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An efficient procedure for lightweight optimal design of composite laminated beams

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Abstract. A simple and efficient numerical optimization approach for the lightweight optimal design of composite laminated beams is presented in this paper. The proposed procedure is a combination between the finite element method (FEM) and a global optimization algorithm developed recently, namely Jaya. In the present procedure, the advantages of FEM and Jaya are exploited, where FEM is used to analyze the behavior of beam, and Jaya is modified and applied to solve formed optimization problems. In the optimization problems, the objective aims to minimize the overall weight of beam; and fiber volume fractions, thicknesses and fiber orientation angles of layers are selected as design variables. The constraints include the restriction on the first fundamental frequency and the boundaries of design variables. Several numerical examples with different design scenarios are executed. The influence of the design variable types and the boundary conditions of beam on the optimal results is investigated. Moreover, the performance of Jaya is compared with that of the well-known methods, viz. differential evolution (DE), genetic algorithm (GA), and particle swarm optimization (PSO). The obtained results reveal that the proposed approach is efficient and provides better solutions than those acquired by the compared methods.

Keywords: Jaya algorithm; lightweight design optimization; laminated composite beams; frequency constraints

1. Introduction

Owing to various remarkable mechanical properties like high stiffness-to-weight ratio, high strength-to-weight ratio, high corrosion resistance, great fatigue properties, and tractability in design, composite materials have been broadly employed in many engineering applications such as automotive industries, civil infrastructures, and aerospace structures, especially in civil aircraft where structural components are required to be light and highly durable. For these applications, a beam is one of the types of structures which has been popularly used (Qatu 1992) such as aircraft wing, helicopter blade, wind turbine blade, robot arm, and space antenna. For example, the floor beams of Boeing 787 are made of composite material, which is the first commercial airplane to use composite floor beams (Liu 2016). Although possessing various exceptional properties and flexibility in design, it is not easy to design a composite structure which can totally exploit these advantages because of its complex mechanical behaviors. Therefore, setting up and solving design optimization problems of composite structures to find optimal solutions for different applications are really necessary and important. However, because of complicated mechanical behaviors, the optimal design procedures of laminated composite structures are usually more challenging than those associated with isotropic material structures.

In recent years, many works have been published for optimization of laminated composite structures. For example, the optimum design of laminated composite plates for maximizing the first natural frequency can be found in Refs. (Apalak *et al*. 2011, Sadr and Ghashochi Bargh 2012, Topal 2012, Apalak *et al*. 2013, Hwang *et al*. 2014), or those for maximizing the buckling load factor in Refs (Hajmohammad *et al*. 2013, Jing *et al*. 2015, Ho-Huu *et al*. 2016), or those for minimizing the weight in Refs. (Cho 2013, Liu and Paavola 2015, Fan *et al*. 2016, Vo-Duy *et al*. 2017b), and or those for maximizing strain energy in Ref. (Le-Anh *et al*. 2015). The optimal design of laminated composite beams to minimize the free vibration frequency was found in Refs. (Roque *et al*. 2016, Tsiatas and Charalampakis 2017), or those to minimize the weight in Refs. (Liu 2015, 2016), or those to maximize the buckling load and minimize the weight at the same time in Ref. (Reguera and Cortínez 2016). So far, the literature review shows that most of the studies focus on the objectives of the fundamental frequency and buckling load factor; and the design variables are often only the fiber orientations which aim to enhance the stiffness of the structure. Nevertheless, the objective of minimizing the structural weight with design variables of thickness and fiber orientations at the same time to save material cost and apply for light structures is still somewhat limited.

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Recently, a novel approach for the lightweight design optimization of laminated composite beams has been proposed by Liu (2015). Compared to the previous studies, besides conventional design variables such as fiber orientation and thickness of layers, Liu (2015) investigated a new kind of a design variable called fiber volume fractions of layers. Also, the constraint on the first natural frequency was also taken into account in the problem formulation. The obtained results indicated that the fiber volume fractions of layers were also potential candidates for the lightweight design of laminated composite structures. In fact, when the fiber volume fractions were considered as design variables, the overall weight of laminated composite structures could be reduced considerably. In this work, however, thicknesses of layers were not considered as design variables, whilst they directly link to the weight of the structures. Furthermore, this study also has two other drawbacks: (1) an analytical method is applied to analyze the behaviour of the laminated composite beam, which is often difficult for extending to structures with complicated boundary conditions; and (2) an linear programming based on linearly approximation of the objective and constraint functions is utilized to solve the design optimization problems, which is hard to obtain a good solution, and is also difficult to cope with discrete design variables or mixed discrete-continuous design variables. To overcome these limitations, Vo-Duy *et al*. (2017a) presented a new formulation with two objectives of the weight and frequency, which considers both the fiber volume fractions and thickness of layers as design candidates, and then the non-dominated sorting genetic algorithm II (NSGA-II) (Deb *et al*. 2002) was applied to solve the multi-objective optimization problems. Although this approach can offer a range of optimal solutions, it is still hard for engineering designers to choose a suitable one; while the single-objective optimal design is a common approach to which engineering designers usually prefer.

Motivated by the success of population-based optimization methods, Venkata Rao (2016), and Venkata Rao and Waghmare (2016) recently proposed a simple parameter-free algorithm, namely Jaya, which only requires common control parameters such as the population size, the number of generations and the elite size. Jaya is, therefore, comparatively simple to understand and implement by structural designers who are with less experience with algorithms. In the previous work by Venkata Rao (2016), Venkata Rao and Waghmare (2016), Jaya has been tested for both unconstrained and constrained benchmark problems. The computational results revealed that Jaya is competitive with or superior to the famous optimization algorithms in the literature such as differential evolution (DE) (Storn and Price 1997), particle swarm optimization (PSO) (Kennedy and Eberhart 1995), teaching -learning-based optimization (TLBO) (Rao *et al*. 2011) and their variations. Although Jaya has been developed for several optimization problems, it has not been yet considered for the complex problems such as the lightweight design optimization of laminated composite beams subjected to frequency constraints. Moreover, it is originally proposed to deal with continuous variables.

From the above considerations, this study aims to present a simple but efficient numerical procedure for the lightweight optimal design of the laminated composite beams with frequency constraints. To surmount the drawbacks of Liu (2015), in this work, the advantages of FEM and Jaya are exploited, where FEM is used to analyze the behavior of beam and Jaya is adjusted and applied to find the optimal solution of the optimization problems with mixed discrete-continuous design variables. In the optimization problem, the objective function is to minimize the overall weight of the laminated composite beam. The design variables are fiber volume fractions, thicknesses and fiber orientation angles of layers in which the former are continuous variables and the latter are discrete variables. The constraints consist of the frequency limitations, fiber volume fractions, thicknesses and fiber orientation angles of the layers. Some numerical examples with different design contexts are carried out, where the influence of various boundary conditions and the different types of design variables on the optimal results is also investigated. The reliability and robustness of the proposed approach are validated and evaluated by comparing the obtained results with those acquired by the well-known methods, differential evolution (DE) (Storn and Price 1997), genetic algorithm (GA) (Goldberg 1989), particle swarm optimization (PSO) (Kennedy and Eberhart 1995), and those available in the literature.

The paper is structured as follows. Section 2 briefly provides FEM for the laminated composite beam. Section 3 presents the formulation of the lightweight design problem of laminated composite beams with frequency constraints. Section 4 presents the Jaya algorithm. Numerical examples are executed in Section 5, and some conclusions are drawn in Section 6.

2. Finite element method for laminated composite beam

Let us consider a laminated composite beam with N layers as shown in Fig. 1. The beam has the length L, the width b, and the thickness h, and is coordinated by a global coordinate system $Oxyz$ at the center, where the x-axis is in the longitudinal direction. In each layer, the fiber orientation angles are defined by $\theta^{(1)}$, $\theta^{(2)}$, $\theta^{(3)}$,...., $\theta^{(N)}$, the fiber volume fractions are denoted by $r_f^1, r_f^2, \ldots, r_f^N$ and vertical coordinates of layers are determined by z_0, z_1,..., $z_{N\text{-}1}$, z_N.

By ignoring the bending of the beam on the yz-plane, the

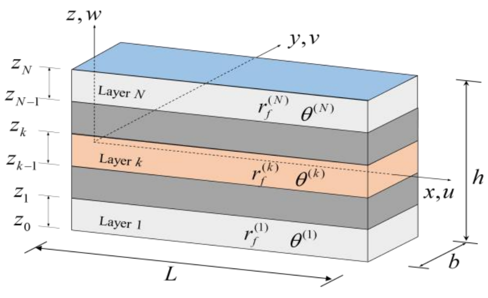


Fig. 1 The laminated composite beam

influence of shear deformation, and rotary inertia, the displacement field of the laminated composite beam obtained by applying Euler-Bernoulli beam theory is given as follows

$$
\begin{aligned}
u(x,z) &= z\beta_x(x) \\
w(x,z) &= w_0(x)
\end{aligned}
\tag{1}
$$

where u and w are, respectively, x-direction and z-direction displacements of the beam; β_x is the rotation of the cross-section, and is defined by $\beta_x = \frac{\partial w}{\partial x}$; and w_0 is the z-direction displacement of the neutral beam axis.

From the relationship between the displacement and strain described by $\varepsilon_x = \frac{\partial u}{\partial x} = z\frac{\partial^2 w}{\partial x^2}$, the stress-strain equations for an element of material in the kth lamina may be written as follows

$$\sigma_x^{(k)} = \bar{Q}_{11}^{(k)}\varepsilon_x \tag{2}$$

where

$$
\begin{aligned}
\bar{Q}_{11}^{(k)} = {} & Q_{11}^{(k)}\cos^4\theta^{(k)} + 2\left(Q_{12}^{(k)} + 2Q_{66}^{(k)}\right)\sin^2\theta^{(k)}\cos^2\theta^{(k)} \\
& + Q_{22}^{(k)}\sin^4\theta^{(k)}
\end{aligned}
\tag{3}
$$

with

$$
\begin{aligned}
Q_{11}^{(k)} &= \frac{E_1^{(k)}}{1-\nu_{12}^{(k)}\nu_{21}^{(k)}}, \qquad Q_{12}^{(k)} = \frac{\nu_{12}^{(k)}E_2^{(k)}}{1-\nu_{12}^{(k)}\nu_{21}^{(k)}}, \\
Q_{66}^{(k)} &= G_{12}^{(k)}, \qquad Q_{22}^{(k)} = \frac{E_2^{(k)}}{1-\nu_{12}^{(k)}\nu_{21}^{(k)}}
\end{aligned}
\tag{4}
$$

where $E_1^{(k)}$ and $E_2^{(k)}$ are the longitudinal and transverse elastic moduli, respectively; $\nu_{12}^{(k)}$ and $\nu_{21}^{(k)}$ are the Poisson constants; $G_{12}^{(k)}$ is the strain modulus. These parameters are computed by (Jones 1998, Liu 2015)

$$
\begin{aligned}
E_1^{(k)} &= E_f r_f^{(k)} + E_m r_m^{(k)} = E_f r_f^{(k)} + E_m\left(1-r_f^{(k)}\right) \\
E_2^{(k)} &= \frac{E_f E_m}{E_f r_m^{(k)} + E_m r_f^{(k)}} = \frac{E_f E_m}{E_f\left(1-r_f^{(k)}\right) + E_m r_f^{(k)}} \\
\nu_{12}^{(k)} &= \nu_{21}^{(k)} = \nu_f r_f^{(k)} + \nu_m r_m^{(k)} = \nu_f r_f^{(k)} + \nu_m\left(1-r_f^{(k)}\right) \\
G_{12}^{(k)} &= \frac{G_f G_m}{G_f r_m^{(k)} + G_m r_f^{(k)}} = \frac{G_f G_m}{G_f\left(1-r_f^{(k)}\right) + G_m r_f^{(k)}} \\
G_f &= \frac{E_f}{2\left(1+\nu_f\right)}, G_m = \frac{E_m}{2\left(1+\nu_m\right)}
\end{aligned}
\tag{5}
$$

where E_f is the elastic modulus of fiber; E_m is the modulus of the matrix; v_f is the Poisson constant of fiber and v_m is the Poisson constant of the matrix.

By applying Hamilton's principle, the governing equation for free vibration of the laminated composite beam can be gained as follows

$$\delta\int_{t_0}^{t_1}\left[\frac{1}{2}\int_V \sigma_x^{(k)}\varepsilon_x \mathrm{d}V - \frac{1}{2}\int_V \rho^{(k)}\left(\dot{u}^2 + \dot{w}^2\right)\mathrm{d}V\right]\mathrm{d}t = 0 \tag{6}$$

where $\rho^{(k)}$ is the mass density of the kth layer and determined by

$$\rho^{(k)} = r_f^{(k)}\rho_f + \left(1-r_f^{(k)}\right)\rho_m \tag{7}$$

where ρ_f and ρ_m are, respectively, the density of fiber and matrix.

With the use of the finite element method, the overall system equations of motion can be obtained as follows

$$\mathbf{M}\ddot{\mathbf{d}} + \mathbf{K}\mathbf{d} = 0 \tag{8}$$

where $\mathbf{d}$ is the nodal displacement vector; $\ddot{\mathbf{d}}$ is the second-order derivative with respect to time of $\mathbf{d}$; $\mathbf{M}$ and $\mathbf{K}$ are the global mass matrix and stiffness matrix, respectively, which are assembled from elemental stiffness matrix ($\mathbf{K}^e$) and elemental mass matrix ($\mathbf{M}^e$), given by

$$
\begin{aligned}
\mathbf{K}^e &= \frac{D_{11}}{l_e^3}\begin{bmatrix} 12 & 6l_e & -12 & 6l_e \\ & 4l_e^2 & -6l_e & 2l_e^2 \\ & & 12 & -6l_e \\ \text{sym.} & & & 4l_e^2 \end{bmatrix}; \\
\mathbf{M}^e &= \frac{I_1 l_e}{420}\begin{bmatrix} 156 & 22l_e & 54 & -13l_e \\ & 4l_e^2 & 13l_e & -3l_e^2 \\ & & 156 & -22l_e \\ \text{sym.} & & & 4l_e^2 \end{bmatrix}
\end{aligned}
\tag{9}
$$

where l_e is the length of the eth element, and D_{11} and I_1 are defined by

$$D_{11} = \frac{1}{3}\sum_{k=0}^{N} b\bar{Q}_{11}^{(k)}\left(z_k^3 - z_{k-1}^3\right);\ I_1 = \sum_{k=0}^{N} b\rho^{(k)}\left(z_k - z_{k-1}\right) \tag{10}$$

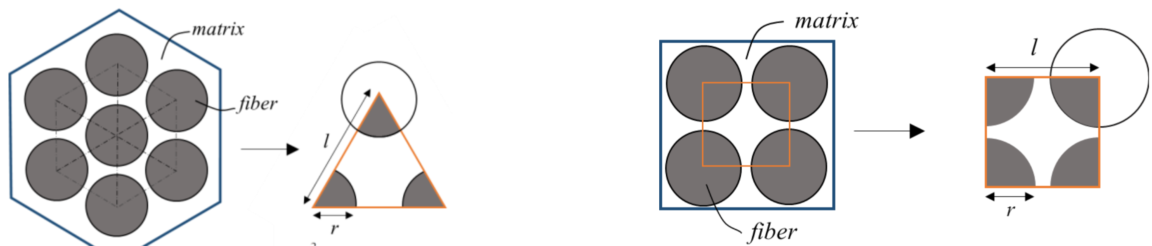
From Eq. (8), the eigenvalue problem can be derived as follows

$$(\mathbf{K} - \omega^2\mathbf{M})\phi = 0 \tag{11}$$

By solving this equation, the natural frequency (ω) and mode shape (ϕ) of the beam are obtained. The values of ω are then used for the formulation of the constraints of the optimization problems.

3. Statement of the optimization problem

As has mentioned above, the main objective of the study is to minimize the overall weight of the laminated composite beams with respect to the design variables of fiber volume fractions, thickness and fiber orientation angles of layers, where the fiber volume fractions are continuous, and thickness and fiber orientation angles are discrete. The optimization problem has a constraint function on the first frequency which must be larger than a predefined value by designers. The problem is mathematically formulated as follows



$$r_f = \frac{\text{Area of fiber}}{\text{Area of matrix}} = \frac{3\frac{\pi r^2}{6}}{\frac{1}{2} l.l \sin\left(\frac{\pi}{3}\right)} = \frac{2\pi r^2}{\sqrt{3} l^2},$$

$$r_f^{\max} = \frac{2\pi r^2}{\sqrt{3} l^2} = \frac{2\pi r^2}{\sqrt{3}.(2r)^2} = \frac{\pi}{2\sqrt{3}} = 0.9069$$

(a)

$$r_f = \frac{\text{Area of fiber}}{\text{Area of matrix}} = \frac{4\frac{\pi r^2}{4}}{l^2} = \frac{\pi r^2}{l^2},$$

$$r_f^{\max} = \frac{\pi r^2}{(2r)^2} = \frac{\pi}{4} = 0.7854$$

(b)

Fig. 2 Fiber arrangements: (a) Hexagonal array; (b) Square array

$$\begin{aligned} \min_{\mathbf{r}_f,\mathbf{t},\boldsymbol{\theta}} \quad & weight\left(\mathbf{r}_f,\mathbf{t}\right) = \sum_{k=1}^{N} \rho^{(k)}\left(r_f^{(k)}\right) \times A_k \times t^{(k)} \\ \text{s.t} \quad & \omega\left(\mathbf{r}_f,\mathbf{t},\boldsymbol{\theta}\right) \geq \underline{\omega}_1 \\ & 0 \leq r_f^{(k)} \leq r_f^{\max} \\ & \theta^{(k)} \in \boldsymbol{\theta}_{\mathrm{D}}, \quad t^{(k)} \in \mathbf{t}_{\mathrm{D}},\ k = 1,\ldots,N \end{aligned} \tag{12}$$

where $weight(\mathbf{r}_f, \mathbf{t})$ and $\omega(\mathbf{r}_f, \mathbf{t}, \boldsymbol{\theta})$ are the weight and first frequency of the beam, respectively; $\mathbf{r}_f$, $\mathbf{t}$, $\boldsymbol{\theta}$ are the design variable vectors of fiber volume fractions $r_f^{(k)}$, thickness $t^{(k)}$ and fiber orientation angle $\theta^{(k)}$ of layers, respectively; $\rho^{(k)}\left(r_f^{(k)}\right)$ and A_k are the mass density and the area of the kth layer, respectively; $\underline{\omega}_1$ is the lower bound of the first frequency; $\boldsymbol{\theta}_{\mathrm{D}}$ is the set of integer values of fiber angles in the range of [-90,90]; $\mathbf{t}_{\mathrm{D}}$ is the set of discrete value of thickness which is defined by manufacturer or designer; N is the total number of layers; and $r_f^{\max}$ is the maximum of r_f in a lamina. In manufacturing, it should be noted that the maximum value of r_f depends on the fiber arrangement in the matrix. As mentioned in Refs. (Robert M. Jones 1998; Altenbach *et al*. 2004), the value of $r_f^{\max}$ can be either 0.7854 if the fiber is arranged as a hexagonal array (Fig. 2(a)) or 0.9069 if the fiber is arranged as a square array (Fig. 2(b)).

It should be noted in the formulated problem in Eq. (12) that although the fiber orientation angles do not directly influence on the objective function, they still have impacts on the frequency constraint. In fact, the change of the fiber orientations will lead to the change in the frequency behavior of beam. Thus, there may be some potential fiber orientation angles that can satisfy the constraint at the lowest weight of beam.

In order to deal with the constraint in Eq. (12), the penalty function method is utilized in this study. It is determined as follows (Kaveh and Zolghadr 2014)

$$\begin{aligned} f_{\text{penalty}}(\mathbf{r}_f,\mathbf{t}) = (1+[\max\{0, g_i(\mathbf{r}_f,\mathbf{t})\}])^{\gamma} \\ \times weight(\mathbf{r}_f,\mathbf{t}),\ g_i(\mathbf{r}_f,\mathbf{t}) = \underline{\omega}_1 - \omega(\mathbf{r}_f,\mathbf{t}) \end{aligned} \tag{13}$$

where $g_i(\mathbf{r}_f,\mathbf{t})$ is the frequency constraint of the optimization problem; the parameters γ_1 and γ_2 are scalars associated with the exploration and exploitation rate of the search space, respectively. In this study, γ is selected to start from 1.5 and then linearly increases to 6 (Kaveh and Zolghadr 2014).

4. Jaya algorithm

Recently, Venkata Rao and Waghmare (2016) have proposed a variant of the teaching-learning-based optimization (TLBO) algorithm called Jaya. Unlike the traditional TLBO algorithm, Jaya is much simpler due to its two advantages: (1) using only one phase instead of two phases as the traditional TLBO algorithm; (2) requiring only common controlling parameters like the population size, and the number of generations. The effectiveness and robustness of the algorithm are demonstrated through unconstrained and constrained benchmark functions and constrained mechanical design problems. The obtained numerical results show that Jaya is superior to or competitive with other well-known optimization algorithms for the considered problems. Nevertheless, Jaya is only tested for benchmark functions and simple mechanical design problems. Moreover, it was originally created to solve continuous design variables. In present work, Jaya is, therefore, extended to handle the constrained optimization problem as presented in Section 3 with mixed discrete-continuous design variables. The details of the Jaya algorithm are presented as follows.

4.1 Algorithm

Firstly, an initial population of N solutions is generated by mean of using randomly sampling from the design space. Each ith solution of the population is a vector containing D design variables $\mathbf{x}_i = (x_1, x_2, \ldots, x_j, \ldots, x_D)$ and is generated by

$$\begin{aligned} x_{j,i} = x_{j,i}^{l} + rand \times \left(x_{j,i}^{u} - x_{j,i}^{l}\right) \\ i = 1,2,\ldots,N; \quad j = 1,2,\ldots,D \end{aligned} \tag{14}$$

where x_j^l and x_j^u are the lower and upper bounds of x_j, respectively; *rand* is a uniformly distributed random number in [0,1]; N is the population size and D is the number of design variables.

Secondly, corresponding to each ith solution $\mathbf{x}_i$, a new solution $\mathbf{u}_i$ is generated by using the information of the current solution $\mathbf{x}_i$, the best solution $\mathbf{x}_{\text{best}}$, and the worst solution $\mathbf{x}_{\text{worst}}$ of the population via operation

$$u_{j,i} = x_{j,i} + rand_{1,j} \times \left(x_{j,\text{best}} - \left|x_{j,i}\right|\right) - rand_{2,j} \times \left(x_{j,\text{worst}} - \left|x_{j,i}\right|\right) \tag{15}$$

where $rand_{1,j}$ and $rand_{2,j}$ are the random numbers within [0,1]; $|x_{j,i}|$ is the absolute value of the solution $x_{j,i}$. In Eq. (15), the component $rand_{1,j} \times (x_{j,\text{best}} - |x_{j,i}|)$ will have a trend to move the new solution closer to the best solution. While the component $rand_{2,j} \times (x_{j,\text{worst}} - |x_{j,i}|)$ will have a trend to move the solution far away from the worst solution. The random numbers $rand_{1,j}$ and $rand_{2,j}$ will guarantee a good exploration of the search space.

After this phase, the jth components $u_{j,i}$ of the vector $\mathbf{u}_i$ are reflected back to the allowable domain if their boundary constraints are violated. This procedure is conducted as follows

$$u_{j,i} = \begin{cases} 2x_j^l - u_{j,i} & \text{if } u_{j,i} < x_j^l \\ 2x_j^u - u_{j,i} & \text{if } u_{j,i} > x_j^u \\ u_{j,i} & \text{otherwise} \end{cases} \tag{16}$$

Finally, based on the value of the objective function, the vector $\mathbf{u}_i$ is compared to the target vector $\mathbf{x}_i$. The better one having lower objective function value will survive to the next generation

$$\mathbf{x}_i = \begin{cases} \mathbf{u}_i & \text{if } \; f(\mathbf{u}_i) \le f(\mathbf{x}_i) \\ \mathbf{x}_i & \text{otherwise} \end{cases} \tag{17}$$

4.2 Handling discrete variables

It should be noted that Jaya is originally proposed for the optimization problems with continuous design variables. Therefore, to cope with the mixed discrete-continuous design variables as stated in Section 3, a simple technique with a rounding function which permits to change the continuous value of a result to the discrete value is utilized in this paper. This technique is depicted as follows

$$\mathbf{x}_i^{\text{discrete}} = round(\mathbf{x}_i^{\text{continuous}}) \tag{18}$$

where *round*($\mathbf{x}$) is the function which rounds each element of $\mathbf{x}$ to the nearest permissible discrete value. By using this approach, after defining the population for the next generation, the thickness discrete design variables of all individuals in the population will be rounded to the nearest discrete values of the set $\mathbf{t}_D$ by using Eq. (18).

By combining between Section 4.1 and 4.2, the Jaya algorithm is briefly presented in Algorithm 1.

Algorithm 1: Jaya algorithm

```
Create the initial population
Evaluate the fitness of each individual in the population
while ( delta > tolerance and MaxIter is not reached ) do
  for i =1 to NP do
    Determine x_best, x_worst
    for j =1 to D do
      if  x_{j,i} is variable r_f^(k)
          u_{j,i} = x_{j,i} + rand_{1,j} × (x_{j,best} − |x_{j,i}|) − rand_{2,j} × (x_{j,worst} − |x_{j,i}|)
        elseif x_{i,i} is variable θ^(k) or t^(k)
          u_{j,i} = x_{j,i} + rand_{1,j} × (x_{j,best} − |x_{j,i}|) − rand_{2,j} × (x_{j,worst} − |x_{j,i}|)
          if x_{i,i} is variable θ^(k)
             u_{j,i} = round(u_{j,i}) to set θ_D
          else
             u_{j,i} = round(u_{j,i}) to set t_D
          end
        end
    end for
    Evaluate the new solution u_i
    if f(u_i) ≤ f(x_i)
       x_i = u_i
    else
       x_i = x_i
    end
  end for
  Determine f_best, f_mean = (1/N) Σ_{i=1}^{N} f_i , and delta = | f_mean / f_best − 1 |
end while
```

where *tolerance* is the allowed error; *MaxIter* is the maximum number of iterations. According to Algorithm 1, the Jaya algorithm will stop the searching progress either when the *delta* is less than or equal to the *tolerance* or when the maximum number of iterations *MaxIter* is achieved.

5. Numerical example

To assess the performance of the components of the proposed approach, this section is divided into four sub-sections. The first section is to evaluate the accuracy of finite element method for free vibration analysis of laminated composite beams compared with analytical solutions. The second section is to study the optimal design problem of the laminated composite beams with one type of design variable, namely fiber volume fractions, as studied in Ref. (Liu 2015). The third section is to study the optimal design problem with two design variables consisting of fiber volume fractions and thickness of the layers. The last section is to investigate the optimization problems with all the three design variables at the same time as presented in Section 3.

For all the considered problems, the beam structures previously studied by Liu (2015) are used. The geometric parameters of the laminated composite beam are given by: length L = 14.4 m, width b = 0.3 m and height h = 0.48 m. The beam has eight layers (N = 8), and all layers have the same material properties, i.e., the fiber material E_f = 294 GPa, v_f = 0.2, ρ_f = 1.81 g/cm^3 and the matrix material E_m = 4.2 GPa, v_m = 0.3, ρ_m = 1.24 g/cm^3. The laminated

Table 1 Comparison of FEM solution ([$0^0/90^0/45^0/45^0$]s)

BC	Method	Mode			
		1	2	3	4
H-H	FEM	2862	45797	231876	733102
	Liu (2015)	2862	45795	231838	732722
	Error %	0	0.004	0.016	0.052
C-C	FEM	14708	111769	429645	1174653
	Liu (2015)	14708	111761	429514	1173680
	Error %	0	0.007	0.030	0.083
C-F	FEM	363	14266	111858	429633
	Liu (2015)	363	14266	111848	429503
	Error %	0	0.000	0.009	0.030
C-H	FEM	6985	73359	319398	934420
	Liu (2015)	6985	73355	319324	933803
	Error %	0	0.005	0.023	0.066

composite beam has two kinds of fiber orientation angles which consist of $[0^0/90^0/45^0/\text{-}45^0]_S$ and $[45^0/0^0/90^0/\text{-}45^0]_S$. It should be noted that these fiber orientations are fixed in the first three sections, while they are treated as design variables in the last section. The beam with four different boundary conditions (BC) is investigated, including hinged-hinged (H-H), clamped-clamped (C-C), clamped-free (C-F) and clamped-hinged (C-H).

For all the optimization problems, the population size *NP*, *tolerance*, and *MaxIter* of Jaya are set to be 20, 10^{-6} and 500, respectively. Since Jaya is a stochastic method, to evaluate its stability as well as effectiveness, five independent runs were performed, and the obtained results are statistical and compared to those gained by DE, GA, and PSO.

5.1 Comparison of FEM solution

The first four dimensionless frequencies of the angle-ply ([$0^0/90^0/45^0/45^0$]s) obtained by FEM and the analytical method gained by Liu (2015) are provided in Table 1. The results in the table show that the FEM solutions agree well with the analytical results.

5.2 Optimal design with only variable r_f

In these optimization problems, only the fiber volume fractions (r_f) are taken as continuous design variables, while the thickness of the beam and the thickness of each layer are kept fixed at 40 mm and 5 mm, respectively. The problem was previously solved by Liu (2015) using linear programming. Moreover, it should also be noted that because the constraint on manufacturing process is not considered in Liu (2015), the upper bound of r_f is always set to be 1.

Tables 2 and 3 compare the optimal results of Jaya, DE, GA, PSO and the linear programming for the cases of [$0^0/90^0/45^0/45^0$]s and [$45^0/0^0/90^0/\text{-}45^0$]s beams, respectively. It can be seen that the best weights obtained by the present

Table 2 Comparison of optimal results of the [$0^0/90^0/45^0/45^0$]s beam

BC	ω	Method	Fiber volume fractions (%)				Weight (kg)	NSA
			$r_f^{(1)}$	$r_f^{(2)}$	$r_f^{(3)}$	$r_f^{(4)}$		
H-H	400	DE	3.7	0	0	0	2582.276	5380
		GA	3.7	0	0	0	2582.335	5220
		PSO	3.7	0	0	0	2582.274	4820
		Jaya	3.7	0	0	0	2582.275	1180
		Liu (2015)	3.7	0	0	0	2582	-
	5652	DE	52.9	100	0	0	3023.188	6260
		GA	100	0	0	0	2866.783	7480
		PSO	52.9	100	0	0	3023.209	10000
		Jaya	99.9	0	0	0	2866.335	1320
		Liu (2015)	90.1	0	100	0	3133	-
C-C	10000	DE	29.2	0	0	0	2657.509	5900
		GA	29.2	0	0	0	2657.538	5220
		PSO	29.2	0	0	0	2657.508	4800
		Jaya	29.2	0	0	0	2657.509	1160
		Liu (2015)	29.2	0	0	0	2658	-
	28900	DE	52.9	100	0	0	3023.116	6220
		GA	100	0	0	0	2866.781	7700
		PSO	91.9	97.2	0	0	3129.985	5300
		Jaya	99.8	0	0	0	2866.267	1180
		Liu (2015)	90.1	0	100	0	3133	-

Table 2 Continued

BC	ω	Method	Fiber volume fractions (%)				Weight (kg)	NSA
			$r_f^{(1)}$	$r_f^{(2)}$	$r_f^{(3)}$	$r_f^{(4)}$		
C-F	400	DE	50.7	0	0	0	2721.041	5580
		GA	50.7	0	0	0	2721.085	5220
		PSO	50.9	3.2	1.1	0.9	2737.062	4800
		Jaya	50.7	0	0	0	2721.042	1220
		Liu (2015)	50.7	0	0	0	2721	-
	729	DE	55.5	100	0	0	3030.838	5180
		GA	55.5	100	0	0	3030.872	21840
		PSO	55.5	100	0	0	3030.988	10000
		Jaya	55.5	100	0	0	3030.839	1900
		Liu (2015)	92.9	0	100	0	3141	-
C-H	2500	DE	13.7	0	0	0	2611.863	5620
		GA	13.7	0	0	0	2611.897	5220
		PSO	13.7	0	0	0	2611.861	5080
		Jaya	13.7	0	0	0	2611.862	1180
		Liu (2015)	13.7	0	0	0	2612	-
	13225	DE	95.5	0	0	0	2853.415	5020
		GA	97.4	0	0	0	2859.110	16280
		PSO	99.6	0	0	0	2865.583	5100
		Jaya	95.5	0	0	0	2853.414	1420
		Liu (2015)	85.4	100	0	0	3119	-

Table 3 Comparison of optimal results of the [$45^0/0^0/90^0/-45^0$]s beam

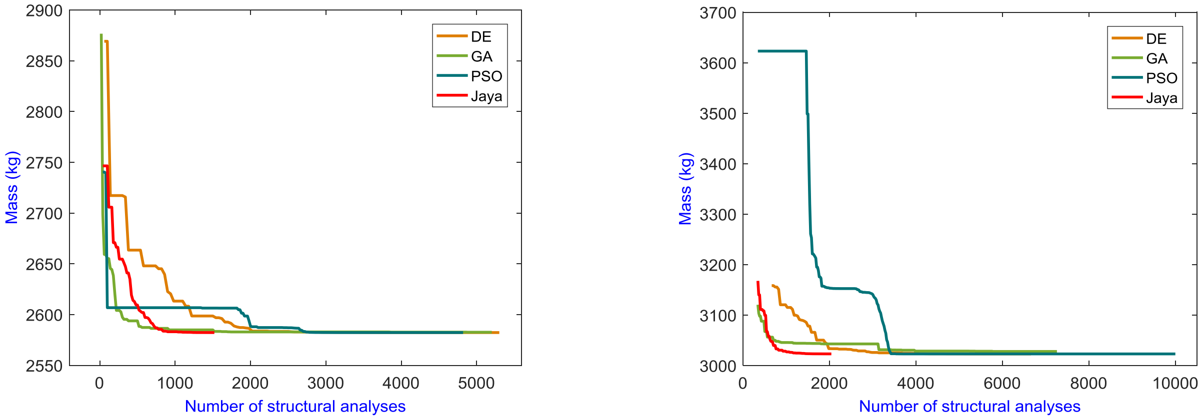
BC	ω	Method	Fiber volume fractions (%)				Weight (kg)	NSA
			$r_f^{(1)}$	$r_f^{(2)}$	$r_f^{(3)}$	$r_f^{(4)}$		
H-H	400	DE	0	7.3	0	0	2592.903	5100
		GA	0	7.3	0	0	2592.923	5220
		PSO	0	7.3	0	0	2592.902	4180
		Jaya	0	7.3	0	0	2592.903	1300
		Liu (2015)	0	7.3	0	0	2593	-
	3600	DE	98.7	0	0	0	2862.908	4820
		GA	98.7	0	0	0	2862.973	8460
		PSO	98.7	0	0	0	2862.906	5000
		Jaya	98.7	0	0	0	2862.908	1520
		Liu (2015)	55.3	100	0	0	3030	-
C-C	10000	DE	0	59.9	0	0	2748.408	5980
		GA	0.8	59.5	0	0	2749.567	5220
		PSO	0	59.9	0	0	2748.406	4960
		Jaya	0	59.9	0	0	2748.407	1320
		Liu (2015)	0	59.9	0	0	2748	-
	16900	DE	98.3	0	0	0	2861.588	5020
		GA	98.5	0	0	0	2862.436	22240
		PSO	98.3	0	0	0	2861.587	5060
		Jaya	98.3	0	0	0	2861.588	1400
		Liu (2015)	28.1	100	0	0	2950	-

Table 3 Continued

BC	ω	Method	Fiber volume fractions (%)				Weight (kg)	NSA
			$r_f^{(1)}$	$r_f^{(2)}$	$r_f^{(3)}$	$r_f^{(4)}$		
C-F	400	DE	98	0	0	0	2860.862	5180
		GA	98	0	0	0	2860.918	9900
		PSO	98	0	0	0	2861.007	5040
		Jaya	98	0	0	0	2860.861	2220
		Liu (2015)	16	100	0	0	2914	-
	900	DE	100	68.7	0	0	3069.627	5420
		GA	100	68.7	0	0	3069.656	15460
		PSO	100	91.2	96.4	5.1	3436.283	10000
		Jaya	100	68.7	0	0	3069.625	1960
		Liu (2015)	100	69	0	0	3070	-
C-H	2500	DE	0	27.5	0	0	2652.45	6220
		GA	0.0	27.6	0	0	2652.950	11420
		PSO	0.0	27.5	0	0	2652.448	4420
		Jaya	0	27.5	0	0	2652.450	1440
		Liu (2015)	0	27	0	0	2652	-
	16900	DE	100	60	0	0	3044.015	6660
		GA	100	64.6	0	0	3057.526	15100
		PSO	99.8	99.9	0.1	0	3161.820	5580
		Jaya	100	60	0	0	3044.014	2320
		Liu (2015)	100	60	0	0	3044	-

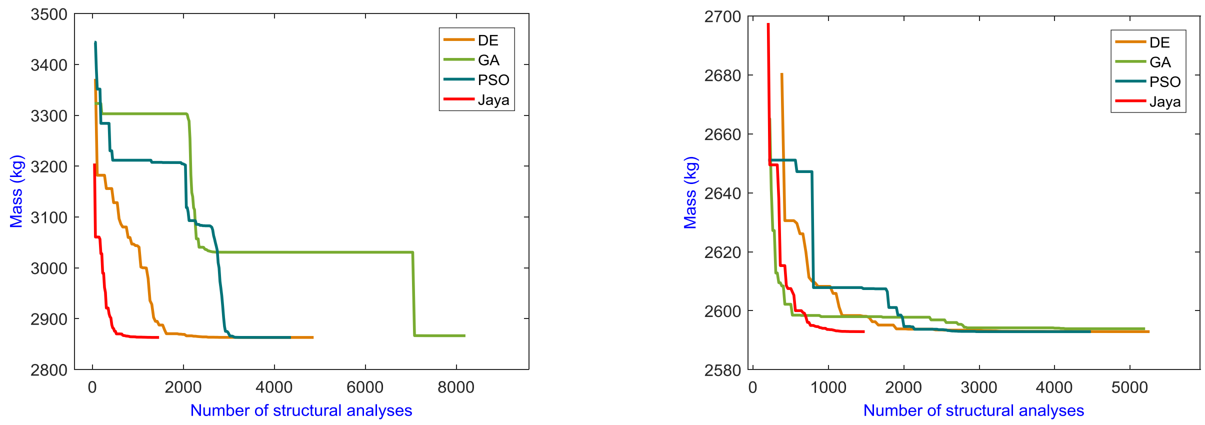
methods are equal or less than those obtained by Liu's method. In addition, Jaya provides the smallest weights in comparison with DE, GA, PSO in some cases of boundary conditions and frequency constraints. For example, the best weights obtained by Jaya, DE, GA, PSO, and Liu's method are, respectively, 2866.267 kg, 3023.116 kg, 2866.781 kg, 3129.985 kg and 3133 kg for the case of the C-C beam ([$0^0/90^0/45^0/45^0$]s) with the frequency constraint of 28900 Hz. It is also noted from the results in the tables that the number of structural analyses (NSA) of Jaya is generally less than those of the compared methods. Convergence histories of Jaya, DE, GA, and PSO are presented in Figs. 3 and 4. It can be seen that Jaya always converges much faster than DE, GA, and PSO.

Table 4 provides statistical results of five independent runs of Jaya, DE, GA, and PSO for H-H laminated composite beams. The results show that Jaya and DE are more stable than GA and PSO. Furthermore, the best weights of these algorithms are mostly the same. Moreover,in all test cases, the average number of structural



(a) (b)

Fig. 3 Convergence histories of DE, GA, PSO and Jaya for the hinged-hinged beam ([$0^0/90^0/45^0/45^0$]s): (a) frequency constraint of 400 Hz; (b) frequency constraint of 5652 Hz



(a) (b)

Fig. 4 Convergence histories of DE, GA, PSO and Jaya for the hinged-hinged beam ([$45^0/0^0/90^0/-45^0$]s): (a) frequency constraint of 400 Hz; (b) frequency constraint of 3600 Hz

Table 4 Statistical results of five independent runs of Jaya, DE, GA and PSO algorithms for hinged-hinged laminated composite beams

Fiber orientation	ω	Method	Best	Worst	Mean	Std.	ANSA
[$0^0/90^0/45^0/45^0$]s	400	DE	2582.276	2582.282	2582.278	0.003	5076
		GA	2582.307	2582.351	2582.332	0.019	7348
		PSO	2582.274	2619.550	2590.998	16.195	5630
		Jaya	2582.275	2582.280	2582.277	0.002	1252
	5652	DE	3023.188	3023.190	3023.189	0.001	6052
		GA	2866.370	3075.051	2971.645	98.072	13592
		PSO	3023.430	3338.630	3158.066	113.421	3158
		Jaya	2866.335	2866.336	2866.336	0.001	1188
[$45^0/0^0/90^0/-45^0$]s	400	DE	2592.903	2592.907	2592.905	0.002	5756
		GA	2592.923	2600.474	2594.472	3.356	6804
		PSO	2592.902	2594.048	2593.189	0.488	4776
		Jaya	2592.903	2592.905	2592.904	0.001	1320
	3600	DE	2862.908	2862.913	2862.910	0.002	4868
		GA	2862.973	3032.112	2910.809	73.901	9800
		PSO	2862.906	3044.289	2900.707	80.287	5988
		Jaya	2862.908	2862.909	2862.908	0.001	1804

analyses (ANSA) of Jaya is much smaller than that of DE, GA, and PSO. This indicates that Jaya is more efficient than DE, GA, and PSO regarding computation cost.

5.3 Optimal design with variables r_f and t

In this section, the optimization problems consider both the fiber volume fractions and the thicknesses of layers as design variables at the same time, in which the thickness of layers is selected from the set of {1,2,3,…,20} (unit: mm). Also, the constraint on manufacturing process is taken into account with fiber arrangements of hexagonal arrays, i.e., the upper bound of the fiber volume fractions is set to be 0.9069.

Tables 5 and 6 compare optimal results of the considered methods for the cases of [$0^0/90^0/45^0/45^0$]s beam and the [$45^0/0^0/90^0/-45^0$]s beam, respectively. In addition, the optimal weights obtained by Liu (2015) in the previous example are also provided to show the effectiveness of considering thicknesses as design variables. From the results, it can be seen that when the thicknesses are considered, the best weight of beams is considerably smaller while the frequency constraint is not violated. In particular, the best weight gained by this problem is around 2785 kg, while that gained by Liu (2015) is 3133 kg for the C-C beam ([$0^0/90^0/45^0/45^0$]s) subjected to the frequency constraint of 28900 Hz. Also, the optimal weights acquired by Jaya and DE are almost the same and often better than those of GA and PSO. From Table 5, it is also recognized that there is still a case with the BC of C-H which the Jaya gives the worse result than DE and PSO. This helps remind that like most of the evolutionary methods, Jaya cannot always guarantee a better solution in all the cases. However, the NSA of Jaya is considerably less than that of DE, GA, and PSO.

5.4 Optimal design with variables r_f, t and θ

This section performs the lightweight design optimization as presented in Section 3. Besides the variables considered in section 5.4, the fiber orientations of layers are also taken as design variables which are integer values in the range of [-90^0, 90^0]. In addition, the upper bound of

Table 5 Optimal results of the [0^0/90^0/45^0/45^0]s beam

BC	ω	Method	Fiber volume fractions (%)				Thickness (cm)				Weight (kg)	NSA
			$r_f^{(1)}$	$r_f^{(2)}$	$r_f^{(3)}$	$r_f^{(4)}$	$t^{(1)}$	$t^{(2)}$	$t^{(3)}$	$t^{(4)}$		
H-H	400	DE	79.2	0	0	0	3	1	1	1	759.899	8660
		GA	73.4	0.1	0.1	0.3	2	1	3	1	822.623	4341
		PSO	83.7	0	87.5	0	3	1	1	1	809.498	4700
		Jaya	79.2	0	0	0	3	1	1	1	759.899	2240
		Liu (2015)	3.7	0	0	0	6	6	6	6	2582	-
	5652	DE	86.9	0	0	0	10	6	1	5	2785.088	13660
		GA	87.0	0	0.1	0.2	10	10	1	1	2785.433	6021
		PSO	86.9	0	0	0.1	10	9	1	2	2785.275	4920
		Jaya	86.9	0	0	0	10	10	1	1	2785.088	2400
		Liu (2015)	90.1	0	100	0	6	6	6	6	3133	-
C-C	10000	DE	85.4	0	0	0	6	2	1	4	1645.072	13500
		GA	82.1	0.9	4.0	1.9	7	3	2	1	1681.929	5001
		PSO	68.8	0	0	0	7	1	2	4	1737.184	4900
		Jaya	85.4	0	0	0	6	5	1	1	1645.072	2120
		Liu (2015)	29.2	0	0	0	6	6	6	6	2658	-
	28900	DE	86.9	0	0	0	10	1	6	5	2784.983	12300
		GA	89.3	0.1	0.1	0	10	10	1	1	2797.368	4581
		PSO	76.5	0	0	0	8	11	1	5	2979.641	5180
		Jaya	86.9	0	0	0	10	2	9	1	2784.982	4240
		Liu (2015)	90.1	0	100	0	6	6	6	6	3133	-
C-F	400	DE	83.9	0	0	0	7	4	2	4	2110.436	14420
		GA	87.4	1.0	0.6	12.7	7	1	8	1	2131.931	2901
		PSO	85.7	60.2	0	21.5	7	1	8	1	2156.897	4840
		Jaya	87.6	0	0	0	9	1	2	4	2102.344	3360
		Liu (2015)	50.7	0	0	0	6	6	6	6	2721	-
	729	DE	89.3	0	0	0	10	7	4	1	2796.660	12900
		GA	89.3	0	0.3	1.2	10	10	1	1	2797.596	5401
		PSO	86.2	0	0	0	11	5	1	5	2824.010	5180
		Jaya	89.3	0	0	0	10	1	10	1	2796.661	3040
		Liu (2015)	92.9	0	100	0	6	6	6	6	3141	-
C-H	2500	DE	78.7	0	0	0	4	3	1	2	1226.474	12140
		GA	78.8	0.1	0.3	0	4	1	4	1	1227.074	5181
		PSO	78.7	0	0	0.2	4	4	1	1	1226.584	4820
		Jaya	78.7	0	0	0	4	1	1	4	1226.474	2640
		Liu (2015)	13.7	0	0	0	6	6	6	6	2612	-
	13225	DE	88.9	0	0	0	11	4	1	5	2731.515	12020
		GA	90.3	0	0.3	0	11	8	1	1	2739.422	3321
		PSO	88.9	0	0	0	11	7	1	2	2731.541	4840
		Jaya	86.9	0	0	0	9	10	2	1	2742.327	2500
		Liu (2015)	85.4	100	0	0	6	6	6	6	3119	-

Table 6 Optimal results of the [$45^0/0^0/90^0/-45^0$]s beam

BC	ω	Method	Fiber volume fractions (%)				Thickness (cm)				Weight (kg)	NSA
			$r_f^{(1)}$	$r_f^{(2)}$	$r_f^{(3)}$	$r_f^{(4)}$	$t^{(1)}$	$t^{(2)}$	$t^{(3)}$	$t^{(4)}$		
H-H	400	DE	9.4	90.7	0	0	1	3	2	1	888.589	10260
		GA	80.4	77.8	2.1	2.5	1	3	2	1	907.783	3761
		PSO	90.7	72.0	0.4	0	1	3	1	2	901.231	10000
		Jaya	9.4	90.7	0	0	1	3	1	2	888.588	2800
		Liu (2015)	0	7.3	0	0	6	6	6	6	2593	-
	3600	DE	47.9	90.7	0	0	1	7	7	4	2371.817	12060
		GA	90.3	90.1	0.1	0	1	9	1	7	2372.178	8001
		PSO	1.3	87.4	0	0	1	8	8	2	2380.583	5160
		Jaya	47.9	90.7	0	0	1	7	1	10	2371.819	3340
		Liu (2015)	55.3	100	0	0	6	6	6	6	3030	-
C-C	10000	DE	17.9	90.7	0	0	1	6	6	1	1776.719	10940
		GA	33.9	89.7	0.5	0.4	1	6	1	6	1782.953	3281
		PSO	0	87.3	0	0	1	7	1	5	1800.829	4940
		Jaya	17.9	90.7	0	0	1	6	1	6	1776.718	2360
		Liu (2015)	0	59.9	0	0	6	6	6	6	2748	-
	16900	DE	53.9	90.7	0	0	1	7	1	9	2267.657	11140
		GA	2.6	84.9	1.4	0.1	1	9	1	7	2306.819	8821
		PSO	49.4	80.7	0	25.9	1	11	4	2	2415.329	10000
		Jaya	53.9	90.7	0	0	1	7	1	9	2267.656	2780
		Liu (2015)	28.1	100	0	0	6	6	6	6	2950	-
C-F	400	DE	0	88.7	0	0	1	7	2	8	2234.372	12740
		GA	0.2	88.7	0	0	1	7	1	9	2234.519	10021
		PSO	57.0	81.1	11.5	0	1	8	1	8	2281.585	5600
		Jaya	0	88.7	0	0	1	7	1	9	2234.371	2640
		Liu (2015)	16	100	0	0	6	6	6	6	2914	-
	900	DE	60.8	90.7	0	0	1	10	9	6	3262.121	14740
		GA	89.8	90.6	0.8	7.4	1	12	11	1	3265.688	1581
		PSO	78.6	86.8	73.6	3.2	1	13	1	11	3698.626	10000
		Jaya	60.8	90.7	0	0	1	10	1	14	3262.123	2700
		Liu (2015)	100	69	0	0	6	6	6	6	3070	-
C-H	2500	DE	0	86.4	0	0	1	4	1	5	1348.796	11260
		GA	2.3	86.3	0.1	0.1	1	4	5	1	1350.126	8421
		PSO	0	78.5	0	0	1	5	3	2	1371.898	4960
		Jaya	0	86.4	0	0	1	4	5	1	1348.795	1960
		Liu (2015)	0	27	0	0	6	6	6	6	2652	-
	16900	DE	90.7	89.8	0	0	1	11	12	1	3209.496	10100
		GA	36.3	88.1	6.7	85.0	1	18	5	1	3535.296	1801
		PSO	52.5	90.2	12.0	0.4	1	9	4	13	3878.647	10000
		Jaya	90.7	89.8	0	0	1	11	12	1	3209.495	5780
		Liu (2015)	100	60	0	0	6	6	6	6	3044	-

fiber volume fractions is set to be 0.9069 with the fiber arrangements of hexagonal arrays.

Table 7 presents the optimal results obtained Jaya, DE, GA and PSO. As can be seen in the table, the optimal weights acquired by Jaya and DE results are quite similar, which are much better than those obtained by GA and PSO. However, Jaya significantly outperforms DE, GA and PSO in terms of the computational cost.

Table 7 Optimal results of the optimization problem with all the three variables

BC	ω	Method	Fiber volume fractions (%)				Thickness (cm)				Fiber orientation (°)				Weight (kg)	NSA
			$r_f^{(1)}$	$r_f^{(2)}$	$r_f^{(3)}$	$r_f^{(4)}$	$t^{(1)}$	$t^{(2)}$	$t^{(3)}$	$t^{(4)}$	$\theta^{(1)}$	$\theta^{(2)}$	$\theta^{(3)}$	$\theta^{(4)}$		
H-H	400	DE	90.7	25.0	0.0	0.0	2	1	1	2	0	0	68	-39	744.464	14940
		GA	79.3	0.0	0.0	0.0	3	1	1	1	1	-34	45	41	760.010	7181
		PSO	84.9	0.0	27.5	0.0	2	3	1	1	-14	88	-62	-87	847.106	5080
		Jaya	90.7	25.0	0.0	0.0	2	1	1	2	0	0	-17	-88	744.464	4620
C-C	10000	DE	90.7	23.8	0.0	0.0	5	1	1	6	0	0	-19	11	1627.803	16220
		GA	85.4	0.3	0.1	0.0	6	1	1	5	0	-11	6	34	1645.258	5241
		PSO	73.8	1.8	0.0	0.0	6	2	3	3	3	57	-13	-90	1719.773	5260
		Jaya	90.7	23.8	0.0	0.0	5	1	1	6	0	0	82	0	1627.803	3880
C-F	400	DE	90.4	0.0	0.0	0.0	8	1	1	6	0	71	-3	51	2070.317	19980
		GA	83.8	1.7	0.0	0.4	7	1	8	1	0	-16	-51	26	2111.438	5501
		PSO	90.7	9.1	0.0	31.7	5	5	5	3	1	-5	51	53	2220.979	6900
		Jaya	90.4	0.0	0.0	0.0	8	1	2	5	0	84	50	-90	2070.317	5060
C-H	2500	DE	90.7	13.0	0.0	0.0	3	1	1	5	0	0	19	-84	1211.746	15300
		GA	90.4	20.1	0.7	0.1	3	1	2	4	0	14	-53	-26	1215.650	3481
		PSO	72.6	32.6	0.0	0.0	5	1	2	2	0	-89	-45	-45	1266.302	4860
		Jaya	90.7	13.0	0.0	0.0	3	1	1	5	0	0	-19	-15	1211.746	4880

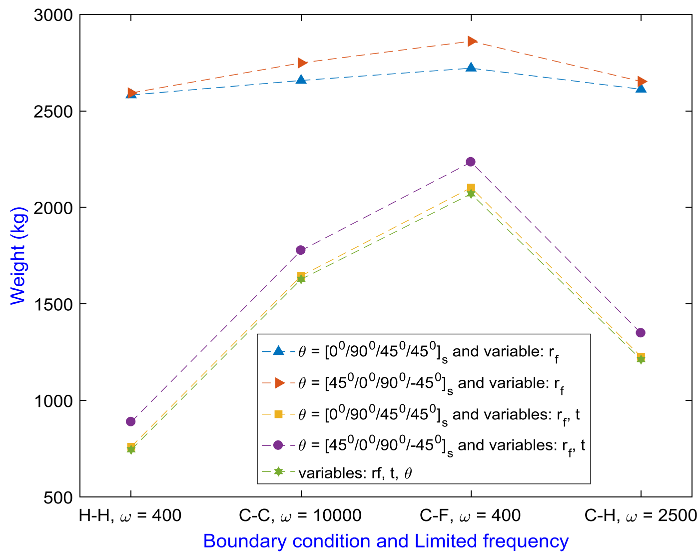


Fig. 5 Comparison of the optimal results obtained by three different design approaches

To evaluate the benefits of considering the fiber orientations as decision variables, Fig. 5 shows a comparison between three different design approaches. In a comparison of the obtained solutions in the two first approaches, it can be observed that those obtained by the fiber orientations of [$0^0/90^0/45^0/45^0$]s is better than that of [$45^0/0^0/90^0/-45^0$]s. This demonstrates that the change of the fiber orientations can help reduce the weight of beam. However, when this kind of variable is considered, the reduction of the weight is not too much significant, which may indicate that the fiber orientations of [$0^0/90^0/45^0/45^0$]s have already been quite close to an optimal solution.

6. Conclusions

In this paper, a simple and efficient numerical approach for the lightweight optimal design of laminated composite beams subjected to frequency constraints is developed. For the formulation of the optimization problem, the whole weight of the laminated composite beam is set to be the objective function, and the fiber volume fractions, the thicknesses and fiber orientation angles of layers are considered as the design variables, where the fiber volume fractions are continuous, while the thicknesses and fiber orientation angles of layers are discrete. The limitation on the first frequency, fiber volume fractions, thicknesses and fiber orientations are set as constraints. For solving the optimal design problems, a global numerical procedure that is a combination of FEM and the Jaya algorithm is developed, where FEM is used to analyze the behavior of composite laminated beams and Jaya is modified and applied for searching the global optimal solution of the optimization problems with mixed discrete-continuous variables. The approach is then applied to solve the optimal design of laminated composite beam with different boundary conditions. A comparison between the present approach and the other existing approaches in the literature is made. The numerical results reveal that the proposed approach is efficient compared with the others in terms of both the quality of solution and computational cost.

With the investigated advantages, Jaya is a promising method which can be extended to the optimization problems of different structures such as truss, frame, plate, and shell structures. Moreover, with a fast convergence rate, it is also a potential method to be applied to reliability-based design optimization problems, where the computational cost is still a major concern.

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