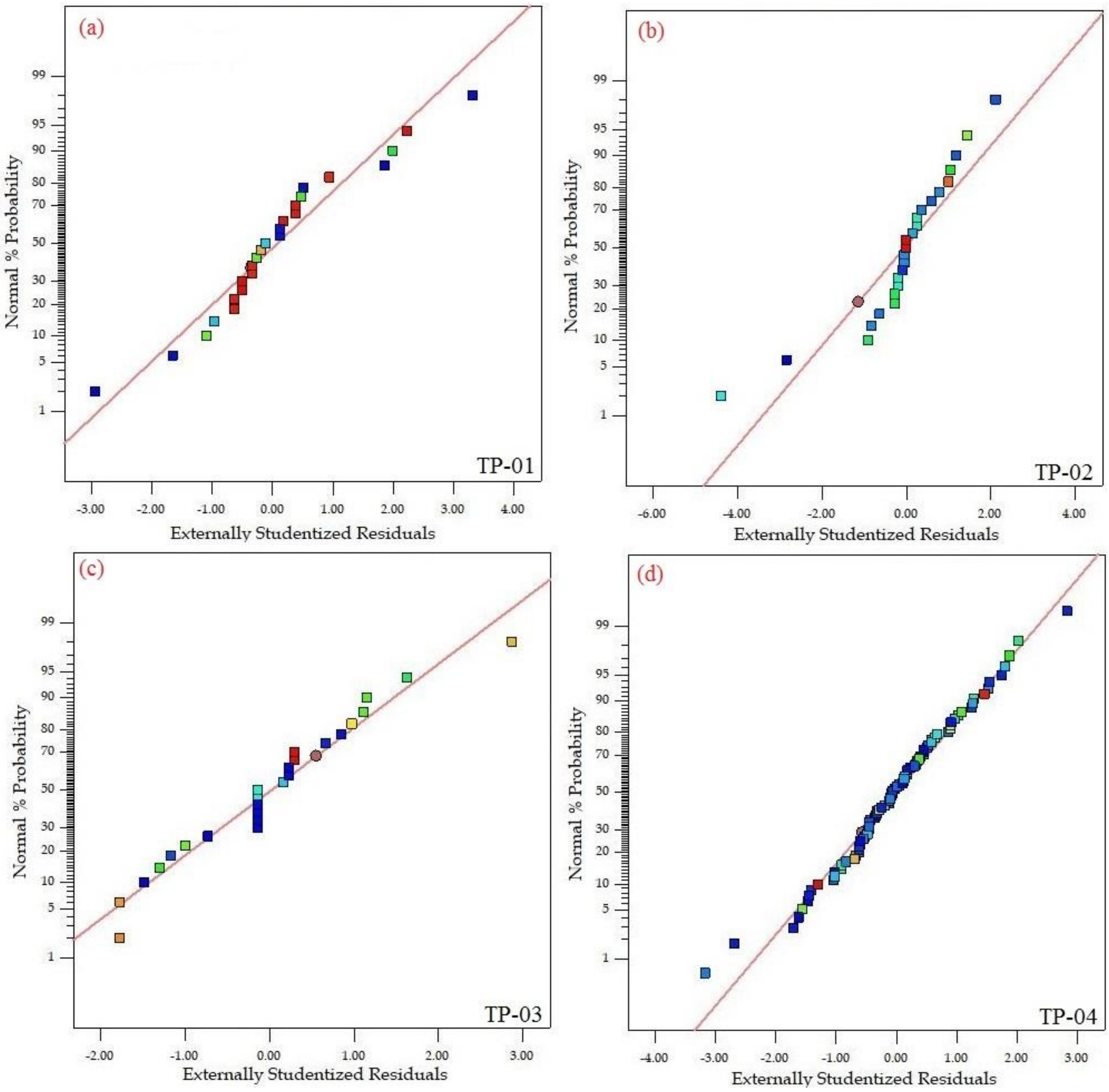

Fig. 4 Normal probability plot of residuals for all λ_1 metamodels

$$v_{t+1}{}^{k} = \omega . v_t{}^{k} + c_1 . r_1 . (pBest^k - x_t{}^k) + c_2 . r_2 . (gBest - x_t{}^k) \quad (10)$$

$$x_{t+1}{}^{k} = x_t{}^{k} + v_{t+1}{}^{k} \quad (11)$$

In Eqs. (10) and (11), the subscripts t and $(t+1)$ represent the current and the next iteration, r_1, r_2 generates random numbers between 0 and 1, c_1 and c_2 are the cognitive and social parameters respectively. Similarly, v and x represent velocity and position respectively, while ω is inertia weight, which controls the influence of the last velocity on the current velocity. The implementation scheme of multi-objective particle swarm optimization (MOPSO) used in the current study is highlighted in Fig. 3.

3. Results and discussion

3.1 Metamodel building & validation

Metamodel building and validation is an iterative process. First, using multiple regression fitting scheme, the sampling data is fitted in simple second-order polynomial forms as reported in Eq. (2). Then by looking at the normal probability plots of the externally studentized residuals, the outliers are identified. In the presence of significant outliers, Box-Cox plots are constructed which helps in identifying suitable data transformations to improve the model.

Normal probability plots of the externally studentized residuals for all λ_1 and λ_{21} metamodels are shown in Figs. 4 and 5 respectively. The residual normality plots do not have any outliers or clusters, implying that the sampling data is appropriate (i.e., they do not contain any ties) and thus the measuring resolution is adequate. In Box-Cox plots, as reported in Figs. 6 and 7, the minimum point of the curve generated by the natural log of the sum of squares of

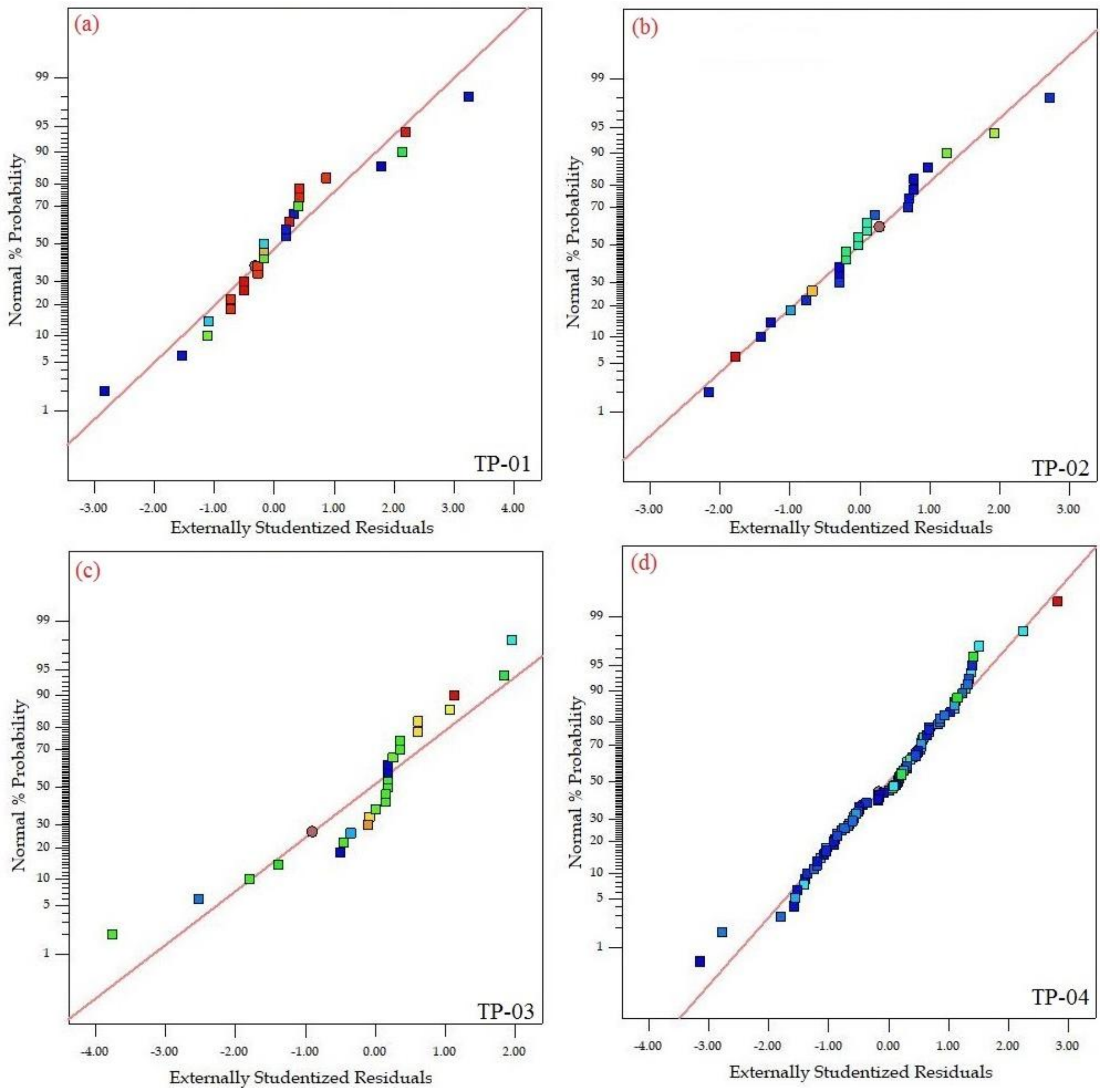

Fig. 5 Normal probability plot of residuals for all λ_{21} metamodels

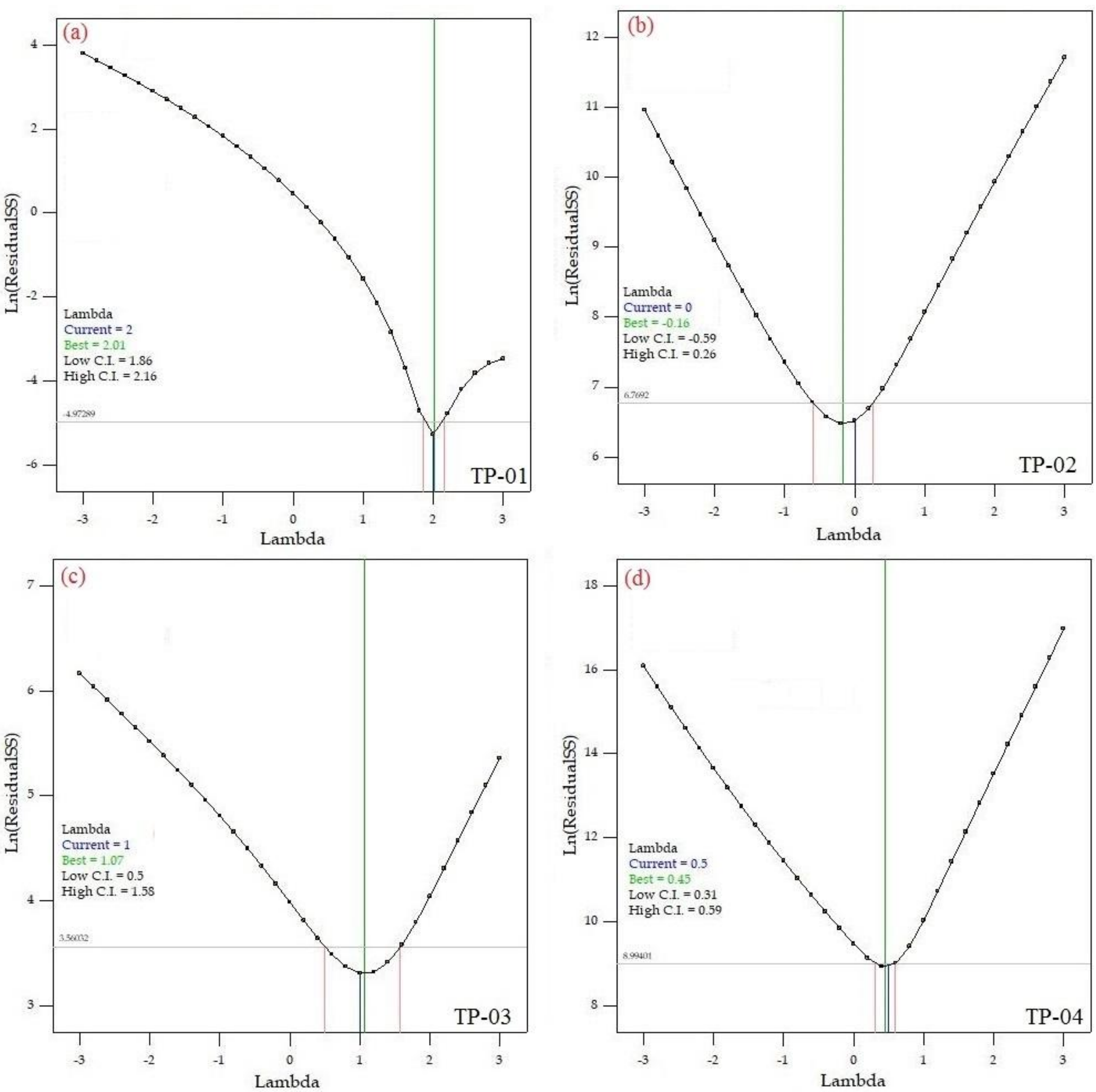

Fig. 6 Box-Cox plots for all λ_1 metamodels

the residuals represents the appropriate Box-Cox transformation parameter. As already discussed in section 2.2, this helps in capturing the high non-linearity in certain design problems. ANOVA is then performed on the metamodels to remove any statistically insignificant terms. The four model accuracy metrics, namely R^2, R^2_{adj}, R^2_{pred} and Q^2_{F3} are reported in Table 2. Another effective way to visually gauge the metamodels' predictive power is by plotting actual versus predicted responses. The predictive performance of all metamodels versus their respective finite element models is shown in Fig. 8. The closer the data points are to the diagonal line of the plot, better are the estimations. Both Table 2 and Fig. 8 reveal that TP-01 and TP-02 show excellent performance on the training as well as the independent test data. TP-03 and TP-04 on the other hand are highly nonlinear problems as well as are modelled for a very large design space and thus it would be unfair to expect near-ideal response surface approximations on them. In fact, simply by looking at Fig. 8(c) it is clear that in TP-03 despite the metamodel having a near-ideal performance on training data, the metamodel is in general over-predicting λ_1 and under-predicting λ_{21}.

3.2 Optimization using MOGA and MOPSO

To predict the Pareto optimal solutions for the 4 test problems, MOGA and MOPSO computer codes are implemented in a MATLAB environment. The second-order non-linear equations developed using the RSM approach for the four test problems are used as the objective functions. Based on a pilot study conducted on TP-04, the various parameters used for MOGA are selected as: population size 500, crossover and mutation probabilities 0.9 and 0.1 respectively. A tournament selection scheme is implemented and the MOGA algorithm is allowed to iterate for 100 generations. The pilot study was run on TP-04 because it is a high-dimensional problem containing 11 variables and thus, is likely to be the most complex amongst all the test problems. Similarly, based on a separate pilot study on TP-04 for the MOPSO algorithm, a swarm of 200 particles

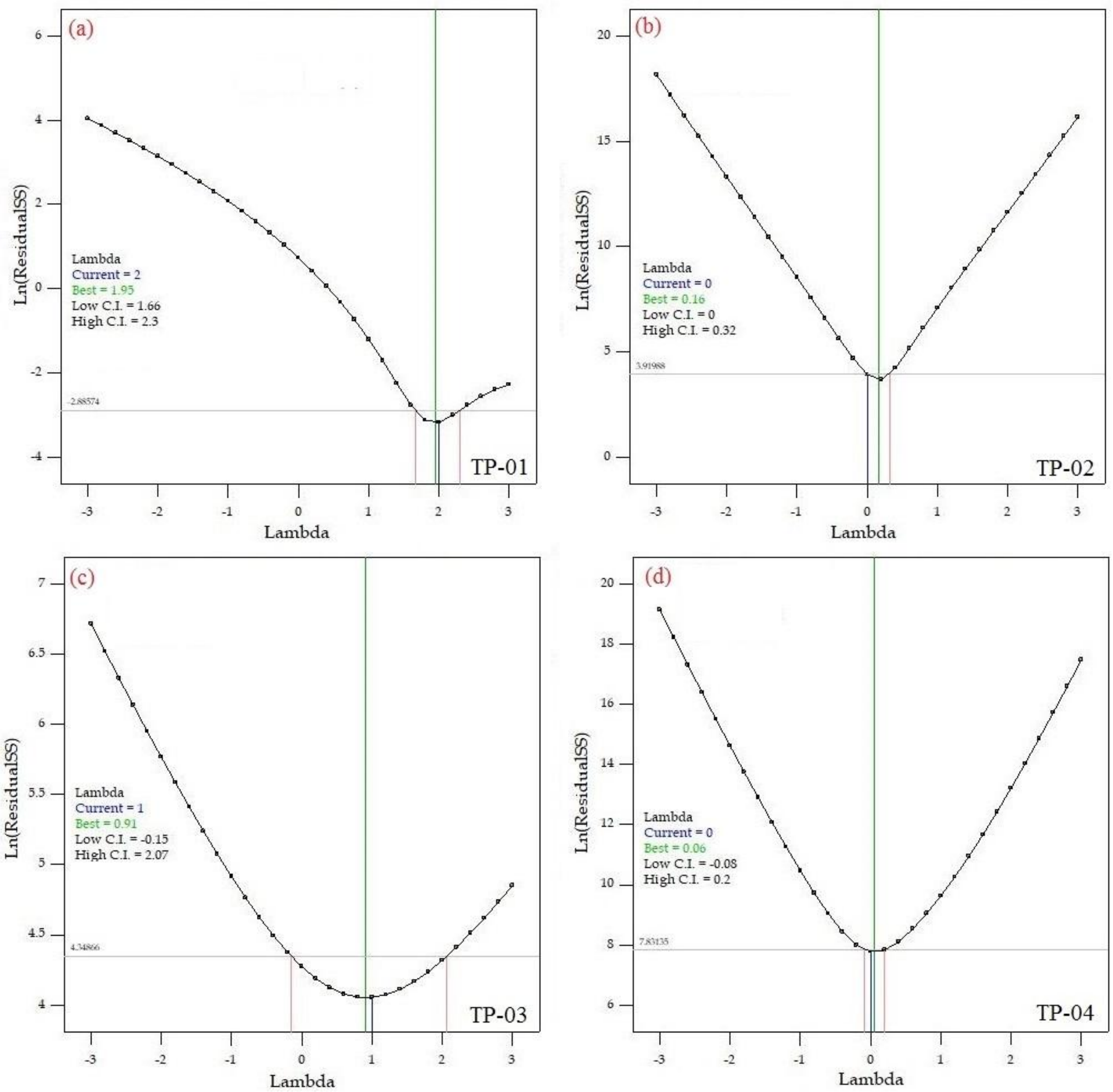

Fig. 7 Box-Cox plots for all λ_{21} metamodels

iterating for 100 generations was found to be the most beneficial. The other MOPSO parameters are taken as: $c_1 = 2,\ c_2 = 2$ and $\omega = 0.4$ as per the suggestion of Ragavendran *et al*. (2018).

The Pareto optimal solutions for the four test problems using MOGA and MOPSO are reported in Fig. 9. However, as reported in Fig. 9(a), a unique optimal solution for TP-01 exists, rather than a Pareto front. A higher orthotropy ratio (E_1/E_2) and a high major shear modulus to Young's modulus ratio (G_{12}/E_2) is beneficial for the simultaneous maximization of the fundamental frequency and frequency separation in TP-01. Except for TP-01, MOPSO has a superior performance as compared to MOGA in all the test problems. The Pareto optimal front obtained using MOPSO appears to be smoother and more continuous compared to the MOGA Pareto front. In TP-02, higher skew angles (α) in general lead to simultaneous maximization of the fundamental frequency and frequency separation. This is possibly due to the increase in stiffness of the plate with an increase in skew angle. Similarly, lower length-to-width ratios (b/a) and thickness-to-width ratios (h/a) are beneficial in increasing the frequency parameters. As the length-to-width ratios (b/a) decreases, the stiffness of the composite plate increases which causes an increase in the frequency parameters. There is a decline in frequency parameters with an increase in thickness-to-width ratios (h/a) due to an increase in the mass of the plate. TP-03 in a highly non-linear NP-hard problem. To the best of authors' knowledge, there exist no closed form equation to directly correlate the effect of fiber angles on frequency parameters. In fact, in the closed-form equations, as reported in (Shooshtari & Razavi, 2010), though the number of layers and the layup angles do not appear directly, they seem to have an effect of increasing the complexity and lengthiness of the closed form equations. As such, particularly for such NP-hard problem MOGA and MOPSO are well suited in finding a range of optimal solutions in the form of a Pareto front as reported in Fig. 9(c). In TP-04 a combination of high orthotropy ratio (E_1/E_2), low minor shear modulus to Young's modulus ratio (G_{23}/E_2), low

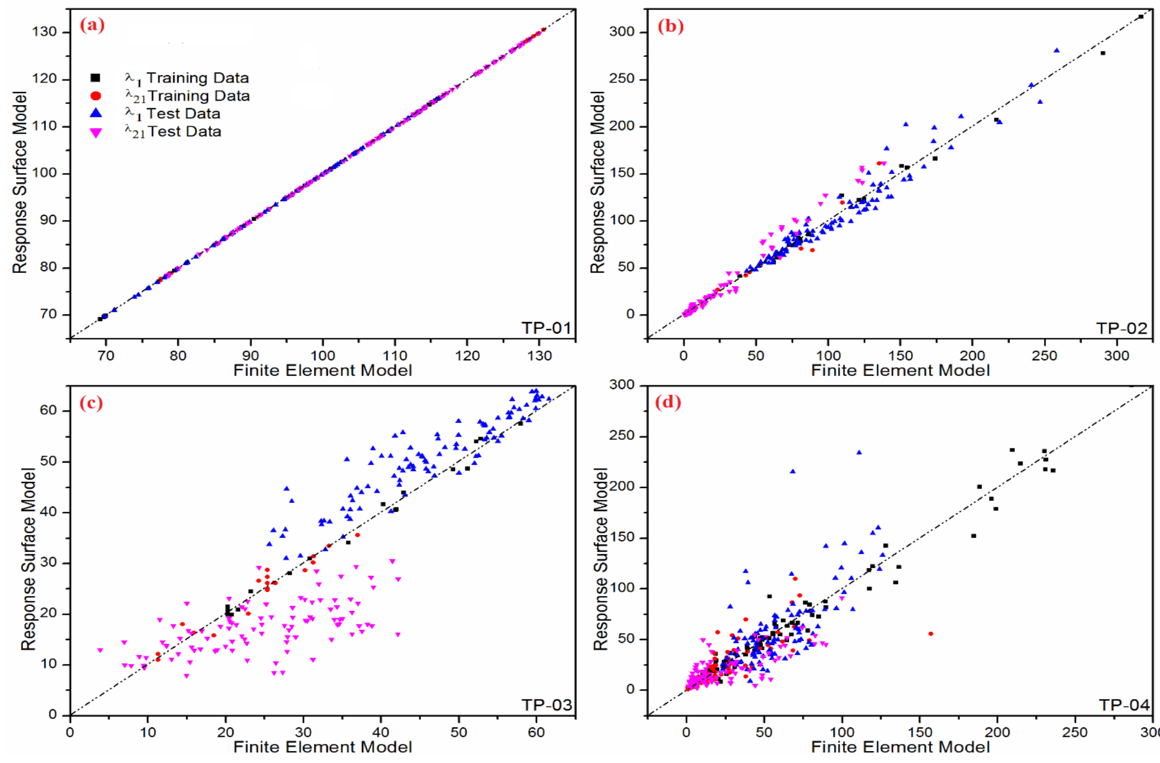

Fig. 8 Predictive performance of all metamodels vs. finite element models

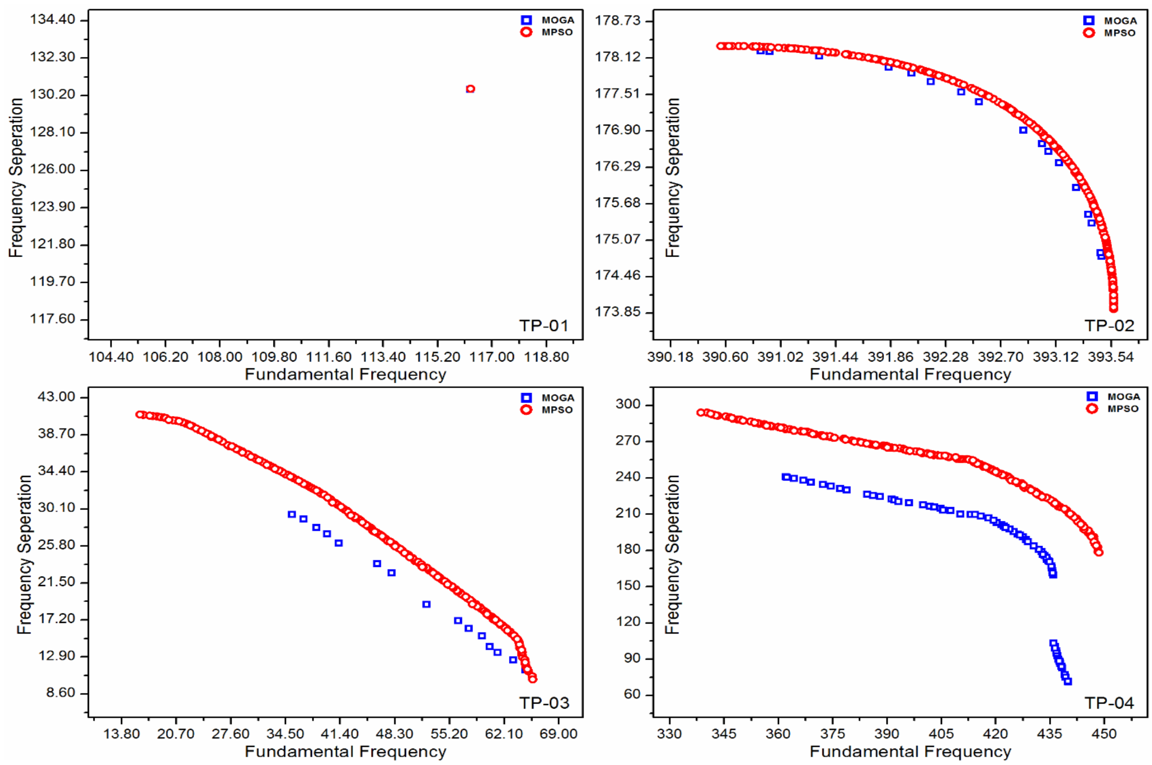

Fig. 9 Pareto front of the optimal solutions for simultaneously maximized λ_1 and λ_{21}

height-to-width ratios (b/a) and high skew angles (α) is advantageous in maximizing both the objective functions. The fiber angles $(\theta_1,\ \theta_2,\ \theta_3,\ \theta_4)$ also seem to have a significant effect on the final solution, with θ_2 being observed to be around $90°$ in most cases.

Though MOPSO predicts a truly continuous Pareto front in TP-02, the superiority of MOPSO over MOGA in terms of magnitude is marginal. The superiority in terms of both continuity of Pareto front and the magnitude of objective functions is more profound for MOPSO in TP-03 and TP-04. One plausible reason is the huge search space for TP-03 and TP-04, where MOPSO, owing to its intrinsic scattering of particles, is better equipped at finding the optimal solution. The Pareto optimal fronts represent a comprehensive set of non-dominated optimal solutions for each test problem. From a mathematical perspective, each Pareto optimal solution is an equally feasible option. The best alternative among these Pareto solutions may be chosen based on the problem at hand or user experience.

4. Conclusions

In this paper, a multi-objective genetic algorithm (MOGA) and a multi-objective particle swarm optimization (MOPSO) is used in designing composite plates for the simultaneous maximization of the fundamental frequency and frequency separation between the first two modes. Instead of a conventional Rayleigh-Ritz or finite element method to carry out intensive numerical calculations, a metamodeling approach is used. The polynomial regression based metamodel is rigorously constructed using statistical techniques. While Box-Cox transformation augments the normality of non-linear data, ANOVA helps remove insignificant terms. Further, the performance of the metamodels is evaluated on the training data and the testing data using residual-based statistical metrics like R^2, R^2_{adj}, R^2_{pred} and Q^2_{F3}.The study successfully applies the metamodel based multi-objective optimization approach for the first time in structural engineering to the prediction of frequency parameters, resulting in a drastic reduction of computation cost with marginal loss of accuracy. Comparison of results on four test problems reveals that MOPSO is superior to MOGA in finding a continuous and smooth Pareto front. Thus, by carefully building an appropriate metamodel and coalescing it with MOGA or MOPSO, the present research work highlights the potential of a robust multi-objective optimization tool that can be developed to optimally engineer the composite laminates for desired maximum frequency and maximum frequency separation.

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