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Rigid plastic analysis for the seismic performance evaluation of steel storage racks

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Abstract. The aim of the paper is the prediction of the seismic collapse mode of steel storage pallet racks under seismic loads. The attention paid by the researchers on the behaviour of the industrial steel storage pallets racks is increased over the years thanks to their high dead-to-live load ratio. In fact, these structures, generally made by cold-formed thin-walled profiles, present very low structural costs but can support large and expensive loads. The paper presents a prediction of the seismic collapse modes of multi-storey racks. The analysis of the possible collapse modes has been made by an approach based on the kinematic theorem of plastic collapse extended to the second order effects by means of the concept of collapse mechanism equilibrium curve. In this way, the dissipative behaviour of racks is determined with a simpler method than the pushover analysis. Parametric analyses have been performed on 24 racks, differing for the geometric layout and cross-section of the components, designed in according to the EN16618 and EN15512 requirements. The obtained results have highlighted that, in all the considered cases, the global collapse mechanism, that is the safest one, never develops, leading to a dangerous situation that must be avoided to preserve the structure during a seismic event. Although the studied racks follow all the codes prescriptions, the development of a dissipative collapse mechanism is not achieved. In addition, also the variability of load distribution has been considered, reflecting the different pallet positions assumed during the in-service life of the racks, to point out its influence on the collapse mechanism. The information carried out from the paper can be very useful for designers and manufacturers because it allows to better understand the racks behaviour in seismic load condition.

Keywords: steel storage pallet racks; thin-walled members; seismic performance; collapse mechanism; TPCM

1. Introduction

An important area of the steel construction industry is represented by thin-walled steel components formed from coil or strips by cold-rolling processes (Pekoz and Winter 1973). These components are widely employed in the retail industries to realize frames used for storing goods and products. One of the most common frames are the adjustable selective storage pallet racks (in the following identified as “racks”), which are the main topic of the present paper. In Fig. 1 the typical racks layout is reported together with its principal components. The vertical elements, named uprights, are jointed with lacings in the transversal direction forming a set of trussed columns and are connected, in the longitudinal direction, by pairs of pallet beams directly supporting the stored goods. The pallets positions change continuously during the in-service life of the racks, leading to very different load situations, like, for example, the one presented in Fig. 1 (load only on the top storage level).

Consequently, owing to the great influence of the masses on the dynamic response of these frames, the performance in seismic zones is strictly dependent on the weight and position of the pallet units. Generally, the condition of fully loaded frame governs the design but different load conditions, with great masses eccentricity, could lead to worse condition (Petrovcic and Kilar 2012). Regarding the seismic behaviour, racks can be considered as moment resisting frames in the down-aisle direction where pallet beams and uprights remain in elastic range and are not able to develop plastic hinges (class 3 or 4 of the EC3 criteria (CEN 2004)) and the plasticity can be concentrated in the beam-to-column and base-plate joints (Rafiqul Haque and Alam 2013). Moreover, this study is valid only for the down-aisle direction. Beam-to-column joints are created by means of tabs and/or hooks located in the brackets at the end of pallet beam, which are hooked in the upright perforation (Shah *et al*. 2012, El Kadi *et al*. 2017). This system is efficient if the owner of the warehouse wants to change the position of the pallet beam during the in-service life. Every type of connections (beam-to-column or base-plate joint) is characterized by a non-linear response that must be experimentally evaluated by the manufacturers (Baldassino and Zandonini 2011), to obtain data for the structural design. Also, quite complex finite element models (Sangle *et al*. 2014, Cardoso and Rasmussen 2016) can be used for the joints characterization (Mohan and Vishnu 2013, Mohan *et al*. 2015, Shah *et al*. 2016), if the models are preliminary validated by an

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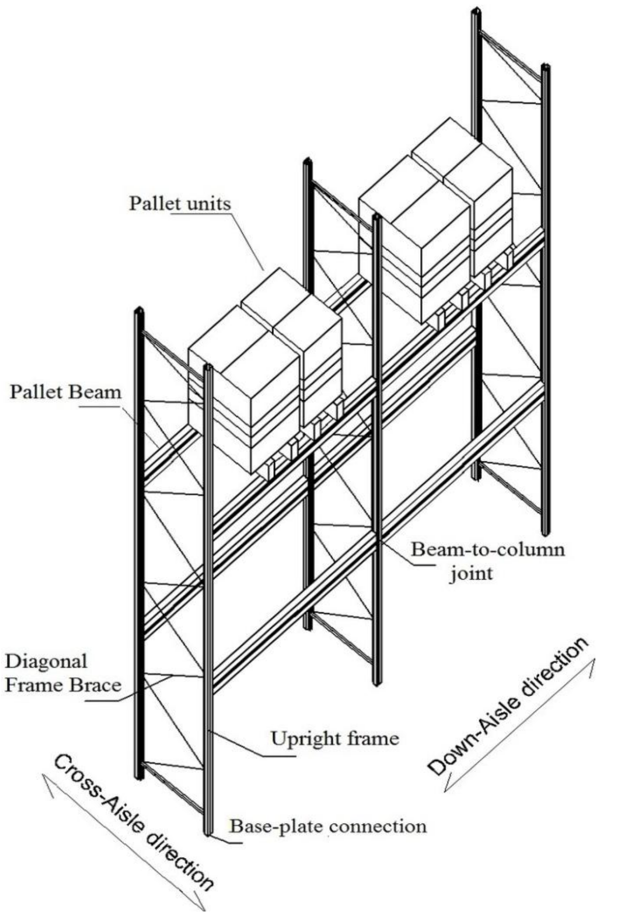


Fig. 1 Typical pallet rack configuration and its components

exhaustive number of experimental activities. In Fig. 2 is depicted an example of the beam-to-column joint cyclic behavior and, in particular, the relationship between the non-dimensional bending moment (m) corresponding to the moment of the bracket of the connection against the elastic resistance moment of the beam and the relative joint rotation (Φ) between the upright and the pallet beam end. It needs to be highlighted that joints provide a limited degree of rotational stiffness and modest flexural resistance if compared with the flexural strength and stiffness of pallet beams (Bernuzzi and Castiglioni 2001, Bernuzzi and Simoncelli 2016, Bernuzzi *et al*. 2017). A direct consequence is the high flexibility of racks to lateral loads that brings to high values of the fundamental period of vibration (T1), up to 3s (typically observed in high-rise and tall steel buildings) (RFCS 2007, Bernuzzi and Simoncelli 2017).

A large number of researches were made in recent years for the study of the behaviour of racks under seismic actions (Filiatrault *et al*. 2006, Bernuzzi and Simoncelli 2016, Bernuzzi *et al*. 2017), especially in the contest of the SEISRACK1 (RFCS 2007) and SEISRACK2 (Castiglioni *et al*. 2014) projects, whose results have remarkably contributed to the development of the actual European code for the seismic design, the EN16618 (CEN 2016). Despite the great effort of these researches there are still several lacks in the standard that need further investigation. For example, the verification formula used for the stability and for the resistance checks of rack elements (postponed to the EN15512 (CEN 2009)) does not consider the bimoment influence when non-symmetric members are employed, despite the great number of researches (Bernuzzi *et al*. 2014, 2016, Dey and Talukdar 2016, Rasmussen and Gilbert 2013) that have demonstrated his non-negligible influence. Another important problem is that these frames even if designed as “seismic frames” and built in seismic areas, may have a local failure mode, like the “soft storey” collapse mode, as demonstrated by push-over experimental tests (Castiglioni *et al*. 2014).

In the framework of a research currently in progress between the Politecnico di Milano, the University of Pavia and the University of Salerno on the seismic behaviour of rack structures, this work is focused on the prediction of the collapse mode of steel racks under seismic loads. As it is known, many dangerous collapse modes can affect structure under destructive seismic events. The most dangerous is the one involving only the top and bottom sections of uprights of a single storey. It is the so called “soft storey” mechanism that together with type-1 and type-2 mechanisms is an undesired mechanism that is preferable to avoid in order to assure an adequate structural safety. In Fig.

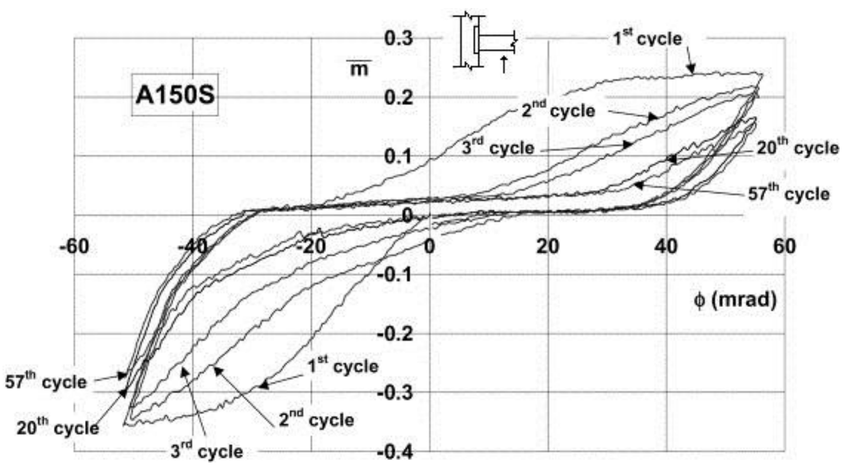


Fig. 2 Definition of the moment-rotation curve for a typical beam-to-column rack joint (Bernuzzi and Castiglioni 2001)

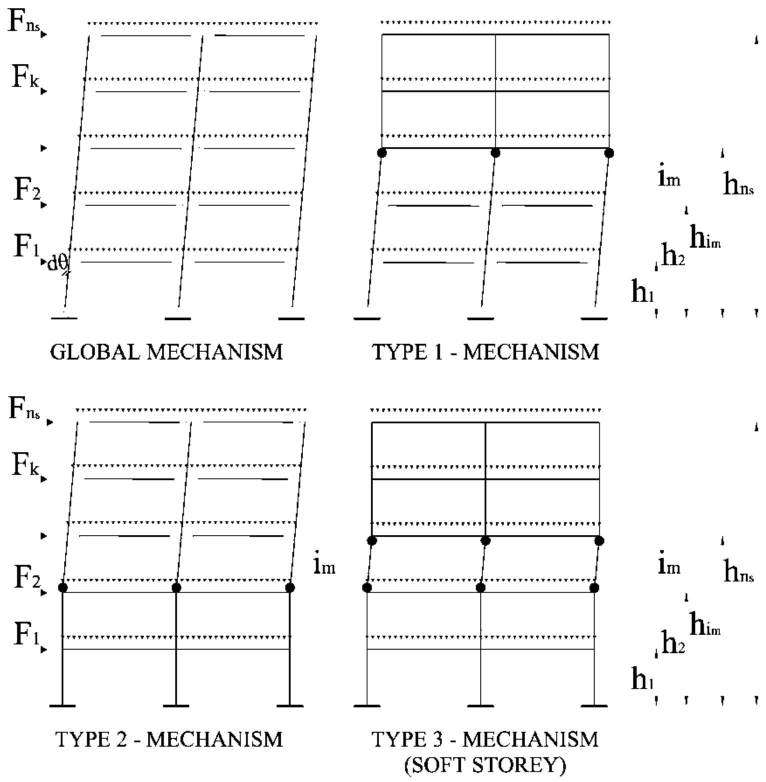


Fig. 3 Collapse mechanism of steel storage pallet racks

3 undesired collapse mechanisms namely type-1, type-2 and type-3 are reported where the rotational springs represent dissipative zones (beam-to-column rack joint) while solid circles represent the achievement of the elastic limit of the upright cross section. In addition, in Fig. 3 also the global type mechanism, which is a type-2 mechanism extended to all storeys, is depicted. The type-1, type 2 and type 3 mechanisms are considered undesired because they are different from the global mechanism. It is due to be underlined that in the case of steel storage pallet racks, all the undesired mechanism cannot really develop. In fact, after the achievement of the elastic limit of the upright cross-sections in case of class 3 section or after the achievement of elastic buckling in case of class 4 section, the collapse is immediately reached due to the lack of ductility (D'Aniello *et al*. 2014, 2015, Calderoni *et al*. 2009), and no mechanism can develop. On the contrary, the only mechanism which can develop is the global one because it is characterized by the fact that the uprights remain in the elastic range, while beam-to-column and base-plate joints (the only parts of the structure that have available ductility) are involved in yielding. So that, in this case, the development of a global mechanism is even more important than in an "ordinary" moment resisting frames (MRFs). In fact, while in the case of a MRFs some resources of ductility are used also when the collapse is characterized by partial mechanism (even if not all the available ductility is used), in the case of steel storage racks if a partial mechanism (i.e., a mechanism different from global one) starts developing, the collapse is immediately reached due to the absence of available ductility of involved members (upright cross-sections of class 3 or 4).

Regarding this issue, the approach presented in this paper has the scope to provide a simple technique to preliminarily determine the collapse mechanism involving a given rack under seismic loads. This approach is based on the kinematic theorem of plastic collapse extended to the concept of collapse mechanism equilibrium curves that accounts for the second order vertical loads effects. It has been already proposed with the name of Theory of Plastic Mechanism Control (TPMC) (Montuori *et al*. 2015) in the framework of the design of all the steel structure typologies (Longo *et al*. 2014a, b, Montuori *et al*. 2015, Piluso *et al*. 2019). In fact, TPMC assures to design structures exhibiting a collapse mechanism of global type (Montuori *et al*. 2015); an ambitious design goal that modern seismic code such as FEM 10.2.08 recommendations (CEN 2010) and EN16681 provisions (CEN 2016), are not able to assure by means of the so-called beam-to-column hierarchy criterion approach.

In this paper, TPMC is used as a verification tool, which can preliminarily check the collapse behavior of racks. In this way push-over analyses, that usually are the only effective tool to determine the actual dissipative behavior of structures both in terms of collapse mechanism developed and in terms of collapse load multiplier, can be substituted by a simpler approach which can be easily carried out by hand calculation. The validation is made by comparing the push-over analysis results with those provided by the proposed approach.

As the position of vertical load can play an important rule during a seismic event, also their distribution is investigated in this paper to point out the more dangerous load positions that should be avoided to preserve the rack during a seismic event.

2. Verification approach by means of TPMC

The approach herein proposed is based on the upper bound theorem of plastic collapse that is used as a tool to verify the seismic capacity of steel racks (Montuori *et al*. 2015). According to the theory of limit analysis, the assumption of a rigid-plastic behaviour is adopted. It means that the attention is focused on the condition that the structure exhibits in the collapse state by neglecting each intermediate condition. However, the simple application of the kinematic theorem of plastic collapse is not sufficient because high horizontal displacements occur before the complete development of the kinematic mechanism. These displacements give rise to significant second order effects that cannot be neglected in the seismic design of structures, particularly in case of steel storage pallet racks whose seismic behavior is deeply affected by large elastic excursions. In light of this, the concept of mechanism equilibrium curve has to be introduced to take in account the vertical load influence on second order effect. The mechanism equilibrium curve is represented by a straight line whose intercept with the vertical axis in a Cartesian diagram is the first order collapse mechanism multiplier $\alpha 0$, while its slope is represented by the parameter γ. The expression of the linearized collapse mechanism equilibrium curve is given by (Montuori *et al*. 2015)

$$\alpha = \alpha_0 - \gamma\delta \tag{1}$$

where α is the multiplier of horizontal forces and δ is the top sway displacement of the structure.

Within the framework of a kinematic approach, for any given collapse mechanism (Fig. 3), the mechanism equilibrium curve can be easily derived requiring that external work is equal to the internal work due to the plastic hinges involved in the collapse mechanism, provided that the external second-order work due to vertical loads is also evaluated (Mazzolani and Piluso 1997).

In fact, with reference to the global mechanism, it is easy to recognize that, for a virtual rotation $d\theta$ of the uprights involved in the mechanism, the internal work can be expressed, as

$$W_i = \left[\sum_{i=1}^{n_c} M_{base,i} + \sum_{k=1}^{n_s}\sum_{j=1}^{n_b}\left(M^{(+)}_{btc,jk} + M^{(-)}_{btc,jk}\right)\right] d\theta \tag{2}$$

where ns is the number of storeys, *nb* is the number of bays, *nc* is the number of columns $M_{base,i}$ is the base connection plastic moment of *i-th* column while $M^{(+)}_{btc,jk}$ and $M^{(-)}_{btc,jk}$ are the sagging and hogging moment of beam-to-column joints of *j-th* bay at *k-th* storey, respectively.

The external work due to the horizontal forces and to the uniform load acting on the beams is given by two quantities: the external work due to seismic horizontal forces (first term of Eq. (3)) and the second-order work due to vertical loads (second term of Eq. (3))

$$W_e = \alpha \sum_{k=1}^{n_s} F_k h_k d\theta + \frac{\delta}{h_{n_s}} \sum_{k=1}^{n_s} V_k h_k d\theta \tag{3}$$

where F_k and h_k are, respectively, the seismic force applied at k-th storey and the k-th storey height with respect to the foundation level, h_{ns} is the value of h_k at the top storey, δ is the top sway displacement and V_k is the total vertical load acting at *k-th* storey.

In order to compute the slope of the mechanism equilibrium curve, it is necessary to evaluate the second-order work due to vertical loads. With reference to Fig. 4, it can be observed that the horizontal displacement of the *k-th* storey involved in the generic mechanism is given by $u_k = r_k \sin\theta$, where r_k is the distance of the *k-th* storey from the centre of rotation C and θ the angle of rotation. The top sway displacement is given by $\delta = h_{ns} \sin\theta$.

The relationship between vertical and horizontal virtual displacements is given by $dv_k \tan\theta \approx du_k \sin\theta$. It shows that, as the ratio dv_k / du_k is independent of the storey, vertical and horizontal virtual displacement vectors have the same shape. In fact, the virtual horizontal displacements are given by $du_k = r_k \cos\theta d\theta \approx r_k\, d\theta$, where r_k defines the shape of the virtual horizontal displacement vector, while the virtual vertical displacements are given by $dv_k = \delta/h_{ns}\, r_k\, d\theta$ and, therefore, they have the same shape r_k of the horizontal ones. It can be concluded that

$$dv_k = \frac{\delta}{h_{n_s}} h_k d\theta \tag{4}$$

where dv_k is the vertical virtual displacement occurring at *k-th* storey.

By equating the internal work (Eq. (2)) to the external one (Eq. (3)), the following relationship is obtained

$$\alpha = \frac{\sum_{i=1}^{n_c} M_{base,i} + \sum_{k=1}^{i_m-1}\sum_{j=1}^{n_b}\left(M^{(+)}_{btc,jk} + M^{(-)}_{btc,jk}\right)}{\sum_{k=1}^{n_s} F_k h_k} - \frac{1}{h_{n_s}} \frac{\sum_{k=1}^{n_s} V_k h_k}{\sum_{k=1}^{n_s} F_k h_k}\delta \tag{5}$$

It is immediately recognized that the mechanism equilibrium curve is a straight line which can be generally expressed in the form

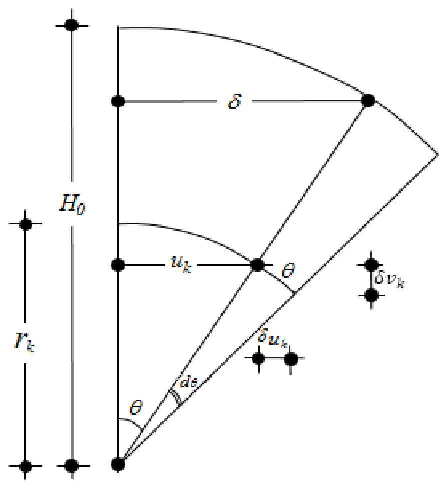


Fig. 4 Second order vertical displacements

$$\alpha = \alpha_{0,i_m}^{(t)} - \gamma_{i_m}^{(t)}\delta \quad \text{with} \quad \begin{matrix} t = 1, 2, 3 \\ i_m = 1, \dots, n_s \end{matrix} \tag{6}$$

where t is the index of undesired mechanisms ranging from 1 to 3 and i_m is the mechanism index equal to the number of storeys. Being all the connection plastic moments, upright sections and gravity loads acting on pallet beams completely defined quantities, it is possible to provide the kinematically admissible multiplier of horizontal forces

$$\alpha_{0,i_m}^{(1)} = \frac{\sum_{i=1}^{n_c} M_{base,i} + \sum_{k=1}^{i_m-1}\sum_{j=1}^{n_b}\left(M_{btc,jk}^{(+)} + M_{btc,jk}^{(-)}\right) + \sum_{i=1}^{n_c} M_{c,ii_m}}{\sum_{k=1}^{n_s} F_k h_k + h_{i_m}\sum_{k=i_m+1}^{n_s} F_k} \tag{7}$$

for type-1 mechanism

$$\alpha_{0,i_m}^{(2)} = \frac{\sum_{i=1}^{n_c} M_{c,ii_m} + \sum_{k=i_m}^{n_s}\sum_{j=1}^{n_b}\left(M_{btc,jk}^{(+)} + M_{btc,jk}^{(-)}\right)}{\sum_{k=i_m}^{n_s} F_k\left(h_k - h_{i_m-1}\right)} \tag{8}$$

for type-2 mechanism

$$\alpha_1^{(3)} = \frac{\sum_{i=1}^{n_c} M_{base,i} + \sum_{i=1}^{n_b} M_{c,i1}}{h_1 \sum_{k=1}^{n_s} F_k} \tag{9}$$

and for type-3 mechanism, where F_k and h_k are, respectively, the seismic force applied at *k-th* storey and the *k-th* storey height with respect to the foundation level; $M_{c,i.i_m}$ is elastic limit moment for class 3 members of *i-th* upright of *k-th* storey reduced due to the contemporary action of the axial force; n_c, n_b and n_s are the number of columns, bays and storeys, respectively.

Regarding the slope γ of the mechanism equilibrium curve, they are given by

$$\gamma_{i_m}^{(1)} = \frac{1}{h_{i_m}} \frac{\sum_{k=1}^{i_m} V_k h_k + h_{i_m}\sum_{k=i_m+1}^{n_s} V_k}{\sum_{k=1}^{i_m} F_k h_k + h_{i_m}\sum_{k=i_m+1}^{n_s} F_k} \tag{10}$$

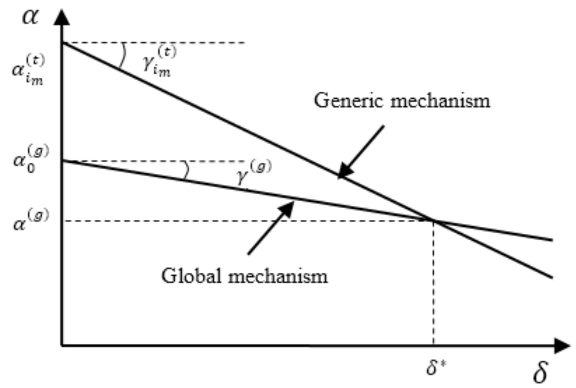


Fig. 5 Collapse mechanism equilibrium curves

for type-1 mechanism

$$\gamma_{i_m}^{(2)} = \frac{1}{h_{n_s} - h_{i_m-1}} \frac{\sum_{k=i_m}^{n_s} V_k\left(h_k - h_{i_m-1}\right)}{\sum_{k=i_m}^{n_s} F_k\left(h_k - h_{i_m-1}\right)} \tag{11}$$

for type-2 mechanism

$$\gamma_{i_m}^{(3)} = \frac{1}{h_{i_m} - h_{i_m-1}} \frac{\sum_{k=i_m}^{n_s} V_k}{\sum_{k=i_m}^{n_s} F_k} \tag{12}$$

where V_k is the total vertical load acting at *k-th* storey.

The collapse mechanism equilibrium curves can be easily represented in a Cartesian diagram (α-δ). The curve that is located below all the other ones is the one governing the collapse mechanism. In other words, if the curve related to the global mechanism is located below all the other ones until the achievement of a generic displacement δ^* the collapse global mechanism develops. However, if one of the other curves representative of a generic undesired mechanism is located below the one related to the global mechanism for a displacement higher than δ^*, the undesired mechanism could start to develop in the structure. In fact, because of the elastic deformation, when δ is equal to δ^* the global mechanism could be not completely developed yet, in this case the mechanism whose equilibrium curve is located below the global one for $\delta > \delta^*$ could start

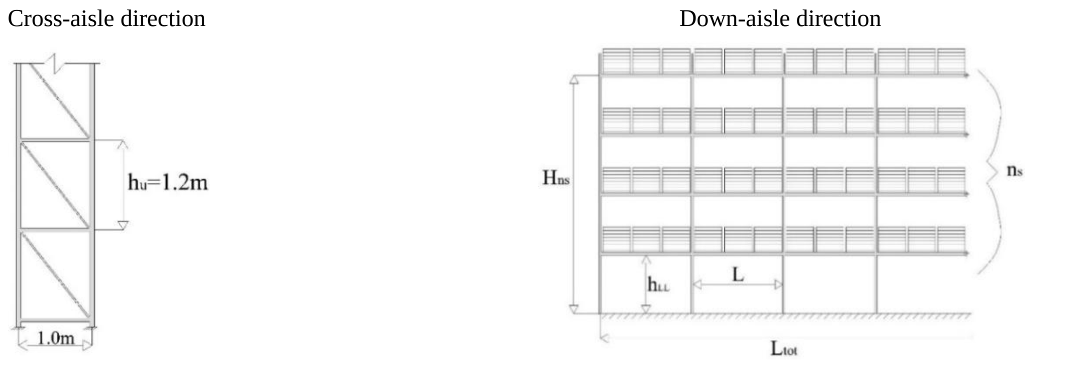


Fig. 6 Detail of considered racks: (a) Transversal view; (b) Longitudinal view

Table 1 Cross-section properties

	Rectangular hollow section 80×80×1.78	Rectangular hollow section 80×120×1.78
Name	M	D
A [mm^2]	560	700
W_y [mm^3]	14205	24400
W_z [mm^3]	14205	19650
I_y [mm^4]	568207	1463520
I_z [mm^4]	568207	786059
I_t [mm^4]	851870	1549664

developing (Fig. 5). This consideration is of paramount importance to understand the results of parametric analyses reported in the next paragraphs.

3. Parametric study on typical racks

A parametric analysis has been developed for medium-rise double-entry racks, unbraced in the down-aisle direction with different number of bays (2.78 m length each) and storage level. The depth is 1 m and upright frames present Z-panels guaranteeing stability in cross-aisle direction. Fig. 6 presents the longitudinal and transversal view of the typical layout with indicated the number of storage level (n_s), the interstorey height (h_{LL}), the bays length (L) and the Z-panel height (h_u).

Two bi-symmetric upright cross-sections (identified in the following as M_ and D_ types) have been considered (Table 1), which belong to class 3 of Eurocode 3, i.e., their behavior is purely elastic. Regarding the gross cross-section, the value of the area, second moment of area and section moduli are reported in Table 1, together with the Saint Venant torsion constant. It can be notice that, in order to avoid the warping influence, in this first study bi-symmetric cross-section have been used, but the results can be extended to all the cross-section typologies. Pallet beams are comprised of rectangular hollow sections (100×50×3 mm RHS) and square hollow sections (35×35×2 mm SHS) are used for the lacings of the upright frames. All these structural components are in S355 steel grade (CEN 2004), with a yielding strength of 355 MPa.

To make an exhaustive parametric analysis, 24 racks with different geometry and loads have been considered. The load is considered as a uniform distributed load acting

Table 2 Main characteristic of the considered frames

Name	**Uprigth section**	**n. of storeys**	**n. of bays**	$\mathbf{T_1}$	α_{cr}	**L**	h_{LL}	q_d	M_{base}	M_{btc}^{+}	M_{btc}^{-}
[-]	[-]	[-]	[-]	[*s*]	[-]	[*mm*]	[*mm*]	[*N/mm*]	[*kNm*]	[*kNm*]	[*kNm*]
M_4_2	M_	4	2	2.843	3.51	2780	1800	3.4	3.60	2.47	3.09
M_4_4			4	3.023	3.18	2780	1800	3.4	3.60	2.47	3.09
M_4_6			6	3.093	3.06	2780	1800	3.4	3.60	2.47	3.09
M_4_8			8	3.131	3.00	2780	1800	3.4	3.60	2.47	3.09
M_5_2		5	2	2.899	3.26	2780	1400	3.9	3.60	2.47	3.09
M_5_4			4	3.073	2.97	2780	1400	3.9	3.60	2.47	3.09
M_5_6			6	3.141	2.88	2780	1400	3.9	3.60	2.47	3.09
M_5_8			8	3.176	2.84	2780	1400	3.9	3.60	2.47	3.09
M_6_2		6	2	2.781	3.44	2780	1200	3.6	3.60	2.47	3.09
M_6_4			4	2.927	3.16	2780	1200	3.6	3.60	2.47	3.09
M_6_6			6	2.983	3.06	2780	1200	3.6	3.60	2.47	3.09
M_6_8			8	3.013	3.02	2780	1200	3.6	3.60	2.47	3.09
D_4_2	D_	4	2	2.926	3.48	2780	1800	5.8	6.25	3.42	4.28
D_4_4			4	3.083	3.21	2780	1800	5.8	6.25	3.42	4.28
D_4_6			6	3.143	3.12	2780	1800	5.8	6.25	3.42	4.28
D_4_8			8	3.175	3.07	2780	1800	5.8	6.25	3.42	4.28
D_5_2		5	2	2.773	3.60	2780	1400	5.8	6.25	3.42	4.28
D_5_4			4	2.906	3.34	2780	1400	5.8	6.25	3.42	4.28
D_5_6			6	2.957	3.25	2780	1400	5.8	6.25	3.42	4.28
D_5_8			8	2.984	3.21	2780	1400	5.8	6.25	3.42	4.28
D_6_2		6	2	2.795	3.56	2780	1200	5.6	6.25	3.42	4.28
D_6_4			4	2.919	3.32	2780	1200	5.6	6.25	3.42	4.28
D_6_6			6	2.966	3.23	2780	1200	5.6	6.25	3.42	4.28
D_6_8			8	2.991	3.19	2780	1200	5.6	6.25	3.42	4.28

on each pallet beam. In Table 2 all the considered parameters have been reported: upright cross-sections, number of storeys, number of bays, the fundamental period of vibration (T_1), the elastic load multiplier (α_{cr}), bay length (L), interstorey height (h_{LL}), vertical loads (W_d), plastic moment of base connection ($M_{base,i}$) and sagging and hogging moment of beam-to-column joints (M_{btc}). As already stated, the non-linear moment-rotation behaviour of the connections, as well as the influence of axial load in the moment resistance of base connections, should be assessed by means of experimental tests. Although experimental tests are mandatory for racks design, this research investigates the accuracy of a numerical methodology to assess the seismic collapse modes, rather than the seismic design of existing racks. Therefore, the aim is to compare rigid plastic and pushover analyses on a set of typical rack configurations with acceptable parameters and connections selected based on authors' experience, avoiding an expensive campaign of experimental tests which will not influence the comparison of the numerical results of this study.

More in detail, the elastic critical load multipliers (α_{cr}) have been obtained from a global elastic buckling analysis of each frames, by using the academic software Śiva (Gabbianelli 2016). From Table 2 it can be noted that the associated values are very low, showing great flexibility of the frames and the relevance played by the second-order effects, that cannot be avoided in the structural design of the selected frames. The high flexibility of the selected frames is underlined by the reported long values of the fundamental period of vibration (ranging from 2.84s to 3.18s). As required by the European Standards, these values have been obtained from a second-order modal analysis, which is a classical modal analysis that considers also the non-linear geometric effects.

As expected, both eigenvectors associated to T_1 and to α_{cr} involve always displacements in the down-aisle direction. With the increasing of the number of bays the period of vibration increases, and the buckling multiplier decrease. It can be remarked that, the considered geometries and cross-section derive strictly from existing racks, designed to meet the seismic performance request by the European code for a design peak ground acceleration equal to 0.15 g and soil category type A.

4. Results of parametric analyses

The verification approach proposed has been applied to check the seismic performances of the racks whose main characteristics are reported in Table 2. In particular, both first order, (Eqs. (7), (8) and (9)) and second order, (Eqs. (10), (11) and (12)) collapse mechanism multiplier have been provided for each structure in order to define the collapse mechanism equilibrium curve. The storey force distribution has been assumed proportional to the mass. It is important to underline that only the seismic force distribution affects the results while the base shear value affects the entity of collapse mechanism multipliers only.

As an example of the results obtained from the analyses Table 3 can be observed, where with reference to the

Table 3 First and second order collapse mechanism multipliers for M_4_4 steel storage rack

Storey (im)	M_4_4 Type-1 $\alpha_{0.1}$ (-)	γ_1 (1/cm)	Type-2 $\alpha_{0.2}$ (-)	γ_2 (1/cm)	Type-3 $\alpha_{0.3}$ (-)	γ_3 (1/cm)
1	4.9316	0.1693	3.9899	0.0353	4.9316	0.1693
2	3.9632	0.0779	5.229	0.0423	6.7258	0.141
3	3.8957	0.0488	7.3551	0.0577	8.9613	0.1209
4	4.2424	0.0353	14.3067	0.1058	16.2287	0.1058

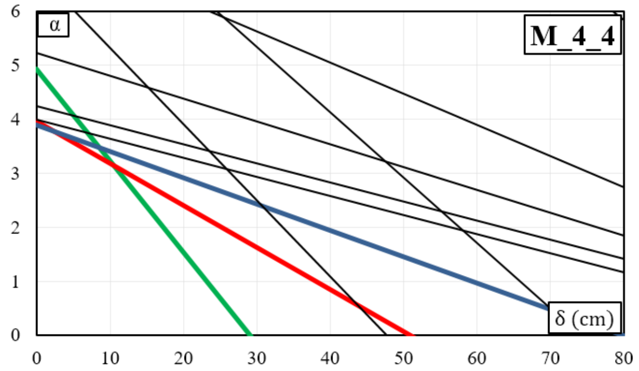


Fig. 7 Collapse mechanism equilibrium curves for D_4_4 steel storage rack

M_4_4 rack only, are reported the collapse mechanism multipliers. In addition, a significant number of collapse mechanism equilibrium curves is reported in Fig. 7 for the same rack, where the sequence of collapse mechanism is pointed out by the colours. The blue line represents the first mechanism shown by the structure because it corresponds to the lower collapse mechanism multiplier. The red line corresponds to the second mechanism exhibited while the green one is related to the final mechanism the structure achieves at the collapse state, in fact, the green curve is located below the other curves. This sequence means that, in a first time, for lower displacement, the M_4_4 rack starts exhibiting a type-1 collapse mechanism at the 3° storey; after that, a type-1 mechanism at the 2° storey starts developing until the achievement of a type-1 mechanism at first storey, that ends up being the final mechanism the structure collapse with. In addition, the same colours used for the curves of Fig. 7 are reported in Table 3 in order to associate the curves with the numerical values

The validation of the approach has been performed by using an academic software, Śiva (Gabbianelli 2016), in order to check the actual mechanism developed by the racks through push-over analyses. Śiva has been already adopted for numerical-experimental works (Bernuzzi *et al.* 2017, Gabbianelli *et al.* 2017), where experimental pushover tests have been performed on steel storage pallet racks, demonstrating a good agreement between the experimental and numerical results.

Push-over analyses have been carried out in displace-ment control accounting for second-order effects. They

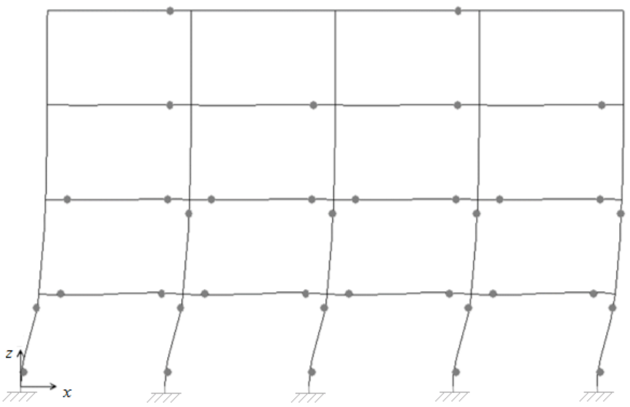


Fig. 8 Pattern of hinges from Śiva for M_4_4 rack

have been performed for all the considered racks but, for sake of shortness, the results in terms of pattern of plastic hinges (collapse mechanism) are herein reported only for the M_4_4 rack. In fact, in Fig. 8, the Śiva screenshot shows that the mechanism actually developed is a type-1 mechanism at first storey as predicted by the proposed approach. Furthermore, other hinges at storey 2° and 3° testify that before the development of type-1 mechanism at first storey, the racks tended to develop a type-1 mechanism at 3° storey followed by the development of a type-1 mechanism at 2° storey as testified by collapse mechanism equilibrium curves reported in Fig. 7.

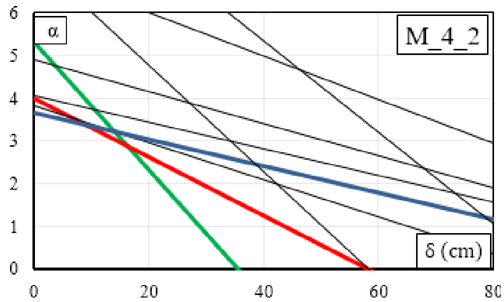
In Tables 4-5 all the results obtained from the analyses are reported, for both M_ and D_ cross-section, respectively.

From these tables, it can be observed that the global mechanism, that is the most favourable one, never develops in all the considered cases. In fact, even if a global mechanism begins developing (see cases M_4_2 in Table 4 and D_5_2 in Table 5), it is immediately followed by other unfavourable mechanisms. In particular, in the aforementioned cases, the final mechanism is the soft storey mechanism at storey 1, that is the most dangerous at all. It means that the considered storage racks, in their full load conditions and with a force distribution assumed proportional to the masses, exhibit always, at the collapse, dangerous mechanisms that could be avoided by a properly design devoted to the development of a collapse mechanism of global type (Montuori *et al*. 2015).

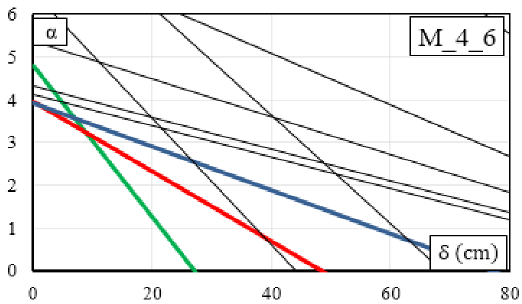
In the cases M_6_2 and M_6_4 for mono-sections and D_6_2 for double-sections the number of mechanisms that starts in sequence is equal to 4. For this reason, another colour, the yellow, is introduced to point out the last mechanism that develops. For a high number of storeys, there are more chances that a large number of mechanisms trigger in sequence until the development of the final one.

Table 4 First and second order collapse mechanism multipliers and equilibrium curves for M_racks

M_4_2						
Storey (im)	Type-1		Type-2		Type-3	
	$\alpha_{0.1}$ (-)	γ_1 (1/cm)	$\alpha_{0.2}$ (-)	γ_2 (1/cm)	$\alpha_{0.3}$ (-)	γ_3 (1/cm)
1	5.3104	0.1497	3.6470	0.0312	5.3104	0.1497
2	4.0046	0.0689	4.9054	0.0374	7.2674	0.1247
3	3.8102	0.0432	7.0244	0.0510	9.6195	0.1069
4	4.0680	0.0312	14.1084	0.0935	17.3171	0.0935



M_4_6						
Storey (im)	Type-1		Type-2		Type-3	
	$\alpha_{0.1}$ (-)	γ_1 (1/cm)	$\alpha_{0.2}$ (-)	γ_2 (1/cm)	$\alpha_{0.3}$ (-)	γ_3 (1/cm)
1	4.7877	0.1772	4.1302	0.0369	4.7877	0.1772
2	3.9516	0.0816	5.3634	0.0443	6.5205	0.1476
3	3.934	0.0511	7.495	0.0604	8.7123	0.1266
4	4.3164	0.0369	14.4051	0.1107	15.8219	0.1107



M_4_8						
Storey (im)	Type-1		Type-2		Type-3	
	$\alpha_{0.1}$ (-)	γ_1 (1/cm)	$\alpha_{0.2}$ (-)	γ_2 (1/cm)	$\alpha_{0.3}$ (-)	γ_3 (1/cm)
1	3.0097	0.1159	2.6869	0.0242	3.0097	0.1159
2	2.5209	0.0534	3.4728	0.029	4.0961	0.0966
3	2.5268	0.0334	4.8363	0.0395	5.4811	0.0828
4	2.7832	0.0242	9.2362	0.0725	9.9683	0.0725

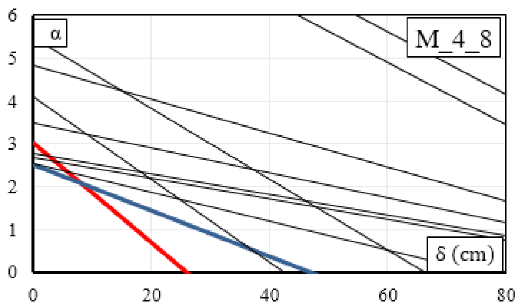
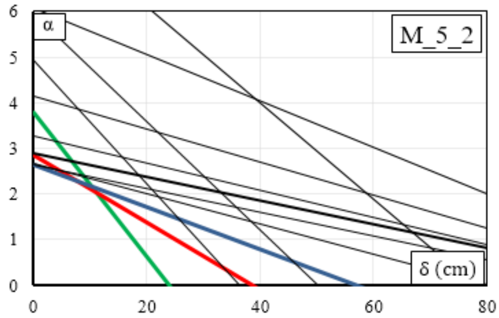
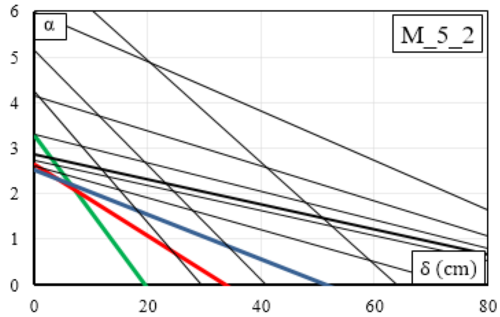


Table 4 Continued

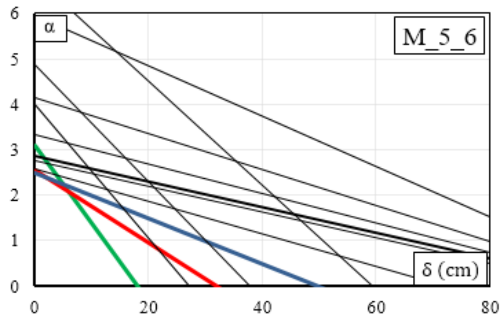
Storey (im)	Type-1 $\alpha_{0.1}$ (-)	Type-1 γ_1 (1/cm)	Type-2 $\alpha_{0.2}$ (-)	Type-2 γ_2 (1/cm)	Type-3 $\alpha_{0.3}$ (-)	Type-3 γ_3 (1/cm)
M_5_2						
1	3.795	0.1584	2.6452	0.0259	3.795	0.1584
2	2.8456	0.0737	3.2761	0.0297	4.9276	0.1357
3	2.637	0.0464	4.1591	0.0365	5.9543	0.1188
4	2.6743	0.0333	6.0881	0.0509	8.213	0.1056
5	2.8966	0.0259	12.4656	0.095	15.2763	0.0950



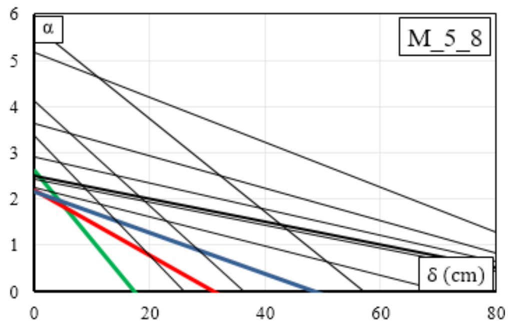
Storey (im)	Type-1 $\alpha_{0.1}$ (-)	Type-1 γ_1 (1/cm)	Type-2 $\alpha_{0.2}$ (-)	Type-2 γ_2 (1/cm)	Type-3 $\alpha_{0.3}$ (-)	Type-3 γ_3 (1/cm)
M_5_4						
1	3.2794	0.1679	2.7276	0.0275	3.2794	0.1679
2	2.6318	0.0781	3.3101	0.0315	4.228	0.1439
3	2.5232	0.0491	4.1541	0.0387	5.1502	0.1259
4	2.6116	0.0352	5.9719	0.0539	7.1565	0.1119
5	2.8674	0.0275	11.8427	0.1007	13.4047	0.1007



Storey (im)	Type-1 $\alpha_{0.1}$ (-)	Type-1 γ_1 (1/cm)	Type-2 $\alpha_{0.2}$ (-)	Type-2 γ_2 (1/cm)	Type-3 $\alpha_{0.3}$ (-)	Type-3 γ_3 (1/cm)
M_5_6						
1	3.0979	0.1715	2.7614	0.0281	3.0979	0.1715
2	2.5583	0.0798	3.3274	0.0322	3.9814	0.147
3	2.4857	0.0502	4.1586	0.0396	4.8675	0.1286
4	2.5928	0.036	5.9394	0.0551	6.7866	0.1143
5	2.8611	0.0281	11.6361	0.1029	12.7498	0.1029



Storey (im)	Type-1 $\alpha_{0.1}$ (-)	Type-1 γ_1 (1/cm)	Type-2 $\alpha_{0.2}$ (-)	Type-2 γ_2 (1/cm)	Type-3 $\alpha_{0.3}$ (-)	Type-3 γ_3 (1/cm)
M_5_8						
1	2.6201	0.1512	2.4235	0.0247	2.6201	0.1512
2	2.1982	0.0704	2.9097	0.0284	3.3613	0.1296
3	2.1511	0.0443	3.6288	0.0349	4.1177	0.1134
4	2.2528	0.0318	5.1651	0.0486	5.7518	0.1008
5	2.4924	0.0247	10.0551	0.0907	10.8243	0.0907



Storey (im)	Type-1 $\alpha_{0.1}$ (-)	Type-1 γ_1 (1/cm)	Type-2 $\alpha_{0.2}$ (-)	Type-2 γ_2 (1/cm)	Type-3 $\alpha_{0.3}$ (-)	Type-3 γ_3 (1/cm)
M_6_2						
1	3.3021	0.1544	2.3007	0.0208	3.3021	0.1544
2	2.4581	0.0725	2.7193	0.0232	4.1583	0.1351
3	2.241	0.0458	3.24	0.027	4.7761	0.1201
4	2.2115	0.0329	4.1767	0.0338	5.9187	0.1081
5	2.2952	0.0254	6.1943	0.0477	8.3256	0.0983
6	2.4893	0.0208	12.8263	0.0901	15.7334	0.0901

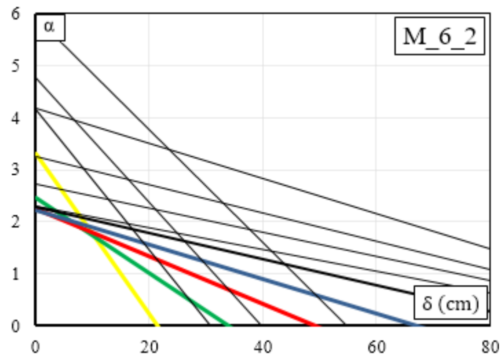
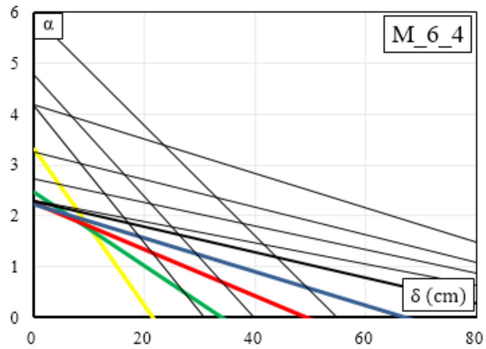
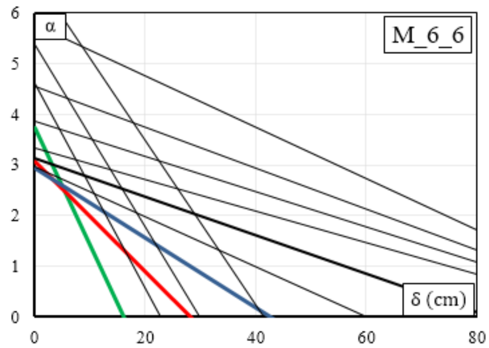


Table 4 Continued

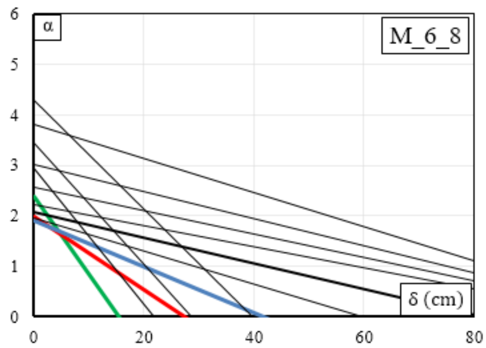
M_6_4						
	Type-1		Type-2		Type-3	
Storey (im)	$\alpha_{0.1}$ (-)	γ_1 (1/cm)	$\alpha_{0.2}$ (-)	γ_2 (1/cm)	$\alpha_{0.3}$ (-)	γ_3 (1/cm)
1	2.685	0.1544	2.2471	0.0208	2.685	0.1544
2	2.1419	0.0725	2.6132	0.0232	3.3481	0.1351
3	2.0212	0.0458	3.0888	0.027	3.8759	0.1201
4	2.0362	0.0329	3.9359	0.0338	4.8377	0.1081
5	2.1425	0.0254	5.7332	0.0477	6.852	0.0983
6	2.3466	0.0208	11.4965	0.0901	13.0298	0.0901



M_6_6						
	Type-1		Type-2		Type-3	
Storey (im)	$\alpha_{0.1}$ (-)	γ_1 (1/cm)	$\alpha_{0.2}$ (-)	γ_2 (1/cm)	$\alpha_{0.3}$ (-)	γ_3 (1/cm)
1	3.7189	0.2317	3.3439	0.0312	3.7189	0.2317
2	3.0548	0.1088	3.867	0.0347	4.6173	0.2027
3	2.9219	0.0687	4.5578	0.0405	5.3639	0.1802
4	2.9668	0.0493	5.7839	0.0507	6.7167	0.1622
5	3.1375	0.0382	8.3699	0.0715	9.5418	0.1474
6	3.4487	0.0312	16.5815	0.1351	18.1949	0.1351



M_6_8						
	Type-1		Type-2		Type-3	
Storey (im)	$\alpha_{0.1}$ (-)	γ_1 (1/cm)	$\alpha_{0.2}$ (-)	γ_2 (1/cm)	$\alpha_{0.3}$ (-)	γ_3 (1/cm)
1	2.3764	0.1544	2.2204	0.0208	2.3764	0.1544
2	1.9838	0.0725	2.5605	0.0232	2.9432	0.1351
3	1.9114	0.0458	3.0136	0.027	3.4261	0.1201
4	1.9487	0.0329	3.8162	0.0338	4.2983	0.1081
5	2.0663	0.0254	5.5035	0.0477	6.1163	0.0983
6	2.2755	0.0208	10.8333	0.0901	11.68	0.0901



5. Load distribution influence on the collapse mechanism

In addition, to point out the more dangerous load conditions that have to be avoided to preserve the structure during a seismic event, also the variability of load distribution has been taken in account. In particular, loads have been distributed in correspondence of one or more storeys or one or more bays by exploiting all the possible configurations. The storey loads have been supposed of the same value. In addition, seismic forces have been computed according to the actual load distribution. The same value of the spectral acceleration has been considered in the computation of storey forces.

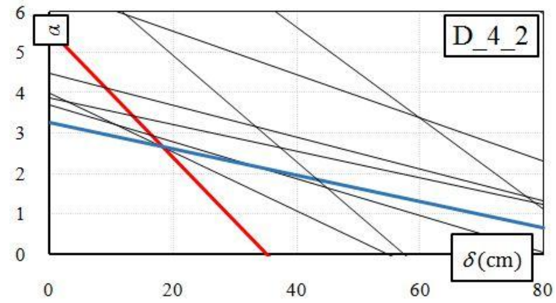
As already done, both first order collapse mechanism multiplier, (Eqs. (7), (8) and (9)) and the slope of mechanism equilibrium curves (Eqs. (10), (11) and (12)) have been provided for each structure and each load distribution in order to provide collapse mechanism equilibrium curves.

Even if all the possible load combinations have been investigated for each structure reported in Table 2, for sake of shortness, being the number of combinations increasing with the structural complexity, the results are herein reported with reference to the M_4_4 steel racks only. In Table 6, first order and second order collapse mechanism multiplier for each load condition are reported. In particular, 17 load combinations are possible for a 4 storey-4 bay racks as described in Tables 7-10. These load combinations are obtained by considering one, two, or three storeys loaded and one, two or three bay loaded. The colours selected to point out the collapse mechanism evolution follow the same order used in the previous paragraph (blue, red and green).

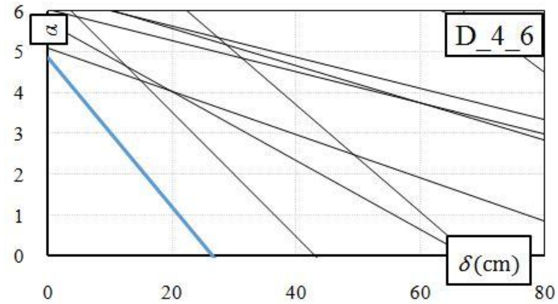
If Table 7 is observed, it can be noted that the storey load distribution, in the case of one storey loaded, deeply affects the structural response in terms of collapse

Table 5 First and second order collapse mechanism multipliers and equilibrium curves for D_racks

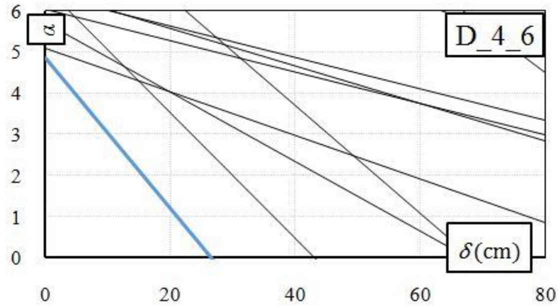
D_4_2						
Storey (im)	**Type-1**		**Type-2**		**Type-3**	
	$\alpha_{0.1}$ (-)	γ_1 (1/cm)	$\alpha_{0.2}$ (-)		$\alpha_{0.1}$ (-)	γ_1 (1/cm)
1	5.5474	0.1576	3.2719	1	5.5474	0.1576
2	3.9839	0.0726	4.4848	2	3.9839	0.0726
3	3.6891	0.0455	6.594	3	3.6891	0.0455
4	3.8712	0.0328	13.8411	4	3.8712	0.0328



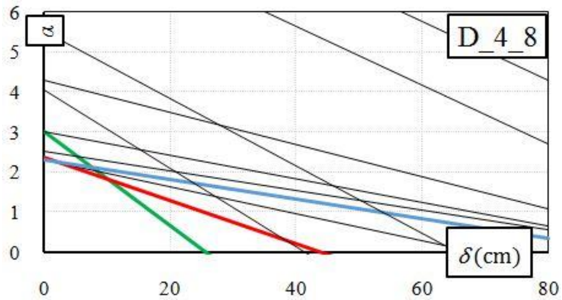
D_4_4						
Storey (im)	**Type-1**		**Type-2**		**Type-3**	
	$\alpha_{0.1}$ (-)	γ_1 (1/cm)	$\alpha_{0.2}$ (-)		$\alpha_{0.1}$ (-)	γ_1 (1/cm)
1	4.8638	0.1692	3.3768	0.0353	4.8638	0.1692
2	3.7026	0.0779	4.4934	0.0423	6.5891	0.141
3	3.5408	0.0488	6.4823	0.0577	8.9031	0.1209
4	3.7925	0.0353	13.1858	0.1058	16.3354	0.1058



D_4_6						
Storey (im)	**Type-1**		**Type-2**		**Type-3**	
	$\alpha_{0.1}$ (-)	γ_1 (1/cm)	$\alpha_{0.2}$ (-)		$\alpha_{0.1}$ (-)	γ_1 (1/cm)
1	4.8642	0.1829	6.0292	0.0381	4.8642	0.1829
2	5.7122	0.0842	6.4805	0.0457	6.5759	0.1524
3	5.077	0.0528	9.954	0.0624	8.9206	0.1307
4	6.3964	0.0381	13.6503	0.1143	16.4276	0.1143



D_4_8						
Storey (im)	**Type-1**		**Type-2**		**Type-3**	
	$\alpha_{0.1}$ (-)	γ_1 (1/cm)	$\alpha_{0.2}$ (-)		$\alpha_{0.1}$ (-)	γ_1 (1/cm)
1	3.000	0.1176	2.2991	0.0245	3.000	0.1176
2	2.3731	0.0542	3.0105	0.0294	4.0511	0.098
3	2.3145	0.0339	4.2969	0.0401	5.5082	0.084
4	2.5088	0.0245	8.5803	0.0735	10.1639	0.0735



D_5_2						
Storey (im)	**Type-1**		**Type-2**		**Type-3**	
	$\alpha_{0.1}$ (-)	γ_1 (1/cm)	$\alpha_{0.2}$ (-)		$\alpha_{0.1}$ (-)	γ_1 (1/cm)
1	3.781	0.1404	2.2356	0.023	3.781	0.1404
2	2.6895	0.0654	2.8057	0.0263	4.8769	0.1204
3	2.4213	0.0411	3.6234	0.0324	5.94	0.1053
4	2.4111	0.0295	5.4422	0.0451	8.2545	0.0936
5	2.5788	0.023	11.6347	0.0843	15.4602	0.0843

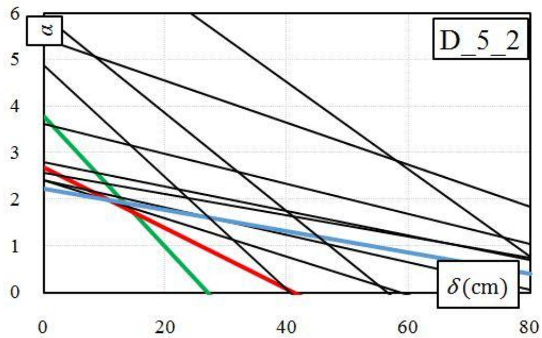
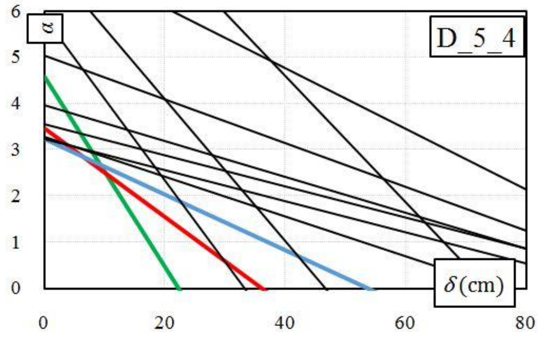
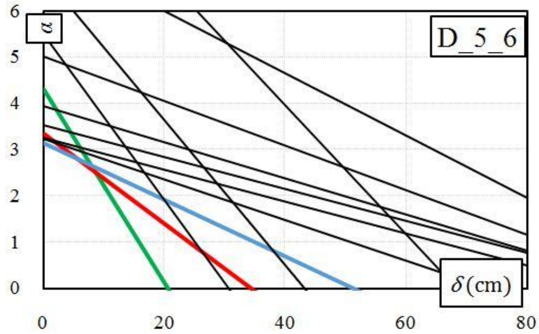


Table 5 Continued

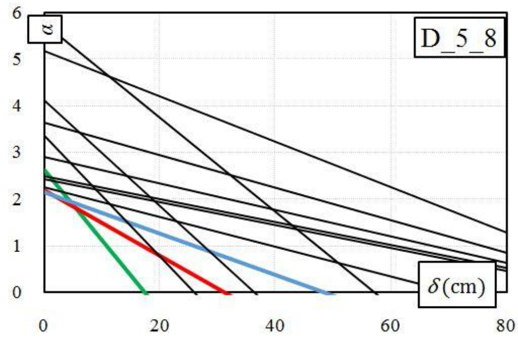
D_5_4						
Storey (im)	Type-1		Type-2		Type-3	
	$\alpha_{0.1}$ (-)	γ_1 (1/cm)	$\alpha_{0.2}$ (-)	γ_2 (1/cm)	$\alpha_{0.3}$ (-)	γ_3 (1/cm)
1	4.5852	0.2058	3.224	0.0337	4.5852	0.2058
2	3.4643	0.0958	3.9559	0.0386	5.8669	0.1764
3	3.2231	0.0602	5.0441	0.0475	7.2116	0.1543
4	3.2767	0.0432	7.4329	0.0661	10.1055	0.1372
5	3.5549	0.0337	15.3946	0.1235	19.0718	0.1235



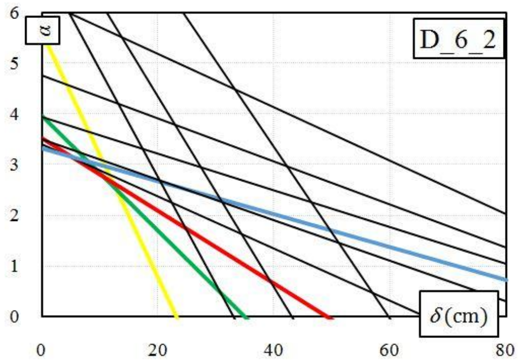
D_5_6						
Storey (im)	Type-1		Type-2		Type-3	
	$\alpha_{0.1}$ (-)	γ_1 (1/cm)	$\alpha_{0.2}$ (-)	γ_2 (1/cm)	$\alpha_{0.3}$ (-)	γ_3 (1/cm)
1	4.3048	0.2094	3.2438	0.0343	4.3048	0.2094
2	3.3384	0.0975	3.9473	0.0393	5.4865	0.1795
3	3.1477	0.0613	5.0101	0.0483	6.7746	0.1571
4	3.226	0.044	7.3309	0.0673	9.5311	0.1396
5	3.5189	0.0343	15.0008	0.1256	18.0529	0.1256



D_5_8						
Storey (im)	Type-1		Type-2		Type-3	
	$\alpha_{0.1}$ (-)	γ_1 (1/cm)	$\alpha_{0.2}$ (-)	γ_2 (1/cm)	$\alpha_{0.3}$ (-)	γ_3 (1/cm)
1	2.6201	0.1512	2.4235	0.0247	2.6201	0.1512
2	2.1982	0.0704	2.9097	0.0284	3.3613	0.1296
3	2.1511	0.0443	3.6288	0.0349	4.1177	0.1134
4	2.2528	0.0318	5.1651	0.0486	5.7518	0.1008
5	2.4924	0.0247	10.0551	0.0907	10.8243	0.0907



D_6_2						
Storey (im)	Type-1		Type-2		Type-3	
	$\alpha_{0.1}$ (-)	γ_1 (1/cm)	$\alpha_{0.2}$ (-)	γ_2 (1/cm)	$\alpha_{0.3}$ (-)	γ_3 (1/cm)
1	5.6041	0.2415	3.3162	0.0325	5.6041	0.2415
2	3.9637	0.1134	3.9455	0.0362	6.9801	0.2113
3	3.5143	0.0716	4.7586	0.0423	8.0911	0.1878
4	3.4077	0.0514	6.24	0.0528	10.1119	0.1691
5	3.4941	0.0398	9.496	0.0746	14.3387	0.1537
6	3.7565	0.0325	20.5315	0.1409	27.3003	0.1409

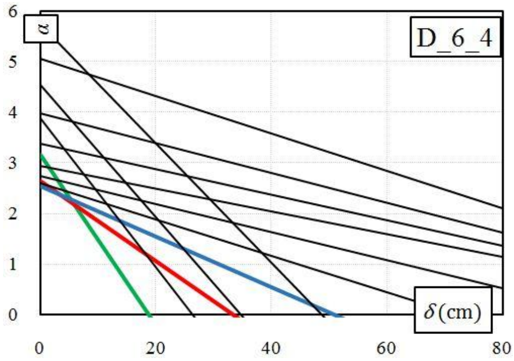


mechanism. When only the first storey is loaded, the type1 mechanism at first storey occurs, conversely, when the load is translated to the other storey, the collapse mechanism becomes better. Same situation is pointed out in the case of 2 or 3 storey loaded (Tables 8-9). It can be concluded that, in order to avoid the begin of a dangerous mechanism, is better to load a rack starting from the top. Conversely, as regards the bay loading, it can be observed that the position of the loaded bay does not affect the seismic response.

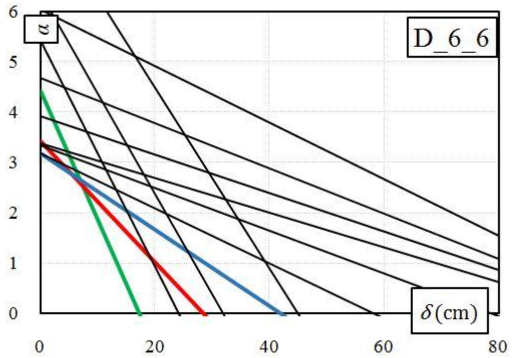
In all the three cases (case 15, 16 and 17) the first mechanism developing is the type1 at third storey, followed by type1 at the second and the first storey. The final mechanism exhibited by the racks in all the possible configurations is type1 at first storey. However, this dangerous mechanism can be possible to trigger depending on the level of load. In other words, when the rack is lightly loaded (one storey or one or two bay) collapse mechanism multipliers of first order are very high and, consequently,

Table 6 Continued

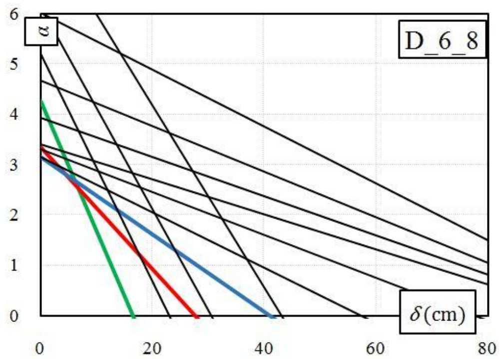
Storey (im)	D_6_4					
	Type-1		Type-2		Type-3	
	$\alpha_{0.1}$ (-)	γ_1 (1/cm)	$\alpha_{0.2}$ (-)	γ_2 (1/cm)	$\alpha_{0.3}$ (-)	γ_3 (1/cm)
1	3.1518	0.1682	2.9495	0.0226	3.1518	0.1682
2	2.6366	0.079	3.379	0.0252	3.8752	0.1471
3	2.5426	0.0499	3.9815	0.0294	4.5391	0.1308
4	2.5936	0.0358	5.0506	0.0368	5.727	0.1177
5	2.751	0.0277	7.3039	0.0519	8.1918	0.107
6	3.0302	0.0226	14.4516	0.0981	15.7181	0.0981



Storey (im)	D_6_6					
	Type-1		Type-2		Type-3	
	$\alpha_{0.1}$ (-)	γ_1 (1/cm)	$\alpha_{0.2}$ (-)	γ_2 (1/cm)	$\alpha_{0.3}$ (-)	γ_3 (1/cm)
1	4.4227	0.2563	3.3874	0.0345	4.4227	0.2563
2	3.4256	0.1203	3.9282	0.0384	5.406	0.2243
3	3.1869	0.076	4.6798	0.0449	6.3624	0.1994
4	3.1838	0.0546	6.0316	0.0561	8.0618	0.1794
5	3.3315	0.0422	8.9431	0.0792	11.576	0.1631
6	3.6346	0.0345	18.5083	0.1495	22.2893	0.1495



Storey (im)	M_6_8					
	Type-1		Type-2		Type-3	
	$\alpha_{0.1}$ (-)	γ_1 (1/cm)	$\alpha_{0.2}$ (-)	γ_2 (1/cm)	$\alpha_{0.3}$ (-)	γ_3 (1/cm)
1	4.2678	0.2585	3.3993	0.0348	4.2678	0.2585
2	3.3561	0.1214	3.9287	0.0388	5.1992	0.2262
3	3.1453	0.0767	4.6727	0.0452	6.1356	0.201
4	3.1561	0.055	6.0082	0.0565	7.7934	0.1809
5	3.3121	0.0426	8.8758	0.0798	11.2147	0.1645
6	3.6209	0.0348	18.2525	0.1508	21.6369	0.1508



difficult to be reached. Conversely, when a rack is deeply loaded, collapse mechanism multiplier of first order becomes closer to the one corresponding to the full load condition. In this last case, the chance to have a dangerous collapse mechanism becomes more realistic. Similar results have been achieved by analysing all the other structures reported in Table 7.

6. Conclusions

In this work, the prediction of the seismic collapse mechanisms of steel storage pallet racks under seismic loads is presented. The proposed verification approach is based on the kinematic theorem of plastic collapse extended to the second order effects by means of the concept of collapse mechanism equilibrium curve and has the scope to predict the seismic collapse of multi-storey racks by varying load and geometry, designed for seismic zones in according to European standards. The response of the structures is given in terms of collapse mechanism typology where all the possible undesired mechanisms are discussed in the paper. Several parametric analyses have been performed on a large number of racks, pointing out that global collapse mechanism, which is the only one which can actually develop, only in few cases occurs, whereas always partial mechanisms start developing. This event is particularly dangerous for steel storage racks because if a partial mechanism (i.e., a mechanism different from global one) starts developing, the collapse is immediately reached due to the absence of available ductility of involved members (upright cross-sections of class 3 or 4). On the contrary, if global mechanism is developed, the ductility available in beam-to-column and base-plate joints can be used, leading to a much more dissipative structure. Of course, these results do not cover all typologies of the racks present on

Table 6 First and second order collapse mechanism multipliers for all the load conditions of M_4_4 rack

	$\alpha_{0.1}$ (-)				$\alpha_{0.2}$ (-)				$\alpha_{0.3}$ (-)			
Storey / Case	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**
1	21.0409	31.4347	41.3895	51.3443	47.8763	10000	10000	10000	21.0409	10000	10000	10000
2	21.0409	15.4978	20.6948	25.6722	23.9381	42.709	10000	10000	21.0409	25.9678	10000	10000
3	21.0409	15.4978	13.6502	17.1148	15.9588	21.3545	32.8006	10000	21.0409	25.9678	25.9678	10000
4	21.0409	15.4978	13.6502	12.7263	11.9691	14.2363	16.4003	22.8923	21.0409	25.9678	25.9678	25.9678
5	10.3009	9.2987	12.4169	15.4033	14.3629	32.0318	10000	10000	10.3009	19.4758	10000	10000
6	10.3009	8.8559	8.1901	10.2689	9.5753	14.2363	21.8671	10000	10.3009	17.3119	17.3119	10000
7	10.3009	8.6099	7.8751	7.4861	7.0406	8.8977	10.2502	14.3077	10.3009	16.2299	16.2299	16.2299
8	10.3009	7.6392	7.8751	9.8739	9.207	13.2094	27.3339	10000	10.3009	12.5449	21.6398	10000
9	10.3009	7.6392	7.6782	7.6358	7.1814	9.0579	12.3002	17.1692	10.3009	12.5449	19.4758	19.4758
10	10.3009	7.6392	6.7519	7.1267	6.7027	8.2192	10.2969	20.0307	10.3009	12.5449	12.5449	22.7218
11	6.7209	5.5558	5.8501	7.3349	6.8395	10.5675	21.8671	10000	6.7209	10.0359	17.3119	10000
12	6.7209	5.4845	5.6207	5.6561	5.3196	7.045	9.5669	13.3538	6.7209	9.7571	15.1479	15.1479
13	6.7209	5.4323	4.9105	5.2211	4.9104	6.2622	7.8452	15.2615	6.7209	9.558	9.558	17.3119
14	6.7209	5.0196	4.8614	5.2661	4.9527	6.2747	8.8259	17.1692	6.7209	8.0706	10.7528	19.4758
15	21.0409	16.3713	15.8346	17.0782	15.9588	21.4094	30.0183	58.0538	21.0409	29.097	37.724	66.5657
16	10.3009	8.099	7.8751	8.5208	7.9794	10.6224	14.9094	28.8897	10.3009	14.1827	18.5484	33.0085
17	6.7209	5.3416	5.2219	5.6683	5.3196	7.0267	9.8731	19.1684	6.7209	9.2112	12.1566	21.8227

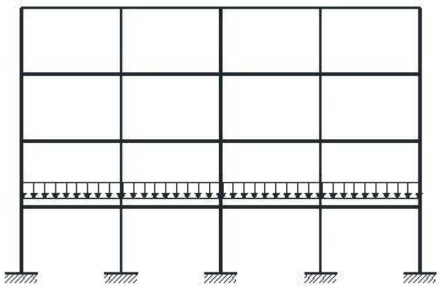
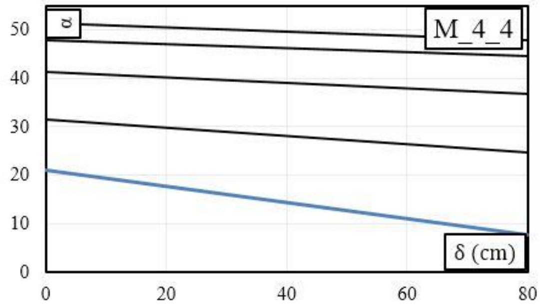
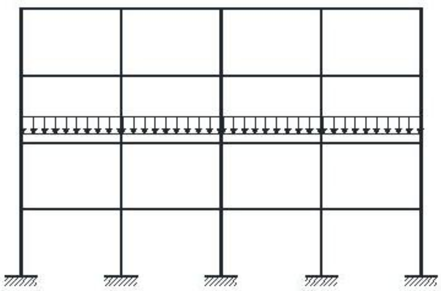
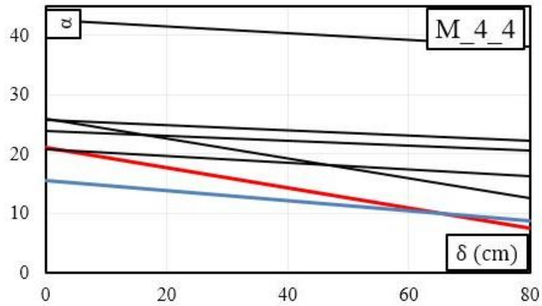
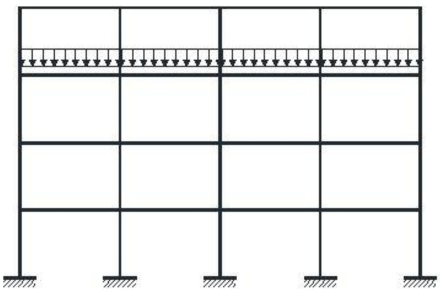
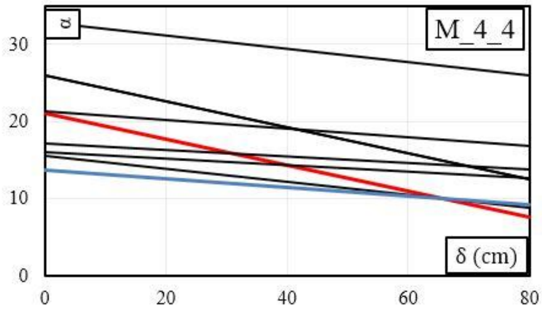
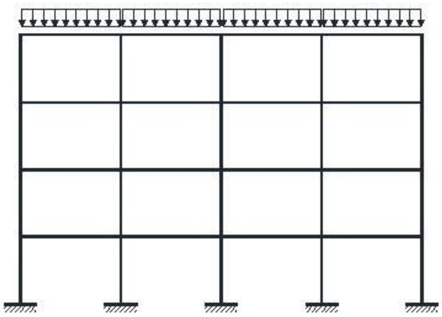
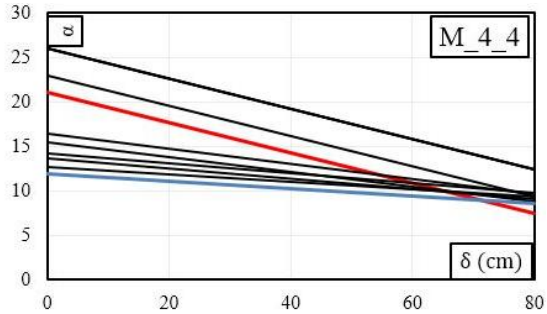
	γ_1 (1/cm)				γ_2 (1/cm)				γ_3 (1/cm)			
Storey / Case	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**
1	0.1692	0.0846	0.0564	0.0423	0.0423	0	0	0	0.1692	0	0	0
2	0.1692	0.0846	0.0564	0.0423	0.0423	0.0564	0	0	0.1692	0.1692	0	0
3	0.1692	0.0846	0.0564	0.0423	0.0423	0.0564	0.0846	0	0.1692	0.1692	0.1692	0
4	0.1692	0.0846	0.0564	0.0423	0.0423	0.0564	0.0846	0.1692	0.1692	0.1692	0.1692	0.1692
5	0.1692	0.0762	0.0508	0.0381	0.0381	0.0423	0	0	0.1692	0.1269	0	0
6	0.1692	0.0725	0.0451	0.0338	0.0338	0.0376	0.0564	0	0.1692	0.1128	0.1128	0
7	0.1692	0.0705	0.0434	0.0311	0.0311	0.0353	0.0529	0.1058	0.1692	0.1058	0.1058	0.1058
8	0.1692	0.0846	0.0542	0.0407	0.0407	0.0529	0.0705	0	0.1692	0.1692	0.141	0
9	0.1692	0.0846	0.0529	0.0381	0.0381	0.0484	0.0635	0.1269	0.1692	0.1692	0.1269	0.1269
10	0.1692	0.0846	0.0564	0.0415	0.0415	0.0548	0.0808	0.1481	0.1692	0.1692	0.1692	0.1481
11	0.1692	0.0769	0.0484	0.0363	0.0363	0.0423	0.0564	0	0.1692	0.1354	0.1128	0
12	0.1692	0.0759	0.0465	0.0329	0.0329	0.0376	0.0494	0.0987	0.1692	0.1316	0.0987	0.0987
13	0.1692	0.0752	0.0479	0.0347	0.0347	0.0418	0.0615	0.1128	0.1692	0.1289	0.1289	0.1128
14	0.1692	0.0846	0.0542	0.0394	0.0394	0.0508	0.0692	0.1269	0.1692	0.1692	0.1451	0.1269
15	0.1692	0.0779	0.0488	0.0353	0.0353	0.0423	0.0577	0.1058	0.1692	0.141	0.1209	0.1058
16	0.1692	0.0779	0.0488	0.0353	0.0353	0.0423	0.0577	0.1058	0.1692	0.141	0.1209	0.1058
17	0.1692	0.0779	0.0488	0.0353	0.0353	0.0423	0.0577	0.1058	0.1692	0.141	0.1209	0.1058

the market, but the proposed methodology is general and can be used always.

In addition, also the variability of load distribution has been taken in account, to point out the more dangerous load conditions that should be avoid preserving the structure during a seismic event. Also in this case, results show that the global mechanism never develops but other unfavourable mechanisms invest the structure leading to low seismic performances. Very important information can be given to the manufacturer from a very deep study in this direction, since some configurations bring to a more dangerous situation than others. The entire logistic flow should be designed considering also this important aspect, especially in seismic zone.

The mechanism exhibited could be improved by designing steel racks with a properly procedure devoted to

Table 7 Load distribution and collapse mechanism equilibrium curves for the racks with one storey loaded

Case	Loads on one storey only	
1 Storey 1		
2 Storey 2		
3 Storey 3		
4 Storey 4		

the development of a collapse mechanism of global type. This procedure, effective and known, is called Theory of Plastic Mechanism Control (Montuori *et al*. 2015), and in further works will be applied to redesign storage racks exhibiting at the collapse a mechanism of global type (Longo *et al*. 2012, 2014a, b, Montuori *et al*. 2014 and 2016a, b).

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Table 8 Load distribution and collapse mechanism equilibrium curves for the racks with two storey loaded

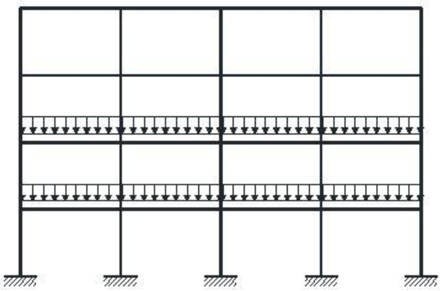
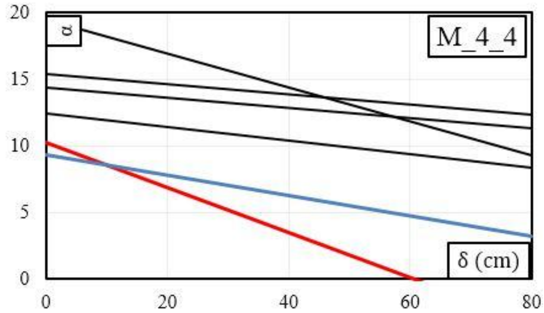
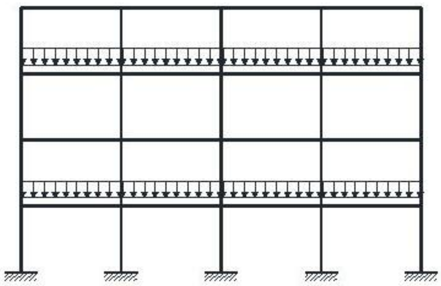
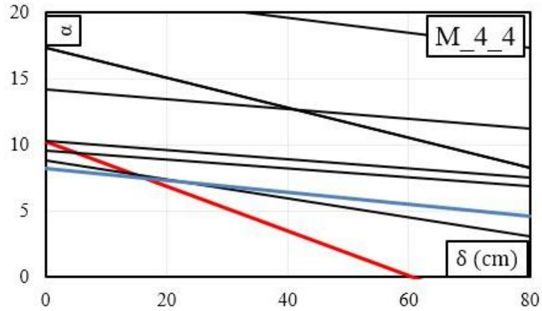
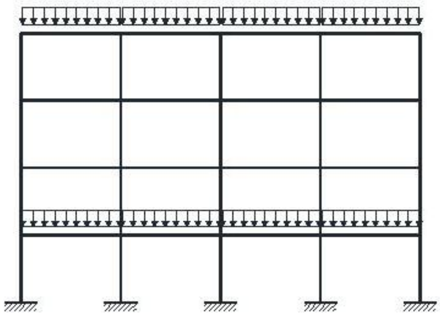
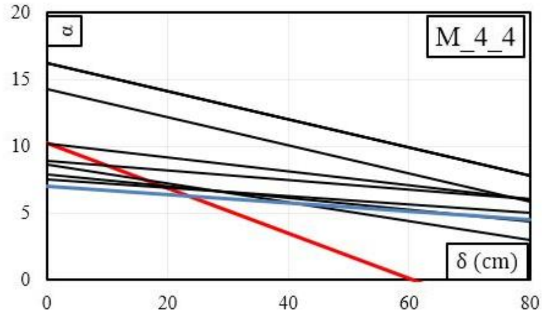
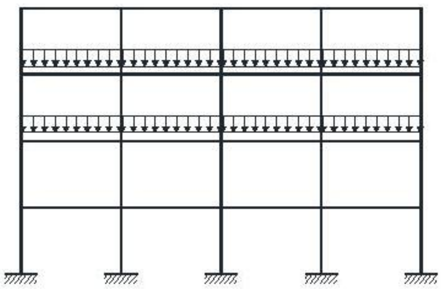
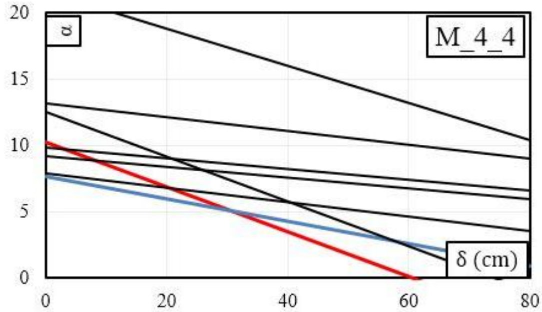
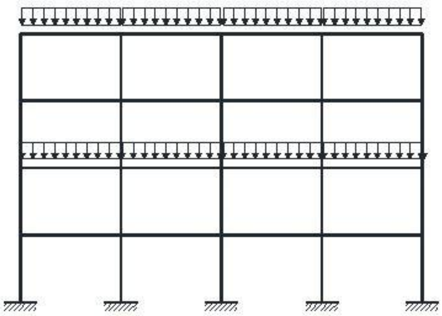
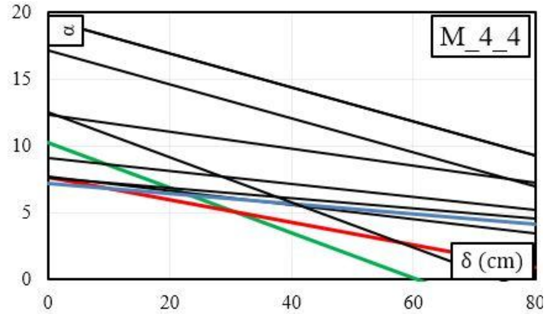
Case	Loads on two storyes	
5 Storeys 1 and 2		
6 Storeys 1 and 3		
7 Storeys 1 and 4		
8 Storeys 2 and 3		
9 Storeys 2 and 4		

Table 8 Continued

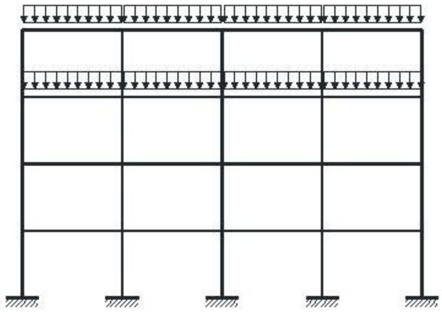
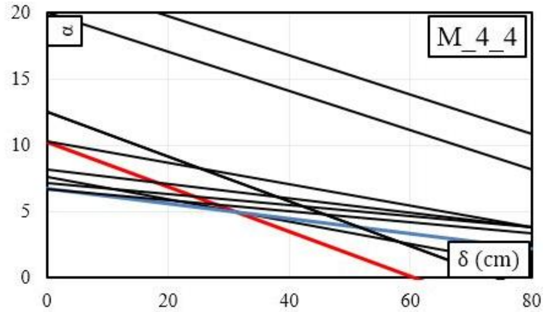
Case	Loads on two storyes	
10 Storeys 3 and 4		

Table 9 Load distribution and collapse mechanism equilibrium curves for the racks with three storey loaded

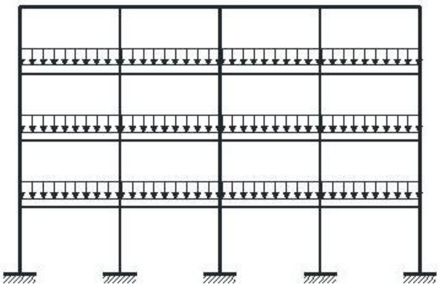
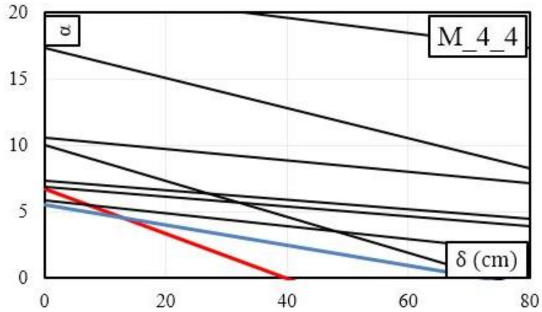
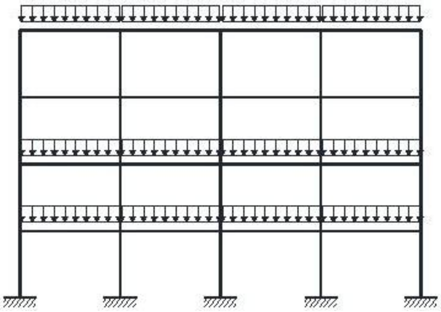
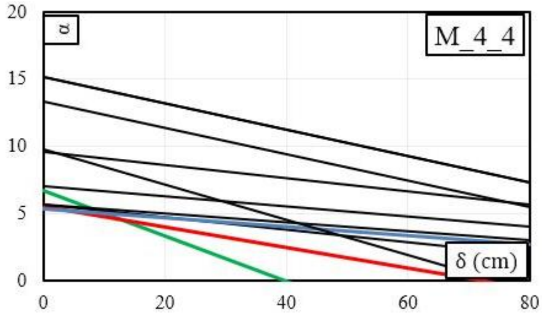
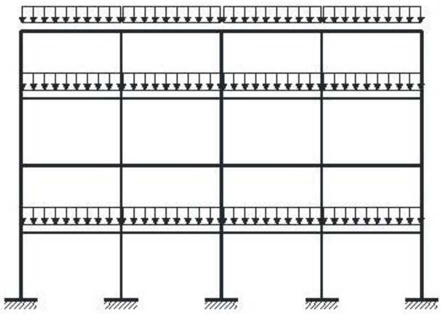
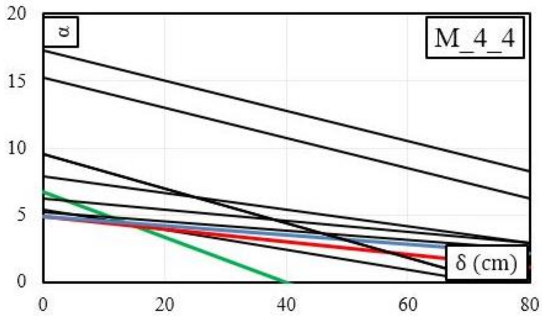
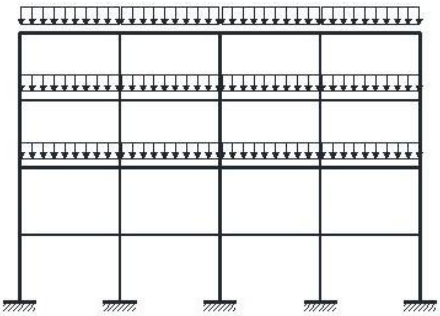
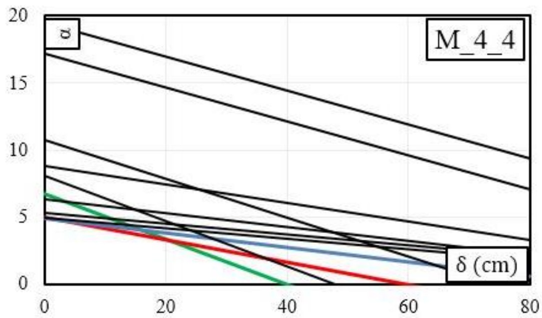
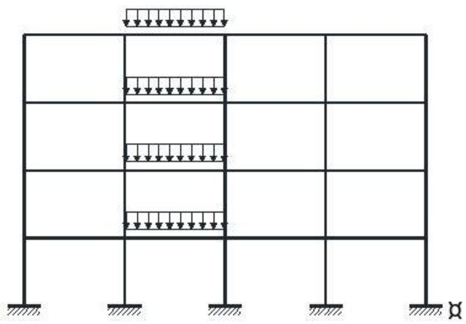
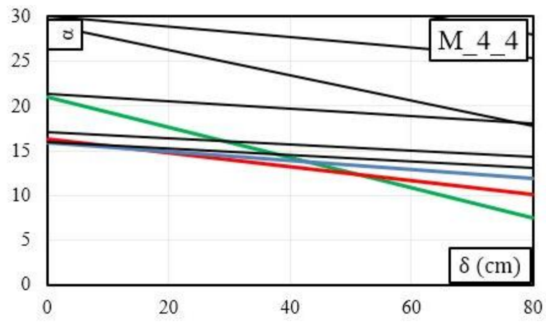
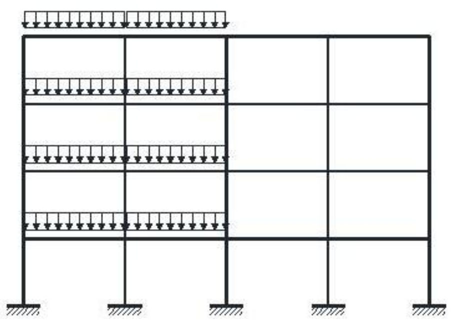
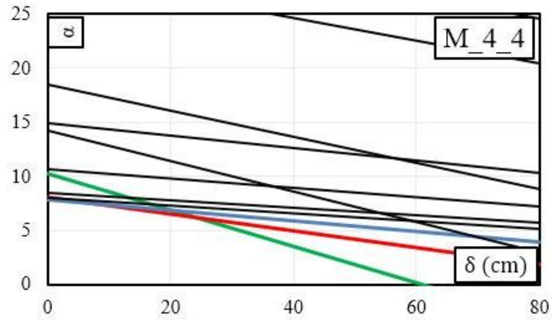
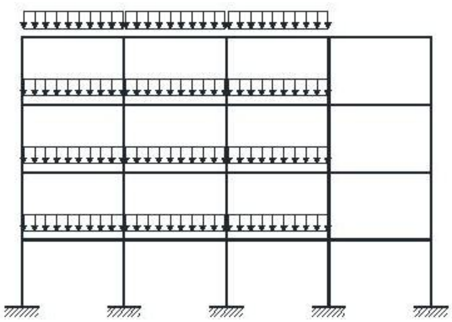
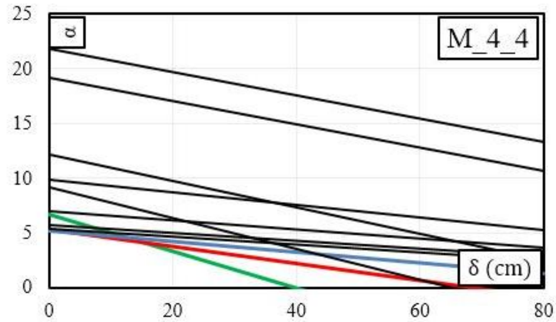
Case	Loads on three storyes	
11 Storeys 1, 2 and 3		
12 Storeys 1, 2 and 4		
13 Storeys 1, 3 and 4		
13 Storeys 2, 3 and 4		

Table 10 Load distribution and collapse mechanism equilibrium curves for the racks with one, two or three bay loaded

Case	Loads on bays	
15 1 bay		
16 2 bays		
17 3 bays		

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