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A new size-dependent shear deformation theory for wave propagation analysis of triclinic nanobeams

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Abstract. For the first time, longitudinal and transverse wave propagation of triclinic nanobeam is investigated via a size-dependent shear deformation theory including stretching effect. Furthermore, the influence of initial stress is studied. To consider the size-dependent effects, the nonlocal strain gradient theory is used in which two small scale parameters predict the behavior of wave propagation more accurately. The Hamiltonian principle is adopted to obtain the governing equations of wave motion, then an analytic technique is applied to solve the problem. It is demonstrated that the wave characteristics of the nanobeam rely on the wave number, nonlocal parameter, strain gradient parameter, initial stress, and elastic foundation. From this paper, it is concluded that the results of wave dispersion in isotropic and anisotropic nanobeams are almost the same in the presented case study. So, in this case, triclinic nanobeam can be approximated with isotropic model.

Keywords: anisotropic materials; wave propagation; stretching effect; initial stress

1. Introduction

Wave propagation includes any kind of ways in which waves travel. According to the direction of the waves, it can be divided into two groups, longitudinal and transverse waves. From another point of view, wave propagation is categorized in several different types such as ground waves, space waves, ionospheric waves. This topic can be studied under several different branches of science such as physics, material science, electrical and mechanical engineering. In mechanical engineering, as our field of interest, the wave propagation in different structures such as beams, plates and shells can be investigated. For studying waves in these types of structures, at least two kinds of wave dispersion are important. First, waves which are studied far from boundaries which are called bulk waves. In other words, we study them before they reach the boundaries and reflected. For the major applications of this type of modeling in the industry, one may mention the non-destructive test (NDT) which can be connected to bulk waves. The second type, known as guided waves, includes the interactions between waves and boundaries and it is more complicated to model them in comparison with the first type but it is obvious that this type is more similar to reality.

The capability to build nano-materials and use of these materials in the engineering structures have been provided in the current century. Nanotechnology is used in a variety of engineering and medical branches such as solar cells, coating, reinforcements, cancer treatment, etc. (Flavel 2018, Subramani *et al*. 2018, Leite *et al*. 2018, Verma *et al*. 2018, Baer *et al*. 2003, Deotare *et al*. 2009, Mehar and Panda 2019). Hence, many scholars have shown their interest in the research and development of this modern technology. A series of experimental tests have shown that classical theories are not able to predict the behavior of materials in nano-dimensions (Lin *et al*. 2013). Therefore, different methods such as non-classical theories, experimental tests and molecular dynamics (MD) simulation were presented to overcome this problem (Walton 1984, Hua *et al*. 2017, Li and Hu 2017, Shahsavari *et al*. 2018c, Mehralian *et al*. 2017, Arefi and Zenkour 2018, Nazemnezhad and Kamali 2018, Farajpour *et al*. 2018a, b, 2019, Nguyen *et al*. 2014, Bessaim *et al*. 2015, Kadari *et al*. 2018, Mehar *et al*. 2018, Apuzzo *et al*. 2018, Ghayesh and Farajpour 2018, Lu *et al*. 2017, Barretta and de Sciarra 2018, Xu *et al*. 2017, Lu *et al*. 2018, Faleh *et al*. 2018, Li *et al*. 2018, Shahverdi and Barati 2017, Arash and Wang 2012). Among the aforementioned methods, non-classical theories are more popular, because it reduces the complexity of other methods. The strain gradient theory, modified couple stress and strain theories, Eringen nonlocal model, as well as nonlocal strain gradient theory have been used extensively in recent years to study nanostructure systems (Gao *et al*. 2019, Barati 2017b, Kaghazian *et al*. 2017, Karami *et al*. 2018c, Zenkour and Abouelregal 2014, Arefi 2018, Mokhtar *et al*. 2018, Mouffoki *et al*. 2017, Sahmani and Aghdam 2017a, b, Sahmani and Aghdam 2018, Lim *et al*. 2015, Aifantis 2009, Askes and Aifantis 2009, Heydari 2018, Ghayesh 2018, Rahmani *et al*. 2018, Ahouel *et al*. 2016, Karami *et al*. 2018e, Shahsavari *et al*. 2018a, Karami *et al*. 2018b, i, 2019d, f, Karami and Janghorban 2019b, Karami and Karami 2019, Karami and Shahsavari 2019, She *et al*. 2019).

Wave propagation of graphene sheets were studied by (Karami and Janghorban 2016) based on a refined plate

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theory and strain gradient theory. Application of wave propagation in functionally graded nanoplates was presented by (Li *et al*. 2015) using nonlocal strain gradient theory. (Barati 2017a) investigated the wave characteristics of nanoporous beams using a general bi-Helmholtz nonlocal strain-gradient elasticity model under the thermal environment. The influences of porosities and thermal conditions on the wave dispersion of porous nanotubes were reported by (She *et al*. 2018) based upon nonlocal strain gradient theory. (Karami *et al*. 2018a) studied the guided wave propagation applications in a fully-clamped porous nameplates using Eringen nonlocal model for the first time. Furthermore, a large number of works have been reported on the dispersion of waves (Arefi and Zenkour 2017, Besseghier *et al*. 2011, Bisheh and Wu 2019, Ebrahimi *et al*. 2018, Gafour *et al*. 2013, Janghorban and Nami 2015, Zeighampour *et al*. 2018, Zhen and Zhou 2017, Karami *et al*. 2017, 2018f, h, 2019a, b, c, e, g, h, Karami and Janghorban, 2019a, Shahsavari *et al*. 2018b). As reviewed in above sentences, there is no study on the wave propagation analysis of nonlocal strain gradient beams made of triclinic material.

In the current work, a new size-dependent shear deformation theory is utilized to investigate the wave phenomena in triclinic nanobeam for the first time. The small-scale effects are captured using nonlocal strain gradient theory. The equations of wave motion are obtained using Hamilton's principle where an analytic technique based on harmonic series is utilized to find the wave frequency and phase velocity as a function of wave number. Furthermore, the influences of small-scale parameters, elastic substrate, initial stress and wave number on the wave characteristics of such nanostructures are presented.

2. Formulation of the problem

Consider a beam with length a, width b and thickness h. The beam is under the initial stress and resting on a two-parameters elastic foundation (see in Fig. 1).

2.1 Kinematics

Based on a hybrid-type higher-order shear defamation theory proposed by Zaoui *et al*. (2017), the following displacement field is expressed

$$u_1(x,z,t) = u(x,t) - z\frac{\partial w}{\partial x} + kf(z)\int \theta(x,t)dx \quad (1)$$

$$u_3(x,z,t) = \mathrm{w}(x,t) + \mathrm{g(z)}\varphi_z(x,t) \quad (2)$$

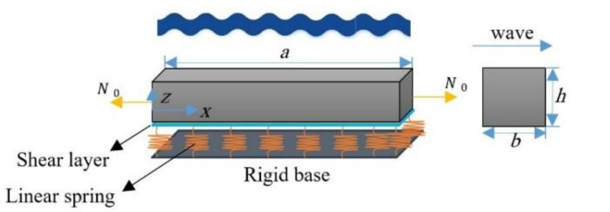


Fig. 1 Geometry of triclinic nanobeam under initial stress

in which u and w are the mid-plane displacements, θ is the rotation of normal to the mid-plane of the beam and φ_z consider the stretching effect. In the current work, the shape function $f(z)$ is chosen as (Zaoui *et al*. 2019)

$$\begin{aligned} f(z) &= \frac{\pi h}{\pi^4 + h^4} e^{(\mathrm{hz}/\pi)} (\pi^2 \sin(\frac{\pi z}{h}) + \mathrm{h}^2 \cos(\frac{\pi z}{h})) \\ &- \frac{\pi h^3}{\pi^4 + h^4} \text{ and } g(z) = \frac{df}{dz} \end{aligned} \quad (3)$$

According to the above displacement field (Eqs. (1)-(2)), the non-zero strains are obtained as follow

$$\begin{aligned} \varepsilon_x &= \varepsilon_x^0 + zk_x^b + f(z)k_x^s \\ \gamma_{xz} &= g(z)\gamma_{xz}^0 \\ \varepsilon_z &= g'(z)\varepsilon_z^0 \end{aligned} \quad (4)$$

where

$$\begin{aligned} &\varepsilon_x^0 = \frac{\partial u}{\partial x},\ k_x^b = -\frac{\partial^2 w_b}{\partial x^2},\ k_x^s = k\theta \\ &\gamma_{xz}^0 = k\int \theta dx + \frac{\partial \varphi_z}{\partial x} \\ &\varepsilon_z^0 = \varphi_z \text{ and } g'(z) = \frac{\partial g(z)}{\partial z} \end{aligned} \quad (5)$$

in which

$$\int \theta dx = A'\frac{\partial \theta}{\partial x} \quad (6)$$

herein coefficient A' (defined according to the type of solutions used) and k (constant depend on the geometry) are given below

$$A' = -\frac{1}{\beta^2}, k = \beta^2 \quad (7)$$

2.2 Triclinic material

"The matrix of elastic constants of a triclinic material can be obtained from that of a transversely isotropic material by appropriate rotations about the x- and the rotated y-axis (Batra *et al*. 2004)". The stress–strain relationship for the triclinic beam is as follows (Batra *et al*. 2004)

$$\begin{Bmatrix} \sigma_x \\ \sigma_z \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{13} & Q_{15} \\ Q_{31} & Q_{33} & Q_{35} \\ Q_{51} & Q_{53} & Q_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_z \\ \gamma_{xz} \end{Bmatrix} \quad (8)$$

in which Q_{11} = 98.84 Gpa, Q_{33} = 87.23 Gpa, Q_{55} = 21.10 Gpa, $Q_{13} = Q_{31}$ = 50.78 Gpa, $Q_{15} = Q_{51}$ = 1.05 Gpa, and Q_{35} = Q_{53} = 1.03 Gpa and ρ = 7750 kg/m^3. In the current investigation, we tried to investigate the possibility and accuracy of replacing present anisotropic model with an isotropic one. Hence, the elastic constants Q_{ij} using the isotropic approach are defined as

$$Q_{11}=Q_{33}=\frac{E(1-\nu)}{(1-2\nu)(1+\nu)}$$
$$Q_{12}=Q_{13}=\frac{\nu E}{(1-2\nu)(1+\nu)} \tag{9}$$
$$Q_{15}=Q_{35}=Q_{51}=Q_{53}=0$$

where the Young's modulus and Poisson's ratio are approximated for the triclinic material as E = 59.63 GPa, and ν = 0.35, respectively.

2.3 Equations of motion

Hamilton's principle is employed to determine the equations of motion

$$\int_0^t \delta(U+V-K)dt=0 \tag{10}$$

where δU is the variation of strain energy; δV is the variation of potential energy of applied forces and δK is the variation of kinetic energy. The variation of strain energy is

$$\begin{aligned}\delta U&=\int_V\left[\sigma_x\delta\varepsilon_x+\sigma_z\delta\varepsilon_z+\tau_{xz}\delta\gamma_{xz}\right]dV\\&=\int_A\begin{bmatrix}N_x\delta\varepsilon_x^0+N_z\delta\varepsilon_z^0+M_x^b\delta k_x^b\\+M_x^s\delta k_x^s+Q_{xz}^s\delta\gamma_{xz}^0\end{bmatrix}dA=0\end{aligned} \tag{11}$$

where the stress resultants N, M, and Q are defined by

$$\begin{aligned}(N_i,M_i^b,M_i^s)&=\int_{-h/2}^{h/2}(1,z,f)\sigma_i dz\\N_z&=\int_{-h/2}^{h/2}g'(z)\sigma_z dz\\Q_{xz}&=\int_{-h/2}^{h/2}g\tau_{xz}dz\end{aligned} \tag{12}$$

The variation of potential energy by the in-plane loads and elastic foundation are given by

$$\delta V=\int_A\left\{K_W w-K_P\frac{\partial^2 w}{\partial x^2}-N_0\frac{\partial^2 w}{\partial x^2}\right\}dA \tag{13}$$

where K_W and K_P are, respectively, the Winkler and the shear stiffness coefficients of the elastic foundation; $N_0 = h\sigma_0$ is the initial load in which σ_0 denotes the initial stress. Noted that, the positive and negative initial stresses represent, respectively, the tension and compressive loads. The variation of kinetic energy of the beam can be written as

$$\begin{aligned}\delta K&=\int_V[\dot u_1\delta\dot u_1+\dot u_3\delta\dot u_3]dV\\&=\int_A\Big\{I_0(\dot u\delta\dot u+\dot w\,\delta\dot w)+J_0(\dot w\,\delta\dot\varphi_z+\dot\varphi_z\delta\dot w)\\&-I_1(\dot u\frac{\partial\delta\dot w}{\partial x}+\frac{\partial\dot w}{\partial x}\delta\dot u)+J_1(kA'(\dot u\frac{\partial\delta\dot\theta}{\partial x}+\frac{\partial\dot\theta}{\partial x}\delta\dot u))\\&+I_2(\frac{\partial\delta\dot w}{\partial x}\frac{\partial\delta\dot w}{\partial x})+K_2((kA')^2(\frac{\partial\dot\theta}{\partial x}\frac{\partial\delta\dot\theta}{\partial x}))\\&-J_2(kA'(\frac{\partial\dot w}{\partial x}\frac{\partial\delta\dot\theta}{\partial x}+\frac{\partial\dot\theta}{\partial x}\frac{\partial\delta\dot w}{\partial x}))+K_0\dot\varphi_z\delta\dot\varphi_z\Big\}dA\end{aligned} \tag{14}$$

herein an over dot designates the differentiation with respect to the time variable t; and I_i, J_i, K_i are mass inertias expressed by

$$\begin{aligned}&\{I_0,I_1,I_2,J_1,J_2,J_0,K_0,K_2\}\\&=\int_{-h/2}^{h/2}\{1,z,z^2,f,zf,g,g^2,f^2\}\rho(z)dz\end{aligned} \tag{15}$$

By substituting Eqs. (11) and (13)-(14) into Eq. (10), the following equation of motion can be obtained

$$\delta u:\frac{\partial N_x}{\partial x}=I_0\ddot u_0-I_1\frac{\partial\ddot w}{\partial x}+J_1kA'\frac{\partial\ddot\theta}{\partial x} \tag{16}$$

$$\begin{aligned}&\delta w:\frac{\partial^2 M_x^b}{\partial x^2}-K_W w+(K_P+N_0)\frac{\partial^2 w}{\partial x^2}\\&=I_0\ddot w+J_0\ddot\varphi_z+I_1\frac{\partial\ddot u}{\partial x}-I_2\frac{\partial^2\ddot w}{\partial x^2}+J_2kA'\frac{\partial^2\ddot\theta}{\partial x^2}\end{aligned} \tag{17}$$

$$\begin{aligned}&\delta\theta:-kM_x^s-kA'\frac{\partial Q_{xz}^s}{\partial x}=-J_1kA'\frac{\partial\ddot u}{\partial x}+J_2kA'\frac{\partial^2\ddot w}{\partial x^2}\\&-K_2(kA')^2\frac{\partial^2\ddot\theta}{\partial x^2}\end{aligned} \tag{18}$$

$$\delta\varphi:\frac{\partial Q_{xz}^s}{\partial x}-N_z=J_0\ddot w+K_0\ddot\varphi_z \tag{19}$$

By inserting Eq. (4) into Eq. (8) and the subsequent results into Eq. (11), the stress resultants are given as

$$\begin{Bmatrix}N_x\\M_x^b\\M_x^s\\N_z\\Q_{xz}^s\end{Bmatrix}=\begin{bmatrix}A_{11}&B_{11}&C_{11}&D_{13}&E_{51}\\A_{11}^1&B_{11}^1&C_{11}^1&D_{13}^1&E_{51}^1\\A_{11}^2&B_{11}^2&C_{11}^2&D_{13}^2&E_{51}^2\\A_{31}^3&B_{31}^3&C_{31}^3&D_{33}^3&E_{35}^3\\A_{51}^4&B_{51}^4&C_{51}^5&D_{53}^4&E_{55}^4\end{bmatrix}\begin{Bmatrix}\frac{\partial u}{\partial x}\\-\frac{\partial^2 w}{\partial x^2}\\k\theta\\\phi_z\\kA'\frac{\partial\theta}{\partial x}+\frac{\partial\varphi_z}{\partial x}\end{Bmatrix} \tag{20}$$

where A_{ij}, B_{ij}…are the beam stiffness defined by

$$\begin{aligned}&\begin{bmatrix}A_{ij}&B_{ij}&C_{ij}&D_{ij}&E_{ij}\\A_{ij}^1&B_{ij}^1&C_{ij}^1&D_{ij}^1&E_{ij}^1\\A_{ij}^2&B_{ij}^2&C_{ij}^2&D_{ij}^2&E_{ij}^2\\A_{ij}^3&B_{ij}^3&C_{ij}^3&D_{ij}^3&E_{ij}^3\\A_{ij}^4&B_{ij}^4&C_{ij}^4&D_{ij}^4&E_{ij}^4\end{bmatrix}\\&=\int_{-\frac{h}{2}}^{\frac{h}{2}}Q_{ij}\begin{bmatrix}1&z&f(z)&g'(z)&g(z)\\z&z^2&zf(z)&zg'(z)&zg(z)\\f(z)&zf(z)&f^2(z)&f(z)g'(z)&f(z)g(z)\\g'(z)&zg'(z)&f(z)g'(z)&g'^2(z)&g(z)g'(z)\\g(z)&zg(z)&g(z)f(z)&g(z)g'(z)&g^2(z)\end{bmatrix}dz\end{aligned} \tag{21}$$

2.4 Nonlocal strain gradient theory

(Askes and Aifantis 2009) presented a model incorporating two different approaches of size-dependency to investigate the nanostructure systems. This model includes two small scale parameters as

$$\sigma_{ij} - \mu^2 \sigma_{ij,mm} = Q_{ijkl}(\varepsilon_{kl} - l^2 \varepsilon_{kl,mm}) \tag{22}$$

where σ_{ij} and ε_{ij} are, respectively, stress and strain tensors; Q_{ijkl} are elastic constants; μ and l indicate internal length scales to be determined by experiment or microscopic models such as MD simulations. Pioneer studies have shown that above constitutive relations can predict the size-dependent phenomena of nano-scale structures well with the results by the MD simulation (Askes and Aifantis 2009) although the exact values of length scale parameters for different cases are still unknown. The equivalent form of Eq. (22) can be rewritten as follows

$$\mathrm{L}_{\mu}\sigma_{ij} = C_{ijkl}\mathrm{L}_{l}\varepsilon_{kl} \tag{23}$$

in which the linear operators are defined as

$$\mathrm{L}_{\mu} = (1-\mu^2\nabla^2), \mathrm{L}_{l} = (1-l^2\nabla^2) \tag{24}$$

where $\nabla^2 = \partial / \partial x^2$.

2.5 Equations of motion in terms of displacements

According to the nonlocal strain gradient stress-strain relation (Eq. (22)), the size-dependent equations of wave motion of the present anisotropic model can be expressed in terms of displacements by substituting Eq. (20) in Eqs. (16)-(19) as follows

$$\begin{aligned} \delta u: &\mathrm{L}_l\left\{A_{11}d_{11}u - B_{11}d_{111}w + kC_{11}d_1\theta + D_{13}d_1\varphi_z\right\} \\ &+\mathrm{L}_l\left\{E_{15}(kA'd_{11}\theta + d_{11}\varphi_z)\right\} = \mathrm{L}_{\mu}\left\{I_0\ddot{u} - I_1 d_1\ddot{w} + (kA')J_1 d_1\ddot{\theta}\right\} \end{aligned} \tag{25}$$

$$\begin{aligned} \delta w: &\mathrm{L}_l\left\{A_{11}^1 d_{111}u - B_{11}^1 d_{1111}w + kC_{11}^1 d_{11}\theta + D_{13}^1 d_{11}\varphi_z\right\} \\ &+\mathrm{L}_l\left\{E_{15}^1(kA'd_{111}\theta + d_{111}\varphi_z)\right\} - \mathrm{L}_{\mu}\left\{K_W w + (K_P + N_0)d_{11}w\right\} \\ &= \mathrm{L}_{\mu}\left\{I_0\ddot{w} + J_0\ddot{\varphi}_z + I_1 \mathrm{d}_1\ddot{u} - I_2 \mathrm{d}_{11}\ddot{w} + (kA')J_2 d_{11}\ddot{\theta}\right\} \end{aligned} \tag{26}$$

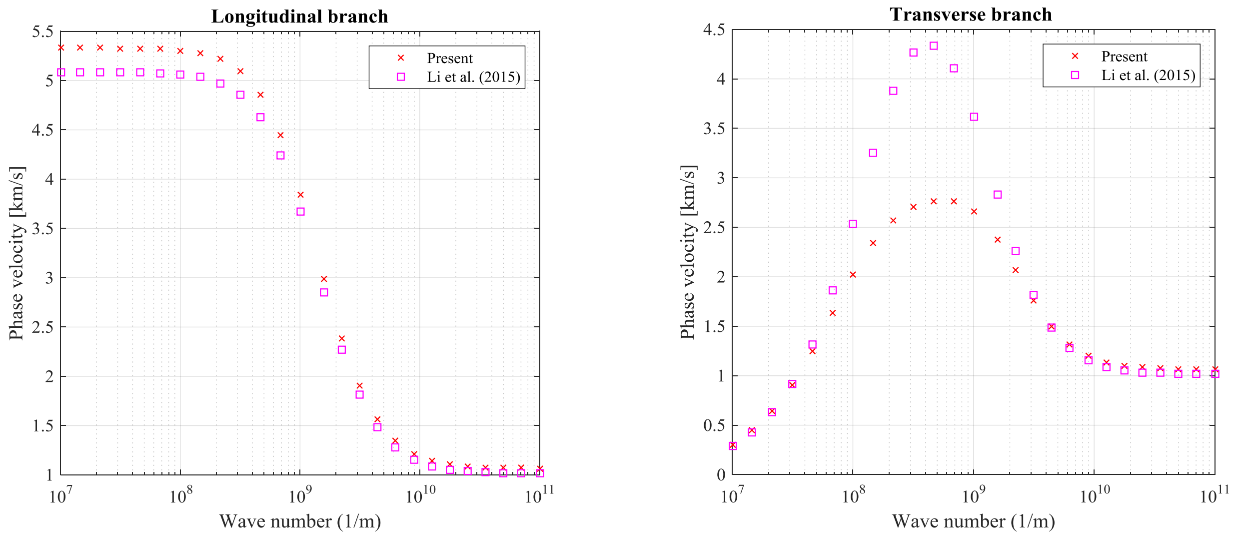
$$\begin{aligned} \delta\theta: &-k\left\langle \mathrm{L}_l\left\{A_{11}^2 d_1 u - B_{11}^2 d_{11}w + kC_{11}^2\theta + D_{13}^2\varphi_z\right\}\right. \\ &\left.+\mathrm{L}_l\left\{E_{15}^2(kA'd_1\theta + d_1\varphi_z)\right\}\right\rangle - kA'\left\langle \mathrm{L}_l\left\{A_{51}^4 d_1 u - B_{51}^4 d_{111}w\right\}\right. \\ &\left.+\mathrm{L}_l\left\{kC_{51}^4 d_{11}\theta + D_{53}^4 d_1\varphi_z + E_{55}^4(kA'd_{11}\theta + d_{11}\varphi_z)\right\}\right. \\ &= \mathrm{L}_{\mu}\left\{-J_1 kA'\ddot{u} + J_2 kA'd_{11}\ddot{w} - K_2(kA')^2 d_{11}\ddot{\theta}\right\} \end{aligned} \tag{27}$$

$$\begin{aligned} \delta\varphi: &\mathrm{L}_l\left\{A_{51}^4 d_{11}u - B_{51}^4 d_{111}w + kC_{51}^4 d_1\theta + D_{53}^4 d_1\varphi_z\right\} \\ &+\mathrm{L}_l\left\{E_{55}^4(kA'd_{11}\theta + d_{11}\varphi_z) - A_{31}^3 d_1 u + B_{31}^3 d_{11}w\right\} \\ &+\mathrm{L}_l\left\{-kC_{31}^3\theta - D_{33}^3\varphi_z - E_{35}^3(kA'd_1\theta + d_1\varphi_z)\right\} \\ &= \mathrm{L}_{\mu}\left\{J_0\ddot{w} + K_0\ddot{\varphi}_z\right\} \end{aligned} \tag{28}$$

Remark 1: *Up to now, the presented displacement field as well as shape function have not been considered to model the anisotropic materials, therefore the use of the employed model for triclinic beams can be a novel work in the open literature. Furthermore, the presented theory has only four-unknown variables for determining fourfold coupled (axial-shear-bending-stretching) wave propagation response of triclinic beams.*

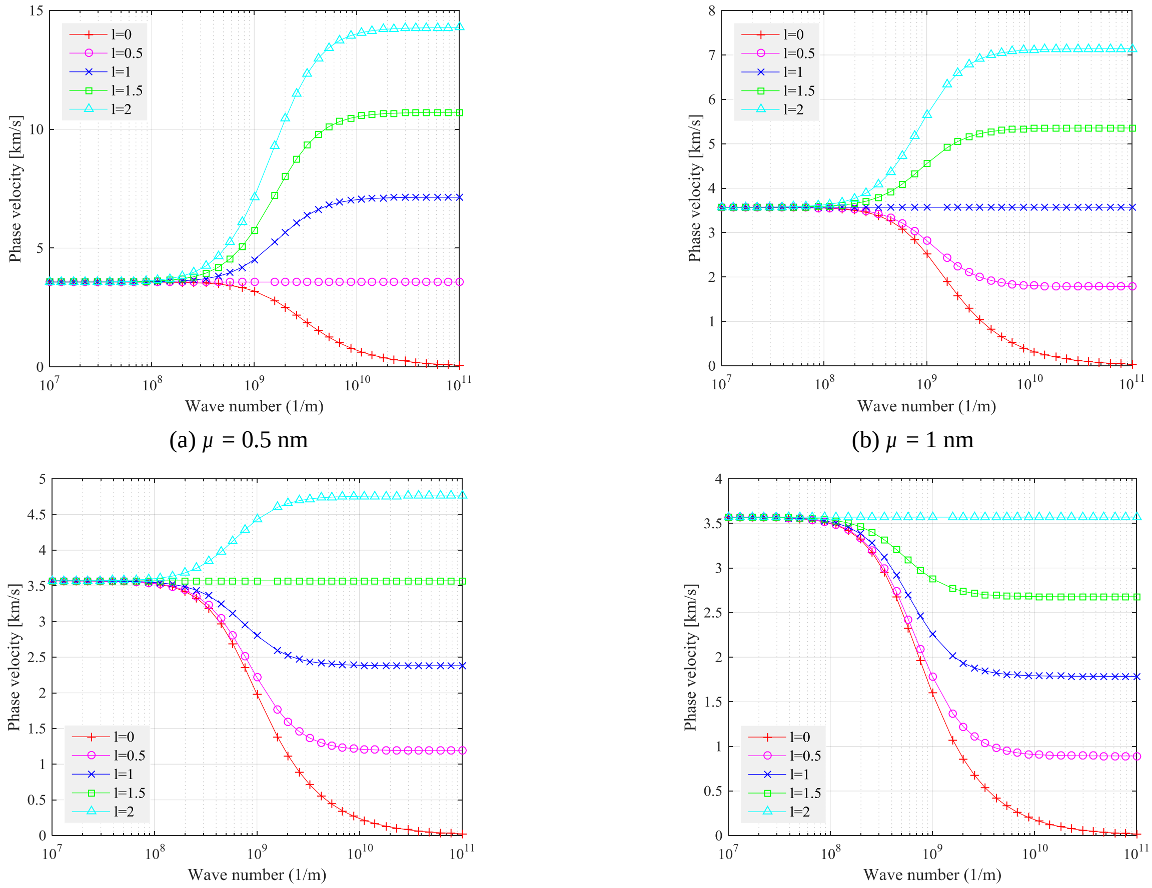
3. Solution procedure

A harmonic solution procedure is employed herein to define the analytical solutions of the partial differential equations (Eqs. (25)-(28)) using following series



(a) (b)

Fig. 2 Comparison of the longitudinal wave propagation for nanobeam (E = 70 GPa, ρ = 2707 kg/m^3, ν = 0.3, h = 20 nm, l = 0.2 nm, μ = 1 nm)



(a) μ = 0.5 nm (b) μ = 1 nm

(c) μ = 1.5 nm (d) μ = 2 nm

Fig. 3 Variation of phase velocity of triclinic nanobeam for different small scale parameters (h=2 nm)

$$\begin{aligned} u &= A_1 \exp(ix\beta - i\omega t) \\ w &= A_2 \exp(ix\beta - i\omega t) \\ \theta &= A_3 \exp(ix\beta - i\omega t) \\ \varphi_z &= A_4 \exp(ix\beta - i\omega t) \end{aligned} \tag{29}$$

where A_1-A_4 are the coefficients of wave amplitude which must be determined; β is the wave number along x-direction, $i = \sqrt{-1}$; and ω is eigenfrequency.

Substituting Eq. (29) into Eqs. (25)-(28) gives

$$\left([K] - \omega^2 [M]\right)\{\Delta\} = 0 \tag{30}$$

in which [K] and [M] are the stiffness matrix and the mass matrix, respectively, and the eigenvector can be given $\Delta = \{A_1, A_2, A_3, A_4\}^T$.

The dispersion relations of wave phenomena in triclinic nano-size beams can be developed by setting the following determinant to zero

$$\left|[K] - \omega^2 [M]\right| = 0 \tag{31}$$

The phase velocity can be defined as

$$C = \frac{\omega}{\beta} \tag{32}$$

Furthermore, the group velocity is defined by

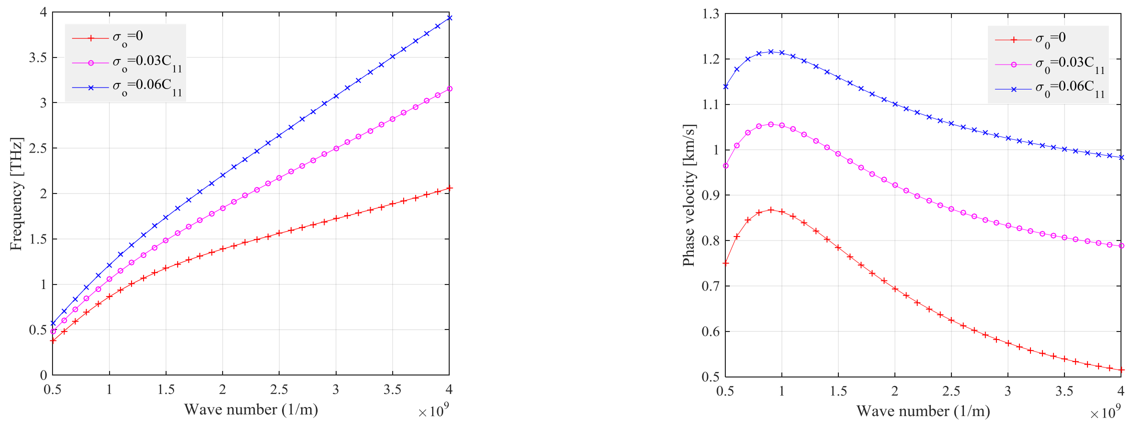
$$G = \frac{d\omega}{d\beta} \tag{33}$$

herein the group velocity $d\omega/d\beta$ is approximated with $\Delta\omega/\Delta\beta$ considering small $\Delta\beta$.

4. Numerical results

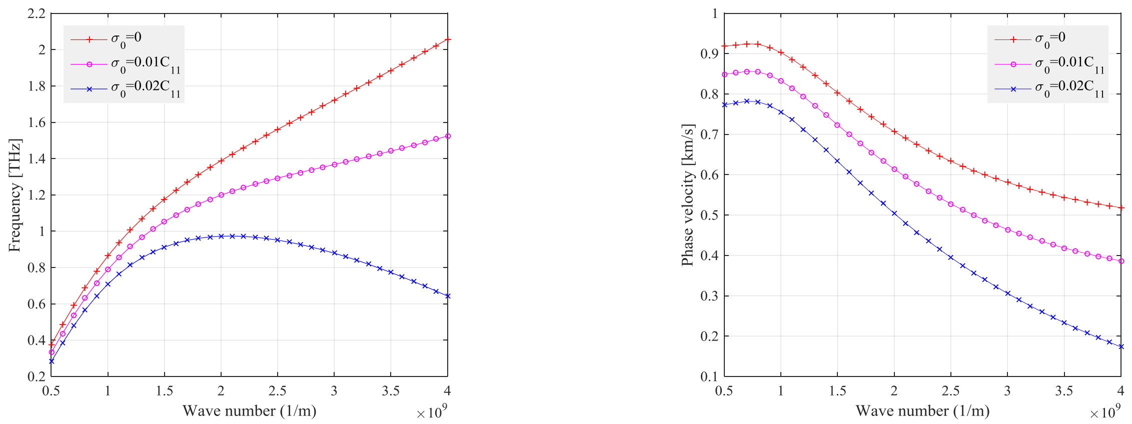
First of all, the accuracy of the present size-dependent anisotropic model is studied. As mentioned previously, there is no study on the wave propagation of triclinic nanobeam. Hence, Fig. 2 shows a comparison between the present methodology and (Li *et al*. 2015) in which a closed-form solution was reported for both longitudinal and transverse wave propagation of isotropic nanobeams. As can be seen, the results are in a good agreement, especially for the longitudinal branch of wave dispersion. In the continuation of the present investigation, the effects of the nonlocal and strain gradient parameters, initial stress, and the elastic substrate are reported as follows.

Size-dependent longitudinal wave characteristics of triclinic nanobeam is analyzed and the numerical results are figured in Fig. 3. It is demonstrated that the phase velocity of the nanobeam is decreased with increasing the nonlocal



(a) (b)

Fig. 4 Wave characteristics of triclinic nanobeam under initial tension loads (h = 2 nm, l = 0.2, μ = 1)



(a) (b)

Fig. 5 Wave characteristics of triclinic nanobeam under initial compression loads (h = 2 nm, l = 0.2, μ = 1)

parameter, however, rising the strain gradient parameter leads to an increment in the results of phase velocity. It worth mentioning that all response of wave phenomena is almost identical when the wave number is smaller than β < 0.1 1/nm. It means that the wave characteristics are not sensitive to small scale parameters when β < 0.1 1/nm. Moreover, it is interesting to note that when β > 0.1 1/nm, longitudinal wave relations are sensitive to small-scale parameters. This phenomenon was reported for different complex structures like shell, plate and also tube and beam in nano-dimension (see in Ref. (Li *et al.* 2015, Barati 2017a, Karami *et al.* 2018d, She *et al.* 2018)).

The impact of initial stress on the wave propgation of triclinic nanobeam under the tension loads is illustrated in Fig. 4 for transverse branch of wave dispersion. It can be seen that both wave frequencies and phase velocities are increased with increment in tension loads. It is important to note that the impact of the elastic substrate is omitted here.

Some vital works have focused on the mechanics of nanostructures under the compressive loads (Androulidakis *et al.* 2014, Xiang and Shen 2016, Karami *et al.* 2018g). Hence, considering a Winkler substrate modulus (k_W = 1.13*10^{18} N/m^3), the transverse branch of wave propagation in triclinic nanobeam for compression loads is studied and the results are figured in Fig. 5. It is observable that the wave dispersion curve of the nanobeam reduces with an increase in the compression loads. This phenomenon is reversed for the tension loads while it was proved for single layer graphene sheets in Ref. (Karami *et al.* 2018g). Particularly, it can be seen that the initial stress causes a key role in wave characteristics of nanostructures. This is for the sake of the compression load decreases the stiffness, however, the tension load provides a stiffness-hardening effect.

The effects of Winkler and Pasternak coefficients on the phase velocity of triclinic nanobeam for the proposed theory are demonstrated in Fig. 6. It should be noted that the phase velocity increased when the electric substrate coefficients growth. Moreover, the Pasternak coefficient has more effect on the results of the nanobeam in comparison with the Winkler one.

Fig. 7 plots to show the longitudinal and transverse wave frequencies as a function of scale factor $c = l/\mu$. The most notable feature is that the value of wave frequency strongly depends on the small-scale parameters. Further, it can be observed that the size-dependent effect is a stiffness-softening effect when the nonlocality value is bigger than the strain gradient size-dependency while it is a stiffness

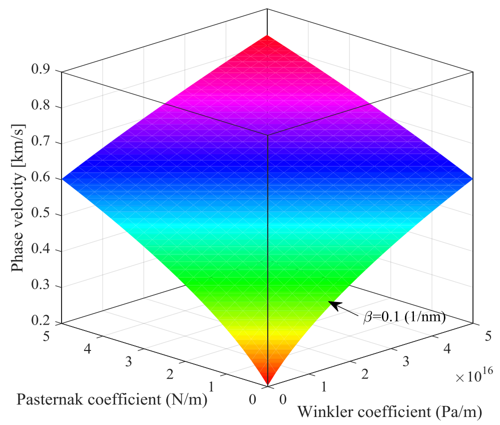


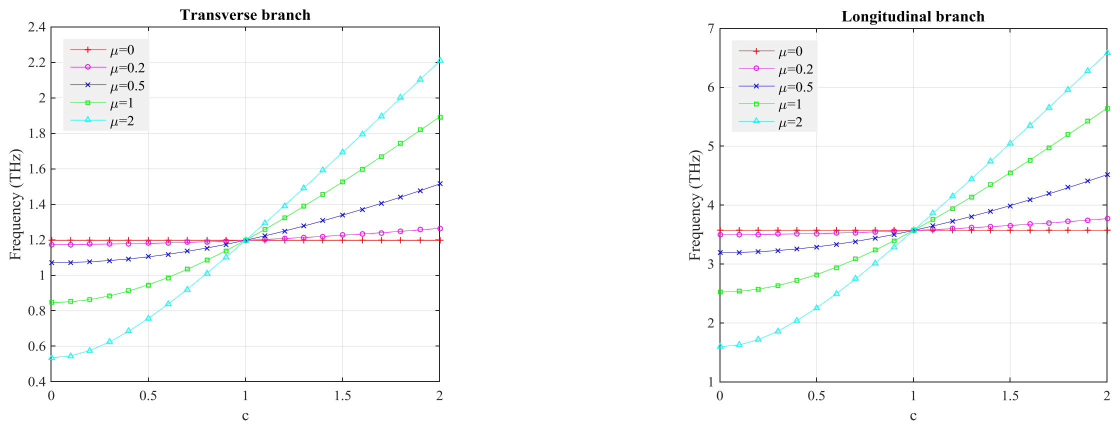
Fig. 6 Phase velocity variation of triclinic nanobeam for different Winkler-Pasternak coefficients (h = 2 nm, l = 0.2, μ = 1)

hardening effect when the relations among mentioned approaches are opposite ($l > \mu$). The size-dependent effect will not be observed when $l = \mu$. Similar phenomena have been reported for some investigations on the dynamics of nanostructures (Li *et al*. 2015, Karami *et al*. 2018g, She *et al*. 2018). In addition, it is noted that some authors (Arash and Wang 2012) present statement that for short nanobeams, there is no need to consider small-scale parameters and classical theories are sufficient. It is suggested to other researchers to investigate whether it is essential to capture size-effects for short nano-beams/tubes or not.

The possibility of replacing present anisotropic model by an isotropic one is studied and the results are plotted in Fig. 8. As can be observed in the present conditions of investigation, there is no prominent differences between the two models.

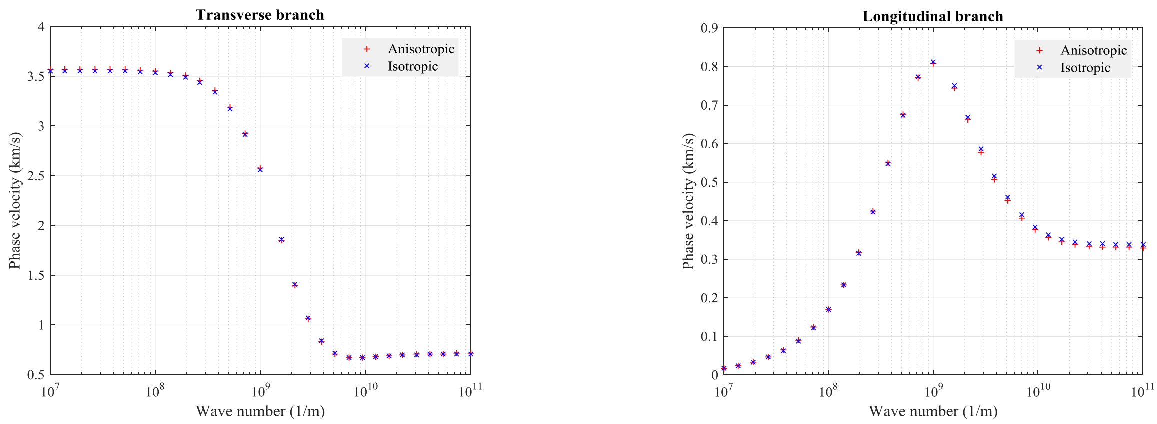
Remark 2: *in the current conditions, the elastic components of the triclinic matrix in comparison with isotropic one do not affect significantly the final trend of results. Thus, it is suggested to use the most simplified equations of wave motion, when the main focus of the analysis is just related to the size-dependent wave behavior of the nano-size triclinic system. So, the equations could disregard any kind of complex equations.*

Furthermore, to show the better accuracy of the present suggestion, the group velocities of studied triclinic nano-



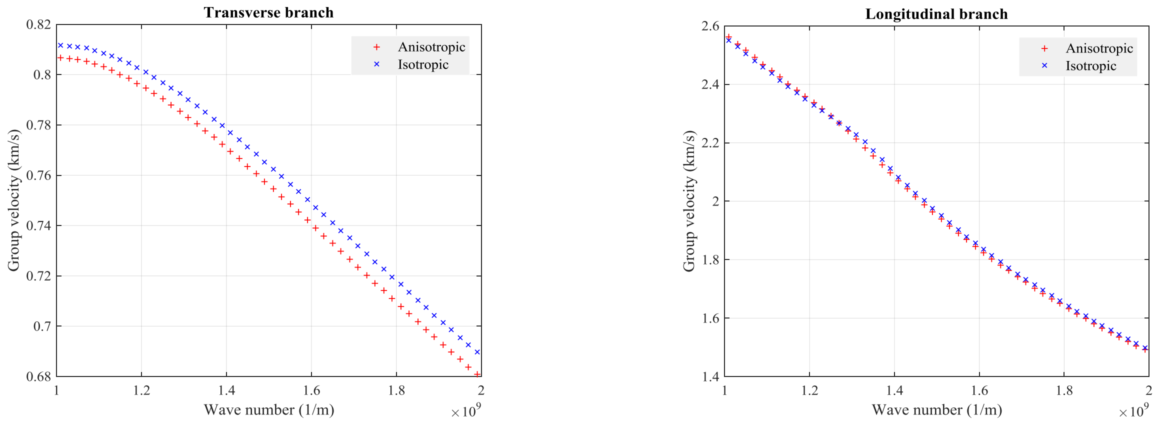
(a) (b)

Fig. 7 Variation of wave frequency as a function of scale factor $c = l/\mu$ (h = 2 nm)



(a) (b)

Fig. 8 Phase velocity of triclinic nanobeam with respect to two different types of stiffness matrix components (h = 2 nm, l = 0.2, μ = 1)



(a) (b)

Fig. 9 Group velocity of triclinic nanobeam with respect to two different types of stiffness matrix components (h = 2 nm, l = 0.2, μ = 1)

beam and isotropic one is extended and the results are plotted in Fig. 9.

5. Conclusions

In the current work, wave propagation characteristic of a triclinic nanobeam is analyzed using a size-dependent shear deformation theory including stretching effect. The equations of wave motion are obtained for the anisotropic materials based on a virtual work of the Hamiltonian principle. Then, these equations are solved for wave frequencies and phase velocities employing harmonic series. Finally, through some parametric study, the effects of different parameters such as nonlocal parameter, strain gradient parameter, tension and compression loads, and wave number on wave propagation behavior of triclinic nanobeam are studied. It is found that increasing the compression load causes a decrement in the wave dispersion while increasing the tension loads leads an increment in the results of wave phenomena. It means that the wave propagation can be tuned by choosing the appropriate values of the compression or tension loads. Furthermore, the phase velocities decreased with growing in nonlocality. In addition, the strain gradient parameter introduces a stiffness-hardening effect on the nanobeam and increases the phase velocities. Moreover, the phase velocity grows by increasing the elastic substrate coefficients. It is also found that the effect of the Pasternak substrate coefficient on the flexural dispersion curve is more significant than that Winkler substrate coefficient.

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