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Inhomogeneous bonding state modeling for vibration analysis of explosive clad pipe

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Abstract. Early detection of damage bonding state such as insufficient bonding strength and interface partial contact defect for the explosive clad pipe is crucial in order to avoid sudden failure and even catastrophic accidents. A generalized and efficient model of the explosive clad pipe can reveal the relationship between bonding state and vibration characteristics, and provide foundations and priory knowledge for bonding state detection by signal processing technique. In this paper, the slender explosive clad pipe is regarded as two parallel elastic beams continuously joined by an elastic layer, and the elastic layer is capable to describe the non-uniform bonding state. By taking the characteristic beam modal functions as the admissible functions, the Rayleigh-Ritz method is employed to derive the dynamic model which enables one to consider inhomogeneous system and any boundary conditions. Then, the proposed model is validated by both numerical results and experiment. Parametric studies are carried out to investigate the effects of bonding strength and the length of partial contact defect on the natural frequency and forced response of the explosive clad pipe. A potential method for identifying the bonding quality of the explosive clad pipe is also discussed in this paper.

Keywords: explosive clad pipe; bonding strength; partial contact defect; Rayleigh-Ritz method

1. Introduction

Explosive clad pipe, in which two parallel pipes with different materials are bonded together under high pressure produced by explosive detonation, have been widely used in submarine oil pipelines and chemical industry owing to their ability to endure poor environment with high temperature, high pressure and strong corrosive (Findik 2011, Si *et al*. 2015, Mendes *et al*. 2013). However, accidents due to explosive clad pipe failures may cause enormous economic loss or even endanger the life safety of field personnel. The insufficient bonding strength and the partial contact defect (PCD) are two main quality problems of the explosive clad pipe in industrial production (Si *et al*. 2017). Therefore, early detection of the interface bonding state is an essential part of preventive strategies that aim to eliminate or at least reduce the risk of operation interruption as well as safety hazards.

Among the studies on detection of the bonding state, destructive inspection and non-destructive evaluation are commonly used for analyzing the explosive clad structures. In the aspect of destructive inspection, Acarer and Demir (2008) detected bonding strength of explosive cladded aluminum-dual phase steel using tensile test and shear test for bonding quality assessment. Kaya and Kahraman (2013) applied impact test and bending test to detect the bonding strength of Grade A ship steel/AISI 316L austenitic stainless steel clad structure. Xia *et al*. (2014) used tensile test and bending test to detect the bonding strength of TA2/2A12 explosive clad structure. Sun *et al*. (2011) detected the bonding quality of Fe/Al clad tube by compression, flattening and compression-shear test. These methods gained certain effect in detection the local area of the structure. However, the destructive inspection approach has the disadvantages of high cost and low efficiency, and cannot be used for global interface bonding state. On the other hand, various non-destructive evaluation techniques such as ultrasonic (Mouritz *et al*. 2000, Guo *et al*. 2012), x-ray (Guo *et al*. 2016a), microscope (Chen *et al*. 2016, Guo *et al*. 2016b) have also been developed to detect damage in piping systems. The techniques are suitable for pore, crack, and other detection items. But it is difficult to apply in the bonding interface of the explosive clad pipe which is close grained but bonding strength shortage.

Explosive clad pipe is a dynamic system consist of mass, stiffness and damping. Vibration characteristics of the explosive clad pipe are sensitive to changes in bonding state, and have a great potential to explore a reliable and cost-effective methodology for bonding state damage detection. An efficient model of the explosive clad pipe not only can reflect the relationship between bonding state and vibration characteristics, but also can provide foundations and priory knowledge for bonding state detection by signal processing technique. Thus, a generalized dynamic modeling of the explosive clad pipe with inhomogeneous

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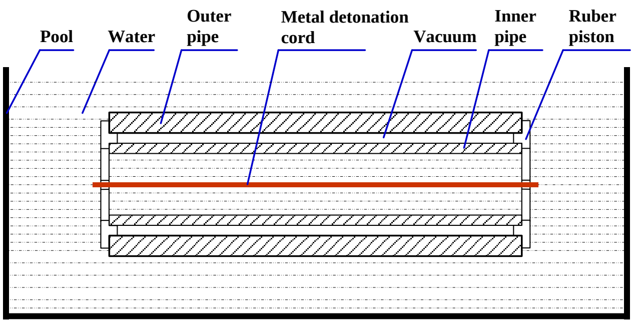
bonding state is of great importance in interface damage detection. Unfortunately, to the best knowledge of the authors, researches efforts on this topic are very limited. El-Gebeily and Khulief (2016) identified the wall-thinning and cracks in pipes utilizing vibration modes and wavelets. Shen *et al*. (2009) studied the vibration properties of a pipe in 3D space. Gao *et al*. (2017) established an analytical model of ground surface vibration due to axisymmetric wave motion in buried fluid-filled pipes. However, these research works are only directed against a single pipe, which structure features are quite different from explosive clad pipe. The conclusions obtained from single pipe may not be suitable to bonded double pipes.

The slender feature of the explosive clad pipe make it possible to consider as a double-beam system, made of two parallel slender beams continuously connected by a Winkler-type elastic layer. Several interesting works have been developed on the dynamics of double-beam systems in recent years. Vu *et al*. (2000) formulated a closed-form solution for the vibration of a double-beam system subjected to harmonic excitations, and the two beams must be identical. Oniszczuk (2000, 2003) presented some analytical expressions for the undamped free and forced vibrations of a simply supported double-beam system. Mao (2012) using the Adomian modified decomposition method to analyze the free vibration of elastically connected multiple-beams. Moreover, other different numerical or analysis methods such as quadrature method (Arani and Amir 2014), nonlocal elasticity theory (Karlicic *et al*. 2015) and spectral element analysis (Li and Hua 2007) have also been used in solving vibration problems of such structures. However, these investigations are assumed that the elastic layer is uniform and continuously. For the explosive clad pipe with the PCD, the elastic layer needs to simulate interface bonding state uneven distribution along the axial direction. Most of previous methods for the modeling of inhomogeneous elastic layer are incapable of action.

Rayleigh-Ritz method, known as one kind of energy methods, has received intensive attention in the vibration of plates and shells of a variety of specifications, configurations and boundary conditions (Pellicano 2007, Bashmal *et al*. 2009, Jeong and Yoo 2019, Emam 2018). Due to its simplicity in implementation and capability to provide satisfactory results, the method is widely acceptable. Based on a convenient choice of admissible functions for the components, the technique allows us to consider inhomogeneous bonding states and any boundary conditions. In this paper, the Rayleigh-Ritz method is employed to study the modeling and vibration characteristics of the explosive clad pipe with inhomogeneous bonding states. The proposed model is validated by numerical simulations and experiment. In what follows the bonding states are considered to parametrically study their effects on the natural frequency and forced response of the explosive clad pipe. Finally, a potential method for identifying the bonding state is discussed.



(a) Installation technology

(b) Formed explosive clad pipe

Fig. 1 Installation technology and formed explosive clad pipe

2. Forming process of explosive clad pipe

The installation technology of underwater explosion forming for the explosive clad pipe is shown in Fig. 1(a). From one end of the metal detonation cord to detonate, the fluid pressure makes enlargement of the inner pipe, and the inner pipe will produce plastic deformation. Furthermore, the inner pipe and outer pipe impact with each other at a certain speed and then expand simultaneously, at this moment the outer pipe produces elastic deformation. After the shock wave pressure disappears, two pipes will be free to rebound, and the rebound amount of the outer pipe is greater than that of the inner pipe. Ultimately, the outer pipe and inner pipe are firmly squeezed together and formed the explosive clad pipe, as shown in Fig. 1(b). Due to the plastic deformation of the inner pipe during the forming process, there will be a little residual stress in the inner pipe after forming. The residual stress is very small and can be neglected compared with the bonding strength of the explosive clad pipe. Thus, the residual stress has little effect on the dynamic analysis of the explosive clad pipe system.

It is worth noting that, the inner pipe and outer pipe are compounded by pressure, and connection interface will not appear metallurgical fusion. This forming process has high production efficiency. Nevertheless, the main shortcoming of this forming process is that it may inevitably lead to inhomogeneous bonding state along the axial direction. In particular, damage bonding state such as the PCD may be also occurred. This will be bringing challenges for modeling and bonding state detection of the explosive clad pipe.

3. Mathematical formulation

For piping systems, in which the diameter of the pipe is far less than the pipe length, usually be treated as a beam

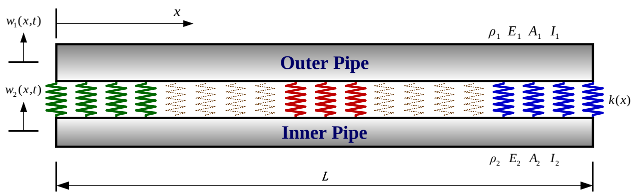


Fig. 2 Dynamic system of explosive clad pipe

with circular hollow cross-section (El-Gebeily and Khulief 2016). In this manner, explosive clad pipe can be considered as a double-beam system that made of two parallel pipes with the same length L and continuously in contact with each other by a linear Winkler-type elastic layer, as shown in Fig. 2. By adjusting the elastic stiffness along axial direction, the elastic layer can describe inhomogeneous bonding state. In this section, the Rayleigh-Ritz method based inhomogeneous bonding state modeling for explosive clad pipe is presented.

Due to slender feature for both outer and inner pipe, the classical Euler-Bernoulli beam theory is adopted in deriving the equation of motion, i.e. the effects of rotational inertia and shear strain are neglected in this study. As shown in Fig. 2, the system coordinates are taken axially in the x-direction and transversely in the z-direction, w_1 (x, t) and w_2 (x, t) are the transverse displacements of outer pipe and inner pipe, respectively. In general, the two pipes have different mechanical properties and are homogeneous. Thus, the outer pipe and inner pipe are fully characterized by modulus of elasticity E_r, mass density ρ_r, cross-sectional area A_r and moment of inertia J_r, where the subscript r = 1, 2 denotes outer and inner pipe, respectively. The elastic layer is allowed to be inhomogeneous, elastic stiffness $k(x)$ could be thought of as interface non-uniform bonding strength along x-direction for the explosive clad pipe.

The purpose of dynamic modeling in this paper is to study the dynamic characteristics of explosive clad pipe under different bonding states. Then, the corresponding relationship between bonding states and dynamic features is revealed, which provides theoretical basis and prior knowledge for the contact defect diagnosis of explosive clad pipe. For free vibration analysis, the system damping has little effect on the natural frequencies and its change trend for explosive clad pipe with different bonding states. For forced vibration response, the system damping directly influences the amplitude of the resonance peak, but has nothing to do with the positions and variation trend of resonance peaks for different bonding states. Therefore, the damping effect is not considered in the system modeling.

3.1 Approximation of the displacement field

In order to build a high degree of model, admissible functions and generalized coordinates are used to introduce the transverse displacements of outer (r = 1) and inner (r = 2) pipes, which can be expressed as

$$w_r(x,t) = \mathbf{\Phi}_r(x)\mathbf{q}_r(t) \tag{1}$$

where $\mathbf{\Phi}_r$ (x) represents the 1 × m matrices consisting of admissible functions in the x-direction, $\mathbf{q}_r$ (t) is m × 1 generalized coordinates vectors of the rth pipe, and m is the number of admissible functions used for the expansion. In this way, we could avoid the complexity of using summation signs in the expression. In fact, $\mathbf{\Phi}_r$ (x) and $\mathbf{q}_r$ (t) are

$$\begin{aligned} \mathbf{\Phi}_r(x) &= \left[\varPhi_{r,1}(x)\ \varPhi_{r,2}(x)\ \cdots\ \varPhi_{r,m}(x)\right] \\ \mathbf{q}_r(t) &= \left[q_{r,1}(t)\ q_{r,2}(t)\ \cdots\ q_{r,m}(t)\right]^{\mathrm{T}} \end{aligned} \tag{2}$$

The admissible functions for the rth uniform pipe are solution of the classical Euler-Bernoulli beam eigenproblem (Oniszczuk 2003)

$$\varPhi''''_{r,i}(x) - \beta_{r,i}^4 \varPhi_{r,i}(x) = 0, \qquad i = 1, 2, \ldots, m \tag{3}$$

where the prime denotes derivative with respect to the spatial coordinate x, while $\Phi_{r,i}$ (x) and $\beta_{r,i}$ are the ith pair of eigenfunction and eigenvalue for the rth pipe. Taking into

Table 1 Admissible functions for outer and inner pipes

Boundary conditions	Eigenfunctions $\varPhi_{r,i}(x)$	Eigenvalue equation
S-S	$\sin(\beta_{r,i}x)$	$\beta_{r,i} = \dfrac{i\pi}{L}$
F-F	$\cos(\beta_{r,i}x)+\cosh(\beta_{r,i}x)-\dfrac{\cos(\beta_{r,i}L)-\cosh(\beta_{r,i}L)}{\sin(\beta_{r,i}L)-\sinh(\beta_{r,i}L)}\left[\sin(\beta_{r,i}x)+\sinh(\beta_{r,i}x)\right]$	$\cos(\beta_{r,i}L)\cosh(\beta_{r,i}L)=1$
C-C	$\cos(\beta_{r,i}x)-1$	$\beta_{r,i} = \dfrac{2i\pi}{L}$
C-S	$\cos(\beta_{r,i}x)-\dfrac{\sin(\beta_{r,i}x)}{\beta_{r,i}L}+\dfrac{x}{L}-1$	$\tan(\beta_{r,i}L)=\beta_{r,i}L$
S-F	$\dfrac{\sinh(\beta_{r,i}L)}{\sin(\beta_{r,i}L)}\sin(\beta_{r,i}x)+\sinh(\beta_{r,i}x)$	$\tan(\beta_{r,i}L)=\beta_{r,i}L$
C-F	$1-\cos(\beta_{r,i}x)$	$\beta_{r,i} = \dfrac{(2i-1)\pi}{2L}$

account classical boundary conditions of the rth pipe, i.e., simply supported-simply supported (S-S), clamped-clamped (C-C), free-free (F-F), clamped-simply supported (C-S), simply supported-free (S-F) and clamped-free (C-F), the non-trivial solutions satisfying Eq. (3) can be obtained. Admissible functions for outer and inner pipes under different boundary conditions are listed in Table 1.

3.2 Expressions of the energy

The total kinetic energy of the explosive clad pipe can be evaluated as the sum of two terms

$$T(t) = T_1(t) + T_2(t) \tag{4}$$

For the outer (r = 1) and inner (r = 2) pipes, the expressions of kinetic energy are given by (Vu *et al*. 2000)

$$T_r(t) = \frac{1}{2}\int_0^L \rho_r A_r \left[\frac{\partial w_r(x,t)}{\partial t}\right]^2 \mathrm{d}x \tag{5}$$

where ρ_r and A_r are density and cross-section area of the rth pipe, respectively.

Substituting expression of the displacement field Eq. (1) into Eq. (5), the kinetic energy of the rth pipe can be written as

$$T_r(t) = \frac{1}{2}\rho_r A_r \dot{\mathbf{q}}_r^{\mathrm{T}} \mathbf{\Phi}_{rr} \dot{\mathbf{q}}_r \tag{6}$$

where the dot · denotes differentiation with respect to time, and

$$\mathbf{\Phi}_{rr} = \int_0^L \mathbf{\Phi}_r^{\mathrm{T}} \mathbf{\Phi}_r \mathrm{d}x$$

Using Eqs. (1) and (5), the total kinetic energy of the explosive clad pipe can be derived in the vector-matrix form

$$T(t) = \frac{1}{2}\dot{\mathbf{q}}^{\mathrm{T}} \mathbf{M} \dot{\mathbf{q}} \tag{7}$$

where $\mathbf{q} = [\mathbf{q}_1^{\mathrm{T}}\ \mathbf{q}_2^{\mathrm{T}}]^{\mathrm{T}}$ is the total generalized coordinate vector, and $\mathbf{M}$ is a $2m \times 2m$ mass matrix of dynamic model for the explosive clad pipe given by

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_1 & \mathbf{0}_{mm} \\ \mathbf{0}_{mm} & \mathbf{M}_2 \end{bmatrix} \tag{8}$$

in which $\mathbf{0}_{mm}$ represents the $m \times m$ zero matrix, and $\mathbf{M}_1 = \rho_1 A_1 \Phi_{11}$ denotes the mass matrix of the outer pipe, while $\mathbf{M}_2 = \rho_2 A_2 \Phi_{22}$ is the mass matrix of the inner pipe.

As shown in Fig. 2, in the coordinate system, the total potential energy of the explosive clad pipe includes potential energy of two pipes and elastic layer, can be evaluated as the sum of three terms

$$V(t) = V_1(t) + V_2(t) + V_k(t) \tag{9}$$

For the outer and inner pipes (r = 1, 2), the expressions of potential energy are defined in integral form as (Li and Hua 2007)

$$V_r = \frac{1}{2}\int_0^L E_r J_r \left[\frac{\partial w_r^2(x,t)}{\partial x^2}\right]^2 \mathrm{d}x \tag{10}$$

where E_r and J_r are the Young's modulus and the moment of inertia for the rth pipe, respectively.

The potential energy of elastic layer take the form

$$V_k = \frac{1}{2}\int_0^L k(x)\left[w_1(x,t) - w_2(x,t)\right]^2 \mathrm{d}x \tag{11}$$

The admissible function matrices and generalized coordinate vectors of Eq. (1) are used to introduce Eqs. (10) and (11), V_r and V_k can be expressed as follows

$$V_r = \frac{1}{2} E_r J_r \mathbf{q}_r^{\mathrm{T}} \mathbf{\Phi}_{rr}'' \mathbf{q}_r \tag{12}$$

$$\begin{aligned} V_k = \frac{1}{2}[&\mathbf{q}_1^{\mathrm{T}} \Delta\mathbf{K}_{11} \mathbf{q}_1 + \mathbf{q}_2^{\mathrm{T}} \Delta\mathbf{K}_{22} \mathbf{q}_2 \\ &- (\mathbf{q}_1^{\mathrm{T}} \Delta\mathbf{K}_{12} \mathbf{q}_2 + \mathbf{q}_2^{\mathrm{T}} \Delta\mathbf{K}_{21} \mathbf{q}_1)] \end{aligned} \tag{13}$$

where $\mathbf{\Phi}_{rr}'' = \int_0^L \mathbf{\Phi}_r''^{T} \mathbf{\Phi}_r''\, \mathrm{d}x$, the prime denotes derivative with respect to the spatial coordinate x, and

$$\Delta\mathbf{K}_{11} = \int_0^L k(x) \mathbf{\Phi}_1^{\mathrm{T}} \mathbf{\Phi}_1 \mathrm{d}x,$$

$$\Delta\mathbf{K}_{22} = \int_0^L k(x) \mathbf{\Phi}_2^{\mathrm{T}} \mathbf{\Phi}_2 \mathrm{d}x,$$

$$\mathbf{K}_{12} = \int_0^L k(x) \mathbf{\Phi}_1^{\mathrm{T}} \mathbf{\Phi}_2 \mathrm{d}x,$$

$$\mathbf{K}_{21} = \int_0^L k(x) \mathbf{\Phi}_2^{\mathrm{T}} \mathbf{\Phi}_1 \mathrm{d}x$$

Therefore, total potential energy of the explosive clad pipe in Eq. (9) can be depicted in the form of matrix as

$$V(t) = \frac{1}{2}\mathbf{q}^{\mathrm{T}} \mathbf{K} \mathbf{q} \tag{14}$$

where the stiffness matrix $\mathbf{K}$ in the potential energy equation of the explosive clad pipe is

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_{11} + \Delta\mathbf{K}_{11} & -\mathbf{K}_{12} \\ -\mathbf{K}_{21} & \mathbf{K}_{22} + \Delta\mathbf{K}_{22} \end{bmatrix} \tag{15}$$

in which $\mathbf{K}_{11} = E_1 J_1 \mathbf{\Phi}_{11}''$ and $\mathbf{K}_{22} = E_2 J_2 \mathbf{\Phi}_{22}''$ are stiffness matrices of outer and inner pipes, respectively. It is worth emphasizing here that the only coupling two pipes is due to the stiffness matrices $\mathbf{K}_{12}$ and $\mathbf{K}_{21}$.

3.3 Lagrangian equations of motion

The generalized force $Q_{r,j}$ (t) associated with the Lagrangian coordinate $q_{r,j}$ (t) can be obtained by projecting the external dynamic loads f_r (x, t), acting on the rth pipe, onto the jth assumed mode for such pipe

Table 2 Comparisons of the first six natural circular frequencies ω (rad/s) for the double-beam system with different boundary conditions

Boundary conditions		Type	Mode index					
Left end	Right end		1	2	3	4	5	6
SS	SS	Present	19.7392	43.4699	78.9568	87.9442	177.6529	181.8256
		Mao (2012)	19.7392	43.4699	78.9568	87.9442	177.6529	181.8256
Clamped	Clamped	Present	44.7466	59.1799	123.3456	129.2832	241.8068	244.8888
		Mao (2012)	44.7466	59.1799	123.3456	129.2832	241.8068	244.8888
SS	Clamped	Present	30.8364	49.5064	99.9297	107.1725	208.4954	212.0621
		Mao (2012)	30.8364	49.5064	99.9297	107.1725	208.4953	212.0620
Clamped	Free	Present	7.0320	39.3630	44.0690	58.6692	123.3944	129.3298
		Mao (2012)	7.0320	39.3630	44.0690	58.6692	123.3944	129.3297

$$\begin{aligned} Q_{r,j}(t) &= \int_0^L f_r(x,t)\frac{\partial}{\partial q_{r,j}(t)} w_r(x,t)\,\mathrm{d}x \\ &= \int_0^L f_r(x,t)\Phi_{r,j}(x)\,\mathrm{d}x \end{aligned} \tag{16}$$

where r = 1, 2 and j = 1, ...,m. Analogously to the array of Lagrangian coordinates $\mathbf{q}_r$ (t) for the rth pipe, the new m-dimensional forcing array $\mathbf{Q}_r$ (t) = $[Q_{r1}(t)\ Q_{r2}(t)\ \ldots\ Q_{rm}(t)]^{\mathrm{T}}$ can be introduced, and $\mathbf{Q}(t) = [\mathbf{Q}_1^{\mathrm{T}}(t)\ \ \mathbf{Q}_2^{\mathrm{T}}(t)]^{\mathrm{T}}$.

Since all the source of kinetic and potential energies are expressed as functions of generalized displacements and velocities, and once the generalized forces are defined, Lagrange's equations ruling the vibration of explosive clad pipe with inhomogeneous bonding state can be formally written as (Pellicano 2007)

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial}{\partial \dot{\mathbf{q}}(t)}\mathcal{L}(t)\right) - \frac{\partial}{\partial \mathbf{q}(t)}\mathcal{L}(t) = \mathbf{Q}(t) \tag{17}$$

where $\mathcal{L}(t) = T(t) - V(\mathrm{t})$ is the so-called Lagrangian function of the whole system, $T(t)$ and $V(t)$ are those of Eqs. (7) and (14).

After some algebra, Eq. (17) can be reduced to the more compact matrix form

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{Q}(t) \tag{18}$$

Especially, for the free vibration analysis of explosive clad pipe, the external force vector $\mathbf{Q}(t)$ in Eq. (18) should be replaced by null vector. Then the natural frequencies and eigenvectors of the piping system can be solved by the introduction of $\mathbf{q} = \mathbf{H}e^{\mathrm{i}\omega t}$ in Eq. (18), and the eigen equation can be written as

$$\left[\mathbf{K} - \omega^2\mathbf{M}\right]\mathbf{H} = \mathbf{0} \tag{19}$$

where ω represents natural frequency. The natural frequencies can be determined easily by finding the nontrivial solutions of Eq. (19).

4. Model validation

4.1 Numerical validation

For the purpose of numerical validation, the proposed dynamic model is initially applied to evaluate natural frequencies of a double-beam system with different boundary condition considered by Mao (2012). In this example, both beams are homogeneous and have the same length L = 10 m, the stiffness of elastic layer is assumed to be uniform distribution, i.e., $k(x)$ = 100 kN/m^2, while the flexural stiffness and mass per unit length of the top beam are E_1I_1 = 4000 kN·m^2 and ρ_1A_1 = 100 kg/m. The bottom beam is more flexible and lighter, whose mechanical parameters are $E_2I_2 = 2E_1I_1$ and $\rho_2A_2 = 2\rho_1A_1$, respectively.

As shown in Table 2, comparisons between the present results and the results obtained from the Adomain modified decomposition method by Mao (2012) are carried out for the double-beam system with different boundary conditions. In this case the first six natural circular frequencies of the proposed approach are in excellent agreement with provided by Mao (2012) for a wide variety of boundary conditions. It indicates that the present method is accurate enough for vibration analysis of double-beam system.

4.2 Experimental validation

To further verify the model, an industrial explosive clad pipe whose materials of outer pipe and inner pipe are separately 20 carbon steel and 316 L stainless steel is employed, and the mechanical parameters of the outer pipe and inner pipe are listed in Table 3. If there is no special statement, the following numerical calculations use the dimensions and material parameters in Table 3.

The experimental setup of explosive clad pipe is shown in Fig. 3, which consists of explosive clad pipe, hammer, accelerometers and data acquisition system. The explosive clad pipe are simply supported at both ends. The collected data from the accelerometer that is mounted on the outer pipe at a point 1 m away from the left end of pipe are selected to analyze. The vibration response signals are measured at a sampling frequency of 680 Hz, and the signals is sampled by the data acquisition system. Hammer

Table 3 Mechanical parameters of outer pipe and inner pipe

Parameters	Outer pipe	Inner pipe
Length L (m)	10	10
Young's modulus E_r (GPa)	206	193
Density ρ_r (kg/m^3)	7850	7980
Outer diameter (m)	0.114	0.094
Wall thickness (m)	0.01	0.002

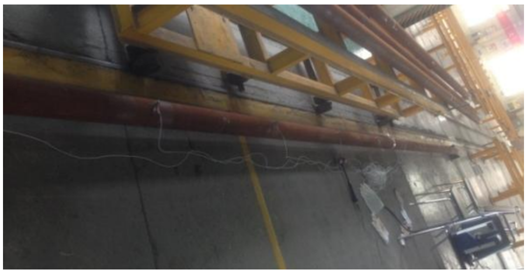


Fig. 3 Experimental setup of explosive clad pipe

excitation is performed manually at a point 2.5 m away from the left end of pipe.

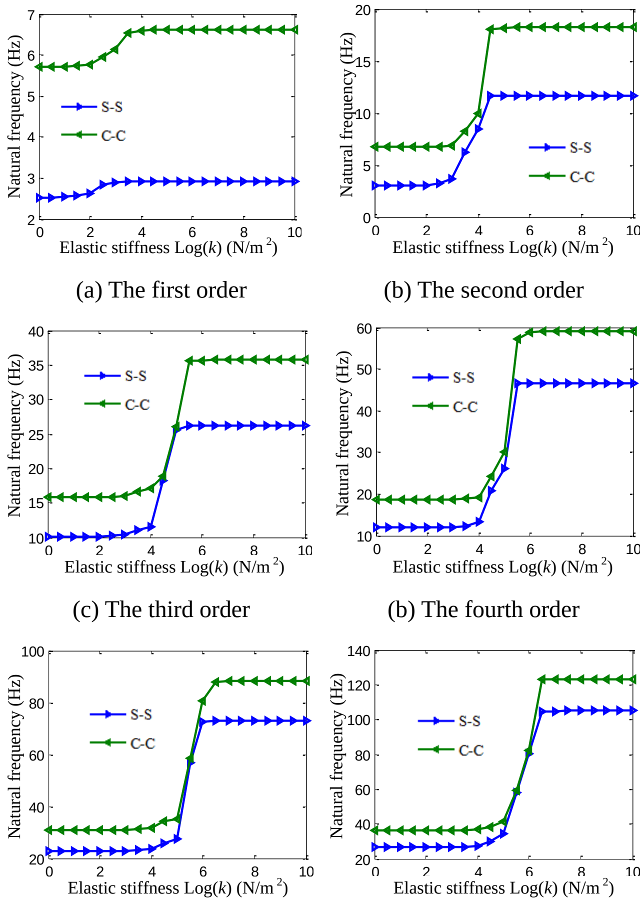
Comparisons of first eight natural frequencies between present approach and experimental for the explosive clad pipe are given in Table 4. Due to the explosive clad pipe used for experiment has a good bonding state on the whole connection interface, the elastic layer of proposed approach is considered to be uniform and set to $k(x)$ = 5000 kN/m^2. As can be seen in Table 4, the maximum relative error of the natural frequencies is below 3% and occurs at the first-order natural frequency. Moreover, the increase of the mode index does not lead to significant relative errors, which means that good accuracy can also be achieved for higher-order natural frequencies. It may indicate that the proposed method has high precision and is capable of solving vibration problems for the explosive clad pipe system.

5. Numerical results

The proposed dynamic model based on Rayleigh-Ritz method has been validated for the vibration analysis of the explosive clad pipe system. In this section, the effects of the interface bonding strength and the PCD on the natural frequencies and forced response of the explosive clad pipe are investigated. For all numerical computations, the number m of admissible functions is determined to be 50, which could satisfy the requirements of calculation accuracy. In the following examples, the mechanical parameters of the outer pipe and inner pipe are consistent with the Table 3.

5.1 Effects of interface bonding strength

In this subsection, it is assumed that the explosive clad pipe has uniform interface bonding strength, and the effects of interface bonding strength on natural frequencies of explosive clad pipe are investigated to explore effective characteristics for the bonding state detection. The interface bonding strength can be considered as elastic stiffness $k(x)$ in the dynamic model, therefore, it is possible to discover



(a) The first order (b) The second order

(c) The third order (b) The fourth order

(e) The fifth order (f) The sixth order

Fig. 4 Effects of elastic stiffness on natural frequency for the explosive clad pipe under S-S and C-C boundary conditions

Table 4 Natural frequencies (Hz) comparison between proposed approach and experimental for the explosive clad pipe

Type	Mode index							
	1	2	3	4	5	6	7	8
Present	2.9155	11.6618	26.2364	46.6300	72.8169	104.7384	141.2689	179.7151
Experimental	3.00	11.88	26.83	46.69	73.11	105.35	142.32	181.80
Error (%)	2.82	1.84	2.21	0.13	0.4	0.58	0.74	1.15

the change trend of vibration characteristics by adjusting the elastic stiffness. Under the framework, the elastic stiffness in the dynamic model can be changed easily during the calculation procedure.

Fig. 4 shows the variation of the first six natural frequencies with the elastic stiffness for the explosive clad pipe under S-S and C-C boundary conditions. It can be seen that, regardless of end boundary conditions, increase of the elastic stiffness causes the natural frequencies to increase. Especially, the natural frequencies increase rapidly as the elastic stiffness k varies from 10 kN/m^2 to 1000 kN/m^2. When the elastic stiffness exceed 5000 kN/m^2 the values of natural frequencies remains relatively stable, because the outer pipe and inner pipe are almost fixed together, which means the explosive clad pipe with excellent interface bonding strength.

Figs. 5 and 6 present the first six modal shapes of the explosive clad pipe with elastic stiffness 100 N/m^2 and 5000 kN/m^2, respectively, both end boundary conditions are set as simply supported. As can be seen in Fig. 5, owing to insufficient elastic stiffness of the dynamic system, outer pipe and inner pipe exhibit different vibration amplitude and direction in certain mode. From Fig. 6, the modal shapes of the outer pipe and inner pipe are demonstrated synchronous vibration, this is due to the elastic stiffness is large enough for the dynamic system, and the explosive clad pipe has healthy interface bonding strength accordingly.

In order to study the effects of interface bonding strength on the forced vibration response of the explosive clad pipe, several results are presented in this part. For simplicity, only simply supported end boundary condition is considered in the following procedure. A vertical force f_1 = 10 N is applied on the outer pipe at x = 4.5 m. The response points are located at x = 1 m on the outer pipe. The range of analysis frequency is from 0 to 100 Hz and the corresponding frequency step is set as 0.1 Hz.

Forced vibration response on the outer pipe of the explosive clad pipe with different elastic stiffness is depicted in Fig. 7. It can be observed that, as the elastic stiffness decreases, the positions of the resonant peaks

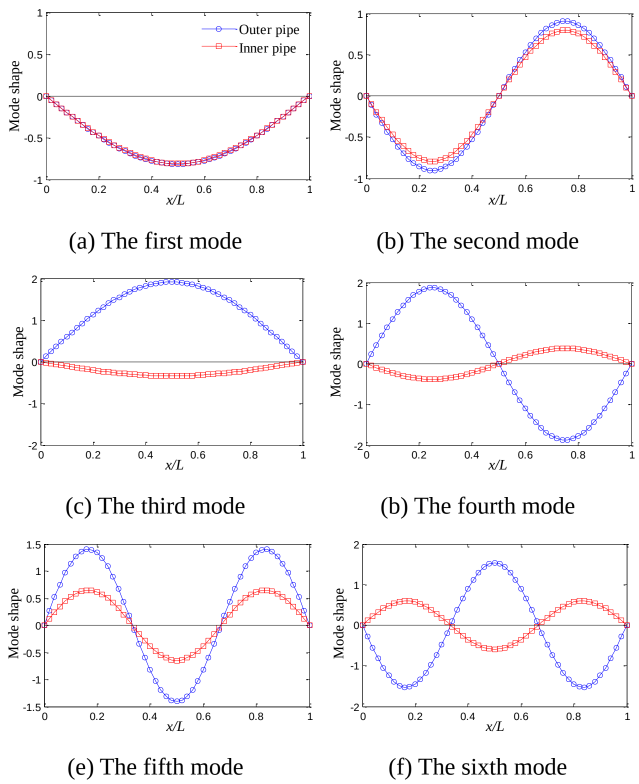


Fig. 5 First six modal shapes of the explosive clad pipe with elastic stiffness k = 100 kN/m^2

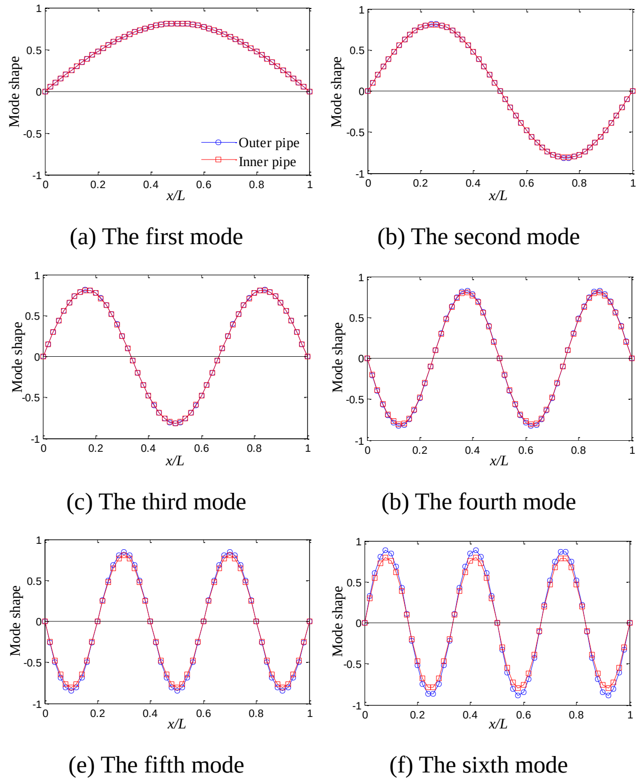


Fig. 6 First six modal shapes of the explosive clad pipe with elastic stiffness k = 5000 kN/m^2

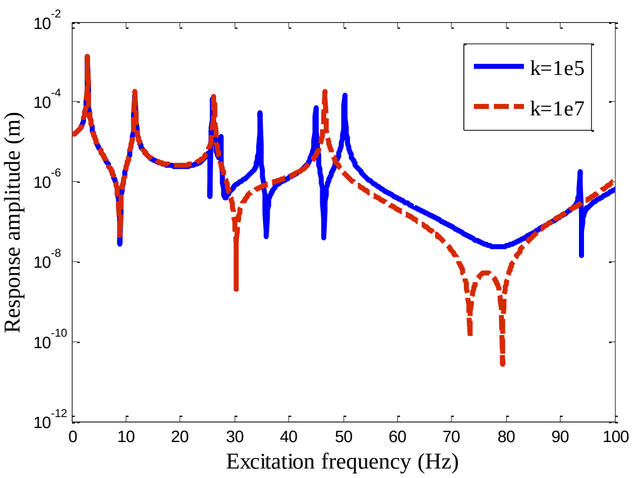


Fig. 7 Transverse displacement response on the outer pipe of the explosive clad pipe with different elastic stiffness

move left in certain high-frequency range for the reason that the stiffness decreases of elastic layer reduce the rigidity of the explosive clad pipe. Moreover, the response amplitudes of explosive clad pipe with k = 10^7 N/m^2 is a little greater than that of k = 10^5 N/m^2. The possible reason is that the low stiffness structure forms more resonance peaks under impulse excitation, which makes the vibration energy more dispersed, resulting in the reduction of the response amplitude of each response frequency.

As can be seen from above analysis, the elastic stiffness has significant effects on the shifts of natural frequencies for the dynamic system. In the same way, natural frequencies are sensitive to the interface bonding strength of the explosive clad pipe. Therefore, indicator based on variation of natural frequencies is a potential technique to identify the bonding state.

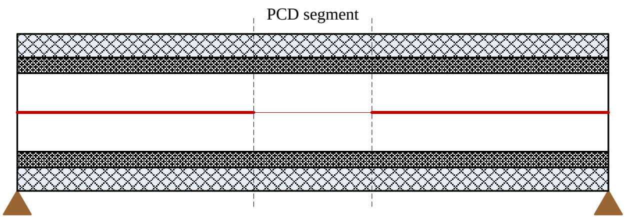


Fig. 8 Schematic diagram of explosive clad pipe with the PCD subjected to S-S boundary conditions

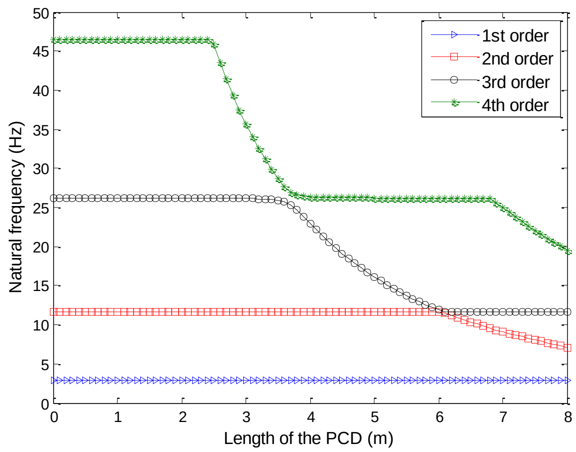
5.2 Effects of the PCD length

In this subsection, the effects of the PCD length on the natural frequencies and forced response of the explosive clad pipe are investigated. The PCD is defined that the contact strength is seriously insufficient or even delamination between outer and inner pipe in local position. For the reason of convenience of the following analysis, we supposed that the PCD is seated in the middle of the explosive clad pipe, as shown in Fig. 8. The two non-PCD segments have same length and excellent bonding strength, i.e., elastic stiffness is 5×10^6 N/m^2, while the elastic stiffness of PCD segment is set as 10 N/m^2. Thus, the elastic stiffness of explosive clad pipe with the PCD can be expressed by

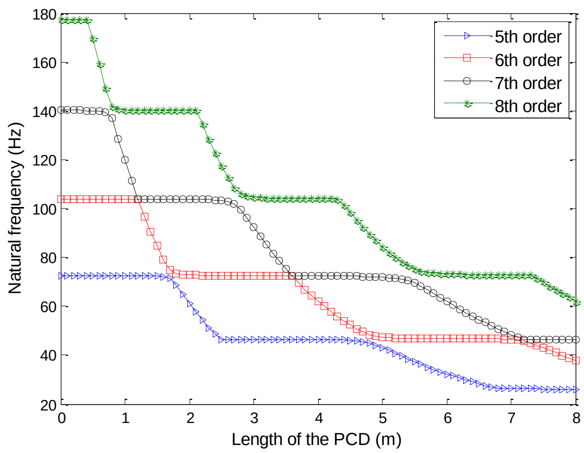
$$k(x)=\begin{cases}5\times10^6\ \text{N/m}^2, & x<(L-\ell)/2\\ 10\ \text{N/m}^2, & (L-\ell)/2\le x\le(L+\ell)/2\\ 5\times10^6\ \text{N/m}^2, & (L+\ell)/2<x<L\end{cases}\qquad(20)$$

where ℓ is the length of the PCD segment.

The shifts of first eight natural frequencies versus length of the PCD are shown in Fig. 9. As can be seen in the figure, all of natural frequencies decrease with the length of the PCD increases, for the reason that the increase of the differentiate the PCD length. Interestingly, the combination

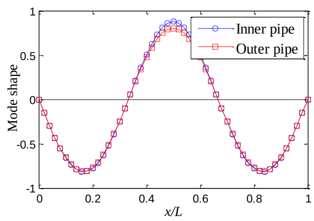


(a) The first four order natural frequencies

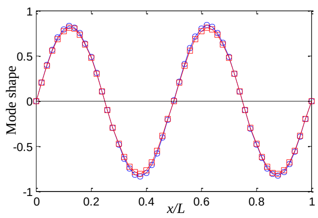


(b) Natural frequencies of the fifth to eighth order

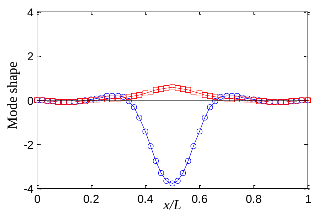
Fig. 9 Shifts of the first eight natural frequencies versus length of the PCD



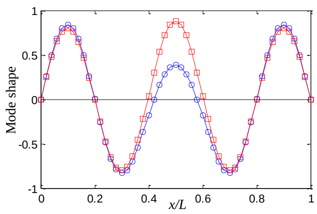
(a) The first mode



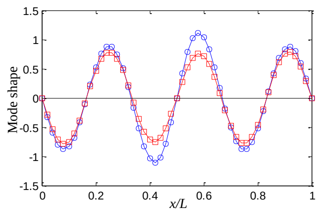
(b) The second mode



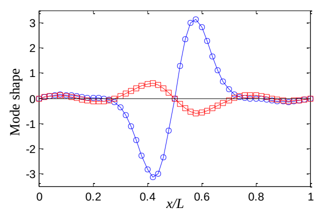
(c) The third mode



(b) The fourth mode



(e) The fifth mode



(f) The sixth mode

Fig. 10 Mode shapes of explosive clad pipe with 2 m PCD of natural frequencies of several orders to identify length of the PCD may achieve preferable effect. In addition, the first and second frequencies change little along the length of the PCD and demonstrate quite different variation characteristic from the others. It is indicated that the first two natural frequencies are insensitive to the PCD for the explosive clad pipe.

Fig. 10 present the mode shapes of the third order to the eighth order for the explosive clad pipe with the PCD of 2 m. It can be observed seen that the outer pipe and inner pipe can achieve approximately synchronous vibration with equal amplitude for the first four mode shapes. Nevertheless, with respect to the mode shapes after five order, the distinct features occur in the PCD segment which located at the middle of the explosive clad pipe, and exhibit quite different mode amplitudes for the outer pipe and inner pipe.

The effects of the PCD length on the forced vibration response of the explosive clad pipe is also studied in this part. A vertical force f_1 = 10 N is applied on the outer pipe at x = 2.5 m. The response point is located at x = 5 m on the outer pipe. The range of excitation frequency is from 0 to 250 Hz and the corresponding frequency step is set as 0.1 Hz.

The frequency response at midspan point of the outer pipe for the explosive clad pipe with different PCD length is shown in Fig. 11. The different color curves in the figure represent the specific PCD length. It is clearly seen that the frequency response of the healthy model (0 m PCD) for the explosive clad pipe in low frequency domain is in good agreement with the other frequency response curves. However, in the high frequency range from 200 to 250 Hz, an obvious feature is that all the resonant peaks are belong to the explosive clad pipe with different PCD length, and the response amplitudes increase with the PCD length increases. From the perspective of bonding state detection, this feature can be employed to determine whether the explosive clad pipe exists the PCD and probably to identify length of the PCD by response amplitude.

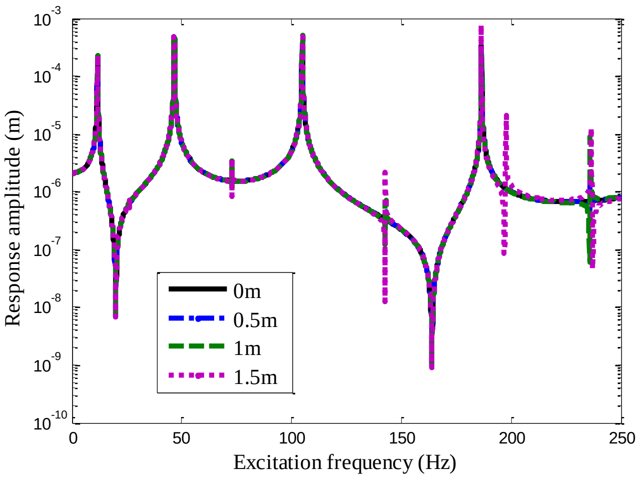


Fig. 11 Frequency response at midspan point of the outer pipe for the explosive clad pipe with different PCD length

5.3 Discussions

Based on the obtained vibration characteristics in previous sections, effective method can be developed to detect the bonding state of the explosive clad pipe. Because of the complexity of the bonding state, the natural frequencies and vibration response should be combined together to identity the interface bonding strength and the PCD of the explosive clad pipe. The outline of a potential method for identifying the bonding state of the explosive clad pipe is presented as follows.

First, the forced response on the outer pipe of the explosive clad pipe can be used to determine whether the PCD exist. If the resonant peaks appear in certain high frequency domain, it can be deemed that there is the PCD in the explosive clad pipe. Otherwise, the explosive clad pipe is thought to be without the PCD or the PCD length is too small to detect. Second, if there is the PCD in explosive clad pipe, the combination of response amplitudes and natural frequencies can be used to determine the length of the PCD. The response amplitude of the sensor near the PCD position is improved clearly compare with that of system without the PCD. Generally, the response amplitudes increase with the PCD length increase. This phenomenon will be more obvious when the excitation points are placed near the possible locations of the PCD. Finally, if there is not the PCD in explosive clad pipe, the natural frequencies can be employed to estimate the interface bonding strength. According to numerical results in section 4.1, we could define the natural frequencies corresponding to elastic stiffness 5×10^6 N/m^2 as the reference frequencies. When the natural frequency of each order is equal or greater than the reference frequency, the explosive clad pipe can be thought to have excellent interface bonding strength.

It should be emphasized that the proposed defect diagnostic method is only applied after explosive clad pipe forming in manufacturing plant of explosive clad pipe, and is not in the actual industrial service process. In practical industries, the explosive clad pipe is filled with medium such as high pressure liquid or gas. Currently, the PCD detection technique in industrial service process of explosive clad pipe has not yet been exploited. Identifying the length and position of the PCD in explosive clad pipe under industrial service process is a challenge work. Acoustic emission technique and ultrasonic guided wave technique developed in recent years are expected to realize the PCD detection of explosive clad pipe in practical industries. Moreover, artificial intelligence algorithms, such as artificial neural network, genetic algorithm and support vector machine, can be employed to develop effective technique for the evaluation the effect on the PCD detection.

6. Conclusions

In this paper, a general method was presented for studying vibrations of the explosive clad pipe, simplified as two parallel Euler-Bernoulli elastic beams continuously connected by a Winkler-type elastic layer. As opposite to other techniques available in the literature, the proposed method can be used also in the general case of inhomogeneous bonding state system and different boundary conditions. By taking the characteristic beam modal functions as the admissible functions, the Rayleigh-Ritz method is employed to derive the dynamic model, which was validated by comparison with numerical results and experimental. The effects of different bonding states on the natural frequencies and forced response were numerical investigated. Natural frequencies of some orders were sensitive to the interface bonding strength, and the explosive clad pipe was considered to possess the excellent bonding strength when the elastic stiffness of dynamic model large enough to keep natural frequencies relative stable. The forced response on the outer pipe will appear resonant peak in certain high frequency domain if the explosive clad pipe suffers from the PCD, and response amplitudes increase with the PCD length increase. From the perspective of bonding state identification, these vibration characteristics provide foundations and priory knowledge for identifying the PCD and insufficient interface bonding strength of the explosive clad pipe.

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References

Acarer, M. and Demir, B. (2008), "An investigation of mechanical and metallurgical properties of explosive welded aluminum-dual phase steel", *Mater. Lett.*, **62**(25), 4158-4160.

Arani, A.G. and Amir, S. (2014), "Flow-induced vibration of double visco-CNTs under magnetic fields considering surface effect", *Comp. Mater. Sci.*, **86**(86), 144-154.

Bashmal, S., Bhat, R. and Rakheja, S. (2009), "In-plane free vibration of circular annular disks", *J. Sound Vib.*, **322**(1), 216-226.

Chen, S., Daehn, G.S., Vivek, A., Liu, B., Hansen, S.R., Huang, J.H. and Lin, S.B. (2016), "Interfacial microstructures and mechanical property of vaporizing foil actuator welding of aluminum alloy to steel", *Mat. Sci. Eng. A*, **659**, 12-21.

El-Gebeily, M. and Khulief, Y.A. (2016), "Identification of wall-thinning and cracks in pipes utilizing vibration modes and wavelets", *Appl. Math. Model*, **40**(9-10), 5335-5348.

Emam, S.A. (2018), "Snapthrough and free vibration of bistable composite laminates using a simplified Rayleigh-Ritz model", *Compos. Struct.*, **206**, 403-414.

Findik, F. (2011), "Recent developments in explosive welding", *Mater. Des.*, **32**(3), 1081-1093.

Gao, Y., Muggleton, J.M., Liu, Y. and Rustighi, E. (2017), "An analytical model of ground surface vibration due to axisymmetric wave motion in buried fluid-filled pipes", *J. Sound Vib.*, **395**, 346-366.

Guo, X., Tao, J., Yuan, Z., Zhang, L. and Sun, X. (2012), "Interface and properties of explosive welded TA1/Al clad tube", *Rare Metal Mat. Eng.*, **41**(1), 139-142.

Guo, X., Fan, M., Wang, L. and Ma, F. (2016a), "Bonding interface and bending deformation of al/316lss clad metal prepared by explosive welding", *J. Mater. Eng. Perform.*, **25**(6), 2157–2163.

Guo, X., Wang, H., Liu, Z. Wang, L. Ma, F. and Tao, J. (2016b), "Interface and performance of CLAM steel/aluminum clad tube prepared by explosive bonding method", *Int. J. Adv. Manuf. Tech.*, **82**(1-4), 543-548.

Jeong, S. and Yoo, H.H. (2019), "Nonlinear structural analysis of a flexible multibody system using the classical Rayleigh-Ritz method", *Int. J. Nonlin. Mech.*, **110**, 69-80.

Karlicic, D., Cajic, M., Murmu, T. and Adhikari, S. (2015), "Nonlocal longitudinal vibration of viscoelastic coupled double-nanorod systems", *Eur. J. Mech. A-Solid*, **49**, 183-196.

Kaya, Y. and Kahraman, N. (2013), "An investigation into the explosive welding/cladding of Grade A ship steel/AISI 316L austenitic stainless steel", *Mater. Des.*, **52**(24), 367-372.

Li, J. and Hua, H. (2007), "Spectral finite element analysis of elastically connected double-beam systems", *Finite Elem. Anal. Des.*, **43**(15), 1155-1168.

Mao, Q. (2012), "Free vibration analysis of elastically connected multiple-beams by using the Adomian modified decomposition method", *J. Sound Vib.*, **331**(11), 2532-2542.

Mendes, R., Ribeiro, J.B. and Loureiro, A. (2013), "Effect of explosive characteristics on the explosive welding of stainless steel to carbon steel in cylindrical configuration", *Mater. Des.*, **51**(51), 182-192.

Mouritz, A.P., Townsend, C. and Shah-Khan, M.Z. (2000), "Non-destructive detection of fatigue damage in thick composites by pulse-echo ultrasonics", *Compos. Sci. Technol.*, **60**(1), 23-32.

Oniszczuk, Z. (2000), "Free transverse vibrations of elastically connected simply supported double-beam system", *J. Sound Vib.*, **232**(2), 387-403.

Oniszczuk, Z. (2003), "Forced transverse vibrations of an elastically connected complex simply supported double-beam system", *J. Sound Vib.*, **264**(2), 273-286.

Pellicano, F. (2007), "Vibration of circular cylindrical shells: Theory and experiments", *J. Sound Vib.*, **303**(1-2), 154-170.

Shen, H., Wen, J., Yu, D. and Wen, X. (2009), "The vibrational properties of a periodic composite pipe in 3D space", *J. Sound Vib.*, **328**(1-2), 57-70.

Si, Y., Zhang, Z., Cheng, W. and Yuan, F. (2015), "State Detection of explosive welding structure by dual-tree complex wavelet transform based permutation entropy", *Steel Compos. Struct., Int. J.*, **19**(3), 569-583.

Si, Y., Zhang, Z., Wang, H. and Yuan, F. (2017), "Feasibility study of bonding state detection of explosive composite structure based on nonlinear output frequency response functions", *Steel Compos. Struct., Int. J.*, **24**(4), 391-397.

Sun, X.J., Tao, J. and Guo, X.Z. (2011), "Bonding properties of interface in Fe/Al clad tube prepared by explosive welding", *T. Nonferr. Metal. Soc.*, **21**(10), 2175-2180.

Vu, H.V., Ordóñez, A.M. and Karnopp, B.H. (2000), "Vibration of a double-beam system", *J. Sound Vib.*, **229**(4), 807-822.

Xia, H., Wang, S. and Ben, H. (2014), "Microstructure and mechanical properties of Ti/Al explosive cladding", *Mater. Des.*, **56**(4), 1014-1019.

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