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An alternative evaluation of the LTB behavior of mono-symmetric beam-columns

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Abstract. Beam-columns are structural members subjected to a combination of axial and bending forces. Lateral-torsional buckling is one of the main failure modes. Beam-columns that are bent about its strong axis may buckle out of the plane by deflecting laterally and twisting as the values of the applied loads reach a limiting state. Lateral-torsional buckling failure occurs suddenly in beam-column elements with a much greater in-plane bending stiffness than torsional or lateral bending stiffness. This study intends to establish a unique convenient closed-form equation that it can be used for calculating critical elastic lateral-torsional buckling load of beam-column in the presence of a known axial load. The presented equation includes first order bending distribution, the position of the loads acting transversely on the beam-column and mono-symmetry property of the section. Effects of axial loads, slenderness and load positions on lateral torsional buckling behavior of beam-columns are investigated. The proposed solutions are compared to finite element simulations where thin-walled shell elements including warping are used. Good agreement between the analytical and the numerical solutions is demonstrated. It is found out that the lateral-torsional buckling load of beam-columns with mono-symmetric sections can be determined by the presented equation and can be safely used in design procedures.

Keywords: lateral-torsional buckling; stability beam-column; mono-symmetric section

1. Introduction

Beam-columns are structural members that combine the beam function of transmitting transverse forces or moments with the compression (or tension) member function of transmitting axial forces. Beam-columns are mostly loaded in the plane of the weak axis so that bending occurs about their strong axis. First order bending moments and in-plane deformations are produced by the end moments and transverse loadings of the beam-column, while axial force will produce second-order moments and additional in-plane deformations. When the values of the loads on the beam-column reach a limiting state, the member will experience out of plane bending and twisting. At this limiting state, the compression flange of the member becomes unstable and bends laterally while the remainder of the cross-section, which is stable, tends to restrain the lateral flexure of the compression flange. The net effect is that the whole section rotates and moves laterally. Lateral-torsional buckling (LTB) failure occurs suddenly in slender beam-columns with a much greater in-plane bending stiffness than their lateral bending or torsional stiffnesses (Torkamani and Roberts 2009). LTB is often the main failure mode controlling the strength of thin-walled structures and should be considered in the design of slender beam-columns with insufficient lateral bracing due to it may occur long before the bending stress at the extreme fiber of the section reaches to yield point. Fig. 1 illustrates LTB of beam-column with I-section. The limit state of the applied loads on the beam-column members is called as the critical elastic LTB load. The cross-section of the member, the unbraced length of the member, the support conditions, the type of loads acting on the member, the vertical positions of the applied loads with respect to the shear center are effective on LTB behavior of mono-symmetric beam-columns. The general concept of flexural buckling and LTB of structural members has been well presented in many textbooks (Timoshenko and Gere 1961, Chen and Lui 1987, Chen and Atsuta 1977, Galambos and Surovek 2008, Trahair 1993).

The differential equilibrium equations obtained for critical LTB load of an axially loaded beam subjected to uniform bending can be solved and presented in closed form by considering the boundary conditions (Trahair 1993, Salvadori 1956, Hill and Clark 1951). However, the analytical solutions are either too complex or involve infinite series for load types where moment gradient is not constant. In a situation like this, the solution of differential equilibrium equations mostly requires use of numerical approaches such as finite difference (Bleich 1952, Chajes 1974, Assadi and Roeder 1985, Suryoatmono and Ho 2002, Serna *et al*. 2006), finite integral (Kitipornchai and Trahair 1975, Kitipornchai and Richter 1978, Kitipornchai *et al*. 1984, 1986), finite element (Barsoum and Gallagher 1970, Powel and Klingner 1970, Hancock and Trahair 1978,

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Fig. 1 LTB of beam-column with I-section

Bradford and Ronagh 1997, Papangelis *et al*. 1998, Lim *et al*. 2003, Lee *et al*. 1994, Park *et al*. 2004, Gu and Chan 2005, Mohri *et al*. 2008a) or finite strip (Bui 2009, 2012, Adany and Schafer 2014, Naderian and Ronagh 2015) methods.

Energy method is based on the equality between the additional strain energy stored during LTB and the additional work done by the applied forces. In this method, the LTB load is calculated by substituting an approximate buckled shape which satisfies the kinematic boundary conditions and corresponds to real mode shape into the energy equation. Kinematic boundary conditions are related to geometrical constraints preventing one or more deflections or rotations at the support of the structural members (Trahair 1993). Extensive studies are carried out in order to describe LTB behavior of thin-walled members using energy method. Wang and Kitipornchai studied LTB of mono-symmetric cantilevers using both the Ritz method and the finite integral method (Wang and Kitipornchai 1986). Aydin *et al*. (2015) presented a compact closed-form equation to calculate LTB loads of simply supported beams with mono-symmetric I-section. Aydin *et al*. (2013) studied the LTB behavior of double angle and tee section cantilevers. Mohri *et al*. (2003) recomputed 3-factor formula, which is commonly used for calculation of elastic LTB loads of beams, and proposed some improvements. Yilmaz and Kirac (2017) introduced an analytical study to evaluate elastic LTB load of both doubly-symmetric and mono-symmetric I-section beams. This study also includes a parametric study where a simplified equation is proposed for calculating the LTB load of the beams with European standard I-section. The LTB of Europen wide flange I-section beam is also studied by Yilmaz *et al*. (2017). Andrade and Camotim (2004) developed an analytical treatment for LTB behavior of doubly-symmetric prismatic and tapered beams. In this study, the effect of pre-buckling deflections on LTB behavior of both simply-supported beams and cantilever beams, which are prismatic and web-tapered, is investigated. LTB behavior of steel beams with simultaneously tapered flanges and web were studied by Kus (2015). This study revealed that tapering of flanges affects much more the critical moments than tapering of the web. The LTB loads of singly and doubly symmetric I-section cantilevers were investigated by Andrade *et al*. (2007). Ozbasaran *et al*. (2015) developed an alternative design procedure for cantilever I-section and introduced a parametric formula based on the energy method to calculate LTB load. The proposed design procedure was compared with code specifications and FEA. Yuan *et al*. (2013) improved an analytical model to determine the LTB behavior of steel web tapered tee-section cantilevers. Kim *et al*. (2016) studied LTB of castellated beams. LTB of the simply supported channel and Z-section purlins with top flange horizontally restrained are investigated by Zhang and Tong (2016). Mohammadi *et al*. (2016) studied the bracing stiffness requirements of mono-symmetric I-beams with discrete torsional braces under pure bending condition. Benyamina *et al*. (2013) introduced an analytical formula to assess LTB behavior of the double-symmetric web tapered I-section beam in function of the classical stiffness terms, the load height level, and the tapering parameter. LTB of the tapered thin-walled beam with arbitrary cross-section and boundary conditions are investigated using the numerical method based on power series by Asgarian *et al*. (2013). The shape optimization of tapered I-beams with lateral-torsional buckling, deflection and stress constraints are investigated by Ozbasaran and Yilmaz (2018). Thai *et al*. (2017) improved novel refined plastic hinge analysis technique including the effect of the LTB and local buckling.

Related to LTB of beam-columns, Wang and Kitipornchai (1989) proposed a set of buckling parameters to describe LTB behavior of mono-symmetric beam-columns under uniform moment or eccentric axial loads. Torkomani and Roberts (2009) derived the energy equations for doubly symmetric beam-column members by expressing in dimensional and non-dimensional forms. Magnucka-Blandzi (2009) investigated beam-columns with I-section subjected to a uniformly distributed transverse load, small axial force and two different moments located at its both ends. Cheng *et al*. (2013) studied flexural buckling and LTB of cold-formed channel section beams under combined compression and biaxial bending. They also concluded the effect of non-symmetric pre-buckling stress due to bending about the minor axis distribution on LTB of channel beams. Kucukler *et al*. (2015) introduced a stiffness reduction method for the flexural-torsional buckling assessment of steel beam-columns subjected to major axis bending and axial compression. Soula *et al*. (2016) demonstrated that the distortion deformations are effective on the elastic lateral buckling behavior of thin-walled box beam elements under combined bending and axial forces. Challamel *et al*. (2010) investigated the elastic flexural-torsional buckling of linearly tapered cantilever strip beam-columns subjected to axial and transversal point loads applied at the tip. In this study, the governing differential equation is integrated into closed form by means of confluent hypergeometric functions for prismatic and wedge-shaped members. For general tapered members, solutions are presented using Frobenius series and numerical treatment. Gu and Chan (2005) developed finite element formulation which can be used the geometrically nonlinear analysis of the space beam-column members allowing for axial–flexural, lateral–torsional and axial–torsional buckling. The quite detailed study where the non-linear stability model for LTB of beam-column elements with doubly-symmetric I-section is established has been introduced by Mohri *et al*. (2008b). The present analytical model included first-order bending distribution, load height level, and presence of axial loads. Then, this analytical model is further improved by Mohri *et al*. (2013) to consider the mono-symmetry property of beam-columns. Besides, with the improvements performed, the three-factor formula which is developed for beam stability Mohri *et al*. (2003) has been extended with the fourth factor for the presence of the axial load. These two advanced studies which are developed for calculation LTB load of doubly-symmetric and mono-symmetric beam columns using Galerkin's approach considers the effect of pre-buckling deflections and also includes the finite element analysis verifications. Tankova *et al*. (2017) introduced a new design proposal for the out of plane buckling resistance of the prismatic beam-columns subject to axial compression and uniaxial major-axis bending that was developed based on the well-known Ayrton-Perry format. The study focused on elastic LTB of tapered beams with doubly-symmetric section subjected to combined bending and axial forces were presented by Osmani and Meftah (2018). The effect of shear deformation is also considered in the study. Optimal design of beam-column beam-columns with I-section considering stress, deflection and stability constraints is studied by Ozbasaran (2018).

This study focused on the effects of axial forces on lateral buckling behavior of beams with mono-symmetric cross sections. Ritz method is utilized in order to establish a unique compact closed-form equation by considering the total potential energy of beam-column elements subjected to a constant axial force and various transverse load cases. The presented equation includes first order bending distribution, the position of the loads acting transversely on the beam-column member respect to the shear center, the mono-symmetry property of the section and the interaction between buckling and lateral-torsional buckling. The effects of axial loads, slenderness and load positions on LTB of both doubly-symmetric and mono-symmetric beam-columns are investigated. The presented solutions are compared to finite element analysis in which beam-column members are modeled with shell elements (S8R5) including

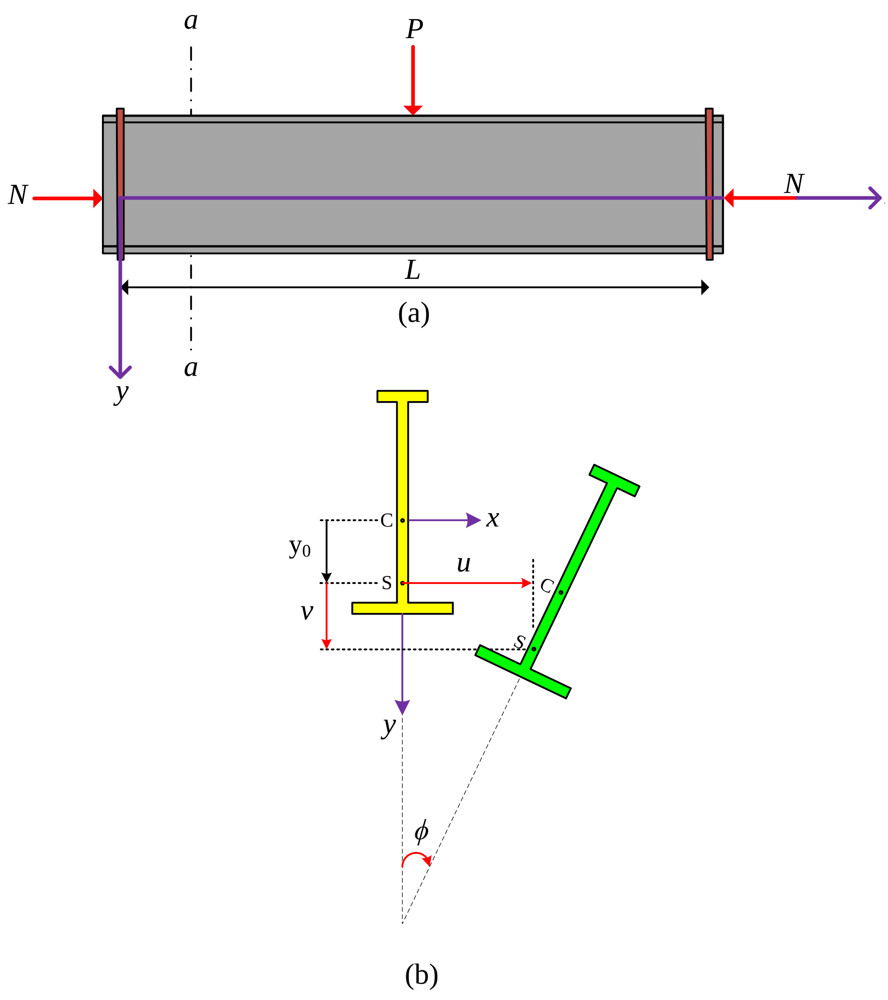

Fig. 2 LTB of mono-symmetric beam-column: (a) side view; (b) a-a section

warping. It is observed that the closed-form solutions are in good agreement with the finite element simulation.

2. Analytical study

The LTB of mono-symmetric beam-columns consists of two stages. First, combined transversely and compressive axially loaded beam-column bends about its major axis, and then it buckles by bending laterally and twisting as the magnitude of the loads acting on the beam-column reaches to a critical level. Fig. 2 shows the LTB of beam-column with mono-symmetric I-section subjected to an axial force that acts through the centroid of the cross section and concentrated force that acts transversely at mid-span.

In Fig. 2(a), L is the beam-column length. a-a section of beam-column is drawn in Fig. 2(b). S and C show the shear center and the center of gravity of the section, respectively. u is the lateral displacement of the shear center, v is vertical displacement of the shear center and φ is the torsional rotation. y_0 is the distance measured from the center of gravity to the shear center. By utilizing Vlassov's model, which assumes that the cross section is rigid in its plane, hence there is no distortion deformation of the section and the shear deformation in the mean surface of the section are negligible (Mohri *et al*. 2003). The total potential energy of the beam-column given in Fig 2, at a slightly buckled configuration can be written as follows by disregarding pre-buckling deflections (Trahair 1993).

$$\begin{aligned}\Pi = {} & \frac{1}{2}\int_0^L EI_y\left(\frac{d^2u}{dz^2}\right)^2 dz + \frac{1}{2}\int_0^L EC_w\left(\frac{d^2\phi}{dz^2}\right)^2 dz \\ & + \frac{1}{2}\int_0^L GJ\left(\frac{d\phi}{dz}\right)^2 dz \\ & - \frac{1}{2}\int_0^L N\left[\left(\frac{du}{dz}\right)^2 + \left(\frac{I_x + I_y}{S} + y_0^2\right)\left(\frac{d\phi}{dz}\right)^2 \right. \\ & \left. + 2y_0\left(\frac{du}{dz}\right)\left(\frac{d\phi}{dz}\right)\right] dz \\ & + \frac{1}{2}\int_0^L M_x\left[2\phi\left(\frac{d^2u}{dz^2}\right) + \beta_x\left(\frac{d\phi}{dz}\right)^2\right] dz + W_h\end{aligned} \quad (1)$$

where E is young modulus, G is shear modulus, S is area of cross-section, I_x is moment of inertia about strong axis, I_y is moment of inertia about weak axis, Cw is warping constant, J is torsional constant and M_x is the bending moment about strong axis and N is constant compressive axial load. Wanger's coefficient β_x associated with a mono-symmetry property of the cross section is defined by Eq. (2) (Trahair 1993).

$$\beta_x = \frac{1}{I_x}\int_S y(x^2 + y^2)dS - 2y_0 \quad (2)$$

where x and y are Cartesian coordinates of the infinitesimal area (dS). y_0 is positive when the shear center below the center of gravity. W_h is work done by loads which are acting outside of the shear center. This work results from changing of the distance between the application points of the loads and the shear center as cross-section rotates. W_h can be calculated by (3) as

$$W_h = \frac{1}{2}\sum PH_p\phi_p^2 + \frac{1}{2}\int_0^L qH_q\phi^2 dz \quad (3)$$

where H_p and H_q are the vertical distance of the acting point of the concentrated (P) and uniformly distributed loads (q) measured from the shear center, respectively. ϕ_p is the torsional rotation at a point in which the concentrated load is applied. In (3), H_p and H_q are positive for loads that act in below the shear center.

Assume that when lateral-torsional buckling occurs, the lateral displacement of beam-column defined at the shear center and the angle of rotation of the cross-section can be described as follows

$$u = a\sin\frac{\pi}{L}z \quad (4)$$

$$\phi = b\sin\frac{\pi}{L}z \quad (5)$$

where A and B are the associated displacement amplitudes. Note that the displacement functions assumed in Eqs. (4)-(5) satisfy the simply supported boundary conditions ($u = \phi = 0$ and $d^2u/dz^2 = d^2\phi/dz^2 = 0$) at supports ($z = 0$ and $z = L$).

The strain energy stored in the beam-column due to lateral bending, warping, and torsion can be calculated using the following formula

$$\begin{aligned}U = {} & \frac{1}{2}\int_0^L EI_y\left(\frac{d^2u}{dz^2}\right)^2 dz + \frac{1}{2}\int_0^L EC_w\left(\frac{d^2\phi}{dz^2}\right)^2 dz \\ & + \frac{1}{2}\int_0^L GJ\left(\frac{d\phi}{dz}\right)^2 dz\end{aligned} \quad (6)$$

Substituting buckling displacements in Eqs. (4)-(5) into Eq. (6) yields

$$U = \frac{a^2\pi^4 EI_y}{4L^3} + \frac{b^2\pi^4 EC_w}{4L^3} + \frac{b^2\pi^2 GJ}{4L} \quad (7)$$

The work done by constant compressive axial forces is as

$$\begin{aligned}V_1 = -\frac{1}{2}\int_0^L N & \left[\left(\frac{du}{dz}\right)^2 + \left(\frac{I_x + I_y}{S} + y_0^2\right)\left(\frac{d\phi}{dz}\right)^2 \right. \\ & \left. + 2y_0\left(\frac{du}{dz}\right)\left(\frac{d\phi}{dz}\right)\right] dz\end{aligned} \quad (8)$$

By substituting Eqs. (4)-(5) into Eq. (8), the work done by the axial force can be written as

$$V_1 = -\frac{N\pi^2\left[a^2S + 2abSy_0 + b^2\left(I_x + I_y + Sy_0{}^2\right)\right]}{4LS} \quad (9)$$

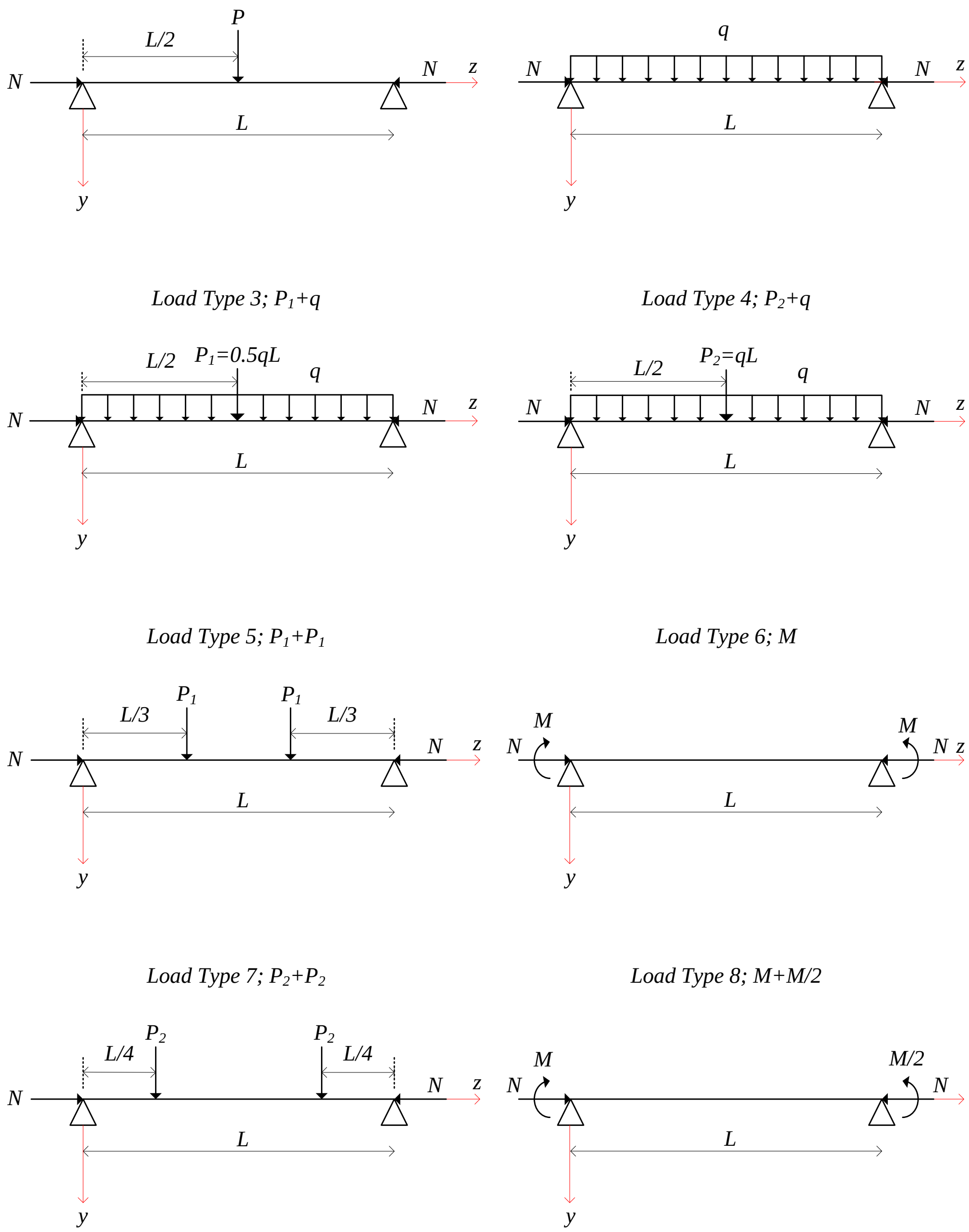

Fig. 3 Load types

The work done by external transverse forces is as

$$V_2 = \frac{1}{2}\int_0^L M_x \left[2\phi\left(\frac{d^2u}{dz^2}\right) + \beta_x\left(\frac{d\phi}{dz}\right)^2\right]dz + \frac{1}{2}\sum P H_p \phi_p^2 + \frac{1}{2}\int_0^L q H_q \phi^2 dz \tag{10}$$

Assuming that the vertical positions of all transverse loads on the beam-column are same (H_p = H_q = H) and substituting displacement functions into Eq. (10), the work done by transverse forces can be written in the following compact form

$$V_2 = D_1 b^2 \beta_x R_{cr} + D_2 ab R_{cr} + D_3 b^2 H R_{cr} \tag{11}$$

Where D_1, D_2, and D_3 are integral parameters depending on moment gradient about the strong axis along the beam length. In this study, D_1, D_2, and D_3 coefficients are calculated for eight load types shown in Fig. 3 and presented in Table 1.

In Eq. (9), R_{cr} is the critical load which can be expressed by Eq. (10) depending on the load type acting on the beam.

$$R_{cr} = P_{cr} = q_{cr} L = \frac{M_{cr}}{L} \tag{12}$$

where P_{cr}, q_{cr} and M_{cr} are critical concentrated load, uniformly distributed load and moment, respectively. It is noted that the critical buckling load type varies according to considered loading case. For load case 3 in Fig. 3, the critical LTB load is in terms of q_{cr} which implies that the critical values of uniformly distributed load and

Table 1 Material ratios for 1 m^3 concrete

Load type	D1	D2	D3
P	0.183425	-0.86685	0.5
q	0.143117	-0.536234	0.25
q+0.5qL	0.234829	-0.969659	0.5
q+qL	0.326542	-1.40308	0.75
P+P (L/3)	0.360811	-1.47162	0.75
M	2.4674	-4.9348	0
P+P (L/4)	0.337638	-1.17528	0.5
M+0.5M	1.85055	-3.7011	0

concentrated load are q_{cr} and $0.5q_{cr}L$, respectively.

Using the flexural buckling load which is presented in Eq. (13), the total potential energy of the beam-column $\Pi = U + V_1 + V_2$ can be written as Eq. (14)

$$N_y = \frac{\pi^2 EI_y}{L^2} \tag{13}$$

$$\begin{aligned}\Pi = \frac{\pi^2}{4L}\Bigg[& b^2 GJ + \frac{b^2 N_y C_w}{I_y} + a^2 N_y \\ & -N\left(a^2 + 2aby_0 + b^2\left(\frac{I_x + I_y}{S} + {y_0}^2\right)\right)\Bigg] \\ & +D_1 b^2 \beta_x R_{cr} + D_2 abR_{cr} + D_3 b^2 HR_{cr}\end{aligned} \tag{14}$$

When buckling occurs, the total energy function reaches a stationary condition, which requires

$$\frac{\partial \Pi}{\partial a} = \frac{\partial \Pi}{\partial b} = 0 \tag{15}$$

Substituting Eq. (14) into Eq. (15) yields

$$bD_2R_{cr} + \frac{\pi^2}{2L}\left[aN_y - N(a + by_0)\right] = 0 \tag{16}$$

$$\begin{aligned}&2bD_1\beta_x R_{cr} + aD_2R_{cr} + 2bD_3HR_{cr} \\ &+\frac{\pi^2}{2L}\left[bGJ + \frac{bC_wN_y}{I_y} - N\left(ay_0 + b\left(\frac{I_x + I_y}{S} + {y_0}^2\right)\right)\right] \\ &= 0\end{aligned} \tag{17}$$

Eqs. (16) and (17) are standard eigenvalue equations and can be expressed as matrix form

$$\begin{bmatrix} \frac{(N_y - N)\pi^2}{2L} & D_2R_{cr} - \frac{N\pi^2 y_0}{2L} \\ D_2R_{cr} - \frac{N\pi^2 y_0}{2L} & \frac{\pi^2}{2L}\left(\begin{matrix} GJ + \frac{C_wN_y}{I_y} - N\left(\frac{I_x + I_y}{S} + {y_0}^2\right) \\ + \frac{4LR_{cr}}{\pi^2}(D_1\beta_x + D_3H) \end{matrix}\right) \end{bmatrix} \begin{Bmatrix} a \\ b \end{Bmatrix} = 0 \tag{18}$$

By using torsional buckling load which is presented in Eq. (19), Eq. (18) can be rearranged to obtain a more compact matrix form as Eq. (20).

$$N_z = \frac{\frac{\pi^2 EC_w}{L^2} + GJ}{\frac{I_x + I_y}{S} + y_0^2} \tag{19}$$

$$\begin{bmatrix} \frac{(N_y - N)\pi^2}{2L} & D_2R_{cr} - \frac{N\pi^2 y_0}{2L} \\ D_2R_{cr} - \frac{N\pi^2 y_0}{2L} & \frac{\pi^2}{2L}\left(\begin{matrix}\frac{I_x + I_y}{S} \\ +{y_0}^2\end{matrix}\right)\left(\begin{matrix} N_z - N \\ + \frac{4L(D_1\beta_x + D_3H)R_{cr}}{\pi^2\left(\frac{I_x + I_y}{S} + {y_0}^2\right)} \end{matrix}\right) \end{bmatrix} \begin{Bmatrix} a \\ b \end{Bmatrix} = 0 \tag{20}$$

Eq. (20) is satisfied when the determinant of the coefficients matrix is equal to zero. Finally, LTB load of beam-column can be calculated following equation in presence of a known axial load

$$R_{cr} = \frac{\pi^2}{L}K_1\left(-K_2 + K_3 + \sqrt{\begin{matrix} {K_2}^2 - 2K_2K_3 \\ + \frac{K_4}{2K_1}(N - N_y)(N - N_z) \end{matrix}}\right) \tag{21}$$

where K_1, K_2, K_3, and K_4 parameters can be defined as follows

$$K_1 = \frac{1}{2{D_2}^2} \tag{22}$$

$$K_2 = (N - N_y)(D_1\beta_x + D_3H) \tag{23}$$

$$K_3 = D_2Ny_0 \tag{24}$$

$$K_4 = \frac{I_x + I_y}{S} + {y_0}^2 \tag{25}$$

At the end of the analytical work, calculating critical LTB of the beam-columns with the mono-symmetric section in presence of a known axial load can be summarized in four steps. First, D_1-D_3 integral parameters can be found for considered load case in Table 1. The flexural buckling load N_y and the torsional buckling load N_z can be calculated using Eqs. (13) and (19), respectively. Then, K_1, K_2, K_3, and K_4 can be calculated by using Eqs. (22)-(25). Finally, critical LTB load can be found by substituting calculated parameters in closed-form Eq. (21).

3. Numerical investigations

In numerical computations, effects of axial loads, slenderness and the load positions on the LTB loads of doubly-symmetric and mono-symmetric beam-columns are investigated. For this purpose, the analytical LTB loads are calculated for different values of the axial load by using the presented formula. The analytical solutions are compared to numerical simulations. ABAQUS finite element software was utilized to validate LTB solutions obtained by the presented equation. Beam-columns were modeled with S8R5 shell elements. S8R5 element has eight-nodes and five degrees of freedom at a node (ABAQUS 2013). Mesh studies have indicated that it would be adequate to use sixty elements along the longitudinal direction, eight elements

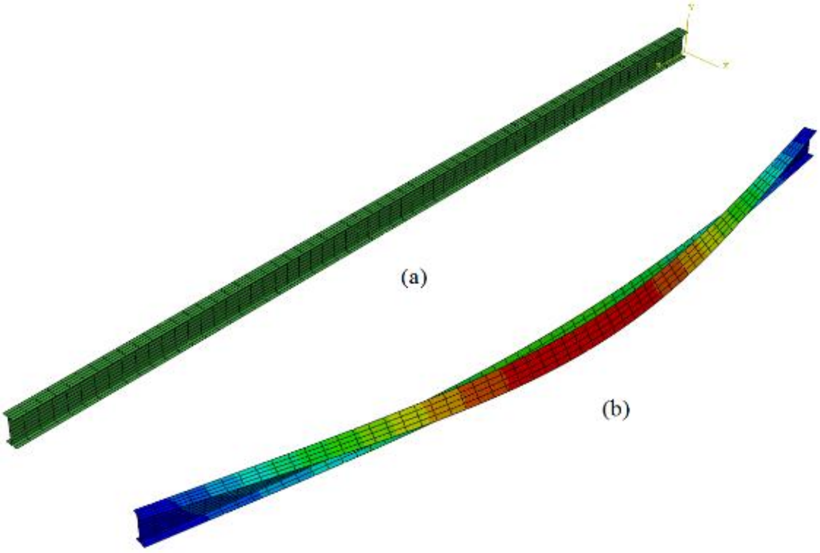

Fig. 4 (a) Shell finite element model; (b) buckled shape

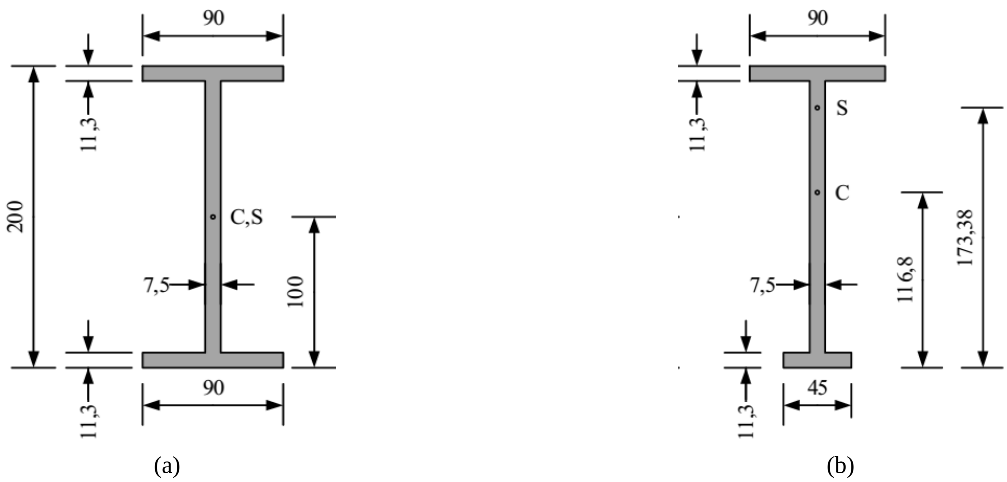

Fig. 5 (a) Section A; (b) Section B

through the depth of the web and four elements across the width of the flange (Aydin *et al*. 2015). Shell finite element model and buckled shape are given in Fig. 4.

The results obtained by the present analytical model were also compared with the results obtained using the equation that exist in the study presented by Mohri *et al*. (2013), which is the most comprehensive and novel work related to lateral-buckling of the beam-column element with the mono-symmetric section. The improved analytical model presented by Mohri *et al*. (2013) includes the first-order bending distribution, load height level, pre-buckling deflection and the presence of axial loads and is based on Galerkin's approach.

Fig. 5 illustrates the sections which are used for numerical examples. Section A is a doubly-symmetric I-shape. Dimensions of Section B is similar to Section A, except the width of the bottom flange. The bottom flange width of Section B is reduced to half of its top flange width in order to design mono-symmetric beam-column for numerical examples. Section properties are shown in Table 2.

Table 2 Section properties

Properties	Section A	Section B	Properties
t_f	11.3 mm	11.3 mm	t_f
t_w	7.5 mm	7.5 mm	t_w
b_{tf}	90 mm	90 mm	b_{tf}
b_{bf}	90 mm	45 mm	b_{bf}
h	200 mm	200 mm	h
E	200000 N/mm^2	200000 N/mm^2	E
G	76923 N/mm^2	76923 N/mm^2	G
J	113110 mm^4	91466.3 mm^4	J
C_w	12.222*10^9 mm^6	2.716*10^9 mm^6	C_w
I_x	21.618*10^6 mm^4	16.280*10^6 mm^4	I_x
I_y	1.379*10^6 mm^4	0.779*10^6 mm^4	I_y
A	3364.5 mm^2	2856 mm^2	A
β_x	0	131.4 mm	β_x
y_0	0	-56.6 mm	y_0

Table 3 Section A and section B under load case 1

Load position	Axial load (N/Ny)	Section A PE	Section A AB	Section A PM	Section A PE/AB	Section A PE/PM	Section B PE	Section B AB	Section B PM	Section B PE/AB	Section B PE/PM
Top Flange	0.00	21.46	20.11	21.24	1.07	1.01	17.13	16.20	17.95	1.06	0.95
	0.10	20.46	19.29	20.20	1.06	1.01	16.42	15.66	17.10	1.05	0.96
	0.20	19.40	18.41	19.09	1.05	1.02	15.66	15.08	16.22	1.04	0.97
	0.30	18.26	17.44	17.92	1.05	1.02	14.85	14.44	15.29	1.03	0.97
	0.40	17.02	16.37	16.65	1.04	1.02	13.98	13.72	14.29	1.02	0.98
	0.50	15.66	15.17	15.27	1.03	1.03	13.01	12.91	13.22	1.01	0.98
	0.60	14.14	13.79	13.74	1.03	1.03	11.94	11.96	12.04	1.00	0.99
	0.70	12.38	12.16	11.99	1.02	1.03	10.71	11.07	10.71	0.97	1.00
	0.80	10.25	10.14	9.89	1.01	1.04	9.23	9.40	9.14	0.98	1.01
	0.90	7.38	7.35	7.10	1.00	1.04	7.26	7.42	7.11	0.98	1.02
Shear Center	0.00	25.26	23.73	24.94	1.06	1.01	17.78	16.83	18.61	1.06	0.96
	0.10	23.90	22.60	23.52	1.06	1.02	17.01	16.25	17.70	1.05	0.96
	0.20	22.47	21.40	22.05	1.05	1.02	16.20	15.61	16.75	1.04	0.97
	0.30	20.96	20.10	20.50	1.04	1.02	15.33	14.92	15.76	1.03	0.97
	0.40	19.35	18.69	18.86	1.04	1.03	14.39	14.15	14.70	1.02	0.98
	0.50	17.61	17.13	17.11	1.03	1.03	13.37	13.27	13.56	1.01	0.99
	0.60	15.71	15.38	15.21	1.02	1.03	12.23	12.26	12.32	1.00	0.99
	0.70	13.56	13.36	13.09	1.01	1.04	10.94	11.06	10.92	0.99	1.00
	0.80	11.04	10.95	10.62	1.01	1.04	9.39	9.56	9.29	0.98	1.01
	0.90	7.79	7.76	7.47	1.00	1.04	7.35	7.51	7.19	0.98	1.02
Bottom Flange	0.00	29.73	28.00	29.30	1.06	1.01	22.58	21.25	23.43	1.06	0.96
	0.10	27.91	26.48	27.40	1.05	1.02	21.35	20.32	22.03	1.05	0.97
	0.20	26.02	24.87	25.46	1.05	1.02	20.07	19.32	20.59	1.04	0.97
	0.30	24.05	23.15	23.45	1.04	1.03	18.74	18.23	19.11	1.03	0.98
	0.40	21.99	21.31	21.36	1.03	1.03	17.34	17.05	17.57	1.02	0.99
	0.50	19.80	19.32	19.17	1.02	1.03	15.86	15.76	15.96	1.01	0.99
	0.60	17.45	17.13	16.84	1.02	1.04	14.25	14.30	14.25	1.00	1.00
	0.70	14.86	14.67	14.30	1.01	1.04	12.49	12.46	12.39	1.00	1.01
	0.80	11.90	11.81	11.42	1.01	1.04	10.46	10.66	10.30	0.98	1.02
	0.90	8.21	8.19	7.86	1.00	1.05	7.93	8.10	7.73	0.98	1.03

Load Case 1, P (kN)

* PE: Present analytical model, AB: Abaqus numerical analysis, PM: Presented by Mohri *et al*. (2013)

In this numerical study, LTB loads of beam-columns were determined for three loading positions, which are the top flange, shear center, and bottom flange in order to examine the effect of load height level on LTB behavior of beam-columns. For the span of 6 m, LTB loads of beam-columns with section A and B subjected to load type 1 are calculated in presence of different axial load varied from N/N_y = 0 to N/N_y = 0.9. The analytical and numerical solutions are presented in Table 3 for section A and section B.

Results in Table 3 have shown that the analytical solutions are in good agreement with both the numerical solutions and the results obtained by the analytical model presented by Mohri *et al*. (2013). The greatest difference between present analytical solutions and finite element solutions are 7%. This difference is quite admissible for present analytical model based on Ritz method where simplified displacement functions (Eqs. (4) and (5)) are used in order to establish a closed-form equation (Trahair 1993, Aydin *et al*. 2015). However, extensive displacement functions with more terms provide better convergence for exact buckling loads (Trahair 1993, Yuan *et al*. 2013). The maximum difference between present analytical solutions and solutions which is obtained using the equation proposed by Mohri *et al*. (2013) are 5%. It is considered that this very small difference stems from the fact that the analytical model presented by Mohri *et al*. (2013) includes the pre-buckling deflection effect. The numerical study also

Table 2 Section A and B under load case 2

Section	Load position	Length (m)	N/Ny = 0.2 PE	AB	PM	PE/AB	PE/PM	N/Ny = 0.8 PE	AB	PM	PE/AB	PE/PM
Load Case 2, q (kN/m)												
Section A	Top Flange	4.00	17.86	17.75	17.97	1.01	0.99	9.35	9.32	9.25	1.00	1.01
		5.00	9.21	9.14	9.27	1.01	0.99	4.81	4.81	4.76	1.00	1.01
		6.00	5.38	5.33	5.41	1.01	0.99	2.80	2.80	2.77	1.00	1.01
		7.00	3.41	3.38	3.44	1.01	0.99	1.77	1.77	1.75	1.00	1.01
		8.00	2.30	2.28	2.32	1.01	0.99	1.19	1.20	1.18	1.00	1.01
		9.00	1.63	1.61	1.64	1.01	0.99	0.84	0.84	0.83	1.00	1.01
		10.00	1.19	1.18	1.20	1.01	0.99	0.61	0.62	0.61	0.99	1.01
	Shear Center	4.00	21.20	21.05	21.38	1.01	0.99	10.22	10.20	10.11	1.00	1.01
		5.00	10.60	10.51	10.69	1.01	0.99	5.17	5.17	5.12	1.00	1.01
		6.00	6.05	6.00	6.11	1.01	0.99	2.98	2.98	2.94	1.00	1.01
		7.00	3.78	3.75	3.81	1.01	0.99	1.87	1.87	1.85	1.00	1.01
		8.00	2.52	2.50	2.54	1.01	0.99	1.25	1.25	1.23	1.00	1.01
		9.00	1.76	1.75	1.78	1.01	0.99	0.88	0.88	0.86	0.99	1.01
		10.00	1.28	1.27	1.29	1.01	0.99	0.64	0.64	0.63	0.99	1.01
	Bottom Flange	4.00	25.16	24.92	25.43	1.01	0.99	11.17	11.15	11.04	1.00	1.01
		5.00	12.20	12.08	12.33	1.01	0.99	5.56	5.55	5.50	1.00	1.01
		6.00	6.82	6.75	6.89	1.01	0.99	3.16	3.16	3.13	1.00	1.01
		7.00	4.19	4.15	4.23	1.01	0.99	1.97	1.97	1.94	1.00	1.01
		8.00	2.76	2.73	2.79	1.01	0.99	1.31	1.31	1.29	1.00	1.01
		9.00	1.91	1.90	1.93	1.01	0.99	0.91	0.92	0.90	0.99	1.01
		10.00	1.38	1.37	1.39	1.01	0.99	0.66	0.67	0.65	0.99	1.01
Section B	Top Flange	4.00	15.23	15.36	16.90	0.99	0.90	9.36	9.74	9.74	0.96	0.96
		5.00	7.59	7.63	8.26	1.00	0.92	4.53	4.68	4.66	0.97	0.97
		6.00	4.32	4.33	4.63	1.00	0.93	2.51	2.59	2.57	0.97	0.98
		7.00	2.68	2.69	2.85	1.00	0.94	1.54	1.58	1.56	0.97	0.98
		8.00	1.78	1.78	1.88	1.00	0.95	1.01	1.03	1.02	0.97	0.99
		9.00	1.24	1.25	1.30	1.00	0.95	0.69	0.71	0.70	0.97	0.99
		10.00	0.90	0.90	0.94	100	0.96	0.50	0.52	0.50	0.97	0.99
	Shear Center	4.00	15.86	16.00	17.58	0.99	0.90	9.56	9.94	9.94	0.96	0.96
		5.00	7.84	7.88	8.53	0.99	0.92	4.60	4.76	4.74	0.97	0.97
		6.00	4.43	4.45	4.76	1.00	0.93	2.55	2.63	2.61	0.97	0.98
		7.00	2.75	2.75	2.92	1.00	0.94	1.55	1.60	1.58	0.97	0.98
		8.00	1.82	1.82	1.92	1.00	0.95	1.02	1.05	1.03	0.97	0.99
		9.00	1.26	1.27	1.33	1.00	0.95	0.70	0.72	0.71	0.97	0.99
		10.00	0.92	0.92	0.96	1.00	0.96	0.50	0.52	0.51	0.97	0.99
	Bottom Flange	4.00	20.43	20.63	22.50	0.99	0.91	10.88	11.28	11.27	0.96	0.97
		5.00	9.64	9.70	10.45	0.99	0.92	5.11	5.27	5.25	0.97	0.97
		6.00	5.28	5.30	5.65	1.00	0.93	2.78	2.86	2.84	0.97	0.98
		7.00	3.19	3.20	3.39	1.00	0.94	1.68	1.72	1.70	0.97	0.98
		8.00	2.07	2.08	2.19	1.00	0.95	1.09	1.12	1.10	0.97	0.99
		9.00	1.42	1.43	1.49	1.00	0.95	0.74	0.77	0.75	0.97	0.99
		10.00	1.02	1.02	1.06	0.99	0.96	0.53	0.55	0.53	0.97	0.99

* PE: Present analytical model, AB: Abaqus numerical analysis, PM: Presented by Mohri *et al*. (2013)

includes the investigation of the effect of the slenderness on LTB behavior of beam-columns. LTB loads of section A and section B subjected to uniformly distributed load is obtained by the present analytical model for different slendernesses varied from L = 4000 mm to L = 10000 mm, in presence of a constant axial load of N/Ny = 0.2 and N/Ny = 0.8. Analytical results and ABAQUS solutions are summarized in Table 4 for section A and section B.

It is observed from Table 4 that LTB loads of section A and section B with different slendernesses subjected to uniformly distributed load calculated by using present closed-form equation are in excellent accordance with both ABAQUS results and the results obtained by the analytical model presented by Mohri *et al*. (2013). It can be suggested that the proposed equation can be safely used to calculate the elastic critical LTB loads of beam-column members with the mono-symmetric cross-section in presence of an axial load.

4. Conclusions

This paper presents a unique compact closed-form equation based on Ritz Method in order to describe the LTB behavior of beam-columns with mono-symmetric section subjected to constant axial force and various transverse load cases. The effect of axial loads, slenderness and load positions on LTB loads of beam-columns are investigated by using the proposed equation. ABAQUS finite element software was utilized to validate LTB loads obtained by the presented equation. It can be concluded that the results obtained by the presented equation are in good accordance with ABAQUS results. It is concluded that the proposed equation can be safely used in the design of mono-symmetric beam-column members.

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