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Hygro-thermal wave propagation in functionally graded double-layered nanotubes systems

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Abstract. In this paper, wave propagation is studied and analyzed in double-layered nanotubes systems via the nonlocal strain gradient theory. To the author's knowledge, the present paper is the first to investigate the wave propagation characteristics of double-layered porous nanotubes systems. It is generally considered that the material properties of nanotubes are related to the porosity and hygro-thermal effects. The governing equations of the double-layered nanotubes systems are derived by using the Hamilton principle. The dispersion relations and displacement fields of wave propagation in the double nanotubes systems which experience three different types of motion are obtained and discussed. The results show that the phase velocities of the double nanotubes systems depend on porosity, humidity change, temperature change, material composition, non-local parameter, strain gradient parameter, interlayer spring, and wave number.

Keywords: wave propagation; double-layered nanotubes systems; phase velocity; hygro-thermal environment

1. Introduction

In recent years, many papers have studied the functionally graded (FG) materials (e.g., Amar *et al*. 2018, Arioui *et al*. 2018, Bellifa *et al*. 2017, Benlahcen *et al*. 2018, Fourn *et al*. 2018, Hachemi *et al*. 2017, Hebbar *et al*. 2018, Houari *et al*. 2018, Meftah *et al*. 2017, Sidhoum *et al*. 2017, Dehrouyeh-Semnani 2017, 2018, Zenkour and Radwan 2019, Zenkour 2018), also many papers have considered the effect of material length scale in the study of solid structures at micro and/or nanometer scales (e.g., Ghayesh 2018, Ghayesh and Farajpour 2018, Ghayesh *et al*. 2018, Malikan *et al*. 2018, Ebrahimi and Habibi 2016, Attia and Rahman 2018, She *et al*. 2017a, b, Attia 2017, Heydari and Shariati 2018, Heydari 2018a, b, Dehrouyeh-Semnani *et al*. 2017, Karami *et al*. 2018c, Radic 2018, Ahouel *et al*. 2016, Akgöz and Civalek 2017b, Ebrahimi and Barati 2018, Arefi and Zenkour 2017, 2018). Among these papers, Eltaher *et al*. (2018a) preformed the analysis of crack occurs under unsteady pressure and temperature in a natural gas facility by applying FGM, Eltaher *et al*. (2018b) also used a modified porosity model in bending analysis of functionally graded porous nanobeams. She *et al*. (2017a) performed the thermal buckling and post-buckling analysis of FG beams using various beam theories.

On the other hand, many non-classical theories have been put forward in the field of nano-mechanics, among which the Eringen nonlocal theory (Eringen 1998) is widely used due to the fact that the nonlocal theory contains only one material parameter and can reasonably explain the mechanical properties of some nanostructures (Numanoğlu *et al*. 2018). However, the Eringen theory has its limitations, which has been discussed by some scholars. For example, Shaat and Abdelkefi (2017) proved that the nonlocal theory is not sufficient to explain the mechanical properties of nanostructures well. Thus, they used a generalized non-local elastic theory with two non-local scale parameters to study the mechanical properties of nanostructures. Besides, Eringen's nonlocal integral law is inapplicable to nanostructures of engineering interest due to confliction between constitutive and equilibrium requirements (Romano and Barretta 2017). All difficulties disappear if a stress-driven nonlocal integral formulation (Barretta and Sciarra 2018, Barretta *et al*. 2018, 2019) is adopted. In addition, the stiffness enhancement effect which can be capture by strain graded theory and modified coupled stress theory (Mindlin 1965, Aifantis 1992, Akgöz and Civalek 2012, 2013a, b, 2017a) cannot be characterized by non-local elastic theory. Since the Eringen nonlocal theory and strain gradient theory describe two completely different physical properties at the nanometer scale. Lim *et al*. (2015) put forward a new high-order non-local strain gradient theory which is a combination of the classical non-local theory and strain gradient theory. This theory extends the classic nonlocal elastic theory, which can characterize two different stiffness effects. Following nonlocal strain gradient theory, many researchers (e.g., Shahverdi and Barati 2017, Xu *et al*. 2017, Sahmani and Aghdam 2017, Lu *et al*. 2017, 2019, Ghayesh and Farajpour 2018, She *et al*. 2018a, b, c, d, 2019, Malikan *et al*. 2018, Ebrahimi and Farazmandnia 2018, Malikan and Nguyen 2018, Faleh *et al*. 2018, Apuzzo *et al*. 2018, Karami *et al*. 2018a, b, Karami and Janghorban 2019, Li *et al*. 2018, Rajasekaran and

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Khaniki 2017, Ebrahimi and Dabbagh 2018, Tang *et al*. 2019, Aria and Biglari 2018, Amiri *et al*. 2018) have studied the mechanical and acoustic properties of nanostructures in detail.

The carbon nanotube is a kind of new carbon material, some papers have studied the properties of nanotubes. For example, Eltaher *et al*. (2018c) presented the vibration analysis of size-dependent carbon nanotubes (CNTs), Eltaher *et al*. (2019a) also discussed the characterization and behaviors of single walled carbon nanotube, Eltaher *et al*. (2019b) also studied the modal participation of fixed–fixed single-walled carbon nanotube with vacancies. Malikan *et al*. (2019) presented the transient response analysis of CNTs with an internal and external damping. Although there is a lot of literature on nanobeams, nanoplates, and nanotubes, relatively little research has been done on double-layered nanostructures.

The double-layered nanotubes systems are composed of two parallel nanotubes connected continuously through a coupled medium. These nanometer systems are widely used in the electromechanical systems of nanometer (Khaniki 2018, Karličić *et al*. 2016, Mehar and Panda 2019). Therefore, it is of great significance to study the wave propagation and vibration behaviors of these structures. For example, Murmu and Adhikari (2010) studied the vibration of the double-nanobeam system. Murmu and Adhikari (2011) also discussed the axial instability of double-nanobeam-systems, and they pointed out that the buckling loads are independent of the stiffness of the springs in the in-phase type buckling. For Murmu and Adhikari' works (Murmu and Adhikari 2010, 2011), the Euler-Bernoulli beam theory in conjunction with the Eringen nonlocal theory is employed to build the size-dependent model, and the properties of the material are independent of porosity, temperature and humidity. Due to the fact that the porous materials have ordered porous structure and high surface area, which are very important for application in the impact damping, noise insulation, catalysis, adsorption, separation, ion exchange and chemical sensing fields because of their unique porous structures and surface performances. Therefore, it is necessary and urgent to research on the mechanical properties of double-nanostructures systems. Based on Euler-Bernoulli beam theory in conjunction with a general bi-Helmholtz nonlocal strain-gradient elasticity model which incorporates three material length parameters, Barati (2017) studied the wave propagation in the porous double-nanobeam systems made of functionally graded materials subjected to the combination of thermal and mechanical loadings, he pointed out that the phase velocity can be decreased as the temperature goes up, besides, the presence of porosity can decrease the phase velocity. For Barati's work (Barati 2017), the shear deformation effects are neglected and hygro-thermal environment is not considered. Therefore, it is necessary that the shear deformation and hygro-thermal environment should be considered to investigate the wave propagation of the double nanobeam systems.

At present, the existing literatures on porous nanotubes are still relatively limited, and the study on wave propagation of porous nanotubes is much scarcer. In a word, there is no study investigating the wave propagation of porous double-layered nanotubes systems in hygro-thermal environment. Therefore, the present paper aims to fill in this gap. To this end, based on nonlocal strain gradient theory, a porosity-dependent and temperature-dependent model is developed for the double-layered nanotubes systems. The nanotubes are made of functionally graded porous materials. Porosity-dependent material properties are estimated via a modified power-law rule. The results show that the wave propagation characteristics of the double-layered nanotubes systems are significantly influenced by porosity, hygro-thermal loadings, material composition, small scaling parameters, interlayer spring, and wave number. The results which are observed here can be served as a benchmark results for study the porous double nanotubes systems.

2. Formulations

Consider a double-FG nanotube-system as shown in Fig. 1, the two nanotubes are referred to as nanotube-1 and nanotube-2, the nanotubes are considered to be of length L, outer radius R_0, inner radius R_i, undergoing flexural vibration of displacement $w(x,t)$. In addition, the material properties (including Young's modulus E, thermal expansion coefficient α, mass density ρ, moisture expansion coefficient γ, Poisson's ratio υ) only vary along the r direction as a power functions (She *et al*. 2017b)

$$\begin{bmatrix} E(r) \\ \alpha(r) \\ \rho(r) \\ \gamma(r) \\ \upsilon(r) \end{bmatrix} = \begin{bmatrix} (E_c - E_m) \\ (\alpha_c - \alpha_m) \\ (\rho_c - \rho_m) \\ (\gamma_c - \gamma_m) \\ (\upsilon_c - \upsilon_m) \end{bmatrix} \left(\frac{r - R_i}{R_0 - R_i} \right)^K + E_m - \frac{\beta}{2} \begin{bmatrix} (E_c + E_m) \\ (\alpha_c + \alpha_m) \\ (\rho_c + \rho_m) \\ (\gamma_c + \gamma_m) \\ (\upsilon_c + \upsilon_m) \end{bmatrix} \tag{1}$$

Here, K is the power law index, β is the porosity volume fraction, the subscript "c" stands for ceramic, "m" is for metal, and the material properties are listed in Table 1.

Zhang and Fu' tube model is employed in this paper with the following displacement field (Zhang and Fu 2013, She *et al*. 2017b, 2018b, c, d, Babaei *et al*. 2019a, b)

$$\begin{aligned} u_1(x,y,z,t) &= u - zw_{,x} + g(y,z)\left[w_{,x} + \psi(x,t)\right] \\ u_2 &= 0 \\ u_3(x,y,z,t) &= w(x,t) \end{aligned} \tag{2}$$

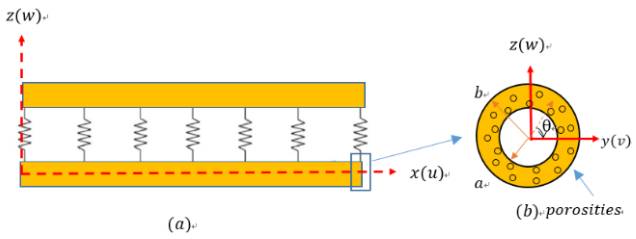


Fig. 1 Configuration of the double nanotubes. The picture on the right is from She et al. (2018b)

Table 1 Material Properties for SUS304 (ν = 0.3262) and Si3N4 (ν = 0.24) (Reddy and Chin 1998, Zhong *et al*. 2016)

Materials	Properties	P_0	P_{-1}	P_1	P_2	P_3
Si_3N_4	E_c(Pa)	348.43e+9	0.0	-3.070e-4	2.160e-7	-8.964e-11
	α_c (1/K)	5.8723e-6	0.0	9.095e-4	0.0	0.0
	ρ_c(Kg/m^3)	2370	0.0	0.0	0.0	0.0
	γ_c(wt%H_2O)$^{-1}$	0.0	0.0	0.0	0.0	0.0
SUS304	E_m(Pa)	201.04e+9	0.0	3.079e-4	-6.543e-7	0.0
	α_m (1/K)	12.33e-6	0.0	8.086e-4	0.0	0.0
	ρ_m(Kg/m^3)	8166	0.0	0.0	0.0	0.0
	γ_m(wt%H_2O)$^{-1}$	0.0005	0.0	0.0	0.0	0.0

in which

$$g(y,z)=z+z\left(R_0^{\,2}R_i^{\,2}r^{-2}-r^2/3\right)\left(R_0^{\,2}+R_i^{\,2}\right)^{-1} \tag{3}$$

In Eq. (2), u_1, u_2 and u_3 are the displacements along the x, y, and z directions, respectively, u is the axial displacement (along the x direction), w is the deflection (along the z direction), ψ is the rotation, and the comma followed by an index signifies partial differentiation with respect to the space coordinate associated with that index. Clearly, for Timoshenko beam model, $g(y, z) = 0$, for Euler beam model, $g(y, z) = z$.

The strains of interest are (She *et al*. 2019)

$$\begin{aligned}\varepsilon_{xx}&=u_{1,x}=u_{,x}-zw_{,xx}+g(y,z)\left(w_{,xx}+\psi_{,x}\right)\\ \gamma_{xz}&=u_{1,z}+u_{3,x}=g_{,z}\left(\psi+w_{,x}\right)\\ \gamma_{xy}&=u_{1,y}+u_{2,x}=g_{,y}\left(\psi+w_{,x}\right)\end{aligned} \tag{4}$$

in which, ε_{xx} is the axial strain, γ_{xz} and γ_{xy} are shear strains. Now, Hamilton's principle can be written as

$$\int_{t_1}^{t_2}\delta\Pi dt=0 \tag{5}$$

where, Π is energy functional, t_1 and t_2 are initial and final time, respectively, and

$$\begin{aligned}\delta\prod&=\int_0^L\left[q_i w_i\delta w_i-\left(N_T+N_C\right)w_{i,x}\delta w_{i,x}\right]dx\\ &+\int_\Omega\left[\sigma_{xx}\delta\varepsilon_{xx}+\left(\sigma_{xz}\delta\varepsilon_{xz}+\sigma_{xy}\delta\varepsilon_{xy}\right)\right]d\Omega\\ &-\int_\Omega\rho(r)\left(\dot{u}_1\delta\dot{u}_1+\dot{u}_3\delta\dot{u}_3\right)dx\\ &=\int_0^L\left[q_i w_i\delta w_i-\left(N_T+N_C\right)w_{i,x}\delta w_{i,x}\right]dx\\ &+\int_0^L\left[\begin{matrix}N_i\delta u_{i,x}-M_i\delta w_{i,xx}+P_i\left(\delta w_{i,xx}+\delta\psi_{i,x}\right)\\ +Q_i\left(\delta w_{i,x}+\delta\psi_i\right)\end{matrix}\right]dx\\ &-\int_0^L\left\{I_0\left(\dot{u}_i\delta\dot{u}_i+\dot{w}_i\delta\dot{w}_i\right)+I_1\dot{w}_{i,x}\delta\dot{w}_{i,x}\right.\\ &\left.+I_2\left(-\dot{\psi}_i\delta\dot{w}_{i,x}-2\dot{w}_{i,x}\delta\dot{w}_{i,x}\right)\right.\\ &\left.+I_2\left(-\dot{w}_{i,x}\delta\dot{\psi}_i\right)+I_3\left[\begin{matrix}\dot{w}_{i,x}\delta\dot{w}_{i,x}+\dot{\psi}_i\delta\dot{w}_{i,x}\\ +\dot{w}_{i,x}\delta\dot{\psi}_i+\dot{\psi}_i\delta\dot{\psi}_i\end{matrix}\right]\right\}dx\end{aligned} \tag{6}$$

in which

$$\begin{cases}q_i=-K_c\left(w_1-w_2\right) & for\ \ i=1\\ q_i=K_c\left(w_1-w_2\right) & for\ \ i=2\end{cases} \tag{7}$$

in which, N_T and N_C are the hygro-thermal resultants, and the stress resultants N_i, M_i, P_i, Q_i, N_{Ti}, K_c is interlayer spring, mass inertias I_0, I_1, I_2, I_3 are evaluated by integration of the direct stress across the cross-sectional area, as

$$\begin{aligned}\left[N_i,M_i,P_i\right]&=\int_A\left[1,z,g\right]\sigma_{xx}dA\\ Q_i&=\int_A\left(\tfrac{\partial g}{\partial z}\tau_{xz}+\tfrac{\partial g}{\partial y}\tau_{xy}\right)dA\\ N_{Ti}&=\int_A E(r)\alpha(r)\Delta T dA\\ N_{Ci}&=\int_A E(r)\gamma(r)\Delta C dA\\ \left[I_0,I_1,I_2,I_3\right]&=\int_A\rho(r)\left[1,z^2,zg,g^2\right]dA\end{aligned} \tag{8}$$

in which, ΔT and ΔC are temperature and humidity changes, respectively, A is the area of the cross section of the nanotube. The motion equations can be obtained by integrating Eq. (6) into Eq. (5), as

$$\delta u: N_{i,x}=I_0\ddot{u}_i \tag{9a}$$

$$\delta\varphi: P_{i,x}-Q_i=-I_2\ddot{w}_{i,x}+I_3\left(\ddot{w}_{i,x}+\ddot{\psi}_i\right) \tag{9b}$$

$$\begin{aligned}\delta w: P_{i,xx}-M_{i,xx}-Q_{i,x}&=q_i-\left(N_{T_i}+N_{C_i}\right)w_{i,xx}-I_0\ddot{w}_i\\ &+I_1\ddot{w}_{i,xx}-I_2\left(2\ddot{w}_{i,xx}+\ddot{\psi}_{i,x}\right)+I_3\left(\ddot{w}_{i,xx}+\ddot{\psi}_{i,x}\right)\end{aligned} \tag{9c}$$

The non-local strain gradient theory is employed. This theory combines two major advantages of strain gradient theory and non-local theory, involving two length scale parameters, one is strain gradient parameter l, and the other

one is non-local parameter *ea*. According to the hypothesis of non-local strain gradient theory, the relation between stress and strain can be expressed as (Lim *et al*. 2015)

$$\left[1-(ea)^2\nabla^2\right]t_{xx}=E\left(1-l^2\nabla^2\right)\varepsilon_{xx} \tag{10}$$

in which, ∇^2 is the Laplace operator.

For hygro-thermal environments, the stress-displacement relations can be given as

$$\begin{aligned}\left[1-(ea)^2\nabla^2\right]\sigma_{xx}&=E(r)\Gamma\begin{bmatrix}u_{i,x}-zw_{i,xx}+g\left(w_{i,xx}+\psi_{i,x}\right)\\-\alpha(r)\Delta T-\gamma(r)\Delta C\end{bmatrix}\\\left[1-(ea)^2\nabla^2\right]\tau_{xz}&=G(r)g_{,z}\Gamma\left(\psi_i+w_{i,x}\right)\\\left[1-(ea)^2\nabla^2\right]\tau_{xy}&=G(r)g_{,y}\Gamma\left(\psi_i+w_{i,x}\right)\end{aligned} \tag{11}$$

in which $\Gamma = 1 - l^2\nabla^2$, $G(r)=E(r)/\left[2\left(1+\upsilon(r)\right)\right]$ is shear modulus, σ_{xx} is the axial stress, γ_{xz} and γ_{xy} are shear stresses. Integration of stresses across the cross-section of the nanotubes gives the stress resultants

$$\begin{bmatrix}N_i\\M_i\\P_i\\Q_i\end{bmatrix}=(ea)^2\begin{bmatrix}N_{i,xx}\\M_{i,xx}\\P_{i,xx}\\Q_{i,xx}\end{bmatrix}+\Gamma\begin{bmatrix}A_{11}u_{,x}-\left(N_T+N_C\right)\\-D_{11}w_{,xx}+E_{11}\left(w_{,xx}+\psi_{,x}\right)\\-E_{11}w_{,xx}+H_{11}\left(w_{,xx}+\psi_{,x}\right)\\B_{11}\left(\psi+w_{,x}\right)\end{bmatrix} \tag{12}$$

in which

$$\begin{aligned}\left(A_{11},D_{11},E_{11},H_{11}\right)&=\int_A E(r)\left(1,z^2,zg,g^2\right)dA\\B_{11}&=\int_A G(r)\left[\left(\tfrac{\partial g}{\partial y}\right)^2+\left(\tfrac{\partial g}{\partial z}\right)^2\right]dA\end{aligned} \tag{13}$$

Substitution of Eq. (12) into Eq. (9) yields the equation of motion for wave propagation of the double nanotubes systems, i.e.

$$A_{11}\Gamma u_{i,xx}=I_0\left[\ddot{u}_i-(ea)^2\ddot{u}_{i,xx}\right] \tag{14a}$$

$$\begin{aligned}&\Gamma\left[-E_{11}w_{i,xxx}+H_{11}\left(w_{i,xxx}+\psi_{i,xx}\right)-B_{11}\left(w_{i,x}+\psi_i\right)\right]\\&=\left[1-(ea)^2\nabla^2\right]\left[-I_2\ddot{w}_{i,x}+I_3\left(\ddot{w}_{i,x}+\ddot{\psi}_i\right)\right]\end{aligned} \tag{14b}$$

$$\begin{aligned}&\Gamma w_{i,xxxx}\begin{bmatrix}\left(-2E_{11}+H_{11}+D_{11}\right)w_{i,xxxx}\\+\left(H_{11}-E_{11}\right)\psi_{i,xxx}-B_{11}\left(w_{i,xx}+\psi_{i,x}\right)\end{bmatrix}\\&=\left[1-(ea)^2\nabla^2\right]\begin{bmatrix}q_i-\left(N_T+N_C\right)w_{i,xx}-I_0\ddot{w}_i\\+\left(I_1-2I_2+I_3\right)\ddot{w}_{i,xx}+\left(I_3-I_2\right)\ddot{\psi}_{i,x}\end{bmatrix}\end{aligned} \tag{14c}$$

Let $A_{11} = A_0$, $-2E_{11} + H_{11} + D_{11} = A_1$, $-E_{11} = H_{11} = A_2$, $B_{11} = A_4$, $I_0 = I_0^*$, $I_1 - 2I_2 + I_3 = I_1^*$, $-I_2 + I_3 = I_2^*$, $I_3 + I_3^*$, then Eq. (14) becomes

$$\begin{aligned}&A_0\,\Gamma u_{1,xx}=I_0^*\left[\ddot{u}_1-(ea)^2\ddot{u}_{1,xx}\right]\\&\Gamma\left[A_1w_{1,xxxx}+A_2\psi_{1,xxx}-A_4\left(w_{1,xx}+\psi_{1,x}\right)\right]\\&+K_c\left[1-(ea)^2\nabla^2\right]\left(w_1-w_2\right)+\left[1-(ea)^2\nabla^2\right]\\&\times\left[\left(N_T+N_C\right)w_{1,xx}+I_0^*\ddot{w}_1-I_1^*\ddot{w}_{1,xx}-I_2^*\ddot{\psi}_{1,x}\right]=0\\&\Gamma\left[A_2w_{1,xxx}+A_3\psi_{1,xx}-A_4\left(w_{1,x}+\psi_1\right)\right]\\&-\left[1-(ea)^2\nabla^2\right]\left(I_2^*\ddot{w}_{1,x}+I_3^*\ddot{\psi}_1\right)=0\end{aligned} \tag{15a}$$

$$\begin{aligned}&A_0\Gamma u_{2,xx}=I_0^*\left[\ddot{u}_2-(ea)^2\ddot{u}_{2,xx}\right]\\&\Gamma\left[A_1w_{2,xxxx}+A_2\psi_{2,xxx}-A_4\left(w_{2,xx}+\psi_{1,x}\right)\right]\\&-K_c\left[1-(ea)^2\nabla^2\right]\left(w_1-w_2\right)+\left[1-(ea)^2\nabla^2\right]\\&\times\left[\left(N_T+N_C\right)w_{2,xx}+I_0^*\ddot{w}_2-I_1^*\ddot{w}_{2,xx}-I_2^*\ddot{\psi}_{2,x}\right]=0\\&\Gamma\left[A_2w_{2,xxx}+A_3\psi_{2,xx}-A_4\left(w_{2,x}+\psi_2\right)\right]\\&-\left[1-(ea)^2\nabla^2\right]\left(I_2^*\ddot{w}_{2,x}+I_3^*\ddot{\psi}_2\right)=0\end{aligned} \tag{15b}$$

3. Dispersion relation

In this section, the dispersion behavior of the double nanotube system will be examined, the dispersion of the wave speed (e.g., phase velocity) will be studied. The double-layer nanotube system experiences three kinds of motion as

$$\text{Out of phase:}\qquad w=w_1-w_2\neq 0 \tag{16a}$$

$$\text{In phase:}\qquad w=w_1-w_2=0 \tag{16b}$$

$$\text{One nanotube fixed:}\qquad w=w_1=0 \tag{16c}$$

The solution of governing equations can be presented by (She *et al*. 2018a, d)

$$u(x,t)=U_m\exp\left[i\left(\kappa x-\omega t\right)\right] \tag{17a}$$

$$\psi(x,t)=\Phi_m\exp\left[i\left(\kappa x-\omega t\right)\right] \tag{17b}$$

$$w(x,t)=W_m\exp\left[i\left(\kappa x-\omega t\right)\right] \tag{17c}$$

in which, $i=\sqrt{-1}$, ω is the circular frequency, κ is the wave number, U_m, Φ_m, and W_m are wave amplitudes.

Case 1: Out of phase. Substituting Eq. (17) into the second and third equations of Eq. (15), one has

$$\begin{aligned}&\left[\gamma_2\left(A_1\kappa^2+A_4\right)-\gamma_1\left(N_T+N_C\right)\right]\kappa^2W_m\\&+2\gamma_1k_0W_m-\gamma_1\left(I_0^{\ *}+I_1^{\ *}\kappa^2\right)\omega^2W_m\\&+i\kappa\left\{-\gamma_2\left(A_2\kappa^2+A_4\right)+\gamma_1I_2^{\ *}\omega^2\right\}\Phi_m=0\end{aligned}\tag{18}$$

$$\begin{aligned}&i\kappa\left\{\gamma_1I_2^{\ *}\omega^2-\gamma_2\left(A_2\kappa^2+A_4\right)\right\}W_m\\&+\left\{\gamma_1I_3^{\ *}\omega^2-\gamma_2\left(A_3\kappa^2+A_4\right)\right\}\Phi_m=0\end{aligned}\tag{19}$$

From Eqs. (18) and (19), we can obtain two positive roots, one of which is a larger one for the shear wave and the other one is for the bending wave. In the following study, only the bending wave will be presented and studied. Upon rearrangement, Eqs. (18)-(19) become

$$\begin{bmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{bmatrix}\begin{bmatrix}W_n\\\Phi_n\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}\tag{20}$$

In which

$$\begin{aligned}a_{11}=&2\gamma_1k_0+\gamma_2\left[A_1\kappa^2+A_4-\frac{\gamma_1}{\gamma_2}\left(N_T+N_c\right)\right]\kappa^2\\&-\gamma_1\left(I_0^{\ *}+I_1^{\ *}\kappa^2\right)\omega^2\end{aligned}\tag{21a}$$

$$a_{12}=a_{21}=i\kappa\left\{-\gamma_2\left(A_2\kappa^2+A_4\right)+\gamma_1I_2^{\ *}\omega^2\right\}\tag{21b}$$

$$a_{22}=-\gamma_2\left(A_3\kappa^2+A_4\right)+\gamma_1I_3^{\ *}\omega^2\tag{21c}$$

The present phase speed $C_p^{present}$ of the flexural wave can be determined as

$$C_p^{present}=\sqrt{\frac{-b-\sqrt{b^2-4ac}}{2a\kappa^2}}\tag{22}$$

In which

$$a=-d_{10}^{\ 2}+b_{10}f_{10}\tag{23a}$$

$$b=-a_{10}f_{10}-b_{10}e_{10}+2c_{10}d_{10}\tag{23b}$$

$$c=-c_{10}^{\ 2}+a_{10}e_{10}\tag{23c}$$

and

$$\begin{aligned}a_{10}&=2\gamma_1k_0+\kappa^2\gamma_2\left[A_1\kappa^2+A_4-\frac{\gamma_1}{\gamma_2}\left(N_T+N_c\right)\right]\\b_{10}&=\gamma_1\left(I_0^{\ *}+I_1^{\ *}\kappa^2\right)\\c_{10}&=\gamma_2\left(A_2\kappa^2+A_4\right)\kappa\\d_{10}&=\gamma_1I_2^{\ *}\kappa\\e_{10}&=\gamma_2\left(A_3\kappa^2+A_4\right)\\f_{10}&=\gamma_1I_3^{\ *}\end{aligned}\tag{24}$$

Case 2: In phase. Substituting Eq. (17) into the second and third equations of Eq. (15)

$$\begin{aligned}&\left[\gamma_2\left(A_1\kappa^2+A_4\right)-\gamma_1\left(N_T+N_C\right)\right]\kappa^2W_m\\&\quad-\gamma_1\left(I_0^{\ *}+I_1^{\ *}\kappa^2\right)\omega^2W_m\\&+i\kappa\left\{-\gamma_2\left(A_2\kappa^2+A_4\right)+\gamma_1I_2^{\ *}\omega^2\right\}\Phi_m=0\end{aligned}\tag{25}$$

$$\begin{aligned}&i\kappa\left\{\gamma_1I_2^{\ *}\omega^2-\gamma_2\left(A_2\kappa^2+A_4\right)\right\}W_m\\&+\left\{\gamma_1I_3^{\ *}\omega^2-\gamma_2\left(A_3\kappa^2+A_4\right)\right\}\Phi_m=0\end{aligned}\tag{26}$$

Hence, the flexural phase velocity, is given by

$$C_p^{present}=\sqrt{\frac{-b-\sqrt{b^2-4ac}}{2a\kappa^2}}\tag{27}$$

In which, *a*, *b*, and *c* have the same forms as Eq. (23), and

$$\begin{aligned}a_{10}&=\kappa^2\gamma_2\left[A_1\kappa^2+A_4-\frac{\gamma_1}{\gamma_2}\left(N_T+N_c\right)\right]\\b_{10}&=\gamma_1\left(I_0^{\ *}+I_1^{\ *}\kappa^2\right)\\c_{10}&=\gamma_2\left(A_2\kappa^2+A_4\right)\kappa\\d_{10}&=\gamma_1I_2^{\ *}\kappa\\e_{10}&=\gamma_2\left(A_3\kappa^2+A_4\right)\\f_{10}&=\gamma_1I_3^{\ *}\end{aligned}\tag{28}$$

Case 3: One nanotube fixed. Substituting Eq. (17) into the second and third equations of Eq. (15)

$$\begin{aligned}&\left[\gamma_2\left(A_1\kappa^2+A_4\right)-\gamma_1\left(N_T+N_C\right)\right]\kappa^2W_m\\&+\gamma_1k_0W_m-\gamma_1\left(I_0^{\ *}+I_1^{\ *}\kappa^2\right)\omega^2W_m\\&+i\kappa\left\{-\gamma_2\left(A_2\kappa^2+A_4\right)+\gamma_1I_2^{\ *}\omega^2\right\}\Phi_m=0\end{aligned}\tag{29}$$

$$\begin{aligned}&i\kappa\left\{\gamma_1I_2^{\ *}\omega^2-\gamma_2\left(A_2\kappa^2+A_4\right)\right\}W_m\\&+\left\{\gamma_1I_3^{\ *}\omega^2-\gamma_2\left(A_3\kappa^2+A_4\right)\right\}\Phi_m=0\end{aligned}\tag{30}$$

Solution of Eqs. (29)-(30) gives

$$C_p^{present}=\sqrt{\frac{-b-\sqrt{b^2-4ac}}{2a\kappa^2}}\tag{31}$$

At this time

$$\begin{aligned}a_{10}&=2\gamma_1k_0+\kappa^2\gamma_2\left[A_1\kappa^2+A_4-\frac{\gamma_1}{\gamma_2}\left(N_T+N_c\right)\right]\\b_{10}&=\gamma_1\left(I_0^{\ *}+I_1^{\ *}\kappa^2\right)\\c_{10}&=\gamma_2\left(A_2\kappa^2+A_4\right)\kappa\\d_{10}&=\gamma_1I_2^{\ *}\kappa\end{aligned}\tag{32}$$

$$e_{10} = \gamma_2\left(A_3\kappa^2 + A_4\right)$$
$$f_{10} = \gamma_1 I_3^* \tag{32}$$

4. For Timoshenko beam model

For Timoshenko beam model, $g(y, z) = 0$, in that case, Eq. (15) becomes

$$\Gamma\left[-\kappa_s A_4\left(w_{1,xx} + \psi_{1,x}\right)\right] + \left[1-(ea)^2\nabla^2\right] \times\left[K_c\left(w_1 - w_2\right) + \left(N_T + N_C\right)w_{1,xx} + I_0^*\ddot{w}_1\right] = 0 \tag{33a}$$

$$\Gamma\left[A_3\psi_{1,xx} - \kappa_s A_4\left(w_{1,x} + \psi_1\right)\right] - \left[1-(ea)^2\nabla^2\right]\left(I_3^*\ddot{\psi}_1\right) = 0 \tag{33b}$$

Using similar steps, we can obtain the expressions of the phase velocities $C_p^{Timos\ henko}$ for Timoshenko beam model.

Case 1: Out of phase

$$C_p^{Timoshenko} = \sqrt{\frac{\left(1+l^2\kappa^2\right)\left\{\kappa^2\left[\kappa_s A_4 - \left(N_T + N_C - \frac{2K_c}{\kappa^2}\right)\frac{1+(ea)^2\kappa^2}{1+l^2\kappa^2}\right]I_3^* + \left(A_3\kappa^2 + \kappa_s A_4\right)I_0^*\right\} - \sqrt{\Delta_1}}{2\left[1+(ea)^2\kappa^2\right]I_0^* I_3^*\kappa^2}} \tag{34}$$

in which, κ_s is shear correction coefficient which have the form $\kappa_s = \frac{6(1+\nu)^2\left[(R_i/R_0)^2+1\right]^2}{\left(7+12\nu+4\nu^2\right)\left[(R_i/R_0)^4+1\right]+\left(34+48\nu+16\nu^2\right)(R_i/R_0)^2}$ (She *et al*. 2017b).

$$\Delta_1 = \left(1+l^2\kappa^2\right)^2\left\{\kappa^2\left[\kappa_s A_4 - \left(N_T + N_C - \frac{2K_c}{\kappa^2}\right)\frac{1+(ea)^2\kappa^2}{1+l^2\kappa^2}\right]I_3^* + \left(A_3\kappa^2 + \kappa_s A_4\right)I_0^*\right\}^2 - 4I_0^* I_3^*\kappa^2\left(1+l^2\kappa^2\right)^2\left[\kappa_s A_4 - \left(N_T + N_C - \frac{2K_c}{\kappa^2}\right)\frac{1+(ea)^2\kappa^2}{1+l^2\kappa^2}\right]\left(A_3\kappa^2 + \kappa_s A_4\right) + 4I_0^* I_3^*\kappa^2\left(1+l^2\kappa^2\right)^2\left(\kappa_s A_4\right)^2 \tag{35}$$

Case 2: In phase

$$C_p^{Timoshenko} = \sqrt{\frac{\left(1+l^2\kappa^2\right)\left\{\kappa^2\left[\kappa_s A_4 - \left(N_T + N_C\right)\frac{1+(ea)^2\kappa^2}{1+l^2\kappa^2}\right]I_3^* + \left(A_3\kappa^2 + \kappa_s A_4\right)I_0^*\right\} - \sqrt{\Delta_2}}{2\left[1+(ea)^2\kappa^2\right]I_0^* I_3^*\kappa^2}} \tag{36}$$

In which

$$\Delta_2 = \left(1+l^2\kappa^2\right)^2\left\{\kappa^2\left[\kappa_s A_4 - \left(N_T + N_C\right)\frac{1+(ea)^2\kappa^2}{1+l^2\kappa^2}\right]I_3^* + \left(A_3\kappa^2 + \kappa_s A_4\right)I_0^*\right\}^2 - 4I_0^* I_3^*\kappa^2\left(1+l^2\kappa^2\right)^2\left[\kappa_s A_4 - \left(N_T + N_C\right)\frac{1+(ea)^2\kappa^2}{1+l^2\kappa^2}\right]\left(A_3\kappa^2 + \kappa_s A_4\right) + 4I_0^* I_3^*\kappa^2\left(1+l^2\kappa^2\right)^2\left(\kappa_s A_4\right)^2 \tag{37}$$

Case 3: One nanotube fixed

$$C_p^{Timoshenko} = \sqrt{\frac{\left(1+l^2\kappa^2\right)\left\{\kappa^2\left[\kappa_s A_4 - \left(N_T + N_C - \frac{K_c}{\kappa^2}\right)\frac{1+(ea)^2\kappa^2}{1+l^2\kappa^2}\right]I_3^* + \left(A_3\kappa^2 + \kappa_s A_4\right)I_0^*\right\} - \sqrt{\Delta_3}}{2\left[1+(ea)^2\kappa^2\right]I_0^* I_3^*\kappa^2}} \tag{38}$$

In which

$$\Delta_3 = \left(1+l^2\kappa^2\right)^2\left\{\kappa^2\left[\kappa_s A_4 - \left(N_T + N_C - \frac{K_c}{\kappa^2}\right)\frac{1+(ea)^2\kappa^2}{1+l^2\kappa^2}\right]I_3^* + \left(A_3\kappa^2 + \kappa_s A_4\right)I_0^*\right\}^2 - 4I_0^* I_3^*\kappa^2\left(1+l^2\kappa^2\right)^2\left[\kappa_s A_4 - \left(N_T + N_C - \frac{K_c}{\kappa^2}\right)\frac{1+(ea)^2\kappa^2}{1+l^2\kappa^2}\right]\left(A_3\kappa^2 + \kappa_s A_4\right) + 4I_0^* I_3^*\kappa^2\left(1+l^2\kappa^2\right)^2\left(\kappa_s A_4\right)^2 \tag{39}$$

5. For Euler-Bernoulli beam model

For Euler-Bernoulli beam model, $g(y, z) = z$, in that case, Eq. (15) becomes

$$\left(1-l^2\nabla^2\right)A_1 w_{1,xxxx} = \left[1-(ea)^2\nabla^2\right] \times\left(K_c\left(w_1 - w_2\right) - \left(N_T + N_C\right)w_{1,xx} - I_0^*\ddot{w}_1 + I_1^*\ddot{w}_{1,xx}\right) \tag{40a}$$

$$\left(1-l^2\nabla^2\right)A_1 w_{2,xxxx} = \left[1-(ea)^2\nabla^2\right] \times\left(K_c\left(w_1 - w_2\right) - \left(N_T + N_C\right)w_{2,xx} - I_0^*\ddot{w}_2 + I_1^*\ddot{w}_{2,xx}\right) \tag{40b}$$

Then, we can obtain the expressions of phase velocities C_p^{Euler} for Euler beam model.

Case 1: Out of phase

$$C_p^{Euler} = \sqrt{\frac{\left(1+l^2\kappa^2\right)A_1\kappa^2 + \left[1+(ea)^2\kappa^2\right]\left[2K_c/\kappa^2 - \left(N_T + N_C\right)\right]}{\left[1+(ea)^2\kappa^2\right]\left(I_0^* + I_1^*\kappa^2\right)}} \tag{41}$$

Case 2: In phase

$$C_p^{Euler} = \sqrt{\frac{\left(1+l^2\kappa^2\right)A_1\kappa^2 + \left[1+(ea)^2\kappa^2\right]\left[-\left(N_T + N_C\right)\right]}{\left[1+(ea)^2\kappa^2\right]\left(I_0^* + I_1^*\kappa^2\right)}} \tag{42}$$

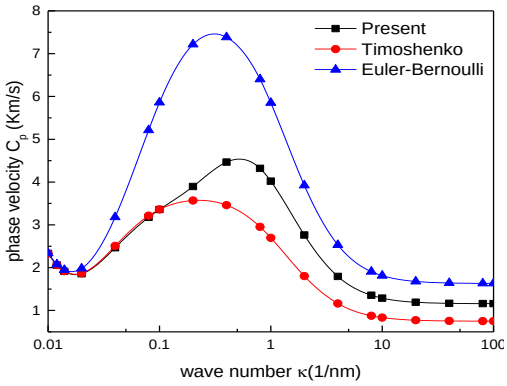
Case 3: One nanotube fixed

$$C_p^{Euler} = \sqrt{\frac{\left(1+l^2\kappa^2\right)A_1\kappa^2 + \left[1+(ea)^2\kappa^2\right]\left[K_c/\kappa^2 - \left(N_T + N_C\right)\right]}{\left[1+(ea)^2\kappa^2\right]\left(I_0^* + I_1^*\kappa^2\right)}} \tag{4}$$

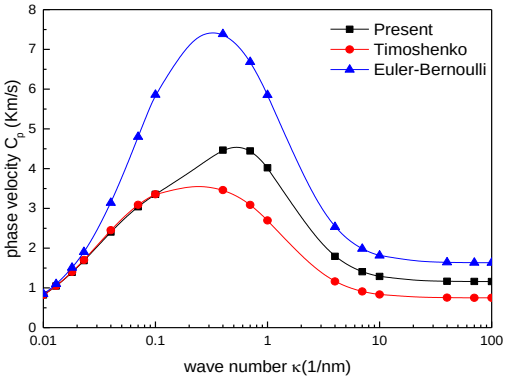
It is worth noting that, for the wave propagation analysis of nanotubes here, we assume that the waves cannot reach the boundary conditions of nanotubes. That is to say, in comparison of wavelength, nanotubes may be viewed as infinite. In this particular case, wave response will be studied far from boundary conditions.

6. Examples and analysis

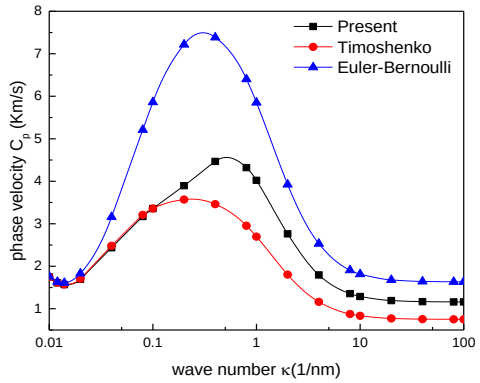
In order to validate the present investigation, numerical results are presented. It can be seen from Fig. 2 that Euler beam model can present reliable results only when the wave number $\kappa < 0.02$, while Timoshenko beam model can still



(a) Out of phase

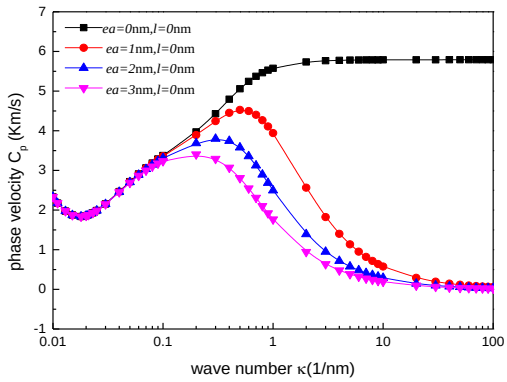


(b) In phase

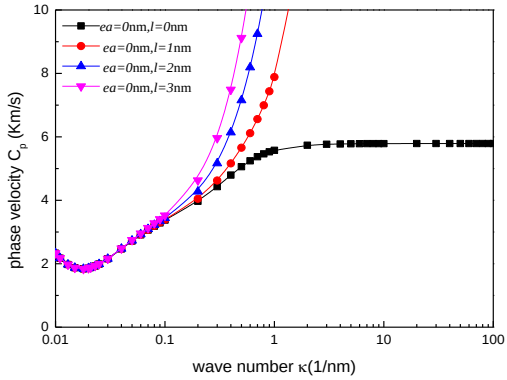


(c) One nanotube fixed

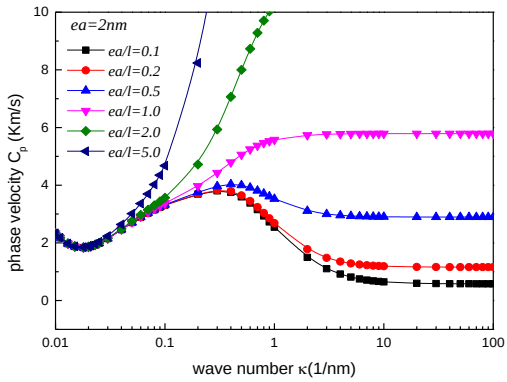
Fig. 2 Flexural phase velocity vs. wavenumber for different theories at $\beta = 0.1$, $R_0 = 20$ nm, $R^i = 10$ nm, $\Delta T = \Delta C = 0$, $K = 1$, $ea = 1$ nm, $l = 0.2$ nm, $k = 10^9$



(a) Effect of nonlocal parameter *ea*



(b) Effect of strain gradient parameter *l*



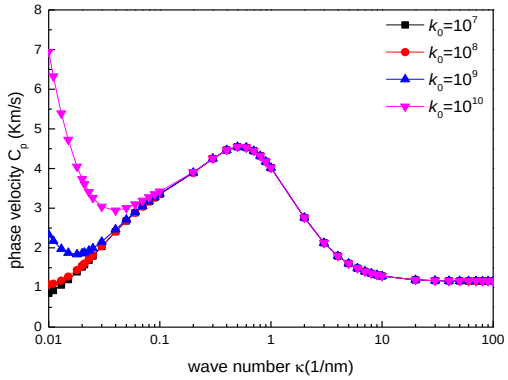
(c) Effect of *ea*/*l*

Fig. 3 Flexural phase velocity vs. wavenumber for different size parameters at $\beta = 0.1$, $R_0 = 20$ nm, $R_i = 10$ nm, $\Delta T = \Delta C = 0$, $K = 1$, $k = 10^9$ (Out of phase)

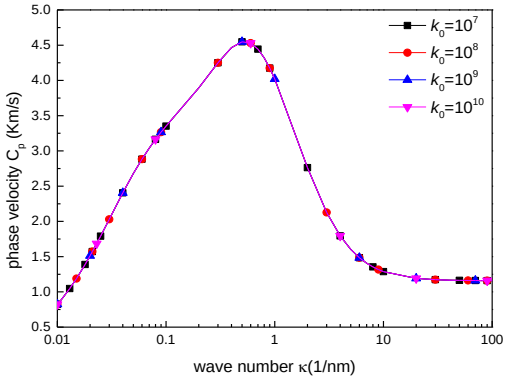
give accurate results when wave number $\kappa = 0.1$. For higher wave numbers, the phase velocity given by the present model is higher than that of Timoshenko beam model, and at the same time, is lower than that of Euler beam model, which indicates considering the shear deformation is necessary. Although different models give different results for large wave numbers, the trends of the curves presented by the three models are consistent.

To study the effects of non-local parameter and strain gradient parameter on phase velocities, the dispersion relations of the double-nanotubes systems are shown in Fig. 2. It can be seen from the figure that before the wave number reaches a certain value, the phase velocity increases with the increase of the wavenumber. According to the established model, after a certain wave number, the change of phase velocity depends on the values of non-local and strain gradient parameters. The results show that the size parameters have no effects on the dispersion relations when the wavenumber is very small. However, at higher wavenumbers, the phase velocity is more affected by the size parameters. The Eringen non-local theory only considers the softening stiffness with the increase of nonlocal parameter.

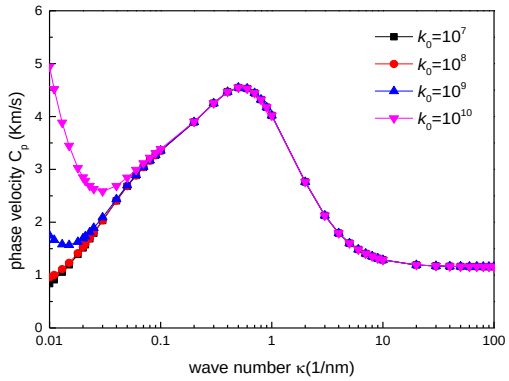
The effect of inter-layer stiffness on the phase of double-layered nanotubes systems is shown in Fig. 4. As can be seen from the figure, for Case 1 and Case 3 (Out of phase



(a) Out of phase

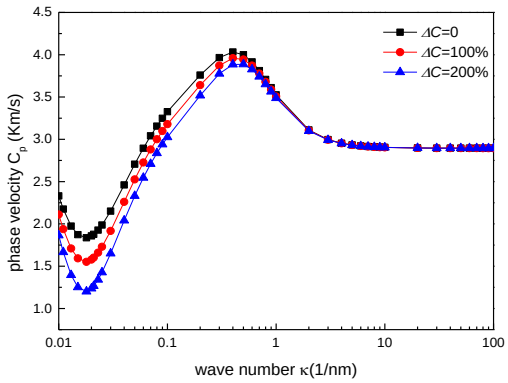


(b) In phase

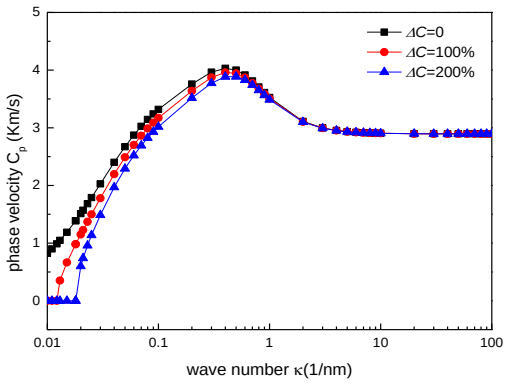


(c) One nanotube fixed

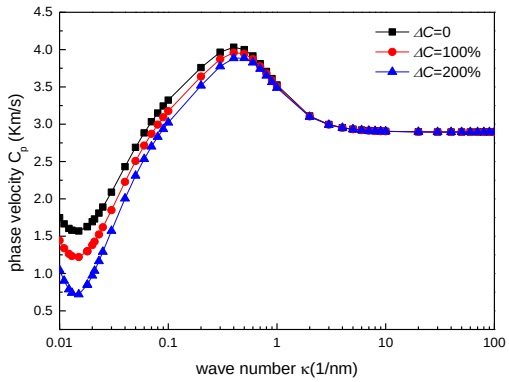
Fig. 4 Flexural phase velocity vs. wavenumber for different foundation stiffness at $\beta = 0.1$, $R_0 = 20$ nm, $R_i = 10$ nm, $\Delta T = \Delta C = 0$, $K = 1$, ea = 1 nm, $l = 0.2$ nm



(a) Out of phase



(b) In phase



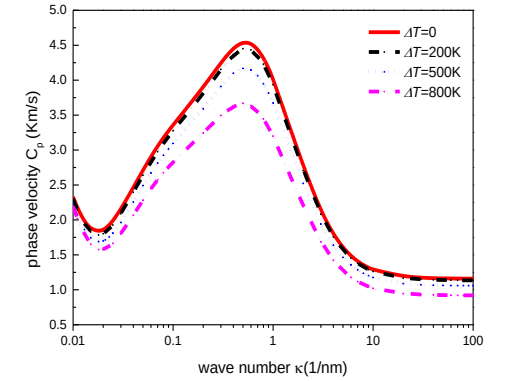
(c) One nanotube fixed

Fig. 5 Flexural phase velocity vs. wavenumber for different moisture rise at $\beta = 0.1$, $K = 1$, $R_0 = 20$ nm, $R_i = 10$ nm, $\Delta T = 0$, $ea = 1$ nm, $l = 0.2$ nm, $k = 10^9$

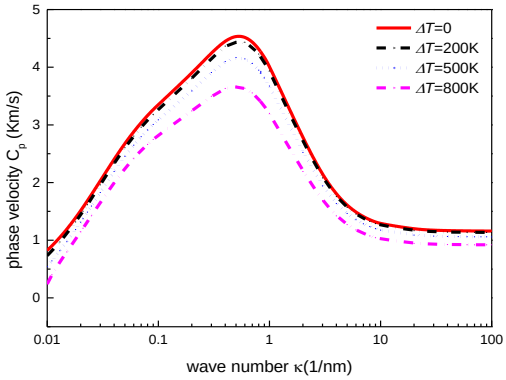
and one nanotube fixed, Fig. 4(a) and (c)), the phase velocity is related to the value of inter-layer stiffness, and the larger the value of inter-layer stiffness is, the larger the value of phase velocity is. Moreover, this effect of inter-layer stiffness on phase velocity is only significant in the case of small wave number, and for the case of large wave number, the effect of inter-layer stiffness on wave velocity is negligible. At the same time, we can see that for the in-phase case (Fig. 4(b)), the inter-layer stiffness has negligible effects on the dispersion curves, and as the inter-layer stiffness goes up, all the dispersion curves show as one curve.

The influences of temperature and humidity on the dispersion relationship are plotted in Figs. 5-6. It can be seen from these figures that the increase of temperature and humidity can reduce the phase velocity of the nanotubes. In other words, the higher the temperature is (the higher the humidity is), the slower the phase velocity is.

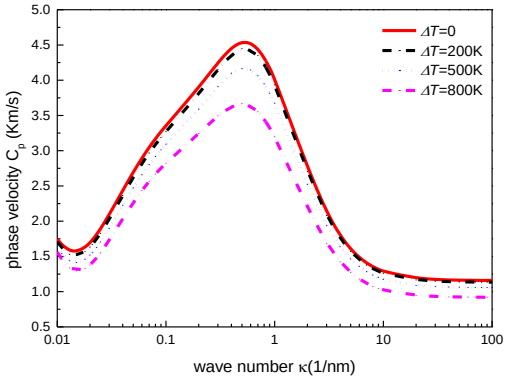
The influences of porosity and power law index on the dispersion relation are plotted in Figs. 7-8. When the wave number κ = 0.01 (See Fig. 7), the phase velocity increases with the increasing of porosity volume fraction, however, when the wave number κ = 1, the phase velocity decreases with the increasing of porosity volume fraction. In other words, as the porosity volume fraction β increases, the phase velocity can either rise or decline, depending on the wavenumber. Meanwhile, we can see from Fig. 8 that, the phase velocities decrease as the power law index K



(a) Out of phase

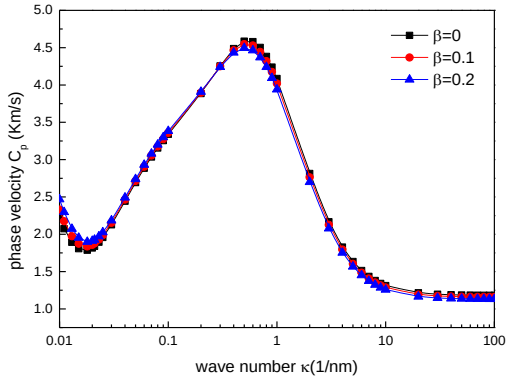


(b) In phase

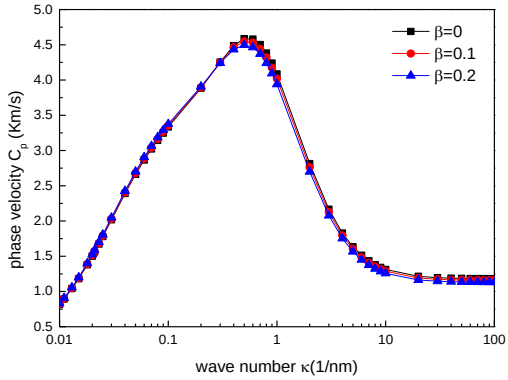


(c) One nanotube fixed

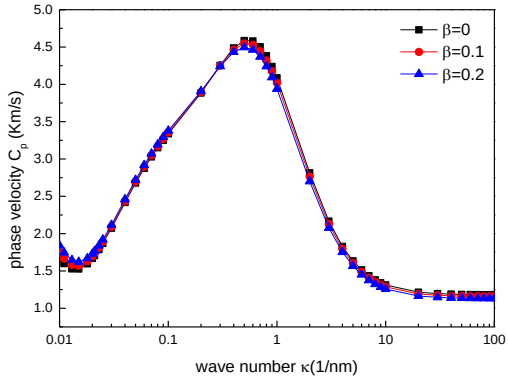
Fig. 6 Flexural phase velocity vs. wavenumber for different temperatures at β = 0.1, R_0 = 20 nm, R_i = 10 nm, ΔC = 0, K = 1, ea = 1 nm, l = 0.2 nm, k = 10^9



(a) Out of phase

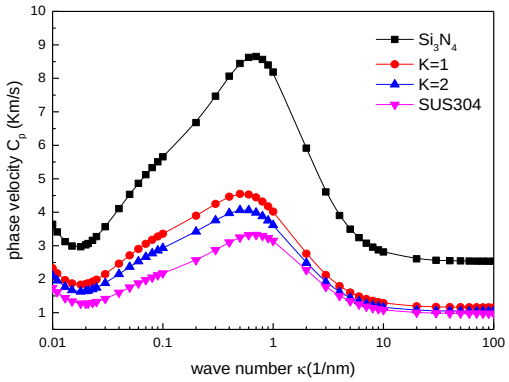


(b) In phase

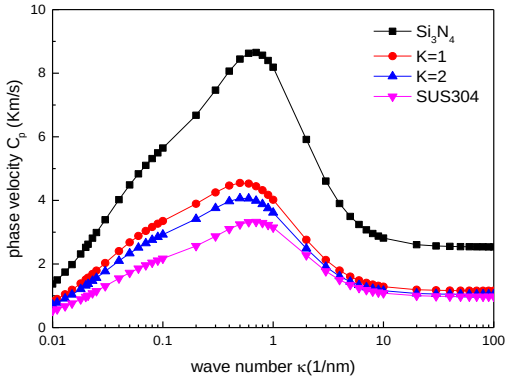


(c) One nanotube fixed

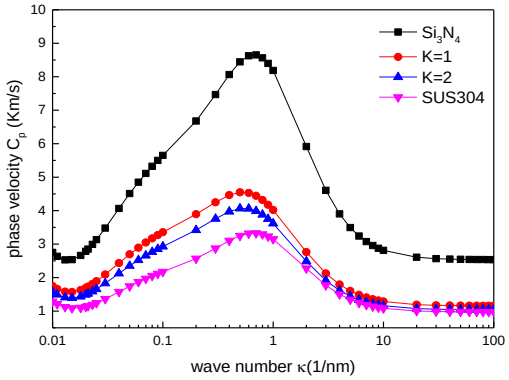
Fig. 7 Flexural phase velocity vs. wavenumber for different porosity volume fraction at K = 1, R_0 = 20 nm, R_i = 10 nm, ΔT = ΔC = 0, ea = 1 nm, l = 0.2 nm, k = 10^9.



(a) Out of phase



(b) In phase



(c) One nanotube fixed

Fig. 8 Flexural phase velocity vs. wavenumber for different power law index at $\beta = 0.1$, $R_0 = 20$ nm, $R_i = 10$ nm, $\Delta T = \Delta C = 0$, $ea = 1$ nm, $l = 0.2$ nm, $k = 10^9$

increases. These phenomena show that the porosity volume fraction and material composition have significant effect on the dispersion relations of double-layered nanotubes.

7. Conclusions

This research work studies the transverse wave propagation in the double-layered nanotubes systems by using the nonlocal strain gradient theory. Based on the above analysis, we can draw the following conclusions

(1) The small scaled parameters on phase velocity are negligible for small wavenumbers, but the effects are remarkable for large wavenumbers.
(2) The power law index have significant influences on the phase velocity.
(3) As the porosity volume fraction β increases, the phase velocity can either rise or decline, depending on the wavenumber.
(4) The presence of humidity and temperature can reduce the phase velocity of the solid waves.
(5) In the case of large wavenumber, the influence of motion type on wave velocity is not obvious. Only for small wavenumber, the in-phase type and the out-of-phase type give the minimum and maximum phase velocities respectively.

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