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A novel four variable refined plate theory for wave propagation in functionally graded material plates

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Abstract. In This work an analysis of the propagation of waves of functionally graduated plates is presented by using a high order hyperbolic (HSDT) shear deformation theory. This theory has only four variables, which is less than the theory of first order shear deformation (FSDT). Therefore, a shear correction coefficient is not required. Unlike other conventional shear deformation theories, the present work includes a new field of displacement which introduces indeterminate integral variables. The properties of materials are supposed classified in the direction of the thickness according to two simple distributions of a power law in terms of volume fractions of constituents. The governing equations of the wave propagation in the functionally graded plate are derived by employing the Hamilton's principle. The analytical dispersion relation of the functionally graded plate is obtained by solving an eigenvalue problem. The convergence and the validation of the proposed theoretical numerical model are performed to demonstrate the efficacy of the model.

Keywords: wave propagation; phase velocity; vibration; functionally graded plate; plate theory; porosity

1. Introduction

Composite structures have been applied in mechanical and civil engineering since the last century. Due to their ease of shaping, it is very used in the civil infrastructure projects, such as industrial buildings and vehicle bridges. In recent years, a new class of composite materials in which the characteristics of the material continuously change between two surfaces, thus eliminating the stress concentration phenomenon characteristic of laminated or conventional composite materials. This type of material is known as Functionally Graduated Materials (FGM).

Generally, these materials consist of a mixture of two materials of different thermal nature; a combination of ceramic and metal. The essential point is to describe precisely the material properties of each material point, its young modulus and its density through the thickness in order to perform a satisfactory analysis of the mechanical behavior of the FGM plates.

For the functionally graded materials, great progress has been made in the theory of elasticity as well as for plates and beams; many studies on FGM structures have been studied in the literature in order to describe the material properties in the thickness direction (Eltaher *et al*. 2012, Bessaim *et al*. 2013, Tounsi *et al*. 2013, Bouderba *et al*. 2013, Bousahla *et al*. 2014, Ahmed 2014, Hebali *et al*. 2014, Arefi 2015, Attia *et al*. 2015, Bourada *et al*. 2015, Hamidi *et al*. 2015, Al-Basyouni *et al*. 2015, Ahouel *et al*. 2016, Bounouara *et al*. 2016, Kar *et al*. 2016, Arani and Kolahchi 2016, Sobhy 2017).

Reissner (1945), Cranch and Adler (1956), Ambartsumyan (1969) and Bresse (1859) were the pioneer investigators in studying the different behavior of structures made with isotropic materials under different stresses. With the development of the FGM concept, many works have been studied in literature.

Reddy (2000) is one of the first to analyzed the static behaviour of FGM rectangular plates based on his plate theory. Cheng and Batra (2000) have found correspondence between eigen values of membranes and functionally graded simply supported polygonal plate. The same membrane analogy was later applied to FGM plate and shell analysis based on a third order theory of plates by Reddy (2002).

Vel and Batra (2004) has come closer to real behavior of structure by studying free vibration of FGM rectangular plates with three-dimensional solution. Zenkour (2006) presented a generalized shear deformation theory in which the membrane displacements are expanded as trigonometric

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function across the thickness. Malekzadeh (2009) studied the analysis of free vibrations of thick plates in FGM on elastic bases with two-parameter. Later some new shape functions were proposed by Ait Atmane *et al*. (2010) Benachour *et al*. (2011), and Ait Amar Meziane *et al*. (2014). Ait Atmane *et al*. (2015) presented a computational shear displacement model for vibrational analysis of FG beams with porosities. Beldjelili *et al*. (2016) analyzed the hygro-thermo-mechanical bending response of S-FGM plates resting on variable elastic foundations using a four-variable trigonometric plate theory. Kolahchi and Bidgoli (2016) developed a size-dependent sinusoidal beam model for dynamic instability of single-walled carbon nanotubes. Bilouei *et al*. (2016) studied buckling of concrete columns retrofitted with Nano-Fiber Reinforced Polymer (NFRP). Recently, Houari *et al*. (2016) proposed a new 3-unknowns sinusoidal plate theory for buckling and vibration of FG sandwich plate. It should be noted that there is an important works on shear deformation theories (Kolahchi *et al*. 2016a, 2017a, b, c, Kolahchi and Cheraghbak 2017, Kolahchi 2017, Hajmohammad *et al*. 2017, Shokravi 2017a, b). Other work can be found for shell structures in the open literature (Panda and Singh 2009, 2013, Belabed *et al*. 2014, Zidi *et al*. 2014, Panda and Mahapatra 2014, Belkorissat *et al*. 2015, Taibi *et al*. 2015, Larbi Chaht *et al*. 2015, Kar and Panda 2015a, b, 2016a, b, Bennoun *et al*. 2016, Draiche *et al*. 2016, Bousahla *et al*. 2016, Zidi *et al*. 2017, Mehar *et al*. 2017, Klouche *et al*. 2017, Bellifa *et al*. 2017a, b, Sahoo *et al*. 2017, Chikh *et al*. 2017, Abdelaziz *et al*. 2017, Benadouda *et al*. 2017, Bouafia *et al*. 2017, Abualnour *et al*. 2018, Attia *et al*. 2018, Benchohra *et al*. 2018, Zine *et al*. 2018).

The wave propagation of structural elements such as plates or beams has been studied in different sectors like aeronautics, medicine, acoustics of the buildings.... Ait Yahia *et al*. (2015) studied wave propagation in order to compare different shear theories and porosities solution in FG plates. Akbaş (2015) investigated the Wave propagation of a functionally graded beam in thermal environments. Boukhari *et al*. (2016) introduced an efficient shear deformation theory for wave propagation of functionally graded material plates. Sharma (2017) studied Vibro-acoustic behaviour of shear deformable laminated composite flat panel using BEM and the higher order shear deformation theory. Han *et al*. (2001) proposed an analytical-numerical method for analyzing the wave characteristics in FGM cylinders.

Sun and Luo (2011a) also studied the wave propagation and dynamic response of rectangular functionally graded material plates with completed clamped supports under impulsive load. Considering the thermal effects and temperature-dependent material properties, Sun and Luo (2011b) investigated the wave propagation of an infinite functionally graded plate using the higher-order shear deformation plate theory.

Abo-Dahab (Abo-Dahab *et al*. 2016) has carried out the quantification of rotational effect on Rayleigh, Love and Stoneley waves in non-homogeneous fibre-reinforced anisotropic general viscoelastic media of higher order. Barati (2017) studied an imperfect nanomaterial by examining the wave propagation in nanoporous materials.

The objective of this study is to develop high order hyperbolic (HSDT) shear deformation theory for the wave propagation of an infinite functionally graded plate. The theory is based on assumption similar to classical theory, the present theory has a new displacement field which introduces undetermined integral variables. The most interesting feature of this theory is that it accounts for a quadratic variation of the transverse shear strains across the thickness and satisfies the zero traction boundary conditions on the top and bottom surfaces of the plate without using shear correction factors. The governing equations of the wave propagation in the functionally graded plate are derived by using the Hamilton's principle, which the effects of shear deformation and the inertia rotation are taken into account. The analytic dispersion relations of the functionally graded plate are obtained by solving an eigenvalue problem. The dispersion, phase velocity and group velocity curves of the wave propagation in the functionally graded plate in thermal environments are plotted. The characteristics of wave propagation of the functionally graded plate are described in detail. The influences of the volume fraction index and the thickness ratio on the dispersion and phase velocity of the wave propagation in the functionally graded plate are clearly discussed.

2. Theory and formulation

The functionally graded (FG) plate is composed by a mixture of ceramic and metal components as shown in Fig. 1. The material characteristics of this plate change across the plate thickness with different power law distributions of the volume fractions of the constituents of the two materials as:

(i) Power law distribution:

$$P(z) = P_m + \left(P_c - P_m\right)\left(\frac{1}{2} + \frac{z}{h}\right)^p \tag{1}$$

(ii) Sigmoid law distribution is defined as two power-law functions

$$g_1(z) = 1 - \frac{1}{2}\left(\frac{h/2 - z}{h/2}\right)^p \quad \text{for} \quad 0 \le z \le h/2 \tag{2a}$$

$$g_2(z) = \frac{1}{2}\left(\frac{h/2 + z}{h/2}\right)^p \quad \text{for} \quad -h/2 \le z \le 0 \tag{2b}$$

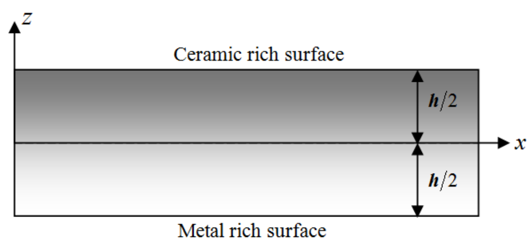


Fig. 1 Coordinates and geometry of functionally graded plate

where g_i, (i = 1, 2) is the volume fraction and p is the power law index which takes values greater than or equal to zero.

By using the rule of mixture, the effective material properties P, such as Young's modulus E, the Poisson ratio υ, and mass density ρ can be expressed as

$$P(z)=g_1(z)P_c+[1-g_1(z)]P_m \quad \text{for} \quad -h/2\leq z\leq 0 \tag{3a}$$

$$P(z)=g_2(z)P_c+[1-g_2(z)]P_m \quad \text{for} \quad -h/2\leq z\leq 0 \tag{3b}$$

Where P denotes the effective material characteristic such as Young's modulus E and mass density ρ subscripts m and c denote the metallic and ceramic components, respectively; and p is the power law exponent. The value of p equal to zero indicates a fully ceramic plate, whereas infinite p represents a fully metallic plate. Since the influences of the variation of Poisson's ratio v on the behavior of FG plates are very small, it is supposed to be constant for convenience.

2.1 Kinematics and strains

In this article, further simplifying supposition are made to the conventional HSDT so that the number of unknowns is reduced. The displacement field of the conventional HSDT is given by (Mahi *et al*. 2015, Bakhadda *et al*. 2018)

$$u(x,y,z,t)=u_0(x,y,t)-z\frac{\partial w_0}{\partial x}+f(z)\varphi_x(x,y,t) \tag{4a}$$

$$v(x,y,z,t)=v_0(x,y,t)-z\frac{\partial w_0}{\partial y}+f(z)\varphi_y(x,y,t) \tag{4b}$$

$$w(x,y,z,t)=w_0(x,y,t) \tag{4c}$$

Where u_0, v_0, w_0, φ_x, φ_y are five unknown displacements of the mid-plane of the plate, f(z) denotes shape function representing the variation of the transverse shear strains and stresses within the thickness. By considering that $\varphi_x=\int\theta(x,y)dx$ and $\varphi_y=\int\theta(x,y)dy$, the displacement field of the present model can be expressed in a simpler form as (Bourada *et al*. 2016, Besseghier *et al*. 2017, Khetir *et al*. 2017, Sekkal *et al*. 2017a, Menasria *et al*. 2017, Yazid *et al*. 2018)

$$u(x,y,z,t)=u_0(x,y,t)-z\frac{\partial w_0}{\partial x}+k_1\,f(z)\int\theta(x,y,t)\,dx \tag{5a}$$

$$v(x,y,z,t)=v_0(x,y,t)-z\frac{\partial w_0}{\partial y}+k_2\,f(z)\int\theta(x,y,t)\,dy \tag{5b}$$

$$w(x,y,z,t)=w_0(x,y,t) \tag{5c}$$

In this work, a comparative study between different higher-order shear deformation plate theories is carried out, the different shape function are sited below. It should noted that there is no need to use shear correction factor in the present theory as the case of the first shear deformation theory (Youcef *et al*. 2018, Zamanian *et al*. 2017, Shokravi 2017c, d, Zarei *et al*. 2017, Arani and Kolahchi 2016, Kolahchi *et al*. 2016b, Madani *et al*. 2016, Bouderba *et al*. 2016, Bellifa *et al*. 2016).

It can be seen that the displacement field in Eq. (5) introduces only four unknowns (u_0, v_0, w_0 and θ). The nonzero strains associated with the displacement field in Eq. (5) are

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}+z\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}+f(z)\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix},\ \begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=g(z)\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix} \tag{6}$$

Where

$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial u_0}{\partial x}\\ \dfrac{\partial v_0}{\partial x}\\ \dfrac{\partial u_0}{\partial y}+\dfrac{\partial v_0}{\partial x}\end{Bmatrix},\quad \begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}=\begin{Bmatrix}-\dfrac{\partial^2 w_0}{\partial x^2}\\ -\dfrac{\partial^2 w_0}{\partial y^2}\\ -2\dfrac{\partial^2 w_0}{\partial x\partial y}\end{Bmatrix} \tag{7a}$$

$$\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix}=\begin{Bmatrix}k_1\theta\\ k_2\theta\\ k_1\dfrac{\partial}{\partial y}\int\theta\,dx+k_2\dfrac{\partial}{\partial x}\int\theta\,dy\end{Bmatrix},\quad \begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix}=\begin{Bmatrix}k_2\int\theta\,dy\\ k_1\int\theta\,dx\end{Bmatrix}$$

$$g(z)=\frac{df(z)}{dz} \tag{7b}$$

And the integrals defined in the above equations shall be resolved by a Navier type method and can be written as follows

$$\begin{aligned}&\frac{\partial}{\partial y}\int\theta\,dx=A'\frac{\partial^2\theta}{\partial x\partial y},\quad \frac{\partial}{\partial x}\int\theta\,dy=B'\frac{\partial^2\theta}{\partial x\partial y}\\ &\int\theta\,dx=A'\frac{\partial\theta}{\partial x},\quad \int\theta\,dy=B'\frac{\partial\theta}{\partial y}\end{aligned} \tag{8}$$

Where the coefficients A' and B' are expressed according to the type of solution used, in this case via Navier. Therefore, A', B', k_1 and k_2 are expressed as follows

$$A'=-\frac{1}{\kappa_1^{\,2}},\quad B'=-\frac{1}{\kappa_2^{\,2}},\quad k_1=\kappa_1^{\,2},\quad k_2=\kappa_2^{\,2} \tag{9}$$

Where κ_1 and κ_2 are the wave numbers of wave propagation along x-axis and y-axis directions respectively.

For elastic and isotropic FGMs, the constitutive relations can be expressed as

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{xz}\end{Bmatrix}=\begin{bmatrix}C_{11}&C_{12}&0&0&0\\ C_{12}&C_{22}&0&0&0\\ 0&0&C_{66}&0&0\\ 0&0&0&C_{44}&0\\ 0&0&0&0&C_{55}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} \tag{10}$$

where (σ_x, σ_y, τ_{xy}, τ_{yz}, τ_{xz}) and (ε_x, ε_y, γ_{xy}, γ_{yz}, γ_{xz}) are the stress and strain components, respectively. Using the material properties defined in Eqs. (1)-(2), stiffness

Coefficients, C_{ij}, can be given as

$$C_{11}=C_{22}=\frac{E(z)}{1-\nu^2},\quad C_{12}=\frac{\nu E(z)}{1-\nu^2},\quad C_{44}=C_{55}=C_{66}=\frac{E(z)}{2(1+\nu)} \tag{11}$$

2.2 Equations of motion

Hamilton's principle is herein utilized to determine the equations of motion (Zemri *et al*. 2015, Mouffoki *et al*. 2017, Sekkal *et al*. 2017b, Meksi *et al*. 2018, Bouhadra *et al*. 2018, Belabed *et al*. 2018)

$$0=\int_0^t(\delta U+\delta V-\delta K)\,dt \tag{12}$$

Where δU is the variation of strain energy; δV is the variation of the external work done by external load applied to the plate; and δK is the variation of kinetic energy.

The variation of strain energy of the plate is given by

$$\begin{aligned}\delta U&=\int_V\left[\sigma_x\delta\varepsilon_x+\sigma_y\delta\varepsilon_y+\tau_{xy}\delta\gamma_{xy}+\tau_{yz}\delta\gamma_{yz}+\tau_{xz}\delta\gamma_{xz}\right]dV\\&=\int_A\left[N_x\delta\varepsilon_x^0+N_y\delta\varepsilon_y^0+N_{xy}\delta\gamma_{xy}^0+M_x^b\delta k_x^b+M_y^b\delta k_y^b+M_{xy}^b\delta k_{xy}^b\right.\\&\left.+M_x^s\delta k_x^s+M_y^s\delta k_y^s+M_{xy}^s\delta k_{xy}^s+S_{yz}^s\delta\gamma_{yz}^s+S_{xz}^s\delta\gamma_{xz}^0\right]dA=0\end{aligned} \tag{13}$$

Where A is the top surface and the stress resultants N, M, and S are defined by

$$\begin{aligned}\left(N_i,M_i^b,M_i^s\right)&=\int_{-h/2}^{h/2}(1,z,f)\sigma_i dz,\qquad (i=x,y,xy)\\\left(S_{xz}^s,S_{yz}^s\right)&=\int_{-h/2}^{h/2}g\left(\tau_{xz},\tau_{yz}\right)dz\end{aligned} \tag{14}$$

The variation of the external work can be expressed as

$$\delta V=-\int_A q\delta w_0 dA-\int_A\left(\begin{array}{l}N_x^0\dfrac{\partial w_0}{\partial x}\dfrac{\partial\delta w_0}{\partial x}+\\2N_{xy}^0\dfrac{\partial w_0}{\partial x}\dfrac{\partial\delta w_0}{\partial y}+N_y^0\dfrac{\partial w_0}{\partial y}\dfrac{\partial\delta w_0}{\partial y}\end{array}\right)dA \tag{15}$$

Where q and (N_x^0,N_y^0,N_{xy}^0) are transverse and in-plane applied loads, respectively.

For the free vibration and wave propagation problems the external work is zero.

The variation of kinetic energy of the plate can be expressed as

$$\begin{aligned}\delta K&=\int_V\left[\dot u\,\delta\dot u+\dot v\,\delta\dot v+\dot w\,\delta\dot w\right]\rho(z)\,dV\\&=\int_A\Big\{I_0\left[\dot u_0\delta\dot u_0+\dot v_0\delta\dot v_0+\dot w_0\delta\dot w_0\right]\\&-I_1\left(\dot u_0\frac{\partial\delta\dot w_0}{\partial x}+\frac{\partial\dot w_0}{\partial x}\delta\dot u_0+\dot v_0\frac{\partial\delta\dot w_0}{\partial y}+\frac{\partial\dot w_0}{\partial y}\delta\dot v_0\right)\\&+J_1\left(\begin{array}{l}(k_1A')\left(\dot u_0\dfrac{\partial\delta\dot\theta}{\partial x}+\dfrac{\partial\dot\theta}{\partial x}\delta\dot u_0\right)\\+(k_2B')\left(\dot v_0\dfrac{\partial\delta\dot\theta}{\partial y}+\dfrac{\partial\dot\theta}{\partial y}\delta\dot v_0\right)\end{array}\right)\\&+I_2\left(\frac{\partial\dot w_0}{\partial x}\frac{\partial\delta\dot w_0}{\partial x}+\frac{\partial\dot w_0}{\partial y}\frac{\partial\delta\dot w_0}{\partial y}\right)\\&+K_2\left((k_1A')^2\left(\frac{\partial\dot\theta}{\partial x}\frac{\partial\delta\dot\theta}{\partial x}\right)+(k_2B')^2\left(\frac{\partial\dot\theta}{\partial y}\frac{\partial\delta\dot\theta}{\partial y}\right)\right)\\&-J_2\left(\begin{array}{l}(k_1A')\left(\dfrac{\partial\dot w_0}{\partial x}\dfrac{\partial\delta\dot\theta}{\partial x}+\dfrac{\partial\dot\theta}{\partial x}\dfrac{\partial\delta\dot w_0}{\partial x}\right)\\+(k_2B')\left(\dfrac{\partial\dot w_0}{\partial y}\dfrac{\partial\delta\dot\theta}{\partial y}+\dfrac{\partial\dot\theta}{\partial y}\dfrac{\partial\delta\dot w_0}{\partial y}\right)\end{array}\right)\Big\}dA\end{aligned} \tag{16}$$

Where dot-superscript convention indicates the differentiation with respect to the time variable t; $\rho(z)$ is the mass density given by Eqs. (1)-(2); and (I_i, J_i, K_i) are mass inertias expressed by

$$\begin{aligned}\left(I_0,I_1,I_2\right)&=\int_{-h/2}^{h/2}\left(1,z,z^2\right)\rho(z)dz\\\left(J_1,J_2,K_2\right)&=\int_{-h/2}^{h/2}\left(f,z\,f,f^2\right)\rho(z)dz\end{aligned} \tag{17}$$

By substituting Eqs. (13), (15) and (16) into Eq. (12), the following can be derived

$$\begin{aligned}\delta u_0:&\ \frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=I_0\ddot u_0-I_1\frac{\partial\ddot w_0}{\partial x}+k_1A'J_1\frac{\partial\ddot\theta}{\partial x}\\\delta v_0:&\ \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=I_0\ddot v_0-I_1\frac{\partial\ddot w_0}{\partial y}+k_2B'J_1\frac{\partial\ddot\theta}{\partial y}\\\delta w_0:&\ \frac{\partial^2M_x^b}{\partial x^2}+2\frac{\partial^2M_{xy}^b}{\partial x\partial y}+\frac{\partial^2M_y^b}{\partial y^2}\\&=I_0\ddot w_0+I_1\left(\frac{\partial\ddot u_0}{\partial x}+\frac{\partial\ddot v_0}{\partial y}\right)\\&-I_2\nabla^2\ddot w_0+J_2\left(k_1A'\frac{\partial^2\ddot\theta}{\partial x^2}+k_2B'\frac{\partial^2\ddot\theta}{\partial y^2}\right)\\\delta\theta:&\ -\mathrm{k}_1M_x^s-\mathrm{k}_2M_y^s-(k_1A'+k_2B')\frac{\partial^2M_{xy}^s}{\partial x\partial y}\\&+k_1A'\frac{\partial S_{xz}^s}{\partial x}+k_2B'\frac{\partial S_{yz}^s}{\partial y}\end{aligned} \tag{18}$$

$$
\begin{aligned}
&=-J_1\left(k_1\,A'\frac{\partial \ddot{u}_0}{\partial x}+k_2\,B'\,\frac{\partial \ddot{v}_0}{\partial y}\right)\\
&-K_2\left((k_1\,A')^2\,\frac{\partial^2\ddot{\theta}}{\partial x^2}+(k_2\,B')^2\,\frac{\partial^2\ddot{\theta}}{\partial y^2}\right)+\\
&J_2\left(k_1\,A'\frac{\partial^2 \ddot{w}_0}{\partial x^2}+k_2\,B'\frac{\partial^2 \ddot{w}_0}{\partial y^2}\right)
\end{aligned}
\tag{18}
$$

Substituting Eq. (6) into Eq. (10) and the subsequent results into Eq. (14), the stress resultants are obtained in terms of strains as following compact form

$$
\begin{Bmatrix} N \\ M^b \\ M^s \end{Bmatrix}=\begin{bmatrix} A & B & B^s \\ B & D & D^s \\ B^s & D^s & H^s \end{bmatrix}\begin{Bmatrix} \varepsilon \\ k^b \\ k^s \end{Bmatrix}, S=A^s\gamma
\tag{19}
$$

in which

$$
N=\{N_x,N_y,N_{xy}\}^t,\qquad M^b=\{M_x^b,M_y^b,M_{xy}^b\}^t,\qquad M^s=\{M_x^s,M_y^s,M_{xy}^s\}^t
\tag{20a}
$$

$$
\varepsilon=\{\varepsilon_x^0,\varepsilon_y^0,\gamma_{xy}^0\}^t,\qquad k^b=\{k_x^b,k_y^b,k_{xy}^b\}^t,\qquad k^s=\{k_x^s,k_y^s,k_{xy}^s\}^t
\tag{20b}
$$

$$
A=\begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66}\end{bmatrix},\qquad B=\begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66}\end{bmatrix},\qquad D=\begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66}\end{bmatrix}
\tag{20c}
$$

$$
B^s=\begin{bmatrix} B_{11}^s & B_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & B_{66}^s\end{bmatrix},\qquad D^s=\begin{bmatrix} D_{11}^s & D_{12}^s & 0 \\ D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & D_{66}^s\end{bmatrix},\qquad H^s=\begin{bmatrix} H_{11}^s & H_{12}^s & 0 \\ H_{12}^s & H_{22}^s & 0 \\ 0 & 0 & H_{66}^s\end{bmatrix}
\tag{20d}
$$

$$
S=\{S_{xz}^s,S_{yz}^s\}^t,\qquad \gamma=\{\gamma_{xz}^0,\gamma_{yz}^0\}^t,\qquad A^s=\begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s\end{bmatrix}
\tag{20e}
$$

and stiffness components are given as

$$
\begin{Bmatrix} A_{11} & B_{11} & D_{11} & B_{11}^s & D_{11}^s & H_{11}^s \\ A_{12} & B_{12} & D_{12} & B_{12}^s & D_{12}^s & H_{12}^s \\ A_{66} & B_{66} & D_{66} & B_{66}^s & D_{66}^s & H_{66}^s \end{Bmatrix}
=\int_{-h/2}^{h/2} C_{11}\left(1,z,z^2,f(z),z\,f(z),f^2(z)\right)\begin{Bmatrix} 1 \\ \nu \\ \dfrac{1-\nu}{2}\end{Bmatrix}dz
\tag{21a}
$$

$$
\begin{aligned}
&\left(A_{22},B_{22},D_{22},B_{22}^s,D_{22}^s,H_{22}^s\right)\\
&=\left(A_{11},B_{11},D_{11},B_{11}^s,D_{11}^s,H_{11}^s\right)
\end{aligned}
\tag{21b}
$$

$$
A_{44}^s=A_{55}^s=\int_{-h/2}^{h/2} C_{44}[g(z)]^2\,dz,
\tag{21c}
$$

Introducing Eq. (19) into Eq. (18), the equations of motion can be expressed in terms of displacements (u_0, v_0, w_0, θ) and the appropriate equations take the form

$$
\begin{aligned}
&A_{11}d_{11}u_0+A_{66}\,d_{22}u_0+(A_{12}+A_{66})d_{12}v_0-B_{11}d_{111}w_0\\
&-(B_{12}+2B_{66})d_{122}w_0+\left(B_{66}^s(k_1A'+k_2\,B')\right)d_{122}\theta\\
&+\left(B_{11}^s k_1+B_{12}^s\,k_2\right)d_1\theta=I_0\ddot{u}_0-I_1\,d_1\ddot{w}_0+J_1\,A'k_1d_1\ddot{\theta},
\end{aligned}
\tag{22a}
$$

$$
\begin{aligned}
&A_{22}\,d_{22}v_0+A_{66}\,d_{11}v_0+(A_{12}+A_{66})\,d_{12}u_0-B_{22}\,d_{222}w_0\\
&-(B_{12}+2B_{66})\,d_{112}w_0+\left(B_{66}^s(k_1A'+k_2\,B')\right)d_{112}\theta\\
&+\left(B_{22}^s k_2+B_{12}^s k_1\right)d_2\theta=I_0\ddot{v}_0-I_1\,d_2\ddot{w}_0+J_1\,B'k_2\,d_2\ddot{\theta},
\end{aligned}
\tag{22b}
$$

$$
\begin{aligned}
&B_{11}\,d_{111}u_0+(B_{12}+2B_{66})d_{122}u_0+(B_{12}+2B_{66})d_{112}v_0\\
&+B_{22}\,d_{222}v_0-D_{11}d_{1111}w_0-2(D_{12}+2D_{66})\,d_{1122}w_0\\
&-D_{22}\,d_{2222}w_0+\left(D_{11}^s\,k_1+D_{12}^s\,k_2\right)d_{11}\theta\\
&+2\left(D_{66}^s(k_1A'+k_2B')\right)d_{1122}\theta+\left(D_{12}^s k_1+D_{22}^s k_2\right)d_{22}\theta\\
&=I_0\ddot{w}_0+I_1\left(d_1\ddot{u}_0+d_2\,\ddot{v}_0\right)-I_2\left(d_{11}\ddot{w}_0+d_{22}\ddot{w}_0\right)\\
&+J_2\left(k_1A'\,d_{11}\ddot{\theta}+k_2\,B'\,d_{22}\ddot{\theta}\right)
\end{aligned}
\tag{22c}
$$

Where d_{ij}, d_{ijl} and d_{ijlm} are the following differential operators

$$
\begin{aligned}
&d_{ij}=\frac{\partial^2}{\partial x_i\partial x_j},d_{ijl}=\frac{\partial^3}{\partial x_i\partial x_j\partial x_l},d_{ijlm}=\frac{\partial^4}{\partial x_i\partial x_j\partial x_l\partial x_m}\\
&d_{ijlm}=\frac{\partial^4}{\partial x_i\partial x_j\partial x_l\partial x_m},d_i=\frac{\partial}{\partial x_i},(i,j,l,m=1,2).
\end{aligned}
\tag{23}
$$

2.3 Dispersion relations

We assume solutions for u_0, v_0, w_0 and θ_0 representing propagating waves in the x-y plane with the form

$$
\begin{Bmatrix} u_0(x,y,t) \\ v_0(x,y,t) \\ w_0(x,y,t) \\ \theta_0(x,y,t)\end{Bmatrix}=\begin{Bmatrix} U\exp[i(\kappa_1 x+\kappa_2 y-\omega\,t)] \\ V\exp[i(\kappa_1 x+\kappa_2 y-\omega\,t)] \\ W\exp[i(\kappa_1 x+\kappa_2 y-\omega\,t)] \\ X\exp[i(\kappa_1 x+\kappa_2 y-\omega\,t)]\end{Bmatrix}
\tag{24}
$$

where U; V; W and X are the coefficients of the wave amplitude, κ_1 and κ_2 are the wave numbers of wave propagation along x-axis and y-axis directions respectively, ω is the frequency, $\sqrt{i}=-1$ the imaginary unit.

Substituting Eq. (24) into Eq. (23), the following problem is obtained

$$\left(\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & 0 & m_{13} & m_{14} \\ 0 & m_{22} & m_{23} & m_{24} \\ m_{13} & m_{23} & m_{33} & m_{34} \\ m_{14} & m_{24} & m_{34} & m_{44} \end{bmatrix}\right) \begin{Bmatrix} U \\ V \\ W \\ X \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (25)$$

where

$$S_{11} = -\left(A_{11}\kappa_1^{\ 2} + A_{66}\kappa_2^{\ 2}\right),$$
$$S_{12} = -\kappa_1\kappa_2\ \left(A_{12} + A_{66}\right),$$
$$S_{13} = \kappa_1 \cdot i \cdot \left(B_{11}\kappa_1^{\ 2} + B_{12}\kappa_2^{\ 2} + 2B_{66}\kappa_2^{\ 2}\right),$$
$$S_{14} = i \cdot \kappa_1\left(k_1 B_{11}^s + k_2 B_{12}^s - \left(k_1 A' + k_2 B'\right) B_{66}^s \kappa_2^{\ 2}\right),$$
$$S_{22} = -\left(A_{66}\kappa_1^{\ 2} + A_{22}\kappa_2^{\ 2}\right),$$
$$S_{23} = i \cdot \kappa_2\left(B_{22}\kappa_2^{\ 2} + B_{12}\kappa_1^{\ 2} + 2B_{66}\kappa_1^{\ 2}\right),$$
$$S_{24} = i \cdot \kappa_2\ \left(k_2 B_{22}^s + k_1 B_{12}^s - \left(k_1 A' + k_2 B'\right) B_{66}^s \kappa_1^{\ 2}\right)$$
$$S_{33} = -\left(D_{11}\kappa_1^{\ 4} + 2(D_{12} + 2D_{66})\kappa_1^{\ 2}\beta^2 + D_{22}\beta^4\right)$$
$$S_{34} = -k_1\left(D_{11}^s\kappa_1^{\ 2} + D_{12}^s\kappa_2^{\ 2}\right) + 2(k_1 A' + k_2 B')\, D_{66}^s \kappa_1^{\ 2}\kappa_2^{\ 2}$$
$$-k_2\left(D_{22}^s\kappa_2^{\ 2} + D_{12}^s\kappa_1^{\ 2}\right), \quad (26)$$
$$S_{44} = -k_1\left(H_{11}^s k_1 + H_{12}^s k_2\right) - (k_1 A' + k_2 B')^2 H_{66}^s \kappa_1^{\ 2}\kappa_2^{\ 2} -$$
$$k_2\left(H_{12}^s k_1 + H_{22}^s k_2\right) - \left(k_1 A'\right)^2 A_{55}^s \kappa_1^{\ 2} - \left(k_2 B'\right)^2 A_{44}^s \kappa_2^{\ 2}$$
$$m_{11} = -I_0,\quad m_{13} = i \cdot \kappa_1 I_1,\quad m_{14} = -i \cdot J_1 k_1 A' \kappa_1,$$
$$m_{22} = -I_0,\ m_{23} = i \cdot \kappa_2\ I_1,\quad m_{24} = -i \cdot k_2 B'\ \kappa_2\ J_1,$$
$$m_{31} = -i \cdot \kappa_1 I_1,\quad m_{32} = -i \cdot \kappa_2\ I_1,$$
$$m_{33} = -I_0 - I_2(\kappa_1^{\ 2} + \kappa_2^{\ 2})$$
$$m_{34} = J_2\left(k_1\ A'\kappa_1^{\ 2} + k_2\ B'\kappa_2^{\ 2}\right),\ m_{41} = i \cdot J_1 k_1 A' \kappa_1$$
$$m_{42} = i \cdot k_2 B'\ \kappa_2\ J_1,\ m_{43} = J_2\left(k_1\ A'\kappa_1^{\ 2} + k_2\ B'\kappa_2^{\ 2}\right),$$
$$m_{44} = -K_2\left(\left(k_1\ A'\right)^2\kappa_1^{\ 2} + \left(k_2\ B'\right)^2\kappa_2^{\ 2}\right)$$

The dispersion relations of wave propagation in the functionally graded beam are given by

$$\left|\left[K\right] - \omega^2\left[M\right]\right| = 0 \quad (27)$$

The roots of Eq. (27) can be expressed as

$$\omega_1 = W_1(\kappa),\ \omega_2 = W_2(\kappa),\ \omega_3 = W_3(\kappa)\ \text{ and }\ \omega_4 = W_4(\kappa) \quad (28)$$

They correspond to the wave modes M_1, M_2, M_3 and M_4 respectively. The wave modes M_1 and M_4 correspond to the flexural wave, the wave mode M_2 and M_3 corresponds to the extensional wave.

The phase velocity of wave propagation in the functionally graded plate can be expressed as

$$C_i = \frac{W_i(\kappa)}{\kappa},\quad (i = 1, 2, 3, 4) \quad (29)$$

3. Numerical results and discussions

In this part, to obtain results of frequencies and velocities one must go through the resolution of a system of four equations on eigenvalues problem; the FG plate made from Si3N4/SUS304, whose material properties are: E = 348.43 GPa, ρ = 2370 kg/m^3, ν = 0.3 for Si3N4 and E = 201.04 GPa, ρ = 8166 kg/m^3, ν = 0.3 for SUS304, are taken from reference Ait Yahia (Ait Yahia *et al.* 2015). The thickness of the functionally graded plate is taken h=0.02 m. Various numerical examples are presented and discussed to check the accuracy of present theory in investigating the wave propagation of FG plates.

The accuracy of the present neutral model involving only four unknown displacement functions is verified by comparing many theories like Reddy (1987), Reissner (1945), Ait Atmane (2010), and Afaq (Afaq *et al.* 2003). The analysis based on the present model is carried out using MAPLE.

3.1 Comparison of theories in P-FGM plate

Relation between power law exponent and wave number

In Fig. 2, the dispersion curves of the different FG plates are represented using different theories of shear deformation plates. It can be noted that the dispersion curves predicted by all the plate theories proposed are almost identical to each other, independently of the power index P and the wave modes (M_1, M_2, M_3 and M_4). For the same value of k, the frequency of the wave propagation in the FG plate increases with the decrease of the power law index p, whatever the wave modes. Moreover, the propagation frequency of the wave becomes maximum in the homogeneous plate (p = 0).

The relationship between the phase velocity of the different FG plates and the number of waves is represented in the Fig. 3, using different shear deformation plate theories. It can be seen that the phase velocity of the wave propagation in the FG plaque increases as the index of power law p decreases for the same wave number k. The phase velocity of the waves modes M_2 and M_3 of the plate (p = 0) is constant, but it is not a constant for the plate ($p \neq$ 0). As well as the frequency, for the homogeneous plate (p = 0), the phase velocity takes the maximum among those of all other compositions. Also, it can be seen that the phase velocity curves predicted by all proposed plate theories are almost identical to each other.

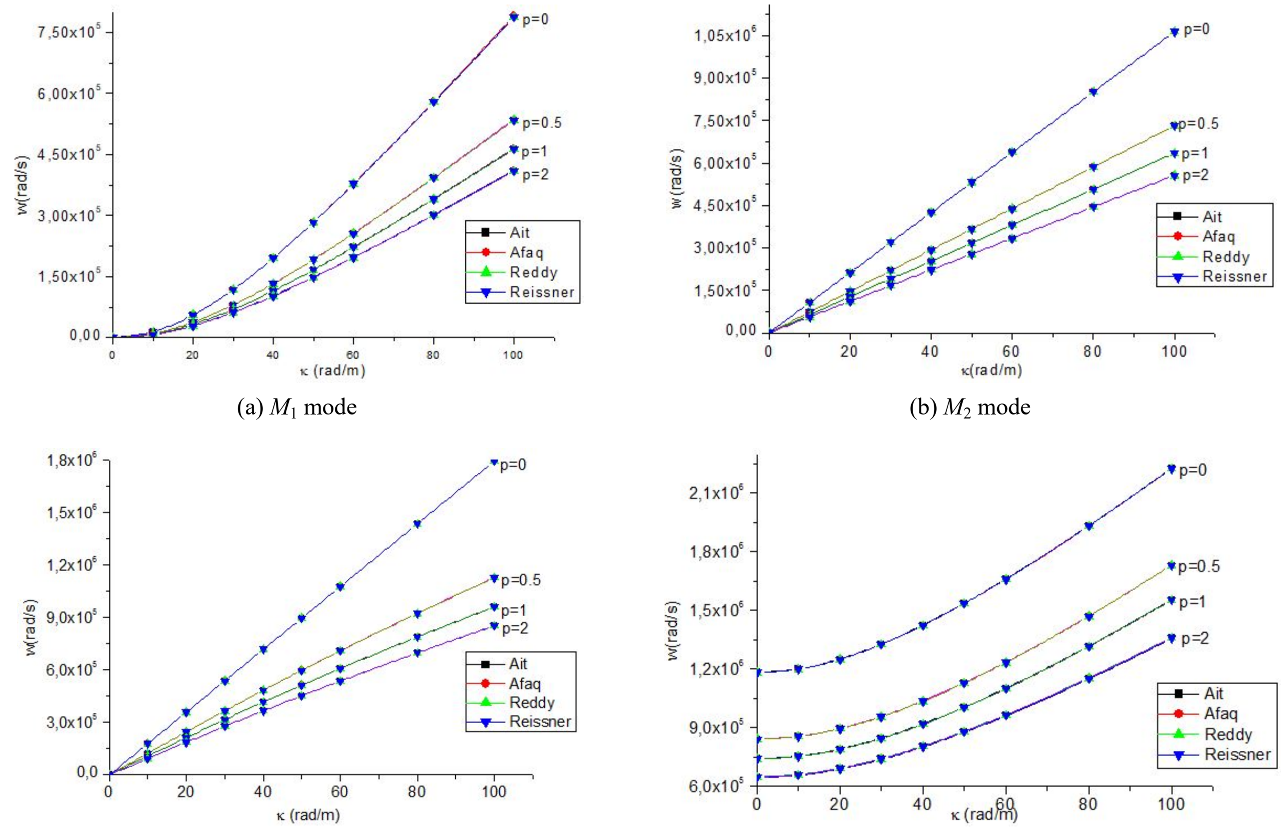
3.2 Parametric study of P-FGM plate

3.2.1 Frequencies

Fig. 4 shows the dispersion curves of different FG plate with p = 2. It can be seen that the thickness of plate has an effect on the frequency of the wave propagation in FG plate for the large wave numbers (κ) and especially for the fundamental mode. Indeed, the frequencies are reduced when the thickness decreases.

3.2.2 Phase velocities

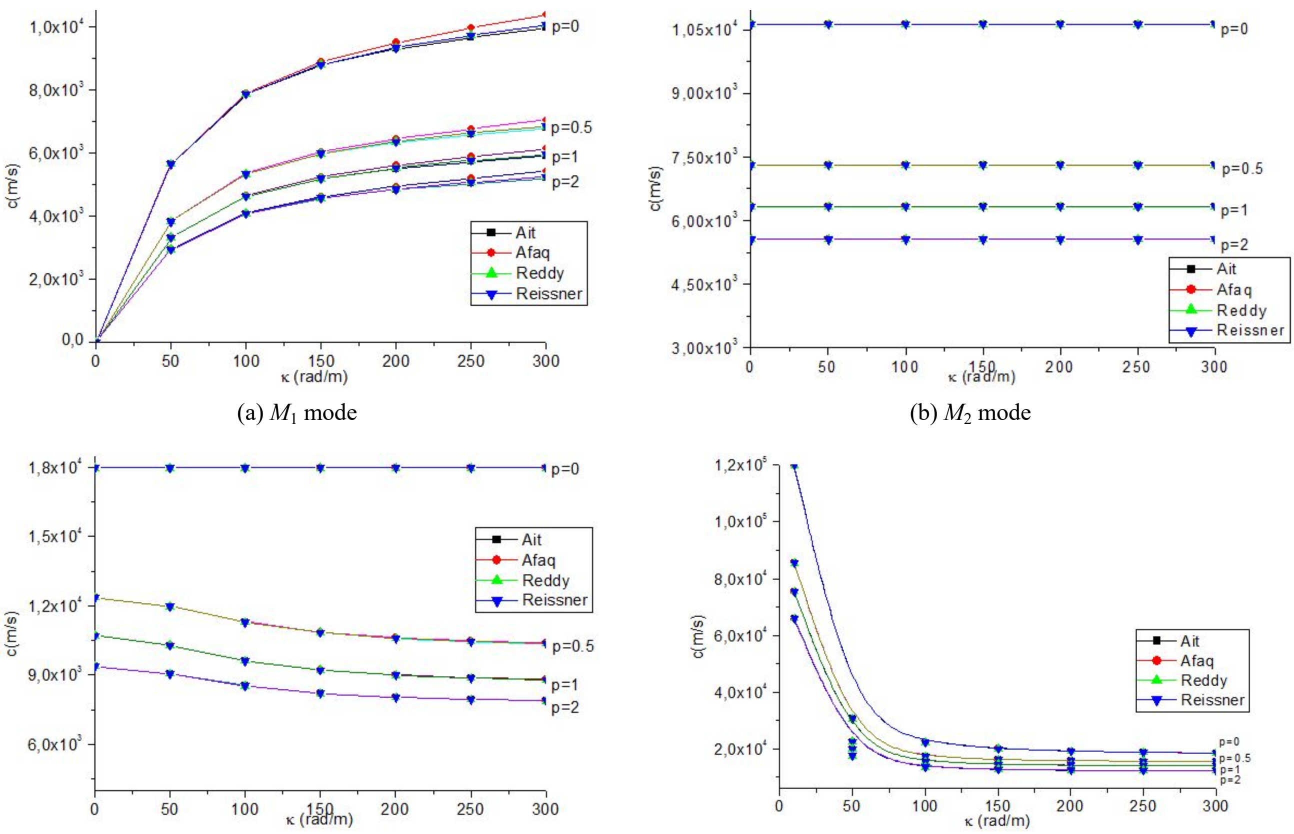
Fig. 5 shows, the phase velocity curves of different FG plate with p = 2. It can be seen from this Figure that of the



(a) M_1 mode (b) M_2 mode

(c) M_3 mode (*d*) M_4 mode

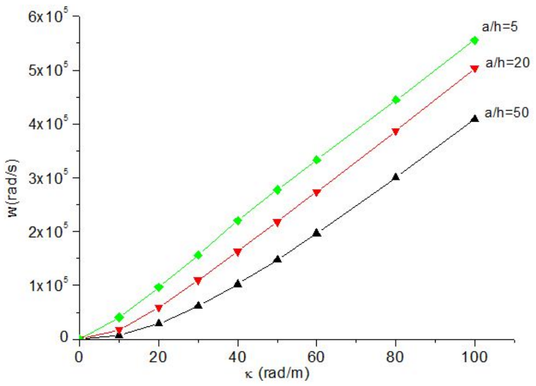
Fig. 2 The dispersion curves of the different functionally graded plates



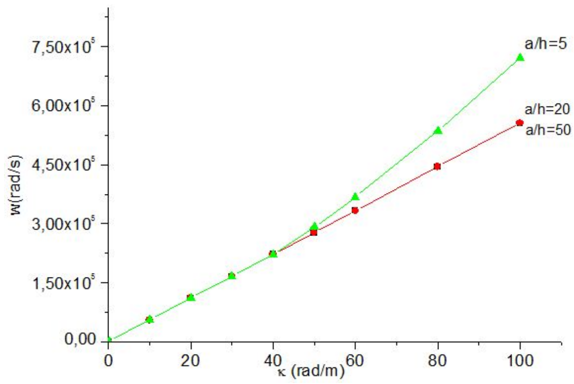
(a) M_1 mode (b) M_2 mode

(c) M_3 mode (*d*) M_4 mode

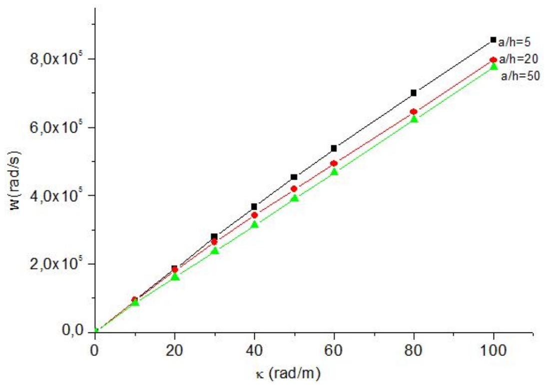
Fig. 3 The phase velocity curves of the different functionally graded plates



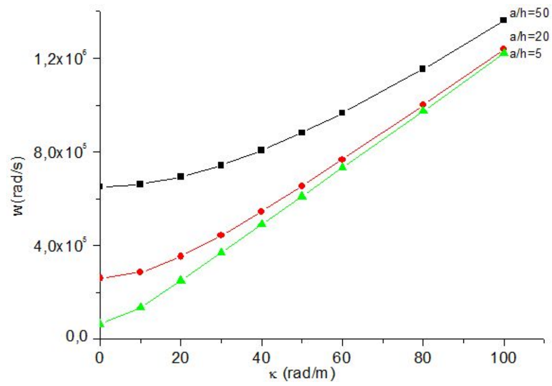
(a) M_1 mode



(b) M_2 mode

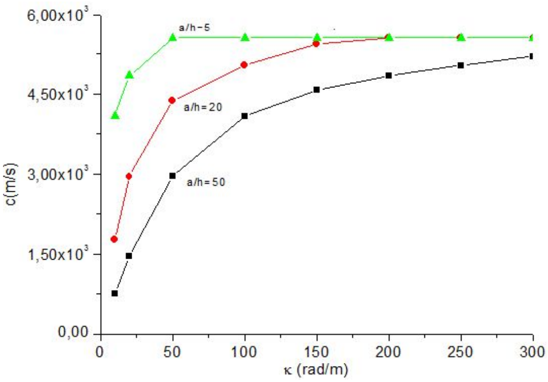


(c) M_3 mode

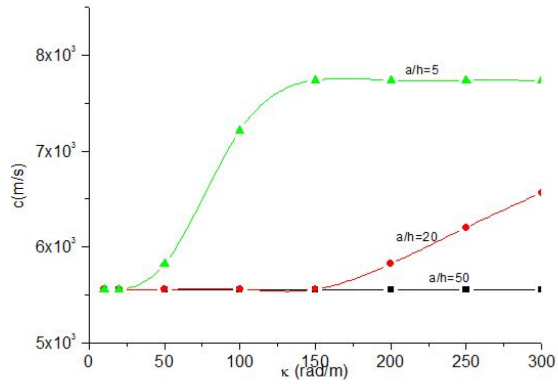


(*d*) M_4 mode

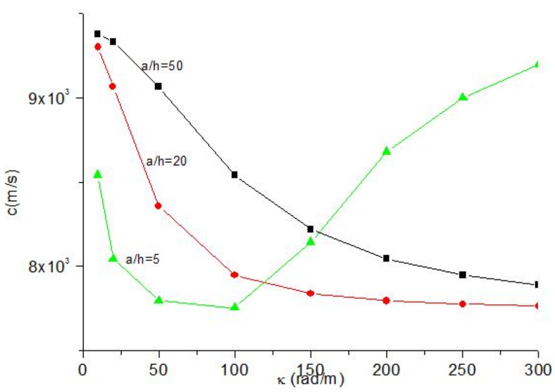
Fig. 4 The dispersion curves of the different functionally graded plates (p = 2)



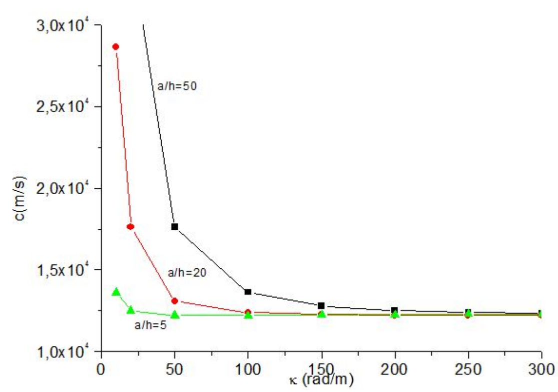
(a) M_1 mode



(b) M_2 mode

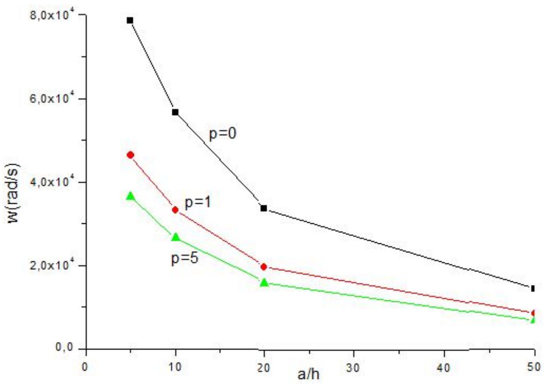


(c) M_3 mode

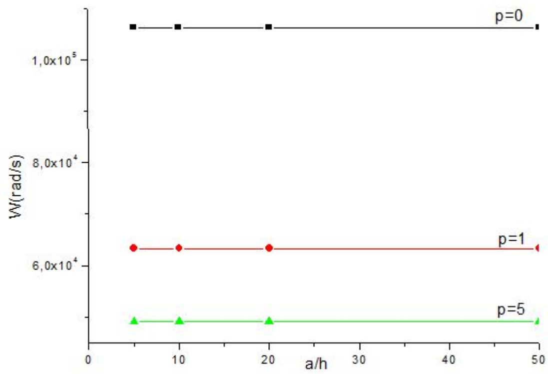


(*d*) M_4 mode

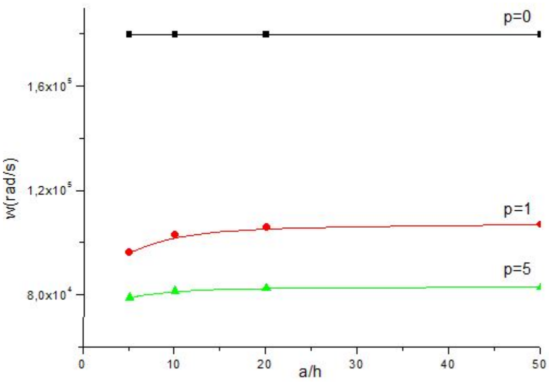
Fig. 5 The phase velocity curves of the different functionally graded plates (p = 2)



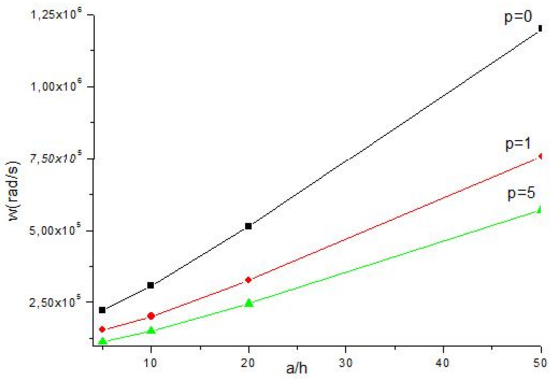
(a) M_1 mode



(b) M_2 mode

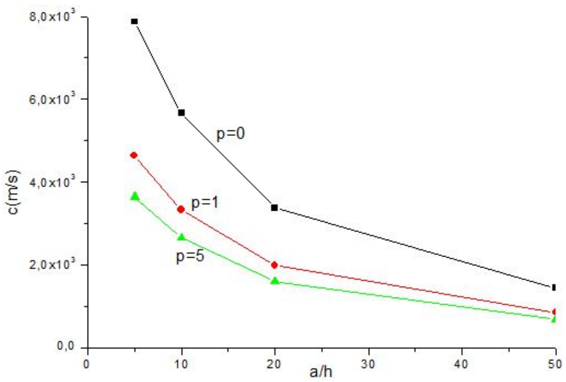


(c) M_3 mode

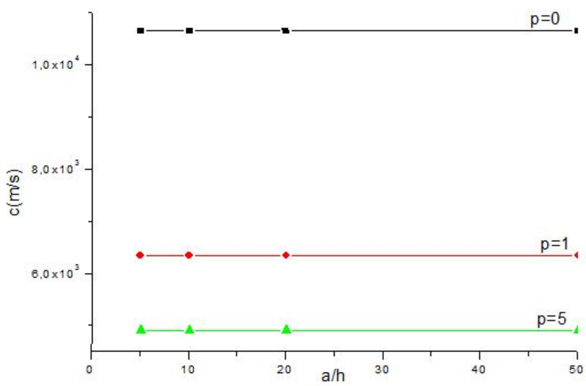


(*d*) M_4 mode

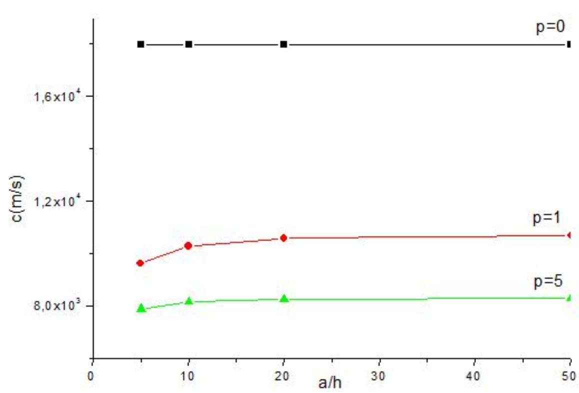
Fig. 6 The dispersion curves of the different functionally graded plates ($\kappa = 10$)



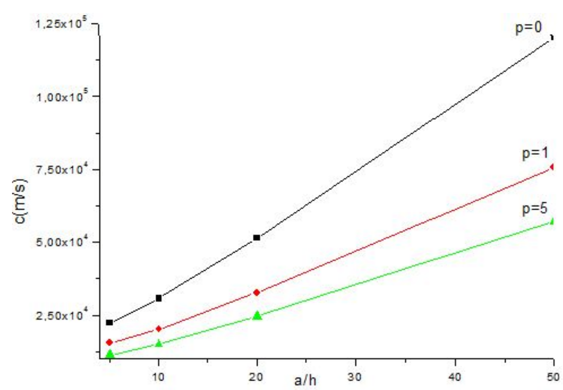
(a) M_1 mode



(b) M_2 mode



(c) M_3 mode



(*d*) M_4 mode

Fig. 7 The phase velocity curves of the different functionally graded plates ($\kappa = 10$)

FG plate decreases as the thickness decreases, except for wave mode M_3, for the large wave number, the increase of thickness involve the increase of phase velocity.

We can also notice that for the fundamental mode, and for the large values of wave number the phase velocities converge whatever the thickness.

Figs. 6-7 present respectively the influence of the dispersion and the phase velocity in the FG plate in function of the length to thickness ratio (a/h), using a hyperbolic shear deformation theory. The wave number is here taken equal to k = 10. From these figures, the similarities in the dispersion and phase velocity evolutions can be put into evidence.

For the M_1 mode, the increase in the plate length to thickness ratio leads to a decrease of the frequency as well as the phase velocity.

For the M_2 and M_3 modes, the increase in the plate length to thickness ratio has no influence on the frequency and the phase velocity.

On the contrary, for the M_4 mode, the increase in the plate length to thickness ratio leads to an increase of both the frequency and the phase velocity.

3.3 Comparisons parametric study of P-FGM and S-FGM plate

From Figs. 8 to 11 it is clear that the SFGM plate shows greater results compared to the PFGM plate for the fundamental mode. This observation is valid for all modes. Thus a smooth distribution of the material properties along the thickness was ensured.

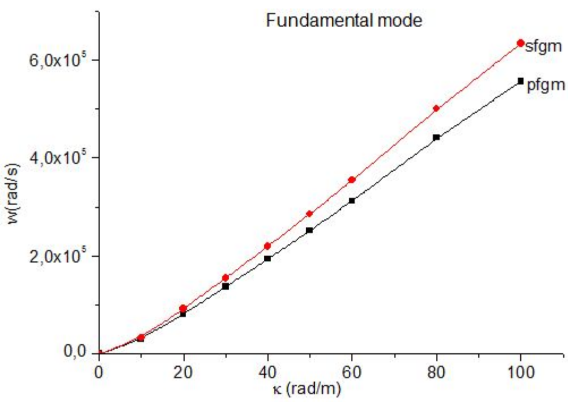


Fig. 8 Comparisons frequencies P-FGM – SFGM (a/h = 10, p = 2)

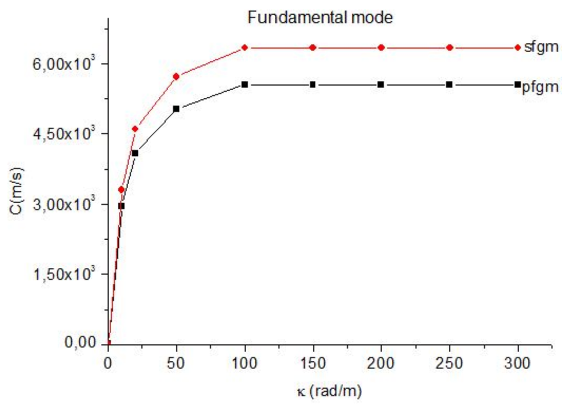


Fig. 9 Comparisons phase velocities P-FGM – SFGM (a/h = 10, p = 2)

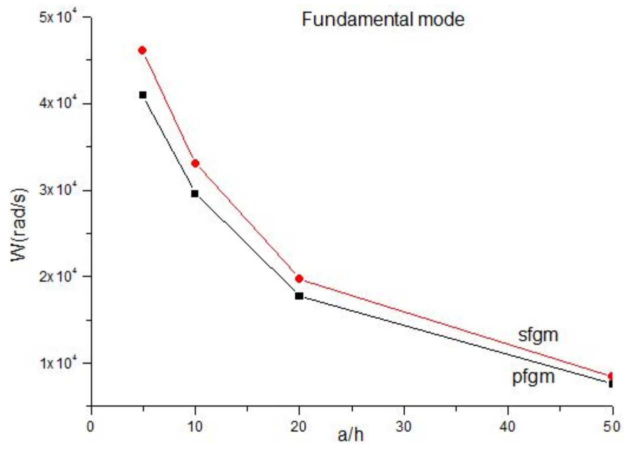


Fig. 10 Comparisons frequencies P-FGM – SFGM (κ = 10, p = 2)

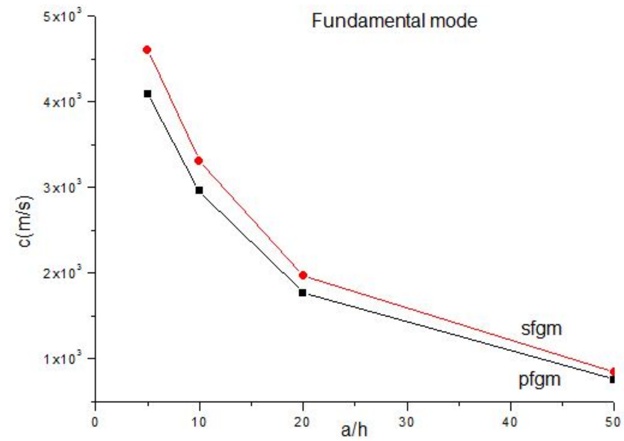


Fig. 11 Comparisons phase velocities P-FGM – SFGM (κ = 10, p = 2)

4. Conclusions

In this paper, a novel higher-order shear deformation theory is used for analyzing wave propagation of a functionally graded plate using various higher-order shear deformation plate theories. This theory incorporates a new field of displacement which introduces indeterminate integral variables. The computational cost can therefore be reduced due to reduced number of unknowns as well as the dispersion relations of wave propagation in the FG plate. The analytic dispersion relation of the functionally graded plate is obtained by solving an eigenvalue problem. From the present work, it can be concluded that the influence of the volume fraction distributions on wave propagation in the FG plate is significant. An amelioration of this formulation is chart in future works by introducing porosity factor and thickness stretching effect.

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