Steel and Composite Structures, *Vol. 29, No. 1* (*2018*) *139-149*
DOI: https://doi.org/10.12989/scs.2018.29.1.139

Damage identification in laminated composite plates using a new multi-step approach

Narges Fallah [a], Seyed Rohollah Hoseini Vaez * and Hossein Fasihi [b]

Department of Civil Engineering, *Faculty of Engineering*, *University of Qom*, *Qom*, *Iran*

(*Received June 8, 2018, Revised July 26, 2018, Accepted August 10, 2018*)

Abstract. In this paper a new multi-step damage detection approach is provided. In the first step, condensed modal residual vector based indicator (CMRVBI) has been proposed to locate the suspected damaged elements of structures that have rotational degrees of freedom (DOFs). The CMRVBI is a new indicator that uses only translational DOFs of the structures to localize damaged elements. In the next step, salp swarm algorithm is applied to quantify damage severity of the suspected damaged elements. In order to assess the performance of the proposed approach, a numerical example including a three-layer square laminated composite plate is studied. The numerical results demonstrated that the proposed CMRVBI is effective for locating damage, regardless of the effect of noise. The efficiency of proposed approach is also compared during both steps. The results demonstrate that in noisy condition, the damage identification approach is capable for the studied structure.

Keywords: multi-step approach; damage identification; CMRVBI; Salp Swarm Algorithm; structures involved rotational DOFs

1. Introduction

The scientific importance of the composite laminates, in structural, mechanical and aerospace engineering, has resulted in performing wide researches upon them. Ever-increasing usage of composite laminates in comparison with general metals is because of their high ratio of strength to weight and stiffness to weight. Damages occurring in composite structures lead to severe decrease of stiffness and tragic consequences. This has resulted in the development of the effective methods of identifying damages in the composite structures (Vo-Duy *et al*. 2016).

In the engineering industries, it is attempted to develop the non-destructive damage detection methods in order to evaluate the integrity of their structures. The basis of using most of these methods lies on the changes that occur in the dynamic parameters after the damage (Laier and Villalba 2015). There are many relatively new approaches of the health monitoring of civil engineering structures such as vibration-based damage detection methods (Xiang and Liang 2012b, Xiang *et al*. 2013, Xiang *et al*. 2014a, Kaveh *et al*. 2016). Also, many researchers have tested different methods to identify damage in engineering structures (Xiang and Liang 2012a, Xiang *et al*. 2012, Xiang *et al*. 2014b, Pedram *et al*. 2017, Fallah *et al*. 2018). Frequencies, mode shapes, and damping are vibration characteristics of a structure that are directly influenced by the physical characteristics of a structure such as mass and stiffness. Damage changes vibration characteristics of a structure and consequently decreases the stiffness of the structure. Therefore, it is possible to detect the location and severity of the damage by measuring and monitoring the vibration characteristics. Through reviews of these damage detection approaches have been presented in Ref. (Doebling *et al*. 1996, Salawu 1997, Carden and Fanning 2004, Fan and Qiao 2011) and their applications for composite structures is reported in Ref. (Zou *et al*. 2000, Montalvao *et al*. 2006).

Some different damage identification methods including wavelet analysis method and optimization-based method employed to identify damage location and severity of composite and other structures. These methods have been implemented in single-step often. These methods, however, include limitations such as measurement errors and low sensitivity of the dynamic parameters to the damage (Humar *et al*. 2006).

The vibration-based damage detection methods of the composite structures are widely used in the studies on structural health monitoring literature. In the recent years, some hybrid or mixed damage detection methods have been provided based on the combination of the transmissibility of structures and the signal processing methods (Yang *et al*. 2017). Zhou *et al*. proposed an approach based on transmissibility together with hierarchical clustering analysis and the similarity measure (Zhou *et al*. 2016, 2017).

Also, some methods mixed with optimization algorithm are considered as appropriate ones and replacement for evaluating damage of structure. These methods are also named two-step or hybrid damage identification approaches. Nobahari *et al*. (2017) have introduced a two-step method to detect damage of truss structures. Xu *et al*.

*Corresponding author, Ph.D.,
E-mail: hoseinivaez@qom.ac.ir
[a] Ph.D. Student, E-mail: nfallah@stu.qom.ac.ir
[b] M.Sc. Student, E-mail: h.fasihi@stu.qom.ac.ir

http://www.techno-press.org/?journal=scs&subpage=6
ISSN: 1229-9367 (Print), 1598-6233 (Online)

(2015) have proposed a new simple and computationally efficient optimization algorithm and combined the Gauss-Newton method with region truncation of each iterative step. Mousavi and Gandomi have provided a new hybrid method which uses only one mode shape and its corresponding eigenvalue of structure (Mousavi and Gandomi 2016). They utilized a transformation matrix of the dynamic condensation technique to carry out damage identification in several structures using incomplete data. Seyedpoor and Montazer have proposed a two-step approach based on Modal Residual Vector Based Indicator (MRVBI) and a Differential Evaluation (DE) algorithm to identify damage of truss structures (Seyedpoor and Montazer 2016). Applying their method to structures with rotational DOFs needs measuring complete modal data while in practice the number of measured DOFs is most of the time restricted.

In the present study, a new mixed method is suggested to detect damages in a laminated composite plate. In which, damage in an element is considered degradation in its stiffness matrix. Consequently, the stiffness reduction of the whole structure has been simulated. In the present work, a Condensed Modal Residual Vector Based Indicator (CMRVBI) is proposed to locate the damaged elements in the first step. This indicator utilized only translational DOFs of mode shapes and is formulated using a transformation matrix of the dynamic condensation technique (Guyan 1965, O'Callahan 1989, Carvalho *et al*. 2007, Mousavi and Gandomi 2016) and MRVBI (Seyedpoor and Montazer 2016). Then in the second step, the reported damaged elements are quantified by Salp Swarm Algorithm (SSA) and Genetic Algorithm (GA). The capability of this method is evaluated through damage identification on the Ferreira example (Ferreira *et al*. 2009). The results illustrate the merits of the proposed approach in indicating damaged and healthy elements in structures with rotational DOFs even with the lack of the measured translational DOFs.

2. Theoretical description

2.1 Dynamic condensation scheme

The equilibrium equation for the analysis of the free vibration of a non-damping system in the time domain is defined as follows

$$M\ddot{X}(t) + KX(t) = 0 \tag{1}$$

where **K** and **M** are the matrices of the mass and stiffness of the structure, which are assumed to be positive semidefinite and positive definite, respectively. In displacement space, the eigen-value of the corresponding problem in this structure is

$$\mathbf{K\Phi} = \mathbf{M\Phi\Lambda} \tag{2}$$

where $\mathbf{\Lambda}$ is a diagonal matrix representing eigen-values, and $\mathbf{\Phi}$ is mode shape matrix in which each column shows a mode shape of the free vibration of the system. Using the dynamic condensation scheme, this equation can be represented as follows

$$\begin{bmatrix} \mathbf{K}_{\mathrm{mm}} & \mathbf{K}_{\mathrm{ms}} \\ \mathbf{K}_{\mathrm{sm}} & \mathbf{K}_{\mathrm{ss}} \end{bmatrix} \begin{Bmatrix} \mathbf{\Phi}_{\mathrm{mm}} \\ \mathbf{\Phi}_{\mathrm{sm}} \end{Bmatrix} = \begin{bmatrix} \mathbf{M}_{\mathrm{mm}} & \mathbf{M}_{\mathrm{ms}} \\ \mathbf{M}_{\mathrm{sm}} & \mathbf{M}_{\mathrm{ss}} \end{bmatrix} \begin{Bmatrix} \mathbf{\Phi}_{\mathrm{mm}} \\ \mathbf{\Phi}_{\mathrm{sm}} \end{Bmatrix} \mathbf{\Lambda}_{\mathrm{mm}} \tag{3}$$

in which, the subscript s and m denote slave and master DOFs that are considered as rotational and translational DOFs in this paper. The slave DOFs are eliminated to compute the reduced system with the remaining master DOFs.

As aforesaid, the main purpose is to calculate the unmeasured part of the mode shape matrix from measured part. By using a transformation matrix, we have (Mousavi and Gandomi 2016)

$$\mathbf{\Phi}_{\mathrm{sm}} = \mathbf{t}\mathbf{\Phi}_{\mathrm{mm}} \tag{4}$$

By substituting Eq. (4) into the second row of Eq. (3) and rearranging it for the transformation matrix we have (Mousavi and Gandomi 2016)

$$\begin{aligned} \mathbf{t} &= -\mathbf{K}_{\mathrm{ss}}^{-1}\mathbf{K}_{\mathrm{sm}} + \mathbf{K}_{\mathrm{ss}}^{-1}(\mathbf{M}_{\mathrm{sm}} + \mathbf{K}_{\mathrm{ss}}\mathbf{t})\mathbf{\Phi}_{\mathrm{mm}}\mathbf{\Lambda}_{\mathrm{mm}}\mathbf{\Phi}_{\mathrm{mm}}^{-1} \\ \mathbf{t}^{(0)} &= -\mathbf{K}_{\mathrm{ss}}^{-1}\mathbf{K}_{\mathrm{sm}} \end{aligned} \tag{5}$$

Using this transformation matrix, the entire field can be calculated from the reduced field corresponding to the master DOFs

$$\begin{bmatrix} \mathbf{\Phi}_{\mathrm{mm}} \\ \mathbf{\Phi}_{\mathrm{sm}} \end{bmatrix} = \begin{bmatrix} \mathbf{I}_{\mathrm{mm}} \\ \mathbf{t} \end{bmatrix} \mathbf{\Phi}_{\mathrm{mm}} = \mathbf{T}\mathbf{\Phi}_{\mathrm{mm}} \tag{6}$$

where $\mathbf{I}_{\mathbf{mm}}$ is the same matrix of size $m \times m$. In this study $\mathbf{t}$ is assumed equal to $\mathbf{t}^{(0)}$ in which **K** is the stiffness matrix of (intact) undamaged structure.

2.2 The proposed condensed modal residual vector based indicator

Modal residual vectors is one type of the vibration-based damage detection methods (Mares and Surace 1996). Many researchers have developed this method and proposed some indicators such as MRVBI (Seyedpoor and Montazer 2016). The methods are based on the complete measurements of natural frequencies and mode shapes of damaged system. However, the measurement of rotational DOFs especially in 3D structures, is not economically feasible due to its very expensive sensors, and is usually very inaccurate and difficult.

In this article, for locating the damaged elements of structures with rotational DOFs, a condensed modal residual vector based indicator (CMRVBI) is proposed. Assuming no change in the matrix before and after damage, the eigen-value equation of a damaged structure is obtained as follows

$$(\mathbf{K}_{\mathrm{d}} - \omega_{\mathrm{di}}^{2}\,\mathbf{M})\,\mathbf{\Phi}_{\mathrm{di}} = 0 \tag{7}$$

where the subscript *d* represents the damaged condition and ω_{di} is the *i*th frequency of damaged structure. The stiffness matrix of the damaged structure can be expressed as follows

(Mares and Surace 1996)

$$\mathbf{K}_{\mathrm{d}} = \mathbf{K} - \Delta\mathbf{K} = \mathbf{K} - \sum_{\mathrm{j}=1}^{n} \beta_{\mathrm{j}} \mathbf{K}_{\mathrm{j}} \tag{8}$$

where $\mathbf{K}_j$ is the stiffness matrix of *j*th element of the structure; *n* is the number of elements. β_j is a reduction factor applied to the stiffness and its value considered in the range [0 1], 0 for an intact element and 1 for a fully damaged element.

By substituting Eq. (9) in Eq. (8), the modal residual vector for *i*th mode of the structure can be represented as (Mares and Surace 1996)

$$\mathbf{R}_{\mathrm{i}} = (\mathbf{K} - \omega_{\mathrm{di}}^{2}\, \mathbf{M})\, \mathbf{\Phi}_{\mathrm{di}} = \Delta\mathbf{K}\mathbf{\Phi}_{\mathrm{di}} \tag{9}$$

In the $\Delta\mathbf{K}$ matrix, zero and non-zero components are related to those DOFs that that are associated to a damaged and intact element, respectively. Also in R_i, the non-zero components will lie along the same DOFs. This connectivity relation between DOFs and elements can be used to determine the location of the damage.

In this study, the condensed modal residual vector for ith mode of the structure with rotational DOFs associated to $\mathbf{\Phi}_i = \mathbf{T}(\mathbf{\Phi}_{di})_{mm}$ can be represented as follows

$$\mathbf{R}_{\mathrm{i}}^{\mathrm{c}} = \mathbf{T}^{\mathrm{T}}\mathbf{R}_{\mathrm{i}} = \mathbf{T}^{\mathrm{T}}(\mathbf{K}_{\mathrm{p}} - \omega_{\mathrm{di}}^{2}\, \mathbf{M}_{\mathrm{p}})\, \mathbf{T}(\mathbf{\Phi}_{\mathrm{di}})_{\mathrm{mm}} \tag{10}$$

where $\mathbf{K}_p$ and $\mathbf{M}_p$ are the partitioned form of the stiffness and mass matrices of intact structure. The condensed absolute sum of the modal residual (CASMR) vectors to *m* modes of the structure and its translational DOFs is given by

$$\mathbf{CASMR} = \sum_{\mathrm{i}=1}^{m} \left| \mathbf{R}_{\mathrm{i}}^{\mathrm{c}} \right| \tag{11}$$

An indicator based on the CASMR values of each element is used to localize the damaged elements

$$L_{\mathrm{j}} = (\mathbf{N}_{\mathrm{j}}^{\mathrm{T}}\mathbf{N}_{\mathrm{j}})^{4} \;\;, j = 1, 2, ..., n \tag{12}$$

where N is a vector containing the CASMR components of the translational DOFs corresponding to *j*th element. Therefore, the size of this vector for the composite plates is 12×1 because every element has four nodes and every node has three translational DOFs (4×3). The mean and standard deviation of the vector are used to normalize the damage index (*L*) as follows

$$L_{\mathrm{j}}^{\mathrm{n}} = \frac{[L_{\mathrm{j}} - \mathrm{mean}(\mathbf{L})]}{\mathrm{std}(\mathbf{L})} \;\;, j = 1, 2, ..., n \tag{13}$$

The condensed modal residual vector based indicator (CMRVBI) is introduced as

$$\mathbf{CMRVBI} = \frac{L_{\mathrm{j}}^{\mathrm{n}}}{\left\| L_{\mathrm{j}} \right\|} \;\;, j = 1, 2, ..., n \tag{14}$$

which ||.|| represents the magnitude of a vector.

This indicator can be employed to locate the suspected damaged elements of the structure with rotational DOFs. In this article, the any element with CMRVBI > 0.1 is considered as a suspected damaged element.

3. Optimization method

To estimate damage severity of suspected damaged elements, an inverse optimization problem should be solved. It should be noted that any metaheuristic algorithm such as grey wolf optimizer algorithm (Mirjalili *et al*. 2014b), binary bat algorithm (Mirjalili *et al*. 2014a), teaching–learning-based optimization algorithm (Xing and Gao 2014) and flower pollination algorithm (Yang *et al*. 2014) etc. can be used and it is possible that the results of these algorithms be better than results of SSA and GA.

Here, an objective function based on changes of structural modal flexibility is defined. Structural flexibility is more sensitive to damage than modal data such as natural frequencies and mode shapes. According to these points, Perera *et al*. proposed following objective function (Perera *et al*. 2009)

$$F = 1 - MACFLEX = 1 - \prod_{\mathrm{j}=1}^{nm} MACFLEX_{\mathrm{j}} \tag{15}$$

where

$$\mathbf{MACFLEX}_{\mathrm{j}} = \frac{|\,\mathbf{F}_{\mathrm{jnum}}^{\mathrm{T}}\mathbf{F}_{\mathrm{jexp}}\,|^{2}}{(\mathbf{F}_{\mathrm{jnum}}^{\mathrm{T}}\mathbf{F}_{\mathrm{jnum}})(\mathbf{F}_{\mathrm{jexp}}^{\mathrm{T}}\mathbf{F}_{\mathrm{jexp}})} \tag{16}$$

in which, $\mathbf{F}_{jnum}$ and $\mathbf{F}_{jexp}$ are computed and measured (experimental) flexibility vectors corresponding to *j*th mode shape respectively, which collect the diagonal terms of the flexibility matrix, *MAC* is a modal assurance criterion which measures correlation between two vector $\mathbf{F}_{jexp}$ and $\mathbf{F}_{jnum}$. Objective function values are normalized between 0 and 1 that low and high values of them indicate low and high correlation, respectively.

The details of used metaheuristic algorithms are

3.1 Salp Swarm Algorithm (SSA)

SSA is a swarm-based technique introduced by Mirjalili *et al*. (2017). The main inspiration of this algorithm is the swarming behavior of salps during foraging and navigating in oceans. Salps form a swarm called salp chain commonly. The behavior is done for receiving better locomotion with using rapid coordinated changes and foraging (Anderson and Bone 1980).

To model the salp chains, the population is divided to two groups including leader and followers firstly. The leader is a salp at the front of the chain and guides swarm. As the swarm's target, there is a food source F in the search space. Also, there is an n-dimensional search space where n is the number of variables of an optimization problem. So the position of all salps are defined in it and stored in a two-dimensional matrix called *x* as follows

$$x_j^1 = \begin{cases} F_j + c_1((ub_j - lb_j)\ c_2 + lb_j) & c_3 < 0.5 \\ F_j - c_1((ub_j - lb_j)\ c_2 + lb_j) & else \end{cases} \tag{17}$$

$$x_j^i = \frac{1}{2}at^2 + v_0 t \quad \forall\ i \ge 2 \tag{18}$$

where x_j^1 and x_j^i are the position of the leader and ith followers in jth dimension, respectively; ub_j and lb_j indicate the upper and lower bound of *j*th dimension, respectively; c_2 and c_3 are random numbers uniformly distributed in the range (0,1). According to Eq. (17), the coefficient c_1 is the most important in SSA as it balances exploitation and exploration and is determined by Eq. (20). Parameter t is time; v_0 is initial speed which is considered equal to 0; $a = v_{final}/v_0$ where $v = (x - x_0)/t$. As in optimization the time is iteration and discrepancy between iterations is equal to 1, Eq. (18) can be expressed by Eq. (19).

$$x_j^i = \frac{1}{2}(x_j^i + x_j^{i-1}) \quad \forall\ i \ge 2 \tag{19}$$

$$c_1 = \exp(-(\frac{4t}{t_{\max}})^2) \tag{20}$$

in which, t and $t_{\max}$ are the current iteration and maximum number of iterations. Using Eqs. (17) to (20), the sap chains can be simulated.

In order to better understand how the SSA and salp chain model are efficient in solving optimization problems, some remarks are expressed as follows:

- The best answer obtained so far is saved by SSA and assigned to the food source variable, so it never gets lost even if all salps deteriorate.
- The position of the leader salp is only updated with respect to the food source which is the best answer obtained so far, therefore the leading salp always explores and exploits the space around it.
- In SSA, the position of follower salps is updated according to each other, so they move towards the leader salp gradually.
- Gradual movements of the followers salps preserve the SSA from easily trapping in local optimum.
- The SSA first explores the search space and then exploits it as the coefficient c1 is decreased adaptively during iterations.
- The coefficient c_1 is the only main controlling parameters of SSA.
- SSA is a simple algorithm and implementing of it is easy.

More information about the pseudo codes, inspiration and the motivation of SSA can be found in Ref. (Mirjalili *et al*. 2017).

3.2 Genetic algorithm

Genetic algorithm (GA) is the most famous and applied evolutionary computational methods which has been proposed in early 1970s by John Holland (Holland 1975). GA has many aspects being done in different ways considering problems such as solution representation (chromosomes), selection scheme, the crossover type, and mutation operators. Crossover is often known as the main variation operator which is included several individuals (mainly two) selected by replacing some of their parts with the others. Moreover, some strategies such as n-point and uniform crossover can do this. Parameter *pc* is the crossover

GA

1. Generate initial population with random individuals
2. Evaluate the cost of every individual
3. Repeat following steps until a termination condition is met
4. Select parents
5. Crossover (recombine) pairs of parents with probability
6. Apply mutation
7. Evaluate new merged individuals
8. select individuals for the next generation
9. Go to 2 or termination if termination condition is met

Fig. 1 The framework of the GA (Hoseini Vaez and Fallah 2017)

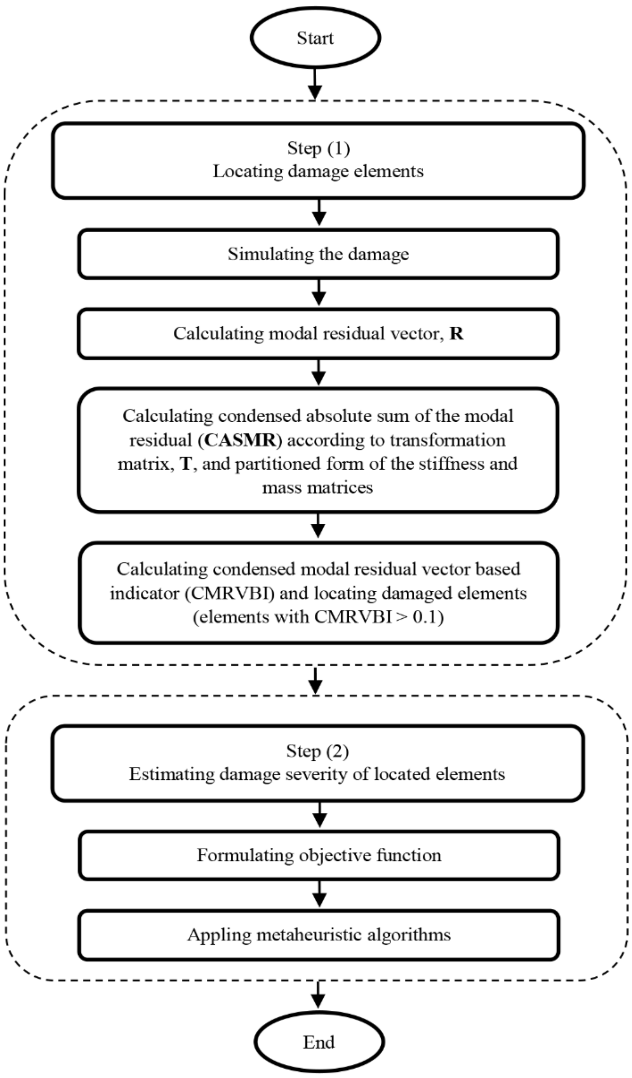
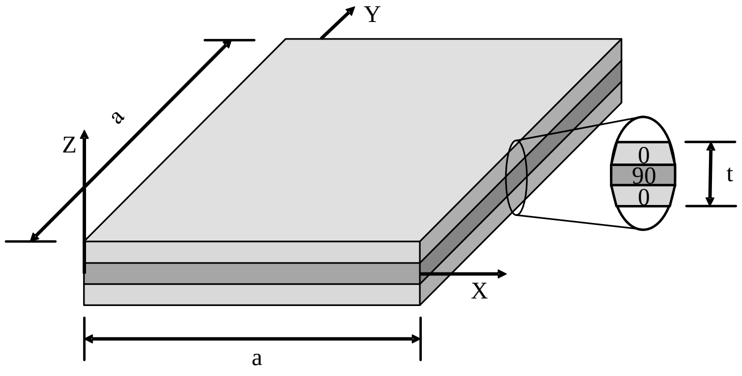
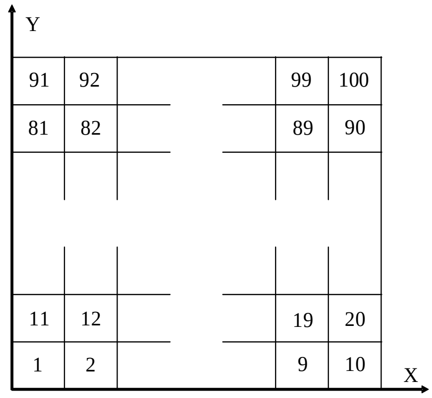


Fig. 2 Flowchart of the proposed approach



(a) A three-layer square laminated composite plate (0°/90°/0°)



(b) Element numbering of the laminated composite plate

Fig. 3 The geometry of the laminated composite plate

rate and shows the probability of being subject to the crossover for each individual and its value is usually between 0.6 and 1 (Bäck and Schwefel 1993). Individuals are evaluated considering their cost value in the selection process and are selected to produce offspring using the objective (cost) function of the optimization problem. Selection schemes include roulette-wheel selection, tournament selection, and ranking selection often. More information about comparisons of selection schemes is introduced in (Blickle and Thiele 1995). The mutation process is applied to the individuals after applying crossover. So new random variables are provided which prohibits the algorithm from trapping in local optima. The mutation rate is *pm* and its value is determined considering optimization problem typically. A complete reference and review of genetic algorithms could be found in (Beasley *et al*. 1993a, b, Michalawicz 1996). The framework of the GA is shown in Fig. 1.

4. Numerical examples

According to details of theoretical description and optimization method in Sections 2 and 3, the proposed approach has two main steps as shown in Fig. 2. In this study, the proposed damage identification approach has been studied on a three-layer square laminated composite plate (0°/90°/0°) with clamped boundaries. The codes of Finite Element (FE) analysis corresponding to the laminated composite plate and the method have been written in MATLAB. Three damage scenarios with 4, 5 and 6 damaged elements have been evaluated in such a way that the damage in the plate has been simulated by reducing the stiffness of chosen elements, i.e. $K_d^e = (1-\beta)K_{di}^e$, in which K_d^e and K_i^e are the stiffness matrices corresponding to eth damaged and intact elements of plate .Also, β is the damage extent where the amount is between 0 and 1 for completely intact and damaged element, respectively.

The material parameters and geometry of the laminated composite plate have been given in Fig. 3 and Table 1. This plate has previously been analyzed in (Ferreira *et al*. 2009). As can be seen from Table 2, the details of three damage scenarios have been summarized.

Table 1 The parameters of the three-layer square laminated composite plate

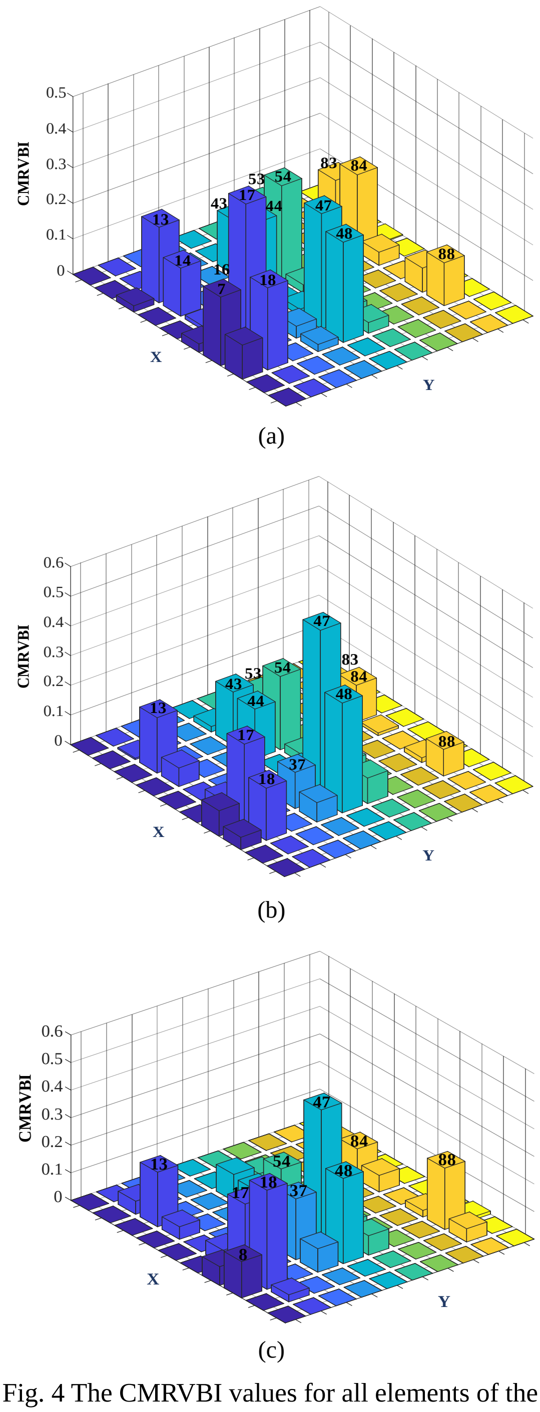
Parameter / unit	Value
The length of a side (a) / m	1
The thickness (t) / m	0.1
Young's Modulus (E_1) / N/m^2	40
Young's Modulus (E_1) / N/m^2	1
Poisson ratio ν_{12}	0.25
Poisson ratio ν_{21}	0.00625

Table 2 Damage scenarios of 100-element laminated composite plate

Scenario 1		Scenario 2		Scenario 3	
Element number	Damage extent	Element number	Damage extent	Element number	Damage extent
47	0.15	17	0.15	17	0.20
48	0.20	18	0.27	48	0.15
54	0.25	57	0.17	53	0.12
84	0.10	58	0.22	57	0.17
		87	0.10	58	0.22
				87	0.10

4.1 Results of CMRVBI in noise-free condition

Figs. 4, 5 and 6 show damage location charts for the first, second and third scenarios of the three-layer square laminated composite plate structure, respectively, in the presence of different amounts of considered modes in noise-free condition. It is worth mentioning that only the values greater than zero have been shown also, the number of elements with CMRVBI > 0.1 are reported in the figures. From the figures, it can be realized that the number of suspected damaged elements identified by CMRVBI in case of lower number of modes (4 modes) is more than the larger ones (5 and 6 modes). Consequently, the approach can accurately localize the damage and the accuracy of the proposed method increases with increasing the number of

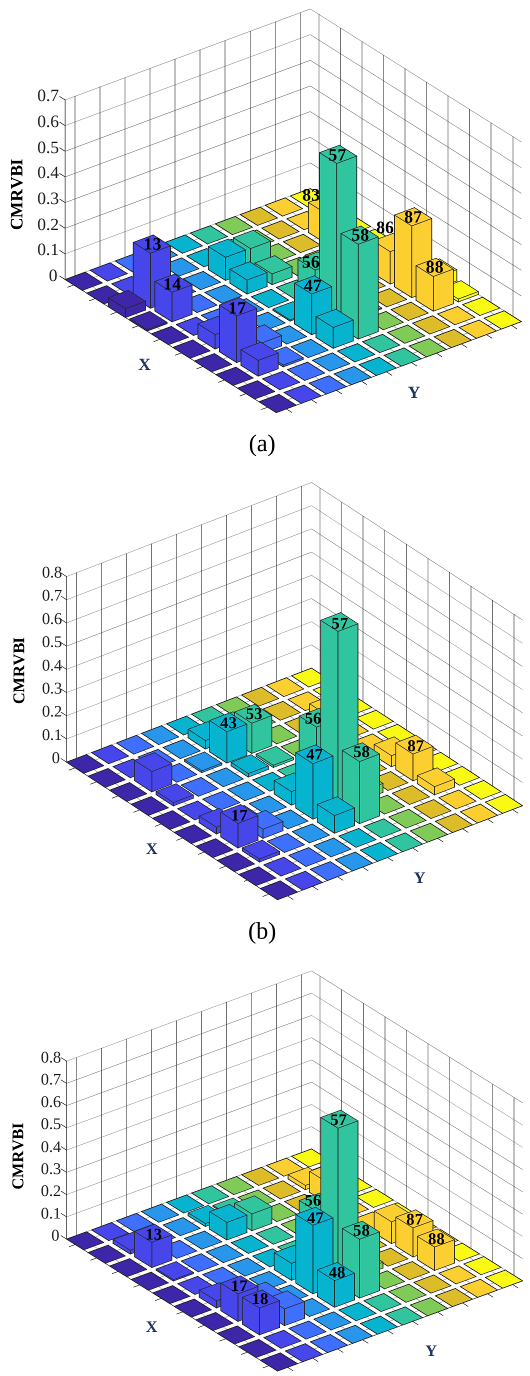


(a)

(b)

(c)

Fig. 4 The CMRVBI values for all elements of the laminated composite plate for scenario 1 with (a) 4; (b) 5; and (c) 6 modes



(a)

(b)

(c)

Fig. 5 The CMRVBI values for all elements of the laminated composite plate for scenario 2 with (a) 4; (b) 5; and (c) 6 modes

considered modes.

4.2 Study noise effect

In order to study noise effect on the performance of the proposed method, the noise by a standard error of ±2% and ±5% (*noise*$_f$ and *noise*$_m$) has been used which influences directly the natural frequencies and mode shapes (Hoseini Vaez and Fallah 2017, 2018)

$$\omega_i^{noisy} = \omega_i^d \times (1 + \alpha \times noise_f) \tag{21}$$

$$\varphi_{i,j}^{noisy} = \varphi_{i,j}^d \times (1 + \alpha \times noise_m) \tag{22}$$

where *noisy* implies noisy condition; α is a uniformly distributed randomly number between –1 and +1.

Figs. 7-9 show the mean of the CMRVBI and MRVBI amounts for 100 independent runs for the three damage scenarios in noisy condition and also considering 4 modes.

As it is seen in the figures, this method can correctly locate the damage even at the presence of the noise effect.

As shown in the Figs. 5-7, those elements having the most damage possibility which have been identified by CMRVBI, are elements 7, 13, 14, 16, 17, 18, 43, 44, 47, 48, 53, 54, 83, 84 and 88 for the first scenario, also elements 13, 14, 17, 47, 56, 57, 58, 83, 86, 87 and 88 for the second scenario 2, and elements 13, 14, 17, 18, 47, 48, 53, 56, 57,

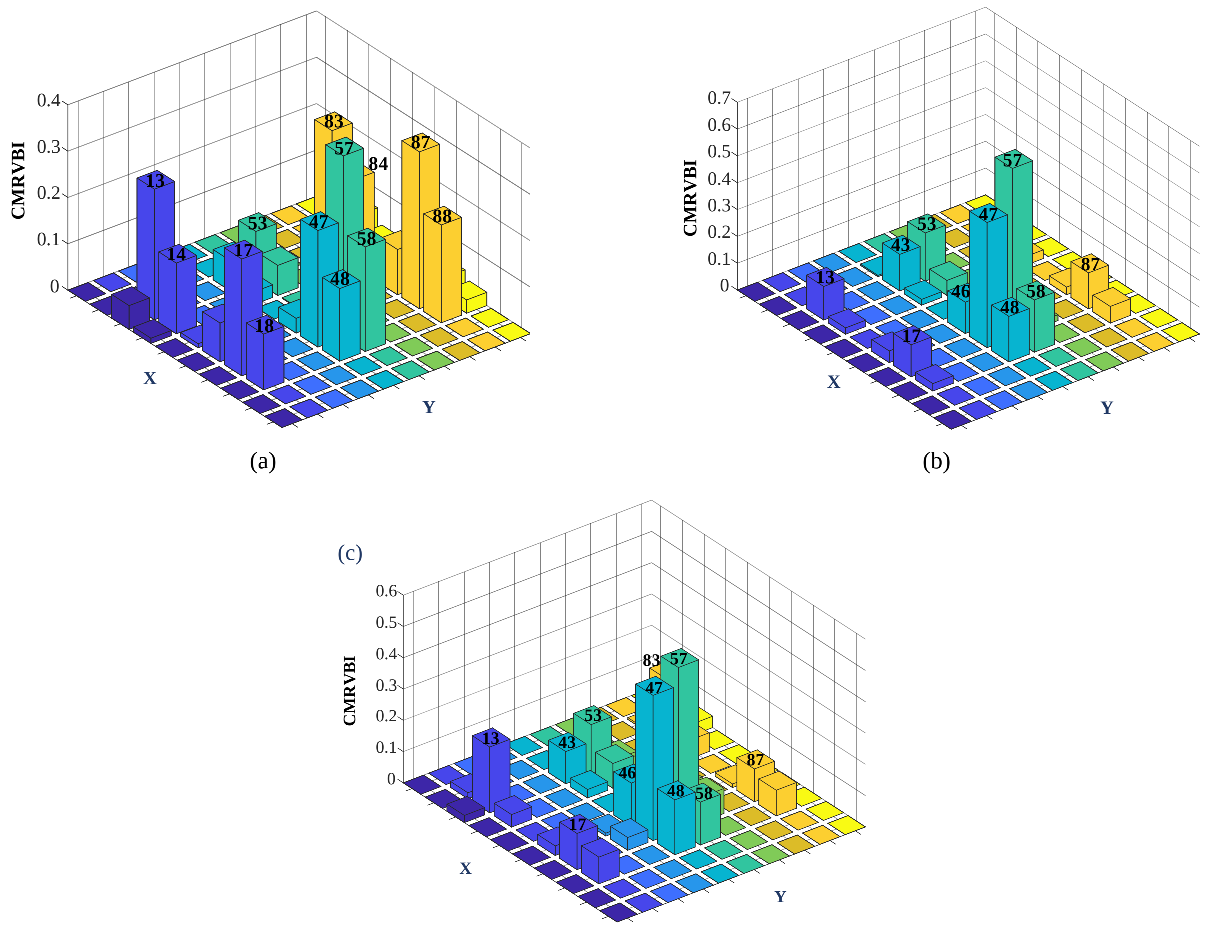


Fig. 6 The CMRVBI values for all elements of the laminated composite plate for scenario 3 with (a) 4; (b) 5; and (c) 6 modes

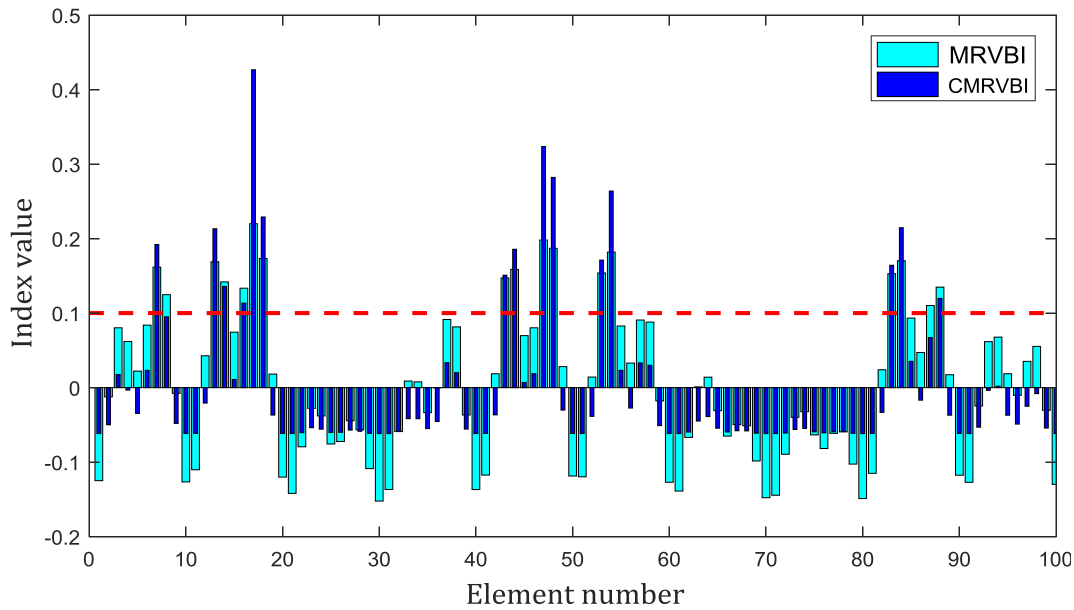


Fig. 7 Damage identification chart for the first scenario considering noise and 4 modes

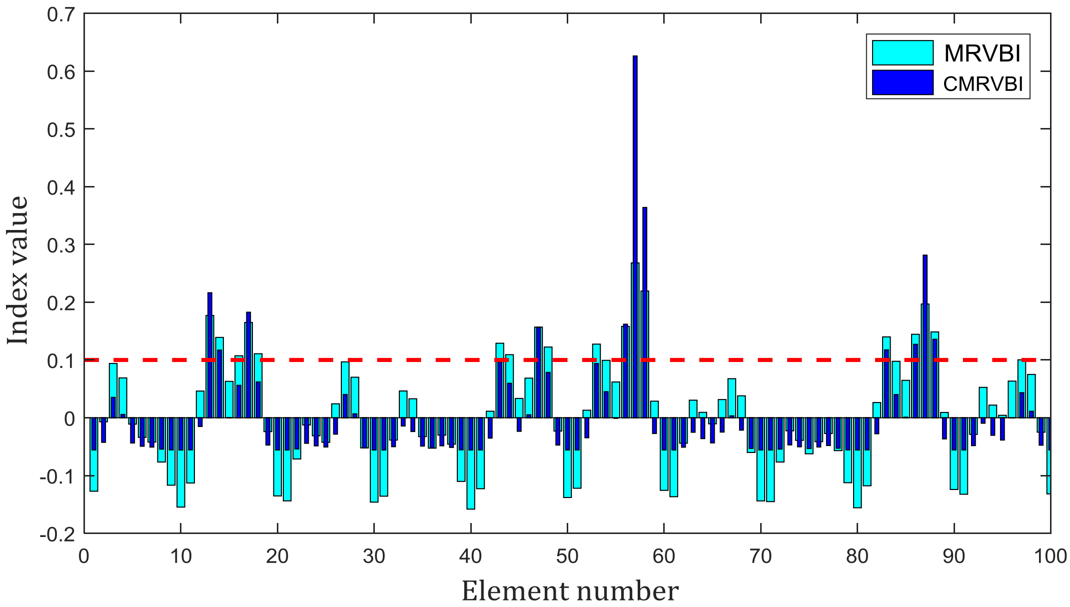


Fig. 8 Damage identification chart for the second scenario considering noise and 4 modes

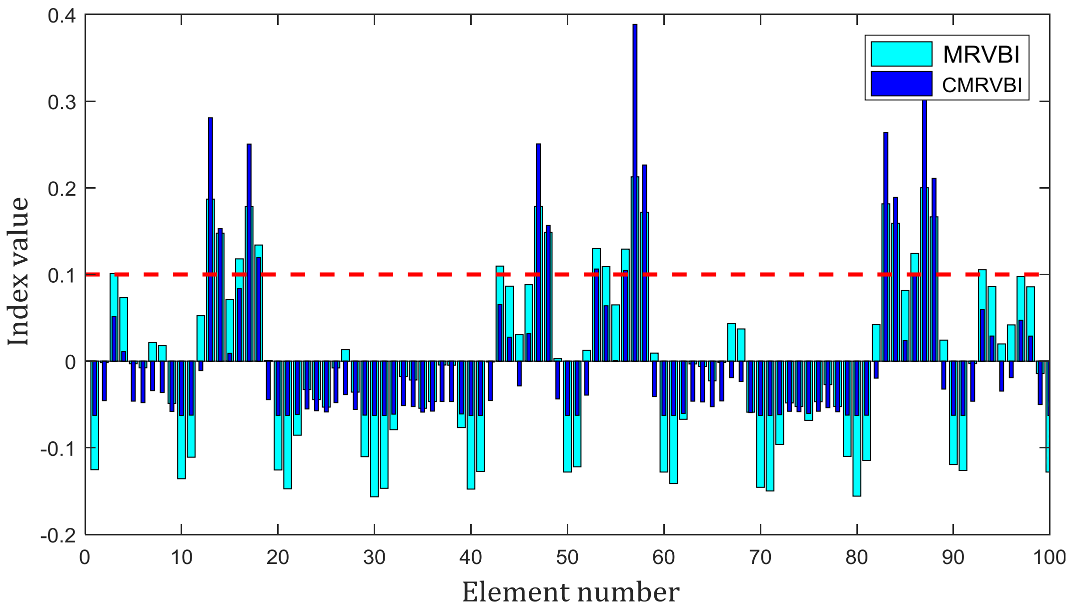


Fig. 9 Damage identification chart for the third scenario considering noise and 4 modes

58, 83, 84, 87 and 88 for the third scenario.

Moreover, the elements having the most damage possibility which have been identified by MRVBI are elements 7, 8, 13, 14, 16, 17, 18, 43, 44, 47, 48, 53, 54, 83, 84, 87 and 88 for the first scenario, also elements 13, 14, 16, 17, 18, 43, 44, 47, 48, 53, 56, 57, 58, 83, 86, 87, 88 and 97 for the third scenario, and elements 3, 13, 14, 16, 17, 18, 43, 47, 48, 53, 54, 56, 57, 58, 83, 84, 86, 87, 88 and 93 for the third scenario. As it is seen, in the all scenarios, the real damaged elements are located by MRVBI. But the element 18 in the scenario 2 is not localized by CMRVBI. While number of misidentified (intact) elements found by MRVBI are much greater than CMRVBI. The scenario 2 is also run using 6 first modes and the results are elements 13, 17, 18, 47, 48, 56, 57, 58, 87 and 88 for CMRVBI and 13, 17, 18, 27, 28, 43, 46, 47, 48, 53, 56, 57, 58, 83, 86, 87 and 88 for MRVBI. So the element 18 is found by CMRVBI with increasing used modes.

Therefore, the accuracy of damage identification corresponding to CMRVBI is more in comparison to MRVBI in this laminated composite plate, despite the inequality of the freedom degrees used in modes.

4.3 Damage estimation by optimization algorithms

In this step, the suspected damaged elements localized by CMRVBI are quantified by the two different optimization algorithms using solving an inverse optimization problem. These algorithms consist of Salp Swarm Algorithms (SSA) and Genetic Algorithm (GA).

Then the capability of the algorithms to estimate damage severity of the found elements by the first step is compared. For all scenarios, 30 independent runs are made in noisy condition and the mean and the best (with least value of objective function) solutions have been reported. For all scenarios, the number of iterations are considered as 200 and population sizes are assumed as 100. The results have been reported to three decimal places and the values less than 0.01 have been assumed equal to zero. In this step, only translational DOFs of the four first modes have been used for all scenarios. Tables 3-5 show damage severities of suspected damaged elements estimated by SSA and GA.

Total amounts of damage for intact elements in mean state of SSA algorithm corresponding to the first, second and third scenarios are equal to 0.032, 0.015 and 0.04, respectively, also about best state are equal to 0.024, 0 and 0, respectively. These values are equal to 0.061, 0.019 and 0.152 for GA algorithm in mean state and are 0, 0.03 and 0 in best state. Also, the sum of the absolute for differences in values of damage corresponding to real damaged elements is calculated. This value in mean state of SSA algorithm corresponding to the first, second and third scenarios is 0.041, 0.021 and 0.029, respectively and 0.012, 0 and 0.011 in best state. These values are equal to 0.048, 0.01 and 0.066 for GA algorithm in mean state and are 0.01, 0 and 0.03 in best state.

Table 3 Damage severities of suspected damaged elements for first scenario of the laminated composite plate

Algorithm		Elements and damage severities
		7, 13, 14, 16, 17, 18, 43, 44, 47, 48, 53, 54, 83, 84, 88
SSA	Mean	0.012, 0, 0, 0, 0, 0, 0, 0, 0.210, 0.182, 0, 0.098, 0.010, 0.251, 0.010
	Best	0.010, 0, 0, 0, 0, 0, 0, 0, 0.201, 0.161, 0, 0.102, 0.014, 0.248, 0
GA	Mean	0.033, 0, 0, 0, 0, 0, 0, 0, 0.210, 0.173, 0.011, 0.092, 0.017, 0.273, 0
	Best	0, 0, 0, 0, 0, 0, 0, 0, 0.196, 0.158, 0, 0.105, 0, 0.247, 0

Table 4 Damage severities of suspected damaged elements for second scenario of the laminated composite plate

Algorithm		Elements and damage severities
		13, 14, 17, 47, 56, 57, 58, 83, 86, 87, 88
SSA	Mean	0, 0, 0.151, 0, 0, 0.169, 0.217, 0, 0, 0.082, 0.015
	Best	0, 0, 0.150, 0, 0, 0.170, 0.220, 0, 0, 0.100, 0
GA	Mean	0, 0, 0.155, 0, 0, 0.145, 0.236, 0, 0, 0.094, 0.019
	Best	0, 0, 0.153, 0, 0, 0.169, 0.221, 0, 0.03, 0.097, 0

Table 5 Damage severities of suspected damaged elements for third scenario of the laminated composite plate

Algorithm		Elements and damage severities
		13, 14, 17, 18, 47, 48, 53, 56, 57, 58, 83, 84, 87, 88
SSA	Mean	0, 0, 0.185, 0.02, 0, 0.148, 0.121, 0, 0.160, 0.228, 0, 0, 0.089, 0.020
	Best	0, 0, 0.199, 0, 0, 0.150, 0.120, 0, 0.172, 0.210, 0, 0, 0.099, 0
GA	Mean	0, 0, 0.163, 0.071, 0.013, 0.135, 0.125, 0, 0.147, 0.266, 0, 0, 0.058, 0.068
	Best	0, 0, 0.193, 0, 0, 0.153, 0.122, 0, 0.164, 0.259, 0, 0, 0.098, 0

In addition, the results of applying CMRVBI index for the second scenario considering six first modes are run using both algorithms. Total amounts of damage for intact elements in mean state are equal to 0.002 and 0.032 for SSA and GA, respectively; and the sum of the absolute values of damage for real damaged elements are 0.002 and 0.109 for SSA and GA, respectively. Also, total amounts of damage for intact elements in best state are equal to 0.001 and 0.043 for SSA and GA, respectively; and the sum of the absolute for differences in values of damage corresponding to real damaged elements are 0 and 0.001 for SSA and GA, respectively.

Figs. 10, 11 and 12 show the mean of variation of the

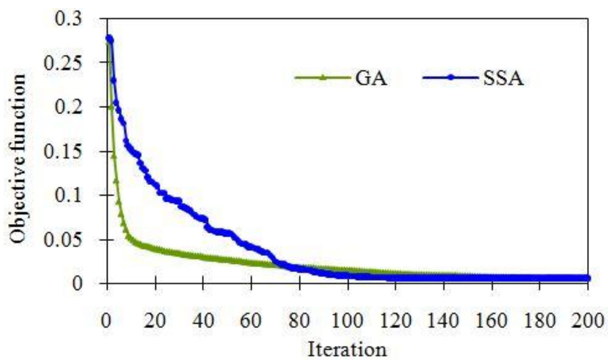


Fig. 10 The variation of the objective function with the number of iterations for the first scenario

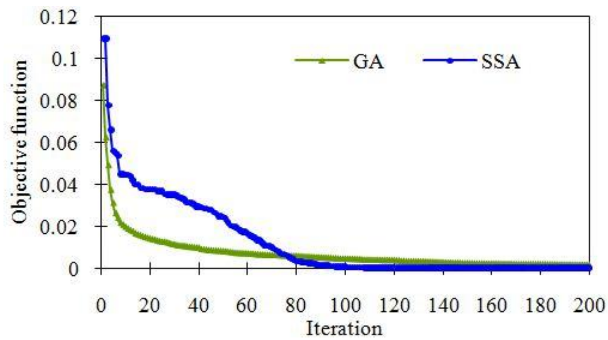


Fig. 11 The variation of the objective function with the number of iterations for the second scenario

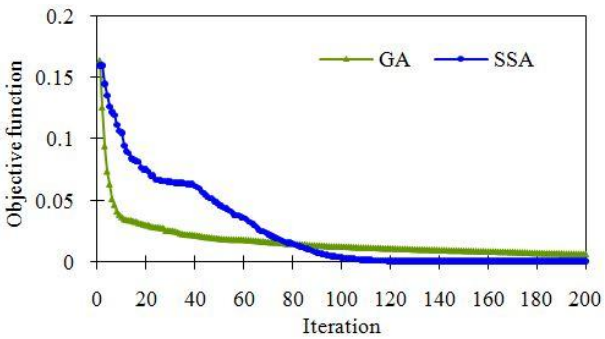


Fig. 12 The variation of the objective function with the number of iterations for the third scenario

objective function with the number of iterations in 30 independent runs for the first, second and third scenario, respectively.

5. Conclusions

In this paper, an efficient multi-step method is proposed to locate and quantify multi-damage scenarios of structures that have rotational DOFs. In the first step, the condensation technique and MRVBI have been combined to introduce CMRVBI. This new indicator finds the suspected damaged elements and reduces the variables of inverse optimization problem. In the next step, SSA and GA algorithms have been utilized to quantify damage severity of the reported elements from the first step. A numerical examples including a three-layer square laminated composite plate has been studied. In both steps, the damage localization and quantification have been performed in noisy condition. The results show that the CMRVBI has located almost all of the actual damaged elements in all considered damage scenarios and also found few intact elements despite using low number of modes and its DOFs. All of these intact elements have been quantified approximately equal to zero by SSA algorithm in the best (with least value of objective function) state, so they have been known as intact elements. Therefore, the results demonstrate that the proposed damage identification is efficient for the studied structure.

References

Anderson, P.A. and Bone, Q. (1980), "Communication between individuals in salp chains. II. Physiology", *Proc. R. Soc. Lond. B*.

Bäck, T. and Schwefel, H.-P. (1993), "An overview of evolutionary algorithms for parameter optimization", *Evolut. Computat.*, **1**(1), 1-23.

Beasley, D., Martin, R. and Bull, D. (1993a), "An overview of genetic algorithms: Part 1. Fundamentals", *Univ. Comput.*, **15**, 58-58.

Beasley, D., Bull, D.R. and Martin, R.R. (1993b), "An overview of genetic algorithms: Part 2, research topics", *Univ. Comput.*, **15**(4), 170-181.

Blickle, T. and Thiele, L. (1995), *A Comparison of Selection Schemes used in Genetic Algorithms*, TIK-Report.

Carden, E.P. and Fanning, P. (2004), "Vibration based condition monitoring: a review", *Struct. Health Monitor.*, **3**(4), 355-377.

Carvalho, J., Datta, B.N., Gupta, A. and Lagadapati, M. (2007), "A direct method for model updating with incomplete measured data and without spurious modes", *Mech. Syst. Signal Process.*, **21**(7), 2715-2731.

Doebling, S.W., Farrar, C.R., Prime, M.B. and Shevitz, D.W. (1996), *Damage Identification and Health Monitoring of Structural and Mechanical Systems from Changes in their Vibration Characteristics: a Literature Review*, Los Alamos National Lab., NM, USA.

Fallah, N., Vaez, S.R.H. and Mohammadzadeh, A. (2018), "Multi-damage identification of large-scale truss structures using a two-step approach", *J. Build. Eng.*, **19**, 494-505.

Fan, W. and Qiao, P. (2011), "Vibration-based damage identification methods: a review and comparative study", *Struct. Health Monitor.*, **10**(1), 83-111.

Ferreira, A., Castro, L.M. and Bertoluzza, S. (2009), "A high order collocation method for the static and vibration analysis of composite plates using a first-order theory", *Compos. Struct.*, **89**(3), 424-432.

Guyan, R.J. (1965), "Reduction of stiffness and mass matrices", *AIAA Journal*, **3**(2), 380.

Holland, J.H. (1975), *Adaptation in Natural and Artificial Systems: an Introductory Analysis with Applications to Biology, Control, and Artificial Intelligence*, University of Michigan Press

Hoseini Vaez, S.R. and Fallah, N. (2017), "Damage Detection of Thin Plates Using GA-PSO Algorithm Based on Modal Data", *Arab. J. Sci. Eng.*, **42**(3), 1251-1263.

Hoseini Vaez, S.R. and Fallah, N. (2018), "Damage identification of a 2D frame structure using two-stage approach", *J. Mech. Sci. Technol.*, **32**(3), 1125-1133.

Humar, J., Bagchi, A. and Xu, H. (2006), "Performance of vibration-based techniques for the identification of structural damage", *Struct. Health Monitor.*, **5**(3), 215-241.

Kaveh, A., Hoseini Vaez, S.R., Hosseini, P. and Fallah, N. (2016), "Detection of damage in truss structures using Simplified Dolphin Echolocation algorithm based on modal data", *Smart Struct. Syst., Int. J.*, **18**(5), 983-1004.

Laier, J. and Villalba, J. (2015), "Ensuring reliable damage detection based on the computation of the optimal quantity of required modal data", *Comput. Struct.*, **147**, 117-125.

Mares, C. and Surace, C. (1996), "An application of genetic algorithms to identify damage in elastic structures", *J. Sound Vib.*, **195**(2), 195-215.

Michalawicz, Z. (1996), *Genetic Algorithms+ Data Structures= Evolution Programs*, Springer-Verlag, Berlin, Germany.

Mirjalili, S., Mirjalili, S.M. and Yang, X.-S. (2014a), "Binary bat algorithm", *Neural Comput. Appl.*, **25**(3-4), 663-681.

Mirjalili, S., Mirjalili, S.M. and Lewis, A. (2014b), "Grey wolf optimizer", *Adv. Eng. Software*, **69**, 46-61.

Mirjalili, S., Gandomi, A.H., Mirjalili, S.Z., Saremi, S., Faris, H. and Mirjalili, S.M. (2017), "Salp Swarm Algorithm: A bio-inspired optimizer for engineering design problems", *Adv. Eng. Software*, **114**, 163-191.

Montalvao, D., Maia, N.M.M. and Ribeiro, A.M.R. (2006), "A review of vibration-based structural health monitoring with special emphasis on composite materials", *Shock Vib Digest.*, **38**(4), 295-324.

Mousavi, M. and Gandomi, A.H. (2016), "A hybrid damage detection method using dynamic-reduction transformation matrix and modal force error", *Eng. Struct.*, **111**, 425-434.

Nobahari, M., Ghasemi, M.R. and Shabakhty, N. (2017), "Truss structure damage identification using residual force vector and genetic algorithm", *Steel Compos. Struct., Int. J.*, **25**(4), 485-496.

O'Callahan, J.C. (1989), "A procedure for an improved reduced system (IRS) model", *Proceedings of the 7th International*

Modal Analysis Conference.

Pedram, M., Esfandiari, A. and Khedmati, M.R. (2017), "Damage detection by a FE model updating method using power spectral density: Numerical and experimental investigation", *J. Sound Vib.*, **397**, 51-76.

Perera, R., Ruiz, A. and Manzano, C. (2009), "Performance assessment of multicriteria damage identification genetic algorithms", *Comput. Struct.*, **87**(1-2), 120-127.

Salawu, O. (1997), "Detection of structural damage through changes in frequency: a review", *Eng. Struct.*, **19**(9), 718-723.

Seyedpoor, S.M. and Montazer, M. (2016), "A two-stage damage detection method for truss structures using a modal residual vector based indicator and differential evolution algorithm", *Smart Struct. Syst., Int. J.*, **17**(2), 347-361.

Vo-Duy, T., Ho-Huu, V., Dang-Trung, H., Dinh-Cong, D. and Nguyen-Thoi, T. (2016), "Damage Detection in Laminated Composite Plates Using Modal Strain Energy and Improved Differential Evolution Algorithm", *Procedia Eng.*, **142**, 182-189.

Xiang, J. and Liang, M. (2012a), "A two-step approach to multi-damage detection for plate structures", *Eng. Fract. Mech.*, **91**, 73-86.

Xiang, J. and Liang, M. (2012b), "Wavelet-Based Detection of Beam Cracks Using Modal Shape and Frequency Measurements", *Comput.-Aid. Civil Infrastruct. Eng.*, **27**(6), 439-454.

Xiang, J., Matsumoto, T., Long, J., Wang, Y. and Jiang, Z. (2012), "A simple method to detect cracks in beam-like structures", *Smart Struct. Syst., Int. J.*, **9**(4), 335-353.

Xiang, J., Matsumoto, T., Wang, Y. and Jiang, Z. (2013), "Detect damages in conical shells using curvature mode shape and wavelet finite element method", *Int. J. Mech. Sci.*, **66**, 83-93.

Xiang, J., Nackenhorst, U., Wang, Y., Jiang, Y., Gao, H. and He, Y. (2014a), "A new method to detect cracks in plate-like structures with though-thickness cracks", *Smart Struct. Syst., Int. J.*, **14**(3), 397-418.

Xiang, J., Liang, M. and He, Y. (2014b), "Experimental investigation of frequency-based multi-damage detection for beams using support vector regression", *Eng. Fract. Mech.*, **131**, 257-268.

Xing, B. and Gao, W.-J. (2014), *Teaching–Learning-based Optimization Algorithm*, Springer.

Xu, Y., Qian, Y., Song, G. and Guo, K. (2015), "Damage detection using finite element model updating with an improved optimization algorithm", *Steel Compos. Struct., Int. J.*, **19**(1), 191-208.

Yang, X.-S., Karamanoglu, M. and He, X. (2014), "Flower pollination algorithm: a novel approach for multiobjective optimization", *Eng. Optimiz.*, **46**(9), 1222-1237.

Yang, Z.-B., Radzienski, M., Kudela, P. and Ostachowicz, W. (2017), "Two-dimensional Chebyshev pseudo spectral modal curvature and its application in damage detection for composite plates", *Compos. Struct.*, **168**, 372-383.

Zhou, Y.-L., Maia, N.M. and Abdel Wahab, M. (2016), "Damage detection using transmissibility compressed by principal component analysis enhanced with distance measure", *J. Vib. Control*, 1077546316674544.

Zhou, Y.-L., Maia, N.M., Sampaio, R.P. and Wahab, M.A. (2017), "Structural damage detection using transmissibility together with hierarchical clustering analysis and similarity measure", *Struct. Health Monitor.*, **16**(6), 711-731.

Zou, Y., Tong, L. and Steven, G.P. (2000), "Vibration-based model-dependent damage (delamination) identification and health monitoring for composite structures—a review", *J. Sound Vib.*, **230**(2), 357-378.

CC