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Evaluation of dynamic increase factor in progressive collapse analysis of steel frame structures considering catenary action

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Abstract. This paper investigates the effects of the tensile catenary action on dynamic increase factor (DIF) in the nonlinear static analysis for progressive collapse of steel-frame buildings. Numerical analyses were performed to verify the accuracy of the empirical and analytical expressions proposed in the literature in cases where the catenary action is activated. For this purpose, nonlinear static and dynamic analyses of a series of steel moment frame buildings with a different number of spans and stories were carried out following the alternate path method. Different column removal scenarios were considered as separate load cases. The dynamic increase factor that approximately compensates for the dynamic effects in the nonlinear static analysis was selected so to match results from the nonlinear dynamic analysis. The study results showed that the many expressions in literature may not work in cases where the catenary stage is fully developed.

Keywords: progressive collapse; nonlinear pushdown analysis; dynamic increase factor

1. Introduction

Progressive collapse refers to the phenomenon whereby the global structural failure initiates by disproportional local damage of structural elements caused by abnormal loads. Numerous studies have been proposed in the literature to investigate the progressive collapse resistance of buildings (Kim and Park 2008, Ellingwood and Dusenberry 2009, Ferraioli and Avossa 2012, Ferraioli *et al.* 2018a, b, Chen *et al.* 2016, Cassiano *et al.* 2016, Mirtaheri and Zoghi 2016). Detailed analysis procedures have been issued by US General Service Administration (GSA 2013) and recommended in the Unified Facilities Criteria UFC 4-023-03 (2013) by Department of Defense (DoD). The Alternate Path Method (APM) is one of the most extensive methods to assess progressive collapse of structures. In this approach, a vertical load-bearing element at the specific location of plan and elevation is removed, and the capability of structure to bridge across the removed element is evaluated. The failure of such structural member under extreme loads events is a nonlinear and dynamic phenomenon. Thus, the analysis should explicitly include dynamic effects, geometric nonlinearity and inelastic material behavior. However, sophisticated nonlinear dynamic approaches, while being capable of providing accurate descriptions of the more significant features of response, are complex and extremely time-consuming. The design guidelines (UFC 2013, GSA 2013) allow using nonlinear static (NS) or linear static (LS) analyses in replace of nonlinear dynamic (ND) analysis with considering some limitations. In order to consider the dynamic effects, the gravity loads in the bays affected by the removed column should be amplified with a dynamic increase factor (DIF). This factor approximately compensates for the dynamic effects corresponding to the real load redistribution and, thus, its value should be selected so that the predicted NS results match well with the ND results. As other studies (Ruth *et al.* 2006, Xu and Ellingwood 2011, Tsai 2012, Liu 2013, Ferraioli 2016, 2017a, Ferraioli *et al.* 2014, 2018b) pointed out, the previous use of 2.0 for the dynamic increase factors (DIF) is too conservative to generate any economical designs. McKay *et al.* (2012) conducted a series of nonlinear analyses for steel and reinforced concrete frame models under various column-loss scenarios to generate regression formulas for dynamic increase factor. These empirical formulas are presently adopted in the latest version of both UFC (2013) and GSA (2013). It should be highlighted that only the structure type, the classification of the structural actions and the rotation capacity of the plastic hinges govern the value of the DIF according to these guidelines. Other parameters playing an important role are neglected, such as, for instance, gravity load, ductility demand of members, damping ratio and post-elastic stiffness ratio. This gave rise to a number of alternative methods to adjust the DIF, so to include the most effective structural parameters and improve the prediction of ND response with NS analysis. Liu (2013) proposed a new DIF that includes gravity loads in affected bays and ductility. Mashhadi *et al.* (Mashhadi *et al.* 2016, 2017, Mashhadi and Saffari 2017) presented new empirical formulas of DIF that includes ductility, gravity loads in affected bays, damping ratio and post-elastic ratio of structural members. Some of these formulas are based on the flexure demand of the beams that are affected by column removal. This parameter depends on DIF. Thus, the authors apply a trial DIF to the extreme

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event load case and compare the flexure demand from NS analysis with the corresponding ratio calculated with the ND analysis. Then, the DIF is modified and a new NS analysis is carried out. Finally, curve fitting of data points is carried out to find the empirical formula. It should be highlighted that the DIF is used in NS analysis as an alternative to ND analysis. Thus, the maximum flexure demand from ND analysis is unknown. This means that the application of these empirical formulas in current practice requires an iterative procedure. At first, an attempt value of DIF is used in NS analysis. Then, the corresponding maximum flexure demand is substituted in the empirical formula and a new attempt value of DIF is calculated. The procedure is iterated until the calculated DIF approximates the initially adopted value with prescribed tolerance.

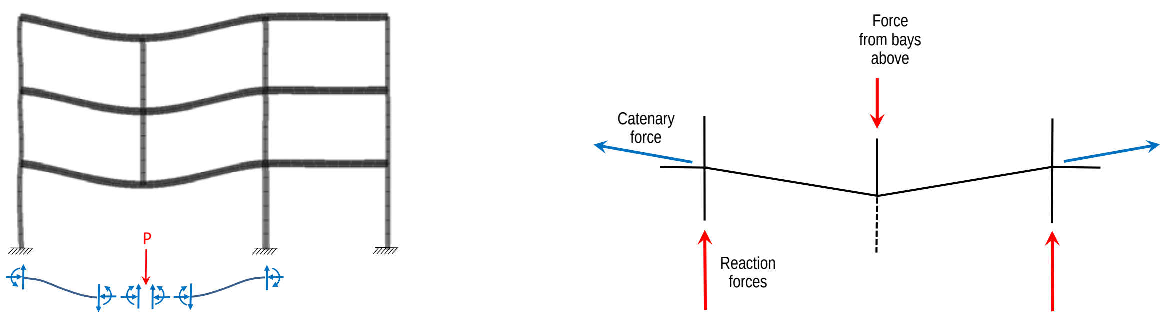
Finally, it should be underlined that many of these methods do not have enough accuracy and efficiency in all probable conditions. This aspect is crucial since different mechanisms are mobilized to resist collapse: (a) Vierendel action of moment-resisting frames (Fig. 1(a)), (b) shear deformation of transfer structures, (c) catenary action in the structural frame (Fig. 1(b)), (d) vertical load-bearing elements acting as suspension, (e) compressive arching in the beams and/or floor slabs, (f) membrane action in structural slabs. The "catenary action" refers to the ability of beams to resist vertical loads by developing a string-like mechanism (Fig. 1(b)) and its activation depends on the geometrical nonlinear behavior of the structure. In fact, this effect can be developed only when the inelastic deformations of the structure are large enough to make possible a transition from the flexural resistance to the tensile catenary resistance. In this case, the nonlinear static response after column removal comprises an initially linear phase followed by significant geometric (large displacements) and material (plasticity) nonlinearities and then by the catenary stiffening effect prior to the ultimate failure.

Even though the design codes recommend the catenary action as one of means for increasing progressive collapse resistance, specific design guideline for this mode of behavior is not yet explicitly considered. Thus, its effect on DIF is often neglected even though it is one of the major research topics. Kim *et al.* (2009) proved that the effect of the catenary action might influence the progressive collapse potential of steel moment frame structures and its contribution increases as the number of stories and bays increases. Yang and Tan (2013) showed the contribution of catenary action to the load resistance of various types of steel beam-column joints. In general, the analytical formulas of DIF are based on a bilinear elastic-plastic model of the load-displacement response. The empirical formulas of DIF are generally derived from case studies in which the catenary stage is not developed because the gravity loads are not high enough or the ductility capacity is significantly restricted since a non-ductile failure mode occurs. Thus, both empirical and analytical formulas proposed in literature can be strongly inaccurate for other nonlinear static responses including the effects of the catenary action. In this case, the bilinear elastic-plastic model becomes inaccurate due to the catenary stiffening effect. Moreover, because of the inevitable differences in various modelling details and analysis procedures in the authors' paper, it is not prudent to compare the values of the DIFs that resulted from different studies.

In this paper, the accuracy of the empirical and analytical formulas proposed in the literature are verified considering buildings with a different number of stories and spans, even in the presence of such high gravity loads to activate catenary action of beams. The effect of catenary action behavior due to axial forces in beams on DIF is evaluated and the applicability limits of various approximate approaches are investigated.

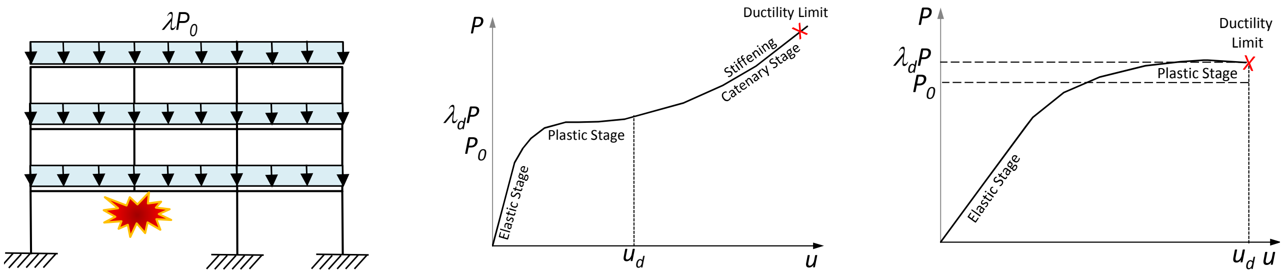
2. Overview of analytical expressions for DIF

The dynamic increase factor (DIF) is conventionally defined as the ratio of the maximum dynamic displacement to the static displacement for an elastic SDOF system under an applied loading. It is also named as "dynamic magnification factor" (Clough and Penzien 1993), "displacement response factor" (Chopra 1995), or "dynamic magnification factor" (Tsai and Lin 2009). For an elastic SDOF system, DIF determines the amplified loading demands under sudden column loss. Thus, from the basic definition of dynamic amplification (Clough and Penzien 1993, Chopra 1995) the DIF may be calculated as the ratio of the nonlinear static (NS) force response and nonlinear dynamic (ND) force response for a structural system subjected to an equal deformation demand. When applied to progressive collapse analysis under sudden column loss scenario, the dynamic increase factor (DIF) determines the



(a) Vierendel action in moment resisting frame (b) Catenary action in structural frame

Fig. 1 Vierendel and catenary action after removal of a supporting column



(a) Loads for nonlinear static analysis

(b) Pushdown curve in a case where energy balance is possible

(c) Pushdown curve in a case where energy balance is not possible

Fig. 2 Energy balance method

amplified loading demands in static analysis procedures. The GSA (2003) and the UFC (2009) progressive collapse design guidelines recommended to apply a DIF of 2.0 to the gravity loads. This value is because the maximum dynamic deflection is twice the static deflection when a structure behaves in a perfectly linear elastic manner. In fact, in progressive collapse, when a column is removed, instant reaction forces, equal and opposite to those supported by the removed column, are applied to the remaining structure. If a step force function is considered, the DIF may be calculated as follows (Chopra 1995)

$$\mathrm{DIF} = 1 + \frac{\left|\sin\left(\pi t_r / T_n\right)\right|}{\pi t_r / T_n} \tag{1}$$

In Eq. (1), T_n is the structural natural period in the force direction force and t_r is the rise time of the step force function. Since the column is removed suddenly, t_r/T_n tends to zero and DIF is approximated to two. However, as other researchers pointed out (Dusenberry and Hamburger 2006, Ruth *et al*. 2006, Xu and Ellingwood 2011, Tsai 2012), the previous use of 2.0 for the dynamic increase factor (DIF) is too conservative to generate any economical designs. First, an amplification factor of 2.0 is correct only if it is assumed that the structure remains linear elastic after the sudden column loss. On the contrary, the real behaviour following sudden column loss is likely to be inelastic since the load initially sustained by the removed column is then transferred to the remaining structure causing an inelastic response. Many studies have revealed that a constant DIF of two may lead to inconsistent results with that obtained from the nonlinear dynamic (ND) approach. In fact, in case of inelastic behaviour, the dynamic effect of the gravity loads on progressive collapse response may be much less than what is predicted by the nonlinear static (NS) analysis using a DIF of 2.0. Thus, several alternative approaches have been proposed in the literature in the recent years. Many of these formulations are obtained under the hypothesis that floor response under gravity loading is elastic-perfectly plastic. By applying this hypothesis, the estimated value of DIF generally decreases as the flexural ductility is developed. However, it should be observed that the failure of vertical members under extreme events might involve the geometrical nonlinear behaviour of the structure and activate the tensile catenary action. In this case, the hypothesis of elastic-perfectly plastic behaviour fails due to the catenary stiffening effect. Thus, the monotonic reduction of DIF with ductility could be not realistic and is currently under discussion.

Actually, two different approaches are used for DIF formulation. The first one develops analytical formulas based on the pseudo-static analysis of simplified inelastic models of single degree-of-freedom (SDOF) systems (Tsai and Lin 2009, Tsai 2012). The second one develops empirical formulas from the regression analysis based on sufficient numerical data of multiple degree-of-freedom (MDOF) systems (McKay *et al*. 2012, Stevens *et al*. 2008, Mashhadi 2016, 2017, Mashhadi and Saffari 2017).

2.1 DIF formulations based on SDOF model

In general, the DIF depends on both the level of gravity loading and the nature of the nonlinear response. A single deformation mode shape generally dominates the response to sudden column loss. Thus, many studies in literature proposed analytical expressions of DIF based on simplified inelastic models of single degree-of-freedom (SDOF) systems. Sasani and Sagiroglu (2008) studied the relationship between applied load and displacement with an equivalent inelastic SDOF system. An alternative simplified approach focus on determining only the maximum dynamic response and the corresponding dynamic load amplification factor. This approach is based on the energy balance method proposed by Izzuddin *et al*. (2008) and on the hypothesis that the sub-structure directly affected by the column removal behaves as a SDOF. At the first step, the nonlinear static (pushdown) analysis of the structure under gravitational loading is carried out. This gives the typical pushdown curve of Fig. 2, which consists of an initial linear elastic segment, followed by a nonlinear transition due to geometric and material nonlinearity, and finally by an eventual stiffening phase due to catenary action, or by a softening phase due to buckling or failure of structural members. The computation of the response implicitly assumes that the response is dominated by a single deformation mode that is the zone directly affected by the column loss behaves as an equivalent SDOF system. The maximum dynamic response of this equivalent SDOF system is achieved when the kinetic energy is reduced to zero and the work done by the gravity loads becomes identical to the energy absorbed by the structure (energy balance). In other words, the equilibrium in the damage state is achieved when the external work (W) is equal to the internal work (U). The level of suddenly applied gravity loading ($P_d = \lambda_d P_0$) that leads to a specific maximum dynamic displacement (u_d) may be calculated from the nonlinear static force-displacement curve, as follows (Fig. 2)

$$W = P_d u_d = \lambda_d P_0 u_d \qquad U = \int_0^{u_d} P(u)du \tag{2}$$

$$W = U \rightarrow P_d = \lambda_d P_0 = \frac{1}{u_d}\int_0^{u_d} P(u)du \tag{3}$$

where P_0 is the value of the static gravity loads. When the work done equals the internal energy, the system reaches equilibrium for a vertical displacement equal to the dynamic displacement. Thus, Eq. (3) gives the value of the equivalent displacement in dynamic equilibrium condition (u_d) and the corresponding equivalent peak dynamic load $P_d = \lambda_d P_0$ obtained from the nonlinear static pushdown response at equilibrium. In cases where the energy balance is not possible, the zero kinetic energy condition is not reached and global structural collapse occurs. Thus, the maximum dynamic displacement is achieved from the ductility limit on the pushdown curve, taken as the minimum displacement at which the ductility demand exceeds the ductility capacity.

Tsai and Lin (2009) proposed an analytical methodology for DIF based on the pseudo-static response of bilinear elastic-plastic SDOF model. In particular, the analytical expression for DIF is derived from the capacity curve of the inelastic SDOF model. From the basic definition of dynamic amplification, the force-based DIF may be estimated as the ratio of the NS force response (P_s) to the ND force response (P_d) for the affected bays of the structure subjected to an equal deformation demand. If consider a bilinear elastic-plastic SDOF model (with elastic stiffness k, yielding displacement u_y and post-stiffness ratio α) for displacements $u < u_y$ the energy balance gives

$$\begin{gathered} W = P_d u \qquad U = \frac{1}{2}ku^2 \\ W = U \rightarrow P_d = \frac{U}{u} = \frac{1}{2}ku = \frac{1}{2}P_s \end{gathered} \tag{4}$$

As expected, for an elastic SDOF system ($u < u_y$) the force-based DIF is $P_s/P_d = 2$. For an inelastic SDOF system (i.e., $u > u_y$) the energy balance after a few steps gives

$$\mathrm{DIF} = \frac{P_s}{P_d} = \frac{2\mu\left[1+\alpha(\mu-1)\right]}{1+\alpha(\mu-1)^2+2(\mu-1)} \tag{5}$$

where μ is the ductility demand in terms of displacement or rotation and α is the post-elastic stiffness ratio. Eq. (5) shows that for $\alpha = 0$ DIF asymptotically approaches 1.0 as the ductility increases towards infinity.

2.2 DIF formulations based on MDOF model

Some studies derived the empirical formulae of DIF from regression analysis of multiple degree-of-freedom (MDOF) systems. Stevens *et al*. (2008) combined the m-factors of life safety condition in ASCE 41-06 (2007) to generate the following empirical formula for steel structures

$$\mathrm{DIF} = 1.44m^{-0.12} \tag{6}$$

where m is the plastic hinge rotation divided by yield rotation of the component or connection in the area which is loaded with the amplified gravity load. The plastic hinge rotation represents the critical structural performance level and is given by the minimum nonlinear acceptance criteria of members. McKay *et al*. (2012) applied the same procedure with framed steel structures subjected to various column-loss scenarios. The generated empirical DIF formula for steel frames that relates only to ductility may be written as follows

$$\mathrm{DIF} = 1.08 + \frac{0.76}{\theta_{pra}/\theta_y + 0.83} \tag{7}$$

where θ_{pra} is the prescribed maximum acceptable plastic hinge rotation angle and θ_y is the yield rotation angle. This formula reveals that DIF is equal to 2.0 if $\theta_{pra}/\theta_y = 0$. This means that when a structure behaves in a perfectly linear elastic manner (that means $\theta_{pra}/\theta_y = 0$), the maximum dynamic deflection is twice the static deflection (that means DIF = 2). The plastic rotation angle θ_{pra} is given in the acceptance criteria tables in ASCE 41-06 (2007) for the appropriate structural response level (Life Safety or Collapse Prevention) while the yield rotation θ_y for steel is given by Equation 5-1 in ASCE 41-06 (2007). In Eq. (7), θ_{pra}/θ_y is defined as the smallest ratio for the structural components (columns excluded) that contribute to progressive collapse resistance and are within the immediately affected bays. Therefore, the parameter θ_{pra}/θ_y reflects, in an overall sense, the ultimate level of inelasticity the damaged frame can experience, and it is assumed that the controlling beam has reached its maximum allowable plastic hinge rotation capacity. Finally, it should be underlined that for the deformation controlled action it is necessary to check that

$$\theta_{pra} \geq \theta_p \tag{8}$$

where θ_p is the plastic rotational demand from the NS analysis. This means that if the focus is on estimating the maximum dynamic response, the plastic rotation demand equals the plastic rotation capacity (that is $\theta_p = \theta_{pra}$). The empirical formula of Eq. (7) is presently adopted in the latest version of both UFC (2013) and GSA (2013). It can be noticed that the value of the DIF defined in both guidelines is governed only by the structure type, the classification of the structural actions and the plastic rotation limit, while other parameters playing an important role (such as structural configuration and axial forces in beams (i.e., catenary effect)) are not considered. Furthermore, even though this formulation for DIF is an upgrading over the standard load factor approach in which a constant DIF is assumed, the monotonic decreasing of DIF with ductility is not generally correct and is currently under discussion. Eq. (7) depends only on the mechanical properties of the structural members and gives the same DIF value regardless of the specific gravity loading. Conversely, the level of gravity loading should influence the dynamic structural responses since the damaged structure that is subjected to greater gravity loads would be more susceptible to progressive collapse. In fact, the damaged structure may not necessarily enter in the inelastic range and, thus, it can remain elastic even after the column has been removed, which is particularly true for frames that were originally designed to withstand large seismic loads. In this case, the structure may have significantly extra capacity against gravity load-induced progressive collapse. On the other side, even if a damaged structure responds inelastically, the actual level of inelasticity is not necessarily so high that the plastic rotation of the controlling structural member reaches θ_{pra}. Under either of these two conditions, the underlying hypotheses of the Eq. (7) are no longer valid and hence the DIF calculated becomes questionable. This has given rise to alternative formulations of DIF based not only on the capacity but also on the flexure demand of the beams that are affected by column removal.

Liu (2013) presented an empirical method that defines the DIF as a function of the maximum ratio of (M_d/M_p). For each affected beam (i.e., those directly adjacent to the removed column) M_d is the member moment demand under unamplified gravity loads (i.e., those unamplified by the DIF), and M_p is the factored plastic moment capacity calculated as $M_p = \Omega_0\phi ZF_y$ (ASCE 41-06 2007), where

Ω_0 = overstrength factor;
ϕ = strength-reduction factor;
Z = beam plastic modulus;
F_y = steel yield stress.

For (M_d/M_p) ≥ 0.5, DIF is given by

$$\mathrm{DIF} = 0.84 + \frac{1.23}{2.95\max\left(M_d/M_p\right) - 0.28} \tag{9}$$

For (M_d/M_p) ≤ 0.5, the following formulas are applied, respectively for exterior and interior column removal scenarios

$$\mathrm{DIF} = 1.15\,max\left(M_d/M_p\right) + 1.12 \tag{10}$$

$$\mathrm{DIF} = 0.58\,max\left(M_d/M_p\right) + 1.55 \tag{11}$$

It is clear that max(M_d/M_p) is an approximate indicator of the residual capacity that dictates how far a damaged frame would enter an inelastic range. The function $1-M_d/M_p$ measures the percentage level of the overall residual capacity of the structure to remain elastic. Moreover, it should be observed that the member moment demand M_d in Eqs. (10)-(11) is a result of the analysis and, thus, it depends on DIF. On the other side, it easily predictable that both damping ratio and post-elastic stiffness influence the vertical displacement and dynamic increase factor (DIF) of the structures against progressive collapse. Mashhadi (2016) proposed an empirical formula including the damping ratio

$$\mathrm{DIF} = \left(2 - 2.54\xi\right) - \frac{\left(0.9 - 1.81\xi\right)\theta_p/\theta_y}{\left(0.84 - 2.15\xi\right) + \theta_p/\theta_y} \tag{12}$$

where θ_p/θ_y is the maximum ratio of the plastic to yield rotation of the beams in the affected bay of the structure. The same authors recently proposed an equation, which relates DIF to ductility θ_p/θ_y and post-elastic stiffness ratio η of the members, as follows

$$\mathrm{DIF} = \left(1.1 + 2\eta\right) + \frac{0.56 - \eta}{0.65 + \theta_p/\theta_y} \tag{13}$$

Finally, it should be underlined that in Eqs. (9)-(13) the DIF depends on the results of analysis since it is based on the flexure demand of the beams that are affected by column removal. In order to obtain these empirical formulas, the authors proposed a two-step procedure. In Liu (2013), the unamplified gravity load are statically applied to the damaged structure and the maximum value of (M_d/M_p) is calculated. Then, DIF is estimated using Eqs. (9)-(11) and additional gravity loads multiplied by a factor (DIF-1) are applied on all affected bays. The resulting deformation demand is finally used to check the acceptance criteria. According to Mashhadi (2016, 2017), in the first step, a nonlinear dynamic (ND) analysis is performed to obtain the plastic rotations of all members of the bays affected by the column removal. In the second step, the nonlinear static (NS) analysis is performed with an iterative approach. A trial DIF is applied to the extreme event load case and the ratio θ_p/θ_y is calculated. This ratio is compared with the corresponding ratio calculated with the ND analysis. Then, the DIF is modified and the NS analysis is carried out. This procedure is iterated until it converges, that is until the maximum ratio θ_p/θ_y obtained from NS analysis approximates the reference ratio from ND analysis with prescribed tolerance. The curve fitting of the data points finally gives the empirical formulas for computing the DIF. As stated, the application of these formulas for progressive collapse analysis requires an iterative procedure. In fact, the NS analysis is applied as an alternative to ND analysis and the maximum ratio θ_p/θ_y from ND analysis is unknown. Thus, a trial DIF is applied in NS analysis and this gives the maximum flexure demand (M_d/M_p or θ_p/θ_y) of the beams that are affected by column removal. Substituting in Eqs. (9)-(13) gives a new attempt value for DIF. The procedure is iterated until the DIF calculated approximates the initially adopted value of DIF with prescribed tolerance.

3. Modelling and analysis procedure

3.1 Steel frame building models

The dynamic increase factor for progressive collapse analysis was investigated using three-, five-, seven- and nine-story models of steel moment frames buildings. Two typical floor plans, one square and the other rectangular,

Table1 Case studies

N.	Number of floors	Plan Type	Perimeter MRFs	Internal MRFs
1	3	Square	3SQ1	3SQ2
2	5	Square	5SQ1	5SQ2
3	7	Square	7SQ1	7SQ2
4	9	Square	9SQ1	9SQ2
5	7	Rectangular	7SR1 - 7SR3	7SR2
6	9	Rectangular	9SR1 - 9SR3	9SR2

were used in the analysis. The details of the perimeter and internal moment-resisting frames (MRFs) of the buildings considered in this study are shows in Figs. 3-4 and Tables 1-2.

Each MRF is named using the number of storeys, the plan configuration of the building and the type of moment frame (for example, 7SQ1 refers to 7-storey building, square plan Q, and moment frame type 1). The buildings were designed according to the Italian Building Code (2018). The interstorey height is 3.5 m for the first floor and 3.0 m for the other floors. The bay length is 5.00 m in both orthogonal directions. The steel material used for all beams and columns is S275, with a lower-bound yield and tensile strength values equal to 275 MPa and 410 MPa, respectively. The moment resisting frames were designed for peak ground acceleration PGA = 0.35 g, soil class A, damping ratio 5% and behavior factor q = 6.5. The self-weight of the interior walls and partitions was applied to the floor slab as a distributed load.

The load redistributions through the floor slabs were neglected and this should provide conservative estimates of the progressive collapse resistance. The plastic hinges were represented by nonlinear moment-curvature and P-M interaction relationships for both beams and columns. The concentrated plastic hinges properties were determined according to ASCE 41-13 (2007). The adopted moment-rotation diagram is defined from the plastic and yield rotations (θ_p and θ_y), and from the plastic and yield moments (M_p and M_y) that are calculated based on ASCE 41-13 (2007). This gives the post-elastic slope of the moment-rotation diagram. The sudden strength degradation was neglected since the acceptable plastic rotation angle of the steel members, as defined in ASCE 41-13 (2007), is always within the first post-yield linear branch of the moment–rotation curve (preceding the strength degradation).

It should be underlined that the catenary action can be developed in the beams that are directly affected by the column removal. The role of the indirectly affected part of

Table 2 Dimensions of structural members of MRFs

Frame	Member		Storey	Section	Storey	Section	Storey	Section	Storey	Section	Storey	Section
3SQ1 3SQ2	Column	Ext.	1	HE180B	2-3	HE160B						
	Column	Int.	1	HE240B	2-3	HE200B						
	Beam		1	IPE270	2-3	IPE270						
5SQ1 5SQ2	Column	Ext.	1-2	HE200B	3	HE160B	4-5	HE160B				
	Column	Int.	1-2	HE260B	3	HE220B	4-5	HE200B				
	Beam		1	IPE300	2-3	IPE270	4-5	IPE270				
7SQ1 7SQ2	Column	Ext.	1-2	HE200B	3-4	HE180B	5	HE160B	6-7	HE160B		
	Column	Int.	1-2	HE260B	3-4	HE240B	5	HE220B	6-7	HE200B		
	Beam		1-2	IPE300	3	IPE300	4-5	IPE270	6-7	IPE270		
9SQ1 9SQ2	Column	Ext.	1-2-3	HE220B	4-5-6	HE200B	7	HE160B	8-9	HE160B		
	Column	Int.	1-2-3	HE280B	4-5-6	HE260B	7	HE220B	8-9	HE200B		
	Beam		1-2-3	IPE330	4-5-6	IPE300	7	IPE270	8-9	IPE270		
7SR1 7SR2	Column	Ext.	1-2	HE200B	3-4	HE180B	5	HE160B	6-7	HE160B		
	Column	Int.	1-2	HE260B	3-4	HE240B	5	HE220B	6-7	HE200B		
	Beam		1-2	IPE270	3-4	IPE270	5	IPE270	6-7	IPE270		
7SR3	Column	Ext.	1-2	HE200B	3-4	HE180B	5	HE160B	6-7	HE160B		
	Column	Int.	1-2	HE200B	3-4	HE180B	5	HE160B	6-7	HE160B		
	Beam		1-2	IPE270	3-4	IPE270	5	IPE270	6-7	IPE270		
9SR1 9SR2	Column	Ext.	1-2	HE220B	3-4	HE200B	5	HE180B	6-7	HE160B	8-9	HE160B
	Column	Int.	1-2	HE280B	3-4	HE240B	5	HE240B	6-7	HE220B	8-9	HE200B
	Beam		1	IPE330	2-3	IPE300	4-5	IPE270	6-7	IPE270	8-9	IPE270
9SR3	Column	Ext.	1-2	HE220B	3-4	HE200B	5	HE180B	6-7	HE160B	8-9	HE160B
	Column	Int.	1-2	HE220B	3-4	HE200B	5	HE180B	6-7	HE160B	8-9	HE160B
	Beam		1	IPE330	2-3	IPE300	4-5	IPE270	6-7	IPE270	8-9	IPE270

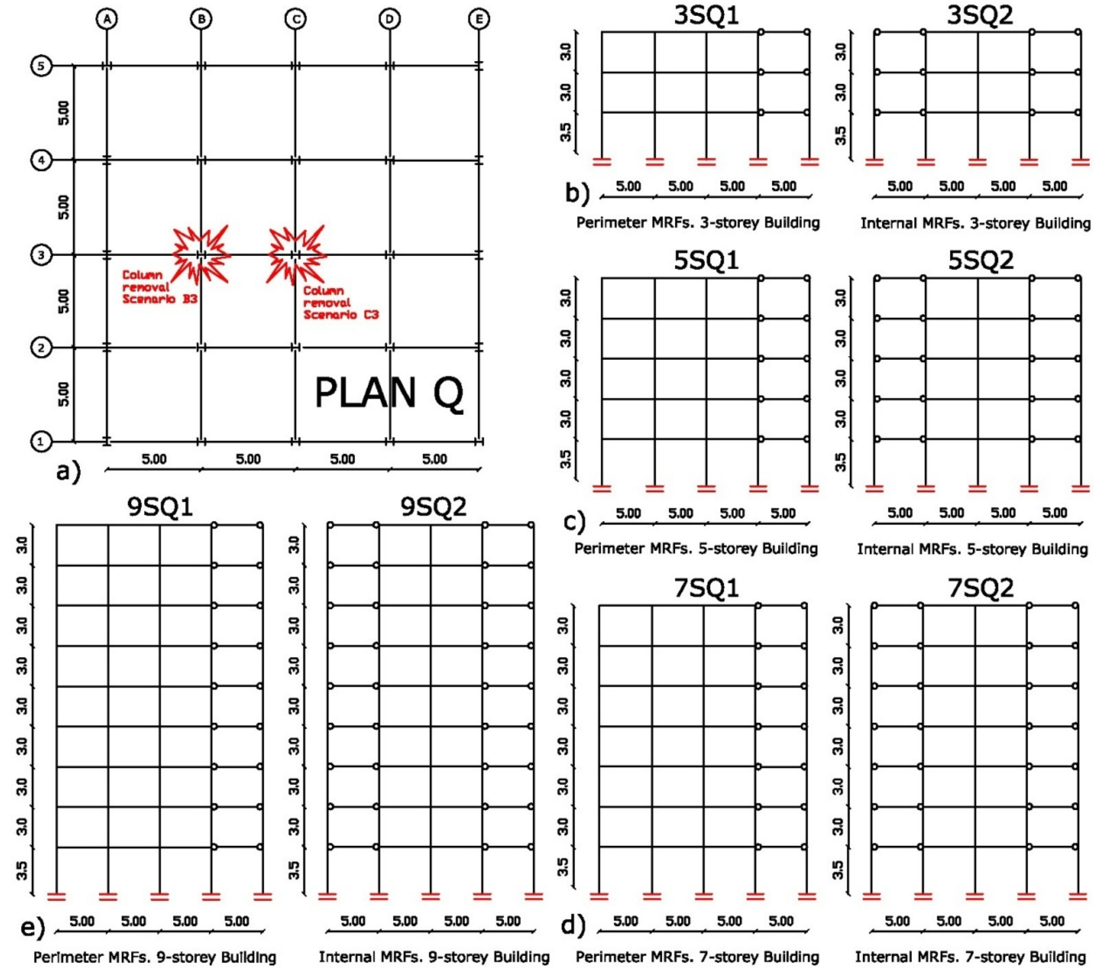


Fig. 3 Square plan buildings: (a) Plan view; (b) MRFs of 3-storey Building; (c) MRFs of 5-storey Building; (d) MRFs of 7-storey Building; (e) MRFs of 9-storey Building

the building is to give a lateral anchorage to these catenary actions. The stiffer the indirectly affected part is the more catenary action will develop in the directly affected part. The floor systems (metal deck with a concrete topping) were assumed non-composite with the steel framing and were modeled as rigid diaphragms in the part of the building non-directly affected by the column removal. Their beneficial tension membrane action at the large deformations was conservatively neglected in the part of the building directly affected by the column removal.

The beams can resist much larger load when the catenary action is fully developed. However, this result is only possible when the beams are strongly connected to the boundaries so that the connections can transfer the increased axial force in the beams for the catenary action. In real structures, the contribution of the catenary action on resisting the applied load depends on the joint conditions since the connections are mainly designed for bending moments. However, the buildings considered in this paper are designed according to the Italian Seismic Code (2018). The connections between the members are classified as fully restrained, based on the strength and stiffness of the connection assembly. Thus, the beam–column joints may be considered strong enough to activate full catenary action of beams that is an assumption normally true in structures with seismic joints. The beam-to-column joints were assumed rigid, full-strength and stronger than the beams. Thus, the model allow plastic hinges to form in beams and columns, not in connections. The effects of the tensile loads transfers to the beam-column connections and their conservative nature were not investigated in the present paper.

The secondary members, such as transverse joist beams and braces, were considered only for the transferring of the gravity loads while they did not directly contribute to the progressive collapse resistance. The model was based on the assumption that the foundation can accommodate the redistributed loads following any column removal, and that connections at the foundations may be modelled as restrained connections. The SAP2000 program (2014) was used as the computational tool to perform the analyses. The model incorporates the effects of geometric nonlinearity for large displacements. This requires an iterative approach. In fact, the element stiffness may change due to large-displacement effect and nonlinear material behavior and should be integrated along the length of the element taking into account the deflection within the element that is assumed cubic for bending. The structural members were subdivided into 20 frame elements to improve the accuracy of results.

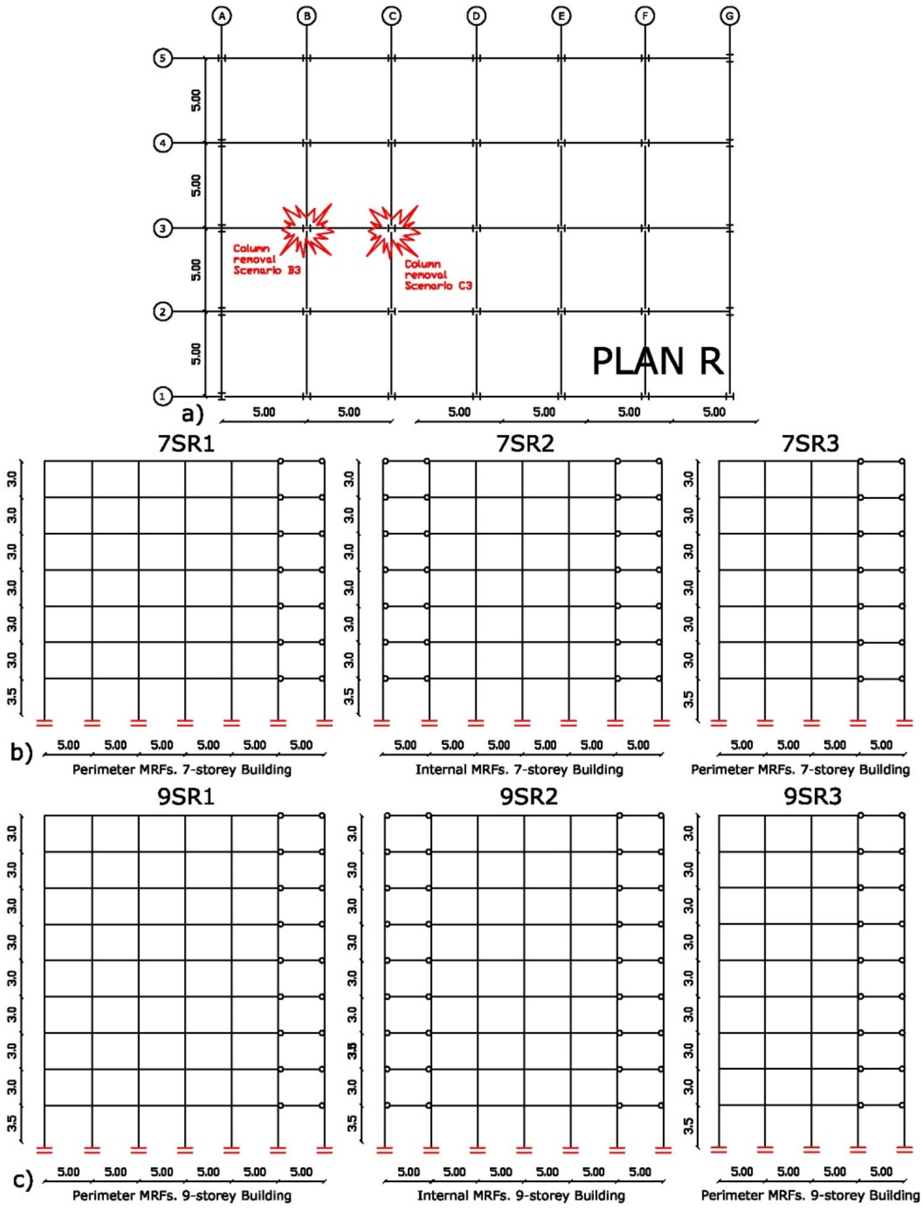


Fig. 4 Rectangular plan buildings: (a) Plan view; (b) MRFs of 7-storey Building; (c) MRFs of 9-storey Building

3.2 Analysis methods

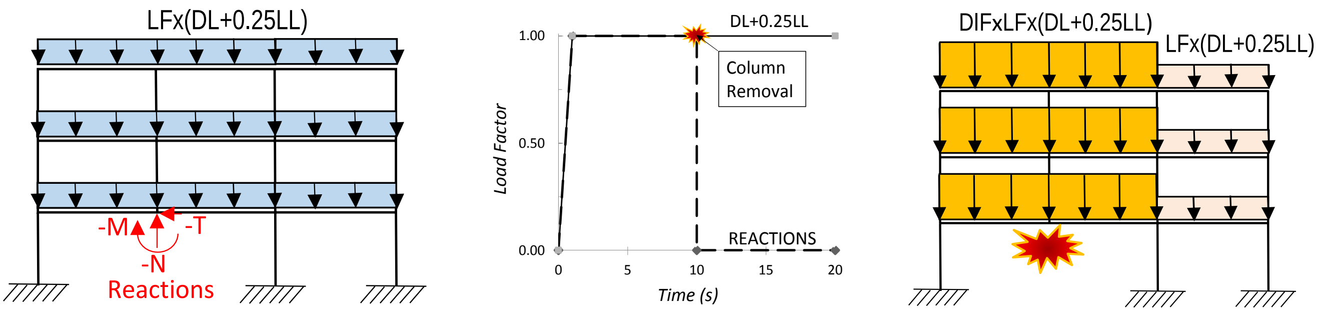
To study the effect of the catenary action on dynamic increase factor, the alternate path method was applied. A critical column is suddenly removed and the capability of the model to successfully absorb the member loss was investigated. Although such a scenario is not same in dynamic effect to column damage resulting from blast or impact, it is intended to represent a situation where an abnormal load or extreme event, taking place over a relatively short duration, destroys the column. Subsequent analysis is carried out under the hypothesis that the strain rates are in the seismic loading range, i.e., rather low, which justifies not accounting for strain rate effects in the analysis model. Both nonlinear static (NS) and dynamic (ND) analyses were used. In ND analysis the load combination was defined as LF×(DL+0.25LL) (Fig. 5(a)), where DL and LL are dead loads and live loads, respectively. Below is described the procedure that dynamically simulates the sudden removal of a column.

Step 1: The gravity loads are statically applied to the undamaged model. The end forces of the to-be-removed target column are determined (i.e., axial force N, bending moment M and shear force V).

Step 2: The reaction forces substitute the column in order to get the displacement configuration at the beginning of the column removal. Practically, the dead and live loads (DL+0.25LL) and the calculated end forces in inverted directions (i.e., -N, -V, -M) are statically applied to the damaged frame (Fig. 5(a)).

This application takes 1s (during which loads are amplified linearly until they reach their full amounts) and then kept unchanged for 9s (Fig. 5(b)) so that the structure can reach a stable condition that reproduces the state of the structure before the column removal.

Step 3: The reaction forces are abruptly brought to zero.



(a) Loads for ND analysis (b) Time-history function for NDA (c) Loads for NS analysis

Fig. 5 Loads for ND and NS analysis

Practically, at 10s the recorded end forces in original directions (i.e., N, V, M) are applied rapidly to the damaged frame to simulate the sudden removal of the column.

Finally, a nonlinear dynamic analysis is carried out using a proportional damping model with an assigned damping ratio for both the first and second mode shapes. The amount of damping depends on the structural joints connecting the steel members that can be made using bolts or by welding. The bolted joints have a damping ratio that is potentially 2 to 3 times higher than the welded joints since the bolted joints can experience greater slip. In this paper, the assumed damping ratio in nonlinear dynamic analysis is 2%. This holds for welded steel structures. The structures under study are usually assembled through bolting and, therefore, a higher damping would be justified. This means that the analysis uses a conservative value of the damping ratio.

The entire process was repeated for increasing values of the vertical load. The magnitude of Load Factor LF was increased until extremely large deflection occurs at the column-removed point. The NS analysis was carried out using the displacement-controlled method. In this approach, the gravity loads, amplified within the affected bays, are increased incrementally after the column removal. This increases displacement to an arbitrary level in the location of the removed column. The maximum value of the displacement is governed by the concentrate hinge properties and acceptance criteria determined according to ASCE 41-13 (2007) chosen as the best metric for approximating structural damage. The NS analysis takes an iterative approach while implementing large-displacement effects. This allowed accounting for the development of catenary action in steel beams that may play a big role in the resistance to progressive collapse. However, the catenary action should be activated before the acceptance criteria are exceeded.

The columns under high axial load ($P/P_{CL} > 0.5$, where P_{CL} is the lower-bound axial load capacity) are classified as force-controlled for both axial loads and flexure. Thus, their plastic hinging becomes a non-ductile failure mode and, thus, the inelastic deformations may be not large enough to make possible a transition from the flexural resistance to the tensile catenary resistance.

3.3 DIF Calculation

For a given column removal scenario, the “exact value” of DIF was obtained such that the structural responses from nonlinear static analysis using the DIF-amplified gravity loads best match those from the nonlinear dynamic analysis. The acceptance criteria for progressive collapse are generally based on the maximum plastic hinge rotation θ_{max} among all beams within the affected bays and the maximum vertical displacement u_{max} at the column removal location. Thus, most the procedures proposed in literature find the DIF that minimizes the differences between NS and ND analysis in terms of maximum plastic hinge rotation or maximum vertical displacement. In this paper, the “exact value” of DIF was calculated in such a way to minimize the following error function that accounts for both the response quantities

$$E = \left(\left| \frac{\theta_{max,NS} - \theta_{max,ND}}{\theta_{max,ND}} \right| + \left| \frac{u_{max,NS} - u_{max,ND}}{u_{max,ND}} \right| \right) \tag{14}$$

Table 3 DIF formulations considered in analysis

N.	Author	Description
1	Izzudin (2008)	Analytical DIF based on energy balance
2	McKay *et al.* (2012) - LS	Empirical DIF based on Life Safety ductility
3	McKay *et al.* (2012) - CP	Empirical DIF based on Collapse Prevention ductility
4	Stevens *et al.* (2008)	Empirical DIF depending on ductility
5	Tsai *et al.* (2009)	Analytical DIF depending on ductility and post-elastic stiffness ratio
6	Liu (2013)	Empirical DIF based on residual capacity of model
7	Mashhadi *et al.* (2016)	Empirical DIF depending on ductility and damping ratio
8	Mashhadi *et al.* (2017a)	Empirical DIF depending on ductility post-elastic stiffness ratio

The “exact value” of DIF was then compared with the DIF estimated using eight different empirical and analytical formulas described in Table 3.

4. Results and comments

Figs. 6-7 show the comparison of pushdown curves (load factor versus vertical displacement in the location of the removed column) for the different case studies considered in this paper (Figs. 3-4). The column removal scenarios were named C3 and B3 according to the grid labels used in Figs. 3-4 for the plan of the building. Two different models were compared. The first model accounts for the geometric non-linearity as well as material non-linearity to consider the catenary action of beams after the column removal (‘catenary’ model). The second model neglects the catenary action since the linear geometric transformation is used for modelling the beam elements and the conventional small displacement analysis is carried out (‘no-catenary’ model). The static response of the ‘catenary’ model comprises three phases (see also Fig. 2(b)): (1)

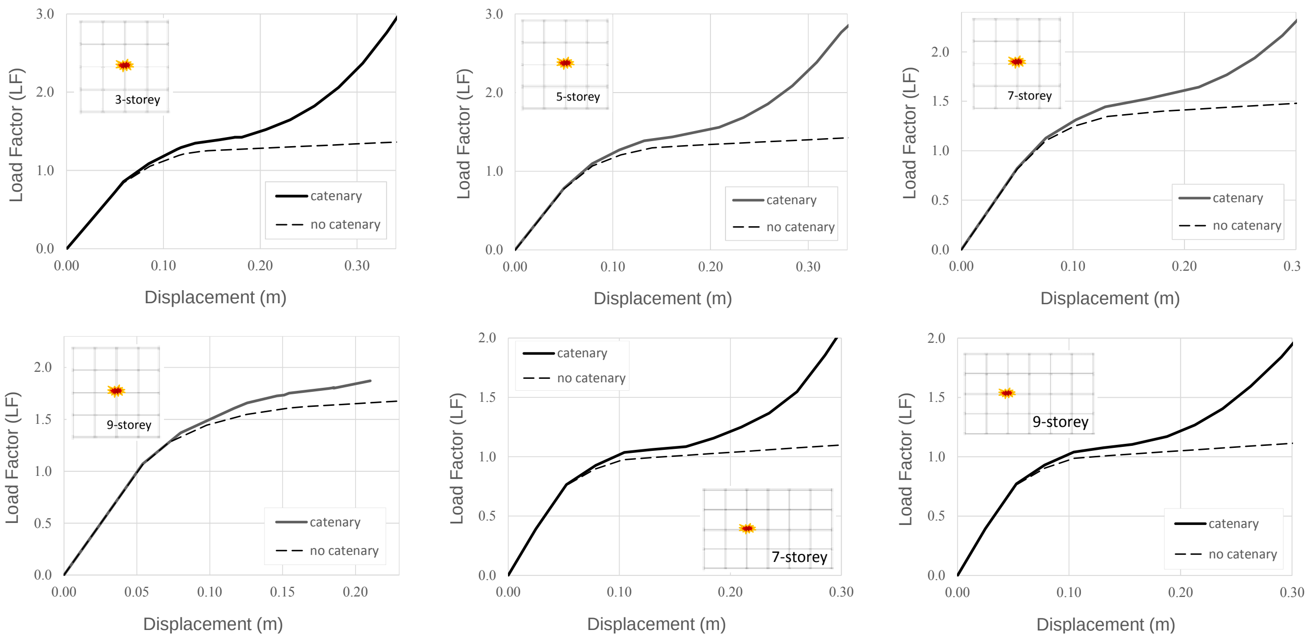


Fig. 6 Pushdown curves under unamplified gravity load with and without considering catenary action. Column removal scenario C3

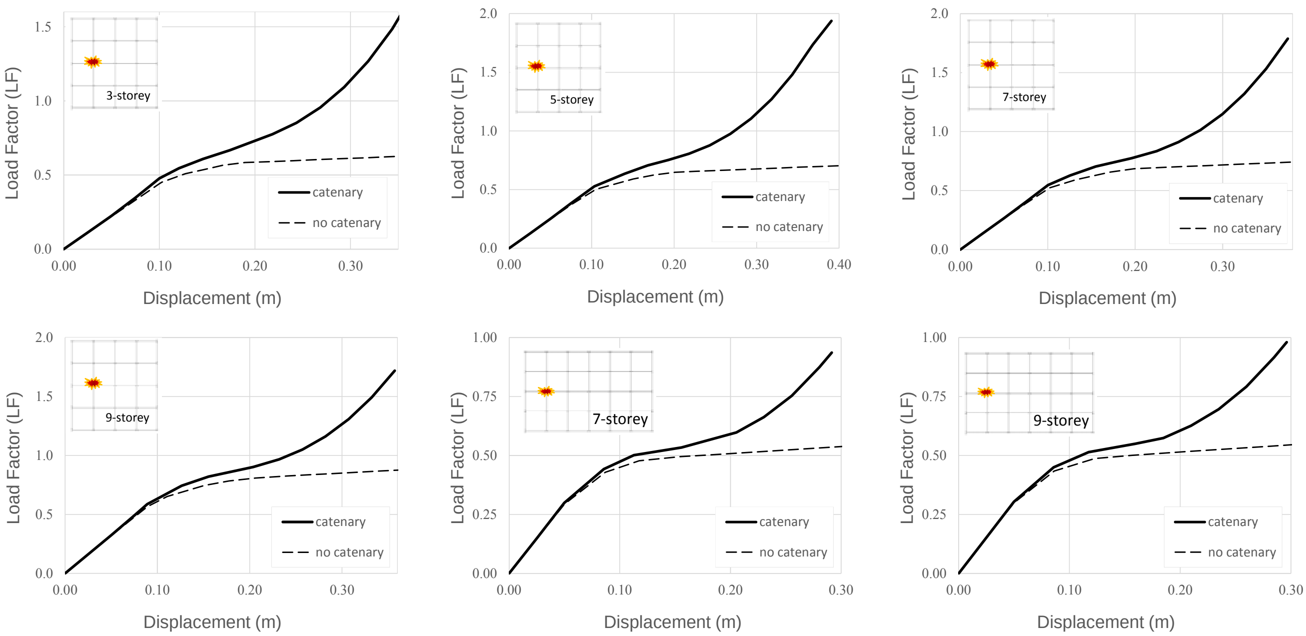


Fig. 7 Pushdown curves under unamplified gravity load with and without considering catenary action. Column removal scenario B3

initially linear phase; (2) plastic phase; (3) catenary phase. During phase 1, the structure after column removal supports the loads coming from the upper storeys in the elastic range of the material. During phase 2, a plastic mechanism develops in the bays affected by the column removal. Each change of slope in the load-displacement curve corresponds to the development of a new hinge, until reaching a complete plastic mechanism. In this phase, the pushdown curve is concave down and the vertical displacement in the location of the removed column increases significantly. Phase 3 starts when this plastic mechanism is formed. In this situation, there is no more first-order stiffness in the structure. Due to the large displacements, the catenary action develops in the beams of the bays affected by the column removal, giving second-order stiffness to the structure. In this phase, the pushdown curve is concave up.

In Figs. 6-7, the static response of the ‘no catenary’ model comprises only two phases (linear and plastic phases) since the catenary effects due to the tensile axial forces in steel beams are neglected. In this case, the pushdown curve is concave down without the inflection point. It can be observed that the pushdown curves of the ‘catenary’ model start to deviate from those of the ‘no-catenary’ models at values of vertical displacement around 10 cm. However, in not all the cases examined the catenary action is activated enhancing the resistance to the progressive collapse of the building. In fact, the 9-storey square plan building under the column removal scenario C3 exhibits only the linear and plastic phases. The catenary stage is not developed because the inelastic deformations are not large enough to fully develop the tensile catenary resistance (Fig. 2(c)). In fact, in this case a non-ductile failure occurs due to the high axial load in the first-storey column affected by the column removal. Comparing Figs. 6-7, different behaviours are observed between the two column removal scenarios. The pushdown curve under the column removal scenario C3 comprises the three phases of the static response (i.e., linear, plastic, catenary). On the contrary, the pushdown curve under the column removal scenario B3 seems not to highlight a significant flexural yielding range since the catenary stiffening effect becomes predominant.

This different behaviour derives from the different end-restraints of the beams affected by the column removal. In fact, the beams of the two bays affected by the column removal C3 are fully restrained. On the contrary, only the beams of one bay affected by the column removal B3 are fully restrained, while the beams of the other bays are hinged to the ends. Thus, the vertical displacement in the location of the removed column increases significantly before the first-order stiffness in the structure becomes zero and, therefore, the catenary stiffening effect develop in beams before the plastic mechanism occurs. Finally, it should be highlighted that the failure load factor is generally greater for the column removal scenario C3 than for column removal scenario B3 since the more internal the column, the more catenary action develop.

In Fig. 8 the axial force induced in beam elements is shown to quantify the effect of catenary action. A steel beam under normal load is mainly subjected to the combined effect of moment and shear force, and the axial force is usually ignored. With the column removal and the increase of load, plastic hinges develop at the beams ends and large deflections occur at the beam midspan. In this situation, the steel beam withstands considerable axial force, which cannot be neglected. As the total cross-section yields, the axial force in the steel beam increases to resist the additional external loading while the bending moment in the plastic hinges decreases. In Fig. 8, the horizontal axis

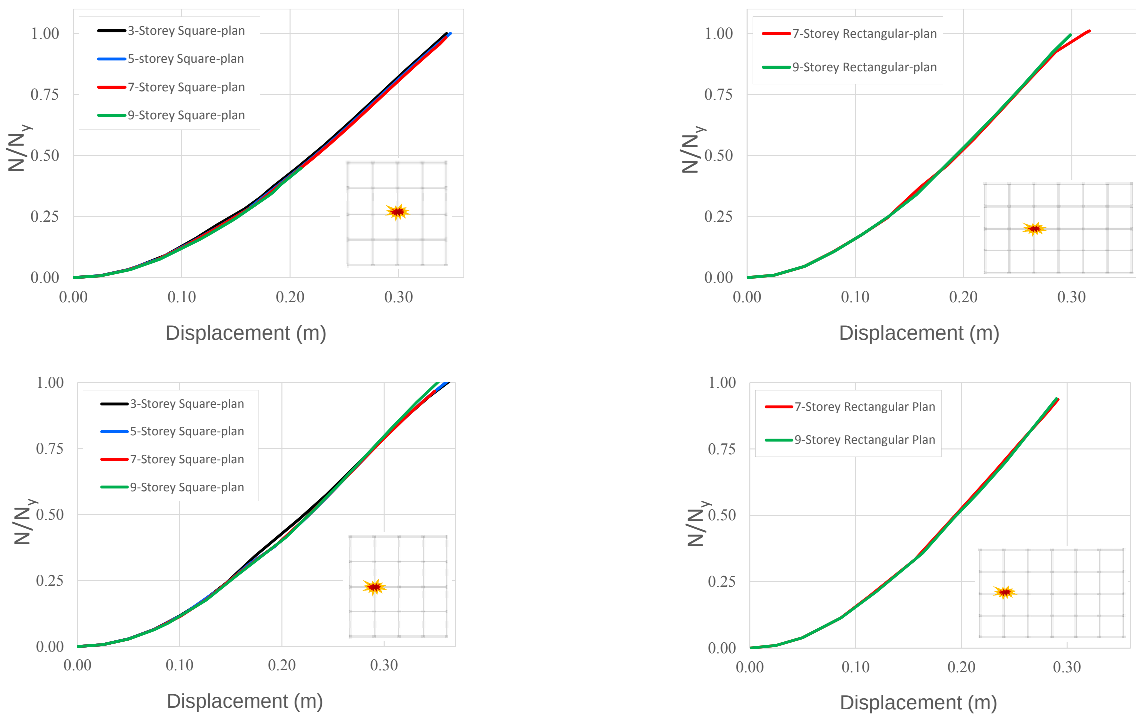


Fig. 8 Variation of axial force from pushdown analysis under unamplified gravity load

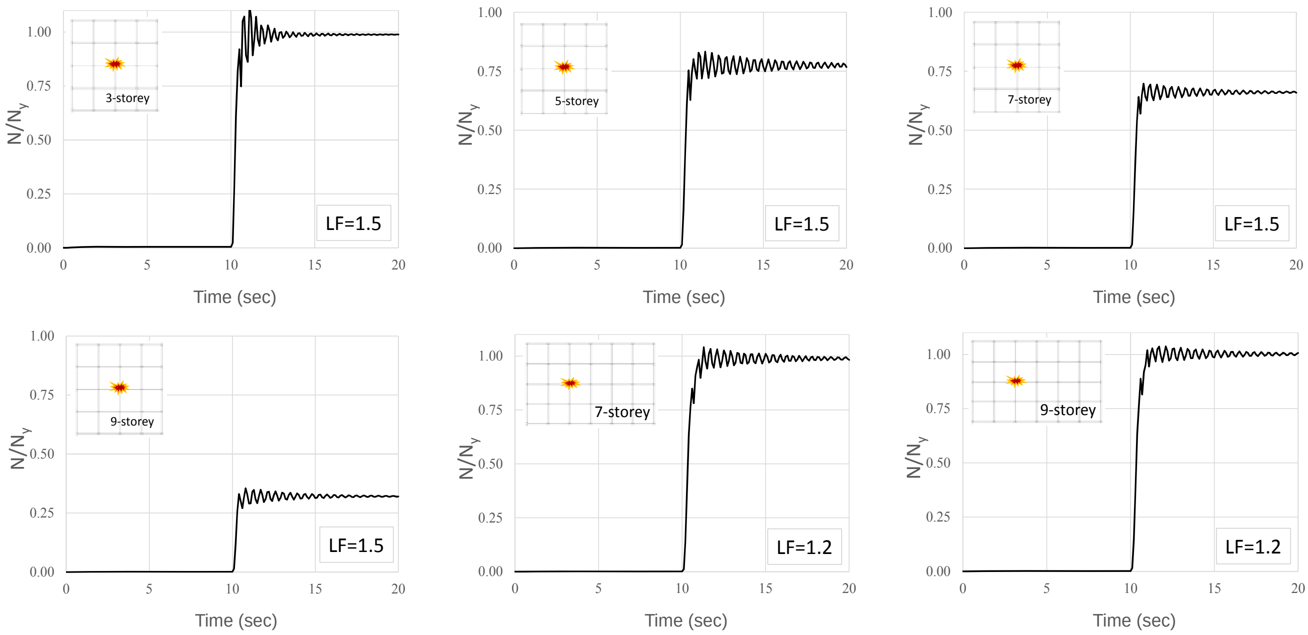


Fig. 9 Axial force time-history from nonlinear dynamic analysis. Column removal scenario C3

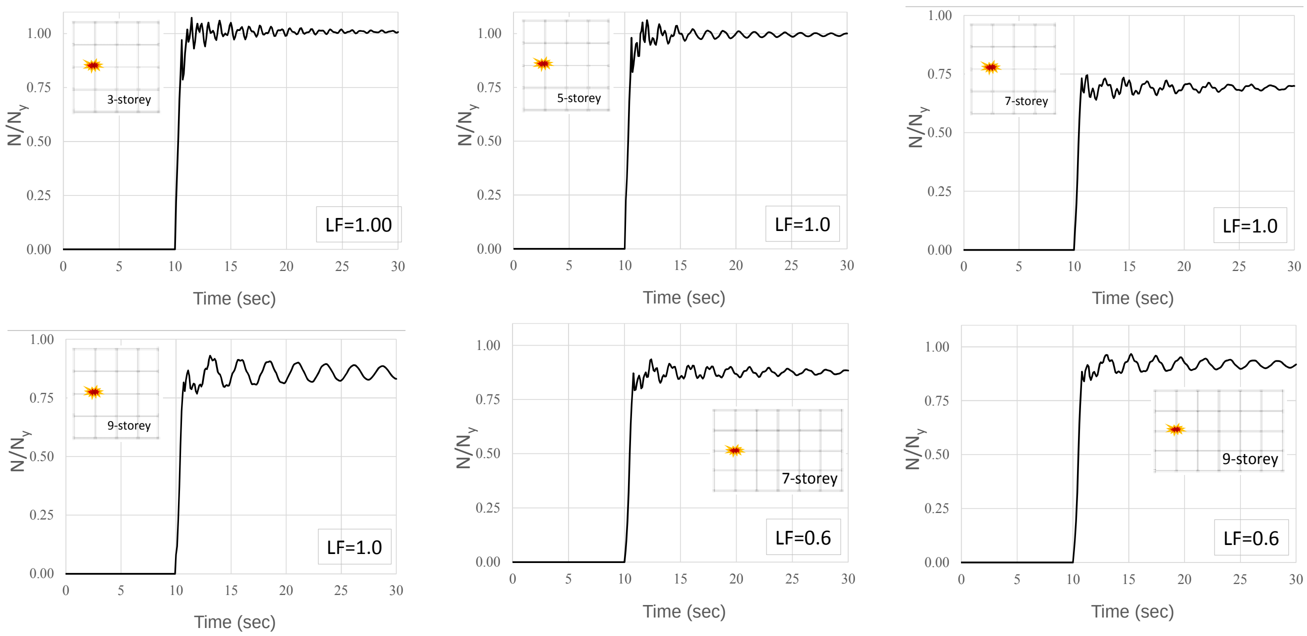


Fig. 10 Axial force time-history from nonlinear dynamic analysis. Column removal scenario B3

represents the vertical displacement in the location of the removed column during the pushdown analysis. The vertical axis represents the axial force normalized by the yield force (N_y) in the first-storey beam adjacent to the removed column. Fig. 8 shows that the axial force increases when the vertical displacement increases. In almost all the case studies here considered, the catenary action is fully developed at the end of the pushdown analysis. In this stage, the axial tensile force in the beam approaches the tensile yield capacity. The 9-storey square plan building represents the only exception. In this case, the pushover analysis ends for the non-ductile failure of the first-storey column and the inelastic deformations of the structure are not large enough to fully develop the tensile catenary resistance.

To study the effects of different levels of gravity loads on the DIF, a gravity load factor (LF) was applied before progressive collapse analysis was carried out. This factor has nothing to do with the DIF. In fact, LF is used to amplify all the GSA-specified combination of the gravity loads, while DIF is used only in NS analysis to multiply the gravity loads in the bays affected by the removed column to approximately reflect the dynamic effect (Figs. 5(a)-(c)). A load factor equal to one (LF = 1) means that the GSA-specified combination of the gravity loads is applied to the structure. In this paper, the value of LF is not kept constant but is changed during the analysis. Practically, LF is used to consider (in a parametric way) actually different levels of vertical displacements that develop when the gravity loads are increased. Strictly speaking, in the case studies here considered greater gravity loads applied to the structure would affect the seismic design of the building and lead to even larger cross-sections of the steel members. However, it should be observed that, in general, the gravity loads actually applied to the structure might be very different from the design gravity loads used to size the cross-sections.

Moreover, the design criteria may be more or less conservative and consider, or do not consider the seismic loads. Many existing buildings are designed to resist only gravity loads or are designed for seismic loads according to old codes and seismic zones.

Other buildings built in recent times are designed according to current seismic design criteria, including ductility and capacity design principles. This means that the gravity loads and the cross-sections of the steel members do not proportionally increase. In case of buildings designed for seismic loads, the cross-sectional area of the steel

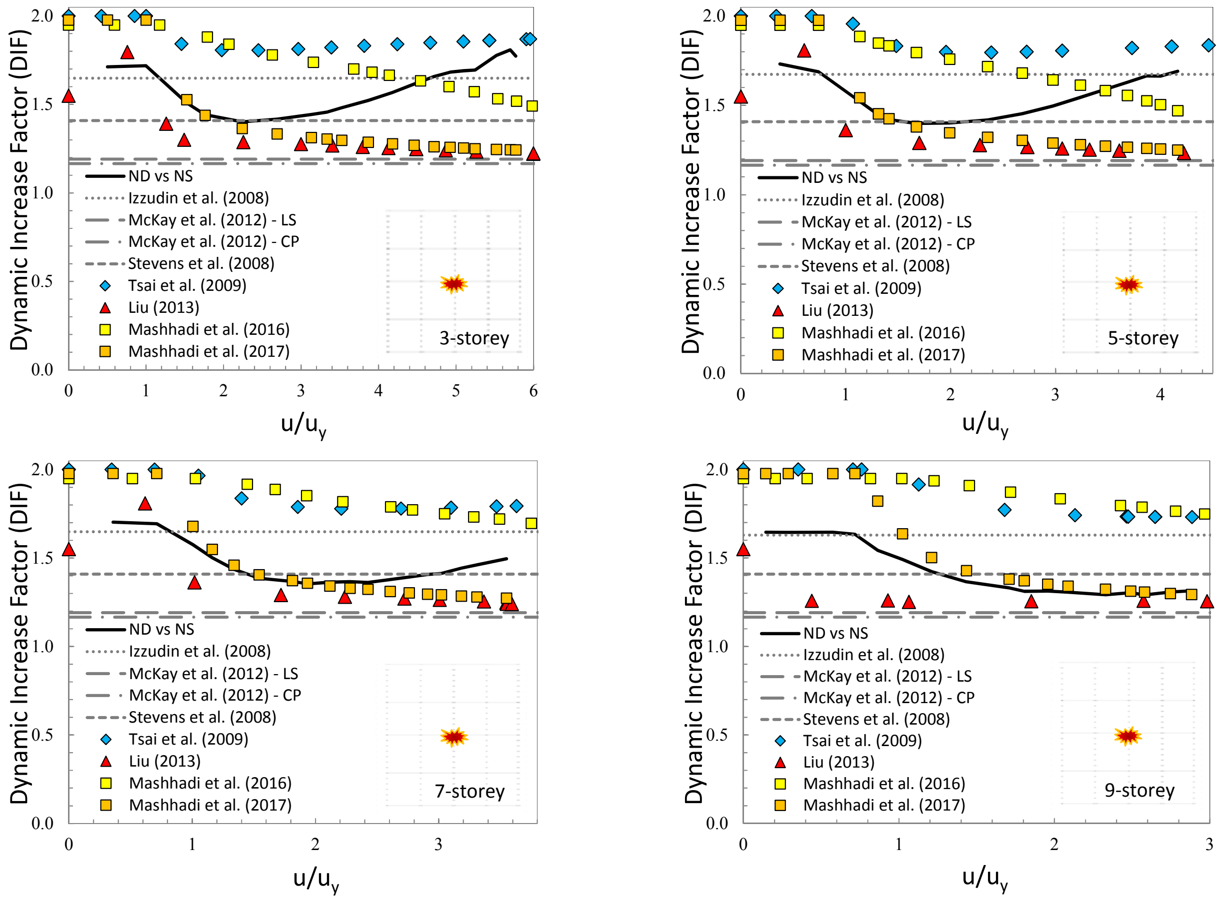


Fig. 11 Variation of DIF with vertical displacement. Square plan buildings. Column removal scenario C3

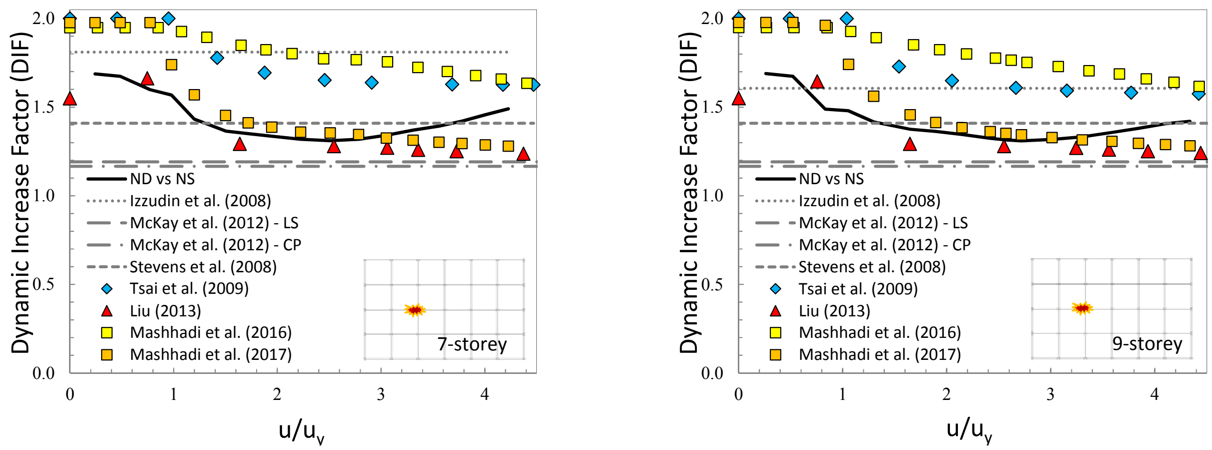


Fig. 12 Variation of DIF with vertical displacement. Rectangular plan buildings. Column removal scenario C3

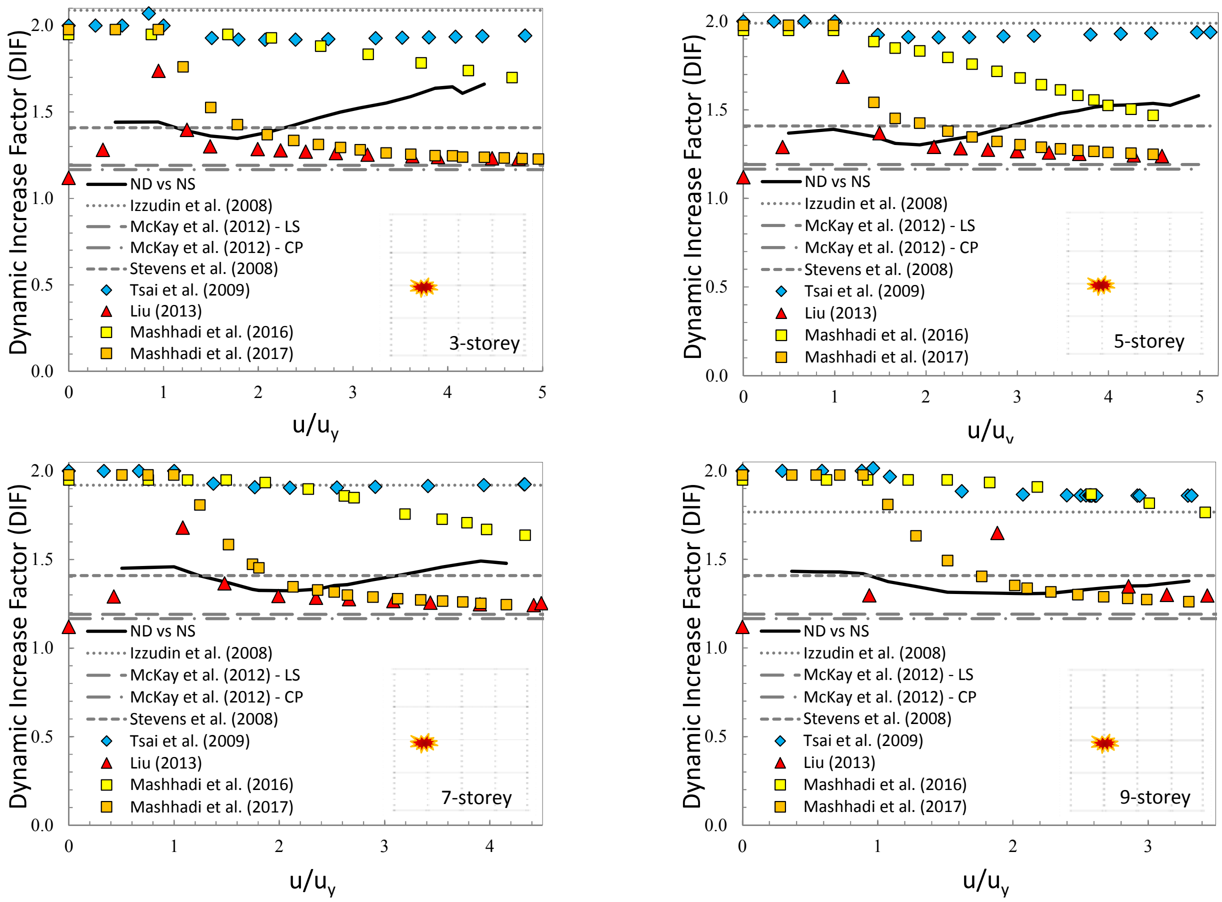


Fig. 13 Variation of DIF with vertical displacement. Square plan buildings. Column removal scenario B3

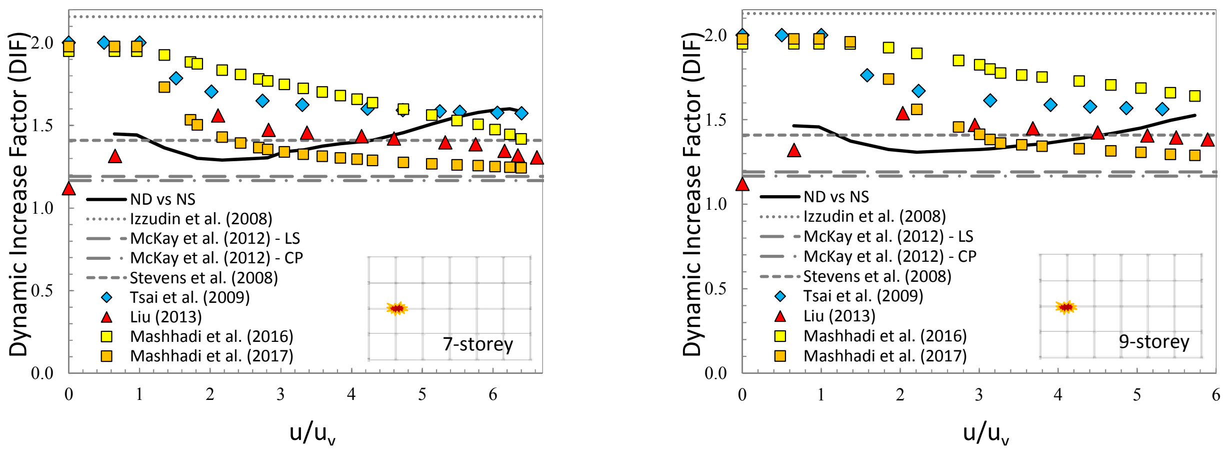


Fig. 14 Variation of DIF with vertical displacement. Rectangular plan buildings. Column removal scenario B3

members is governed by the seismic design combination and, thus, it increases as the intensity of seismic action increases even if the gravity loads are constant. In case of buildings designed only for gravity loads, the cross-sectional area increases as gravity loads increases. In order to evaluate this effect, the parametric analysis is carried out by varying the load factor while the cross-sections of the steel members are kept constant. In this way, different types of buildings are implicitly considered. The cases with a low gravity load amplifier are representative of buildings designed to resist not only gravity loads but also lateral loads. The cases with a high gravity load factor are representative of buildings originally intended to resist primarily gravity loads. As the load factor increases, the damaged building shows higher levels of inelasticity and activates the catenary action. Thus, the DIF that generates the best match of ND analysis depends on the vertical displacement.

In Figs. 11-14 the DIF is plotted as a function of vertical displacement (u) divided the yield displacement (u_y)

obtained from the pushdown curve under unamplified gravity loads (Figs. 6-7). In this paper, the yield displacement is determined as follows. At first, only the curve composed of the initially linear phase and the plastic phase of the pushdown curve is considered. Then, this curve is transformed in a bilinear curve based on equal energy criterion. In Figs. 11-14, the "exact value" of DIF that gives the best match of ND analysis (named ND vs NS) is compared with the values estimated using eight different analytical and empirical formulas proposed in the literature (Table 3). It can be observed that some of the formulations here considered reflect only the effect of structural deformation capacity on the DIF while the specific gravity load level (and thus the real plastic deformation level) is not accounted for.

The DIF proposed by McKay *et al*. (2012) (see Eq. (7)) and included in the GSA Guidelines (2013) depends only on the maximum acceptable plastic hinge rotation angle of the structural members (for Life Safety (LS) or Collapse Prevention (CP) Limit States). As shown in Figs. 11-14, Eq. (7) gives a constant DIF of 1.19 and 1.17, respectively for LS and CP limit states based on $\max(\theta_{pra}/\theta_y) = 6$ for LS and $\max(\theta_{pra}/\theta_y) = 8$ for CP. In the same way, the DIF in Eq. 6 (Stevens *et al*. 2008) is a function only of plastic hinge rotation divided by yield rotation. Finally, the analytical formulas in Eqs. (2)-(3) based on energy balance (Izzuddin *et al*. 2008) provides a single value of DIF. Thus, in Figs. 11-14 each of the formulas proposed by McKay *et al*. (2012), Stevens *et al*. (2008) and Izzuddin *et al*. (2008) gives a constant DIF represented by a horizontal line. This prevents them from providing accurate solutions for different levels of gravity loads since the "exact value" depends on the load factor and, therefore, on the level of vertical displacement in the location of the removed column. The other formulas (Tsai and Liu 2009, Liu 2013, Mashhadi 2016, 2017) gives a DIF that changes with the vertical displacement.

In the linear phase ($u/u_y < 1$) the structure remains linear elastic after the column removal and this gives the higher values of DIF. The formula proposed by Tsai and Liu (2009) gives a DIF of 2.0 that is the value traditionally recommended by the design guidelines for progressive collapse analysis. Even the formulas of Mashhadi (2016, 2017) provide a value very close to 2. On the contrary, the formulas of Liu (2013) gives much lower and non-conservative values and, particularly, 1.12 (Eq. (10)) and 1.55 (Eq. (11)), respectively for perimeter and corner column removal scenarios.

In the nonlinear phase (i.e., $u/u_y > 1$) two cases should be distinguished:

(1) Plastic phase without catenary stage;
(2) Plastic phase followed by catenary phase.

In the first case (i.e., plastic phase without catenary phase), the acceptance criterion for columns is exceeded before the catenary action is fully activated. This happens, for example, in the 9-storey square plan building (Fig. 6). In this case, a monotonic reduction of DIF with vertical displacement is observed. The formula of Mashhadi (2017) gives a very good match of the exact value of DIF since it is able to depict the almost inversely proportional relationship between the gravity load level and the DIF. The formulation proposed by Liu (2013) provides accurate solutions since the parameter $\max(M_d = M_p)$ reflects not only the structural capacity (via M_p) but also the gravity load level (via M_d). Thus, $\max(M_d/M_p)$ is highly correlated to the actual DIF. However, this formulation tends to become not accurate for structures with a high level of nonlinearity. Finally, it should be observed that the DIF corresponding to the failure condition tends to be similar to the constant value defined from the GSA formulation of DIF proposed by McKay *et al*. (2012).

In the second case (i.e., plastic phase followed by catenary phase), the "exact value" of DIF first decreases and then increases with the vertical displacement. The cause for this difference is the beginning of the phase where the catenary action is fully developed. In fact, the curve of DIF decreases with the vertical displacement in the plastic phase. On the contrary, when the catenary phase is fully developed, the curve of DIF increases with the vertical deflection. This means that a monotonic reduction of DIF with actual vertical displacement is not realistic and it becomes inaccurate for nonlinear static responses involving the tensile catenary action. Moreover, the "exact value" of DIF is sensibly greater than the DIFs calculated with the formulas proposed by McKay *et al*. (2012), Liu (2013) and Mashhadi (2017). This means that the pushdown analysis conducted with these DIFs underestimates the vertical displacement in the location of the removed column when compared to the ND analysis. Thus, a greater load factor is required to obtain the vertical displacement corresponding to the collapse mechanism, which implicates a non-conservative estimation of the progressive collapse resistance. On the contrary, the analytical formulas proposed by Tsai and Lin (2009) and Mashhadi (2016) are generally conservative. However, they give a great overestimation of DIF, with values very close to the conventional value of 2.0 originally proposed in the literature.

5. Conclusions

Results show that the DIF should be based on the actual (as opposed to maximum allowable) plastic deformation level that the damaged building experiences, thus accounting for the specific gravity load level. In fact, due to the specific magnitude of gravity loads, the building could respond elastically or inelastically. Moreover, if the structure has sufficient ductility, the catenary action in steel beams could be fully developed at large deflection stage.

Some of the formulas available in the literature reflect only the effect of structural deformation capacity on DIF, while the specific gravity load level (and thus the actual plastic deformation level) is not accounted for. As a result, these formulas may not depict the actual relationship between the gravity load level and the DIF. In fact, the damaged building may undergo limited plastic deformation or even stay essentially elastic under lower gravity loads.

This may be particularly true for buildings that were originally designed to withstand large lateral loads (such as those from earthquakes or strong winds). In this case, the formulas proposed by Tsai and Lin (2009), Mashhadi (2016, 2017) gives a conservative value of 2 for DIF. On the contrary, the formulas based on the hypothesis that the controlling beam has reached its maximum allowable plastic hinge rotation capacity (Stevens *et al*. 2008, McKay *et al*. 2012) may give a strong underestimation of DIF. When the specific magnitude of gravity loads increases, the damaged buildings enters into the inelastic range. However, the actual level of inelasticity is not necessarily so high that the plastic rotation of the controlling structural member reaches its prescribed maximum acceptable value. As a consequence, the formulas based on the maximum allowable plastic hinge rotation capacity (Stevens *et al*. 2008, McKay *et al*. 2012) tend to underestimate the DIF for structures that experience a modest level of nonlinearity. On the contrary, both the formula of Liu (2013) and the formula of Mashhadi (2017) give accurate solutions in the intermediate range of plastic deformations since they account for not only structural capacity but also ductility demand of structural members. Finally, higher gravity loads can drive the damaged building well into the inelastic range. In this situation, two cases may occur: plastic phase without catenary action and plastic phase with catenary action.

In the first case (i.e., plastic phase without catenary phase), the catenary stage is not developed because the plastic rotation capacity is exceeded, typically in one of the next to the removed columns. Both the formula of Liu (2013) and the formula of Mashhadi (2017) are able to give accurate solutions since an almost inversely proportional relationship occurs between the level of plastic deformation (i.e., the levels of gravity load) and the exact value of DIF. The DIF corresponding to the failure condition tends to the constant value defined by the formula of McKay *et al*. (2012) presently adopted in the latest version of both UFC (2013) and GSA (2013).

In the second case (i.e., plastic phase with fully development of catenary action), it was found that the catenary stiffening effect leads to an increase in the DIF with the vertical deflection. Thus, the assumption of a monotonic decreasing of DIF with ductility is no longer valid and the formulas proposed by McKay *et al*. (2012), Liu (2013) and Mashhadi (2017) become questionable and not conservative since they can underestimate significantly the DIF.

Finally, it must be observed that for this analysis some simplifying assumptions were made. At first, the foundations were assumed capable of withstanding the redistribution of forces that occur when individual columns are removed. Then, the strain rate effects were neglected in the analysis model and imperfections of the members were not explicitly modelled. Finally, the beam–column joints were considered strong enough to activate full catenary action of beams that is an assumption normally true only in structures with seismic joints. In spite of the aforementioned simplifying assumptions, the results observed are expected to be reproduced in a complete approach.

Acknowledgments

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