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Effect of fatigue crack propagation on natural frequencies of system in AISI 4140 Steel

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Abstract. In this study, we investigated the effect of fatigue crack propagation of the beams which have a vital importance in engineering applications, on the natural frequency of the system. Beams which have a wide range of applications, are used as fundamental structural elements in engineering structures. Therefore, early detection of any damages in these structures is of vital importance for the prevention of possible destructive damages. One of the widely used methods of early detection of damages is the vibration analysis of the structure. Hence, it is of vital importance to detect and monitor any changes in the natural frequencies of the structure. From this standpoint, in this study we experimentally investigated the effect of fatigue crack propagation on beams produced from 4140 steel, of the natural frequency of the beam. A crack was opened on the 8×16×500 mm beam using a 3 mm long and 0.25 mm wide wire erosion. The beam, then, underwent 3 point bending tests at 10 Hz with a dynamic fatigue device and its natural frequencies were measured in scheduled intervals and any changes taking place on the natural frequencies of the beam were measured. This data allowed us to identify and measure the crack occurring on the beam subjected to dynamic loading, during the propagation phase. This method produced experimental data. The experimental data showed that the natural frequency of the beam decreased with the propagation of the fatigue crack on the beam.

Keywords: vibration analysis; cracked beam; fatigue crack propagation; natural frequency

1. Introduction

One of the most important issues for the industry is the suspension of manufacturing activities due to unexpected malfunctions in the machinery. Such an event causes losses in manufacture, prevents manufacture schedules from being followed, increases the costs and decreases the profits. The unexpected malfunctions in the machinery not only cause the manufacturing activities to be suspended, but they also require a significant amount of effort, funds and time for maintenance and repairs. Matters such as timely response from the authorized services or personnel for maintenance and repairs, and the procurement of spare parts, may take a long time depending on the location of the company and the magnitude of the malfunction. As a result, such a situation not only translates into a loss of manufacture and profits, but also increases the costs. In short, the number one thing that manufacturers wish to avoid, is the suspension of manufacturing activities dues to an unexpected malfunction. As a result, it is an increasingly common view among the industrial organizations that malfunctions must be detected early and be remedied as soon as possible.

Cracks are the most common type of damage seen in structures. The structural cracks may pose a threat due to static or dynamic loads, therefore crack detection plays a vital role for the proper functioning of the structure. Beam type structures are widely used in steel construction and machine industries. There are various studies in the literature, regarding the structural safety of beams and, especially, the detection of cracks by closely monitoring the structure. Studies regarding the detection of cracks through close monitoring of the structure, focus on the changes in the natural frequencies and modal form of the beam. While Orhan (2007) focus on the dynamic response of the beam as a result of harmonic force. Rezaee and Hassannejad (2011), Caddemi and Calio (2009), Wang *et al*. (2017), Ahmad *et al*. (2015), Agarwalla and Parhi (2013), Matsuda and Gotoh (2015), Lorenzino and Navarro (2015), Pei *et al*. (2016), Sutar *et al*. (2015), Tao *et al*. (2017), Aid *et al*. (2014), Xie *et al*. (2014), Okura and Ishikawa (2002) modelled the crack as linear torsion curve and assumed the rigidity, crack depth and cross section geometry of the beam, which are related to the mechanical characteristics of the beam, to be stable in order to study the effect of the crack during the vibration of the beams. They acquired the natural frequencies and modal forms of the cracked beam employing analytical or numerical techniques.

Yao *et al*. (2017) propose a model to predict fatigue crack propagation with reasonable accuracy in metals. Theoretical predictions are very similar with experimental data. Zong *et al*. (2017) studied mix-mode crack

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Table 1 The chemical compound of the beam material

C	Si	Mn	P_{max}	S_{max}	Cr	Mo	V
0,38-0,45	0,15-0,40	0,50-0,80	0,035	0,035	0,90-1,20	0,15-0,30	-

propagation analysis on load carrying welded joints. Mousavinasab and Blais (2017) suggested in their study that fatigue crack propagation rates as a function of microstructural content in a nickel PM steel. Tang *et al.* (2015) examined the strain-strengthening factor on the notch-crack fatigue propagation in austenitic stainless steel. This factor has no effects on the crack growthpath. Daneshmehr *et al.* (2013) and Jassim *et al.* (2013) studied free vibration analysis of cracked composite beam subjected to coupled bending–torsion loading. Masserey and Fromme (2017) suggested a 3D finite difference model to detect fatigue crack propagation at fastener holes. Liu *et al.* (2016) developed a new three segment beam model to investigate crack length, depth, and location. Gawande and More (2016) studied dynamic properties of cantilever beams subjected to free vibration under the influence of notch depth and location. Moezi *et al.* (2015) propose a modified cuckoo optimization algorithm to crack detection. Kumar *et al.* (2016) used extended finite element method (XFEM) for cracked panels subjected to tensile fatigue loading.

As seen in the literature, there are many studies focused on the crack detection, and crack location and size measurement, on beams. Moreover, studies have been carried out regarding the detection and measurement of multiple cracks on beams. Studies performed on machinery elements with different cross sections and purposes such as functional gradient material (FGM), composite beams, mills and turbine flaps bring a new perspective to the matter. Additionally, studies carried out on welded beams and materials provide insight into the production and construction aspect of the matter. Studies carried out on fatigue crack propagation rarely indicate the fatigue life-span of the material. In this study we investigated the effect of crack propagation in beams made from 4140 steel, on the natural frequency of the system. In the experimental study, beams with 500 mm length and 8×16 mm^2 cross-sectional area were prepared. In order to resemble the real crack, an artificial crack was opened using a 0.25 mm wide wire. The beam was subjected to a 3 point bending test with 10 Hz at the dynamic fatigue device and the natural frequencies of the cracked beam were measured periodically. The size of the crack on the beam and the natural frequencies of the beam were measured for each fatigue cycle, which served as a basis for evaluation. Thus, the behavior of a cracked beam under dynamic working conditions has been investigated experimentally.

2. Experimental setup

2.1 Sample preparation

The beams used in this study are made from 4140 steel. The chemical compound of this type of steel is suitable for reinforcement in terms of its carbon content. This type of steel is alloy structure steel which exhibits high-toughness under certain loads after tempering. It has good induction quality. This type of tempered steel is used in the manufacture of automobiles and planes, crank shaft, axle shaft and box, spline shaft, construction and agricultural machinery, machine tools, parts such as bolts and nuts that have high ductility, and also in gears and reels. The approximate chemical compound of the material can be seen in Table 1.

Fig. 1 8×16×500 mm beam produced from 4140 steel

The beam was subjected to tension test to identify the mechanical characteristics of its material The yield tension of the material was determined as (σ_{yield}) 670 MPa after the tension test. The commercial 4140 steel usually has a circular cross section. Therefore we acquired a 20 mm diameter and 500 mm long cylindrical material and machined it on a milling table into a beam. We determined the width (B) and height (W) values of the beam according to the ASTM E-399 standard. The size of the machined beam is 8×16×500 mm samples machined into beams on milling table are shown in Fig. 1.

The artificial crack mechanism is different from the actual fatigue crack mechanism. Therefore, when the artificial crack was opened, 0.25-mm wire that the thinnest wire, was used to simulate the actual crack as much as possible. We opened a dovetail and a 3 mm long artificial crack on the beam shown in Fig. 1 using wire erosion. This process was performed with a 0.25 mm wire on wire erosion table. The dovetail was opened to properly mount the extensometer on the beam. The dovetail and the artificial crack are shown in Fig. 2.

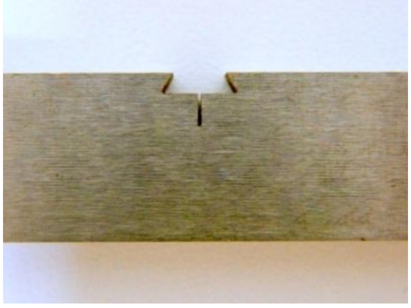


Fig. 2 The beam with 2 mm dovetail and 3 mm artificial crack opened with wire erosion

Table 2 The thickness and height values of the three measured beams

Beam number	Thickness (B) mm	Height (W) mm
1	8.015	16.05
2	8.022	16.03
3	8.018	16.08

Before the experiment, the surface of the beams where the crack will be monitored was treated with 320, 1200 and 2000 mesh sand paper and alumina fluid with maximum 0.3 micron sized polisher, initially manually and then using a portable gravure blasting machine.

The thickness and height of the polished beams were measured with a micrometer. We performed comparisons to see whether the data complied with the thresholds acknowledged in ASTM-E399 standards. The thickness (B) and height (W) values measured for each of the three beams are shown in Table 2. It is assumed to be as $B = W/2 \pm (0{,}010\ W)$ in ASTM E-399 standards.

2.2 Fatigue tests

We used a dynamic fatigue test device (Instron 8801) which has a 100 kN load inflicting capacity in order to evaluate the change in the system’s natural frequency with the propagation of the fatigue crack. Three point bending apparatus was mounted on the fatigue device to perform the bending test on the beams. A millimetric ruler was used to measure the propagation of the crack on the beam. An extensometer was used to measure the opening of the crack’s tip during the test.

2.3 Vibration data collection

The vibration data collection set up used to measure the natural frequencies of cracked beams are made up of four fundamental parts. These are; accelerometer, impact hammer, signal conditioner and software. The vibrations of the beam as a result of the impact was measured using an accelerometer that had a sensitivity of 100 mV/g. An impact was applied to the beam using an impact hammer that has a sensitivity of 2,27 mv/N. The signals coming from the impact hammer and the accelerometer are imported to the computer through a signal conditioner that has 4 BNC inputs. The system’s natural frequencies and modal forms were obtained using the vibration analysis software.

2.4 Fatigue crack propagation and natural frequency

Beams whose crack monitoring surfaces were polished and on which a millimetric rulers was glued, were placed on the three point bending test apparatus on the fatigue test device. The loading frequency of the fatigue device was 10 Hz as a sinus curve. The loading frequency was selected as 10 Hz throughout the experiment. Force ratio (P_{max}/P_{min}) to be applied to the test sample was determined as R=0.1. The distance between supports (s) was 64 mm. The fatigue load condition on test specimen is shown in Fig. 3.

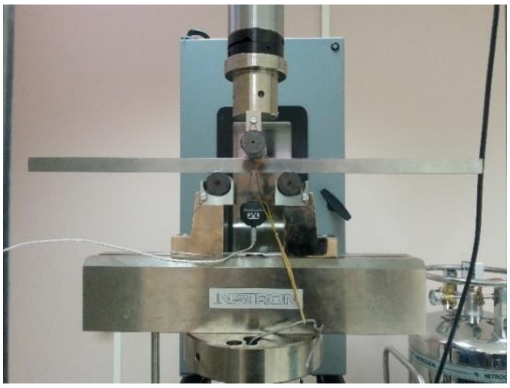


Fig. 3 The fatigue load condition on test specimen

As P_{max} maximum load will remain fixed throughout the experiment, and as the stress will exceed the flow stress as the crack propagates, plastic deformation will start on the sample after fatigue crack propagation. Therefore, when calculating the maximum force to be applied to the beam, 30% of the flow stress of 4140 steel, which is 670 MPa, was taken as a basis, which equals to 201 MPa. The calculations based on this flow value led to a maximum force of 2026.75 N to be applied to the beam. As a result of the calculations, we determined that in addition to the initial crack opened with 5 mm wire erosion, a 6 mm fatigue crack would propagate, and as the stress during the experiment would exceed the flow stress of the beam, plastic deformation would begin on the beam.

As the purpose of the experiment was to determine the interaction with fatigue crack propagation and the system’s natural frequency, the fatigue load on the beam was suspended from time to time to measure the natural frequencies of the beam after a certain amount of crack propagation. With this in mind, one beam was loaded constantly to determine the fatigue life span of the beams and the beam broke after approximately 168000 cycles. Based on the results, for the other two beams we decided to measure the vibrations initially once in 15000 cycles and, towards the end of the fatigue life span and as crack would propagate faster according to Paris-Erdogan graph, once in 10000 and once in 5000 cycles. The examination of the first beam that broke showed that fatigue crack propagation distance was approximately 6.5 mm, and that the calculations regarding crack propagation and loading were accurate. The surface of the beam that broke, is shown in Fig. 4.

After the set number of cycles, the required data was obtained by measuring the amount of crack propagation and the natural frequency of the beam. The fatigue crack propagation on the beam is shown in Fig. 5. Moreover, throughout the experiment, the number of load cycles applied to the beam and the values of the crack size seen on the extensometer were recorded. The fatigue crack propagation was measured by the extensometer as seen from the Fig. 3 during the experiment. Pakdil *et al*. (2011) and Koçak *et al*. (2002) used an extensometer to measure fatigue crack length. When the following similar studies were examined, it was seen that extensometer was found to be sufficient for the crack length measurement.

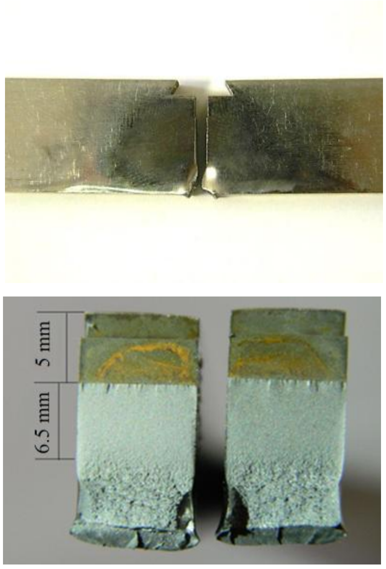


Fig. 4 The surface of the beam that broke

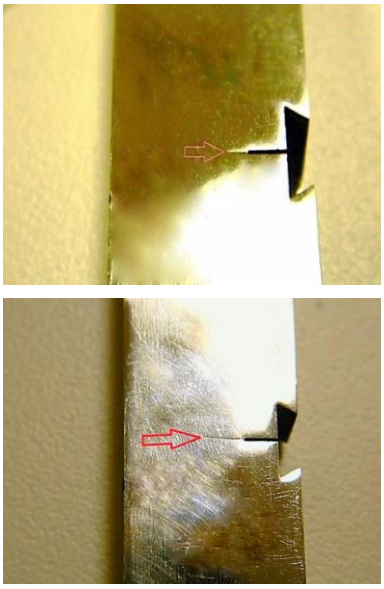


Fig. 5 Fatigue crack propagation

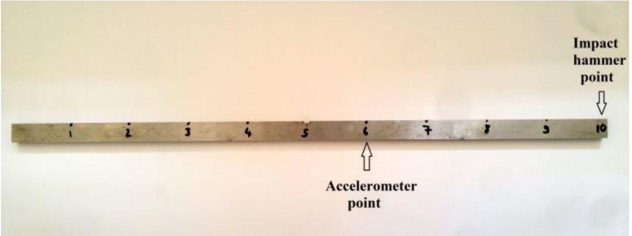


Fig. 6 The locations where the accelerometer was mounted and the impact hammer was hit

Table 3 The cycle and extensometer values obtained for beam

Applied load cycles	Values on extensometer (mm)	Measured crack length (mm)
0	0.01532	5.248
15000	0.02948	5.479
30000	0.03760	5.611
45000	0.04742	5.77
60000	0.04848	5.787
75000	0.05008	5.813
90000	0.05325	5.865
105000	0.06814	6.107
120000	0.06988	6.135
130000	0.07250	6.178
140000	0.08690	6.412
150000	0.09780	6.589
155000	0.10770	6.75
160000	0.12700	7.063
163000	0.14390	7.338
165000	0.17820	7.895
167000	0.240	8.9
168000	0.400	11.5

Beams were removed from the test device in intervals of set number of cycles in order to determine the interaction between the fatigue crack propagation on the cracked beams subjected to fatigue test and the system’s natural frequency, and they were supported from one end to perform the measurements. In order to obtain the accurate natural frequencies of cracked beam, the accelerometer was connected to the beam at the same point in each measurement and the force was applied to the beam at the the same point. The accelerometer was mounted on point 6 on the beam for each measurement and force was applied to the beam on point 10 during each measurement with the impact hammer. The points where the accelerometer was mounted and the impact hammer was hit are shown in the Fig. 6. Then, the signals received by the signal analyzer from the accelerometer and the impact hammer were processed, digitized and transferred to the computer. The beam’s natural frequency value was determined and made into a graph using the software developed by Brüel & Kjaer.

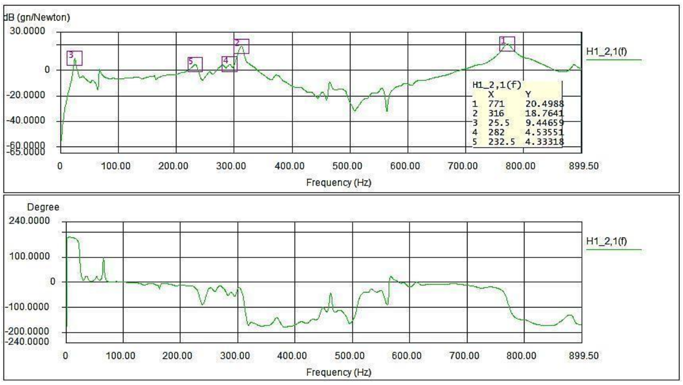
3. Results

The length of the crack propagating on the beam during the crack propagation test, the values read on the extensometer and the cycle information of the load were recorded. The extensometer values obtained from beam per number of cycles are shown in Table 3.

The natural frequencies as a result of the cycles specified in Table 3 for beam no 2 were measured. The natural frequency values of beam no 2 without the fatigue test are shown in Fig. 7.

2nd beam’s natural frequency values obtained after 15000 fatigue cycles are shown in Fig. 8, 120000 fatigues cycles in Fig. 9, and 168000 fatigue cycles in Fig. 10.

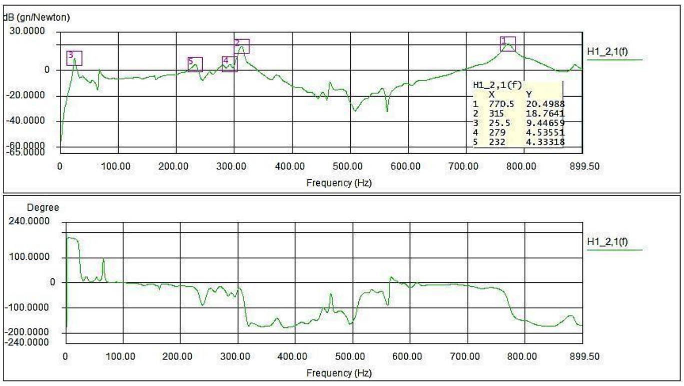
The natural frequencies obtained as a result of fatigue



H1_2,1(f)

	X	Y
1	771	20.4988
2	316	18.7641
3	25.5	9.44659
4	282	4.53551
5	232.5	4.33318

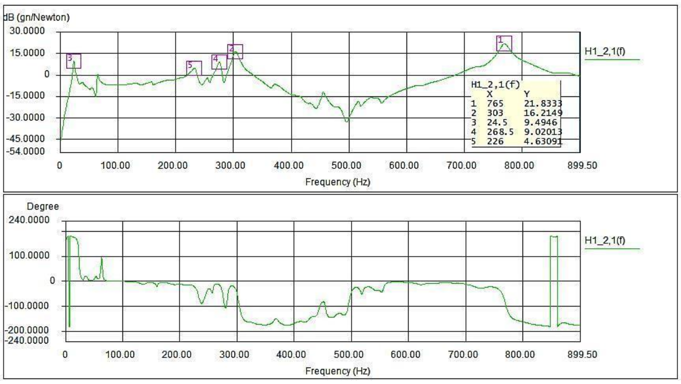
Fig. 7 The natural frequencies of beam without the fatigue test



H1_2,1(f)

	X	Y
1	770.5	20.4988
2	315	18.7641
3	25.5	9.44659
4	279	4.53551
5	232	4.33318

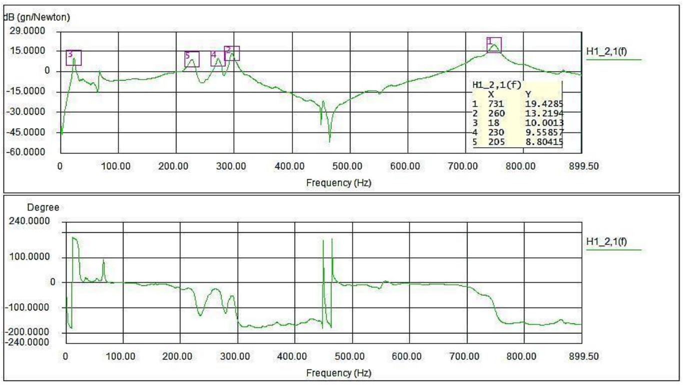
Fig. 8 The natural frequencies of beam after 15000 fatigue cycles



H1_2,1(f)

	X	Y
1	765	21.8333
2	303	16.2149
3	24.5	9.4946
4	268.5	9.02013
5	226	4.63091

Fig. 9 The natural frequencies of beam after 120000 fatigue cycles



H1_2,1(f)

	X	Y
1	731	19.4285
2	260	13.2194
3	18	10.0013
4	230	9.55857
5	205	8.80415

Fig. 10 The natural frequencies of beam after 168000 fatigue cycles

Table 4 Natural frequencies of beam after fatigue cycles

Number of fatigue cycles	Measured crack length (mm)	1. Natural frequency (ω_1)	2. Natural frequency (ω_2)	3. Natural frequency (ω_3)	4. Natural frequency (ω_4)	5. Natural frequency (ω_5)
0	5.24895	25.5	232.5	282	316	771
15000	5.47905	25.5	232	279	315	770.5
30000	5.611	25.5	231.5	278.5	314	770
45000	5.7705	25	231	278	313	769.5
60000	5.7878	25	230	277.5	312	769
75000	5.8138	25	229	277	310	768.5
90000	5.8653	24.5	228	269.5	308	768
105000	6.107275	24.5	227	269	306	767
120000	6.13555	24.5	226	268.5	303	765
130000	6.178125	24	224.5	268	301	763
140000	6.412125	24	223	267	298	761
150000	6.58925	23.5	221.5	266	295	759
155000	6.750125	23.5	220	264	291	756
160000	7.06375	23	218	261	286	752
163000	7.3383	22	216	258	281	748
165000	7.89575	21	214	254	275	744
167000	8.9	20	211	245	268	739
168000	11.5	18	205	230	260	731

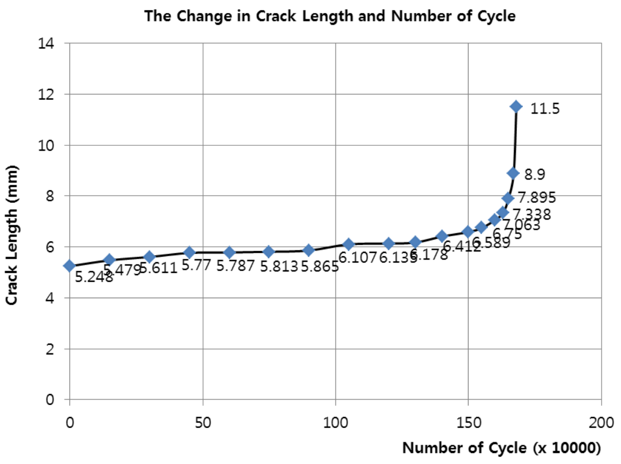


Fig. 11 The change in crack length and fatigue cycle

cycles and the measured crack lengths are shown in Table 4.

The change in crack length per fatigue cycle is shown in Fig. 11.

The changes in the system's 1st, 2nd and 3rd natural frequencies by crack length are shown in Figs. 12, 13 and 14.

4. Conclusions

According to the vibration data obtained from fatigue crack propagation tests in cracked beams, as the crack in the beam propagates, the system's natural frequency decreases. This provides a clue for the detection of cracks in machinery and structural elements in working conditions.

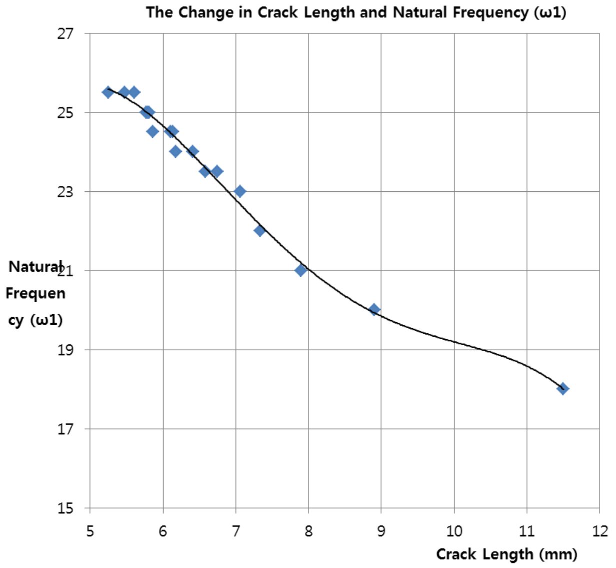


Fig. 12 The change in crack length and system's 1st natural frequency (ω1)

Beams are the fundamental part of many structural elements and any damage sustained by the beams, may inflict damage to the entire structure and may cause the structure to sustain a damage preventing its functionality. Therefore, the early detection of damages of fundamental structural elements is vital to maintain the integrity of the structure. We examined the interaction between the fatigue cracks which represent the most common type of damage seen in beams, and the system's natural frequencies, and concluded that crack propagation decreased the structure's natural

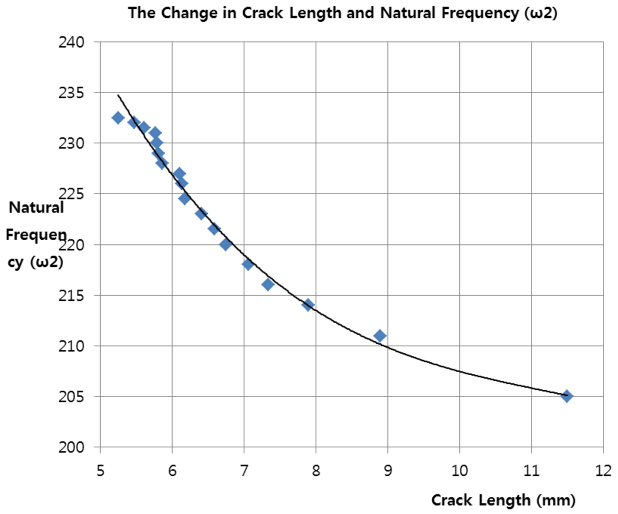


Fig. 13 The change in crack length and system's 2nd natural frequency (ω2)

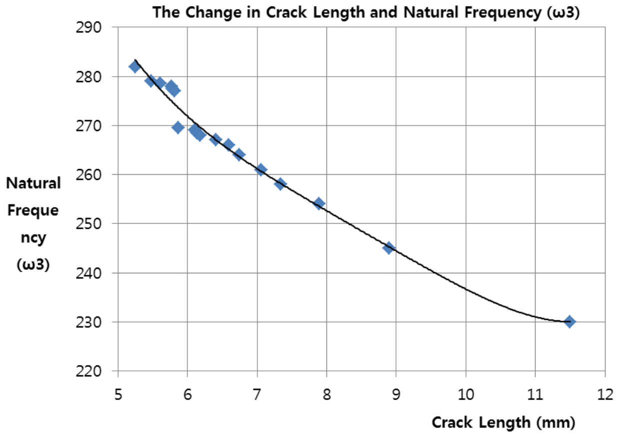


Fig. 14 The change in crack length and system's 3rd natural frequency (ω3)

frequencies. The resulting data complied with previous studies.

Monitoring the structure's natural frequencies, which is one of the most common methods for damage detection, has been a topic of interest for many investigators and numerous studies have been performed thereon. Systems capable of monitoring the system's natural frequencies on-line, should receive more attention for the early detection of damages that can occur in the structure under actual working conditions. In this study, we underlined the importance of the matter and tried to exhibit the possible benefits for the actual working conditions. Further studies have been planned for the determination of the structure's natural frequencies via vibration measurement on the beams under actual working conditions, and the online detection and measurement of cracks depending thereon. We also plan to establish an analytical connection between the propagation speed of the crack and the fatigue cycles applied to the system. The aim of this, is to enable the calculation of the maximum time that the system may continue safe operations after a crack in its structure. The fact that this matter focuses on the interaction between the fatigue crack propagation in alloy steels and the system's natural frequency, not only sheds light on many aspects, but also will underline the effects of alloy elements on fatigue crack propagation, which will prove to be useful for the production industry. The expansion of the study to include various lengths and thicknesses, will widen the scope of the determination of the interaction between the length/thickness ratio (L/h), and crack propagation and system's natural frequencies, and works towards related analytic solutions and finite element modelling, and contribute to a practical know-how source for the production industry.

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