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A simple quasi-3D HSDT for the dynamics analysis of FG thick plate on elastic foundation

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Abstract. This work presents a dynamic investigation of functionally graded (FG) plates resting on elastic foundation using a simple quasi-3D higher shear deformation theory (quasi-3D HSDT) in which the stretching effect is considered. The culmination of this theory is that in addition to taking into account the effect of thickness extension ($\varepsilon_z \neq 0$), the kinematic is defined with only 4 unknowns, which is even lower than the first order shear deformation theory (FSDT). The elastic foundation is included in the formulation using the Pasternak mathematical model. The governing equations are deduced through the Hamilton's principle. These equations are then solved via closed-type solutions of the Navier type. The fundamental frequencies are predicted by solving the eigenvalue problem. The degree of accuracy of present solutions can be shown by comparing it to the 3D solution and other closed-form solutions available in the literature.

Keywords: vibration; FG plate; Quasi-3D HSDT

1. Introduction

Functionally graded materials (FGMs) are a class of heterogeneous composite material in which properties vary progressively in one or more directions. This material is fabricated by mixing two or more materials in a certain volume ratio (e.g., metal and ceramic). Conventional composite structures such as fiber-reinforced plastics suffer from a discontinuity in the properties of materials at the interface of plies and constituents. These problems can be mitigated by progressively modifying the volume fraction of the constituent materials and adapting the material to the desired application (Li and Batra 2013, Ahmed 2014, Zidi *et al*. 2014, Wattanasakulpong and Ungbhakorn 2014, Kar and Panda 2015, Kolahchi *et al*. 2016a, b, Bilouei *et al*. 2016, Bousahla *et al*. 2016, Madani *et al*. 2016, Zamanian *et al*. 2017, Aldousari 2017, Khetir *et al*. 2017, Karami *et al*. 2017, Fahsi *et al*. 2017, Kolahchi and Cheraghbak 2017, Shokravi 2017a, b, Sekkal *et al*. 2017a, Avcar and Mohammed 2018, Selmi and Bisharat 2018, Soliman *et al*. 2018, Hajmohammad *et al*. 2018a, b, c, d, Golabchi *et al*. 2018, Karami *et al*. 2018a, b, 2019, Bakhadda *et al*. 2018, Amnieh *et al*. 2018, Bensattalah *et al*. 2018, Hadji *et al*. 2019, Gao *et al*. 2019, Avcar 2019).

Several investigations have been established to examine the vibration of FG plates. Vel and Batra (2004), employed a 3D exact solution for free and forced vibrations of simply supported FG plates. Ferreira *et al*. (2006), discussed the dynamic responses of FG plates by using a global collocation technique, the FSDT and the third-order shear deformation plate theory. Qian *et al*. (2004), studied bending, and free and forced vibrations of a thick FG plate by employing a higher order shear and normal deformable plate model. Matsunaga (2008), analyzed the vibration and buckling of FG plates by considering the influences of transverse shear and normal deformations and rotatory inertia. Zhao *et al*. (2009), presented a dynamic investigation of metal and ceramic FG plates by utilizing the element-free kp-Ritz procedure. Chen *et al*. (2009), studied the vibration and stability of FG plates based on a HSDT. Neves *et al*. (2012a, b), used a sinusoidal shear deformation model and a hybrid quasi-3D hyperbolic shear deformation theory for static and dynamic analysis of FG plates. It should be noted that several shear deformation theories have been developed by different researchers to study the mechanical behavior of structures (Bouderba *et al*. 2013, Belkorissat *et al*. 2015, Kolahchi and Moniri Bidgoli 2016, Sayyad *et al*. 2016, Bellifa *et al*. 2016, 2017a, b, Arani and Kolahchi 2016, Ahouel *et al*. 2016, Kolahchi *et al*. 2016a, b, 2017a, b, c, Bouderba *et al*. 2016, Kolahchi

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2017, Benadouda *et al*. 2017, Hajmohammad *et al*. 2017, Abdelaziz *et al*. 2017, Shokravi 2017c, d, Bensaid *et al*. 2018, Belabed *et al*. 2018, Kadari *et al*. 2018, Bourada *et al*. 2018, Zine *et al*. 2018, Bouadi *et al*. 2018, Karami *et al*. 2018c, Mokhtar *et al*. 2018, Shi *et al*. 2018, Karami and Karami 2019). In addition, other theories called "quasi 3D HSDTs" are also developed for the same aim (Belabed *et al*. 2014, Bousahla *et al*. 2014, Hebali *et al*. 2014, Bourada *et al*. 2015, Hamidi *et al*. 2015, Akavci 2016, Draiche *et al*. 2016, Adim and Daouadji 2016, Bennoun *et al*. 2016, Karamanlı 2017, Benahmed *et al*. 2017, Bouafia *et al*. 2017, Sekkal *et al*. 2017b, Hadji *et al*. 2018, Wu and Yu 2018, Bouhadra *et al*. 2018, Karami *et al*. 2018d, e, Younsi *et al*. 2018, Benchohra *et al*. 2018, Abualnour *et al*. 2018, Yu *et al*. 2019, Zaoui *et al*. 2019, Khiloun *et al*. 2019, Tlidji *et al*. 2019).

On the other hand, elastic foundations have many applications in engineering. Winkler (1867) proposed a one-parameter model for describing the mechanical response of elastic foundations, while Pasternak (1954) proposed a two-parameter model. In addition, this model takes into account the shear deformation between the springs. Therefore, the Winkler model can be seen as a particular case of the Pasternak model by considering the shear modulus to zero.

From this decade, Hosseini-Hashemi *et al*. (2010) proposed an analytical solution for dynamic analysis of moderately thick plates composed of FGMs and resting on Winkler or Pasternak elastic foundations. Lu *et al*. (2009), presented dynamic analysis of FG thick plates on elastic foundation based on 3D elasticity. Malekzadeh (2009), analyzed the free vibration of thick FG plates resting by two-parameter elastic foundation based on the 3D elasticity model. Hasani Baferani *et al*. (2011) gave a dynamic investigation of FG plates resting on Pasternak elastic foundation employing the Reddy's HSDT. Thai and Choi (2012) employed a HSDT for dynamic of FG plates on elastic foundation. Sheikholeslami and Saidi (2013) investigated the dynamic behavior of FG plates resting on two-parameter elastic foundation using a HSDT and an analytical approach. Akavci (2014) presented a dynamic study of FG plates on elastic foundation using a non-polynomial HSDT and an optimization procedure. Beldjelili *et al*. (2016) discussed the hygro-thermo-mechanical bending of S-FGM plates resting on variable elastic foundations. Akavci (2016) used a new hyperbolic shear and normal deformation plate theory to examine the bending, free vibration and buckling response of the simply supported FG sandwich plates on elastic foundation. Besseghier *et al*. (2017) analyzed the free vibration behavior of nanosize FG plates resting on elastic foundations. Attia *et al*. (2018) presented a thermoelastic analysis of FG plates resting on variable elastic foundations.

In the current work, a generalized approach with stretching influence for the dynamic analysis of FG plates on elastic foundation is provided. The highlight of this formulation is that, in addition to introducing the thickness stretching influence ($\varepsilon_z \neq 0$), the kinematic is modeled with only 4 variables, which is even less than the FSDT and do not use shear correction factor. The mechanical properties of the structures are considered to change in the thickness direction according to a power law variation in terms of the volume fractions of the constituents. The governing equations of FG plates supported by the elastic foundation are obtained by using the Hamilton's principle. These motion equations are then solved via Navier method. As a result, fundamental frequencies are determined by solving eigenvalue problem. The accuracy of the present code is checked by comparing it with HSDT's solutions available in literature.

2. Theoretical formulation

2.1 Functionally graded plates

A rectangular plate of uniform thickness "*h*", length "*a*" and width "*b*", made of an FGM and resting on an elastic base is considered in this study. The rectangular Cartesian coordinate system *x*, *y*, *z* has the plane *z* = 0, which coincides with the median surface of the plate. The properties of the material vary across the thickness according to a power law distribution, which is expressed below (Tounsi *et al*. 2013, Yahia *et al*. 2015, El-Haina *et al*. 2017, Mouffoki *et al*. 2017)

$$P_{(z)} = \left(P_t - P_b\right) V_{(z)} + P_b \tag{1a}$$

$$V_{(z)} = \left(\frac{z}{h} + \frac{1}{2}\right)^p \quad -\frac{h}{2} \le z \le \frac{h}{2} \tag{1b}$$

where *P* presents the effective material characteristic, P_t and P_b present the property of the upper and lower faces of the structure, respectively, and *p* is the material index that defines the material distribution profile across the thickness. The effective material characteristics of the plate, such as "*Young's modulus*", *E*, and shear modulus, *G*, vary according to Eq. (1), and Poisson ratio, *v* is considered to be constant.

2.2 Displacement field

The kinematic respecting the conditions of transverse shear stresses disappearing at a point (*x*, *y*, ±*h*/2) on the external and internal surfaces of the plate, is expressed as follows (Menasria *et al*. 2017, Chikh *et al*. 2017, Fourn *et al*. 2018, Yazid *et al*. 2018, Meksi *et al*. 2019)

$$u(x,y,z) = u_0(x,y) - z\frac{\partial w_0}{\partial x} + k_1\, f(z)\int \theta(x,y)\,dx \tag{2a}$$

$$v(x,y,z) = v_0(x,y) - z\frac{\partial w_0}{\partial y} + k_2\, f(z)\int \theta(x,y)\,dy \tag{2b}$$

$$w(x,y,z) = w_0(x,y) + g(z)\theta(x,y) \tag{2c}$$

The coefficient k_1 and k_2 depends on the geometry. It can be observed that the kinematic in Eq. (2) includes only four unknowns (u_0, v_0, w_0 and θ) with considering the thickness stretching influence. In this work, the proposed quasi-3D HSDT is obtained by setting

- Model 1:

$$f(z) = -\left[\frac{3\pi}{2} z\,\mathrm{sech}^2\left(\frac{1}{2}\right)\right] + \frac{3\pi}{2} h \tanh\left(\frac{z}{h}\right); \qquad g(z) = \frac{2}{15}\frac{df}{dz} \tag{3a}$$

- Model 2:

$$f(z) = \frac{4h}{5}\sinh\left(\frac{5}{4h}z\right) + z\left[-\cosh\left(\frac{5}{8}\right) + \frac{3}{20}\cos\left(\frac{5}{8}\right)\right]; \quad g(z) = -\frac{3}{20}\cos\left(\frac{5}{4h}z\right) \tag{3b}$$

2.3 Kinematic and constitutive relations

In this part, small strains assumption is considered for the derivation of the needed. The linear strain relations obtained from the kinematic model of Eq. (2), are as follows

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} = \begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} + z\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix} + f(z)\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix}, \quad \begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} = f^{'}(z)\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix} + g(z)\begin{Bmatrix}\gamma_{yz}^1\\ \gamma_{xz}^1\end{Bmatrix}, \qquad \varepsilon_z = g'(z)\,\varepsilon_z^0 \tag{4}$$

where

$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} = \begin{Bmatrix}\dfrac{\partial u_0}{\partial x}\\ \dfrac{\partial v_0}{\partial x}\\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x}\end{Bmatrix}; \qquad \begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix} = \begin{Bmatrix}-\dfrac{\partial^2 w_0}{\partial x^2}\\ -\dfrac{\partial^2 w_0}{\partial y^2}\\ -2\dfrac{\partial^2 w_0}{\partial x \partial y}\end{Bmatrix}; \quad \begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix} = \begin{Bmatrix}k_1\theta\\ k_2\theta\\ k_1\dfrac{\partial}{\partial y}\displaystyle\int\theta\,dx + k_2\dfrac{\partial}{\partial x}\displaystyle\int\theta\,dy\end{Bmatrix} \tag{5a}$$

$$\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix} = \begin{Bmatrix}k_2\displaystyle\int\theta\,dy\\ k_1\displaystyle\int\theta\,dx\end{Bmatrix}, \qquad \begin{Bmatrix}\gamma_{yz}^1\\ \gamma_{xz}^1\end{Bmatrix} = \begin{Bmatrix}\dfrac{\partial\theta}{\partial y}\\ \dfrac{\partial\theta}{\partial x}\end{Bmatrix}, \qquad \varepsilon_z^0 = \theta \quad \text{and} \quad g'(z) = \frac{dg(z)}{dz} \tag{5b}$$

The integrals employed in the "*above equations*" can be resolved by a Navier type procedure and are expressed as follows

$$\frac{\partial}{\partial y}\int\theta\,dx = A'\frac{\partial^2\theta}{\partial x\partial y}, \qquad \frac{\partial}{\partial x}\int\theta\,dy = B'\frac{\partial^2\theta}{\partial x\partial y}, \qquad \int\theta\,dx = A'\frac{\partial\theta}{\partial x}, \qquad \int\theta\,dy = B'\frac{\partial\theta}{\partial y} \tag{6}$$

In Eq. (6), the coefficients A' and B' are utilized according to the type of solution employed, in this case via Navier method. Hence, A', B', k_1 and k_2 are expressed as follows

$$A' = -\frac{1}{\alpha^2}, \quad B' = -\frac{1}{\beta^2}, \quad k_1 = -\alpha^2, \quad k_2 = -\beta^2 \tag{7}$$

Where α and β are defined in expression (23).

For the functionally graded plates, the stress-strain relationships for plane-stress state can be expressed as

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{xy}\\ \tau_{xz}\\ \tau_{yz}\end{Bmatrix} = \begin{bmatrix}C_{11} & C_{12} & C_{13} & 0 & 0 & 0\\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0\\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & C_{66} & 0 & 0\\ 0 & 0 & 0 & 0 & C_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & C_{44}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{Bmatrix} \tag{8}$$

The engineering constants C_{ij} are given as follows

$$C_{11} = C_{22} = C_{33} = \frac{(1-\nu)E(z)}{(1-2\nu)(1+\nu)}, \tag{9a}$$

$$C_{12} = C_{13} = C_{23} = \frac{\nu\,E(z)}{(1-2\nu)(1+\nu)}, \tag{9b}$$

$$C_{44} = C_{55} = C_{66} = \frac{E(z)}{2(1+\nu)}, \tag{9c}$$

2.4 Equations of motion

Hamilton's principle is herein utilized to determine the equations of motion (Meziane *et al*. 2014, Al-Basyouni *et al*. 2015, Zemri *et al*. 2015, Attia *et al*. 2015, Mahi *et al*. 2015, Larbi Chaht *et al*. 2015, Boukhari *et al*. 2016, Houari *et al*. 2016, Bounouara *et al*. 2016, Youcef *et al*. 2018, Zidi *et al*. 2017, Hachemi *et al*. 2017, Klouche *et al*. 2017, Cherif *et al*. 2018, Kaci *et al*. 2018, Bourada *et al*. 2019)

$$0 = \int_t^0 (\delta U + \delta V_e - \delta K)\,dt \tag{10}$$

where δU is the variation of strain energy, δV_e is the variation of potential energy of elastic foundation and δK is the variation of kinetic energy.

The variation of stain energy of structure is expressed by

$$\delta U = \int_V \begin{pmatrix}\sigma_{xx}\delta\varepsilon_{xx} + \sigma_{yy}\delta\varepsilon_{yy} + \sigma_{zz}\delta\varepsilon_{zz} + \tau_{xy}\delta\gamma_{xy}\\ +\tau_{yz}\delta\gamma_{yz} + \tau_{xz}\delta\gamma_{xz}\end{pmatrix}dV = \int_\Omega \begin{bmatrix}N_x\delta\varepsilon_x^0 + N_y\delta\varepsilon_y^0 + N_{xy}\delta\varepsilon_{xy}^0 + M_x\delta k_x^b + M_y\delta k_y^b\\ +M_{xy}\delta k_{xy}^b + S_x\delta k_x^s + S_y\delta k_y^s + S_{xy}\delta k_{xy}^s + Q_{zz}\varepsilon_z^0\\ +G_{yz}\gamma_{yz}^1 + G_{xz}\gamma_{xz}^1 + Y_{yz}\gamma_{yz}^0 + Y_{xz}\gamma_{xz}^0\end{bmatrix}dA \tag{11}$$

Where Ω is the top surface and the stress resultants N, M, S and Q are defined by

$$(N_i, M_i, S_i) = \int_{-h/2}^{h/2} (1, z, f)\sigma_i d_z, (i = x, y, xy) \tag{12a}$$

$$Q_z = \int_{-h/2}^{h/2} g^{'}(z)\sigma_z dz \tag{12b}$$

$$\left(G_{xz}, G_{yz}\right) = \int_{-h/2}^{h/2} g(\tau_{xz}, \tau_{yz})dz \tag{12c}$$

$$(Y_{xz}, Y_{yz}) = \int_{-h/2}^{h/2} f^{'}(\tau_{xz}, \tau_{yz})dz \tag{12d}$$

The variation of kinetic energy of the structure can be written as

$$\begin{aligned}
\delta K &= \frac{1}{2}\int_V \rho(z)\left[\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial v}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2\right]dV \\
&= \int_V \left[\dot{u}\delta\dot{u} + \dot{v}\delta\dot{v} + \dot{w}\delta\dot{w}\right]\rho(z)dV \\
&= \int_A \ \{I_1\left[\dot{u}_0\delta\dot{u}_0 + \dot{v}_0\delta\dot{v}_0 + \dot{w}_0\delta\dot{w}_0\right] \\
&-I_2\left(\dot{u}_0\frac{\partial\delta\dot{w}_0}{\partial x} + \frac{\partial\dot{w}_0}{\partial x}\delta\dot{u}_0 + \dot{v}_0\frac{\partial\delta\dot{w}_0}{\partial y} + \frac{\partial\dot{w}_0}{\partial y}\delta\dot{v}_0\right) \\
&+I_3\left(\frac{\partial\dot{w}_0}{\partial x}\frac{\partial\delta\dot{w}_0}{\partial x} + \frac{\partial\dot{w}_0}{\partial y}\frac{\partial\delta\dot{w}_0}{\partial y}\right) \\
&+I_4\begin{bmatrix}(k_1A^{'})\left(\dot{u}_0\frac{\partial\delta\dot{\theta}_0}{\partial x} + \frac{\partial\dot{\theta}_0}{\partial x}\delta\dot{u}_0\right) \\ +(k_2B^{'})\left(\dot{v}_0\frac{\partial\delta\dot{\theta}_0}{\partial y} + \frac{\partial\dot{\theta}_0}{\partial y}\delta\dot{v}_0\right)\end{bmatrix} \\
&+I_5\begin{bmatrix}(k_1A^{'})\left(\frac{\partial\dot{w}_0}{\partial x}\frac{\partial\delta\dot{\theta}_0}{\partial x} + \frac{\partial\dot{\theta}_0}{\partial x}\frac{\partial\delta\dot{w}_0}{\partial x}\right) \\ +(k_2B^{'})\left(\frac{\partial\dot{w}_0}{\partial y}\frac{\partial\delta\dot{\theta}_0}{\partial y} + \frac{\partial\dot{\theta}_0}{\partial y}\frac{\partial\delta\dot{w}_0}{\partial y}\right)\end{bmatrix} \\
&+I_6\left[(k_1A^{'})^2\left(\frac{\partial\dot{\theta}_0}{\partial x}\frac{\partial\delta\dot{\theta}_0}{\partial x}\right) + (k_2B^{'})^2\left(\frac{\partial\dot{\theta}_0}{\partial y}\frac{\partial\delta\dot{\theta}_0}{\partial y}\right)\right] \\
&+I_7\left(\dot{w}_0\delta\dot{\theta}_0 + \dot{\theta}_0\delta\dot{w}_0\right) + I_8\dot{\theta}_0\delta\dot{\theta}_0\}dA
\end{aligned} \tag{13}$$

where dot- superscript convention presents the differentiation with respect to the time variable t, $\rho(z)$ is the mass density given by Eq. (1) ; and I_j are mass inertias expressed by

$$\begin{aligned}
&(I_1, I_2, I_3, I_4, I_5, I_6, I_7, I_8) = \\
&\int_{-h/2}^{h/2} \rho(z)(1, z, z^2, f, zf, f^2, g, g^2)dz
\end{aligned} \tag{14}$$

The variation of potential energy of elastic foundation can be written as

$$\begin{aligned}
\delta V_e &= \int_A \left[k_0^w w\delta w + k_1^w\left(\frac{\partial w}{\partial x}\frac{\partial\delta w}{\partial x} + \frac{\partial w}{\partial y}\frac{\partial\delta w}{\partial y}\right)\right]dA \\
&= \int_A \begin{bmatrix} k_0^w\left[w_0\delta w_0 + q^*\left(w_0\delta\theta + \theta\delta w_0\right) + q^{*2}\theta\delta\theta\right] + \\ k_1^w \begin{bmatrix} \frac{\partial w_0}{\partial x}\frac{\partial\delta w_0}{\partial x} + \frac{\partial w_0}{\partial y}\frac{\partial\delta w_0}{\partial y} \\ +q^*\begin{pmatrix}\frac{\partial\theta}{\partial x}\frac{\partial\delta w_0}{\partial x} + \frac{\partial\theta}{\partial y}\frac{\partial\delta w_0}{\partial y} + \\ \frac{\partial w_0}{\partial x}\frac{\partial\delta\theta}{\partial x} + \frac{\partial w_0}{\partial y}\frac{\partial\delta\theta}{\partial y}\end{pmatrix} \\ +q^{*2}\left(\frac{\partial\theta}{\partial x}\frac{\partial\delta\theta}{\partial x} + \frac{\partial\theta}{\partial y}\frac{\partial\delta\theta}{\partial y}\right)\end{bmatrix}\end{bmatrix} dA
\end{aligned} \tag{15}$$

Where k_0^w and k_1^w are the Winkler foundation stiffness and the shear stiffness of the elastic foundation and $q^* = -g(h/2)$.

By substituting Eqs. (11), (13) and (15) into Eq. (10) the following equations of motion can be obtained

$$\begin{aligned}
&\delta u_0 : \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_1\ddot{u}_0 - I_2\frac{\partial\ddot{w}_0}{\partial x} + I_4k_1A^{'}\frac{\partial\ddot{\theta}}{\partial x} \\
&\delta v_0 : \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_1\ddot{v}_0 - I_2\frac{\partial\ddot{w}_0}{\partial y} + I_4k_2B^{'}\frac{\partial\ddot{\theta}}{\partial y} \\
&\delta w_0 : \frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_y}{\partial y^2} + 2\frac{\partial^2 M_{xy}}{\partial x\partial y} - k_0^w\left(w_0 + q^*\theta\right) \\
&+k_1^w\left(\frac{\partial^2 w_0}{\partial x^2} + q^*\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2 w_0}{\partial y^2} + q^*\frac{\partial^2\theta}{\partial y^2}\right) = I_1\ddot{w}_0 + I_2\left(\frac{\partial\ddot{u}_0}{\partial x} + \frac{\partial\ddot{v}_0}{\partial y}\right) \\
&-I_3\left(\frac{\partial^2\ddot{w}_0}{\partial x^2} + \frac{\partial^2\ddot{w}_0}{\partial y^2}\right) + I_5\left(k_1A^{'}\frac{\partial^2\ddot{\theta}}{\partial x^2} + k_2B^{'}\frac{\partial^2\ddot{\theta}}{\partial y^2}\right) - I_7\ddot{\theta} \\
&\delta\theta : -k_1A^{'}\frac{\partial^2 S_x}{\partial x^2} - k_2B^{'}\frac{\partial^2 S_y}{\partial y^2} - \left(k_1A^{'} + k_2B^{'}\right)\frac{\partial^2 S_{xy}}{\partial x\partial y} - Q_{zz} \\
&+k_1A^{'}\frac{\partial Y_{xz}}{\partial x} + k_2B^{'}\frac{\partial Y_{yz}}{\partial y} + \frac{\partial G_{xz}}{\partial x} + \frac{\partial G_{yz}}{\partial y} - q^*k_0^w w_0 - q^{*2}k_0^w\theta \\
&+q^*k_1^w\frac{\partial w_0}{\partial x^2} + q^{*2}k_1^w\frac{\partial^2\theta}{\partial x^2} + q^*k_1^w\frac{\partial w_0}{\partial y^2} + q^{*2}k_1^w\frac{\partial^2\theta}{\partial y^2} = \\
&-I_4\left(k_1A^{'}\frac{\partial^2\ddot{u}_0}{\partial x^2} + k_2B^{'}\frac{\partial^2\ddot{v}_0}{\partial y^2}\right) + I_5\left(k_1A^{'}\frac{\partial^2\ddot{w}_0}{\partial x^2} + k_2B^{'}\frac{\partial^2\ddot{w}_0}{\partial y^2}\right) \\
&-I_6\left[\left(k_1A^{'}\right)^2\frac{\partial^2\ddot{\theta}}{\partial x^2} + \left(k_2B^{'}\right)^2\frac{\partial^2\ddot{\theta}}{\partial y^2}\right] + I_7\ddot{w}_0 + I_8\ddot{\theta}
\end{aligned} \tag{16}$$

By substituting Eq. (4) into Eq. (8) and the subsequent results into Eq. (12), the stress resultants are obtained in terms of strains as following compact form

$$\begin{aligned}
&\begin{Bmatrix} N \\ M^b \\ M^s \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}\begin{Bmatrix} \varepsilon \\ k^b \\ k^s \end{Bmatrix} + \varepsilon^0\begin{Bmatrix} L \\ L^a \\ R \end{Bmatrix}, \\
&\begin{Bmatrix} Q \\ S \end{Bmatrix} = \begin{bmatrix} F^s & X^s \\ X^s & A^s \end{bmatrix}\begin{Bmatrix} \gamma^0 \\ \gamma^1 \end{Bmatrix}
\end{aligned} \tag{17}$$

In which

$$
N=\left\{N_x,N_y,N_{xy}\right\}^t,\quad M^b=\left\{M_x^b,M_y^b,M_{xy}^b\right\}^t,\\
M^s=\left\{M_x^s,M_y^s,M_{xy}^s\right\}^t \tag{18a}
$$

$$
\varepsilon=\left\{\varepsilon_x^0,\varepsilon_y^0,\gamma_{xy}^0\right\}^t,\quad k^b=\left\{k_x^b,k_y^b,k_{xy}^b\right\}^t\\
k^s=\left\{k_x^s,k_y^s,k_{xy}^s\right\}^t \tag{18b}
$$

$$
A=\begin{bmatrix} A_{11} & A_{12} & 0\\ A_{12} & A_{22} & 0\\ 0 & 0 & A_{66}\end{bmatrix},\quad B=\begin{bmatrix} B_{11} & B_{12} & 0\\ B_{12} & B_{22} & 0\\ 0 & 0 & B_{66}\end{bmatrix},\\
D=\begin{bmatrix} D_{11} & D_{12} & 0\\ D_{12} & D_{22} & 0\\ 0 & 0 & D_{66}\end{bmatrix} \tag{18c}
$$

$$
B^s=\begin{bmatrix} B_{11}^s & B_{12}^s & 0\\ B_{12}^s & B_{22}^s & 0\\ 0 & 0 & B_{66}^s\end{bmatrix},\quad D^s=\begin{bmatrix} D_{11}^s & D_{12}^s & 0\\ D_{12}^s & D_{22}^s & 0\\ 0 & 0 & D_{66}^s\end{bmatrix},\\
H^s=\begin{bmatrix} H_{11}^s & H_{12}^s & 0\\ H_{12}^s & H_{22}^s & 0\\ 0 & 0 & H_{66}^s\end{bmatrix} \tag{18d}
$$

$$
Q=\left\{Q_{xz}^s,Q_{yz}^s\right\}^t,\quad S=\left\{S_{xz}^s,S_{yz}^s\right\}^t\\
\gamma^0=\left\{\gamma_{xz}^0,\gamma_{yz}^0\right\}^t,\quad \gamma^1=\left\{\gamma_{xz}^1,\gamma_{yz}^1\right\}^t \tag{18e}
$$

$$
F^s=\begin{bmatrix} F_{55}^s & 0\\ 0 & F_{44}^s\end{bmatrix},\quad X^s=\begin{bmatrix} X_{55}^s & 0\\ 0 & X_{44}^s\end{bmatrix},\\
A^s=\begin{bmatrix} A_{55}^s & 0\\ 0 & A_{44}^s\end{bmatrix} \tag{18f}
$$

$$
\begin{Bmatrix} L\\ L^a\\ R\\ R^a\end{Bmatrix}=\int_z \lambda(z)\begin{Bmatrix} 1\\ z\\ f(z)\\ g'(z)\dfrac{1-\nu}{\nu}\end{Bmatrix}g'(z)dz \tag{18g}
$$

and stiffness components are given as

$$
\begin{Bmatrix} A_{11} & B_{11} & D_{11} & B_{11}^s & D_{11}^s & H_{11}^s\\ A_{12} & B_{12} & D_{12} & B_{12}^s & D_{12}^s & H_{12}^s\\ A_{66} & B_{66} & D_{66} & B_{66}^s & D_{66}^s & H_{66}^s\end{Bmatrix}=\\
\int_z \lambda(z)\left(1,z,z^2,f(z),z\,f(z),f^2(z)\right)\begin{Bmatrix}\dfrac{1-\nu}{\nu}\\ \nu\\ \dfrac{1-2\nu}{2\nu}\end{Bmatrix}dz \tag{19a}
$$

$$
\left(A_{22},B_{22},D_{22},B_{22}^s,D_{22}^s,H_{22}^s\right)\\
=\left(A_{11},B_{11},D_{11},B_{11}^s,D_{11}^s,H_{11}^s\right) \tag{19b}
$$

$$
\left(F_{44}^s,X_{44}^s,A_{44}^s\right)=\\
\int_{-h/2}^{h/2}\frac{E(z)}{2(1+\nu)}\left([f'(z)]^2,f'(z)g(z),g^2(z)\right)dz \tag{19c}
$$

$$
\left(F_{55}^s,X_{55}^s,A_{55}^s\right)=\left(F_{44}^s,X_{44}^s,A_{44}^s\right) \tag{19d}
$$

Introducing Eq. (17) into Eq. (16), the equations of motion can be expressed in terms of displacements (u_0, v_0, w_0, θ) and the appropriate equations take the form

$$
A_{11}d_{11}u_0+A_{66}\,d_{22}u_0+\left(A_{12}+A_{66}\right)d_{12}v_0-B_{11}d_{111}w_0\\
-\left(B_{12}+2B_{66}\right)d_{122}w_0+\left(B_{66}^s\left(k_1A'+k_2\,B'\right)+B_{12}^s k_2\,B'\right)d_{122}\theta\\
+B_{11}^s k_1A'\,d_{111}\theta+Ld_1\theta=I_1u_0-I_2\frac{\partial\ddot{w}_0}{\partial x}+I_4k_1A'\frac{\partial\ddot{\theta}}{\partial x} \tag{20a}
$$

$$
A_{22}\,d_{22}v_0+A_{66}\,d_{11}v_0+\left(A_{12}+A_{66}\right)d_{12}u_0-B_{22}\,d_{222}w_0\\
-\left(B_{12}+2B_{66}\right)d_{112}w_0+\left(B_{66}^s\left(k_1A'+k_2\,B'\right)+B_{12}^s k_1\,A'\right)d_{112}\theta\\
+B_{22}^s k_2\,B'd_{222}\theta+Ld_2\theta=I_1v_0-I_2\frac{\partial\ddot{w}_0}{\partial y}+I_4k_2\,B'\frac{\partial\ddot{\theta}}{\partial y} \tag{20b}
$$

$$
B_{11}\,d_{111}u_0+\left(B_{12}+2B_{66}\right)d_{122}u_0+\left(B_{12}+2B_{66}\right)d_{112}v_0+\\
B_{22}\,d_{222}v_0-D_{11}d_{1111}w_0-2\left(D_{12}+2D_{66}\right)d_{1122}w_0\\
-D_{22}\,d_{2222}w_0+D_{11}^s k_1\,A'd_{1111}\theta\\
+\left((D_{12}^s+2D_{66}^s)\left(k_1A'+k_2B'\right)\right)d_{1122}\theta+D_{22}^s k_2B'd_{2222}\theta\\
+L^a(d_{11}\theta+d_{22}\theta)-k_0^w\left(w_0+q^*\theta\right)\\
+k_1^w\left(\frac{\partial^2w_0}{\partial x^2}+q^*\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2w_0}{\partial y^2}+q^*\frac{\partial^2\theta}{\partial y^2}\right)\\
=I_1\ddot{w}_0+I_2\left(\frac{\partial\ddot{u}}{\partial x}+\frac{\partial\ddot{v}}{\partial y}\right)-I_3\left(\frac{\partial^2\ddot{w}_0}{\partial x^2}+\frac{\partial^2\ddot{w}_0}{\partial y^2}\right)\\
+I_5\left(k_1\,A'\frac{\partial^2\ddot{\theta}}{\partial x^2}+k_2\,B'\frac{\partial^2\ddot{\theta}}{\partial y^2}\right)+I_7\ddot{\theta} \tag{20c}
$$

$$
-k_1A'B_{11}^s d_{111}u_0-\left(B_{12}^s k_2B'+B_{66}^s\left(k_1A'+k_2B'\right)\right)d_{122}u_0\\
-\left(B_{22}^s k_1A'+B_{66}^s\left(k_1A'+k_2B'\right)\right)d_{112}v_0-B_{22}^s k_2B'd_{222}v_0\\
+D_{11}^s k_1A'd_{1111}w_0+\left((D_{12}^s+2D_{66}^s)\left(k_1A'+k_2B'\right)\right)d_{1122}w_0\\
+D_{22}^s k_2\,B'd_{2222}w_0-H_{11}^s\,(k_1A')^2\,d_{1111}\theta-H_{22}^s\,(k_2B')^2\,d_{2222}\theta\\
-\left(2\,H_{12}^s\,k_1\,k_2A'B'+\left(k_1A'+k_2B'\right)^2H_{66}^s\right)d_{1122}\theta\\
+\left((k_1A')^2F_{55}^s+2k_1A'X_{55}^s+A_{55}^s\right)d_{11}\theta\\
+\left((k_2B')^2F_{44}^s+2k_2B'X_{44}^s+A_{44}^s\right)d_{22}\theta\\
-2R(k_1A'd_{11}\theta+k_2B'd_{11}\theta)-L(d_1u_0+d_2v_0)\\
+L^a(d_{11}w_0+d_{22}w_0)-R^a\theta\\
-k_0\left(q^*w_0+q^{*2}\theta\right)+k_1^w\left(q^*\frac{\partial^2w_0}{\partial x^2}+q^{*2}\frac{\partial^2\theta}{\partial x^2}+q^*\frac{\partial^2w_0}{\partial y^2}+q^{*2}\frac{\partial^2\theta}{\partial y^2}\right) \tag{20d}
$$

$$=-I_4\left(k_1A'\frac{\partial \ddot{u}_0}{\partial x}+k_2B'\frac{\partial \ddot{v}_0}{\partial y}\right)+I_5\left(k_1A'\frac{\partial^2 \ddot{w}_0}{\partial x^2}+k_2B'\frac{\partial^2 \ddot{w}_0}{\partial y}\right)$$
$$-I_6\left[(k_1A')^2\frac{\partial^2\ddot{\theta}}{\partial x^2}+(k_2B')^2\frac{\partial^2\ddot{\theta}}{\partial y^2}\right]+I_7\ddot{w}_0+I_8\ddot{\theta} \tag{20d}$$

where d_{ij}, d_{ijl} and d_{ijlm} are the following differential operators

$$d_{ij}=\frac{\partial^2}{\partial x_i \partial x_j},\ d_{ijl}=\frac{\partial^3}{\partial x_i \partial x_j \partial x_l},\ d_{ijlm}=\frac{\partial^4}{\partial x_i \partial x_j \partial x_l \partial x_m},$$
$$d_i=\frac{\partial}{\partial x_i},\ (i,j,l,m=1,2). \tag{21}$$

3. Analytical solution

The Navier solution method is considered to determine the analytical solutions for which the displacement variables are expressed as product of arbitrary parameters and known trigonometric functions to respect the equations of motion and boundary conditions.

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta \end{Bmatrix}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\begin{Bmatrix} U_{mn}\cos(\alpha\, x)\sin(\beta\, y) \\ V_{mn}\sin(\alpha\, x)\cos(\beta\, y) \\ W_{mn}\sin(\alpha\, x)\sin(\beta\, y) \\ X_{mn}\sin(\alpha\, x)\sin(\beta\, y) \end{Bmatrix} \tag{22}$$

Where ω is the frequency of free vibration of the plate. With

$$\alpha=m\pi/a \quad \text{and} \quad \beta=n\pi/b \tag{23}$$

Substituting Eq. (22) into Eq. (20), the following problem is obtained

$$\left(\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix}-\omega^2\begin{bmatrix} m_{11} & 0 & m_{13} & m_{14} \\ 0 & m_{22} & m_{23} & m_{24} \\ m_{13} & m_{23} & m_{33} & m_{34} \\ m_{14} & m_{24} & m_{34} & m_{44} \end{bmatrix}\right)\begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ X_{mn} \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{24}$$

Where

$$S_{11}=A_{11}\alpha^2+A_{66}\beta^2, \qquad S_{12}=\alpha\beta\left(A_{12}+A_{66}\right),$$
$$S_{13}=-\alpha\left(B_{11}\alpha^2+(B_{12}+2B_{66})\beta^2\right),$$
$$S_{14}=\alpha\left((k_2B'B_{12}^s+\left(k_1A'+k_2B'\right)B_{66}^s)\beta^2+k_1A'B_{11}^s\alpha^2-L\right),$$
$$S_{22}=A_{66}\alpha^2+A_{22}\beta^2, \qquad S_{23}=-\beta\left(B_{22}\beta^2+(B_{12}+2B_{66})\alpha^2\right),$$
$$S_{24}=\beta\left((k_1A'B_{12}^s+\left(k_1A'+k_2B'\right)B_{66}^s)\alpha^2+k_2B'B_{22}^s\beta^2-L\right),$$
$$S_{33}=D_{11}\alpha^4+2(D_{12}+2D_{66})\alpha^2\beta^2+D_{22}\beta^4+k_0^w$$
$$+k_1^w\left(\alpha^2+\beta^2\right),$$
$$S_{34}=-k_1A'D_{11}^s\alpha^4-D_{12}^s(k_1A'+k_2B')\alpha^2\beta^2$$
$$-2D_{66}^s(k_1A'+k_2B')\alpha^2\beta^2-k_2B'D_{22}^s\beta^4+L^a(\alpha^2+\beta^2)$$
$$+q^*k_0^w+q^*k_0^w(\alpha^2+\beta^2),$$
$$S_{44}=(k_1A')^2H_{11}^s\alpha^4+(2k_1k_2A'B'H_{12}^s+(k_1A'+k_2B')^2H_{66}^s)\alpha^2\beta^2$$
$$+\left((k_1A')^2F_{55}^s+2k_1A'X_{55}^s+A_{55}^s\right)\alpha^2+(k_2B')^2H_{22}^s\beta^4+R^a$$
$$-2R(k_1A'\alpha^2+k_2B'\beta^2)+\left((k_2B')^2F_{44}^s+2k_2B'X_{44}^s+A_{44}^s\right)\beta^2$$
$$+q^{*2}k_0^w+q^{*2}k_1^w\left(\alpha^2+\beta^2\right),$$
$$m_{11}=I_1, m_{13}=-\alpha I_2, m_{14}=I_4k_1A'\alpha,$$
$$m_{22}=I_1, m_{23}=-\beta I_2, -m_{24}=I_4k_2B'\beta$$
$$m_{33}=I_1+I_3(\alpha^2+\beta^2), -m_{34}=-I_5\left(k_1A'\alpha^2+k_2B'\beta^2\right)+I_7,$$
$$-m_{44}=-I_6\left(\left(k_1A'\right)^2\alpha^2+\left(k_2B'\right)^2\beta^2\right)+I_8 \tag{25}$$

4. Numerical results and discussions

In this section, the accuracy of the current quasi-3D HSDT for a dynamic analysis is checked. Numerical examples for dynamic investigation of FG plates with different material indexes specifying the material variation profile across the thickness and several values of the thickness ratio (a/h) and the aspect ratio (a/b) are also examined. The used mechanical characteristics of ceramics and metals in FG plates are reported in Table 1. In the computations, both, homogeneous isotropic structures and FG structures are investigated. The effect of the material, thickness ratio "a/h", aspect ratio "a/b", the material index "p", Winkler and Pasternak parameters on the natural frequency is also examined.

4.1 Investigation of homogeneous isotropic plates

In this section, we analyze the particular case of homogeneous isotropic materials. Tables 2 and 3 present the eigenfrequency results obtained by the current theory for square plates. Unless otherwise indicated, for this work, the following relationships were used for the presentation of non-dimensional natural frequencies and non-dimensional foundation coefficients

$$\hat{\omega}=\omega a^2\sqrt{\rho h/D_0},\ \ \bar{K}_0=k_0^w a^4/D_0,\ \ \bar{K}_1=k_1^w a^2/D_0 \tag{26}$$

with $D_0 = Eh^3 / [12(1-v^2)]$.

Table 2 shows the results determined for the first eight non-dimensional eigenfrequencies. These results are compared to those obtained by different authors. 3D exact solutions by Leissa (1973), Zhou *et al*. (2002), Nagino *et al*. (2008), a FSDT using DQM by Liu and Liew (1999), and the HSDTs by Shufrin and Eisenberger (2005), Hosseini-Hashemi *et al*. (2011), Akavci (2014) and Mantari (2015). It

Table 1 Material properties of the used FG plate

Material	Properties		
	E (GPa)	v	ρ (kg/m^3)
Aluminum (Al)	70	0.3	2702
Alumina (Al_2O_3)	380	0.3	3800
Zirconia (ZrO_2)	200	0.3	5700

Table 2 Dimensionless fundamental frequencies $\hat{\omega} = \sqrt{\rho h / D_0}$ for simply supported isotropic square plates

a/h	Theory	Mode							
		(1,1)	(1,2)	(2,1)	(2,2)	(1,3)	(3,1)	(2,3)	(3,2)
1000	Leissa (1973)	19.7392	49.3480	49.3480	78.9568	98.6960	98.6960	128.3021	128.3021
	Zhou *et al*. (2002)	19.7115	49.3470	49.3470	78.9528	98.6911	98.6911	128.3048	128.3048
	Akavci (2014)	19.7391	49.3476	49.3476	78.9557	98.6943	98.6943	128.3020	128.3020
	Mantari (2015)	19.7405	49.3486	49.3486	78.9580	98.6967	98.6967	128.3049	128.3049
	Model 1	19.7548	49.3868	49.3868	79.0185	98.7728	98.7728	128.4041	128.4041
	Model 2	19.7398	49.3492	49.3492	79.9584	98.6977	98.6977	128.3064	128.3064
100	Liu and Liew (1999)	19.7319	49.3027	49.3027	78.8410	98.5150	98.5150	127.9993	127.9993
	Nagino *et al*. (2008)	19.7320	49.3050	49.3050	78.8460	98.5250	98.5250	128.0100	128.0100
	Akavci (2014)	19.7322	49.3045	49.3045	78.8456	98.5223	98.5223	128.0120	128.0120
	Mantari (2014)	19.7332	49.3086	49.3086	78.8550	98.5365	98.5365	128.0346	128.0346
	Model 1	19.7484	49.3469	49.3469	78.9164	98.6134	98.6134	128.1347	128.1347
	Model 2	19.7332	49.3084	49.3084	78.8539	98.5344	98.5344	128.0307	128.0307
10	Liu and Liew (1999)	19.0584	45.4478	45.4478	69.7167	84.9264	84.9264	106.5154	106.5154
	Nagino *et al*. (2008)	19.0653	45.4869	45.4869	69.8093	85.0646	85.0646	106.7350	106.7350
	Akavci (2014)	19.0850	45.5957	45.5957	70.0595	85.4315	85.4315	107.3040	107.3040
	Mantari (2014)	19.1190	45.7339	45.7339	70.3148	85.7622	85.7622	107.7376	107.7376
	Model 1	19.1356	45.7791	45.7791	70.3912	85.8603	85.8603	107.8696	107.8696
	Model 2	19.1109	45.7047	45.7047	70.2646	85.6991	85.6991	107.6572	107.6572
5	Shufrin and Eisenberger (2005)	17.4524	38.1884	38.1884	55.2539	65.3130	65.3130	78.9864	78.9864
	Hosseini-Hashemi *et al*. (2011)	17.4523	38.1883	38.1883	55.2543	65.3135	65.3135	78.9865	78.9865
	Akavci (2014)	17.5149	38.4722	38.4722	55.8358	66.1207	66.1207	80.1637	80.1637
	Mantari (2014)	17.5899	38.6582	38.6582	56.0674	66.3474	66.3474	80.3365	80.3365
	Model 1	17.5978	38.6792	38.6792	56.1025	66.3922	66.3922	80.3963	80.3963
	Model 2	17.5661	38.5969	38.5969	55.9711	66.2278	66.2278	80.1810	80.1810

can be observed in this table that for the value of the thickness ratio "a/h = 1000", the results obtained by model 2 are very close to the values provided by Leissa (1973). For the thickness ratio "a/h = 100", the values of model 2 are slightly higher than those given by Akavci (2014) and Nagino *et al*. (2008). By decreasing the thickness ratio "a/h", there is a "*slight over-prediction*" compared to the solutions of the other formulations.

4.2 Investigation of homogeneous isotropic plates

FG plates are examined by considering two kinds of FGM (Al/Al_2O_3 and Al/ZrO_2). Unless otherwise indicated, here, the following relationships of non-dimensional natural frequencies and foundation coefficients were used:

$$\bar{\omega} = \omega h\sqrt{\rho_M / E_M}\,, \qquad \beta = \omega h\sqrt{\rho_C / E_C}\,,$$
$$\tilde{\omega} = \left(\omega a^2 / h\right)\sqrt{\rho_M / E_M}\,, \qquad \tilde{\beta} = \left(\omega a^2 / h\right)\sqrt{\rho_C / E_C}\,, \tag{27}$$
$$\bar{K}_0^w = k_0^w a^4 / A, \qquad \bar{K}_1^w = k_1^w a^2 / A$$

where

$$A = \left[h^3/12\left(1-\nu^2\right)\right][p\left(8+3p+p^2\right)E_M$$
$$+3\left(2+p+p^2\right)E_C]/\left[\left(1+p\right)\left(2+p\right)\left(3+p\right)\right]$$

Table 3 presents the nondimensional fundamental frequencies of a simply supported plate. The results are compared with those of the 3D exact solution given by Jin *et al*. (2014) and HSDT of Mantari (2015). The computed results have good accuracy for square plates. In rectangular plates, the results are close to the reference value in cases where the thickness ratio $a/h \geq 5$.

4.3 Parameter studies

Fig. 1 presents the variation of dimensionless natural frequency of "moderately thick plates" (a/h = 10) versus the gradient index for two types of FGMs (Al/Al_2O_3 and Al/ZrO_2). The results for a homogeneous plate (fully metal; Al) are also considered as a reference case. It can be observed that for each value of the material index, the fundamental frequency is important when "Al_2O_3" is utilized in the top face, we also remarked that the fundamental frequency is higher for FG plates than for homogeneous structure. For high "values" of the material index, the fundamental frequency does not change too much.

Table 3 Dimensionless fundamental frequencies $\bar{\omega} = \omega h \sqrt{\rho_M / E_M}$ of Al/Al_2O_3 FG plates

b/*a*	*a*/*h*	*p*	Theory			
			Jin *et al.* (2014)	Mantari (2014)	Model 1	Model 2
1	10	0	0.1135	0.1137	0.1137	0.1136
		1	0.0870	0.0883	0.0883	0.0882
		2	0.0789	0.0806	0.0806	0.0805
		5	0.0741	0.0756	0.0756	0.0756
	5	0	0.4169	0.4183	0.4185	0.4177
		1	0.3222	0.3271	0.3271	0.3266
		2	0.2905	0.2965	0.2965	0.2962
		5	0.2676	0.2726	0.2727	0.2728
	2	0	1.8470	1.8543	1.8588	1.8539
		1	1.4687	1.4803	1.4836	1.4800
		2	1.3095	1.3224	1.3254	1.3226
		5	1.1450	1.1565	1.1577	1.1574
2	10	0	0.0719	0.0719	0.0719	0.0718
		1	0.0550	0.0558	0.0558	0.0557
		2	0.0499	0.0510	0.0510	0.0509
		5	0.0471	0.0480	0.0479	0.0479
	5	0	0.2713	0.2721	0.2721	0.2717
		1	0.2088	0.2121	0.2121	0.2118
		2	0.1888	0.1928	0.1928	0.1926
		5	0.1754	0.1789	0.1789	0.1789
	2	0	0.9570	1.3075	1.3101	1.3070
		1	0.7937	1.0371	1.0389	1.0367
		2	0.7149	0.9297	0.9314	0.9298
		5	0.6168	0.8248	0.8252	0.8254

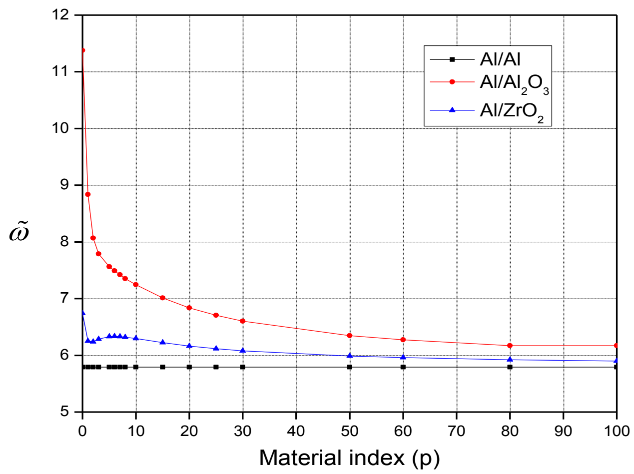

Fig. 1 Variation of dimensionless fundamental frequency $\tilde{\omega} = (\omega a^2 / h)\sqrt{\rho_M / E_M}$ of FG square plate versus material index (*a*/*h* = 10)

In Fig. 2 the variation of dimensionless frequency of moderately thick structures with respect to the ratio "*a*/*b*" for two types of FG plates (Al/Al_2O_3 and Al/ZrO_2) and for a homogeneous plate is presented. From this figure, it can be observed that for each value of the ratio "*a*/*b*" the dimensionless frequency is considerable in the case when "Al/Al_2O_3" is employed in the top face. We again note that

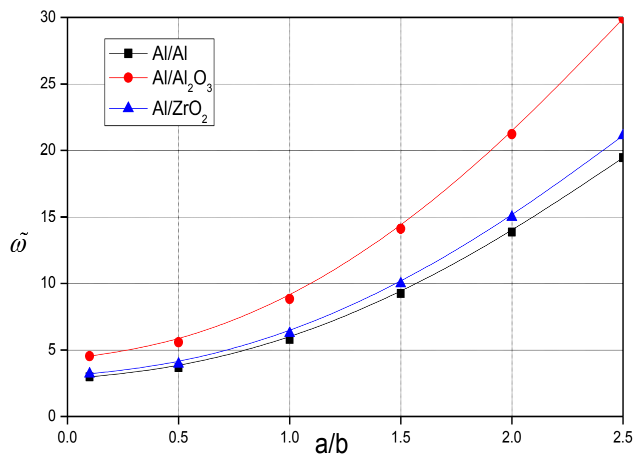

Fig. 2 Variation of dimensionless fundamental frequency $\tilde{\omega} = (\omega a^2 / h)\sqrt{\rho_M / E_M}$ of FG plates versus the aspect ratio (*a*/*h* = 10, *p* = 1)

the dimensionless frequency is higher for FG plates, however, for small values of the ratio "*a*/*b*", the results of Al/ZrO_2 are very close to the natural frequency of a homogeneous plate. It can be also seen that for small values of the ratio ($a/b \leq 5$), the dimensionless frequency does not vary enough.

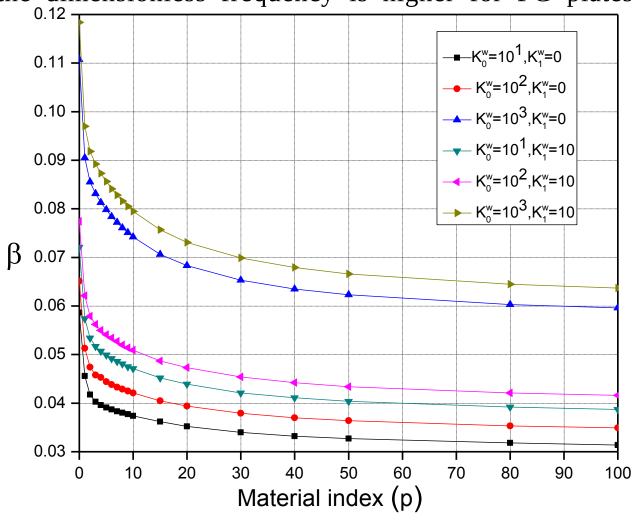

Fig. 3 Variation of dimensionless fundamental frequency $\beta = \omega h \sqrt{\rho_C / E_C}$ of Al/Al_2O_3 FG square plates resting on elastic foundation with material index ($a/h = 10$)

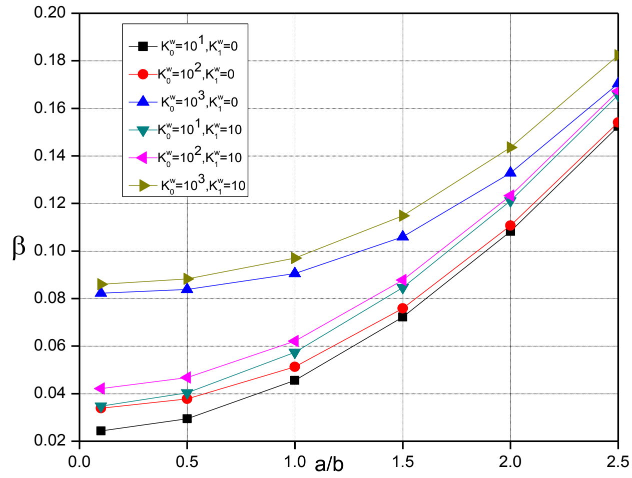

Fig. 4 Variation of dimensionless fundamental frequency $\beta = \omega h \sqrt{\rho_C / E_C}$ of Al/Al_2O_3 FG rectangular plates resting on elastic foundation with geometric ratio *a*/*b* ($a/h = 10$, $p = 1$)

Fig. 3 demonstrates the variation of dimensionless fundamental frequency of FG plates supported by elastic foundation versus the gradient index for different values of the coefficients of the "Pasternak model" ($\bar{K}_0^w$, $\bar{K}_1^w$). It can be seen that for each value of "*p*" and a coefficient of the "Pasternak model", as the other coefficient increases, the dimensionless fundamental increases. Again, we can observed that for the considerable values of "*p*", the dimensionless frequency does not vary enough.

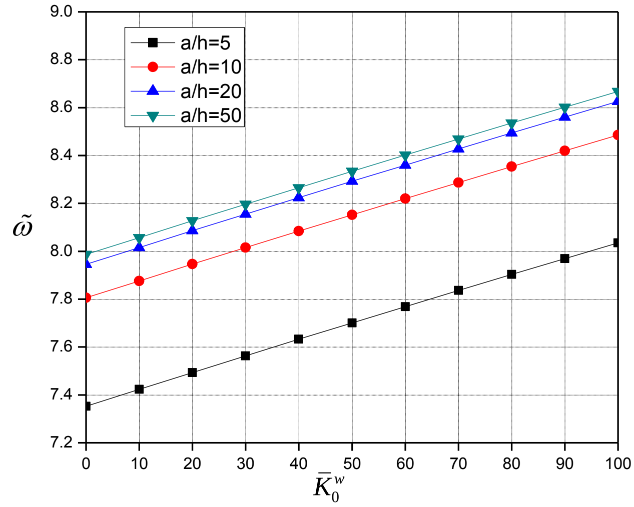

Fig. 5 Variation of dimensionless fundamental frequency $\tilde{\omega} = \left(\omega a^2 / h\right)\sqrt{\rho_M / E_M}$ of Al/ZrO_2 FG square plates resting on elastic foundation with $\bar{K}_0^w$ ($\bar{K}_1^w = 10$, $p = 1$)

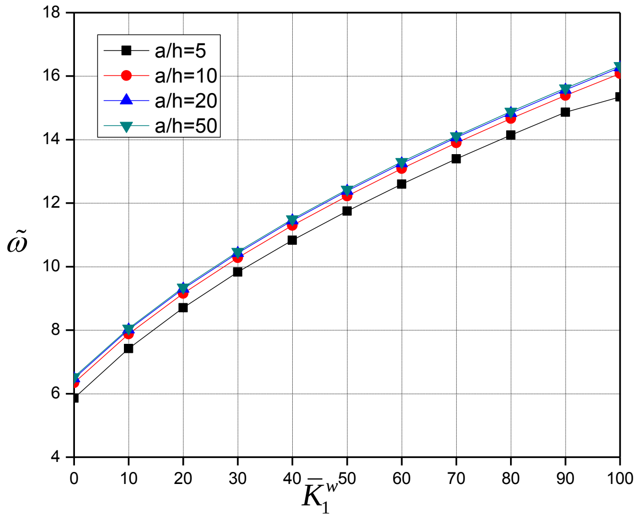

Fig. 6 Variation of dimensionless fundamental frequency $\tilde{\omega} = \left(\omega a^2 / h\right)\sqrt{\rho_M / E_M}$ of Al/ZrO_2 FG square plates resting on elastic foundation with $\bar{K}_1^w$ ($\bar{K}_0^w = 10$, $p = 1$)

In Fig. 4 the variation of the fundamental frequency versus the geometric ratio "*a*/*b*" of FG plate ($p = 1$) for different values of the coefficients of the "Pasternak model" ($\bar{K}_0^w$, $\bar{K}_1^w$) is presented. It can be noted that for a provided value of the ratio "*a*/*b*" and the coefficient "$\bar{K}_0^w$", when the parameter $\bar{K}_1^w$ becomes high, the natural frequency increases. Note also that for small values of the ratio "*a*/*b*" and the constant "$\bar{K}_0^w$", the curves tend to be closer to the same value, this can be shown more clearly for high values of constant "$\bar{K}_0^w$". When the parameter "$\bar{K}_1^w$" is constant for a provided value of the geometric ratio "*a*/*b*", as the value of parameter "$\bar{K}_0^w$" becomes high, the value of the fundamental frequency increases. It is also remarked that the curves come closer when the ratio "a/b" increases.

Figs. 5 and 6 present the variation of the fundamental frequency as a function of the coefficients "$\bar{K}_0^w$" and "$\bar{K}_1^w$", respectively. Fig. 5 demonstrates that the curves have a linear trend for the given range of "$\bar{K}_0^w$". The curves shown in Fig. 6 have a higher slope than those of Fig. 5, that is, the coefficient "$\bar{K}_1^w$" has a higher effect on the fundamental frequency than the coefficient "$\bar{K}_0^w$".

5. Conclusions

This work deals with a dynamic investigation for FG plates supported by elastic foundation utilizing a hyperbolic 4-variables quasi-3D type HSDT with stretching influence. The equations of motion are determined within the Hamilton's principle. The equations obtained are solved via the Navier procedure; then, the natural frequencies can be deduced from the problem of eigenvalues. The results are compared with the CPT, FSDT and other HSDTs with higher number of variables and a good accuracy is observed.

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