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Behaviour and stability of prestressed steel plate girder for torsional buckling

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Abstract. A higher level of engineering standard in the field of construction, is the use of prestressing in building structures. The concept of prestressing steel structures has only recently been widely considered, despite a long and successful history of prestressing concrete members. Several analytical studies of prestressed steel girders were reported in literatures, but much of the work was not studied with reference to the optimal design and behaviour of the prestressed steel plate girder. A plate girder prestressed eccentrically, will behave as a beam-column, which is subjected to axial compression and bending moment which will cause the beam to buckle out. The study of buckling of the prestressed steel plate girder is necessary for stability criteria. This paper deals with the stability of prestressed steel plate girder using concept of *"Vlasov's Circle of Stability*" under eccentric prestressing force.

Key words: prestressing; plate girder; torsion, buckling, stability; Vlasov's circle.

1. Introduction

The structural efficiency, economy and design flexibility of prestressed steel plate girder can be reflected in their extensive use in engineering work. For the optimal design of a simply supported prestressed steel plate girder (Fig. 1), the flange plates welded to a relatively thin web plate, are normally varied in size and thickness satisfying the bending requirement with the tendons provided below the bottom flange. Introduction of a tendon converts a girder to a statically indeterminate system. Under the service loads the bearing capacity of the girder increases, firstly because the prestresses are cancelled initially, a fact which extends the elastic service range of the material, and secondly because a girder with a tendon behaves as a statically indeterminate beam.

Theory for twisting deformation is explained by Gemmerling (1960) in which he has suggested working method for checking the stability of prestressed beams on the basis of the general theory worked out by Vlasov. The basic theory of bending and torsion of thin-walled elastic beams has been treated in detail in Timoshenko and Gere (1951) from which the basic equations are studied. Bradford (1991) in his paper presents design charts for the elastic buckling load induced by stressing an eccentric

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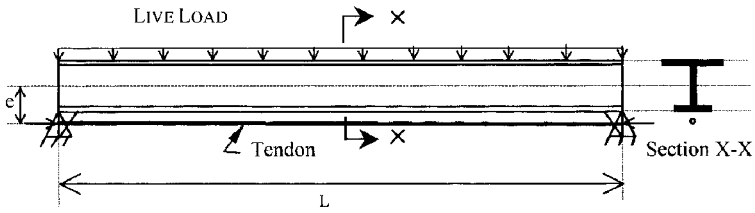

Fig. 1 Straight tendon for full span

tendon, and uses this to obtain a design buckling strength in accordance with the LRFD Specification. Theory of Beam-Column is fully explaind by Chen and Atsuta (1977).

The purpose of this paper is to study the behaviour of prestressed steel girder and to check the optimal designed girder for torsional buckling. The ratio of depth of neutral axis from top & bottom flanges also plays an important role in the optimal design. The depth of web has been assumed equal to effective span by 10. The position of the tendon will also be important to avoid buckling of the plate girder.

2. Location of tendon

Behavior of a girder, its design and economic performance depend to a great degree on the location of the tendon. Tendons may be rectilinear for full or partial span and curved. Tendons may be located within the depth of the girder or built out.

The effectiveness of a tendon increases with its distance to the center of gravity of the girder cross section, and it may be of smaller cross section. Girders with tendons built out at a considerable distance beyond their cross section may be used when there is no limit to the depth of the structure. However, the disadvantage is complicated anchor design and the girder handling. If the tendon is located inside the girder, it gives easier handling of the girder and protection against corrosion. But the demerit is, the location of anchors and jacks, which is a big design problem. However reducing the depth of the girder near the tendon anchoring can solve the problem.

The connection of tendon depends upon the profile of tendon along the length of girder. A straight tendon is simple and convenient as regards tensioning and anchoring as it is connected at the ends and is free all along the length. In case of other profile of tendons, the tendons are connected at specific intervals by means of diaphragms, ribs, clamps and other types of grips which allow longitudinal movement but prevent buckling of girder during prestressing. Curved tendons are advantageous as they create a prestress whose value varies over the length of the girder. A point where a tendon bends creates the vertical components, which additionally ease the stresses in the girder. However, the disadvantages of curved tendons is their complicated manufacturing, as it involves greater consumption of costly high strength material and requires the use of special guides for laying tendons. Friction arises between the tendon and the guide in the course of prestressing, thus increasing the tendon force, all in all, the introduction of curvilinear tendons involves problems in anchor bearing plate design and tensioning device location.

3. Behaviour of prestressed steel plate girder as a beam-column

Due to introduction of prestressing force, the steel girder will be subjected to axial compression, bending moment and shear. Thus, the steel girder will behave as a beam-column, which is under compressive, bending and shear stresses. Columns and beams are special cases of beam-column. A beam-column is likely to buckle under a particular combination of compression and bending moment. In order to determine the strength of a beam-column, it is necessary to obtain a particular combination of loads under elastic and inelastic behaviour of the material and the interaction of flexural and torsion buckling modes complicates the behaviour of beam-column.

To obtain the optimum cross sectional area of the steel girder, the girder will be designed as a beam-column which is subjected to axial compression and the bending moments, and the interaction formula that must be satisfied (as per clause 7.1.1.of I.S. 800-1984) is

$$\frac{\sigma_{ac,\,cal}}{\sigma_{ac}} + \frac{\sigma_{bc,\,cal}}{\left(1 - \dfrac{\sigma_{ac,\,cal}}{0.6 \cdot f_{cc}}\right)} \leq 1 \tag{1}$$

Also, prestressed girders subjected to both shear and bending stresses are proportional accordingly to clause 7.1.4 of IS 800-1984. The equivalent stress $\sigma_{e,cal.}$ obtained from the following formula, shall not exceed the value $\sigma_e = 0.9\, f_y$ is

$$\sigma_{e,\,cal} = \sqrt{\sigma_{bc,\,cal}^2 + 3 \cdot \tau_{vm,\,cal}^2} \tag{2}$$

where, $\sigma_{ac,cal.,}$ $\sigma_{bc,cal.}$ = calculated average axial and bending compressive stressed.

$\sigma_{ac,}$ σ_{bc} = permissible axial and bending compressive stressed.

f_y = yield stress, τ_{vm} = maximum permissible shear stress.

4. Problem formulation

A simply supported prestressed steel girder of I-section is considered for the design. The girder compression flange is assumed to be laterally unsupported. The tendon is provided below the bottom flange to have maximum advantage of prestressing, connected at supports and is free all along the length of the girder. The optimal ratio of depth of neutral axis from top and bottom flanges, to obtain optimum cross-section, is 2.1. The depth of web is taken equal to span/10 with best regard to have economical effectiveness in minimum cross sections.

For the various loading cases and considering the tendon profile straight over full span of the girder, two separate loading conditions are taken into account;

viz. a) Dead Load + Prestressing load, and

b) Dead Load + Live Load + Prestressing load + Self-stressing Load

To reach the required objective the design parameters are assumed and then the girder is designed to obtain the optimized cross section.

4.1. Permissible prestressing force

To obtain the optimum cross-section, the permissible prestressing force is obtained by keeping the prestressing force unknown in the equation and equating it by 1, the permissible prestressing force is calculated by trial & error method for the assumed cross-sectional parameters.

$$\left[\frac{P_{per}/A}{\sigma_{ac}} + \frac{(P_{per}\cdot e - M_{DL})/z_2}{\left(1 - \dfrac{P_{per}/A}{0.6\cdot f_{cc}}\right)\cdot \sigma_{bc}}\right] = 1 \tag{3}$$

∴ *Total prestressing force*

$$P = P_{per} + \Delta P$$

where, M_{DL} = bending moment due to dead weight of plate girder.
e = eccentricity.
Z_2 = section modulous along the bottom flange.
A = cross-sectional area of the plate girder.
P_{per} = permissible prestresseing force.
ΔP = self-stressing force in the tendon due to external load.
f_{cc} = elastic critical stress in compression.
σ_{ac}, σ_{bc} = permissible axial and bending compressive stressed.

For this total prestressing force and assumed cross-section, the combined stresses are checked to the limiting value.

5. Torsional buckling of members

Torsional buckling of a member may occur either by twisting or by combination of bending and twisting. Such torsional buckling failures occurs if torsional rigidity of a section is very low. The cross section may be solid or thin walled section. The thin walled members may be open or closed. Generally, non-circular sections warp when the members get twisted. If the end sections are free to warp, the member is said to be subjected to uniform or pure torsion or St. Venants torsion, resulting in pure shear stresses. If an end section of a member is restrained, it is said to be under non-uniform or warping torsion, resulting in shear and warping stresses.

The assumption that plane section remains plane after deformation is valid for the following:

- Round bars and cylindrical tubes.
- Open sections having two thin rectangles whose lines meet at a point (angles and tees).
- Thin walled hollow sections the resultants of sides that intersect in one point.

All other solid or hollow sections, including rectangular hollow sections of constant wall thickness, rolled steel beams and channels do not remain plain after deformation that is, when the member get twisted the sections do not remain plane, but get warped. When the sections are free to warp, only shear stresses are produced due to torsion, which is called pure or St. Venants torsion. When warping of the section is restrained, the torsion produces shear and warping stresses due to warping torsion.

If the plane section remains plain at the restrained end, the St. Venants torsion constant is taken as the polar moment of inertia of section.

6. Stability of beams in the prestressing range for torsional buckling

A beam-column is likely to buckle under a particular combination of compression and bending moments. Gemmerling (1960) considered the problem of plan shape bending with no allowance for bending and twisting deformation.

The value of the critical force in the tendon, which causes the loss of stability by the beam in twisting deformation, is:

$$N_{cr} = \frac{\frac{\pi^2}{L^2} \cdot E \cdot I_w + G \cdot J}{rd^2 - X_{td}^2 - Y_{td}^2 + \frac{U_x}{I_x} \cdot Y_{td} + \frac{U_y}{I_y} \cdot X_{td}} \tag{4}$$

where, I_x, I_y - axial moment of inertia

I_w and J - warping and torsion constant.

X_{td} and Y_{td} - coordinates of the c.g. of tendon w.r.t. axes x and y.

G - shear modulus, taking $G = 0.4E$

E - Young's modulus for steel, $E = 2.1 \times 10^5$ N/mm^2

r_d - polar radius of gyration.

$$r_d = \sqrt{\frac{I_x + I_y}{A}} \tag{5}$$

$$\left.\begin{aligned} U_x &= \int_A (x^2 + y^2) \cdot y \cdot dA \\ U_y &= \int_A (x^2 + y^2) \cdot x \cdot dA \end{aligned}\right\} \tag{6}$$

where, x and y are the current coordinates of the cross section w.r.t. the principal axes.

It may be shown that for some values of tendon coordinates, the denominator of Eq. (4) turns to zero, and the critical force thus becomes infinitely great, which means that the loss of stability is impossible. Equating the denominator in Eq. (4) to zero, we obtain the expression for a curve characteristic of which is that the tensioning of a tendon located in any point of the curve or beyond it cannot cause the beam to buckle. This curve is Vlasov's circle of stability.

The coordinates of the circle of stability and its radius are determined as below:

$$K_x = X_m; \; K_y = Y_m; \;\; R^2 = K_x^2 + K_y^2 + r_d^2 \tag{7}$$

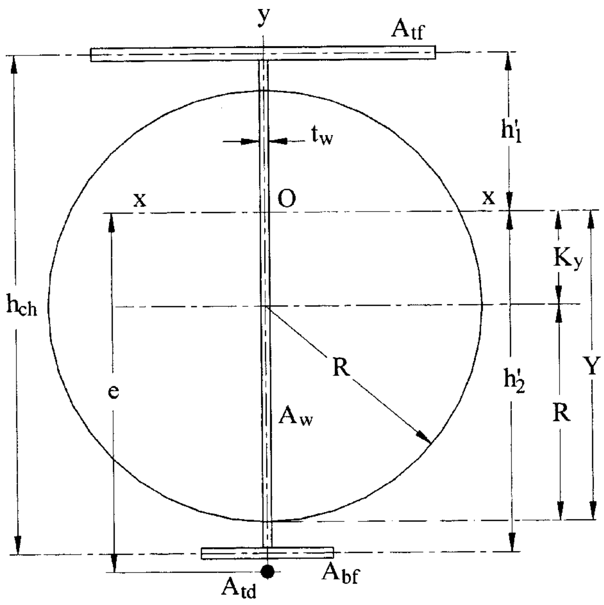

Fig. 2 Beam cross-section with Vlasov's circle of stability

A beam may lose stability if the tendon is located inside the circle of stability only. The minimum value of the critical force will answer to a tendon located in the point of coordinates-

$$X_m = \frac{U_y}{2I_y}; \quad Y_m = \frac{U_x}{2I_x} \tag{8}$$

In this case value of the critical force is

$$N_{cr} = \frac{\frac{\pi^2}{L^2} \cdot E \cdot I_w + G \cdot J}{rd^2 + \frac{U_x}{I_x} \cdot Y_{td} + \frac{U_y}{I_y} \cdot X_{td}} \tag{9}$$

The maximum value of the critical forces will be in a tendon located on the boundary of the circle of stability. When the tendon is located outside the circle of stability, buckling is impossible, since the critical force in the tendon which is required for the girder to buckle is not tensile, but compressive.

Therefore, no checks for buckling are necessary, if a tendon is located at a distance y from the beam center of gravity, as the physical property of a tendon is to be in tension always and not compression. where,

$$y = (K_y \pm R) = \left(\frac{U_x}{2I_x} \pm R\right) \tag{10}$$

thus, to have the prestressed steel plate girder safe for torsional buckling, the tendon should be placed on or outside the circle of stability so that buckling is impossible.

For an I-beam asymmetric cross section, the parameter U_x is

$$U_x = (I_{x1} + I_{y1}) \cdot h_1' + (I_{x2} + I_{y2}) \cdot h_2' + \frac{t_w \cdot h_1'^4}{4} + \frac{t_w \cdot h_2'^4}{4} \tag{11}$$

where, I_{x1}, I_{y1}, I_{x2} and I_{y2} = moments of inertia of top and bottom flanges w.r.t. the principal axis.

h_1' and h_2' = distance between the centre of top and bottom flanges from the centre of gravity of the beam cross section.

Table 1 Critical buckling force in tendon for h_2 / h_1=2.1, e / h_2 = 1.0 for the optimal crosssections

Sr. No.	Span (m)	Load (kN/m)	*C/S* Area (Sq. mm)	Total Prestressing Force P(kN)	Eccen-Tricity (mm)	Dist. *Y* (mm)	N_{CR} (kN)
1	10	60	23270	586.5	749	710	-5310
2	10	70	24500	681.7	749	726	-10900
3	10	80	25770	768.3	749	741	-41500
4	10	90	27100	866.9	752	758	+47900
5	10	100	28330	962.6	752	773	+20200
1	12	60	28180	722	888	855	-6790
2	12	70	29640	834	888	874	-19200
3	12	80	31910	946	888	892	+97900
4	12	90	33460	1060	888	909	+24800
5	12	100	35860	1180	890	924	+21600
1	15	60	36260	928	1089	1069	-16900
2	15	70	39110	1075	1091	1094	+163000
3	15	80	41390	1213	1091	1114	+38000
4	15	90	43400	1368	1091	1133	+28100
5	15	100	47090	1508	1096	1154	+23500
1	18	60	45720	1144	1294	1294	-2047000
2	18	70	49460	1330	1297	1322	+44400
3	18	80	51930	1500	1297	1344	+32600
4	18	90	55630	1679	1300	1368	+25000
5	18	100	58050	1866	1300	1386	+22300
1	20	60	52480	1299	1432	1444	+77900
2	20	70	56700	1507	1434	1477	+37600
3	20	80	59380	1692	1436	1499	+25000
4	20	90	63070	1887	1438	1520	+27000
5	20	100	65660	2087	1437	1538	+27700

Note: -ve sign indicates compressive critical force in tendon and +ve sign indicates tensile critical force in tendon.

6.1. Illustration

Considering steel plate girder with a transverse uniformly distributed load, simply supported, with the compression flange laterally unsupported. As the section is not restrained, the ends are free to warp and the applied torsion is resisted by St. Venant's torsion constant '*J*'. The tendon is located just below the bottom flange along the girder, considering ratio $e / h_2 = 1.0$. The plate girder is symmetric along *y*-axis, and the ratio $h_2 / h_1 = 2.1$ for optimal cross-section.

Therefore, we can take co-ordinates $X_m = 0$ and $X_{td} = 0$, Also, $U_y = 0$

For an unsymmetrical I-section,

Torsion constant,

$$J = \frac{(b_{tf} \cdot t_{tf}^2 + b_{bf} \cdot t_{bf}^3 + h \cdot t_w^3)}{3} \tag{12}$$

Warping constant,

$$I_w = \left(\frac{t_{bf} \cdot h^2}{12}\right) \cdot \left(\frac{b_{tf}^3 \cdot b_{bf}^3}{b_{tf}^3 \cdot b_{bf}^3}\right) \tag{13}$$

where, t_{tf}, t_{bf} - the thickness of top and bottom flange.
b_{tf}, b_{bf} - the width of top and bottom flange.
t_w and h - the thickness and depth of web.
h_1, h_2 - the depth of neutral axis from top and bottom flanges.

Using the above equations from Eqs. (4) to (13), the critical load for a single span girder for various loads and span can be calculated. For uniformly distributed loads, the critical loads are calculated and tabulated in Table 1.

7. Conclusions

The following conclusions are made from the above study.

1) Steel plate girder under external and prestreesed load will behave as a beam-column.

2) Limiting the ratio depth of web / span = 10 and placing the tendon below the bottom flange, a large critical force (Ncr) is required for torsional buckling of girder, making the girder safe against twisting deformation. Hence the concept of Vlasovs circle of stability can be satisfactorily used.

3) To make the girder safe against torsional bucking, the tendon should be placed on the circle of stability or beyond it, even if the girder is not restrained.

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References

Belenya, E. (1977), *Prestressed Load-Bearing Metal Structures*, Mir Publishers, Moscow.

Gemmerling, A.V. (1960), Ob Vstoichivosti predvaritelno-napry a zhennykh balok (*on stability of prestrssed beams.*), "stroitelnaya mekhanika I raschet sooruzheniy", 1960 No. 3.

IS 800 (1984), *Code of Practice for General Construction in Steel*. Indian Standards Institution, New Delhi.

Bradford, Mark A. (1991), "Buckling of prestressed steel girders", *Engineering journal of American Institute of Steel Construction*, **28**, Third quarter 1991, 98-101.

Timoshenko, S.P. and Gere, J.M. (1961), *Theory of Elastic Stability*, McGraw-Hill, Inc.

Troitsky, M.S. and Rabbani, N.F. (1987), "Tendon configurations of prestressed steel girder bridges", *Proceedings of Centennial Conference, Canadian Society of Civil Engineering*, Montreal, Quebec, Canada, 172 182.

Chen, Wai-Fah and Atsuta, Toshi (1977), *Theory of Beam-Columns*, McGraw-Hill, Inc.

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