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# Free vibration of symmetrically laminated quasi-isotropic super-elliptical thin plates

Erkin Altunsaray*

*Institute of Marine Science and Technology, Dokuz Eylul University, Baku Bulvari 100, TR-35340, İzmir, Turkey*

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**Abstract.** Free vibration analysis of super-elliptical composite thin plates was investigated. Plate is formed by symmetrical quasi-isotropic laminates. Rayleigh-Ritz method was used for parametric analysis based on the governing differential equations of Classical Laminated Plate Theory (CLPT). Simply supported and clamped boundary conditions at the periphery of plates were considered. Parametric study was performed for the effect of different lamination type, aspect ratio, thickness and super-elliptical power on natural frequencies. Convergence study and validation of isotropic case were achieved. A number of design parameters like different dimensions, structure systems, panel sizes, panel thicknesses, lamination sequences, boundary conditions and loading conditions must be considered in the production of composite ships. The number of possible combinations practically may be so high that a parametric study should be carried out in order to determine the optimum design parameters rapidly during the preliminary design stage. The use of Rayleigh-Ritz method could make this parametric study possible. Thereby it might be decreasing the consumption of time, material and labor. Certain results for some different super-elliptical powers presented in tabulated form in Appendix for designers as well.

**Keywords:** super-elliptical composite thin plates; free vibration; parametric study; Rayleigh-Ritz method

## 1. Introduction

Composite thin plates are widely used as structural elements of ships, aircrafts and civil engineering applications. Applications of composite materials in marine, transportation and aerospace industries were given by Mouritz *et al*. (2001), Eric Green Associates (1999) and Jones (1999). Ship hull and its supporting structural elements have to be designed to avoid from resonance vibrations, which are mainly originated from propeller, main and auxiliary engines during operating conditions (TL Rules 2017).

Vibration of plates with various geometry (rectangular, circular and elliptical) has been studied by hundreds of researchers and given in some review articles (Kreja 2011, Sayyad and Ghugal 2015, Kumar 2018). A few of them were mentioned in this study because of the restriction of this paper. Vibration of rectangular laminated composite plates has been examined by Bui and Nguyen (2011), Singhatanadgid and Wetchayanon (2014) and Sadoune *et al*. (2014). Bui and Nguyen (2011) presented a novel meshfree model for buckling and vibration of rectangular orthotropic plates. Singhatanadgid and Wetchayanon studied free vibration analyses of laminated composite and isotropic plate using the extended Kantorovich method and Finite Element Method (2014). A novel first order shear deformation theory (FSDT) for laminated antisymmetric cross-ply and angle-ply composite rectangular plate was developed by Sadoune *et al*. (2014). One of the experimental studies about vibration of composite rectangular plates was employed by Mishra and Sahu (2012); another was achived by Nayak *et al*. (2013). Kim (2003), Nallim and Grossi (2008) and Afsharmanesh *et al*. (2014) analysed vibration of laminated composite elliptical or circular plates. Kim (2003) studied natural frequency of elliptical orthotropic (glass/epoxy, boron/epoxy, carbon/epoxy and kevlar/epoxy) plates. Nallim and Grossi (2008) investigated natural frequencies of elastically restrained solid and annular cross-ply and angle-ply elliptical laminated plates based on Rayleigh-Ritz Method. Afsharmanesh *et al*. (2014) dealt with buckling and vibration of laminated ange-ply plates on winkler-type foundation problem and solved the problem by Ritz Method based on CLPT. Ghaheri *et al*. (2014) analyzed buckling and vibration of thin, symmetrically laminated, elliptical angle-ply composite plate under initial in-plane edge loads and resting on a Winkler-type elastic foundation based on CLPT using by Ritz energy method. In this study authors indicated that critical buckling load and natural frequencies are affected by the aspect ratio, fiber orientation, layup sequence, in-plane load and foundation parameter. Authors compared their results with FEM and existing results and they obtained good convergence.

Super-elliptical plate has a geometry between an ellipse and a rectangle, which enables the structure to diffuse and to dilute the stress at the corner of rectangular plates (Liew *et al*. 1998). It is this advantage making them preferred in engineering applications such as naval and aerospace industries. However, static and dynamic analysis of super-elliptical plates has been dealt with in academic fields for

*Corresponding author, Ph.D., E-mail: erkin.altunsaray@deu.edu.tr


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last few decades. One of the early studies was performed by Irie *et al*. (1983). They studied naturel frequencies of square membrane and square isotropic plates with rounded corners. A number of researchers studied vibration of super-elliptical thin isotropic plates based on Kirchhoff–Love plate theory using Rayleigh- Ritz method (Wang *et al*. 1994, Çeribaşı 2009, Zhangh and Zhou 2014). There dimensional free vibration analyses of super-elliptical isotropic plates using Chebyshev–Ritz Method have been studied by Zhou *et al*. (2004). Liew and Feng (2001) carried out 3-D vibration analysis of perforated type super-elliptical isotropic plates via p-Ritz method. Çeribaşı (2012) investigated static and dynamic analysis of super-elliptical FGM (functionally graded materials) based on Kirchhoff plates by Galerkin Method and Ritz Method.

Altekin (2017) examined free transverse vibration of shear deformable super-elliptical moderately thick plates based on Mindlin plate theory by Finite Element Method (FEM). With this work, the author contributed to the literature large amount of data about influence of boundary condition, super-elliptical power, aspect ratio and thickness of the plate on the frequency parameter of shear deformable super-elliptical plates.

A few studies about special orthotropic, cross-ply and angle-ply super-elliptical plates are available in literature (Lim *et al*. 1998, Chen *et al*. 1999, Altekin 2009, Ghaheri *et al*. 2016). Altekin (2009) investigated free vibration of orthotropic super-elliptical plates on intermediate supports by Ritz method based on Kirchhoff–Love plate theory. The results were compared with those in the open source for various plate shapes and different boundary conditions.

The importance of using the quasi-isotropic plates in practice is given by Altunsaray and Bayer (2013). In today's maritime industry the composite structure system, which is formed by the combination of thermosetting resins and multi-axial fibers, is widely applied. Due to the limitations in material production, the fibers commonly used in the structure are at 0°, + 45°, -45° and 90° angles. In symmetric layered structure, the angles of the layers are symmetrical with respect to the middle axis and the plate remains flat despite the thermal stresses occurring during production. As the number of layers increases, the angle between the adjacent laminae decreases such that $\Delta\theta = 180°/N$ where N is the number of layers. For instance; [-45°/0°/45°/90°] ≡ $\pi/4$. Shear modulus of the quasi-isotropic plates consisting of 0°, +45°,-45° and 90° angles is greater than that of cross-ply plates. Moreover, it is stated that quasi-isotropic structure is preferred with few exceptions in the aircraft in NASA [Aran 1990, Jones 1999, Harper 2002]. Therefore, the free vibration of the symmetrically laminated quasi-isotropic super-elliptical plates composed of 24 different combinations of 0°, +45°, -45° and 90° angles was investigated. Moreover, it is chosen because present investigation for this particular configuration of layups has not previously been found in the literature. The authors investigated deflection and free vibration of symmetrically laminated quasi-isotropic thin rectangular plates based on Classical Laminated Plate Theory using some weighted residual methods and Method of Finite Elements.

Recently, static deflections of symmetrically laminated quasi-isotropic super-elliptical thin plates have been presented by using Rayleigh-Ritz method (Altunsaray 2017). Now this particular study is extended to the free vibration of quasi-isotropic super-elliptical plates, because it seems that it has not yet been published in the literature.

The main motivation of this paper is to attempt to fill the gap, which is the free vibration of symmetrically laminated quasi-isotropic super-elliptical thin plates, in the literature. It is also aimed to present practical data for designers involved in concept design stage of composite ships. In this study, the above named problem based on a Classical Laminated Plate Theory was investigated using Rayleigh-Ritz method. The effect of some parameters such as lamination type, super-elliptical power ($n$), aspect ratio ($a/b$ and $b/a$) and boundary condition (clamped and simply supported) on natural frequencies of symmetrically laminated quasi-isotropic super-elliptical thin plates was studied. Validation study was performed for isotropic plate because it was only available in open literature.

As mentioned in the above paragraphs, the novelty of this paper is the investigation of the free vibration frequencies of quasi-isotropic super-elliptical thin plates which still seems to be lacking in the literature. By this way, at the preliminary design stage of composite ships, which is quite complicated because there are a number of parameters like different geometries, boundary conditions, thicknesses, lamination sequences, etc., possible better alternative design options may be obtained. Convergence study of free vibrations of LT1 ($[-45_2/0_2/45_2/90_2]s$) plate was carried out up to 10 terms to obtain reasonable accuracy. In this parametric study, some selected results for different parameters such as lamination types, boundary conditions, super-elliptical powers and aspect ratios were given with tables.

## 2. Analysis

### 2.1 Main parameters

Geometry looking like a rectangular plate with rounded corners is named super-elliptical plate (Fig. 1).

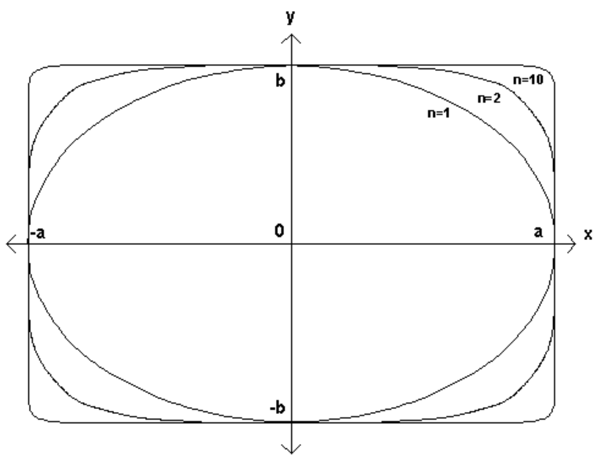


Fig. 1 Geometry of super-elliptical plate in the Cartesian co-ordinates

$$\frac{x^{2n}}{a^{2n}} + \frac{y^{2n}}{b^{2n}} = 1, \qquad n = 1, 2..\infty \tag{1}$$

$a$ and $b$ are the half lengths of the plate and $n$ is the power of the super ellipse. If $n$ is equal to 1, shape is an ellipse, while n goes to infinity (∞) the shape becomes a rectangle.

Material properties of carbon/epoxy are given in Table 1 (Tsai 1988). Aspect ratios are given in Table 2.

In this study, twenty four different types of lamination of plates are shown in Table 3. Each plate consists of four different stacking sequences (−45°, 0°, 45° and 90°). Thickness of a laminate ($t$) is equal to 0.2 mm and total thickness of a plate is 3.2 mm (each plate contains 16 laminates).

### 2.2 Method of solution

Rayleigh-Ritz Method used for calculation based on Classical Laminated Plate Theory (CLPT) in this study. The use of CLPT for this study is considered appropriate, since the ratio of the plate thickness to the length of short edge is sufficiently small. Main assumptions of CLPT were given in (Altunsaray 2017).

Table 1 Material properties of T300-934 coded carbon/epoxy, Tsai (1988)

| | |
|---|---|
| Longitudinal Young Modulus ($E_{11}$) | 148×10$^9$ (N/m$^2$) |
| Transversal Young Modulus ($E_{22}$) | 9.65×10$^9$ (N/m$^2$) |
| Longitudinal Shear Modulus ($G_{12}$) | 4.55×10$^9$ (N/m$^2$) |
| Longitudinal Poisson ratio ($v_{12}$) | 0.3 |
| Laminate thickness ($t$) | 0.185×10$^{-3}$ – 0.213×10$^{-3}$ (m) |
| Density ($\rho_0$) | 1.5×10$^3$ (kg/m$^3$) |

Table 2 Aspect ratios

| $a/b$ | 1 | 1.2 | 1.4 | 1.6 | 1.8 | 2 |
|---|---|---|---|---|---|---|
| $b/a$ | 1 | 1.2 | 1.4 | 1.6 | 1.8 | 2 |

Table 3 Symmetrically laminated quasi-isotropic super-elliptical plate types

| | | | |
|---|---|---|---|
| LT1 | $[-45_2/0_2/45_2/90_2]_s$ | LT13 | $[45_2/-45_2/0_2/90_2]_s$ |
| LT2 | $[-45_2/0_2/90_2/45_2]_s$ | LT14 | $[45_2/-45_2/90_2/0_2]_s$ |
| LT3 | $[-45_2/45_2/0_2/90_2]_s$ | LT15 | $[45_2/0_2/-45_2/90_2]_s$ |
| LT4 | $[-45_2/45_2/90_2/0_2]_s$ | LT16 | $[45_2/0_2/90_2/-45_2]_s$ |
| LT5 | $[-45_2/90_2/0_2/45_2]_s$ | LT17 | $[45_2/9_{02}/-45_2/0_2]_s$ |
| LT6 | $[-45_2/90_2/45_2/0_2]_s$ | LT18 | $[45_2/90_2/0_2/-45_2]_s$ |
| LT7 | $[0_2/-45_2/45_2/90_2]_s$ | LT19 | $[90_2/-45_2/0_2/45_2]_s$ |
| LT8 | $[0_2/-45_2/90_2/45_2]_s$ | LT20 | $[90_2/-45_2/45_2/0_2]_s$ |
| LT9 | $[0_2/45_2/-45_2/90_2]_s$ | LT21 | $[90_2/0_2/-45_2/45_2]_s$ |
| LT10 | $[0_2/45_2/90_2/-45_2]_s$ | LT22 | $[90_2/0_2/45_2/-45_2]_s$ |
| LT11 | $[0_2/90_2/-45_2/45_2]_s$ | LT23 | $[90_2/45_2/-45_2/0_2]_s$ |
| LT12 | $[0_2/90_2/45_2/-45_2]_s$ | LT24 | $[90_2/45_2/0_2/-45_2]_s$ |

The strain energy ($U$) of the symmetrically laminated plate is given below (Eq. (2)) (Reddy 2004)

$$U = \frac{1}{2}\int_{-a}^{a}\int_{-b\sqrt[2n]{1-\frac{x^{2n}}{a^{2n}}}}^{b\sqrt[2n]{1-\frac{x^{2n}}{a^{2n}}}} \begin{bmatrix} D_{11}(\frac{\partial^2 w}{\partial x^2})^2 + 2D_{12}(\frac{\partial^2 w}{\partial x^2})(\frac{\partial^2 w}{\partial y^2}) \\ +4D_{66}(\frac{\partial^2 w}{\partial x \partial y})^2 + D_{22}(\frac{\partial^2 w}{\partial y^2})^2 \\ +4D_{16}(\frac{\partial^2 w}{\partial x^2})(\frac{\partial^2 w}{\partial x \partial y}) \\ +4D_{26}(\frac{\partial^2 w}{\partial y^2})(\frac{\partial^2 w}{\partial x \partial y}) \end{bmatrix} dxdy \tag{2}$$

$w$ is deflection function, the elements of bending stiffness matrix $D_{ij}$ are calculated as given in (Reddy 2004)

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{N}\overline{Q}_{ij}^{k}(z_{k+1}^3 - z_k^3) \tag{3}$$

The members of transformed reduced stiffness matrix ($\bar{Q}_{ij}$) are calculated for each lamina as given below (Eq. (4)). $N$ is the total number of plies in the laminate and $z_k$ and $z_{k\text{-}1}$ are the distance from the reference plane to the two surfaces of the $k_{\text{th}}$ lamina (Reddy 2004). Coordinate locations of plies in a typical laminated plate is demonstrated in Fig. 2.

$$\begin{aligned} \overline{Q}_{11} &= Q_{11}c^4(\theta) + 2(\theta_{12} + 2\theta_{66})s^2(\theta)c^2(\theta) \\ &\quad + Q_{22}s^4(\theta) \\ \overline{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66}s^2(\theta) + c^2(\theta) \\ &\quad + Q_{12}(s^4\theta) + c^4(\theta)) \\ \overline{Q}_{22} &= Q_{11}s^4(\theta) + 2(Q_{12} + 2Q_{66})s^2(\theta)c^2(\theta) \\ &\quad + Q_{22}c^4(\theta) \\ \overline{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})s(\theta)c^3(\theta) \\ &\quad + (Q_{11} - Q_{22} + 2Q_{66})s^3(\theta)c(\theta) \\ \overline{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})s^3(\theta)\cos(\theta) \\ &+ (Q_{11} - Q_{22} + 2Q_{66})s(\theta)c^3(\theta) \\ \overline{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})s^2(\theta)c^2(\theta) \\ &\quad + Q_{66}(s^4(\theta)c^4(\theta)) \end{aligned} \tag{4}$$

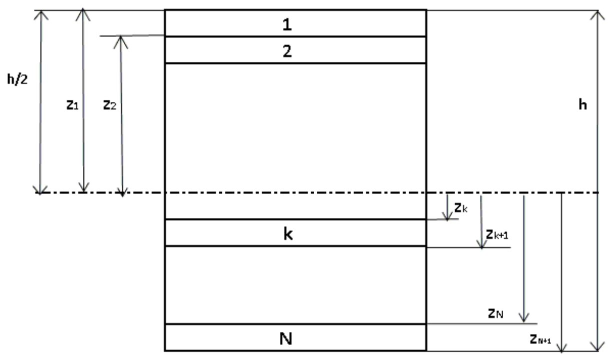


Fig. 2 Coordinate locations of plies in a typical laminated plate

where $s$ is sine, $c$ is cosine and $\theta$ is the angle of the lamina. The reduced stiffness matrix elements $(Q_{ij})$ are determined as follows

$$
\begin{aligned}
Q_{11} &= E_{11}/(1-\nu_{12}\nu_{21}),\\
Q_{12} &= \nu_{12}E_{11}/(1-\nu_{12}\nu_{21}),\\
Q_{22} &= E_{22}/(1-\nu_{12}\nu_{21}),\\
Q_{66} &= G_{12},
\end{aligned} \tag{5}
$$

The kinetic energy ($T$) of the symmetrically laminated plate is given (Reddy 2004)

$$T = \frac{1}{2}\omega^2 \int_{-a}^{a}\int_{-b\sqrt[2n]{1-\frac{x^{2n}}{a^{2n}}}}^{b\sqrt[2n]{1-\frac{x^{2n}}{a^{2n}}}} I_0 w^2 dxdy \tag{6}$$

Where $\omega$ is natural angular frequency and I0 is mass moments of inertia as calculated as given in Eq. (7) (Reddy 2004).

$$I_0 = \sum_{k=1}^{N} \rho_0^{(k)}(z_{k+1} - z_k) \tag{7}$$

The total potential energy functional is given below (Eq. (8))

$$F = \mathrm{U} + \mathrm{T} \tag{8}$$

Substituting Eqs. (2) and (6) into Eq. (8), the total potential energy is given below (Eq. (9))

$$
\begin{aligned}
F = \frac{1}{2}\int_{-a}^{a}\int_{-b\sqrt[2n]{1-\frac{x^{2n}}{a^{2n}}}}^{b\sqrt[2n]{1-\frac{x^{2n}}{a^{2n}}}}
&\begin{bmatrix}
D_{11}(\frac{\partial^2 w}{\partial^2 x})^2 + 2D_{12}(\frac{\partial^2 w}{\partial^2 x})(\frac{\partial^2 w}{\partial^2 y}) \\
+4D_{66}(\frac{\partial^2 w}{\partial x \partial y})^2 + D_{22}(\frac{\partial^2 w}{\partial^2 y})^2 \\
+4D_{16}(\frac{\partial w}{\partial x})^2(\frac{\partial^2 w}{\partial x \partial y}) \\
+4D_{26}(\frac{\partial w}{\partial y})^2(\frac{\partial^2 w}{\partial x \partial y})
\end{bmatrix} dxdy \\
&- \frac{1}{2}\omega^2 \int_{-a}^{a}\int_{-b\sqrt[2n]{1-\frac{x^{2n}}{a^{2n}}}}^{b\sqrt[2n]{1-\frac{x^{2n}}{a^{2n}}}} I_0 w^2 dxdy
\end{aligned} \tag{9}
$$

The boundary conditions at the plate edges are given as follows

$$w = 0 \quad \text{and} \quad \frac{\partial w}{\partial n_i} = 0 \quad \text{(for clamped)} \tag{10}$$

$$w = 0 \quad \text{and} \quad M = 0 \quad \text{(for simply supported)} \tag{11}$$

where $M$ denotes bending moment, $\partial n_i$ is the outward normal of the boundary. Trial function used is given below

$$\phi_{mn} = \begin{pmatrix} c_{00} + c_{02}y^2 + c_{20}x^2 + c_{22}x^2y^2 + c_{24}x^2y^4 \\ + c_{04}y^4 + c_{40}x^4 + c_{42}x^4y^2 + c_{06}y^6 + c_{60}x^6 \end{pmatrix} \tag{12}$$

where are the unknown coefficients and order of polynomial ($r$) is 6.

Deflection function which satisfies the boundary conditions is given below

$$w(x,y) = \left(\frac{x^{2n}}{a^{2n}} + \frac{y^{2n}}{b^{2n}} - 1\right)^p \phi_{mn} \tag{13}$$

where $p$ is equal to 1 for simply supported condition, 2 for clamped condition.

In order to find a least upper bound on the frequency, Eq. (9) is minimized with respect to the coefficients $c_{ij}$

$$\frac{\partial F}{\partial c_{ij}} = 0 \tag{14}$$

Then, the following equation is obtained

$$[K - \lambda^2 M]\{c_{ij}\} = 0 \tag{15}$$

where $\lambda^2$ is the fundamental frequency parameter including material properties, characteristic dimensions and natural angular frequency of the plate. $K$ is the stiffness matrix related with the strain energy and $M$ is the mass matrix related to kinetic energy. $K$ and $M$ are symmetric matrices both of which are of the order ten (at most) and $M$ is positive definite. This is a generalized eigenvalue problem. For a non-trivial solution, the determinant of the coefficient matrix should be equal to zero

$$|K - \lambda^2 M| = 0 \tag{16}$$

Solution of Eq. (16) leads to a characteristic equation involving a polynomial of tenth degree in $\lambda^2$, from which the fundamental natural frequency ($\omega$) may be found.

Table 4 Comparison of the fundamental frequency parameter of super-elliptical isotropic plates ($\lambda^2 = \omega b^2\sqrt{\rho h/D}$, $\nu$ = 0.3, $r$ = 6)

| $n$ | $a/b$ | Simply Supported | | Clamped | |
|---|---|---|---|---|---|
| | | Present | Çeribaşı *et al.* (2009) | Present | Çeribaşı *et al.* (2009) |
| 1 | 1 | 4.93515 | 4.9351 | 10.2158 | 10.2158 |
| | 1.2 | 4.21276 | 4.2128 | 8.71781 | 8.7178 |
| | 1.4 | 3.81495 | 3.8149 | 7.89103 | 7.8910 |
| | 1.6 | 3.57431 | 3.5743 | 7.39275 | 7.3928 |
| | 1.8 | 3.41583 | - | 7.06884 | - |
| | 2 | 3.30336 | 3.3034 | 6.84436 | 6.8444 |
| 10 | 1 | 5.17031 | 5.181 | 9.25884 | 9.3763 |
| | 1.2 | 4.3798 | 4.3883 | 7.90807 | 8.0022 |
| | 1.4 | 3.90019 | 3.908 | 7.17222 | 7.2459 |
| | 1.6 | 3.58665 | 3.5933 | 6.73597 | 6.7934 |
| | 1.8 | 3.36836 | - | 6.45755 | - |
| | 2 | 3.21059 | 3.2158 | 6.27002 | 6.3068 |

Table 5 Convergence study of free vibrations of LT1 ($[-45_2/0_2/45_2/90_2]_s$ ) plate with increasing terms

| Shape function | Fundamental natural frequency $\omega$ (Hz) | | | |
|---|---|---|---|---|
| | Simply Supported | | Clamped | |
| | $n$ = 1 | $n$ = 10 | $n$ = 1 | $n$ = 10 |
| $c_{00}$ | 533.190 | 8371.87 | 979.911 | 10152.500 |
| $c_{00}+c_{20}x^2$ | 495.309 | 4796.28 | 975.220 | 5910.770 |
| $c_{00}+c_{20}x^2+c_{02}y^2$ | 470.594 | 2444.73 | 968.730 | 3435.320 |
| $c_{00}+c_{20}x^2+c_{02}y^2+c_{22}x^2y^2$ | 470.098 | 789.967 | 968.704 | 1602.380 |
| $c_{00}+c_{20}x^2+c_{02}y^2+c_{22}x^2y^2+c_{40}x^4$ | 470.010 | 716.840 | 968.673 | 1367.940 |
| $c_{00}+c_{20}x^2+c_{02}y^2+c_{22}x^2y^2+c_{40}x^4+c_{04}y^4$ | 469.882 | 645.946 | 968.601 | 1215.490 |
| $c_{00}+c_{20}x^2+c_{02}y^2+c_{22}x^2y^2+c_{24}x^2y^4+c_{40}x^4+c_{04}y^4$ | 469.882 | 619.138 | 968.601 | 1144.540 |
| $c_{00}+c_{02}x^2+c_{20}y^2+c_{22}x^2y^2+c_{24}x^2y^4+c_{40}x^4+c_{04}y^4+c_{42}x^4y^2$ | 469.881 | 516.067 | 968.601 | 873.786 |
| $c_{00}+c_{02}x^2+c_{20}y^2+c_{22}x^2y^2+c_{24}x^2y^4+c_{40}x^4+c_{04}y^4+c_{42}x^4y^2+c_{60}x^6$ | 469.881 | 512.302 | 968.601 | 870.714 |
| $c_{00}+c_{02}x^2+c_{20}y^2+c_{22}x^2y^2+c_{24}x^2y^4+c_{40}x^4+c_{04}y^4+c_{42}x^4y^2+c_{60}x^6+c_{06}y^6$ | 469.880 | 510.757 | 968.601 | 869.746 |

## 3. Results and discussion

In this section, natural frequencies of isotropic and quasi-isotropic super-elliptical plates computed by Rayleigh-Ritz method are given.

Convergence study, effect of super-elliptical power ($n$) variation, thickness, boundary condition, aspect ratio and super elliptical power on natural frequencies are also included.

### *3.1 Validation of the isotropic case*

For validation, fundamental natural frequency parameters of super-elliptical thin isotropic plates were compared with the results of Çeribaşı *et al*. (2009) given in Table 4. It seems that the results are in good agreement with each other. It can be also seen from the results that fundamental frequency parameter decreases with the increase of the aspect ratio ($a/b$) and super-elliptical power (Selected trial functions were given in Eq. (12).).

### *3.2 Convergence study of free vibrations of symmetrically laminated quasi-isotropic super-elliptical plates*

Convergence study was carried out for free vibrations of simply supported and clamped LT1 ($[-45_2/0_2/45_2/90_2]_s$) plate with different shape function (number of terms from 1 to 10) and super elliptical power ($n$ = 1 and 10). From the results when the number of terms reach 10 ($r$ = 6), accuracy seems to be enough for simply supported and clamped conditions (Table 5).

### *3.3 Effect of super-elliptical power (n) variation on natural frequencies (Hz) of super-elliptical plates*

In this study, the natural frequencies of super-elliptical plate LT1 ($[-45_2/0_2/45_2/90_2]_s$) are compared with the natural frequencies of circular ($n$ = 1, $a/b$ = 1), elliptical ($n$ = 1, $a/b$ = 1.2, 1.4, 1.6, 1.8 and 2) and super-elliptical plates ($n$ = 2, 4, 6, 8 and 10) given in Tables 6-7. The natural frequencies ($\omega$) decrease with the increase of the aspect ratios ($a/b$, $b/a$). It can be seen that the natural frequencies for clamped boundary conditions cases are higher than the ones for simply supported boundary conditions.

### *3.4 Effect of thickness variation on natural frequencies (Hz) of super-elliptical plates*

Selected three different thicknesses, aspect ratios ($a/b$ and $b/a$), boundary conditions (simply supported and clamped) and super-elliptical powers ($n$ = 1 and 10) on natural frequency of super-elliptical plates were examined in this section and presented (Tables 8-11).

From the results natural frequency decreases as the

Table 6 Fundamental natural frequencies $\omega$ (Hz) of super-elliptical LT1 ($[-45_2/0_2/45_2/90_2]_s$) plate (simply supported)

| | Present - Rayleigh-Ritz ($r$ = 6) | | | | | |
|---|---|---|---|---|---|---|
| $a/b$ | $n$ = 1 | $n$ = 2 | $n$ = 4 | $n$ = 6 | $n$ = 8 | $n$ = 10 |
| 1.0 | 469.88 | 453.51 | 477.99 | 490.25 | 500.23 | 510.75 |
| 1.2 | 383.43 | 371.74 | 392.85 | 403.27 | 411.59 | 420.23 |
| 1.4 | 333.70 | 322.46 | 340.12 | 348.92 | 356.06 | 363.53 |
| 1.6 | 302.86 | 290.50 | 305.01 | 312.45 | 318.67 | 325.43 |
| 1.8 | 282.49 | 268.60 | 280.40 | 286.69 | 292.16 | 298.42 |
| 2.0 | 268.30 | 252.93 | 262.45 | 267.78 | 272.60 | 278.35 |
| $b/a$ | $n$ = 1 | $n$ = 2 | $n$ = 4 | $n$ = 6 | $n$ = 8 | $n$ = 10 |
| 1.0 | 469.88 | 453.51 | 477.99 | 490.25 | 500.23 | 510.75 |
| 1.2 | 415.91 | 396.75 | 415.08 | 424.71 | 432.95 | 442.16 |
| 1.4 | 384.78 | 362.54 | 376.03 | 383.60 | 390.49 | 398.73 |
| 1.6 | 364.91 | 340.33 | 350.09 | 356.09 | 361.86 | 369.26 |
| 1.8 | 351.15 | 325.08 | 331.98 | 336.78 | 341.62 | 348.31 |
| 2.0 | 340.99 | 314.14 | 318.85 | 322.72 | 326.80 | 332.80 |

Table 7 Fundamental natural frequencies $\omega$ (Hz) of super-elliptical LT1 ($[-45_2/0_2/45_2/90_2]_s$) plate (clamped)

| Present - Rayleigh-Ritz ($r$ = 6) | | | | | | |
|---|---|---|---|---|---|---|
| $a/b$ | $n$ = 1 | $n$ = 2 | $n$ = 4 | $n$ = 6 | $n$ = 8 | $n$ = 10 |
| 1.0 | 968.60 | 859.53 | 852.43 | 857.13 | 863.08 | 869.75 |
| 1.2 | 790.48 | 700.28 | 694.67 | 698.57 | 703.54 | 709.08 |
| 1.4 | 687.90 | 610.15 | 605.16 | 608.51 | 612.76 | 617.54 |
| 1.6 | 624.26 | 555.33 | 550.56 | 553.49 | 557.19 | 561.34 |
| 1.8 | 582.32 | 520.01 | 515.27 | 517.86 | 521.10 | 524.74 |
| 2.0 | 553.22 | 496.15 | 491.35 | 493.65 | 496.48 | 499.65 |
| $b/a$ | $n$ = 1 | $n$ = 2 | $n$ = 4 | $n$ = 6 | $n$ = 8 | $n$ = 10 |
| 1.0 | 968.60 | 859.53 | 852.43 | 857.13 | 863.08 | 869.75 |
| 1.2 | 857.30 | 764.43 | 757.61 | 761.51 | 766.41 | 771.92 |
| 1.4 | 793.42 | 711.79 | 704.88 | 708.16 | 712.20 | 716.73 |
| 1.6 | 753.19 | 680.05 | 672.96 | 675.72 | 679.02 | 682.70 |
| 1.8 | 725.93 | 659.58 | 652.30 | 654.64 | 657.33 | 660.28 |
| 2.0 | 706.35 | 645.66 | 638.22 | 640.22 | 642.40 | 644.75 |

Table 8 Fundamental natural frequency $\omega$ (Hz) of different thinner or thicker super-elliptical plates (simply supported, $r$ = 6, $n$ = 1)

| $a/b$ | Fundamental natural frequency $\omega$ (Hz) | | |
|---|---|---|---|
| | $[-45_2/0_2/45_2/90_2]_s$ | $[-45_3/0_3/45_3/90_3]_s$ | $[-45_4/0_4/45_4/90_4]_s$ |
| | $t$ = 3.2 mm | $t$ = 4.8 mm | $t$ = 6.4 mm |
| 1 | 469.880 | 704.820 | 939.701 |
| 1.2 | 383.432 | 575.148 | 766.815 |
| 1.4 | 333.700 | 500.551 | 667.358 |
| 1.6 | 302.860 | 454.29 | 605.681 |
| 1.8 | 282.494 | 423.741 | 564.951 |
| 2 | 268.297 | 402.445 | 536.557 |
| $b/a$ | Fundamental natural frequency $\omega$ (Hz) | | |
| | $[-45_2/0_2/45_2/90_2]_s$ | $[-45_3/0_3/45_3/90_3]_s$ | $[-45_4/0_4/45_4/90_4]_s$ |
| | $t$ = 3.2 mm | $t$ = 4.8 mm | $t$ = 6.4 mm |
| 1 | 469.880 | 704.820 | 939.701 |
| 1.2 | 415.912 | 623.868 | 831.77 |
| 1.4 | 384.775 | 577.163 | 769.499 |
| 1.6 | 364.908 | 547.362 | 729.763 |
| 1.8 | 351.151 | 526.726 | 702.249 |
| 2 | 340.986 | 511.478 | 681.917 |

aspect ratio increases, while it increases with the increase in thickness. It seems that the natural frequencies for simply supported plates are lower than clamped plates at all aspect ratios. Natural frequencies increase with the increase of the super-elliptical power ($n$).

### 3.5 Effect of lamination types, aspect ratios and boundary conditions on natural frequencies (Hz) of super-elliptical plates

Fundamental natural frequencies (Hz) of super-elliptical quasi-isotropic plates for clamped and simply supported boundary condition with the periphery of the plates for $r$ = 6, $n$ = 10 and 24 different lamination types are presented in Tables 12-15.

It seems that the fundamental natural frequencies depend on lamination types. It can be seen from the results the fundamental natural frequencies decrease with the increase of the aspect ratios ($a/b$ and $b/a$). Another important parameter is the selection of the short edge of the plate in $x$ or $y$ direction ($a$ or $b$). It corresponds to longitudinal or lateral framing system in ship construction. The fundamental natural frequencies for clamped boundary conditions are higher than those for the simply supported boundary conditions. The results of plates having different super-elliptical powers ($n$ = 1 and 4) are given in Appendix B. From the results that natural frequencies increase as the super elliptical powers ($n$) increases for simply supported boundary condition. On the contrary natural frequencies decrease as the super elliptical power ($n$) increases for clamped boundary condition.

From Table 8 (simply supported case) LT3 ($[-45_2/45_2/0_2/90_2]_s$), LT4 ($[-45_2/45_2/90_2/0_2]_s$), LT13 ($[45_2/-45_2/0_2/90_2]_s$) and LT14 ($[45_2/-45_2/90_2/0_2]_s$) have the highest natural frequencies for aspect ratio $a/b$ = 1.

For aspect ratio $a/b$ = 2, LT20 ($[90_2/-45_2/45_2/0_2]_s$) and LT23 ($[90_2/45_2/-45_2/0_2]_s$) have the highest natural frequencies.

From Table 9 (simply supported case) LT3 ($[-45_2/45_2/0_2/90_2]_s$), LT4 ($[-45_2/45_2/90_2/0_2]_s$), LT13 ($[45_2/-45_2/0_2/90_2]_s$) and LT14 $[45_2/-45_2/90_2/0_2]_s$ have the highest natural frequencies for aspect ratio $b/a$ = 1. For aspect ratio $b/a$ = 2, LT7 ($[0_2/-45_2/45_2/90_2]_s$) and LT9 ($[0_2/45_2/-45_2/90_2]_s$) have the highest natural frequencies.

Table 9 Fundamental natural frequency $\omega$ (Hz) of different thinner or thicker super-elliptical plates (clamped, $r$ = 6, $n$ = 1)

| $a/b$ | Fundamental natural frequency $\omega$ (Hz) | | |
|---|---|---|---|
| | $[-45_2/0_2/45_2/90_2]_s$ | $[-45_3/0_3/45_3/90_3]_s$ | $[-45_4/0_4/45_4/90_4]_s$ |
| 1 | 968.601 | 1452.900 | 1936.650 |
| 1.2 | 790.475 | 1185.71 | 1580.480 |
| 1.4 | 687.898 | 1031.85 | 1375.400 |
| 1.6 | 624.263 | 936.394 | 1248.190 |
| 1.8 | 582.315 | 873.473 | 1164.350 |
| 2 | 553.219 | 829.828 | 1106.19 |
| $b/a$ | Fundamental natural frequency $\omega$ (Hz) | | |
| | $[-45_2/0_2/45_2/90_2]_s$ | $[-45_3/0_3/45_3/90_3]_s$ | $[-45_4/0_4/45_4/90_4]_s$ |
| | $t$ = 3.2 mm | $t$ = 4.8 mm | $t$ = 6.4 mm |
| 1 | 968.601 | 1452.9 | 1936.65 |
| 1.2 | 857.296 | 1285.94 | 1714.16 |
| 1.4 | 793.42 | 1190.13 | 1586.49 |
| 1.6 | 753.193 | 1129.79 | 1506.1 |
| 1.8 | 725.931 | 1088.9 | 1451.62 |
| 2 | 706.353 | 1059.53 | 1412.5 |

Table 10 Fundamental natural frequency $\omega$ (Hz) of different thinner or thicker super-elliptical plates (simply supported, $r$ = 6, $n$ = 10)

| $a/b$ | Fundamental natural frequency $\omega$ (Hz) | | |
|---|---|---|---|
| | $[-45_2/0_2/45_2/90_2]_s$ | $[-45_3/0_3/45_3/90_3]_s$ | $[-45_4/0_4/45_4/90_4]_s$ |
| | $t$ = 3.2 mm | $t$ = 4.8 mm | $t$ = 6.4 mm |
| 1 | 510.747 | 766.152 | 1021.02 |
| 1.2 | 420.234 | 630.335 | 840.053 |
| 1.4 | 363.531 | 545.266 | 726.724 |
| 1.6 | 325.428 | 488.059 | 650.593 |
| 1.8 | 298.419 | 447.6 | 596.351 |
| 2 | 278.347 | 417.515 | 556.452 |
| $b/a$ | Fundamental natural frequency $\omega$ (Hz) | | |
| | $[-45_2/0_2/45_2/90_2]_s$ | $[-45_3/0_3/45_3/90_3]_s$ | $[-45_4/0_4/45_4/90_4]_s$ |
| | $t$ = 3.2 mm | $t$ = 4.8 mm | $t$ = 6.4 mm |
| 1 | 510.747 | 766.152 | 1021.02 |
| 1.2 | 442.155 | 663.209 | 883.86 |
| 1.4 | 398.730 | 598.061 | 797.09 |
| 1.6 | 369.262 | 553.893 | 738.281 |
| 1.8 | 348.313 | 522.444 | 696.379 |
| 2 | 332.804 | 499.23F | 665.439 |

Table 11 Fundamental natural frequency $\omega$ (Hz) of different thinner or thicker super-elliptical plates (clamped, $r$ = 6, $n$ = 10)

| $a/b$ | Fundamental natural frequency $\omega$ (Hz) | | |
|---|---|---|---|
| | $[-45_2/0_2/45_2/90_2]_s$ | $[-45_3/0_3/45_3/90_3]_s$ | $[-45_4/0_4/45_4/90_4]_s$ |
| 1 | 869.749 | 1304.63 | 1739.11 |
| 1.2 | 709.082 | 1063.64 | 1417.81 |
| 1.4 | 617.537 | 926.292 | 1234.77 |
| 1.6 | 561.342 | 842.024 | 1122.43 |
| 1.8 | 524.743 | 787.111 | 1049.28 |
| 2 | 499.649 | 749.502 | 999.124 |
| $b/a$ | Fundamental natural frequency $\omega$ (Hz) | | |
| | $[-45_2/0_2/45_2/90_2]_s$ | $[-45_3/0_3/45_3/90_3]_s$ | $[-45_4/0_4/45_4/90_4]_s$ |
| | $t$ = 3.2 mm | $t$ = 4.8 mm | $t$ = 6.4 mm |
| 1 | 869.749 | 1304.63 | 1739.11 |
| 1.2 | 771.921 | 1157.88 | 1543.52 |
| 1.4 | 716.729 | 1075.1 | 1433.22 |
| 1.6 | 682.696 | 1024.05 | 1365.2 |
| 1.8 | 660.279 | 990.412 | 1320.41 |
| 2 | 644.750 | 967.125 | 1289.36 |

Table 12 Fundamental natural frequencies $\omega$ (Hz) of super-elliptical plates (simply supported) $r$ = 6, $n$ = 10

| Plate types | Aspect ratios ($a/b$). Short half side, $b$, is on $y$ direction | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 1.2 | 1.4 | 1.6 | 1.8 | 2 |
| LT1 | 510.747 | 420.234 | 363.531 | 325.428 | 298.419 | 278.347 |
| LT2 | 500.939 | 416.080 | 363.847 | 329.111 | 304.812 | 286.946 |
| LT3 | 531.118 | 443.923 | 388.027 | 349.523 | 321.792 | 301.040 |
| LT4 | 531.123 | 450.921 | 399.278 | 363.748 | 338.044 | 318.847 |
| LT5 | 500.938 | 431.022 | 387.733 | 358.778 | 338.418 | 323.496 |
| LT6 | 510.804 | 442.138 | 398.710 | 369.183 | 348.308 | 332.818 |
| LT7 | 477.525 | 380.824 | 322.328 | 284.214 | 258.013 | 239.201 |
| LT8 | 467.479 | 376.504 | 322.782 | 288.706 | 265.728 | 249.611 |
| LT9 | 477.525 | 380.824 | 322.328 | 284.214 | 258.013 | 239.201 |
| LT10 | 467.479 | 376.504 | 322.782 | 288.706 | 265.728 | 249.611 |
| LT11 | 445.277 | 366.967 | 323.293 | 297.208 | 280.202 | 268.629 |
| LT12 | 445.277 | 366.967 | 323.293 | 297.208 | 280.202 | 268.629 |
| LT13 | 531.118 | 443.923 | 388.027 | 349.523 | 321.792 | 301.040 |
| LT14 | 531.123 | 450.921 | 399.278 | 363.748 | 338.044 | 318.847 |
| LT15 | 510.747 | 420.234 | 363.531 | 325.428 | 298.419 | 278.347 |
| LT16 | 500.939 | 416.080 | 363.847 | 329.111 | 304.812 | 286.946 |
| LT17 | 510.804 | 442.138 | 398.710 | 369.183 | 348.308 | 332.818 |
| LT18 | 500.938 | 431.022 | 387.733 | 358.778 | 338.418 | 323.496 |
| LT19 | 467.495 | 416.712 | 386.525 | 366.969 | 353.156 | 343.425 |
| LT20 | 477.637 | 427.667 | 397.042 | 376.510 | 362.397 | 351.907 |
| LT21 | 445.288 | 392.548 | 363.351 | 345.565 | 333.463 | 324.969 |
| LT22 | 445.288 | 392.548 | 363.351 | 345.565 | 333.463 | 324.969 |
| LT23 | 477.637 | 427.667 | 397.042 | 376.510 | 362.397 | 351.907 |
| LT24 | 467.495 | 416.712 | 386.525 | 366.969 | 353.156 | 343.425 |

Table 13 Fundamental natural frequencies $\omega$ (Hz) of super-elliptical plates (simply supported) $r$ = 6, $n$ = 10

| Plate types | Aspect ratios ($b/a$). Short half side, $a$, is on $x$ direction | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 1.2 | 1.4 | 1.6 | 1.8 | 2 |
| LT1 | 510.747 | 442.155 | 398.730 | 369.262 | 348.313 | 332.804 |
| LT2 | 500.939 | 431.053 | 387.736 | 358.784 | 338.377 | 323.399 |
| LT3 | 531.118 | 450.935 | 399.270 | 363.703 | 338.056 | 318.882 |
| LT4 | 531.123 | 443.945 | 387.987 | 349.544 | 321.792 | 301.022 |
| LT5 | 500.938 | 416.047 | 363.782 | 329.095 | 304.774 | 286.978 |
| LT6 | 510.804 | 420.249 | 363.534 | 325.423 | 298.361 | 278.356 |
| LT7 | 477.525 | 427.709 | 396.961 | 376.556 | 362.250 | 351.922 |
| LT8 | 467.479 | 416.789 | 386.547 | 366.862 | 353.269 | 343.411 |
| LT9 | 477.525 | 427.709 | 396.961 | 376.556 | 362.250 | 351.922 |
| LT10 | 467.479 | 416.789 | 386.547 | 366.862 | 353.269 | 343.411 |
| LT11 | 445.277 | 392.651 | 363.437 | 345.545 | 333.556 | 325.124 |
| LT12 | 445.277 | 392.651 | 363.437 | 345.545 | 333.556 | 325.124 |
| LT13 | 531.118 | 450.935 | 399.270 | 363.703 | 338.056 | 318.882 |
| LT14 | 531.123 | 443.945 | 387.987 | 349.544 | 321.792 | 301.022 |
| LT15 | 510.747 | 442.155 | 398.730 | 369.262 | 348.313 | 332.804 |
| LT16 | 500.939 | 431.053 | 387.736 | 358.784 | 338.377 | 323.399 |
| LT17 | 510.804 | 420.249 | 363.534 | 325.423 | 298.361 | 278.356 |
| LT18 | 500.938 | 416.047 | 363.782 | 329.095 | 304.774 | 286.978 |
| LT19 | 467.495 | 376.538 | 322.823 | 288.729 | 265.756 | 249.543 |
| LT20 | 477.637 | 380.858 | 322.307 | 284.193 | 258.023 | 239.193 |
| LT21 | 445.288 | 366.924 | 323.361 | 297.150 | 280.234 | 268.650 |
| LT22 | 445.288 | 366.924 | 323.361 | 297.150 | 280.234 | 268.650 |
| LT23 | 477.637 | 380.858 | 322.307 | 284.193 | 258.023 | 239.193 |
| LT24 | 467.495 | 376.538 | 322.823 | 288.729 | 265.756 | 249.543 |

Table 14 Fundamental natural frequencies $\omega$ (Hz) of super-elliptical plates (clamped) $r$ = 6, $n$ = 10

| Plate types | Aspect ratios ($a/b$). Short half side, $b$, is on $y$ direction | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 1.2 | 1.4 | 1.6 | 1.8 | 2 |
| LT1 | 869.749 | 709.082 | 617.537 | 561.342 | 524.743 | 499.649 |
| LT2 | 874.358 | 724.780 | 641.255 | 590.929 | 558.557 | 536.613 |
| LT3 | 862.603 | 722.896 | 642.068 | 591.518 | 557.879 | 534.409 |
| LT4 | 862.602 | 743.963 | 675.434 | 632.470 | 603.808 | 583.742 |
| LT5 | 874.333 | 766.580 | 707.021 | 670.954 | 647.568 | 631.575 |
| LT6 | 869.743 | 771.894 | 716.705 | 682.707 | 660.269 | 644.786 |
| LT7 | 878.296 | 685.819 | 576.588 | 510.523 | 468.407 | 440.319 |
| LT8 | 883.395 | 702.303 | 602.211 | 543.195 | 506.399 | 482.274 |
| LT9 | 878.296 | 685.819 | 576.588 | 510.523 | 468.407 | 440.319 |
| LT10 | 883.395 | 702.303 | 602.211 | 543.195 | 506.399 | 482.274 |
| LT11 | 891.338 | 732.860 | 649.577 | 602.628 | 574.334 | 556.259 |
| LT12 | 891.338 | 732.860 | 649.577 | 602.628 | 574.334 | 556.259 |
| LT13 | 862.603 | 722.896 | 642.068 | 591.518 | 557.879 | 534.409 |
| LT14 | 862.602 | 743.963 | 675.434 | 632.470 | 603.808 | 583.742 |
| LT15 | 869.749 | 709.082 | 617.537 | 561.342 | 524.743 | 499.649 |
| LT16 | 874.358 | 724.780 | 641.255 | 590.929 | 558.557 | 536.613 |
| LT17 | 869.743 | 771.894 | 716.705 | 682.707 | 660.269 | 644.786 |

Table 14 Continued

| Plate types | Aspect ratios ($a/b$). Short half side, $b$, is on $y$ direction | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 1.2 | 1.4 | 1.6 | 1.8 | 2 |
| LT18 | 874.333 | 766.580 | 707.021 | 670.954 | 647.568 | 631.575 |
| LT19 | 883.392 | 806.059 | 764.916 | 740.601 | 725.163 | 714.772 |
| LT20 | 878.302 | 810.420 | 773.184 | 750.611 | 736.109 | 726.038 |
| LT21 | 891.347 | 794.951 | 746.031 | 718.529 | 701.600 | 690.605 |
| LT22 | 891.347 | 794.951 | 746.031 | 718.529 | 701.600 | 690.605 |
| LT23 | 878.302 | 810.420 | 773.184 | 750.611 | 736.109 | 726.038 |
| LT24 | 883.392 | 806.059 | 764.916 | 740.601 | 725.163 | 714.772 |

From Table 10 (clamped case) LT11 ($[0_2/90_2/-45_2/45_2]_s$), LT12 ($[0_2/90_2/45_2/-45_2]_s$), LT21 ($[90_2/0_2/-45_2/45_2]_s$) and LT22 ($[90_2/0_2/45_2/-45_2]_s$) have the highest natural frequencies for aspect ratio $a/b = 1$. For aspect ratio $a/b = 2$, LT20 ($[90_2/-45_2/45_2/0_2]_s$) and LT23 ($[90_2/45_2/-45_2/0_2]_s$) have the highest natural frequencies.

From Table 11 (clamped case) LT11 ($[0_2/90_2/-45_2/45_2]_s$), LT12 ($[0_2/90_2/45_2/-45_2]_s$), LT21 ($[90_2/0_2/-45_2/45_2]_s$) and LT22 ($[90_2/0_2/45_2/-45_2]_s$) have the highest natural frequencies for aspect ratio $b/a = 1$. For aspect ratio $b/a = 2$, LT7 ($[0_2/-45_2/45_2/90_2]_s$) and LT9 ($[0_2/45_2/-45_2/90_2]_s$) have the highest natural frequencies.

These results are similar to those obtained by (Altunsaray and Bayer 2013), which was about the deflection and free vibration of symmetrically laminated quasi-isotropic thin rectangular plates.

## 4. Conclusions

Within this parametric study free vibration analysis of super-elliptical quasi-isotropic plates has been examined. Plates are considered as simply supported and clamped around the periphery. Effect of the lamination types, aspect

Table 15 Fundamental natural frequencies ω (Hz) of super-elliptical plates (clamped) $r = 6$, $n = 10$

| Plate types | Aspect ratios ($b/a$). Short half side, $a$, is on $x$ direction | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 1.2 | 1.4 | 1.6 | 1.8 | 2 |
| LT1 | 869.749 | 771.921 | 716.729 | 682.696 | 660.279 | 644.750 |
| LT2 | 874.358 | 766.572 | 706.997 | 670.962 | 647.569 | 631.553 |
| LT3 | 862.603 | 743.959 | 675.427 | 632.478 | 603.813 | 583.760 |
| LT4 | 862.602 | 722.887 | 642.070 | 591.518 | 557.892 | 534.405 |
| LT5 | 874.333 | 724.766 | 641.248 | 590.919 | 558.571 | 536.614 |
| LT6 | 869.743 | 709.100 | 617.527 | 561.353 | 524.741 | 499.666 |
| LT7 | 878.296 | 810.413 | 773.169 | 750.620 | 735.991 | 726.012 |
| LT8 | 883.395 | 806.042 | 764.891 | 740.624 | 725.177 | 714.758 |
| LT9 | 878.296 | 810.413 | 773.169 | 750.620 | 735.991 | 726.012 |
| LT10 | 883.395 | 806.042 | 764.891 | 740.624 | 725.177 | 714.758 |
| LT11 | 891.338 | 794.897 | 746.016 | 718.484 | 701.609 | 690.596 |
| LT12 | 891.338 | 794.897 | 746.016 | 718.484 | 701.609 | 690.596 |
| LT13 | 862.603 | 743.959 | 675.427 | 632.478 | 603.813 | 583.760 |
| LT14 | 862.602 | 722.887 | 642.070 | 591.518 | 557.892 | 534.405 |
| LT15 | 869.749 | 771.921 | 716.729 | 682.696 | 660.279 | 644.750 |
| LT16 | 874.358 | 766.572 | 706.997 | 670.962 | 647.569 | 631.553 |
| LT17 | 869.743 | 709.100 | 617.527 | 561.353 | 524.741 | 499.666 |
| LT18 | 874.333 | 724.766 | 641.248 | 590.919 | 558.571 | 536.614 |
| LT19 | 883.392 | 702.316 | 602.211 | 543.220 | 506.418 | 482.286 |
| LT20 | 878.302 | 685.817 | 576.581 | 510.524 | 468.410 | 440.296 |
| LT21 | 891.347 | 732.875 | 649.549 | 602.608 | 574.333 | 556.245 |
| LT22 | 891.347 | 732.875 | 649.549 | 602.608 | 574.333 | 556.245 |
| LT23 | 878.302 | 685.817 | 576.581 | 510.524 | 468.410 | 440.296 |
| LT24 | 883.392 | 702.316 | 602.211 | 543.220 | 506.418 | 482.286 |

ratios, thicknesses and super-elliptical power ($n$) on natural frequencies of super-elliptical plates has been investigated by Rayleigh-Ritz Method based on Classical Lamination Plate Theory (CLPT).

- Validation study was performed with the isotropic case only available in open literature. Results are in good agreement with each other. From the convergence study of LT1 $[-45_2/0_2/45_2/90_2]_s$ plate that when the number of terms in trial function is 10, the accuracy obtained seems to be appropriate. From the results natural frequency of symmetrically laminated quasi-isotropic super-elliptical plates is influenced by the boundary conditions, thickness, aspect ratios, lamination types and super-elliptical power. Natural frequency for simply supported plates is less than clamped plates. Fundamental natural frequency increases with the increase in thickness. Some lamination types are more favorable than others given in Chapter 3.
- In ship structures, plates and shells are generally supported with primary and secondary structural members. The construction system is named transverse or longitudinal depending on the position of secondary supporting members mainly called as stiffeners. The use of different aspect ratios ($a/b$ or $b/a$) actually corresponds to the different constructional systems either transverse or longitudinal. Many different parameters play a part in complicated composite ship design process such as constructional systems, plate thickness, lamination types, boundary conditions etc. It can be deduced that Rayleigh-Ritz method is one of the suitable calculation methods for such problems as avoiding from resonance at the preliminary design process of composite ship structures. The effect of different super-elliptical powers ($n$ = 1 and $n$ = 4) on fundamental natural frequency of composite plate is presented with tables for the use of ship designers in Appendix A.

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## Appendix A

Table A1 Fundamental natural frequencies ω (Hz) of super-elliptical plates (simply supported) $r = 6$, $n = 1$

| Plate types | Aspect ratios ($a/b$). Short half side, $b$, is on $y$ direction | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 1.2 | 1.4 | 1.6 | 1.8 | 2 |
| LT1 | 469.880 | 383.432 | 333.700 | 302.860 | 282.494 | 268.297 |
| LT2 | 470.245 | 389.866 | 344.528 | 316.824 | 298.670 | 286.016 |
| LT3 | 469.859 | 393.654 | 349.056 | 320.792 | 301.689 | 288.071 |
| LT4 | 469.859 | 404.572 | 366.316 | 341.895 | 325.191 | 313.100 |
| LT5 | 470.245 | 411.460 | 378.314 | 357.623 | 343.546 | 333.271 |
| LT6 | 469.880 | 415.912 | 384.775 | 364.908 | 351.151 | 340.986 |
| LT7 | 468.591 | 366.743 | 308.531 | 273.075 | 250.279 | 234.885 |
| LT8 | 469.210 | 373.564 | 320.323 | 288.674 | 268.689 | 255.313 |
| LT9 | 468.591 | 366.743 | 308.531 | 273.075 | 250.279 | 234.885 |
| LT10 | 469.210 | 373.564 | 320.323 | 288.674 | 268.689 | 255.313 |
| LT11 | 469.572 | 386.199 | 342.024 | 316.733 | 301.020 | 290.396 |
| LT12 | 469.572 | 386.199 | 342.024 | 316.733 | 301.020 | 290.396 |
| LT13 | 469.859 | 393.654 | 349.056 | 320.792 | 301.689 | 288.071 |
| LT14 | 469.859 | 404.572 | 366.316 | 341.895 | 325.191 | 313.100 |
| LT15 | 469.880 | 383.432 | 333.700 | 302.860 | 282.494 | 268.297 |
| LT16 | 470.245 | 389.866 | 344.528 | 316.824 | 298.670 | 286.016 |
| LT17 | 469.880 | 415.912 | 384.775 | 364.908 | 351.151 | 340.986 |
| LT18 | 470.245 | 411.460 | 378.314 | 357.623 | 343.546 | 333.271 |
| LT19 | 469.210 | 426.658 | 402.837 | 387.485 | 376.456 | 367.938 |
| LT20 | 468.591 | 430.504 | 408.326 | 393.604 | 382.864 | 374.527 |
| LT21 | 469.572 | 417.958 | 390.804 | 374.311 | 362.887 | 354.164 |
| LT22 | 469.572 | 417.958 | 390.804 | 374.311 | 362.887 | 354.164 |
| LT23 | 468.591 | 430.504 | 408.326 | 393.604 | 382.864 | 374.527 |
| LT24 | 469.210 | 426.658 | 402.837 | 387.485 | 376.456 | 367.938 |

Table A2 Fundamental natural frequencies $\omega$ (Hz) of super-elliptical plates (simply supported) $r = 6$, $n = 1$

| Plate types | Aspect ratios ($b/a$). Short half side, $a$, is on $x$ direction | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 1.2 | 1.4 | 1.6 | 1.8 | 2 |
| LT1 | 469.880 | 415.912 | 384.775 | 364.908 | 351.151 | 340.986 |
| LT2 | 470.245 | 411.460 | 378.314 | 357.623 | 343.546 | 333.271 |
| LT3 | 469.859 | 404.572 | 366.316 | 341.895 | 325.191 | 313.100 |
| LT4 | 469.859 | 393.654 | 349.056 | 320.792 | 301.689 | 288.071 |
| LT5 | 470.245 | 389.866 | 344.528 | 316.824 | 298.670 | 286.016 |
| LT6 | 469.880 | 383.432 | 333.700 | 302.860 | 282.494 | 268.297 |
| LT7 | 468.591 | 430.504 | 408.326 | 393.604 | 382.864 | 374.527 |
| LT8 | 469.210 | 426.658 | 402.837 | 387.485 | 376.456 | 367.938 |
| LT9 | 468.591 | 430.504 | 408.326 | 393.604 | 382.864 | 374.527 |
| LT10 | 469.210 | 426.658 | 402.837 | 387.485 | 376.456 | 367.938 |
| LT11 | 469.572 | 417.958 | 390.804 | 374.311 | 362.887 | 354.164 |
| LT12 | 469.572 | 417.958 | 390.804 | 374.311 | 362.887 | 354.164 |
| LT13 | 469.859 | 404.572 | 366.316 | 341.895 | 325.191 | 313.100 |
| LT14 | 469.859 | 393.654 | 349.056 | 320.792 | 301.689 | 288.071 |
| LT15 | 469.880 | 415.912 | 384.775 | 364.908 | 351.151 | 340.986 |

Table A2 Continued

| Plate types | Aspect ratios (*b*/*a*). Short half side, *a*, is on *x* direction | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 1.2 | 1.4 | 1.6 | 1.8 | 2 |
| LT16 | 470.245 | 411.460 | 378.314 | 357.623 | 343.546 | 333.271 |
| LT17 | 469.880 | 383.432 | 333.700 | 302.860 | 282.494 | 268.297 |
| LT18 | 470.245 | 389.866 | 344.528 | 316.824 | 298.670 | 286.016 |
| LT19 | 469.210 | 373.564 | 320.323 | 288.674 | 268.689 | 255.313 |
| LT20 | 468.591 | 366.743 | 308.531 | 273.075 | 250.279 | 234.885 |
| LT21 | 469.572 | 386.199 | 342.024 | 316.733 | 301.020 | 290.396 |
| LT22 | 469.572 | 386.199 | 342.024 | 316.733 | 301.020 | 290.396 |
| LT23 | 468.591 | 366.743 | 308.531 | 273.075 | 250.279 | 234.885 |
| LT24 | 469.210 | 373.564 | 320.323 | 288.674 | 268.689 | 255.313 |

Table A3 Fundamental natural frequencies $\omega$ (Hz) of super-elliptical plates (simply supported) $r = 6$, $n = 4$

| Plate types | Aspect ratios (*a*/*b*). Short half side, *b*, is on *y* direction | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 1.2 | 1.4 | 1.6 | 1.8 | 2 |
| LT1 | 477.985 | 392.854 | 340.116 | 305.009 | 280.397 | 262.454 |
| LT2 | 468.662 | 389.168 | 340.771 | 309.092 | 287.225 | 271.503 |
| LT3 | 496.290 | 414.935 | 363.275 | 328.185 | 303.181 | 284.706 |
| LT4 | 496.290 | 422.064 | 374.862 | 342.838 | 320.072 | 303.296 |
| LT5 | 468.662 | 404.276 | 365.077 | 339.457 | 321.806 | 309.135 |
| LT6 | 477.985 | 415.083 | 376.025 | 350.087 | 331.983 | 318.849 |
| LT7 | 448.452 | 356.498 | 301.442 | 265.963 | 241.813 | 224.661 |
| LT8 | 438.307 | 352.270 | 302.038 | 270.507 | 249.581 | 235.062 |
| LT9 | 448.452 | 356.498 | 301.442 | 265.963 | 241.813 | 224.661 |
| LT10 | 438.307 | 352.270 | 302.038 | 270.507 | 249.581 | 235.062 |
| LT11 | 416.639 | 343.162 | 302.858 | 279.096 | 264.220 | 254.422 |
| LT12 | 416.639 | 343.162 | 302.858 | 279.096 | 264.220 | 254.422 |
| LT13 | 496.290 | 414.935 | 363.275 | 328.185 | 303.181 | 284.706 |
| LT14 | 496.290 | 422.064 | 374.862 | 342.838 | 320.072 | 303.296 |
| LT15 | 477.985 | 392.854 | 340.116 | 305.009 | 280.397 | 262.454 |
| LT16 | 468.662 | 389.168 | 340.771 | 309.092 | 287.225 | 271.503 |
| LT17 | 477.985 | 415.083 | 376.025 | 350.087 | 331.983 | 318.849 |
| LT18 | 468.662 | 404.276 | 365.077 | 339.457 | 321.806 | 309.135 |
| LT19 | 438.307 | 392.662 | 366.397 | 349.999 | 339.105 | 331.506 |
| LT20 | 448.452 | 403.940 | 377.441 | 360.428 | 348.866 | 340.652 |
| LT21 | 416.639 | 368.580 | 342.893 | 327.904 | 318.517 | 312.292 |
| LT22 | 416.639 | 368.580 | 342.893 | 327.904 | 318.517 | 312.292 |
| LT23 | 448.452 | 403.940 | 377.441 | 360.428 | 348.866 | 340.652 |
| LT24 | 438.307 | 392.662 | 366.397 | 349.999 | 339.105 | 331.506 |

Table A4 Fundamental natural frequencies ω (Hz) of super-elliptical plates (simply supported) $r = 6$, $n = 4$

| Plate types | Aspect ratios (*b*/*a*). Short half side, *a*, is on *x* direction | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 1.2 | 1.4 | 1.6 | 1.8 | 2 |
| LT1 | 477.985 | 415.083 | 376.025 | 350.087 | 331.983 | 318.849 |
| LT2 | 468.662 | 404.276 | 365.077 | 339.457 | 321.806 | 309.135 |

Table A4 Continued

| Plate types | Aspect ratios ($b/a$). Short half side, $a$, is on $x$ direction | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 1.2 | 1.4 | 1.6 | 1.8 | 2 |
| LT3 | 496.290 | 422.064 | 374.862 | 342.838 | 320.072 | 303.296 |
| LT4 | 496.290 | 414.935 | 363.275 | 328.185 | 303.181 | 284.706 |
| LT5 | 468.662 | 389.168 | 340.771 | 309.092 | 287.225 | 271.503 |
| LT6 | 477.985 | 392.854 | 340.116 | 305.009 | 280.397 | 262.454 |
| LT7 | 448.452 | 403.940 | 377.441 | 360.428 | 348.866 | 340.652 |
| LT8 | 438.307 | 392.662 | 366.397 | 349.999 | 339.105 | 331.506 |
| LT9 | 448.452 | 403.940 | 377.441 | 360.428 | 348.866 | 340.652 |
| LT10 | 438.307 | 392.662 | 366.397 | 349.999 | 339.105 | 331.506 |
| LT11 | 416.639 | 368.580 | 342.893 | 327.904 | 318.518 | 312.292 |
| LT12 | 416.639 | 368.580 | 342.893 | 327.904 | 318.518 | 312.292 |
| LT13 | 496.290 | 422.064 | 374.862 | 342.838 | 320.072 | 303.296 |
| LT14 | 496.290 | 414.935 | 363.275 | 328.185 | 303.181 | 284.706 |
| LT15 | 477.985 | 415.083 | 376.025 | 350.087 | 331.983 | 318.849 |
| LT16 | 468.662 | 404.276 | 365.077 | 339.457 | 321.806 | 309.135 |
| LT17 | 477.985 | 392.854 | 340.116 | 305.009 | 280.397 | 262.454 |
| LT18 | 468.662 | 389.168 | 340.771 | 309.092 | 287.225 | 271.503 |
| LT19 | 438.307 | 352.270 | 302.038 | 270.507 | 249.581 | 235.062 |
| LT20 | 448.452 | 356.498 | 301.442 | 265.963 | 241.813 | 224.661 |
| LT21 | 416.639 | 343.163 | 302.858 | 279.096 | 264.220 | 254.422 |
| LT22 | 416.639 | 343.163 | 302.858 | 279.096 | 264.220 | 254.422 |
| LT23 | 448.452 | 356.498 | 301.442 | 265.963 | 241.813 | 224.661 |
| LT24 | 438.307 | 352.270 | 302.038 | 270.507 | 249.581 | 235.062 |

Table A5 Fundamental natural frequencies $\omega$ (Hz) of super-elliptical plates (clamped) $r = 6$, $n = 1$

| Plate types | Aspect ratios ($a/b$). Short half side, $b$, is on $y$ direction | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 1.2 | 1.4 | 1.6 | 1.8 | 2 |
| LT1 | 968.601 | 790.475 | 687.898 | 624.263 | 582.315 | 553.219 |
| LT2 | 968.992 | 803.375 | 709.908 | 652.877 | 615.712 | 590.090 |
| LT3 | 969.195 | 811.974 | 719.855 | 661.501 | 622.188 | 594.347 |
| LT4 | 969.195 | 834.366 | 755.375 | 705.132 | 671.040 | 646.678 |
| LT5 | 968.992 | 847.831 | 779.795 | 737.829 | 709.876 | 690.071 |
| LT6 | 968.601 | 857.296 | 793.420 | 753.193 | 725.931 | 706.353 |
| LT7 | 966.307 | 756.266 | 636.269 | 563.124 | 516.084 | 484.375 |
| LT8 | 967.435 | 770.201 | 660.430 | 595.179 | 554.069 | 526.744 |
| LT9 | 966.307 | 756.266 | 636.269 | 563.124 | 516.084 | 484.375 |
| LT10 | 967.435 | 770.201 | 660.430 | 595.179 | 554.069 | 526.744 |
| LT11 | 968.706 | 796.689 | 705.630 | 653.730 | 621.911 | 600.971 |
| LT12 | 968.706 | 796.689 | 705.630 | 653.730 | 621.911 | 600.971 |
| LT13 | 969.195 | 811.974 | 719.855 | 661.501 | 622.188 | 594.347 |
| LT14 | 969.195 | 834.366 | 755.375 | 705.132 | 671.040 | 646.678 |
| LT15 | 968.601 | 790.475 | 687.898 | 624.263 | 582.315 | 553.219 |
| LT16 | 968.992 | 803.375 | 709.908 | 652.877 | 615.712 | 590.090 |
| LT17 | 968.601 | 857.296 | 793.420 | 753.193 | 725.931 | 706.353 |
| LT18 | 968.992 | 847.831 | 779.795 | 737.829 | 709.876 | 690.071 |
| LT19 | 967.435 | 880.278 | 832.590 | 803.177 | 783.275 | 768.869 |

Table A5 Continued

| Plate types | Aspect ratios ($a$/$b$). Short half side, $b$, is on $y$ direction | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 1.2 | 1.4 | 1.6 | 1.8 | 2 |
| LT20 | 966.307 | 888.504 | 844.401 | 816.402 | 797.072 | 782.894 |
| LT21 | 968.706 | 862.565 | 807.561 | 775.398 | 754.495 | 739.754 |
| LT22 | 968.706 | 862.565 | 807.561 | 775.398 | 754.495 | 739.754 |
| LT23 | 966.307 | 888.504 | 844.401 | 816.402 | 797.072 | 782.894 |
| LT24 | 967.435 | 880.278 | 832.590 | 803.177 | 783.275 | 768.869 |

Table A6 Fundamental natural frequencies $\omega$ (Hz) of super-elliptical plates (clamped) $r$ = 6, $n$ = 1

| Plate types | Aspect ratios ($b$/$a$). Short half side, $a$, is on $x$ direction | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 1.2 | 1.4 | 1.6 | 1.8 | 2 |
| LT1 | 968.601 | 857.296 | 793.420 | 753.193 | 725.931 | 706.353 |
| LT2 | 968.992 | 847.831 | 779.795 | 737.829 | 709.876 | 690.071 |
| LT3 | 969.195 | 834.366 | 755.375 | 705.132 | 671.040 | 646.678 |
| LT4 | 969.195 | 811.974 | 719.855 | 661.501 | 622.188 | 594.347 |
| LT5 | 968.992 | 803.375 | 709.908 | 652.877 | 615.712 | 590.090 |
| LT6 | 968.601 | 790.475 | 687.898 | 624.263 | 582.315 | 553.219 |
| LT7 | 966.307 | 888.504 | 844.401 | 816.402 | 797.072 | 782.894 |
| LT8 | 967.435 | 880.278 | 832.590 | 803.177 | 783.275 | 768.869 |
| LT9 | 966.307 | 888.504 | 844.401 | 816.402 | 797.072 | 782.894 |
| LT10 | 967.435 | 880.278 | 832.590 | 803.177 | 783.275 | 768.869 |
| LT11 | 968.706 | 862.565 | 807.561 | 775.398 | 754.495 | 739.754 |
| LT12 | 968.706 | 862.565 | 807.561 | 775.398 | 754.495 | 739.754 |
| LT13 | 969.195 | 834.366 | 755.375 | 705.132 | 671.040 | 646.678 |
| LT14 | 969.195 | 811.974 | 719.855 | 661.501 | 622.188 | 594.347 |
| LT15 | 968.601 | 857.296 | 793.420 | 753.193 | 725.931 | 706.353 |
| LT16 | 968.992 | 847.831 | 779.795 | 737.829 | 709.876 | 690.071 |
| LT17 | 968.601 | 790.475 | 687.898 | 624.263 | 582.315 | 553.219 |
| LT18 | 968.992 | 803.375 | 709.908 | 652.877 | 615.712 | 590.090 |
| LT19 | 967.435 | 770.201 | 660.430 | 595.179 | 554.069 | 526.744 |
| LT20 | 966.307 | 756.266 | 636.269 | 563.124 | 516.084 | 484.375 |
| LT21 | 968.706 | 796.689 | 705.630 | 653.730 | 621.911 | 600.971 |
| LT22 | 968.706 | 796.689 | 705.630 | 653.730 | 621.911 | 600.971 |
| LT23 | 966.307 | 756.266 | 636.269 | 563.124 | 516.084 | 484.375 |
| LT24 | 967.435 | 770.201 | 660.430 | 595.179 | 554.069 | 526.744 |

Table A7 Fundamental natural frequencies $\omega$ (Hz) of super-elliptical plates (clamped) $r$ = 6, $n$ = 4

| Plate types | Aspect ratios ($a$/$b$). Short half side, $b$, is on $y$ direction | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 1.2 | 1.4 | 1.6 | 1.8 | 2 |
| LT1 | 852.434 | 694.672 | 605.158 | 550.555 | 515.267 | 491.353 |
| LT2 | 855.746 | 709.253 | 627.962 | 579.376 | 548.519 | 527.907 |
| LT3 | 846.372 | 709.363 | 630.492 | 581.517 | 549.265 | 527.007 |
| LT4 | 846.372 | 730.428 | 663.999 | 622.827 | 595.715 | 576.985 |
| LT5 | 855.746 | 751.095 | 694.006 | 660.090 | 638.524 | 624.044 |
| LT6 | 852.434 | 757.610 | 704.884 | 672.955 | 652.298 | 638.219 |
| LT7 | 860.223 | 670.663 | 563.618 | 499.167 | 458.316 | 431.266 |
| LT8 | 863.278 | 685.534 | 587.843 | 530.628 | 495.283 | 472.398 |

Table A7 Continued

| Plate types | Aspect ratios ($a/b$). Short half side, $b$, is on $y$ direction | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 1.2 | 1.4 | 1.6 | 1.8 | 2 |
| LT9 | 860.223 | 670.663 | 563.618 | 499.167 | 458.316 | 431.266 |
| LT10 | 863.278 | 685.534 | 587.843 | 530.628 | 495.283 | 472.398 |
| LT11 | 868.148 | 713.545 | 632.980 | 588.153 | 561.700 | 545.236 |
| LT12 | 868.148 | 713.545 | 632.980 | 588.153 | 561.700 | 545.236 |
| LT13 | 846.372 | 709.363 | 630.492 | 581.517 | 549.265 | 527.007 |
| LT14 | 846.372 | 730.428 | 663.999 | 622.827 | 595.715 | 576.985 |
| LT15 | 852.434 | 694.672 | 605.158 | 550.555 | 515.267 | 491.353 |
| LT16 | 855.746 | 709.253 | 627.962 | 579.376 | 548.519 | 527.907 |
| LT17 | 852.434 | 757.610 | 704.884 | 672.955 | 652.298 | 638.219 |
| LT18 | 855.746 | 751.095 | 694.006 | 660.090 | 638.524 | 624.044 |
| LT19 | 863.278 | 789.589 | 751.522 | 729.821 | 716.444 | 707.672 |
| LT20 | 860.223 | 795.926 | 761.655 | 741.516 | 728.764 | 720.209 |
| LT21 | 868.148 | 775.605 | 729.814 | 704.993 | 690.448 | 681.349 |
| LT22 | 868.148 | 775.605 | 729.814 | 704.993 | 690.448 | 681.349 |
| LT23 | 860.223 | 795.926 | 761.655 | 741.516 | 728.764 | 720.209 |
| LT24 | 863.278 | 789.589 | 751.522 | 729.821 | 716.444 | 707.672 |
| LT20 | 860.223 | 795.926 | 761.655 | 741.516 | 728.764 | 720.209 |

Table A8 Fundamental natural frequencies $\omega$ (Hz) of super-elliptical plates (clamped) $r$ = 6, $n$ = 4

| Plate types | Aspect ratios ($b/a$). Short half side, $a$, is on $x$ direction | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 1.2 | 1.4 | 1.6 | 1.8 | 2 |
| LT1 | 852.434 | 757.610 | 704.884 | 672.955 | 652.298 | 638.219 |
| LT2 | 855.746 | 751.095 | 694.006 | 660.090 | 638.524 | 624.044 |
| LT3 | 846.372 | 730.428 | 663.999 | 622.827 | 595.715 | 576.985 |
| LT4 | 846.372 | 709.363 | 630.492 | 581.517 | 549.265 | 527.007 |
| LT5 | 855.746 | 709.253 | 627.962 | 579.376 | 548.519 | 527.907 |
| LT6 | 852.434 | 694.672 | 605.158 | 550.555 | 515.267 | 491.353 |
| LT7 | 860.223 | 795.926 | 761.655 | 741.516 | 728.764 | 720.209 |
| LT8 | 863.278 | 789.589 | 751.522 | 729.821 | 716.444 | 707.672 |
| LT9 | 860.223 | 795.926 | 761.655 | 741.516 | 728.764 | 720.209 |
| LT10 | 863.278 | 789.589 | 751.522 | 729.821 | 716.444 | 707.672 |
| LT11 | 868.148 | 775.605 | 729.814 | 704.993 | 690.448 | 681.349 |
| LT12 | 868.148 | 775.605 | 729.814 | 704.993 | 690.448 | 681.349 |
| LT13 | 846.372 | 730.428 | 663.999 | 622.827 | 595.715 | 576.985 |
| LT14 | 846.372 | 709.363 | 630.492 | 581.517 | 549.265 | 527.007 |
| LT15 | 852.434 | 757.610 | 704.884 | 672.955 | 652.298 | 638.219 |
| LT16 | 855.746 | 751.095 | 694.006 | 660.090 | 638.524 | 624.044 |
| LT17 | 852.434 | 694.672 | 605.158 | 550.555 | 515.267 | 491.353 |
| LT18 | 855.746 | 709.253 | 627.962 | 579.376 | 548.519 | 527.907 |
| LT19 | 863.278 | 685.534 | 587.843 | 530.628 | 495.283 | 472.398 |
| LT20 | 860.223 | 670.663 | 563.618 | 499.167 | 458.316 | 431.266 |
| LT21 | 868.148 | 713.545 | 632.980 | 588.153 | 561.700 | 545.236 |
| LT22 | 868.148 | 713.545 | 632.980 | 588.153 | 561.700 | 548.236 |
| LT23 | 860.223 | 670.663 | 563.618 | 499.167 | 458.316 | 431.266 |
| LT24 | 863.278 | 685.534 | 587.843 | 530.628 | 495.283 | 472.398 |