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Statistical properties of the maximum elastoplastic story drift of steel frames subjected to earthquake load

Gang Li†

Department of Engineering Mechanics, State Key Laboratory of Structural Analysis of Industrial Equipment, Dalian University of Technology, Dalian, 116024, China

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Abstract. The concept of performance based seismic design has been gradually accepted by the earthquake engineering profession recently, in which the cost-effectiveness criterion is one of the most important principles and more attention is paid to the structural performance at the inelastic stage. Since there are many uncertainties in seismic design, reliability analysis is a major task in performance based seismic design. However, structural reliability analysis may be very costly and time consuming because the limit state function is usually a highly nonlinear implicit function with respect to the basic design variables, especially for the complex large-scale structures for dynamic and nonlinear analysis. Understanding statistical properties of the structural inelastic deformation, which is the aim of the present paper, is helpful to develop an efficient approximate approach of reliability analysis. The present paper studies the statistical properties of the maximum elastoplastic story drift of steel frames subjected to earthquake load. The randomness of earthquake load, dead load, live load, steel elastic modulus, yield strength and structural member dimensions are considered. Possible probability distributions for the maximum story are evaluated using K-S test. The results show that the choice of the probability distribution for the maximum elastoplastic story drift of steel frames is related to the mean value of the maximum elastoplastic story drift. When the mean drift is small (less than 0.3%), an extreme value type I distribution is the best choice. However, for large drifts (more than 0.35%), an extreme value type II distribution is best.

Key words: steel frames; elastoplastic analysis; seismic design; probability distribution; K-S test.

1. Introduction

The concept of performance based seismic design was proposed in the early 1990s in the USA, and has been gradually accepted by the earthquake engineering profession recently (ATC 1996, BRI 2000, FEMA 1996, SEAOC 1995). Compared with the current seismic design codes, performance based seismic design takes cost-effectiveness criterion as its important principle, and pays more attention to the structural performance in the inelastic stage. A popular measure of performance is the maximum story drift. In general, there are many uncertainties in structural design, including the uncertainties of external loads, structural capacity, analysis models and structural performance, and uncertainty is even more serious under earthquake hazard environment. Engineers must deal with these kinds of uncertainties in the structural design process, and the theory and methodology of structural reliability is a useful tool to help the designers realize such a purpose. Therefore, the reliability analysis of the structural performance,

†Associate Professor

usually measured by inelastic deformation, is a major procedure in performance based seismic design (Cheng and Li 2000, Collins 1998).

The structural reliability theories and applications have undergone a lot of developments during the last few decades (Melchers 1999), and a number of design codes in many countries have adopted reliability criteria usually with the partial safety format (Ellingwood 1994, Mrazik and Krizma 1997). However, structural reliability analysis is potentially very challenging because the limit state function is usually a highly nonlinear implicit function with respect to the basic design variables, especially for the complex large-scale structures and the mathematical method used for nonlinear analysis. Thus developing an efficient approximate approach of reliability analysis with the accepted accuracy is an important task and challenge to the researchers and engineers, and research to determine the probability distribution of structural deformation, which is the main purpose of the present paper, is helpful to develop such an approximate method.

2. Statistical properties of concerned random variables

The randomness of earthquake load, dead load, live load, steel elastic modulus, steel yield strength and structural member dimensions is considered in the present paper. According to Chinese seismic design code (GBJ 2001), the dynamic earthquake load (a stochastic process) is treated as the equivalent static earthquake load (a random variable), which follows an extreme value type I distribution for a given seismic intensity (Gao and Bao 1985). Based on Chinese Uniform Standards for Building Structure Design (GBJ 1984), dead load and live load follow normal distribution and extreme value type I distribution, respectively. The structural member dimensions and steel elastic modulus have normal distributions, and the steel yield strength has a log-normal distribution (Li *et al.* 1990).

The probability distribution and statistical parameters of the concerned random variables are summarized in Table 1, in which the standard value means the design value of the random variable, and the coefficient of variation is the ratio of the standard deviation to the mean.

According to the data in Table 1, the cumulative distribution functions (CDFs) of live load and earthquake load, which have extreme value type I distribution, can be stated as

$$F_{I-L}(L) = \exp\left[-\exp\left(-\frac{L-0.423L_k}{0.084L_k}\right)\right] \quad (1)$$

$$F_{I-V}(V) = \exp\left[-\exp\left(-\frac{V-0.917V_k}{0.248V_k}\right)\right] \quad (2)$$

Table 1 Probability distribution and statistical parameters of the random variables

	Probability distribution	Ratio of mean to standard value	Coefficient of variation
Earthquake load	Extreme I	1.06	0.30
Dead load	Normal	1.06	0.070
Live load	Extreme I	0.471	0.229
Area of steel member	Normal	1.00	0.05
Steel elastic modulus	Normal	1.08	0.08
Steel yield strength	Log-normal	1.21	0.15

where L_k and V_k are the standard values of live load and earthquake load, respectively

3. Generation of random variables

The inverse transform method is a general technique to generate random variables from a specific distribution whose CDF can be inverted (Kennedy and Gentle 1980, Gao 1995). First, generate a uniformly distributed number R within [0,1] (James 1990), then set it equal to the CDF of the random variable

$$F(x) = R \quad \text{or} \quad x = F^{-1}(R) \tag{3}$$

where $F(x)$ is the CDF of the random variable x, R is the uniformly distributed random number within [0,1].

The random variables (such as earthquake load and live load) that have the extreme value type I distribution are generated by the inverse transform method, by which the simple equations can be deduced from Eqs. (1) and (2) as follows

$$L = 0.423L_k - 0.084L_k\, Ln(-Ln(R)) \tag{4}$$

$$V = 0.917V_k - 0.248V_k\, Ln(-Ln(R)) \tag{5}$$

It should be pointed out that although the inverse transform method can also be used to generate normally distributed random variables, it is not an efficient way. The CDF of the normal random variable can not be inverted directly with the explicit equation, because the CDF contains an integral that must be calculated by some simulation methods. Therefore, some approximate methods that can generate the standard normal random variables with high efficiency were developed (Kennedy and Gentle 1980, Payne 1977, Atkinson and Pearce 1976, Kinderman and Ramage 1976), such as central limit theorem approximation, Box-Muller transformation, Marsaglia’s Polar method, rectangle-wedge-tail method, Kinderman-Ramage procedure, Forsythe’s method and modifications.

Use of the central limit theorem on the uniformly distributed random numbers can provide a simple and efficient method for closely approximating normal random variables, which can be stated as (Kennedy and Gentle 1980, Gao 1995, Xu 1992).

$$x_s = \sqrt{\frac{n}{12}}\left(\sum_{i=1}^{n} R_i - n/2\right) \tag{6}$$

where x_s is the standard normal random variable, R_i is the uniform random variables within [0,1]. Because the uniform distribution is a well-behaved distribution, the approximation is fairly good even for small n. Choosing $n = 12$ leads to the simple form that is often used:

$$x_s = \left(\sum_{i=1}^{12} R_i - 6\right) \tag{7}$$

The central limit theorem approximation is used in the present paper to generate normal random variables (such as structural member dimension, steel elastic modulus and dead load) as follows

$$x = \mu + \sigma\left(\sum_{i=1}^{12} R_i - 6\right) \tag{8}$$

where x is the normally distributed random variable, μ and σ are the mean and standard deviation.

The log-normally distributed random variable sequence (such as steel yield strength) can be generated based on the relation to normal random variables. That is, the normally distributed random variable x is generated by Eq. (8) first, then the log-normally distributed random variable y can be obtained by the following relation

$$y = \exp(x) \tag{9}$$

4. Test of goodness-of-fit

In the present paper, we try to find the probability distribution of the maximum elastoplastic story drift of steel frames subjected to earthquake load, which belongs to the nonparametric problem in testing hypotheses, called test of goodness-of-fit (DeGroot 1986, Kendall and Stuart 1979, Wu and Wang 1996). The test of goodness-of-fit is to test the simple null hypothesis that the unknown distribution function $F(x)$ is actually a particular continuous distribution function $F_0(x)$ against the general alternative that the actual distribution function is not $F_0(x)$. The major steps of test of goodness-of-fit are as follows:

(1) Suppose the hypothesis of test of distribution function,

$$H_0{:}F(x) = F_0(x) \tag{10}$$

(2) Generate a random sample (the sample size is n) and calculate the sample distribution function $F_n^*(x)$;

(3) Define a proper statistic, which can evaluate the difference between the distributions of the sample and the hypothesis; Calculate the value of this statistic based on the observed values of the generated sample;

(4) For a given level of significance, judge whether the null hypothesis H_0 is true or rejected.

The χ^2 test and the Kolmogorov-Smirnov test (K-S test) are two well-known methods of test of goodness-of-fit to a particular continuous distribution. In the χ^2 test of goodness-of-fit, the entire real line or any particular interval that has the probability 1 must be partitioned into a finite number k of disjoint subintervals first, say, $(-\infty, a_1), [a_1, a_2),..., [a_{k-1}, +\infty)$, then the following statistic is calculated.

$$\chi^2 = \sum_{i=1}^{k} \frac{(N_i - np_i^0)^2}{np_i^0} \tag{11}$$

where N_i denotes the number of trials that fall within the i^{th} subinterval, and $\sum_{i=1}^{k} N_i = n$; p_i^0 is the corresponding probability calculated based on the hypothesized distribution. The difference between the actual number of trials N_i and the expected number np_i^0 will tend to be smaller when H_0 is true than when H_0 is not true.

It should be pointed out that a particular feature of the χ^2 test of goodness-of-fit is that the procedure is designed to test the null hypothesis H_0' that $p_i = p_i^0$ (p_i denotes the probability of the trials within the i^{th} subinterval) for i=1,…, k. That is, the hypothesis in the χ^2 test of goodness-of-fit is actually the following hypothesis.

$$H_0' : F(a_i) - F(a_{i-1}) = F_0(a_i) - F_0(a_{i-1}) \quad i = 1, \dots k \tag{12}$$

and not the original hypothesis Eq. (10). In some cases, the hypothesis H_0' is true, however, the hypothesis H_0 may not be true. Furthermore, the way in which the subintervals are chosen may affect the final solution of test of goodness-of-fit (DeGroot 1986, Kendall and Stuart 1979, Wu and Wang 1996).

The K-S test overcomes the shortcomings of the χ^2 test discussed above to some extent by using the following statistic at each trial of the sample.

$$D_n = \sup_{x \in R} \{ |F_n^*(x) - F_0(x)| \} = \max_i \{ \max[|F_n^*(x_i) - F_0(x_i)|, |F_n^*(x_{i+1}) - F_0(x_i)|] \} \tag{13}$$

where $F_n^*(x)$ is the sample distribution.

For a given level of significance a, check the upper quantile values of the distribution of the K-S static $D_{n,\alpha}$ ($P\{D_n \le D_{n,\alpha}\} = 1 - a$) shown in Table 2. If $D_n \le D_{n,\alpha}$, then the null hypothesis H_0 is true; else the null hypothesis H_0 is rejected. The K-S test was used in the study of probability distributions of external loads (GBJ 1984), structural deformation capacity (Gao 1990) and elastic deformation of RC frames (Li and Cheng 2002), etc. In the present paper, the K-S test is also employed.

In addition to the above probabilistic-statistical tests, it is also possible to use the tests of the geometrical contiguousness, such as through the use of the quadrate deviations sum of the hypothesized distribution from the obtained sample distribution, by means of the graphical assessment of the contiguousness of the hypothesized distribution and the experimental one.

5. Probability distribution of the maximum elastoplastic story drift of steel frames

The procedure to study the probability distributions of the maximum story drift of steel frames in the present paper is as follows

(1) Select the steel frames for sampling, which are designed according to the Chinese seismic design code (GBJ 2001). Obtain the main initial design data (standard values) of the steel frames (such as steel elastic modulus, yield strength and structural member dimensions) and live load and dead load.

(2) Calculate the standard value of the total earthquake force using the response spectra method

Table 2 Upper quantile values of the distribution of the K-S static $D_{n,\alpha}$

a	$D_{n,\alpha}$	$D_{n,\alpha}$ (n = 500)
0.20	$1.07 / \sqrt{n}$	.479E-01
0.10	$1.22 / \sqrt{n}$	.546E-01
0.05	$1.36 / \sqrt{n}$	.608E-01
0.01	$1.63 / \sqrt{n}$	.729E-01

provided in Chinese seismic design code (GBJ 2001), based on the initial data (standard values) of steel frames, live load and dead load.

(3) According the probability distributions and statistical parameters in Table 1 and the standard values obtained in step 1 and 2 of the concerned random variables, generate the trials (the number is n) of the sample frames with the central limit theorem approximation and the inverse transform method. Each trial contains a set of observed values, which are earthquake load, dead load, live load, steel elastic modulus, steel yield strength and structural member areas.

(4) Perform the seismic analysis to each trial of the random variables, which is actually a static inelastic analysis process herein, and obtain the observed values for the random sample of the maximum story drift. In this step, the APDL (ANSYS Parametric Design Language) is used to perform the seismic analysis of all the trials with ANSYS program within a batch job (ANSYS Inc. 2000a, 2000b).

(5) Study the probability distribution of the maximum story drift with the K-S test.

The flowchart of the procedure is shown in Fig. 1.

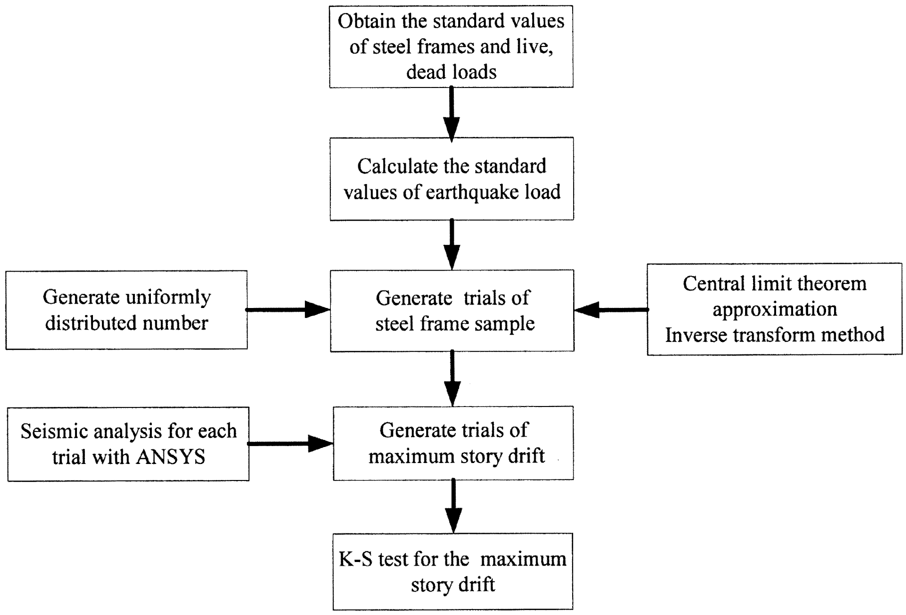
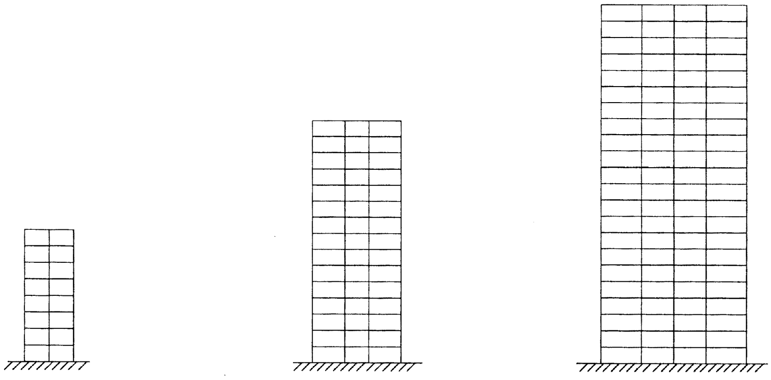


Fig. 1 Flowchart of the K-S test of maximum story drift

Three types of steel frame samples are considered to study the probability distribution of the maximum inelastic story drift of steel frames, which are two-bay eight-story frame, three-bay fifteen-story frame and four-bay twenty-two-story frame, as shown in Fig. 2. The standard values of the initial data of steel frames and live load, dead load and seismic parameters are shown in Tables 3 and 4, in which the standard value of live load and dead load are evaluated approximately according to the Chinese uniform standards (GBJ 1984), the values of the seismic intensity, the parameters for the site where the structures are located are taken based on the Chinese seismic design code (GBJ 2001). The standard values of the elastic modulus and the yield strength of steel frames are .206E+06 N / mm^2 and 225 N / mm^2, respectively. The column section of steel frames is square or H shape, and the beam section is I shape.



two-bay eight-story three-bay fifteen-story four-bay twenty-two-story

Fig. 2 Steel frames (two-bay eight-story, three-bay fifteen-story, four-bay twenty-two-story)

Table 3 Standard values of the steel frames

	Standard story	Column section (middle and side)/mm	Beam section / mm	Story height / m	Frame span / m
2-bay 8-story	1~8	H 350×350×23×15 H 350×350×21×13	I 600×200 ×16×12	3.3×8	5.0, 5.0
3-bay 15-story	1~7 8~15	□ 380×380×25 □ 380×380×22 □ 380×380×20 □ 380×380×17	I 600×200 ×19×12	3.3×15	6.0, 5.0, 6.0
4-bay 22-story	1~8 9~15 16~22	□ 450×450×28 □ 450×450×25 □ 450×450×25 □ 450×450×22 □ 450×450×22 □ 450×450×18	I 650×200 ×22×12	3.3×22	7.5, 6.0, 6.0, 7.5

Table 4 Standard values of the live, dead loads and seismic parameters

	Story dead load /kN	Story live Load /kN	Seismic intensity	Site type
2-bay 8-story	300	150	8	Type 2 group 2
3-bay 15-story	450	200	8	Type 2 group 2
4-bay 22-story	600	260	8	Type 3 group 2

To provide additional insight into how much the earthquake uncertainty determines the problem, two combinations of random variables are studied. In Case 1 (combination 1), only the randomness of earthquake load is considered. In Case 2 (combination 2) the randomness of all the variables is considered. The probability distributions of the maximum story drift obtained from the elastic and inelastic analysis are compared. Generally in the seismic analysis, when the structure is given, the magnitude of the earthquake load is the dominant factor that determines the structure performance whether in elastic stage or inelastic stage. Thus, for all the 500 trials of the steel frames, the story drifts

Table 5 Statistical parameters of 8-story steel frame

Random variables	Obtained by initial data			by generated trials	
	Standard value	Mean	Coefficient of variation	Mean	Coefficient of variation
Middle column area mm^2	.207E+05	.207E+05	0.050	.206E+05	0.050
Side column area mm^2	.187E+05	.187E+05	0.050	.187E+05	0.050
Beam area mm^2	.132E+05	.132E+05	0.050	.132E+05	0.051
Elastic modulus N / mm^2	.206E+06	.223E+06	0.080	.222E+06	0.081
Dead load N	.300E+06	.318E+06	0.070	.318E+06	0.072
Live load N	.150E+05	.707E+05	0.229	.709E+05	0.220
Yield strength N / mm^2	.225E+03	.272E+03	0.150	.274E+03	0.149
Earthquake load Case-1 N	.260E+06	.276E+06	0.300	.276E+06	0.301
Earthquake load Case-2 N	.260E+06	.276E+06	0.300	.263E+06	0.305

Table 6 D_n values of the maximum story drift of 8-story steel frame

		Extreme type I	Extreme type II	Normal	Log-normal
Case-1	Elastic	.176E-01	.956E-01	.800E-01	.272E-01
	Elastoplastic (1.5)	.839E-01	.539E-01	.130E+00	.825E-01
	Elastoplastic (1.8)	.116E+00	.354E-01	.158E+00	.109E+00
	Elastoplastic (2.0)	.139E+00	.422E-01	.156E+00	.130E+00
Case-2	Elastic	.199E-01	.101E+00	.853E-01	.292E-01
	Elastoplastic (1.5)	.300E-01	.941E-01	.913E-01	.373E-01
	Elastoplastic (1.8)	.117E+00	.466E-01	.147E+00	.111E+00
	Elastoplastic (2.0)	.144E+00	.562E-01	.157E+00	.137E+00

Table 7 D_n values of the maximum story drift of 15-story steel frame

		Extreme type I	Extreme type II	Normal	Log-normal
Case-1	Elastic	.175E-01	.957E-01	.798E-01	.270E-01
	Elastoplastic (1.5)	.393E-01	.819E-01	.913E-01	.458E-01
	Elastoplastic (1.8)	.813E-01	.539E-01	.125E+00	.810E-01
	Elastoplastic (2.0)	.133E+00	.608E-01	.168E+00	.127E+00
Case-2	Elastic	.217E-01	.107E+00	.777E-01	.236E-01
	Elastoplastic (1.5)	.204E-01	.106E+00	.777E-01	.234E-01
	Elastoplastic (1.8)	.422E-01	.853E-01	.969E-01	.457E-01
	Elastoplastic (2.0)	.110E+00	.700E-01	.134E+00	.104E+00

obtained by the seismic analysis keep elastic for some trials and inelastic for the other trials. In order to study the effect of different number of the trials with inelastic performance on the probability distribution of the maximum story drift, a factor (the value of 1.5, 1.8 and 2.0 are taken herein) is introduced to magnify the observed value of the earthquake load for each trial. In the elastoplastic analysis, the ideal elastoplastic relation between stress and strain is assumed.

The statistical parameters of the random variables of 8-story frame obtained by the initial standard values and by the generated trials are listed in Table 5. Some of the results are shown in Tables 6~11

Table 8 D_n values of the maximum story drift of 22-story steel frame

		Extreme type I	Extreme type II	Normal	Log-normal
Case-1	Elastic	.175E-01	.957E-01	.797E-01	.270E-01
	Elastoplastic (1.0)	.290E-01	.883E-01	.874E-01	.375E-01
	Elastoplastic (1.2)	.631E-01	.670E-01	.112E+00	.657E-01
	Elastoplastic (1.5)	.114E+00	.467E-01	.149E+00	.109E+00
Case-2	Elastic	.281E-01	.956E-01	.856E-01	.320E-01
	Elastoplastic (1.0)	.275E-01	.949E-01	.858E-01	.320E-01
	Elastoplastic (1.2)	.300E-01	.851E-01	.861E-01	.369E-01
	Elastoplastic (1.5)	.990E-01	.470E-01	.134E+00	.935E-01

Table 9 Statistical parameters of maximum story drift of 8-story steel frame

		Standard value	Mean	Ratio of mean to standard value	Coefficient of variation
Case-1	Elastic	.211E-02	.224E-02	1.06	0.300
	Elastoplastic (1.5)	.316E-02	.344E-02	1.09	0.389
	Elastoplastic (1.8)	.379E-02	.419E-02	1.10	0.438
	Elastoplastic (2.0)	.422E-02	.468E-02	1.11	0.473
Case-2	Elastic	.211E-02	.200E-02	0.947	0.315
	Elastoplastic (1.5)	.316E-02	.301E-02	0.952	0.328
	Elastoplastic (1.8)	.379E-02	.371E-02	0.979	0.445
	Elastoplastic (2.0)	.422E-02	.413E-02	0.978	0.483

Table 10 Statistical parameters of maximum story drift of 15-story steel frame

		Standard value	Mean	Ratio of mean to standard value	Coefficient of variation
Case-1	Elastic	.184E-02	.195E-02	1.06	0.301
	Elastoplastic (1.5)	.276E-02	.295E-02	1.07	0.331
	Elastoplastic (1.8)	.331E-02	.357E-02	1.08	0.381
	Elastoplastic (2.0)	.368E-02	.400E-02	1.09	0.451
Case-2	Elastic	.184E-02	.176E-02	0.958	0.314
	Elastoplastic (1.5)	.276E-02	.264E-02	0.958	0.316
	Elastoplastic (1.8)	.331E-02	.320E-02	0.967	0.351
	Elastoplastic (2.0)	.368E-02	.359E-02	0.975	0.435

and Figs. 3 and 4, in which Elastoplastic (1.5) denotes the elastoplastic analysis with the magnification factor 1.5 to the observed value of earthquake load. Tables 6~8 are D_n values of the K-S test for different probability distribution hypotheses and Tables 9~11 are statistical parameters of the maximum story drift, in which the standard value of story drift is calculated by the seismic analysis using the initial standard values of steel frames and external loads. Fig. 3 shows the curves of the probability density functions of maximum story drift based on the histogram and the hypothesized distribution functions of extreme value type I and type II distributions, normal and log-normal distributions. Fig. 4

Table 11 Statistical parameters of maximum story drift of 22-story steel frame

		Standard value	Mean	Ratio of mean to standard value	Coefficient of variation
Case-1	Elastic	.254E-02	.270E-02	1.06	0.301
	Elastoplastic (1.0)	.254E-02	.271E-02	1.07	0.319
	Elastoplastic (1.2)	.305E-02	.328E-02	1.08	0.360
	Elastoplastic (1.5)	.381E-02	.411E-02	1.08	0.422
Case-2	Elastic	.254E-02	.243E-02	0.959	0.311
	Elastoplastic (1.0)	.254E-02	.244E-02	0.959	0.313
	Elastoplastic (1.2)	.305E-02	.294E-02	0.964	0.329
	Elastoplastic (1.5)	.381E-02	.374E-02	0.983	0.418

shows the linear relation between the mean and coefficient of variation of the elastoplastic maximum story drift by regression.

It should be noted that in elastoplastic analyses (particularly for the trials with the large magnification factor to the earthquake load), the following two situations may occur in several trials in the calculation process: (1) the analysis process does not convergence due to large story drift; (2) the maximum story drift exceeds 2.5%. In fact, the structure could collapse in the above two situations which are not allowed in practical design according to the design code (GBJ 2001). Therefore, the trials with the above two situations (only few trials in the whole trials) are eliminated in the process of statistical analysis of observed value for the of maximum story drift, which is the reason that the mean of the maximum elastoplastic story drift seems small to some extent.

The following observations can be obtained from the above results:

(1) Probability distribution of maximum story drift (see Tables 6~8 and Fig. 3). In elastic analysis of steel frame samples, the maximum story drift has extreme value distribution type I or log-normal distribution, in which the extreme value type I distribution fits better. In elastoplastic analysis, the choice of probability distribution of the maximum elastoplastic story drift of steel frames is related to the mean of the maximum elastoplastic story drift. It follows extreme value type I distribution when the mean value is small, less than 0.3% (that is, there are relatively few samples in elastoplastic stage). This is similar to the probability distribution obtained by elastic analysis. It has extreme value type II distribution when the mean is large, say, more than 0.35%. By the way, the maximum story drift among all stories of the trials of the steel frames always occurs in two particular stories, with very slight difference between the drift values of these two stories. For example, for 8-story steel frame, the maximum story drift always occurs in the 2nd or 3rd story (few cases of the 3rd story); for 15-story frame, the 3rd or 4th story and for 22-story frame, the 4th or 5th story.

(2) Coefficient of variation of maximum story drift (see Tables 9~11). In elastic analysis of steel frame samples, for Case-1 (only the randomness of earthquake load is treated), the coefficient of variation of maximum story drift is 0.30, the same as that of earthquake load; for Case-2 (the randomness of all variables is treated), the coefficient of variation of maximum story drift is 0.31, a little larger than that of earthquake load, which shows that the coefficient of variation of maximum story drift is mainly determined by that of earthquake load. In other words, the uncertainty of earthquake load is the dominant factor to the uncertainty of structural responses, which has also been observed by others (Wen 2001a 2001b, Song and Ellingwood

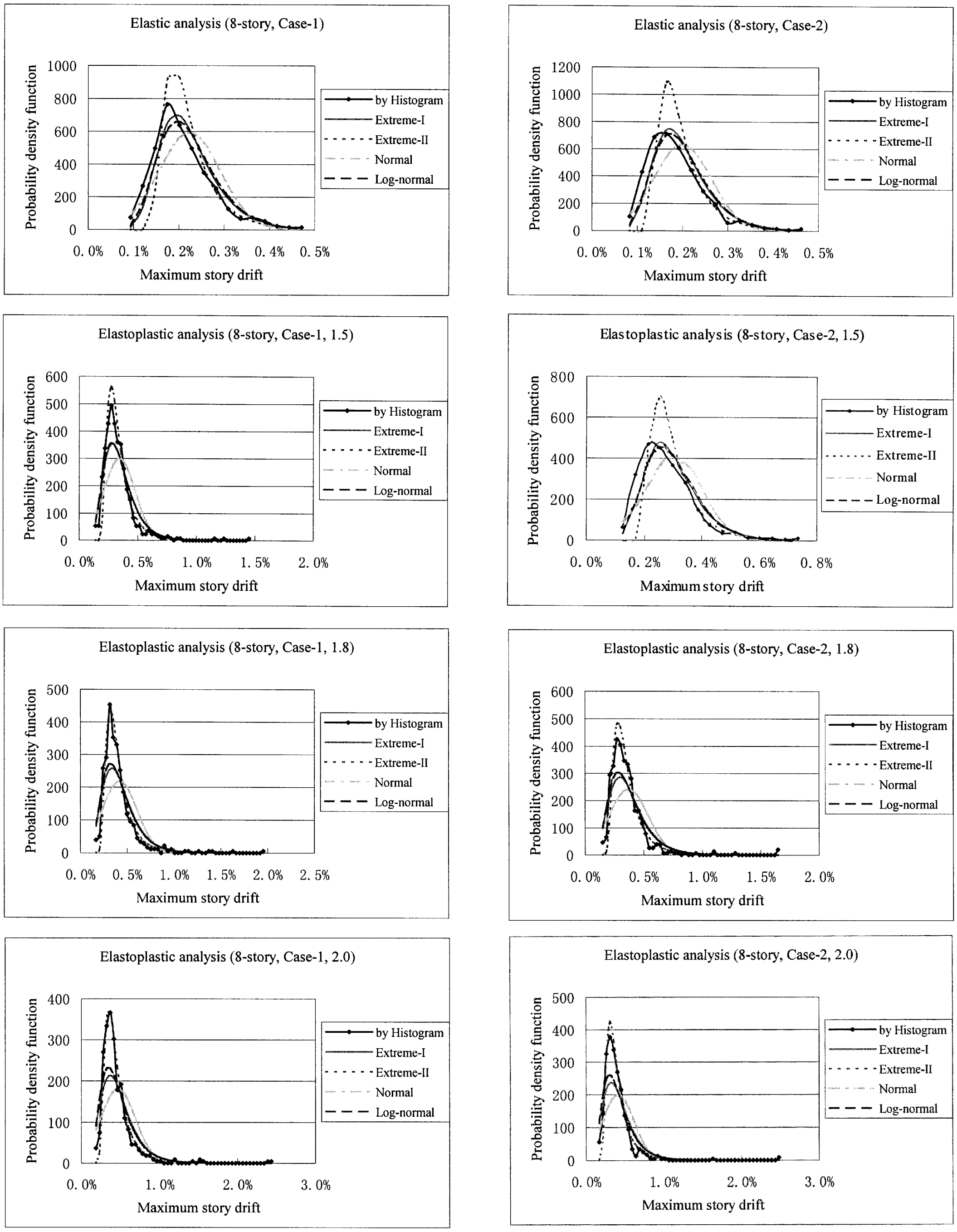


Fig. 3 Different probability density functions of maximum story drift of 8-story steel frame

1999, Li and Cheng 2002). In elastoplastic analysis, the coefficient of variation of maximum story drift is related to the mean, in other words, the number of trials of steel frame samples in elastoplastic stage will affect the value of the coefficient of variation of maximum story

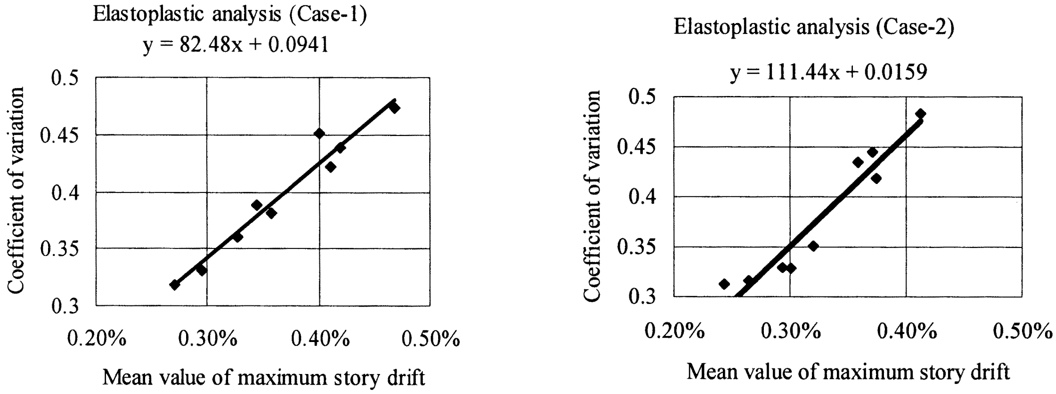


Fig. 4 Relation between mean and coefficient of variation of maximum story drift

drift. When the mean of the maximum story drift becomes large, the coefficient of variation also becomes large. The approximate relation between the mean and the coefficient of variation of the maximum story drift can be obtained by linear regression as follows $y = 82.5x + 0.0941$ (Case-1) and $y = 111x + 0{,}0159$ (Case-2) (x denotes the mean value and y the coefficient of variation of maximum story drift). The data are shown in Tables 9~11 of the results of the elastoplastic analysis, with three couples of data for each frame type of Case-1 and Case-2, respectively.

(3) Ratio of mean to standard value of the maximum story drift (see Tables 9~11). In elastic analysis, the ratio of mean to standard value of the maximum story drift for Case-1 is about 1.06, the same as that of earthquake load; the ratio for Case-2 is about 0.947~0.959 due to the effect of randomness of other variables besides earthquake load. In elastoplastic analysis, the standard value of steel frame is generally in elastic stage, some trials of steel frame samples are in elastoplastic stage and others keep in elastic stage, thus, the ratio of mean to standard value of the maximum story drift has little difference from that of elastic analysis, which are 1.07~1.11 for Case-1 and 0.952~0.983 for Case-2.

6. Conclusions

The present paper studies the statistical properties of the maximum elastoplastic story drift of steel frames considering the randomness of earthquake load, dead load, live load, steel elastic modulus, steel yield strength and structural member dimensions. The results show that the probability distribution of the maximum elastoplastic story drift of steel frames is related to the mean. An extreme value type I distribution is best when the mean is small, and an extreme value type II distribution is best when the mean value is relatively large.

This solution is helpful to develop an efficient approximate approach of reliability analysis with the limit state function in terms of the maximum story drift. For example, in general structural reliability analysis, the limit state function can be written as $f(X, P) = u_p - u(X, P)$ in which u_p is the allowable story drift, $u(X, P)$ is the maximum story drift, which is the high nonlinear and implicit function with respect to the basic random variables X (random variables related to structural properties) and P

(random variables related to external loads), and needs time-consuming FEM reanalysis in each iteration step of reliability analysis. If we obtained the probability distribution and statistical parameters of the maximum story drift of steel frames by only several FEM analysis based on the solution in the present paper, then the limit state function simplifies into the following explicit equation $f(u_p, u_s) = u_p - u_s$, in which u_s is the random variable of the maximum story drift with the known probability distribution and statistical parameters, and thus any available algorithm of reliability analysis can be employed to solve such a simple problem.

It should be pointed out that the solution obtained based on 500 samples in the present paper sometimes may not guarantee the good approximation of the tails of probability distribution of the maximum story drift, which is very important for small probability cases in structural reliability analysis. And the predictions of seismic story drift are based on static analyses and not dynamic, thus, the uncertainty associated with using equivalent static force analysis is not reflected in the results in the present paper. These problems will be studied further in the future.

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