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Dynamic stability and nonlinear vibration of rotating sandwich cylindrical shell with considering FG core integrated with sensor and actuator

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Abstract. In this research, the dynamic stability and nonlinear vibration behavior of a smart rotating sandwich cylindrical shell is studied. The core of the structure is a functionally graded material (FGM) which is integrated by functionally graded piezoelectric material (FGPM) layers subjected to electric field. The piezoelectric layers at the inner and outer surfaces used as actuator and sensor, respectively. By applying the energy method and Hamilton's principle, the governing equations of sandwich cylindrical shell derived based on first-order shear deformation theory (FSDT). The Galerkin method is used to discriminate the motion equations and the equations are converted to the form of the ordinary differential equations in terms of time. The perturbation method is employed to find the relation between nonlinear frequency and the amplitude of vibration. The main objective of this research is to determine the nonlinear frequencies and nonlinear vibration control by using sensor and actuator layers. The effects of geometrical parameters, power law index of core, sensor and actuator layers, angular velocity and scale transformation parameter on nonlinear frequency-amplitude response diagram and dynamic stability of sandwich cylindrical shell are investigated. The results of this research can be used to design and vibration control of rotating systems in various industries such as aircraft, biomechanics and automobile manufacturing.

Keywords: dynamic stability; nonlinear vibration; rotating sandwich cylindrical shell; functionally graded core; functionally graded piezoelectric layers, perturbation method

1. Introduction

The use of piezoelectric materials as sensors and actuators for controlling vibration and sound has increased significantly in recent years. Ceramics and polymers are two classes of piezoelectric materials that used to control vibrations. The most famous piezoelectric is PZT, which consists of lead, zirconium and titanium. By changing the proportion of zirconium to titanium, its properties can be optimized for various applications. PZT is widely used for wide frequency range and also includes ultrasonic applications. Vibration and dynamic response of shells have been studied by many researchers.

Shu (1995) investigated the free vibrations of isotropic cone skins by using the general differential quadrature method (GDQM). He obtained the natural frequencies of the system with four different boundary conditions and compared the results with Irie *et al*. (1982). The influence of the angular velocity on natural frequency of rotating cylindrical shell is investigated by Hua and Lam (1998). Chen *et al*. (2004) examined the vibrations of a functionalized graded piezoelectric (FGP) orthotropic cylindrical. The effect of the fluid is considered as a relationship between fluid pressure and radial displacement. Finally, the influence of various parameters such as velocity of sound in the fluid, as well as the ratio of thickness to the radius of the cylindrical on the natural frequency, have been investigated. Liew *et al*. (2004) examined the vibration control of functionally graded (FG) cylindrical shell by using sensor and actuator. They used the first-order shear deformation theory (FSDT) and finite element method (FEM) for deriving and solving governing equation of motion. Proportional derivative (PD) controller used for controlling the system and the performance of the control system has been evaluated in reducing the vibration amplitude. Patel *et al*. (2005) studied the vibrations of a FG cylindrical shell with elliptical cross-section. Also, the equilibrium equations using the FEM are solved and finally evaluated the natural frequencies and the mode shape of the vibrational systems. Tornabene (2009) studied the dynamic behavior of thick FG conical, cylindrical shells and annular plates. His results indicated the influence of the power-law exponent, of the power-law distribution on the mechanical behavior. Sheng and Wang (2009b) examined the nonlinear dynamic response of sandwich cylindrical shell subjected to lateral load by considering the FG core and piezoelectric layers. Their results showed the effect of lateral load on dynamic response of sandwich cylindrical shell. Active vibration control of sandwich cylindrical shell subjected to thermal and mechanical load is studied by Sheng and Wang (2009a). The Galerkin's method for solving the governing equation of motion is used. The effect of piezoelectric material on dynamic response of system is investigated. Ghorbanpour Arani *et al*. (2011) presented dynamic stability of the double-walled carbon nanotube under axial

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loading embedded in an elastic medium using the energy method. By using FSDT and generalized differential quadrature (GDQ) method, Tornabene *et al*. (2011) studied the dynamic behavior of FGMs and laminated doubly curved shells and panels. By drawing the Campbell diagram, Sun *et al*. (2012) analyzed the influence the geometric parameters and boundary conditions on free vibration of rotating cylindrical shell. Malekzadeh and Heydarpour (2012) analyzed the dynamic response of rotating FG cylindrical shell subjected to thermal load, and using FSDT and Hamilton's principle, the governing equation of motion is obtained. Arefi and Rahimi (2012) analyzed the dynamic response of FG cylindrical shell by employing FSDT and Hamilton's principles. The dynamic response of system by considering the various boundary conditions is investigated by them. Tornabene and Reddy (2013) applied generalized differential quadrature (GDQ) method for static analysis of functionally graded material (FGM) and laminated doubly-curved shells and panels resting on nonlinear and linear elastic foundations. Cao *et al*. (2013) presented the active control vibration of sandwich cylindrical shell. The FSDT for modeling displacement field of system and also for control vibration by the PD controller is employed. Their results showed the effect of controlling power factor and piezoelectric material on dynamic response of system. Mohammadimehr and Rahmati (2013) studied small scale effect on electro-thermo-mechanical vibration analysis of single-walled boron nitride nanorods under electric excitation based on nonlocal elasticity theory. Kumar *et al*. (2013) studied the vibrations of a sandwich shell based on zigzag theory. The finite element method (FEM) is used to solve the system equations. It can be seen that FEM based on the zigzag theory is more accurate than FEMs based on first order and higher order theory. Heydarpour *et al*. (2014) studied the free vibration of FG incomplete cone shell by employing the Hamilton's principle and FSDT and DQM is used to solve the governing equation of motion. Their results showed the effect of angular velocity on natural frequency of system. By discarding Coriolis Effect and using Lagrange method, Dey *et al*. (2014) obtained the governing equation of FG cone cylindrical shell. Also, by drawing the shape mode of system, the influence of various parameters on dynamic response of system is studied. Mohammadimehr *et al*. (2014) presented the vibration and buckling analysis of FG piezoelectric of cylindrical shell. Their results showed the effect of various parameters such as elastic foundation coefficient, type of piezoelectric material on natural frequency and shape mode of system. Using Sanders shell theory, Assaee and Hasani (2015) presented the forced vibration analysis of composite thin-walled cylindrical shell. For the calculation of natural frequencies, Fourier transform method has been used in their works. Also for validate, the results of this method are compared with the results of FEM. Mohammadimehr *et al*. (2015) considered surface stress effect on the nonlocal biaxial bending and buckling analysis of polymeric piezoelectric nanoplate reinforced by CNT using Eshelby-Mori-Tanaka approach. Tornabene *et al*. (2015a) used the generalized zigzag displacement field and the carrera unified formulation (CUF) to study stress and strain recovery for FG free-form and doubly-curved sandwich shells. Active vibration control of cylindrical shell by considering the piezoelectric disk is studied by Loghmani *et al*. (2015) and used the Donnel-Mushtari shell theory and Rayleigh-Ritz method to model and solve the equation of system. Zhang *et al*. (2015) investigated the dynamics response of cylindrical shell subjected to thermal load. Tornabene *et al*. (2015b) investigated the effect of carbon nanotube (CNT) agglomeration on the free vibrations of laminated composite doubly-curved shells and panels reinforced by CNTs. Das and Karmakar (2016) examined the effect of rotation and pre-twist angle on free vibration characteristics of FG conical shells. Ghorbanpour Arani *et al*. (2016) illustrated surface stress and agglomeration effects on nonlocal biaxial buckling polymeric nanocomposite plate reinforced by CNT using various approaches. Using finite element method, Mohammadimehr and Alimirzaei (2016) presented nonlinear static and vibration analysis of Euler-Bernoulli composite beam model reinforced by FG-SWCNT with initial geometrical imperfection. Razavi *et al*. (2016) examined the free vibration of FG piezoelectric nano-cylindrical shell and derived the equilibrium equation of system by using couple stress theory and size-dependent piezoelectric theory. Influence of geometric and material parameters on natural frequency and dynamic response of system are studied. Tornabene *et al*. (2017) used the higher order shear deformation theory (HSDT) and differential quadrature method (DQM) to study the effect of the curvilinear fiber path on the modal response of foam core composite sandwich plates and shells. Mohammadimehr *et al*. (Mohammadimehr and Shahedi 2017, Mohammadimehr and Mehrabi 2017, Mohammadimehr *et al*. 2017) investigated buckling and vibration analysis of sandwich structures. Sheng and Wang (2017) presented the nonlinear vibration of rotating FG cylindrical shell and employed FSDT as well as large deformation terms (von Karman) of strain vector to derive the governing equation. Higher order shear deformation theory and Maxwell's equation used to analyzing the free flexural vibration behavior of doubly curved complete and incomplete sandwich shells with functionally graded (FG) porous core, FG carbon nanotube reinforced composite (FG-CNTRC) face sheets and integrated piezoelectric layers by Setoodeh *et al*. (2018). Zghal *et al*. (2018) analyzed the buckling of FG composite plates and curved panels reinforced by carbon nanotubes and employed the high order shear deformation theory to derive the governing equation of motion. Das and Karmakar (2018) investigated the influence rotation and pre-twist angle on free vibration characteristics of FG conical shells. Based on first-order shear deformation theory (FSDT), Mohammadimehr *et al*. (2018a) presented free vibration analysis of magneto-electro-elastic cylindrical composite panel reinforced by various distributions of CNTs with considering open and closed circuits boundary conditions. Frikha *et al*. (2018) investigated the dynamic behavior of functionally graded carbon nanotubes reinforced composite shell structures (FG-CNTRC). Their results showed the influence of volume fraction of carbon nanotube, various boundary conditions and other geometrical parameters on dynamic behavior of FG-CNTRC shells. Trabelsi *et al*. (2019) used the modified first order shear deformation theory to analysis the thermal buckling behavior of FG plates and cylindrical shells. Some researchers worked about bending, vibration and active control of sandwich beam, plate and shell (Yazdani *et al*. 2019, Rajabi and Mohammadimehr 2019, Emdadi *et al*. 2019, Akhavan Alavi *et al*. 2019, Rostami *et al*. 2019).

The nanotechnology and micro scale analysis for smart structures were reviewed in the introduction. According to reviewed literature, a combination of these topics for nonlinear vibration and dynamic stability of a rotating sandwich shell with considering FG core integrated with sensor and actuator is a lack of study. Thus, the dynamic stability and nonlinear vibration behavior of a smart rotating sandwich cylindrical shell will be analyzed with FGM core which is integrated by functionally graded piezoelectric material (FGPM) layers subjected to electric field as sensor and actuator based on first-order shear deformation theory (FSDT) is studied in the present research. The governing equations and the corresponding boundary conditions are established through the Hamilton's principle. The Galerkin method is used to discriminate the motion equations and the equations are converted to the form of the ordinary differential equations in terms of time. The perturbation method is employed to find relation between nonlinear frequency and amplitude response. The effects of geometrical parameters, power law index of core, sensor and actuator layers, angular velocity and scale transformation parameter on nonlinear frequency-amplitude response diagram and dynamic stability of sandwich cylindrical shell are investigated. The main objective of this research is to determine the nonlinear frequencies and nonlinear vibration control by using sensor and actuator layers.

2. Governing equations of motion

The rotating sandwich cylindrical shell is considered, as shown in Fig. 1. L and R are length and radius of the shell, respectively. h is the thickness of FG core. h_a and h_s represent the thicknesses of actuator and sensor layers, respectively. F is lateral force and Ω denotes a constant rotating speed about the x-axis. The coordinate system (x, θ, z) is considered on the middle layer of the central layer.

2.1 Displacement field of cylindrical shell

Based on FSDT, the general displacement field of sandwich cylindrical shell is defined as Lam and Qian (2007)

$$u(x,\theta,z,t)=u_0(x,\theta,t)+z\,\varphi_x(x,\theta,t) \tag{1}$$

$$v(x,\theta,z,t)=v_0(x,\theta,t)+z\,\varphi_\theta(x,\theta,t) \tag{2}$$

$$w(x,\theta,z,t)=w_0(x,\theta,t) \tag{3}$$

(u, v, w) is the displacement of a point in direction of (x, θ, z) that including middle surface displacement (u_0, v_0, w_0) and the image of the normal vector rotation on the middle surface around x, θ axes (φ_x, φ_θ).

The strain vector is defined as Oh and Lee (2007)

$$\{\varepsilon\}=\{e\}+z\,\{K\} \qquad \begin{Bmatrix}\varepsilon_x\\ \varepsilon_\theta\\ \gamma_{x\theta}\\ \gamma_{\theta z}\\ \gamma_{xz}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_x^{(0)}\\ \varepsilon_\theta^{(0)}\\ \gamma_{x\theta}^{(0)}\\ \gamma_{z\theta}^{(0)}\\ \gamma_{xz}^{(0)}\end{Bmatrix}+z\begin{Bmatrix}K_x\\ K_\theta\\ K_{x\theta}\\ 0\\ 0\end{Bmatrix} \tag{4}$$

Based on FSDT and considering the nonlinear Von-Karman terms, the components of strain vector are expressed as follows

$$\varepsilon_x^{(0)}=\frac{\partial u}{\partial x}+0.5\left(\frac{\partial w}{\partial x}\right)^2 \tag{5}$$

$$\varepsilon_\theta^{(0)}=\frac{1}{R}\left(\frac{\partial v}{\partial\theta}+w\right)+0.5\left(\frac{1}{R}\frac{\partial w}{\partial\theta}\right)^2 \tag{6}$$

$$\gamma_{x\theta}^{(0)}=\frac{\partial u}{R\partial\theta}+\frac{\partial v}{\partial x}+\frac{1}{R}\frac{\partial w}{\partial x}\frac{\partial w}{\partial\theta} \tag{7}$$

$$\gamma_{\theta z}^{(0)}=\varphi_\theta+\frac{1}{R}\frac{\partial w}{\partial\theta} \tag{8}$$

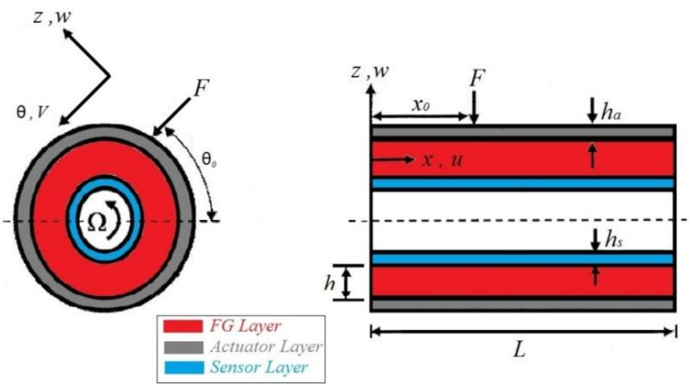

Fig. 1 A schematic view of a sandwich cylindrical shell considering FG core and FGPM sensor and actuator layers

$$\gamma_{xz}^{(0)} = \varphi_\theta + \frac{\partial w}{\partial x} \tag{9}$$

$$K_x = \frac{\partial \varphi_x}{\partial x} \tag{10}$$

$$K_\theta = \frac{1}{R}\frac{\partial \varphi_\theta}{\partial \theta} \tag{11}$$

$$K_{x\theta} = \frac{\partial \varphi_\theta}{\partial x} + \frac{1}{R}\frac{\partial \varphi_x}{\partial \theta} \tag{12}$$

2.2 The definition of functionally graded material properties

Variation properties in FG core and FGPM layers are considered as Mohammadimehr *et al*. (2016)

$$\begin{cases} E_f(z) = (E_c - E_m)(\frac{1}{2}+\frac{z}{h})^{g_f} + E_m \\ \rho_f(z) = (\rho_c - \rho_m)(\frac{1}{2}+\frac{z}{h})^{g_f} + \rho_m \end{cases} \quad -\frac{h}{2} \le z \le \frac{h}{2} \tag{13}$$

$$Y_i = Y_b^{\,i}(\frac{2z}{h})^{g_i} \qquad i = a,s \qquad |z| \ge \frac{h}{2} \tag{14}$$

where indeces m and c are related to metal and ceramic materials of the functionally graded (FG) core, respectively. Also, Eq. (13) shows the variation of Young's modulus and density in thickness direction of FG core. Eq. (14) illustrates the change of Young's modulus, density, and also the matrix of stiffness of the FGPM layers. g_f, g_a and g_s are the power law indeces of core, actuator and sensor layers, respectively.

2.3 Stress-strain relations

Due to the change in properties along the thickness of the FG layer and also considering the effect of electric field and electrical displacement in piezoelectric layers, the stress-strain relation of sandwich cylindrical shell is expressed as Dehghan *et al*. (2016)

$$\sigma_i = \left\{ \sigma_x^i \quad \sigma_\theta^i \quad \sigma_{x\theta}^i \quad \sigma_{\theta z}^i \quad \sigma_{xz}^i \right\} \qquad i = f, a, s \tag{15}$$

$$\sigma_f = C_f(z)\varepsilon \qquad -\frac{h}{2} \le z \le \frac{h}{2} \tag{16}$$

$$\begin{cases} \sigma_i = C_i\varepsilon - e_i E_i \\ D_i = e_i^T\varepsilon + \zeta_i E_i \end{cases} \quad i = s, a \qquad |z| \ge \frac{h}{2} \tag{17}$$

σ, ε, C, E, D, e and ζ are stress, strain, stiffness matrix, electric field, electric displacement, piezoelectric effective matrix and dielectric matrix, respectively.

$$\begin{cases} C_i = \begin{bmatrix} C_{1i_{2\times2}} & 0_{2\times3} \\ 0_{3\times2} & C_{2i_{3\times3}} \end{bmatrix} \\ C_{1i} = \begin{bmatrix} Q_{11i} & Q_{12i} \\ Q_{21i} & Q_{22i} \end{bmatrix} \qquad i = f, a, s \\ C_{2i} = diag(Q_{66i}, Q_{44i}, Q_{55i}) \end{cases} \tag{18}$$

$$\begin{cases} Q_{11f} = Q_{22f} = \dfrac{E_f}{1-\nu^2} \\ Q_{12f} = \dfrac{\nu E_f}{1-\nu^2} \\ Q_{44f} = Q_{55f} = Q_{66f} = \dfrac{E_f}{2(1+\nu)} \end{cases} \tag{19}$$

$$\begin{cases} \zeta_i = diag(\zeta_{11i}, \zeta_{22i}, \zeta_{33i}) \\ e_i = \begin{bmatrix} 0 & 0 & e_{31i} \\ 0 & 0 & e_{32i} \\ 0 & 0 & 0 \\ 0 & e_{24i} & 0 \\ e_{15i} & 0 & 0 \end{bmatrix} \qquad i = a, s \end{cases} \tag{20}$$

By substituting the components of strain vector and stiffness matrix into stress-strain relation the stress and electric displacement of piezoelectric layer obtained.

2.4 Stress-strain relations

The vector of electric field is defined as Lia and Pan (2015)

$$\begin{Bmatrix} E_a \\ E_s \end{Bmatrix} = -\begin{Bmatrix} \nabla\varphi_a \\ \nabla\varphi_s \end{Bmatrix} \tag{21}$$

The created electrical potential by the elastic deformation in the sensor layer is stated as Rouzegar and Abad (2015)

$$\varphi_s = [(z + \frac{h+h_s}{2})^2 - (\frac{h_s}{2})^2]\psi_s(x,\theta,z) \tag{22}$$

ψ_s is the component of the electric field in the sensor layer.

In the actuator layer, there is also an external potential that considering for calculation the electric potential of actuator layer

$$\begin{aligned} \varphi_a &= [(z - \frac{h+h_a}{2})^2 - (\frac{h_a}{2})^2]\psi_a(x,\theta,z) \\ &+ \frac{1}{h_a}(2z - h + h_a)U(x,\theta,z) \end{aligned} \tag{23}$$

ψ_a is the component of the electric field in the actuator layer. U is the external potential applied to actuator layer. By substituting the Eqs. (22) and (23) into Eq. (21) the electric field of sensor and actuator layer obtained.

2.5 Hamilton's principle

The governing equation of system is derived by employing Hamilton's principle, in the first step, the kinetic energy is obtained. The velocity vector of cylindrical shell will be obtained from the sum of the relative velocities and the angular velocity of shell.

$$\vec{V} = \vec{V}_{rel} + \vec{\Omega} \times \vec{r} \tag{24}$$

$\vec{V}_{rel}$, $\vec{\Omega}$ and $\vec{r}$ are relative velocity, angular velocity and location vector, respectively. The location vector of system based on the FSDT is presented as follows

$$\vec{r} = u_1\vec{e}_1 + v_1\vec{e}_2 + w_1\vec{e}_3 \tag{25}$$

$$\vec{V}_{rel} = \dot{\vec{r}} = \dot{u}_1\vec{e}_1 + \dot{v}_1\vec{e}_2 + \dot{w}_1\vec{e}_3 \tag{26}$$

$$\vec{\Omega} = -\Omega\vec{i} \tag{27}$$

By substituting Eqs. (25)-(27) into Eq. (24), the vector velocity of sandwich cylindrical shell is obtained as follows

$$\vec{V} = V_1\vec{e}_1 + V_2\vec{e}_2 + V_3\vec{e}_3 \tag{28}$$

$$V_1 = \dot{u} + z\,\dot{\varphi}_x \tag{29}$$

$$V_2 = \dot{v} + z\,\dot{\varphi}_\theta + \Omega w_1 \tag{30}$$

$$V_3 = \dot{w} - \Omega v - zv\,\varphi_\theta \tag{31}$$

Kinetic energy of sandwich cylindrical shell could be derived as Li *et al*. (2008)

$$\begin{aligned} &K = K_f + K_a + K_s \\ &K_i = \frac{1}{2}\int \rho_i (V_1^2 + V_2^2 + V_3^2)dV \qquad i = f, a, s \end{aligned} \tag{32}$$

Next step is calculation strain energy of sandwich cylindrical shell

$$H = H_f + H_a + H_s \tag{33}$$

$$H_f = \frac{1}{2}\int \sigma_{ijf}\,\varepsilon_{ijf}\,dV \tag{34}$$

$$H_a = \frac{1}{2}\int (\sigma_{ija}\varepsilon_{ija} - D_i E_i)dV \tag{35}$$

$$H_s = \frac{1}{2}\int (\sigma_{ijs}\varepsilon_{ijs} - D_i E_i)dV \tag{36}$$

The external force for the rotating sandwich shell is defined as follows

$$F = F_0\cos(\omega t)\delta(x - x_0)\delta(\theta - \theta_0) \tag{37}$$

The work of the external force is the product of the multiplication of force in the displacement

$$V_N = F\,w \tag{38}$$

Final step for deriving equilibrium equation of sandwich cylindrical shell, Hamilton's principle is applied as Jafari *et al*. (2014) and Mohammadimehr *et al*. (2018b)

$$\int_0^t (\delta K - \delta H - \delta V_N)dt = 0 \tag{39}$$

By substituting Eqs. (32), (33) and (38) into Eq. (39), the governing equations of rotating sandwich shell under lateral loading are presented in the form of a non-homogeneous nonlinear partial differential equation as follows

$$\begin{aligned}
&M'\ddot{X} + C'\dot{X} + (K' - \Omega^2 J')X + H' + G' = (D' + N')U + B'F \\
&X = \begin{bmatrix} u & v & w & \varphi_x & \varphi_\theta & \psi_a & \psi_s \end{bmatrix}^T \\
&M' = \begin{bmatrix} I_1 & 0 & 0 & I_2 & 0 & 0 & 0 \\ 0 & I_1 & 0 & 0 & I_2 & 0 & 0 \\ 0 & 0 & I_1 & 0 & 0 & 0 & 0 \\ I_1 & 0 & 0 & I_3 & 0 & 0 & 0 \\ 0 & I_2 & 0 & 0 & I_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
&C' = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_1 & 0 & 0 & 0 & 0 \\ 0 & c_2 & 0 & 0 & c_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
&J' = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I_1 & 0 & 0 & I_2 & 0 & 0 \\ 0 & 0 & I_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I_2 & 0 & 0 & I_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
&K' = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} & k_{16} & k_{17} \\ k_{21} & k_{22} & k_{23} & k_{24} & k_{25} & k_{26} & k_{27} \\ k_{31} & k_{32} & k_{33} & k_{34} & k_{35} & k_{36} & k_{37} \\ k_{41} & k_{42} & k_{43} & k_{44} & k_{45} & k_{46} & k_{47} \\ k_{51} & k_{52} & k_{53} & k_{54} & k_{55} & k_{56} & k_{57} \\ k_{61} & k_{62} & k_{63} & k_{64} & k_{65} & k_{66} & 0 \\ k_{71} & k_{72} & k_{73} & k_{74} & k_{75} & 0 & k_{77} \end{bmatrix}
\end{aligned} \tag{40}$$

$$\begin{aligned} H' &= \begin{bmatrix} H_1' & H_2' & H_3' & H_4' & H_5' & H_6' & H_7' \end{bmatrix}^T \\ G' &= \begin{bmatrix} 0 & 0 & G_3' & 0 & 0 & 0 & 0 \end{bmatrix}^T \\ D' &= \begin{bmatrix} D_1' & D_2' & D_3' & D_4' & D_5' & D_6' & 0 \end{bmatrix}^T \\ N' &= \begin{bmatrix} 0 & 0 & N_3' & 0 & 0 & 0 & 0 \end{bmatrix}^T \\ B' &= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T \end{aligned} \tag{40}$$

M' and C' are inertia and gyroscope matrices, respectively. Matrix datasets of K' and D' are linear operators that contain different location derivatives. H' and G' contains nonlinear second and third order semantics of vector components of system variables $\{X\}$.

3. Discretization of the governing equations

The simply supported boundary conditions are assumed in the two ends of the cylindrical shell. Using Galerkin's method, the spatial and temporal terms of the system variables should be separated so that variables vector of the system according to the relationship as follows

$$u = \sum_{m=0}^{H} \sum_{n=0}^{I} u_{mn}(t) \cos(\lambda_m x) \cos(n\theta) \tag{41}$$

$$v = \sum_{m=0}^{H} \sum_{n=0}^{I} v_{mn}(t) \sin(\lambda_m x) \sin(n\theta) \tag{42}$$

$$w = \sum_{m=0}^{H} \sum_{n=0}^{I} w_{mn}(t) \sin(\lambda_m x) \cos(n\theta) \tag{43}$$

$$\varphi_x = \sum_{m=0}^{H} \sum_{n=0}^{I} \varphi_{x\,mn}(t) \cos(\lambda_m x) \cos(n\theta) \tag{44}$$

$$\varphi_\theta = \sum_{m=0}^{H} \sum_{n=0}^{I} \varphi_{\theta mn}(t) \sin(\lambda_m x) \sin(n\theta) \tag{45}$$

$$\psi_a = \sum_{m=0}^{H} \sum_{n=0}^{I} \psi_{a\,mn}(t) \sin(\lambda_m x) \cos(n\theta) \tag{46}$$

$$\psi_S = \sum_{m=0}^{H} \sum_{n=0}^{I} \psi_{s\,mn}(t) \sin(\lambda_m x) \cos(n\theta) \tag{47}$$

By substituting Eqs. (41)-(47) into Eq. (40), the spatial and temporal terms of the system variables are separated from each other, and for removing the spatial terms used Galerkin's method as follows

$$\int_\theta \int_x \sum_{j=1}^{7} (I_{1j} \ddot{Z}_j + c_{1j} \dot{Z}_j + K_{1j} Z_j) \cos(\lambda_m x) \cos(n\theta) dx d\theta = 0 \tag{48}$$

$$\int_\theta \int_x \sum_{j=1}^{7} (I_{2j} \ddot{Z}_j + c_{2j} \dot{Z}_j + K_{2j} Z_j) \sin(\lambda_m x) \sin(n\theta) dx d\theta = 0 \tag{49}$$

$$\int_\theta \int_x \sum_{j=1}^{7} (I_{3j} \ddot{Z}_j + c_{3j} \dot{Z}_j + K_{3j} Z_j) \sin(\lambda_m x) \cos(n\theta) dx d\theta = 0 \tag{50}$$

$$\int_\theta \int_x \sum_{j=1}^{7} (I_{4j} \ddot{Z}_j + c_{4j} \dot{Z}_j + K_{4j} Z_j) \cos(\lambda_m x) \cos(n\theta) dx d\theta = 0 \tag{51}$$

$$\int_\theta \int_x \sum_{j=1}^{7} (I_{5j} \ddot{Z}_j + c_{5j} \dot{Z}_j + K_{5j} Z_j) \sin(\lambda_m x) \sin(n\theta) dx d\theta = 0 \tag{52}$$

$$\int_\theta \int_x \sum_{j=1}^{7} (I_{6j} \ddot{Z}_j + c_{6j} \dot{Z}_j + K_{6j} Z_j) \sin(\lambda_m x) \cos(n\theta) dx d\theta = 0 \tag{53}$$

$$\int_\theta \int_x \sum_{j=1}^{7} (I_{7j} \ddot{Z}_j + c_{7j} \dot{Z}_j + K_{7j} Z_j) \sin(\lambda_m x) \cos(n\theta) dx d\theta = 0 \tag{54}$$

By using the above method, the governing equations of the system are converted into ordinary differential equations.

$$\begin{gathered} M\ddot{Z} + (K - \Omega^2 J) Z + G = DU + B \cos(\omega t) \cos(n\Omega t) \\ K = \begin{bmatrix} T_{11} & T_{12} & T_{13} & T_{14} & T_{15} & T_{16} & T_{17} \\ T_{22} & T_{22} & T_{23} & T_{24} & T_{25} & T_{26} & T_{27} \\ T_{31} & T_{32} & T_{33} & T_{34} & T_{35} & T_{36} & T_{37} \\ T_{41} & T_{42} & T_{43} & T_{44} & T_{45} & T_{46} & T_{47} \\ T_{51} & T_{52} & T_{53} & T_{54} & T_{55} & T_{56} & T_{57} \\ T_{61} & T_{62} & T_{63} & T_{64} & T_{65} & T_{66} & 0 \\ T_{71} & T_{72} & T_{73} & T_{74} & T_{75} & 0 & T_{77} \end{bmatrix} \\ Z = \begin{bmatrix} u_{mn}(t) & v_{mn}(t) & w_{mn}(t) & \varphi_{x\,mn}(t) & \varphi_{\theta mn}(t) \\ \psi_{a\,mn}(t) & \psi_{s\,mn}(t) \end{bmatrix}^T \\ G = \begin{bmatrix} 0 & 0 & G_3 & 0 & 0 & 0 & 0 \end{bmatrix}^T \\ B = \begin{bmatrix} 0 & 0 & B_3 & 0 & 0 & 0 & 0 \end{bmatrix}^T \\ D = \begin{bmatrix} D_1 & D_2 & D_3 & D_4 & D_5 & D_6 & 0 \end{bmatrix}^T \\ M = \begin{bmatrix} I_1 & 0 & 0 & I_2 & 0 & 0 & 0 \\ 0 & I_1 & 0 & 0 & I_2 & 0 & 0 \\ 0 & 0 & I_1 & 0 & 0 & 0 & 0 \\ I_1 & 0 & 0 & I_3 & 0 & 0 & 0 \\ 0 & I_2 & 0 & 0 & I_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ J = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I_1 & 0 & 0 & I_2 & 0 & 0 \\ 0 & 0 & I_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I_2 & 0 & 0 & I_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{gathered} \tag{55}$$

The last two equations of the Eq. (55) are algebraic equations, which uses these equations to obtain the electric field generated in the piezoelectric actuator and sensor as follows

$$\psi_{a\,mn} = \frac{-1}{T_{66}}(T_{61}u_{mn} + T_{62}v_{mn} + T_{63}w_{mn} + T_{64}\varphi_{x\,mn} + T_{65}\varphi_{\theta mn} - D_6 U_{mn}) \quad (56)$$

$$\psi_{s\,mn} = \frac{-1}{T_{77}}(T_{71}u_{mn} + T_{72}v_{mn} + T_{73}w_{mn} + T_{74}\varphi_{x\,mn} + T_{75}\varphi_{\theta mn}) \quad (57)$$

By substituting Eqs. (56) and (57) into Eq. (55), the final equations of rotating sandwich cylindrical shell under lateral loading are obtained as five non-linear ordinary differential equation in terms of time derivatives.

The most important vector component of the system variables is w, which founds the third row of the governing equations of the system. For this reason, in order to study the dynamic stability, the governing equations of the system become an equation in the lateral direction (w). For this reason, inertial terms are neglected in other equations, which lead to algebraic equations. By solving algebraic equation the variable vector of system consist of u_{mn}, v_{mn}, $\varphi_{x\,mn}$ and $\varphi_{\theta\,mn}$ is obtained as follow

$$u_{mn}(t) = f_{uw}(w_{mn}(t)) + f_{uU}(U_{mn}(t)) \quad (58)$$

$$v_{mn}(t) = f_{uw}(w_{mn}(t)) + f_{vU}(U_{mn}(t)) \quad (59)$$

$$\varphi_{x\,mn}(t) = f_{\varphi_x w}(w_{mn}(t)) + f_{\varphi_x U}(U_{mn}(t)) \quad (60)$$

$$\varphi_{\theta mn}(t) = f_{\varphi_\theta w}(w_{mn}(t)) + f_{\varphi_\theta U}(U_{mn}(t)) \quad (61)$$

By substituting Eqs. (58)-(61) into third row of Eq. (55), new form of equilibrium equation of rotating sandwich cylindrical shell under lateral forced is obtained as follows

$$b_1\ddot{w} + b_2 w + b_3 w^3 + b_4 U + b_5 F_0 \cos(\omega t)\cos(n\Omega t) = 0 \quad (62)$$

By employing perturbation method and adding the scale transformation parameter ς, the new deferential equation by considering linear damping forces obtained as follows

$$\ddot{w} + \omega_0^2 w + \frac{b_3}{b_1}\varsigma w^3 + \frac{b_4}{b_1}\varsigma U + \frac{b_5}{b_1}\varsigma F_0 \cos(\omega t)\cos(n\Omega t) = 0 \quad (63)$$

where

$$\omega_0 = \omega + \Omega + \varsigma\eta \quad (64)$$

η is detuning parameter.

By substituting the uniform solution of Eq. (63) and separating different order of ς as Liu and Chu (2012), we have

$$w(t,\varsigma) = w_0(T_0,T_1) + \varsigma w_1(T_0,T_1) + \ldots \quad (65)$$

order ς^0

$$D_0^2 w_0 + \omega_0^2 w_0 = 0 \quad (66)$$

order ς^1

$$\begin{aligned} D_0^2 w_1 + \omega_0^2 w_1 &= -2D_0 D_1 w_0 \\ &- \frac{b_3}{b_1} w_0^3 - \frac{b_4}{b_1} U - \frac{b_5}{b_1} F_0 \cos(\omega t)\cos(n\Omega t) \end{aligned} \quad (67)$$

The solution of the Eq. (66) is expressed as follows

$$w_0 = A_1(T_1)e^{i\omega_0 T_0} + \bar{A}_1(T_1)e^{-i\omega_0 T_0} \quad (68)$$

Substituting (68) into (67) yields the following equation

$$\begin{aligned} D_0^2 w_1 + \omega_0^2 w_1 &= (-2i\,\omega_0 D_1 A_1 \\ &- 3\frac{b_3}{b_1} A_1^2 \bar{A}_1 - \frac{b_5}{b_1}\frac{F_0}{2} e^{i\eta T_1}) e^{i\omega_0 T_0} + CC + NST \end{aligned} \quad (69)$$

where *NST* represents all non-secular terms, the symbol *CC* represents the complex conjugate terms. The secular terms should be zero

$$2i\,\omega_0 D_1 A_1 + 3\frac{b_3}{b_1} A_1^2 \bar{A}_1 + \frac{b_5}{b_1}\frac{F_0}{2} e^{i\eta T_1} = 0 \quad (70)$$

By considering A_1 as Eq. (69) and substituting into Eq. (70) and then separating the real and imaginary parts from the resulting equations.

$$D_1 a = -\frac{b_5 F}{2b_1\omega_0}\sin\varphi \quad (71)$$

$$aD_1\varphi = a\eta - 3\frac{b_3 a^3}{8b_1\omega_0} - \frac{b_5 F}{2b_1\omega_0}\cos\varphi \quad (72)$$

where $\varphi = \eta T_1 - \beta$.

For finding steady solution of system, it should become $D_1 a = 0$, $D_1\phi = 0$. By employing these equations, the algebraic equations of amplitude-frequency could be written as follows

$$(\omega + \Omega - \omega_0 - 3\frac{b_3}{b_1}\frac{\varepsilon a^2}{8\omega_0})^2 = (\frac{b_5 F}{2b_1\omega_0})^2 \quad (73)$$

The used method is one of the best and most widely used methods to find the relationship between nonlinear frequency and displacement of system.

4. Discretization of the governing equations

In this section, the influence of various parameters on nonlinear vibration and dynamic response of sandwich cylindrical shell is investigated. Both piezoelectric layers (sensor and actuator) are considered to be PZT-4. Geometric parameters, mechanical properties of FG core and mechanical properties and electrical coefficients of piezoelectric layers are presented in Tables 1, 2 and 3.

Table 1 The geometrical parameters of sandwich cylindrical shell

h_s (m)	h_a (m)	h (m)	L (m)	R (m)
0.001	0.001	0.01	2	1

Table 2 The mechanical properties of FG core

ν	ρ_m (Kg/m^3)	ρ_c (Kg/m^3)	E_m (GPa)	E_c (GPa)
0.33	2700	3800	70	380

Table 3 The mechanical properties and electrical coefficients of piezoelectric layers

e_{31} (C/m^2)	e_{32} (C/m^2)	e_{24} (C/m^2)	e_{15} (C/m^2)	ζ_{11} (10^{-11} F/m)	ζ_{22} (10^{-11} F/m)	ζ_{33} (10^{-11} F/m)
-5.2	-5.2	12.7	12.7	650	650	560
Q_{11} (10^{10}N/m^2)	Q_{12} (10^{10}N/m^2)	Q_{22} (10^{10}N/m^2)	Q_{44} (10^{10}N/m^2)	Q_{55} (10^{10}N/m^2)	Q_{66} (10^{10}N/m^2)	ρ (Kg/m^3)
13.9	7.8	13.9	2.56	2.56	3.05	7500

4.1 Validation of results

In order to ensure the accuracy of the obtained results in this study, we first compared the results with past research. For cylindrical shell with simply supported boundary conditions, a comparison between the results of this study and the obtained results presented by Liu and Chu (2012) for various ω and C. There is a good agreement between the results of this research and the results of Liu and Chu (2012).

Table 4 Amplitude–frequency response of rotating thin circular cylindrical shell for various ω and C

Method	ω	C		
		0.01	0.05	0.1
Liu and Chu (2012)	0.8	0.55	0.25	0.22
	1	1.1	0.9	0.8
	1.2	0.52	0.2	0.18
Present study	0.8	0.58	0.22	0.24
	1	1.15	0.93	0.86
	1.2	0.55	0.19	0.2

4.2 Nonlinear vibration of sandwich cylindrical shell

In this article, the nonlinear vibration and dynamic stability analysis of a smart rotating sandwich cylindrical shell with considering a functionally graded (FG) core integrated by piezoelectric layers subjected to electric field is investigated. The piezoelectric layers at the inner and outer surfaces used as actuator and sensor, respectively. By applying the energy method and Hamilton's principle, the governing equations of sandwich cylindrical shell are derived based on first-order shear deformation theory (FSDT). The Galerkin's method is used to discriminate the motion equations and then these equations are converted to the form of the ordinary differential equations in terms of time. The perturbation method is employed to find relation between nonlinear frequency and the amplitude of vibraion. The used perturbation method in this article is one of the best and most widely methods to find the relationship between nonlinear frequency and displacement of system. The most important advantage of this method is a very short time for solving the equation.

Heterogeneous coefficient (power law index) of FGM layer (g_f), heterogeneous coefficient of sensor layer (g_s) and heterogeneous coefficient of sensor layer (g_a) are studied as the first parameter on nonlinear frequency of sandwich cylindrical shell. Fig. 2 shows by increasing the shell heterogeneity coefficient (power law index) of FGM layer, in constant amplitude, the nonlinear frequency of system reduces. It is due to decreasing the stiffness of sandwich cylindrical shell. On the other hands, by increasing the power law index, the sandwich structure becomes softer. Also Figs. 3 and 4 show the influence of heterogeneous coefficient of sensor and actuator layers on nonlinear frequency of sandwich cylindrical shell. It can be seen from Fig. 5 that by increasing the angular velocity of sandwich

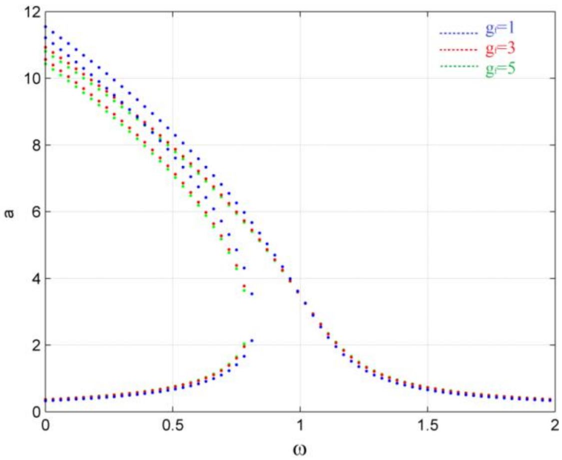

Fig. 2 The effect of heterogeneity coefficient (power law index(g_f)) of FGM layer on amplitude–frequency response (L/R = 5, R/h/20, h/h_a = 25, h_a/h_s = 1, g_a = 0, g_s = 0, Ω = 1000 rad/s, ω_0 = 1, ς = –0.5)

cylindrical shell, in constant amplitude the nonlinear vibration of system decreases. Figs. 6 and 7 indicate the effect of geometric parameter such as thickness to radius ratio (h/R) and aspect ratio (L/R) on nonlinear vibration of smart sandwich cylindrical shell. Fig. 6 shows by increasing the h/R the nonlinear frequency of system increases because of the stiffness of structures increases. The stability zone of the system is also predictable from this shape. The existence of three different displacements at a nonlinear frequency indicates that the system was unstable. In Fig. 7, we can see

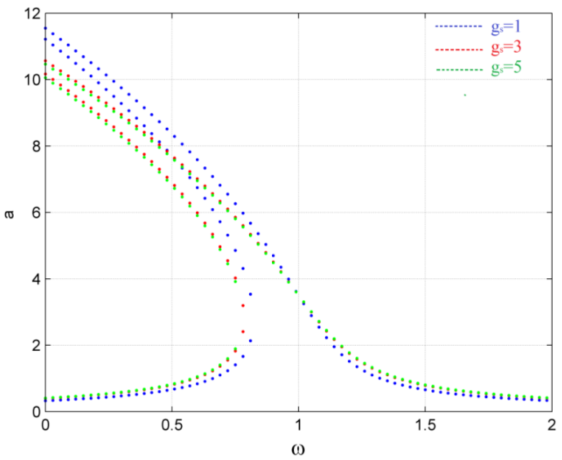

Fig. 3 The effect of heterogeneity coefficient of sensor layer on amplitude–frequency response (L/R = 5, R/h/20, h/h_a = 25, h_a/h_s = 1, g_f = 1, g_a = 0, Ω = 1000 rad/s, ω_0 = 1, ς = –0.5)

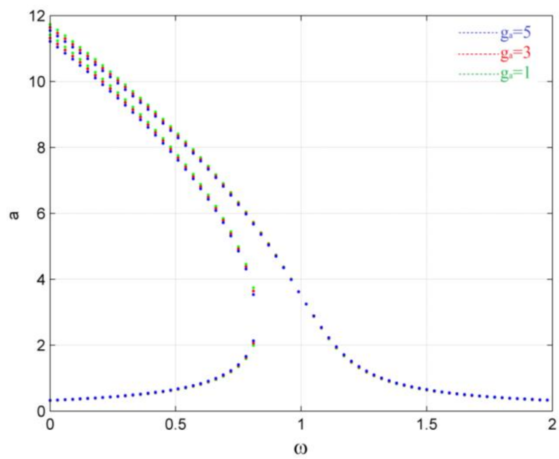

Fig. 4 The effect of heterogeneity coefficient of actuator layer on amplitude–frequency response (L/R = 5, R/h/20, h/h_a = 25, h_a/h_s = 1, g_f = 1, g_s = 0, Ω = 1000 rad/s, ω_0 = 1, ς = –0.5)

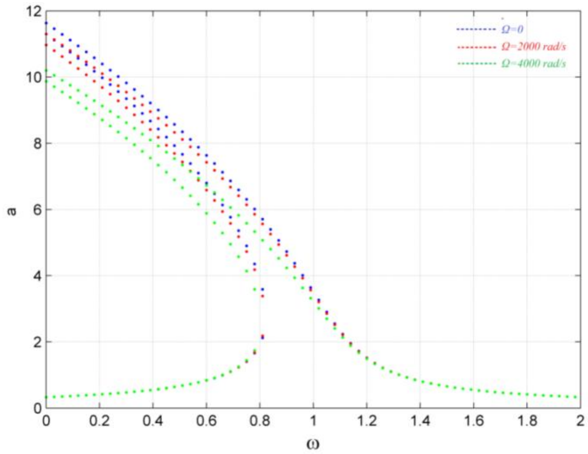

Fig. 5 The effect of angular velocity on amplitude–frequency response (L/R = 5, R/h/20, h/h_a = 25, h_a/h_s = 1, g_a = 0, g_s = 0, ω_0 = 1, ς = –0.5)

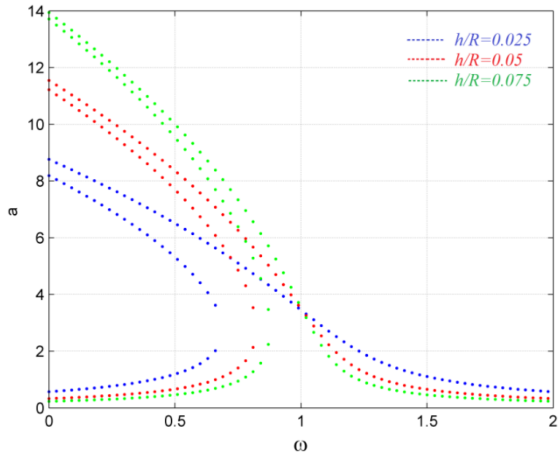

Fig. 6 The effect of thickness to radius ratio (h/R) on amplitude–frequency response (L/R = 5, h/h_a = 25, h_a/h_s = 1, g_a = 0, g_s = 0, Ω = 1000 rad/s, ω_0 = 1, ς = –0.5)

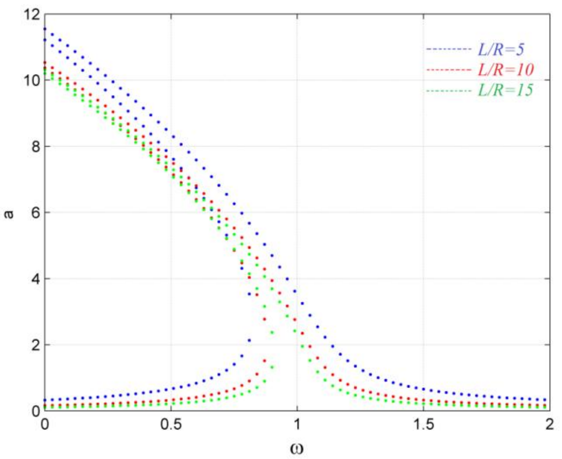

Fig. 7 The effect of length to radius ratio (L/R) on amplitude–frequency response (R/h/20, h/h_a = 25, h_a/h_s = 1, g_a = 0, g_s = 0, Ω = 1000 rad/s, ω_0 = 1, ς = –0.5)

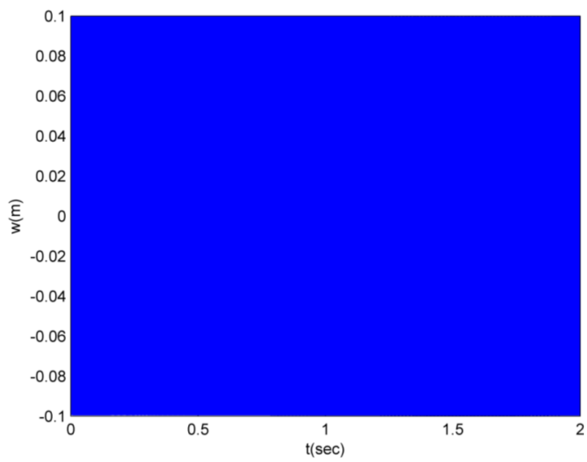

Fig. 8 Dynamic response of sandwich cylindrical shell (V = 0, K_v = 1000, L/R = 5, R/h/20, h/h_a = 25, h_a/h_s = 1, g_a = 0, g_s = 0, Ω = 1000 rad/s)

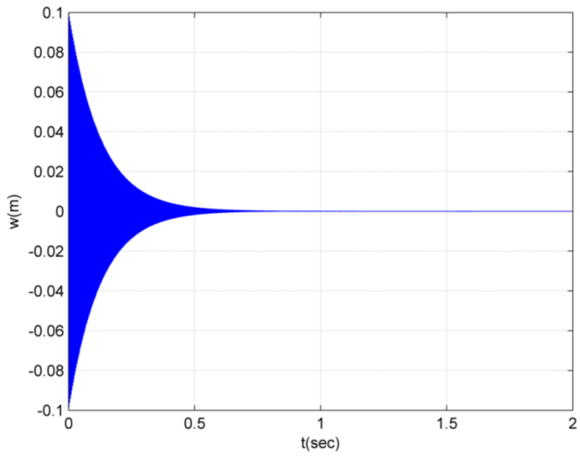

Fig. 9 Dynamic response of sandwich cylindrical shell (V = 24v, K_v = 1000, L/R = 5, R/h/20, h/h_a = 25, h_a/h_s = 1, g_a = 0, g_s = 0, Ω = 1000 rad/s)

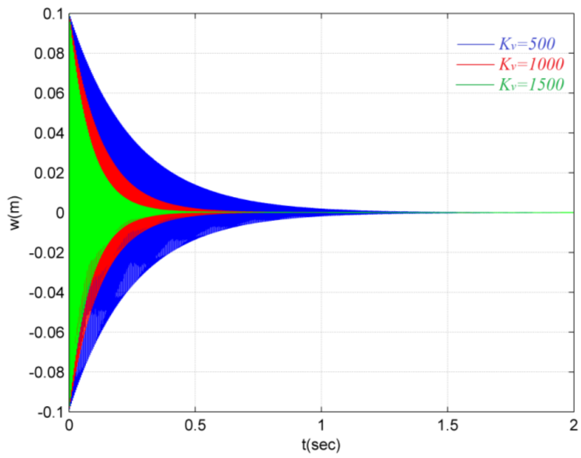

Fig. 10 The effect of voltage factor on dynamic response of sandwich cylindrical shell (V = 24v, L/R = 5, R/h = 20, h/h_a = 25, h_a/h_s = 1, g_a = 0, g_s = 0, Ω = 1000 rad/s)

the effects of L/R on stability region of smart sandwich cylindrical shell. With increasing of the L/R, the stability region of system decreases.

Piezoelectric layers have been used to control and stabilize the system. As soon as the sandwich cylinder is deformed, the sensor layer sends a pulse to the designed controller. Then controller applies the required voltage to the actuator according to the amount of deformation. Fig. 8 illustrates the uncontrolled dynamic response of system. The displacement of the midpoint of the sandwich shell is a harmonic response with constant amplitude. As shown in Fig. 9, after controlling force, the system response is controlled and after a short time, the system vibrations disappear. Therefore, by comparing the controlled response and the uncontrolled response of the system, it can be concluded that the control system is completely successful in eliminating the system vibrations and, after deployment, the system returns to equilibrium. Fig. 10 shows the effect of the voltage coefficient on the system's sustainability time. As it's obvious, increasing the voltage factor reduces the time it takes to stabilize the smart sandwich cylindrical shell.

5. Conclusions

In this paper, the dynamic stability and nonlinear vibration behaviors of a smart rotating sandwich cylindrical shell are studied. The first-order shear deformation theory (FSDT) is used to model the sandwich cylindrical shell. The core of the structure is a functionally graded material (FGM) which is integrated by functionally graded piezoelectric material (FGPM) layers subjected to electric field. The piezoelectric layers at the inner and outer surfaces used as actuator and sensor, respectively. The Galerkin's method is used to discriminate the motion equations and the equations are converted to the form of the ordinary differential equations in terms of time. The perturbation method is employed to find relation between nonlinear frequency and the amplitude of vibration. The used perturbation method in this article is one of the best and most widely methods to find the relationship between nonlinear frequency and displacement of system. The most important advantage of this method is a very short time for solving the equation. The main objective of this research is to determine the nonlinear frequencies and nonlinear vibration control by using sensor and actuator layers.

In this study, we try to analysis the influence the heterogeneity coefficient (power law index) of FG, sensor and actuator layers, angular velocity, thickness to radius ratio, length to radius ratio, and control voltage on frequency responses of steady solution and amplitude-nonlinear frequency diagrams.

The results show that:

- By increasing the heterogeneity coefficient (power law index) of FGM layer, in constant amplitude, the nonlinear frequency of system reduces because the stiffness of system decreases. On the other hands, the sandwich structure becomes softer.
- Increasing the angular velocity could be reduce the stiffness and, consequently, reduce the nonlinear frequency of the system.
- The bifurcation is clearly seen with several solutions for the system vibration amplitude at a given nonlinear frequency.
- Increasing the angular velocity will increase the stability region of the system.
- According to the results obtained, it can be seen that reducing the ratio of h/R and L/R will increase the stability region of the system.
- The best way to reduce time control of system is using a controller with higher voltage factor.

Thus it is suggested that for reducing the nonlinear vibration, the softer material properties, higher angular velocity are used. Also, controlling dynamic response of system could be done successfully by applying external voltage. Designing the controller and changing its parameters will help to stabilize the system and achieve the desired stable time in different system conditions. The results of this research can be used to design and vibration control of rotating systems in various industries such as aircraft, biomechanics and automobile manufacturing.

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