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Evaluation of vibroacoustic responses of laminated composite sandwich structure using higher-order finite-boundary element model

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Abstract. In this paper, the vibroacoustic responses of baffled laminated composite sandwich flat panel structure under the influence of harmonic excitation are studied numerically using a novel higher-order coupled finite-boundary element model. A numerical scheme for the vibrating plate has been developed in the frame work of the higher-order mid-plane kinematics and the eigen frequencies are obtained by employing suitable finite element steps. The acoustic responses are then computed by solving the Helmholtz wave equation using boundary element method coupled with the structural finite elements. The proposed scheme has been implemented via an own MATLAB base code to compute the desired responses. The validity of the present model is established from the conformance of the current natural frequencies and the radiated sound power with the available benchmark solutions. The model is further utilized to scrutinize the influence of core-to-face thickness ratio, modular ratio, lamination scheme and the support condition on the sound radiation characteristics of the vibrating sandwich flats panel. It can be concluded that the present scheme is not only accurate but also efficient and simple in providing solutions of the coupled vibroacoustic response of laminated composite sandwich plates.

Keywords: laminated composite sandwich plate; vibroacoustic response; HSDT; FEM-BEM scheme; MATLAB computer code

1. Introduction

Owing to ever pressing demand from the growing spectra of weigh sensitive and high performance engineering applications, there is a strong demand for the lightweight and high strength structures that could not only withstand environmental hostilities but at the same time possess excellent dynamic properties and a small acoustic signature. In this respect, the laminated composite sandwich structures are exhaustively and favorably being used in a wide variety of applications as they have most of the desired traits. However, when subjected to external loading, the sound emitted by the structure propagates into the surrounding medium and interacts with the other structures in its path, thereby leaving an acoustic signature of the structure. This renders the tailoring of the vibration induced acoustic radiation responses of the structures extremely crucial in the applications such as aircraft, aerospace, defense, naval and nuclear sectors where stealth is of prime importance. It is well known that, insertion of sandwiched core between two faces influences the overall stiffness, toughness and other dynamic properties of the structure. However, to accurately predict the vibration and acoustic responses of the panels, it is important to model the structure using a kinematics/formulation that allows handling the material discontinuities, stresses developed and slipping (if at all there is any) at the core-face interfaces. Several mathematical models (theories) have been developed and used in past in an attempt to model the mid-plane kinematics of the structures more accurately (Alijani and Amabili 2014). We note various equivalent single layer theories for instance the classical laminate theory (CLT), the first-order shear deformation theory (FOSDT) and the higher-order shear deformation theory (HOSDT) are commonly utilized. This is due their simplicity in formulation and ease of implementation alongside the reasonable accuracy that they offer to compute the structural responses of the laminated composite and sandwich structures including the effect of delamination (Bousahla *et al*. 2014, Bui *et al*. 2016, Do *et al*. 2017a, Khalfi *et al*. 2014, Mahapatra *et al*. 2015, Moradi-Dastjerdi *et al*. 2017, Nikrad *et al*. 2016, 2017, Nikrad and Asadi 2015, Panda and Mahapatra 2014, Parhi and Singh 2017, Yin *et al*. 2014, 2016). Other theories such as the layerwise theory (Ferreira *et al*. 2013, Tornabene *et al*. 2017), the refined theories (Belabed *et al*. 2014, Bourada *et al*. 2015, Houari *et al*. 2016, Noor and Burton 1990), non-local theories (Kolahchi 2017) and and the theories incorporating geometric nonlinearities (Mahapatra *et al*. 2016) have also been proposed and used from time to time as each has some benefit over the others. It is

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imperative that the accuracy of the computation of acoustic radiation responses is significantly reliant on the on the correctness in obtaining the free vibration responses. This is due to the fact that the modal values obtained are utilized in predicting the acoustic radiation responses of the vibrating structure. It has been established that the HSDT renders more realistic modeling of the flexure of the structure so far as the shear deformable laminated composite structures are concerned (Sharma *et al*. 2018a, b). Several authors have studied and analyzed the vibration induced acoustic responses of isotropic, multilayered composite, functionally graded flat and curved panels under the ambient and/or unlike environment. Layered orthotropic flat panels radiating sound in an ambient environment (Sharma *et al*. 2017), subjected to thermal loading (Li *et al*. 2016) and hygroscopic loading (Zhao *et al*. 2013) have been studied from time to time. The commercially available software packages have also been methodically utilized by many researchers to investigate the sound radiation emanating from vibrating isotropic flat panels (Jeyaraj *et al*. 2008), laminated composite flat panels (Geng and Li 2014, Jeyaraj *et al*. 2009) in ambient as well as in elevated thermal environment (Atalla *et al*. 1996, Holmström 2001, Johnson and Cunefare 2002, Tournour and Atalla 1998). Moreover, the coupled FE-BE (finite element-boundary element) technique has been proved to be a versatile scheme to for computing the acoustic radiation emitted by the structures (Geng and Li 2012, Jeyaraj *et al*. 2011b).

Numerous studies related to the dynamic (Bui *et al*. 2013, Do *et al*. 2017b, Moradi-Dastjerdi and Payganeh 2017) and vibroacoustic responses of sandwich composite panel structures have been conducted by several researchers in past. Liu and Li (2013) studied analytically the vibration and acoustic radiation responses of sandwich flat panels having face and core made up of isotropic material and exposed to uniform temperature loads using the equivalent classical theory. Li and Yu (2015) analyzed sandwich flat panels of orthotropic materials for their sound radiation behavior under thermal environment. The panel kinematics was modeled using the piecewise low order shear deformation theory and Rayleigh integral formulation was used to obtain the sound pressure level in the surroundings. Further, simply supported rectangular sandwich flat panels with functionally graded (FG) core and metal-ceramic faces have been analyzed numerically for their acoustic radiation responses using the simplified FOSDT in conjunction with the element radiator approach (Chandra *et al*. 2015). Tong *et al*. (2017) utilized coupled FEM-BEM approach to study the sound radiation characteristics of simply supported sandwich composite cylindrical shell having viscoelastic core. Additionally, the influence of thermal environment on the sound emission characteristics of the layered sandwich panel with the viscoelastic core has also been investigated (Jeyaraj *et al*. 2011a) using commercial FE software and FEM-BEM scheme. Larbi et al. (2015) developed a novel FE scheme to study the sound insulation property of double-wall sandwich panels with viscoelastic core. Several works including the active control of sound power through soft-cored sandwich panels using volume velocity cancellation (Sahu and Tuhkuri 2014, 2015) and through multiple piezoelectric actuators (Sahu *et al*. 2015) have also been reported.

Literature review affirms that although the vibroacoustic behavior of composite as well as the isotropic shell panel structures has been extensively studied for several types of mechanical excitations and in-situ environments, the laminated composite sandwich structure have got less attention. Further, numerical studies related to the vibroacoustic behavior of laminated composite sandwich structure by utilizing the HOSDT mid-plane kinematic based structural model has not yet been reported. The present research targets to fill this research gap by studying the acoustic radiation responses of laminated composite sandwich flat panels in the framework of the HOSDT. In addition, a MATLAB script has been written to perform the modal (eigen frequency) analysis and the coupled FE-BE analysis for computing the sound radiation responses. The validity of the proposed model is established by comparing present results with the available benchmark solutions. Appropriate numerical examples have been solved and discussed in detail to bring out the influence of core-to-face thickness ratio, modular ratio, lamination scheme and the support condition on the sound radiation characteristics of the vibrating sandwich flat panel.

2. Mathematical formulation

The lamination scheme and the geometry of the composite sandwich flat panel analysed in this work are depicted in Fig. 1. The composite sandwich is considered to be made up of an isotropic core layer of thickness 't_c' and two laminated composite face layers each of thickness 't_f' sandwiching the core from the top and the bottom.

In the present study, for precise incorporation of the effect of shear deformation (as the laminated sandwich structures are more susceptible to shear failure over failure due to tension and fatigue), the mathematical model of the flat shell panels has been developed based on the higher-order shear deformation theory. The in-plane displacements are expanded as cubic functions of the thickness coordinate while the transverse displacement is assumed to be independent (remain constant) of the plate thickness (Kant and Swaminathan 2001)

$$\left.\begin{aligned} p(x_1,x_2,x_3,t) &= p_0(x_1,x_2,t) + x_3 p_1(x_1,x_2,t) + x_3^2 p_2(x_1,x_2,t) + x_3^3 p_3(x_1,x_2,t) \\ q(x_1,x_2,x_3,t) &= q_0(x_1,x_2,t) + x_3 q_1(x_1,x_2,t) + x_3^2 q_2(x_1,x_2,t) + x_3^3 q_3(x_1,x_2,t) \\ r(x_1,x_2,x_3,t) &= r_0(x_1,x_2,t) \end{aligned}\right\} \quad (1)$$

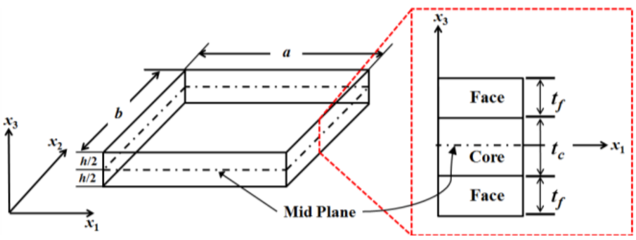

Fig. 1 Definition of lamination scheme and geometry of composite sandwich flat shell panel

where, p, q and r denote the axial translations of any point along the x_1, x_2 and x_3 coordinate axes, respectively; t denotes the time; p_0, q_0 and r_0 represent the translations of any point lying on the mid-plane; p_1 and q_1 represent the angular displacement of the mid-surface normal *w.r.t* the x_2 and x_1- axes, respectively. The remaining terms p_2, q_2, p_3 and q_3 are defined as the higher order terms in the Taylor series expansion.

Eq. (2) represents the relationship between stress and the strain tensors, $\{\sigma\}$ and $\{\varepsilon\}$, respectively for any k^{th} lamina having an orientation 'θ' *w.r.t* any arbitrary axis

$$\{\sigma\} = \left[\bar{Q}\right]\{\varepsilon\} \tag{2}$$

where, $[\bar{Q}]$ represents the reduced stiffness matrix. The strain tensor for any sandwich flat shell panel can also be stated as Mahapatra and Panda (2015)

$$\left.\begin{aligned}\{\varepsilon\} &= \left\{\varepsilon_{x_1x_1} \quad \varepsilon_{x_2x_2} \quad \varepsilon_{x_1x_2} \quad \varepsilon_{x_1x_3} \quad \varepsilon_{x_2x_3}\right\}^T \\ &= \left\{\left(\frac{\partial p}{\partial x_1}\right) \quad \left(\frac{\partial q}{\partial x_2}\right) \quad \left(\frac{\partial p}{\partial x_2}+\frac{\partial q}{\partial x_1}\right) \quad \left(\frac{\partial p}{\partial x_3}+\frac{\partial r}{\partial x_1}\right) \quad \left(\frac{\partial q}{\partial x_3}+\frac{\partial r}{\partial x_2}\right)\right\}^T\end{aligned}\right\} \tag{3}$$

The structural model of the sandwich flat panel has been discretized using a nine-noded quadrilateral Lagrangian isoparametric element (Cook *et al*. 2000) with nine degrees of freedom at each node. The displacement (δ) of any point located on the mid-plane can be written as

$$\delta = \sum_{i=1}^{n} N_i\left(x_1, x_2\right)\delta_i \tag{4}$$

where, δ_i represents the nodal displacement vector, corresponding to the i^{th} node, defined as $\{\delta_i\}$ $=\{\delta_i\} = \{p_{0_i}\ q_{0_i}\ r_{0_i}\ p_{1_i}\ q_{1_i}\ p_{2_i}\ q_{2_i}\ p_{3_i}\ q_{3_i}\}^T$ and N_i is the corresponding shape function.

The matrix form of strain vector $\{\varepsilon\}$ in terms of the matrix of thickness coordinates $[T_e]$ and the strain vector $\{\bar{\varepsilon}\}$ corresponding to the mid-plane is given as Mahapatra and Panda (2015)

$$\{\varepsilon\} = [T_e]\{\bar{\varepsilon}\} \tag{5}$$

The expression in Eq. (5) can further be written as Mahapatra and Panda (2015)

$$\{\bar{\varepsilon}\} = \left[B_L\right]\{\delta\} \tag{6}$$

where, $[B_L]$ is the matrix representing the strain-displacement relation. The global displacement vector can be formulated as a function of thickness coordinate ($[I]$)

$$\{\bar{\delta}\} = \begin{Bmatrix} p \\ q \\ r \end{Bmatrix} = [\mathrm{I}]\{\delta\} \tag{7}$$

$q_0 = r_0 = q_1 = q_2 = q_3 = 0$ at x_1=0 and a;The expression for the total strain energy of the laminated sandwich flat panel is given as Mahapatra and Panda (2015)

$$E_1 = \frac{1}{2}\int_V \{\varepsilon\}^T\{\sigma\}dV = \frac{1}{2}\iint\left[\int_{-h/2}^{+h/2}\{\varepsilon\}^T\{\sigma\}dx_3\right]dx_1dx_2 \tag{8}$$

Eq. (8) can be further reformulated by using Eq. (2) and Eq. (5)

$$E_1 = \frac{1}{2}\iint\left(\{\bar{\varepsilon}\}^T[D]\{\bar{\varepsilon}\}\right)dx_1dx_2 \tag{9}$$

where, $[D] = \int_{-h/2}^{+h/2}[T_e]^T[Q_{ij}][T_e]dx_3$ (refer (Mahapatra and Panda 2015) for details).

The flat panel kinetic energy can be written as Cook *et al*. (2000), Mahapatra and Panda (2015)

$$E_2 = \frac{1}{2}\int_V \rho\{\dot{\bar{\delta}}\}^T\{\dot{\bar{\delta}}\}dV \tag{10}$$

where, ρ represents the density and $\{\dot{\bar{\delta}}\}$ represents first order time derivative of the global displacement vector. Eq. (7) can also be rewritten using Eq. (10) as

$$E_2 = \frac{1}{2}\int_A\left(\sum_{k=1}^{n}\int_{x_{3,k-1}}^{x_{3,k}}\{\dot{\delta}\}^T[\mathrm{I}]^T\rho^k[\mathrm{I}]\{\dot{\delta}\}dx_3\right)dA = \frac{1}{2}\int_A\{\dot{\delta}\}^T[\varsigma]\{\dot{\delta}\}dA \tag{11}$$

where, $[\varsigma] = \sum_{k=1}^{n}\int_{x_{3,k}=1}^{x_{3,k}}([I]^T\rho^k[I])dx_3$ is the inertia matrix and n is the number of layers.

The final form of T is derived by replacing Eq. (4) in Eq. (11) and written as

$$E_2 = \frac{1}{2}\int_A\sum_{i=1}^{n}\left(\left[N_i\right]^T[\varsigma][N_i]dA\right)\{\dot{\delta}_i\} \tag{12}$$

The stiffness and mass matrices, $[K]$ and $[M]$, respectively, of the sandwich flat shell panel can be stated as

$$\left.\begin{aligned}[K] &= \int_A\Big(\sum_{k=1}^{n}\int_{x_{3,k-1}}^{x_{3,k}}[B_L]^T[D][B_L]dx_3\Big)dA \\ [M] &= \int_A\Big(\sum_{k=1}^{n}\int_{x_{3,k-1}}^{x_{3,k}}[N]^T[\varsigma][N]dx_3\Big)dA\end{aligned}\right\} \tag{13}$$

The final form of the system governing equation for the freely composite sandwich flat shell panel is obtained by using Hamilton's principle (Bedford 1985) and expressed as

$$\delta\int_{t_1}^{t_2}(E_2 - E_1)dt = 0 \tag{14}$$

The governing equation is finally expressed in the form of stiffness and mass matrices and conceded as

$$[K]\{\delta\} + [M]\{\ddot{\delta}\} = 0 \tag{15}$$

where, $\ddot{\delta}$ is the second-order time derivative of the displacement.

The eigenvalue form of Eq. (15) is derived as in Eq. (16) and the modal parameters (frequency ω and mode shape$\{\Phi\}$) are computed.

$$\left(\left[K\right]-\omega^2\left[M\right]\right)\{\Phi\}=0 \tag{16}$$

The equation of motion of any elastic structure vibrating under the action of external load is given by Atalla and Sgard (2015)

$$[K]\{\delta\}+[C]\{\dot{\delta}\}+[M]\{\ddot{\delta}\}=\Gamma(t) \tag{17}$$

where, the damping matrix $[C]=\frac{2\xi}{\omega_c}[\mathrm{K}]$ is dependent on the excitation frequency (ω_c) and the panel stiffness, ξ is the structural damping ratio and Γ represents the excitation load vector. The vibroacoustic analysis of the sandwich panels acted upon by harmonic point excitation is conducted to obtain the sound radiation responses via a coupled FE-BE technique. The Helmholtz wave equation dictates the dynamics of acoustic radiation in stationary fluid contiguous to the vibrating panels and is given as

$$\nabla^2 p+k^2 p=0 \tag{18}$$

where, k is the wave number and p is the sound pressure at any point in the domain. The discreet form of the Helmholtz equation using the boundary elements is conceded as in Eq. (19)

$$Z_1 p=i\rho_s\omega_c Z_2 v_p \tag{19}$$

Here, Z_1 and Z_2 represent the boundary influence matrices, p is the matrix comprising the sound pressure values, v_p holds the nodal velocity of the flat panel along x_3 axis and ρ_s denotes the density of the contiguous medium. The panel velocity, v_p, can be expressed as in Eq. (20), in the form of displacement vector using the matrix $\bar{T}$ that maps the nodal displacement to nodal velocity.

$$v_p=i\omega_c\delta\bar{T} \tag{20}$$

The final representation of the equation of coupled structural finite-elements and acoustic boundary-elements (FE-BE) can be stated as (Atalla and Sgard 2015, Mariem and Hamdi 1987)

$$\begin{pmatrix} K+i\omega_c C-\omega_c{}^2 M & L \\ \rho_s\omega_c{}^2 Z_2\bar{T} & Z_1 \end{pmatrix}\begin{Bmatrix}\delta\\ p\end{Bmatrix}=\begin{Bmatrix}\Gamma\\ 0\end{Bmatrix} \tag{21}$$

where, the matrix L couples the structural finite and acoustic boundary-elements. The panel displacement, δ and the sound pressure, p at any field point is obtained by solving Eq. (21). Once, the displacement field and the sound pressure values corresponding to each excitation frequency of interest are known, the acoustic response indicators can also be computed as follows:

The radiated sound power can be computed by evaluating the following integral over the surface S of the panel

$$W_{rad}=\frac{1}{2}\mathrm{real}\left[\iint_s p v_p\, dS\right] \tag{22}$$

The sound power level W_L can be obtained as

$$W_L=10\times\log\left(\frac{W_{rad}}{W_{ref}}\right) \tag{23}$$

The reference power W_{ref} equal to 10^{-12} W.

The average normal mean square velocity ($\langle v_n^2\rangle$) of the panel can be written as

$$\left\langle v_n^2\right\rangle(\omega_c)=\frac{1}{2S}\int_S\left|v_p^2\right|dS \tag{24}$$

The radiation efficiency (Ω) is computed as

$$\Omega=\frac{W_{rad}}{\rho_s cS\left\langle v_n^2\right\rangle} \tag{25}$$

The sound pressure level (SPL) at any point in terms of corresponding pressure p can be written as

$$\mathrm{SPL}=20\times\log\left(\frac{p}{p_{ref}}\right) \tag{26}$$

where, the reference pressure p_{ref} is taken equal to 20 μPa.

3. Results and discussion

In this section, the vibroacoustic responses of sandwich composite flat panels are investigated and discussed in detail. A FE model for the sandwich composite flat panel is developed based on the HOSDT mid-plane kinematics. A MATLAB script is composed to implement the proposed formulation. The modal frequencies and the corresponding mode shapes are computed by finding the solution to Eq. (16) after subjecting the panel model to appropriate constraints at the supports. The natural frequencies and radiated sound power thus obtained using the current numerical formulations are matched with the available benchmark solutions. The fundamental frequency in non-dimensional form is expressed as $\varpi=\omega b^2\sqrt{(\rho/E_2)_f/h^2}$, where ω is the natural frequency in rad/s. In order to reduce the number of unknowns the various support conditions imposed are as follows:

Simply supported edge (S):
$q_0=r_0=q_1=q_2=q_3=0$

at $x_1 = 0$ and a;

$p_0 = r_0 = p_1 = p_2 = p_3 = 0$
at $x_2 = 0$ and b.

Clamped edge (C):
$p_0 = q_0 = r_0 = p_1 = q_1 = p_2 = q_2 = p_3 = q_3 = 0$
at $x_1 = 0$ and a; at $x_2 = 0$ and b.

Free edge (F):
$p_0 = q_0 = r_0 = p_1 = q_1 = p_2 = q_2 = p_3 = q_3 \neq 0$
at $x_1 = 0$ and a; at $x_2 = 0$ and b.

The support conditions at the edges (taken in order) at $x_1 = 0$, $x_2 = 0$, $x_1 = a$ and $x_2 = b$ are chosen to have the following combinations: SSSS, CCCC, SCSC, SFSF, CFCF and CFFF.

In the present analysis, the sandwich panels are realized by sandwiching an isotropic core in between two faces of equal thickness made up of laminated composite material. The properties of the face and the core materials utilized in the numerical examples are as follows and remain same unless specified otherwise:

Core (Liu and Li 2013):

$E_c = 7$ GPa, $v_c = 0.3$, $\rho_c = 1000$ kg/m^3

Face (Kant and Swaminathan 2001):

$E_{f,1} = 131$ GPa, $E_{f,2} = 10.34$ GPa,

$G_{f,23} = G_{f,13} = 6.895$ GPa, $G_{f,23} = 6.205$ GPa,

$v_{f,12} = v_{f,13} = 0.22$, $v_{f,23} = 0.49$ GPa, $\rho_f = 1627$ kg/m^3.

The sandwich flat panel is considered to be resting on an infinite rigid baffle with geometrical parameters as: $a = 0.4$ m, $b = 0.3$ m and $h = 0.01$ m where the panel thickness h is defined in terms of the core thickness (t_c) and face thickness (t_f) as $h = t_c + 2t_f$. The panels are assumed to be vibrating in air and subjected to point harmonic excitation equal to 1 N at (0.1 m, 0.1 m, 0) on the surface of the panel *w.r.t.* the reference system mentioned in Fig. 1. The SPL is obtained at a field location directly above the point of excitation at a distance of 2 m along the x_2 direction. The sound pressure level directivity is observed at the points on a semicircle of radius 1 m lying in the plane $x_2 = b/2$ and centred at the central point of the vibrating flat panel. A damping ratio $\xi = 0.01$ has been considered throughout.

3.1 Convergence and validation of natural frequency

A simply supported sandwich composite flat panel example available in (Kant and Swaminathan 2001) is solved to establish the validity of natural frequencies. The non-dimensional fundamental frequency parameters are computed for different thickness ratio (a/h) values with increasing mesh size and the results are listed in Table 1. The result of (Kant and Swaminathan 2001) is also listed for comparison. It is evident that the present values converge well with mesh enhancement. Also, the present results agree well with the corresponding reference results thus establishing the validity of the proposed structural model. Consequently, a (10×10) mesh is used for the computation of desired responses all the way through the analysis.

3.2 Validation of radiated sound power

In this section, the correctness of the present numerical solutions of sound radiation responses of composite sandwich panels has been demonstrated by matching the obtained results with those available in the published literature. In order to do so, the sound power emitted by a clamped laminated composite (no core) panel as considered by Li *et al*. (2016) and simply supported sandwich composite flat panel as considered by Liu and Li (2013) is reproduced using the present approach and the comparison is illustrated in Figs. 2(a) and (b), respectively. Additionally, the results are also computed via simulation model using commercially available FE and BE packages. In the current simulation model, the free vibration responses are first obtained using ANSYS expending an eight-noded isoparametric element 6 *dof* at each node. The modal values contained in ANSYS results file (*.*rst* file) is then exported to LMS Virtual. Lab environment wherein an indirect BEM is used to obtain the sound power radiated by the panels.

It is clearly observed that the present numerical results closely follow the reference values both in the case of the laminated composite as well as the sandwich composite panels. It is essential to mark that the current HOSDT based numerical model yields values lesser than the simulation (NASTRAN and VA.One) model of Li *et al*. (2016) and the same is evident from Fig. 2(a). However, in the case of sandwich composite panels, the present numerical values are higher than the reference (Liu and Li 2013) values and

Table 1 Convergence and comparison of non-dimensional fundamental frequency parameter of simply supported anti-symmetric laminate composite sandwich panel ($a/b = 1$, $t_c/t_f = 10$)

a/h	Non-dimensional fundamental frequency parameter								
	Mesh size							Kant and Swaminathan (2001)	
	2×2	4×4	6×6	8×8	10×10	12×12	14×14	HOSDT (12 dof)	HOSDT (9 dof)
10	4.9921	4.9539	4.9519	4.9515	4.9513	4.9512	4.9512	4.8594	4.8519
40	12.9295	12.7113	12.7043	12.7037	12.7038	12.7039	12.7039	12.6821	12.6555
70	15.2151	14.7359	14.7160	14.7137	14.7134	14.7134	14.7135	14.7977	14.7583
100	16.1331	15.4229	15.3849	15.3795	15.3784	15.3781	15.3781	15.5093	15.4647

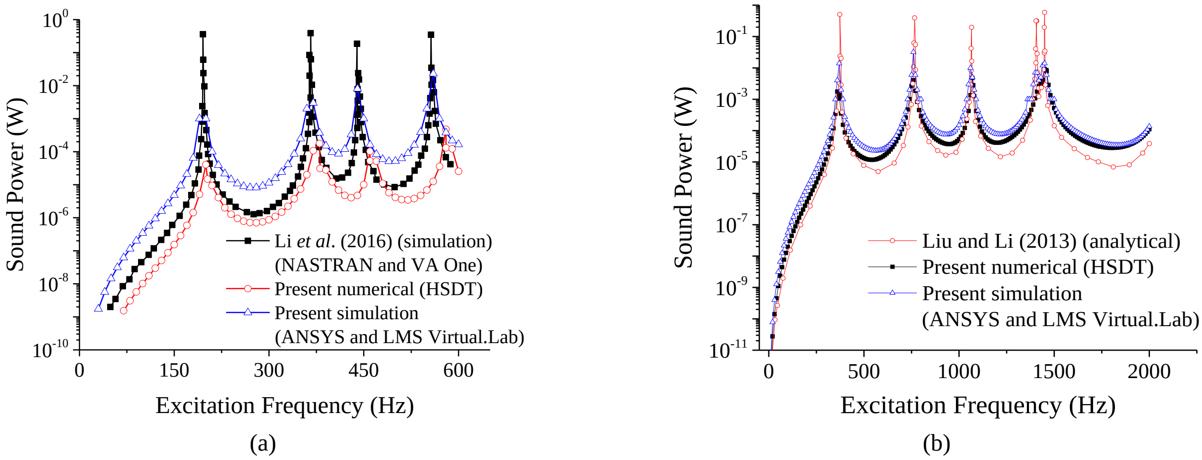

Fig. 2 Comparison of sound power: (a) Simply supported laminated composite panel; (b) Clamped sandwich composite panel

lesser than the current simulation results as depicted in Fig. 2(b). This is due to the fact that the reference utilized the analytical formulations based on equivalent non-classical theory (under estimates displacement) and the current simulation model is based on the FOSDT, whereas the more general (for the structural modelling purpose) HOSDT based coupled FEM-BEM technique has been employed in the present scheme. Indeed, the novelty of the current research lies in the similar line as well.

3.3 Numerical illustrations

The necessity and accuracy of the proposed scheme to obtain the numerical solutions of the acoustic radiation responses of vibrating laminated composite sandwich structure has been established in the previous section. Now, several numerical examples are solved using the present scheme to investigate the influence of various geometrical parameters, material property and support conditions on the sound radiation behaviour of sandwich composite flat panels.

Firstly, the influence of core-to-face thickness ratio (t_c/t_f) is investigated. Simply supported composite sandwich panels having [0°/90°/*C*/90°/0°] scheme are considered. The thickness of core and the thickness of the face are varied keeping the total thickness of the panel (*h*) constant. The influence of four different core-to-face thickness ratios (t_c/t_f = 2, 5, 8 and 20) on the natural frequency is shown in Fig. 3. It is observed that as the t_c/t_f values increases, the core becomes thicker at the expense of the thickness of the faces.

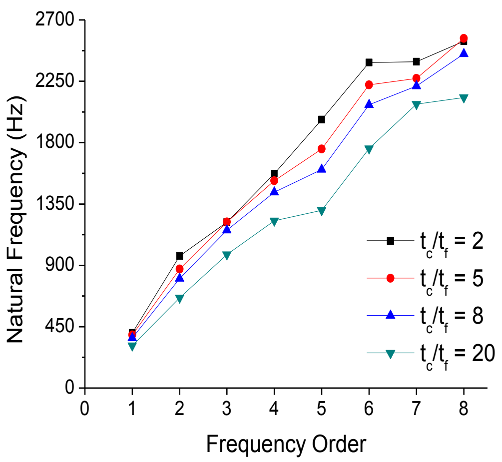

Fig. 3 Influence of core-to-face thickness ratio on natural frequency

Consequently, the stiffness of the panels decreases leading to reduction in the natural frequencies. It is worthy to note that for a 10-fold increase in t_c/t_f from 2 to 20, the first eight natural frequencies decrease by 22.98%, 31.62%, 19.55%, 21.97%, 33.81%, 26.41%, 12.95%, and 16.24%, respectively.

Fig. 4 depicts the variation of acoustic response indicators with core-to-face thickness ratio (t_c/t_f). Due to decreasing stiffness with the increasing t_c/t_f, the displacement corresponding to the excitation location increases as is evident from the Fig. 4(a). Also, the resonance peaks in the response curves of displacement, average root mean square (RMS) velocity and radiated sound power shift to lower frequencies as evident from Figs. 4(a), (b) and (d), respectively. Further, the average RMS velocity of the panels follows a trend similar to that of the displacement. The velocity corresponding to first resonance peak for t_c/t_f = 20 exceeds by 36.57%. the corresponding velocity for t_c/t_f = 2.

Consequently, the panels tend to radiate less efficiently with increasing core thickness. The radiation efficiency fluctuates more for the panels with a thicker core indicating increased sensitivity of the acoustic radiation caused by them at higher frequencies. Also, the sound power radiated is higher for the panels with a thicker core and the same is evident from Fig. 4(d).

Next, clamped composite sandwich panels with [0°/90°/*C*/90°/0°] scheme are considered for studying the influence of core-to-face modular ratio (E_c/E_f) on the sound radiation caused by the vibrating panels. The young's modulus ($E_{f,1}$) of the face is kept constant and the young's modulus of the core (E_c) is varied such that $E_c/E_{f,1}$ = 0.05, 0.5, 0.1 and 1. The acoustic responses of the panels with varying modular ratio are shown in Fig. 5. The panels become increasingly stiffer with increasing modular ratio

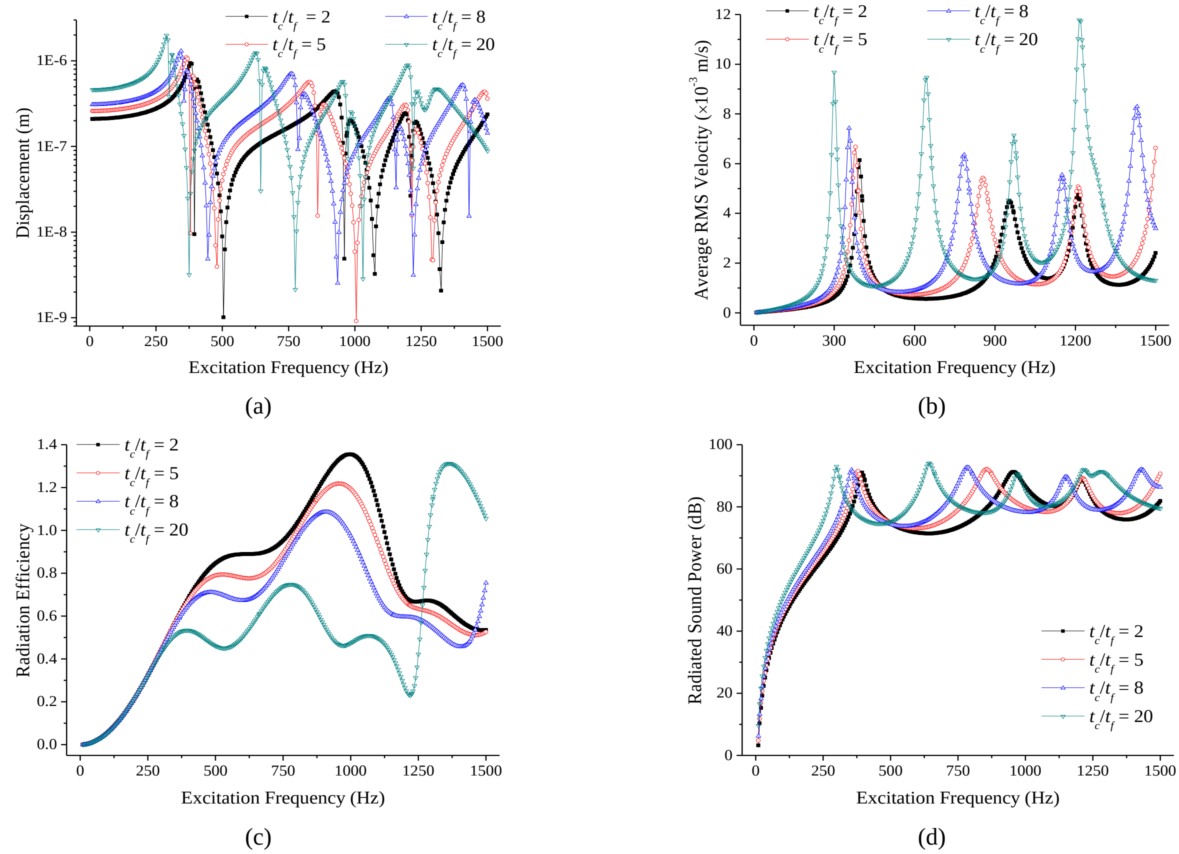

Fig. 4 Influence of core-to-face thickness ratio on the acoustic responses: (a) Displacement; (b) Average RMS velocity; (c) Radiation efficiency; (d) Radiated sound power

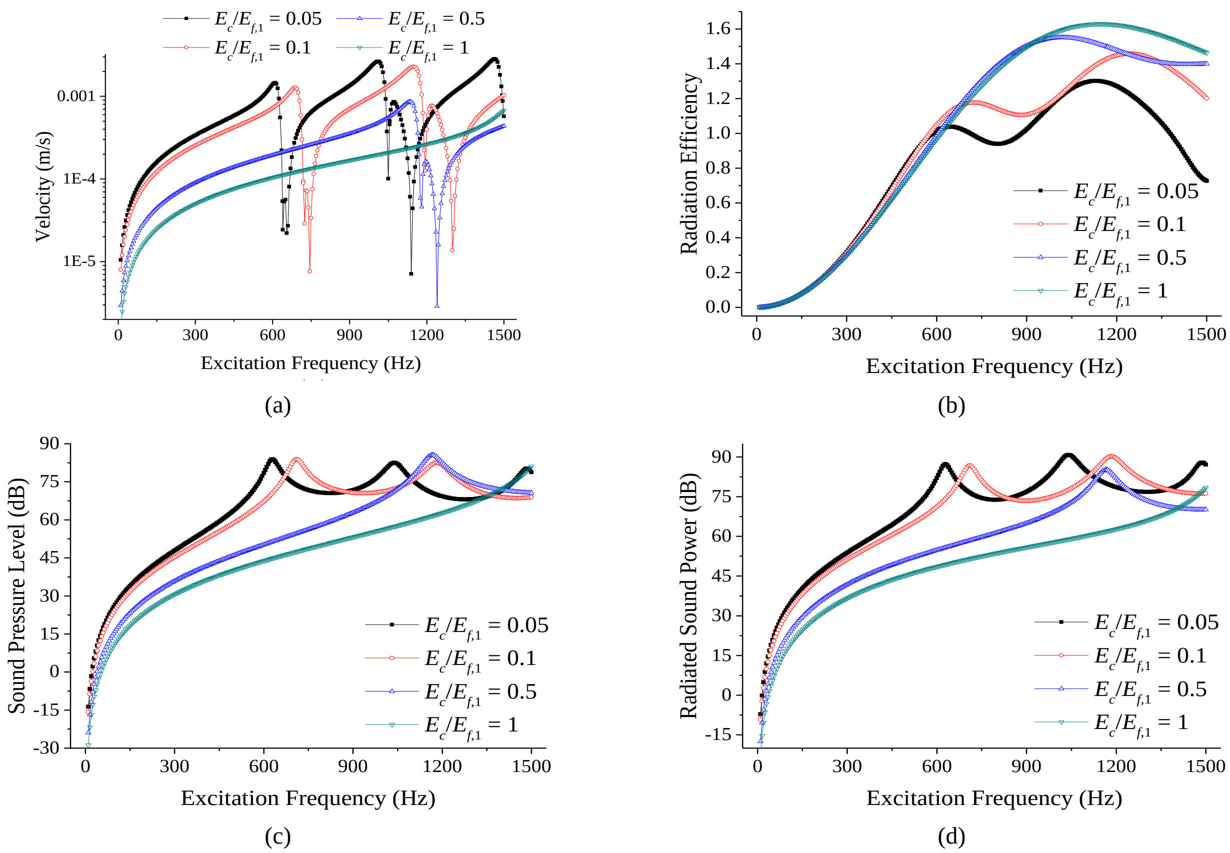

Fig. 5 Influence of core-to-face modular ratio on the acoustic responses: (a) Velocity; (b) Radiation efficiency; (c) Sound pressure level; (d) Radiated sound power

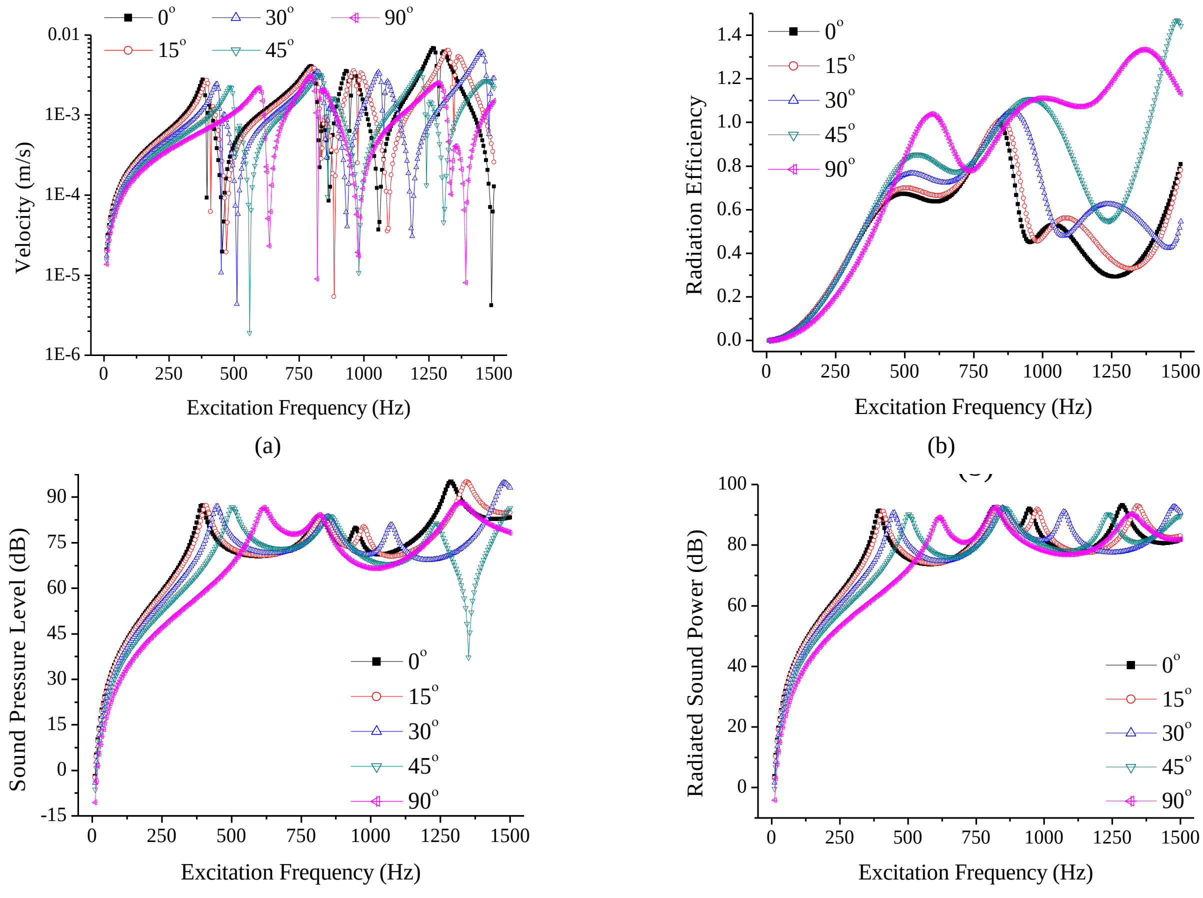

Fig. 6 Influence of lamination scheme (0º/θ/-θ/C/-θ/θ/0º): (a) Velocity; (b) Radiation efficiency; (c) Sound pressure level; (d) Radiated sound power

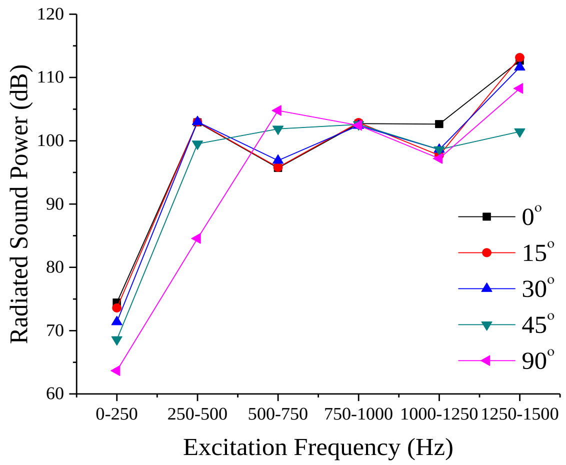

Fig. 7 Deviation of overall radiated sound power with fibre orientation [0°/θ/-θ/C/-θ/θ/0°] in different frequency bands

values. It is useful to note that the velocity of the point of excitation decreases with increasing modular ratio. However, the peaks in the velocity response curves shift to higher frequencies for increasing modular ratio. The radiation efficiency of the panels increases with increasing $E_c/E_{f,1}$ values with $E_c/E_{f,1}$ = 1 case being the most efficient radiator. The radiated sound power and the SPL at the field point follow similar variation patterns with the excitation frequency. The tendency to radiate sound is higher for the panels with lesser core modulus and the $E_c/E_{f,1}$ = 0.05 case causes the most SPL and the radiated sound power in the considered frequency range.

In order to investigate the influence of fibre orientation on the sound radiation characteristics laminated composite sandwich flat panels under CSCS constraint are considered. The lay-up is defined, to have a symmetric configuration, as [0°/θ/-θ/C/-θ/θ/0°] where θ takes the values as 0°, 15°, 30°, 45° and 90°. Fig. 6 depicts the dynamic and sound emission characteristics of the panels for varying lamination scheme. It is strikingly visible that the resonance peaks in the curves cascade to right in the response figures indicating that the panels become increasingly stiff with increasing value of the fibre angle θ. The nodal velocity of the excitation location generally decreases with increasing θ. This is in direct contrast to radiation efficiency that increases with increasing θ and is the highest for θ = 90°. Also, the SPL at the field point decreases with increasing value of the fibre angle θ. A similar trend is observed in the variation of radiated sound power and the same is evident from Fig. 6(d).

Further, to have a more clear indication of the radiation behaviour, the overall radiated sound power in frequency

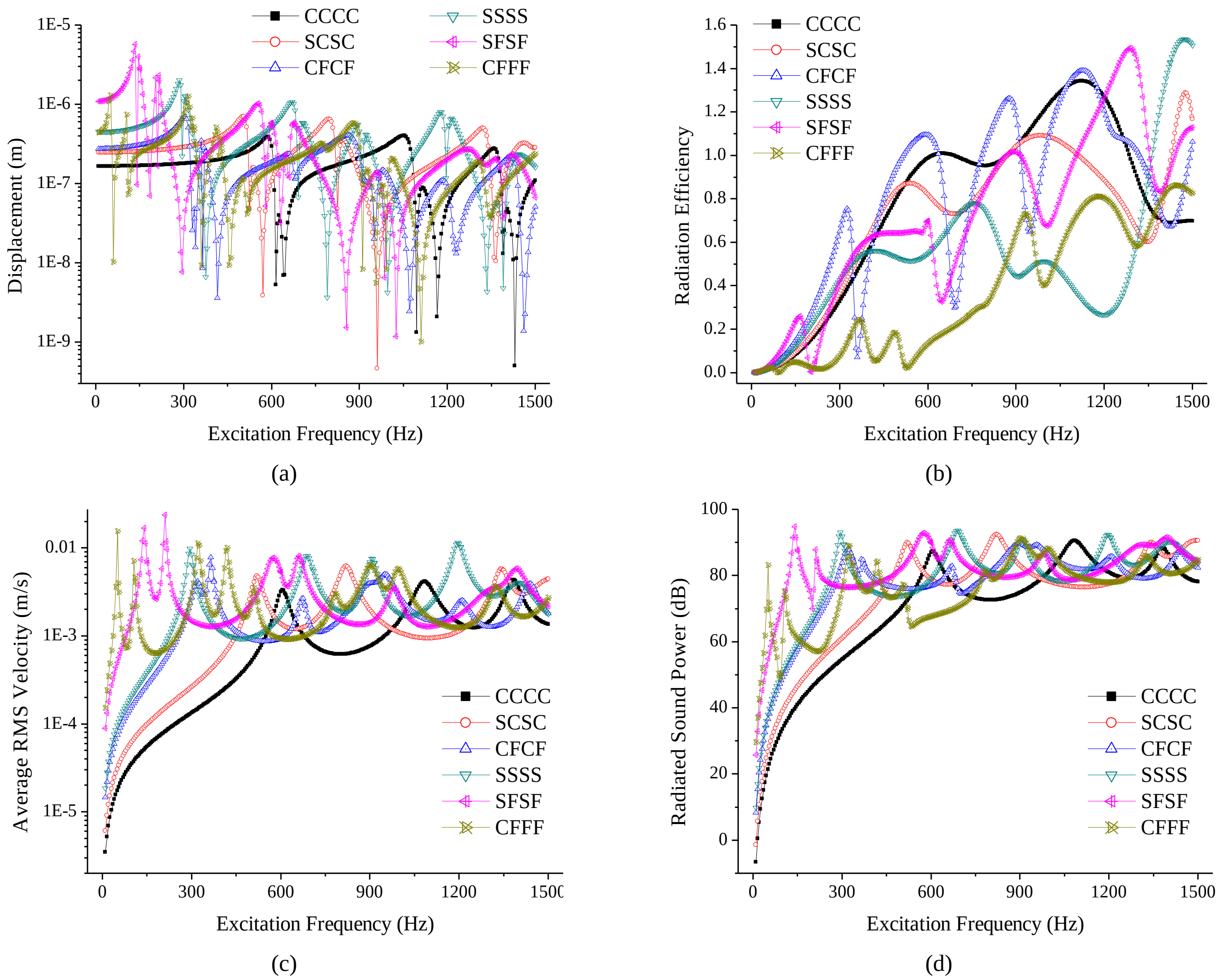

Fig. 8 Influence of support condition on the acoustic responses: (a) Displacement; (b) Radiation efficiency; (c) Average RMS velocity; (d) Radiated sound power

bands of equal width is computed and shown in Fig. 7. It is evident that the different schemes radiate differently in different frequency ranges. The radiated sound power decreases with increasing fibre angle in (0-250) Hz whereas, a reverse trend is observed in the frequency range (500-750) Hz. However, it is interesting to note equal sound radiation in (750-1000) Hz excitation frequency range irrespective of the lay-up schemes.

Moreover, the sound pressure level directivity patterns for different support conditions and corresponding to two frequencies of 400 Hz and 1200 Hz are shown in Figs. 9(a) and (b), respectively. We note a significant difference in the radiation patterns. This is because of the fact that, corresponding to 400 Hz only first few modes contribute to the vibration, whereas at 1200 Hz the larger numbers of modes are excited and hence the pattern is complex. It is worthy to note that the CFCF case causes highest and CCCC case the least amount of sound pressure in the surroundings for 1200 Hz and 400 Hz, respectively.

4. Conclusions

Sound radiation characteristics of composite sandwich flat panels under the influence of harmonic point load have been investigated with the aid of a novel methodology utilizing the HSDT based coupled finite and boundary element method using a customized MATLAB script. Firstly, the numerical results of the natural frequency and the radiated sound power obtained using the present formulation is matched with the available benchmark solutions to establish its validity. Subsequently, extensive numerical experimentations are performed to comprehend the influence of core-to-face thickness ratio, core-to-face modular ratio and support condition on the acoustic radiation responses of composite sandwich flat panels. The validation study revealed the necessity of incorporation of the higher-order mid-plane kinematics in providing accurate numerical solutions of the sound emission responses of shear deformable composite sandwich structure. From the numerical illustrations it is observed that the natural frequency of the panels decrease as the core becomes thicker and tend to radiate more sound in comparison to the panels with a thinner core. The increasing core-to-face modular ratio is found to have a stiffening influence on the panels and the panels with core-to-face modular ratio equal to unity radiate the least sound power of all the other cases considered within the considered excitation frequency

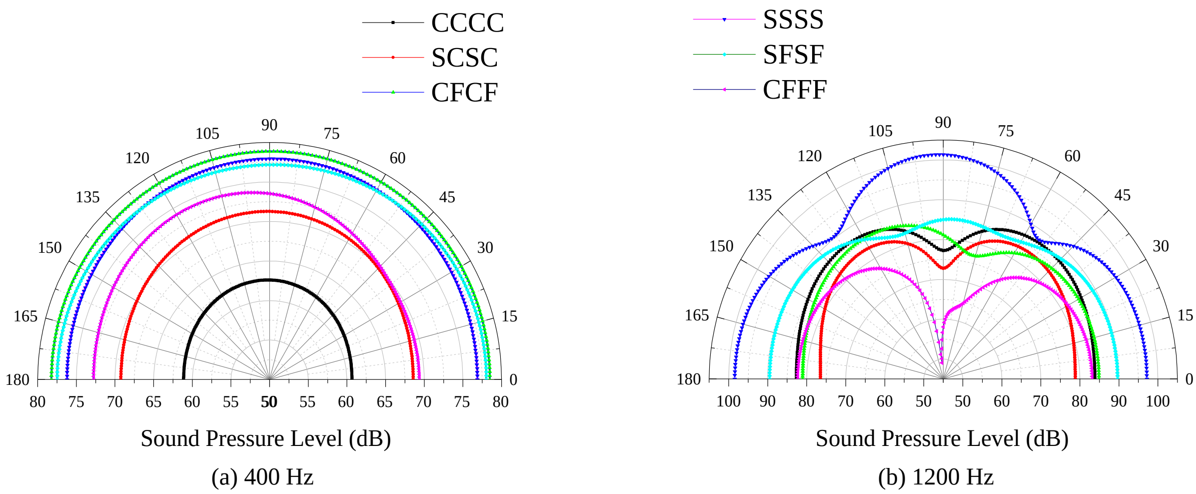

Fig. 9 Sound pressure level directivity patterns for different frequencies: (a) 400 Hz, (b) 1200 Hz

range. The radiation efficiency of the panels decreases with decreasing fibre angle and the resonance peaks in the response curves shift towards higher frequencies. Additionally, the panels radiate less efficiently with the lesser constraints at the supports.

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