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A novel higher-order shear deformation theory for bending and free vibration analysis of isotropic and multilayered plates and shells

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Abstract. In this work, the bending and free vibration analysis of multilayered plates and shells is presented by utilizing a new higher order shear deformation theory (HSDT). The proposed involves only four unknowns, which is even less than the first shear deformation theory (FSDT) and without requiring the shear correction coefficient. Unlike the conventional HSDTs, the present one presents a novel displacement field which incorporates undetermined integral variables. The equations of motion are derived by using the Hamilton's principle. These equations are then solved via Navier-type, closed form solutions. Bending and vibration results are found for cylindrical and spherical shells and plates for simply supported boundary conditions. Bending and vibration problems are treated as individual cases. Panels are subjected to sinusoidal, distributed and point loads. Results are presented for thick to thin as well as shallow and deep shells. The computed results are compared with the exact 3D elasticity theory and with several other conventional HSDTs. The proposed HSDT is found to be precise compared to other several existing ones for investigating the static and dynamic response of isotropic and multilayered composite shell and plate structures.

Keywords: layered structures; bending; vibration; higher order shear deformation theory

1. Introduction

Shells structures have improved structural stiffness compared to plates. The interest use of shell structures is their capability in supporting forces and moments by a coupled membrane and bending action because of their curvature. On the other hand, composite materials present high performance and reliability because of their well-known properties. Consequently, shell structures fabricated with composite materials will continue being widely employed for many years in various engineering areas such as aerospace, naval, construction industries, automotive and for sporting goods, medical devices and many other fields (Swanson 1997, Allen 1969, Jones 1999, Mouritz *et al*. 2001, Benachour *et al*. 2011, Ait Atmane *et al*. 2015, Larbi Chaht *et al*. 2015, Mahi *et al*. 2015, Bakora and Tounsi 2015, El-Hassar *et al*. 2016, Barati and Shahverdi 2016, Bounouara *et al*. 2016, Laoufi *et al*. 2016, Bousahla *et al*. 2016, Chikh *et al*. 2016, Bellifa *et al*. 2017, Meksi *et al*. 2017, El-Haina *et al*. 2017).

It is important to indicate that the investigation of shell models permit the understanding of plates, curved beams, and flat beams as special cases. In the past four decades, a considerable number of theories for composited laminated shells have been proposed. These models can be classified in different theories, such as equivalent single layer, quasi-layerwise and layerwise theories (Demasi 2009a, b). Detailed explanation for the above indicated models may be consulted in the articles presented by Carrera (2000, 2001 and 2002), Demasi (2009a, b, c, d and e) and Mantari *et al*. (2012). According to Carrera's unified formulation (CUF) (Carrera 2003) or the unified generalized formulation (GUF) proposed by Demasi (2009a, b, c, d, e, 2008), among equivalent single layer theories, there are many available class of models which CUF or GUF can be reproduced. However, the use of some of these models to layered anisotropic composite shells can produce errors up to 30% in deflections, stresses and frequencies (Reddy 2004), as it will also be demonstrated in this article. Thus, a careful selection of the appropriate shear deformation theory is crucial.

Supposing the usual classification of the shell and plate deformation theories (other classifications of models may be found in paper (Demasi 2009a)), there are mainly three important theories: namely the classical lamination theory (CLT), the first order shear deformation theory (FSDT) and the higher shear deformation theory (HSDT). First order shear deformation theory (FSDT) is based on the kinematic field supposition defined by Mindlin (1951), in which the uniform transverse shear strain distribution within the thickness accounted (Reissner 1945, 1975, Reissner and Wan 1982, Alieldin *et al*. 2011, Adda Bedia *et al*. 2015, Meksi *et al*. 2015, Bellifa *et al*. 2016, Bouderba *et al*.

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2016). Higher order shear deformation theories (HSDTs) were proposed to improve the investigation of shell and behaviors and extensively employed by many scientists (Lo *et al*. 1977, Murthy 1981, Reddy 1984a, b, 1986, Reddy and Liu 1985, Librescu and Khdeir 1988, Librescu *et al*. 1987, Bert 1984, Murakami 1986, Kant 1993, Kant and Swaminathan 2002, Swaminathan and Ragounadin 2004, Khare *et al*. 2003, Batra and Vidoli 2002, Ferreira *et al*. 2005, Kim and Cho 2007, Bhaskar and Sivaram 2008, Oktem and Chaudhuri 2007, 2009a, b, Oktem and Soares 2011, Shimpi and Ghugal 1999 and 2001, Arya *et al*. 2002, Shimpi and Aynapure 2001, Roque *et al*. 2005, Levinson 1980, Touratier 1991, Ambartsumian 1958, Xiang *et al*. 2009, Soldatos 1992, Idibi *et al*. 1997, Matsunaga 2000, Karama *et al*. 2003, 2009). Interesting higher order models were developed by Kant and Swaminathan (2002), Reddy and Liu (1985), Touratier (1991), Soldatos (1992), Karama *et al*. (2003, 2009), Mantari *et al*. (2012, 2011), Swaminathan and Naveenkumar (2014), Ahmed (2014); Duc and Cong (2013), Kar and Panda (2015), Kar *et al*. (2015), Bourada *et al*. (2015) and very recently Tounsi *et al*. (2016), Eltaher *et al*. (2016) and Akavci (2016). Normally, these models verify the free surface boundary conditions and consider approximately parabolic variation of shear stresses within the thickness of the shell. More advanced higher shear deformation theories consider the continuity of the transverse shear stresses, and give improved results (Demasi 2009c, d and e). It should be noted that FGM, FGCNT and laminated structures are investigated recently using HSDT kinematics (Sahoo *et al*. 2017a, b, c, d, Mehar *et al*. 2017a, b, Mehar and Panda 2017a, b, c, d, Benahmed *et al*. 2017, Benbakhti *et al*. 2016, Benchohra *et al*. 2018, Chikh *et al*. 2017, Hirwani *et al*. 2016a, b, 2017, Besseghier *et al*. 2017, Klouche *et al*. 2017, Bouafia *et al*. 2017, Fahsi *et al*. 2017, Zidi *et al*. 2017, Mouffoki *et al*. 2017, Draiche *et al*. 2016, Menasria *et al*. 2017, Dutta *et al*. 2017, Kar *et al*. 2016, Singh *et al*. 2016, Kar and Panda 2016a, Ahouel *et al*. 2016, Boukhari *et al*. 2016, Abdelbari *et al*. 2016, Barka *et al*. 2016, Abdelhak *et al*. 2016, Benferhat *et al*. 2016, Bennoun *et al*. 2016, Sahoo *et al*. 2016, Mehar and Panda 2016, Katariya and Panda 2016, Hebali *et al*. 2016, Houari *et al*. 2016, Beldjelili *et al*. 2016, Kar and Panda 2015, Attia *et al*. 2015, Taibi *et al*. 2015, Ait Yahia *et al*. 2015, Meradjah *et al*. 2015, Hamidi *et al*. 2015, Panda and Mahapatra 2014, Fekrar *et al*. 2014, Bousahla *et al*. 2014, Belabed *et al*. 2014, Zidi *et al*. 2014, Hebali *et al*. 2014, Ait Amar Meziane *et al*. 2014, Tounsi *et al*. 2013, Bessaim *et al*. 2013, Bouderba *et al*. 2013). Recently, Kar *et al*. (2017) discussed effect of different temperature load on thermal postbuckling behavior of FG shallow curved shell panels. Kar and Panda (2017) studied the postbuckling response of shear deformable FG shallow spherical shell panel under nonuniform thermal environment. Kar and Panda (2016b) investigated the post-buckling behavior of shear deformable FG curved shell panel under edge compression. Also, Kar and Panda (2016c) examined nonlinear thermomechanical behavior of FGM cylindrical/ hyperbolic/elliptical shell panel with temperature-dependent and temperature-independent properties

In this paper, the present HSDT, first developed for plates by Hebali *et al*. (2016) and Bourada *et al*. (2016), extended to doubly curved shells for the first time, is simple in the sense that it contains the same dependent variables as in the FSDT. It is based on a kinematic in which the integral term is included leading to a reduction of the number of unknowns and equations of motion. This theory considers an approximate parabolic variation of the transverse shear strains within the shell thickness and the tangential stress-free boundary conditions on the shell surface, hence a shear correction factor is not needed. The governing equations are obtained by employing the principle of virtual work. These equations are then solved via a Navier-type, closed form solution. Bending and dynamic results are given for cylindrical and spherical shells for simply supported boundary conditions. Shells and plates are subjected to sinusoidal, distributed and point loads. Numerical results are provided for thick to thin as well as shallow and deep shells. The accuracy of the present method is ascertained by comparing it with various available results in the literature. It is confirmed that the present theory gives accurate results compared to those of other higher order theories in literature.

2. Statement of the problem

The aim of this paper is to extend the HSDT developed by Hebali *et al*. (2016) and Bourada *et al*. (2016) for plates to the bending and dynamic analysis of shells. The displacement field of the conventional HSDTs for shells is defined by

$$\bar{u}(\xi_1,\xi_2,\xi_3;t)=\left(1+\frac{\xi_3}{R_1}\right)u-\xi_3\frac{\partial w}{a_1\partial\xi_1}+f(\xi_3)\varphi_1 \quad (1a)$$

$$\bar{v}(\xi_1,\xi_2,\xi_3;t)=\left(1+\frac{\xi_3}{R_2}\right)v-\xi_3\frac{\partial w}{a_2\partial\xi_2}+f(\xi_3)\varphi_2 \quad (1b)$$

$$\bar{w}(\xi_1,\xi_2;t)=w \quad (1c)$$

where $u(\xi_1,\xi_1,t), v(\xi_1,\xi_1,t), w(\xi_1,\xi_1,t), \varphi_1(\xi_1,\xi_1,t)$ and $\varphi_2(\xi_1, \xi_1, t)$ are the five unknown displacement functions of the middle surface of the panel, h the thickness of the multilayered shell and $f(z)$ represents shape function defining the variation of the transverse shear strains and stresses across the thickness. In this article a novel displacement field with four unknowns is proposed

$$\bar{u}(\xi_1,\xi_2,\xi_3;t)=\left(1+\frac{\xi_3}{R_1}\right)u-\xi_3\frac{\partial w}{a_1\partial\xi_1}+k_1 f(\xi_3)\int\theta\, d\xi_1 \quad (2a)$$

$$\begin{aligned}&\bar{v}(\xi_1,\xi_2,\xi_3;t)\\&=\left(1+\frac{\xi_3}{R_2}\right)v-\xi_3\frac{\partial w}{a_2\partial\xi_2}+k_2 f(\xi_3)\int\theta\, d\xi_2\end{aligned} \quad (2b)$$

$$\bar{w}(\xi_1,\xi_2;t)=w \quad (2a)$$

Where u_0, v_0, w_0 and θ are the four unknown displacement functions of middle surface of the panel.

In this work, the present HSDT is obtained by setting (Mantari *et al*. 2012)

$$f(\xi_3) = \sin\left(\frac{\pi\xi_3}{h}\right)e^{\frac{1}{2}\cos\left(\frac{\pi\xi_3}{h}\right)} + \frac{\pi}{2h}\xi_3 \tag{3}$$

In the derivation of the necessary equations, a set of additional suppositions is considered:

- Small elastic deformations are supposed (i.e., displacements and rotations are small, and obey Hooke's law).
- The structure is composed of a number of layers which are supposed to be perfectly bonded.

The starting point of the current thick shell theory is the three dimensional elasticity theory, expressed in general curvilinear (reference) surface-parallel coordinates; while the thickness coordinate is normal to the reference (middle) surface as shown in Fig. 1.

The strain–displacement relations, based on this formulation, are expressed as follows (Reddy 2004)

$$\varepsilon_1 = \frac{1}{A_1}\left(\frac{\partial \bar{u}}{\partial \xi_1} + \frac{1}{a_2}\frac{\partial a_1}{\partial \xi_2}\bar{v} + \frac{a_1}{R_1}\bar{w}\right) \tag{4a}$$

$$\varepsilon_2 = \frac{1}{A_2}\left(\frac{\partial \bar{v}}{\partial \xi_2} + \frac{1}{a_1}\frac{\partial a_2}{\partial \xi_1}\bar{u} + \frac{a_2}{R_2}\bar{w}\right) \tag{4b}$$

$$\varepsilon_4 = \frac{1}{A_2}\frac{\partial \bar{w}}{\partial \xi_2} + A_2\frac{\partial}{\partial \xi_3}\left(\frac{\bar{v}}{A_2}\right) \tag{4c}$$

$$\varepsilon_5 = \frac{1}{A_1}\frac{\partial \bar{w}}{\partial \xi_1} + A_1\frac{\partial}{\partial \xi_3}\left(\frac{\bar{u}}{A_1}\right) \tag{4d}$$

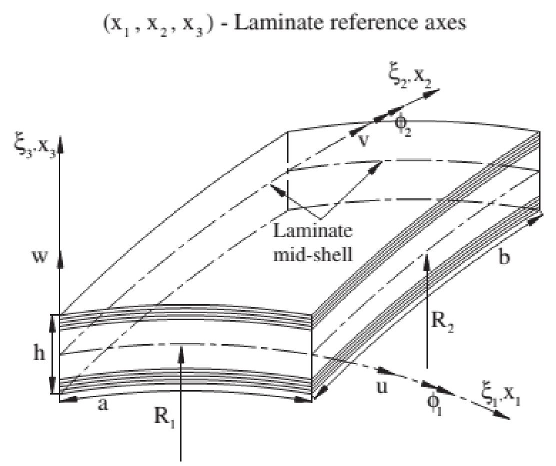

Fig. 1 Laminate geometry with positive set of lamina/laminate reference axes, displacement components and fiber orientation

$$\varepsilon_6 = \frac{A_2}{A_1}\frac{\partial}{\partial \xi_1}\left(\frac{\bar{v}}{A_2}\right) + \frac{A_1}{A_2}\frac{\partial}{\partial \xi_2}\left(\frac{\bar{u}}{A_1}\right) \tag{4e}$$

$$A_1 = \left(1 + \frac{\xi_3}{R_1}\right)a_1, \qquad A_2 = \left(1 + \frac{\xi_3}{R_2}\right)a_2 \tag{4f}$$

And $\xi_i (i = 1,....,6)$ represent strain components $\bar{u}$, $\bar{v}$ and $\bar{w}$ are the displacements on the surface (ξ_1, ξ_2, ξ_3) and a_1 and a_2 the vectors tangent to the ξ_1 and ξ_2 coordinate lines. Introduction of Eqs. (2) into the relations given in Eqs. (4) of a moderately shallow and deep shell supplies the following strain–displacement relations, valid for a doubly-curved panel under consideration

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{Bmatrix} = \begin{Bmatrix} \varepsilon_1^0 \\ \varepsilon_2^0 \\ \varepsilon_6^0 \end{Bmatrix} + z\begin{Bmatrix} \varepsilon_1^1 \\ \varepsilon_2^1 \\ \varepsilon_6^1 \end{Bmatrix} + f(\xi_3)\begin{Bmatrix} \varepsilon_1^2 \\ \varepsilon_2^2 \\ \varepsilon_6^2 \end{Bmatrix}, \qquad \begin{Bmatrix} \varepsilon_4 \\ \varepsilon_5 \end{Bmatrix} = g(\xi_3)\begin{Bmatrix} \varepsilon_4^3 \\ \varepsilon_5^3 \end{Bmatrix} \tag{5}$$

where

$$\begin{Bmatrix} \varepsilon_1^0 \\ \varepsilon_2^0 \\ \varepsilon_6^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u}{\partial x_1} + \dfrac{w}{R_1} \\ \dfrac{\partial v}{\partial x_2} + \dfrac{w}{R_2} \\ \dfrac{\partial u}{\partial x_2} + \dfrac{\partial v}{\partial x_1} \end{Bmatrix}, \quad \begin{Bmatrix} \varepsilon_1^1 \\ \varepsilon_2^1 \\ \varepsilon_6^1 \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w}{\partial x_1^2} \\ -\dfrac{\partial^2 w}{\partial x_2^2} \\ -2\dfrac{\partial^2 w}{\partial x_1 \partial x_2} \end{Bmatrix},$$
$$\begin{Bmatrix} \varepsilon_1^2 \\ \varepsilon_2^2 \\ \varepsilon_6^2 \end{Bmatrix} = \begin{Bmatrix} k_1\theta \\ k_2\theta \\ k_1\dfrac{\partial}{\partial x_2}\displaystyle\int \theta dx_1 + k_2\dfrac{\partial}{\partial x_1}\displaystyle\int \theta dx_2 \end{Bmatrix} \tag{6a}$$

$$\begin{Bmatrix} \varepsilon_4^3 \\ \varepsilon_5^3 \end{Bmatrix} = \begin{Bmatrix} k_2\int \theta dx_2 \\ k_1\int \theta dx_1 \end{Bmatrix} \tag{6b}$$

and

$$g(\xi_3) = \frac{df(\xi_3)}{d\xi_3} \tag{6c}$$

The integrals used in the above relations shall be resolved by a Navier solution and can be expressed by

$$\frac{\partial}{\partial x_2}\int \theta dx_1 = A'\frac{\partial^2\theta}{\partial x_1 \partial x_2}, \qquad \frac{\partial}{\partial x_1}\int \theta dx_2 = B'\frac{\partial^2\theta}{\partial x_1 \partial x_2}$$
$$\int \theta dx_1 = A'\frac{\partial\theta}{\partial x_1}, \qquad \int \theta dx_2 = B'\frac{\partial\theta}{\partial x_2} \tag{7}$$

where the parameters A' and B' are defined according to the type of solution employed, in this case via Navier. Hence, A' and B' are expressed by

$$A' = -\frac{1}{\alpha^2}, \quad B' = -\frac{1}{\beta^2}, \quad k_1 = \alpha^2, \quad k_2 = \beta^2 \tag{8}$$

Where α and β are defined in expression (19), and x_i denote the Cartesian coordinates $(dx_1 = a_i d\xi_i,\ i = 1,2)$. The stress–strain relations for the kth lamina are given by Reddy (2004)

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \\ \sigma_4 \\ \sigma_5 \end{Bmatrix}^{(k)} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} & 0 & 0 \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} & 0 & 0 \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \overline{Q}_{44} & 0 \\ 0 & 0 & 0 & 0 & \overline{Q}_{55} \end{bmatrix}^{(k)} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \\ \varepsilon_4 \\ \varepsilon_5 \end{Bmatrix}^{(k)} \tag{9}$$

In which $(\sigma_1, \sigma_2, \sigma_6, \sigma_4, \sigma_5)$ are the stress and $(\varepsilon_1, \varepsilon_2, \varepsilon_6, \varepsilon_4, \varepsilon_5)$ are the strain components, and $\overline{Q}_{ij}$ are the material constants of the kth lamina in the laminate coordinate system. Hamilton's principle is applied to the present case, and the following expressions are obtained

$$\begin{aligned} 0 = &\int_0^t \left[\int_{-h/2}^{h/2} \int_\Omega \left(\sigma_1 \delta\ \varepsilon_1^{(k)} + \sigma_2 \delta\ \varepsilon_2^{(k)} + \sigma_6 \delta\ \varepsilon_6^{(k)} \right.\right. \\ &\left.\left. + \sigma_4 \delta\ \varepsilon_4^{(k)} + \sigma_5 \delta\ \varepsilon_5^{(k)} \right) dx_1 dx_2 d\xi_3 \right] dt \\ &- \int_0^t \int_\Omega q \delta w dx_1 dx_2 dt \\ &- \int_0^t \delta \left\{ \int_{-h/2}^{h/2} \int_\Omega \rho \left[\left(\dot{u}\right)^2 + \left(\dot{v}\right)^2 + \left(\dot{w}\right)^2 \right] dx_1 dx_2 d\xi_3 \right\} dt \end{aligned} \tag{10}$$

$$\begin{aligned} = &\int_0^t \left\{ \int_\Omega \left(N_1 \delta\varepsilon_1^0 + N_2 \delta\varepsilon_2^0 + N_6 \delta\varepsilon_6^0 + M_1 \delta\varepsilon_1^1 + M_2 \delta\varepsilon_2^1 \right.\right. \\ &+ M_6 \delta\varepsilon_6^1 + P_1 \delta\varepsilon_1^2 + P_2 \delta\varepsilon_2^2 + P_6 \delta\varepsilon_6^2 + Q_1 \delta\varepsilon_4^3 + Q_2 \delta\varepsilon_5^3 \\ &- q\delta w + \left(\left(I_1 + \frac{2I_2}{R_1} \right) \ddot{u} - \left(I_2 + \frac{I_3}{R_1} \right) \frac{\partial \ddot{w}}{\partial x_1} + k_1 A' \left(I_4 + \frac{I_5}{R_1} \right) \frac{\partial \ddot{\theta}}{\partial x_1} \right) \delta u \\ &+ \left(\left(I_1 + \frac{2I_2}{R_2} \right) \ddot{v} - \left(I_2 + \frac{I_3}{R_2} \right) \frac{\partial \ddot{w}}{\partial x_2} + k_2 B' \left(I_4 + \frac{I_5}{R_2} \right) \frac{\partial \ddot{\theta}}{\partial x_2} \right) \delta v \\ &+ \left(\left(I_2 + \frac{I_3}{R_1} \right) \frac{\partial \ddot{u}}{\partial x_1} + \left(I_2 + \frac{I_3}{R_2} \right) \frac{\partial \ddot{v}}{\partial x_2} - I_3 \left(\frac{\partial^2 \ddot{w}}{\partial x_1^2} + \frac{\partial^2 \ddot{w}}{\partial x_2^2} \right) \right. \\ &\left. + I_5 \left(k_1 A' \frac{\partial^2 \ddot{\theta}}{\partial x_1^2} + k_2 B' \frac{\partial^2 \ddot{\theta}}{\partial x_2^2} \right) + I_1 w \right) \delta w \\ &+ \left(k_1 A' \left(I_4 + \frac{I_5}{R_1} \right) \frac{\partial \ddot{u}}{\partial x_1} + k_2 B' \left(I_4 + \frac{I_5}{R_1} \right) \frac{\partial \ddot{v}}{\partial x_1} \right. \\ &\left.\left. - I_5 \left(k_1 A' \frac{\partial^2 \ddot{w}}{\partial x_1^2} + k_2 B' \frac{\partial^2 \ddot{w}}{\partial x_2^2} \right) + I_6 \left(k_1^2 A'^2 \frac{\partial^2 \ddot{\theta}}{\partial x_1^2} + k_2^2 B'^2 \frac{\partial^2 \ddot{\theta}}{\partial x_2^2} \right) \right) \delta\theta \right) \\ &dx_1 dx_2 \Big\} dt \end{aligned} \tag{11}$$

Where q is the distributed transverse load, N_i, M_i, P_i and R_i are the resultants of the following integrations

$$\begin{aligned} &\left(N_i, M_i, P_i \right) = \sum_{k=1}^{n} \int_{\xi_3^{(k-1)}}^{\xi_3^{(k)}} \sigma_i^{(k)} \left(1, \xi_3, f(\xi_3) \right) d\xi_3 \\ &i = 1,2,6; \\ &Q_1 = \sum_{k=1}^{n} \int_{\xi_3^{(k-1)}}^{\xi_3^{(k)}} \sigma_4^{(k)} g(\xi_3) d\xi_3 \\ &Q_2 = \sum_{k=1}^{n} \int_{\xi_3^{(k-1)}}^{\xi_3^{(k)}} \sigma_5^{(k)} g(\xi_3) d\xi_3 \end{aligned} \tag{12}$$

And the inertia terms I_i (i = 1, 2, 3, 4, 5, 6) are defined by the following equations

$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{pmatrix} = \sum_{k=1}^{n} \int_{\xi_3^{(k-1)}}^{\xi_3^{(k)}} \rho^{(k)} Q_{ij}^{(k)} \begin{pmatrix} 1 \\ \xi_3 \\ \xi_3^2 \\ f(\xi_3) \\ \xi_3 f(\xi_3) \\ \left[f(\xi_3) \right]^2 \end{pmatrix} d\xi_3 \tag{13}$$

The governing equations of motion can be derived from Eq. (11) by integrating the displacement gradients by parts and setting the coefficients of $\delta u, \delta v, \delta w, \delta\theta$ to zero separately, and the following equation can be obtained

$$\begin{aligned} &\delta u: \ \frac{\partial N_1}{\partial x_1} + \frac{\partial N_6}{\partial x_2} = \left(I_1 + \frac{2I_2}{R_1} \right) \ddot{u} \\ &- \left(I_2 + \frac{I_3}{R_1} \right) \frac{\partial \ddot{w}}{\partial x_1} + k_1 A' \left(I_4 + \frac{I_5}{R_1} \right) \frac{\partial \ddot{\theta}}{\partial x_1} \\ &\delta v: \ \frac{\partial N_2}{\partial x_2} + \frac{\partial N_6}{\partial x_1} = \left(I_1 + \frac{2I_2}{R_2} \right) \ddot{v} \\ &- \left(I_2 + \frac{I_3}{R_2} \right) \frac{\partial \ddot{w}}{\partial x_2} + k_2 B' \left(I_4 + \frac{I_5}{R_2} \right) \frac{\partial \ddot{\theta}}{\partial x_2} \\ &\delta w: -\frac{N_1}{R_1} - \frac{N_2}{R_2} + \frac{\partial^2 M_1}{\partial x_1^2} + 2\frac{\partial^2 M_6}{\partial x_1 \partial x_2} + \frac{\partial^2 M_2}{\partial x_2^2} \\ &= \left(I_2 + \frac{I_3}{R_1} \right) \frac{\partial \ddot{u}}{\partial x_1} + \left(I_2 + \frac{I_3}{R_2} \right) \frac{\partial \ddot{v}}{\partial x_2} \\ &- I_3 \left(\frac{\partial^2 \ddot{w}}{\partial x_1^2} + \frac{\partial^2 \ddot{w}}{\partial x_2^2} \right) + I_5 \left(k_1 A' \frac{\partial^2 \ddot{\theta}}{\partial x_1^2} + k_2 B' \frac{\partial^2 \ddot{\theta}}{\partial x_2^2} \right) \\ &+ I_1 w - q \\ &\delta\theta: \ \mathrm{k}_1 A' \frac{\partial^2 P_1}{\partial x_1^2} + \mathrm{k}_2 B' \frac{\partial^2 P_2}{\partial x_2^2} + \left(\mathrm{k}_1 A' + \mathrm{k}_2 B' \right) \frac{\partial^2 P_6}{\partial x_1 \partial x_2} \\ &- \mathrm{k}_1 A' \frac{\partial Q_1}{\partial x_1} - \mathrm{k}_2 B' \frac{\partial Q_2}{\partial x_2} = k_1 A' \left(I_4 + \frac{I_5}{R_1} \right) \frac{\partial \ddot{u}}{\partial x_1} \end{aligned} \tag{14}$$

$$+k_2 B'\left(I_4+\frac{I_5}{R_1}\right)\frac{\partial \ddot{v}}{\partial x_1}-I_5\left(k_1 A'\frac{\partial^2 \ddot{w}}{\partial x_1^2}+k_2 B'\frac{\partial^2 \ddot{w}}{\partial x_2^2}\right)$$
$$+I_6\left(k_1^2 A'^2\frac{\partial^2 \ddot{\theta}}{\partial x_1^2}+k_2^2 B'^2\frac{\partial^2 \ddot{\theta}}{\partial x_2^2}\right) \tag{14}$$

By substituting the stress–strain relations into the definitions of force and moment resultants of the present theory given in Eq. (12) the following constitutive equations are obtained

$$\begin{Bmatrix} N \\ M \\ P \end{Bmatrix}=\begin{bmatrix} A & B & E \\ B & D & F \\ E & F & H \end{bmatrix}\begin{Bmatrix} \varepsilon^0 \\ \varepsilon^1 \\ \varepsilon^2 \end{Bmatrix},$$

$$N=\{N_1,N_2,N_6\}, \quad M=\{M_1,M_2,M_6\}$$

$$P=\{P_1,P_2,P_6\}, \quad \varepsilon^0=\{\varepsilon_1^0,\varepsilon_2^0,\varepsilon_6^0\}$$

$$\varepsilon^1=\{\varepsilon_1^1,\varepsilon_2^1,\varepsilon_6^1\}, \quad \varepsilon^2=\{\varepsilon_1^2,\varepsilon_2^2,\varepsilon_6^2\}$$

$$A=\begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{61} & A_{62} & A_{66} \end{bmatrix}, \quad B=\begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{21} & B_{22} & B_{26} \\ B_{61} & B_{62} & B_{66} \end{bmatrix} \tag{15a}$$

$$D=\begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{21} & D_{22} & D_{26} \\ D_{61} & D_{62} & D_{66} \end{bmatrix}, \quad E=\begin{bmatrix} E_{11} & E_{12} & E_{16} \\ E_{21} & E_{22} & E_{26} \\ E_{61} & E_{62} & E_{66} \end{bmatrix}$$

$$F=\begin{bmatrix} F_{11} & F_{12} & F_{16} \\ F_{21} & F_{22} & F_{26} \\ F_{61} & F_{62} & F_{66} \end{bmatrix}, \quad H=\begin{bmatrix} H_{11} & H_{12} & H_{16} \\ H_{21} & H_{22} & H_{26} \\ H_{61} & H_{62} & H_{66} \end{bmatrix}$$

and

$$\begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix}=\begin{bmatrix} A_{44}^S & A_{45}^S \\ A_{45}^S & A_{55}^S \end{bmatrix}\begin{Bmatrix} \varepsilon_4^3 \\ \varepsilon_5^3 \end{Bmatrix} \tag{15b}$$

Here the stiffness coefficients are defined as

$$\begin{pmatrix} A_{ij} \\ B_{ij} \\ D_{ij} \\ E_{ij} \\ F_{ij} \\ H_{ij} \end{pmatrix}=\sum_{k=1}^{n}\int_{\xi_3^{(k-1)}}^{\xi_3^{(k)}} Q_{ij}^{(k)}\begin{pmatrix} 1 \\ \xi_3 \\ \xi_3^{\,2} \\ f(\xi_3) \\ \xi_3 f(\xi_3) \\ [f(\xi_3)]^2 \end{pmatrix} d\xi_3 \quad i,j=1,2,6 \tag{16a}$$

$$A_{ij}^s=\sum_{k=1}^{n}\int_{\xi_3^{(k-1)}}^{\xi_3^{(k)}} Q_{ij}^{(k)}[g(z)]^2 d\xi_3, \quad i,j=4,5 \tag{16b}$$

In what follows, the following simply supported boundary conditions are considered here prescribed at all four edges

$$\begin{aligned}
&u(x_1,0)=u(x_1,b)=v(0,x_2)=v(a,x_2)=0;\\
&w(x_1,0)=w(x_1,b)=w(0,x_2)=w(a,x_2)=0;\\
&N_2(x_1,0)=N_2(x_1,b)=N_1(0,x_2)=N_1(a,x_2)=0;\\
&M_2(x_1,0)=M_2(x_1,b)=M_1(0,x_2)=M_1(a,x_2)=0;\\
&P_2(x_1,0)=P_2(x_1,b)=P_1(0,x_2)=P_1(a,x_2)=0;\\
&\theta(x_1,0)=\theta(x_1,b)=\theta(0,x_2)=\theta(a,x_2)=0;
\end{aligned} \tag{17}$$

3. Solution procedure

For the analytical solution of the partial differential equations given in Eq. (14), the Navier method, based on double Fourier series, is used under the specified boundary conditions. For anti-symmetric cross-ply laminated plates, the following stiffness components are identically zero

$$\begin{aligned}
&A_{i6}=B_{i6}=D_{i6}=E_{i6}=F_{i6}=H_{i6}=0; \ i=1,2\\
&B_{12}=E_{12}=B_{66}=E_{66}=A_{45}^s=A_{54}^s=0
\end{aligned} \tag{18}$$

Using Navier's procedure, the solution of the displacement variables satisfying the simple supported boundary conditions can be expressed in the following Fourier series

$$\begin{Bmatrix} u(x_1,x_2,t) \\ v(x_1,x_2,t) \\ w(x_1,x_2,t) \\ \theta(x_1,x_2,t) \end{Bmatrix}=\sum_{r=1}^{\infty}\sum_{s=1}^{\infty}\begin{Bmatrix} U_{rs}\cos(\alpha x_1)\sin(\beta x_2)e^{j\omega t} \\ V_{rs}\sin(\alpha x_1)\cos(\beta x_2)e^{j\omega t} \\ W_{rs}\sin(\alpha x_1)\sin(\beta x_2)e^{j\omega t} \\ \Theta_{rs}\sin(\alpha x_1)\sin(\beta x_2)e^{j\omega t} \end{Bmatrix} \tag{19}$$

Where $U_{rs}, V_{rs}, W_{rs}, \Theta_{rs}$ are coefficients, and α and β are expressed as

$$\alpha=r\pi/a; \quad \beta=s\pi/b \tag{20}$$

Substituting Eqs. (15)-(20) into Eq. (14), the following equations are obtained

$$\begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{12} & M_{22} & M_{23} & M_{24} \\ M_{13} & M_{23} & M_{33} & M_{34} \\ M_{14} & M_{24} & M_{34} & M_{44} \end{bmatrix}\begin{Bmatrix} \ddot{U}_{rs} \\ \ddot{V}_{rs} \\ \ddot{W}_{rs} \\ \ddot{\Theta}_{rs} \end{Bmatrix}+\begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{12} & K_{22} & K_{23} & K_{24} \\ K_{13} & K_{23} & K_{33} & K_{34} \\ K_{14} & K_{24} & K_{34} & K_{44} \end{bmatrix}\begin{Bmatrix} U_{rs} \\ V_{rs} \\ W_{rs} \\ \Theta_{rs} \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \\ Q_{rs} \\ 0 \end{Bmatrix} \tag{21}$$

Where Q_{rs} are the coefficients in the double Fourier expansion of the transverse load

$$q(x,y)=\sum_{r=1}^{\infty}\sum_{s=1}^{\infty} Q_{rs}\sin(\alpha x)\sin(\beta y) \tag{22}$$

And the elements of $[M_{ij}]$ and $[K_{ij}]$ in Eq. (21) are given

$$\begin{aligned}
&M_{11} = -\left(I_1 + 2\frac{I_2}{R_1}\right), \quad M_{12} = 0, \\
&M_{13} = \left(I_2 + \frac{I_3}{R_1}\right)\alpha, \quad M_{14} = -k_1 A'\left(I_4 + \frac{I_5}{R_1}\right)\alpha, \\
&M_{22} = -\left(I_1 + 2\frac{I_2}{R_2}\right), \quad M_{23} = \left(I_2 + \frac{I_3}{R_2}\right)\beta, \\
&M_{24} = -k_2 B'\left(I_4 + \frac{I_5}{R_2}\right)\beta, \\
&M_{33} = -I_1 - I_3\left(\alpha^2 + \beta^2\right), \\
&M_{34} = I_5\left(k_1 A'\alpha^2 + k_2 B'\beta^2\right), \\
&M_{44} = I_6\left(k_1^2 A'^2 \alpha^2 + k_2^2 B'^2 \beta^2\right)
\end{aligned} \tag{23}$$

and

$$\begin{aligned}
&K_{11} = -\left(\alpha^2 A_{11} + \beta^2 A_{66}\right), \quad K_{12} = -\alpha\beta\left(A_{12} + A_{66}\right) \\
&K_{13} = \alpha\left(\frac{A_{11}}{R_1} + \frac{A_{12}}{R_2} + \alpha^2 B_{11}\right), K_{14} = \alpha k_1 \mathrm{E}_{11} \\
&K_{24} = \beta k_2 \mathrm{E}_{22} \\
&K_{22} = -\left(\alpha^2 A_{66} + \beta^2 A_{22}\right), \; K_{24} = \beta k_2 \mathrm{E}_{22} \\
&K_{23} = \beta\left(\frac{A_{12}}{R_1} + \frac{A_{22}}{R_2} + \beta^2 B_{22}\right) \\
&K_{33} = -\frac{1}{R_2}\left(\frac{A_{12}}{R_1} + \frac{A_{22}}{R_2} + 2\beta^2 B_{22}\right) \\
&-2\alpha^2\beta^2\left(D_{12} + 2D_{66}\right) \\
&-\frac{1}{R_1}\left(\frac{A_{11}}{R_1} + \frac{A_{12}}{R_2} + 2\alpha^2 B_{11}\right) - \left(\alpha^4 D_{11} + \beta^4 D_{22}\right) \\
&K_{34} = -\left(k_1\frac{E_{11}}{R_1} + k_2\frac{E_{22}}{R_2}\right) \\
&-\left(k_2\alpha^2 + k_1\beta^2\right)\left(F_{12} + 2F_{66}\right) - k_1\alpha^2 F_{11} - k_2\beta^2 F_{22} \\
&K_{44} = -k_1 A'\left(k_1 A'\alpha^4 H_{11} + k_2 B'\alpha^2\beta^2 H_{12}\right) \\
&-\left(k_1 A' + k_2 B'\right)\left(k_2 B'\alpha^2\beta^2 H_{66} + k_1 A'\alpha^2\beta^2 H_{66}\right) \\
&-k_2^2 B'^2 \beta^2 A_{44}^s - k_2 B'\left(k_2 B'\beta^4 H_{22} + k_1 A'\alpha^2\beta^2 H_{12}\right) \\
&-k_1^2 A'^2 \alpha^2 A_{55}^s
\end{aligned} \tag{24}$$

4. Numerical results and discussion

4.1 Bending analysis

For this section, the mechanical properties of each layer are given as follows (Reddy and Liu 1985)

$$\begin{aligned}
&E_1 = 25E_2, \qquad G_{12} = G_{13} = 0.5E_2, \\
&\nu_{12} = \nu_{13} = 0.25\,, \qquad G_{23} = 0.2E_2
\end{aligned} \tag{25}$$

The following normalized quantity is defined for deflection

$$\bar{w} = \frac{10^2 E_2 h^3}{a^4 q_0} w\left(\frac{a}{2}, \frac{b}{2}, 0\right) \tag{26}$$

For the uniform and point loads, the values of "*r*" and "*s*" in the series are taken equal to 101 as in Reddy and Liu (1985) and Giunta *et al*. (2011).

Tables 1 to 3 provide the non-dimensional center deflection of cross-ply (0°/90°) laminated spherical shells under sinusoidal, uniform, and point loads, respectively. The computed results are compared with the FSDT and HSDT reported from Reddy and Liu (1985) and the HSDT developed by Mantari *et al*. (2012), for variousv values given in Tables 1, 2 and 4. The values are also compared with the trigonometric shear deformation theory and multiquadricsby Ferreira *et al*. (2011), the HSDT developed

Table 1 Normalized central deflections of cross-ply two layer (0/90) square laminated spherical shells under sinusoidal load

R/*a*	Theory	*a*/*h* = 10	*a*/*h* = 100
5	Present	11.0078	1.1938
	HSDT (Mantari *et al*. 2012)	11.0221	1.1940
	HSDT (Mantari *et al*. 2011)	11.1080	1.1940
	HSDT (Touratier 1991)	11.1532	1.1940
	HSDT (Reddy and Liu 1985)	11.1660	1.1937
	FSDT (Reddy and Liu 1985)	11.4290	1.1948
10	Present	11.7280	3.5745
	HSDT (Mantari *et al*. 2012)	11.7321	3.5750
	HSDT (Mantari *et al*. 2011)	11.8296	3.5751
	HSDT (Touratier 1991)	11.8810	3.5751
	HSDT (Reddy and Liu 1985)	11.8960	3.5733
	FSDT (Reddy and Liu 1985)	12.1230	3.5760
20	Present	11.9231	7.1287
	HSDT (Mantari *et al*. 2012)	11.9241	7.1291
	HSDT (Mantari *et al*. 2011)	12.0249	7.1295
	HSDT (Touratier 1991)	12.0780	7.1298
	HSDT (Reddy and Liu 1985)	12.0940	7.1236
	FSDT (Reddy and Liu 1985)	12.3090	7.1270
50	Present	11.9789	9.8790
	HSDT (Mantari *et al*. 2012)	11.9790	9.8791
	HSDT (Mantari *et al*. 2011)	12.0807	9.8800
	HSDT (Touratier 1991)	12.1343	9.8804
	HSDT (Reddy and Liu 1985)	12.1500	9.8692
	FSDT (Reddy and Liu 1985)	12.3620	9.8717
100	Present	11.9869	10.4552
	HSDT (Mantari *et al*. 2012)	11.9869	10.4553
	HSDT (Mantari *et al*. 2011)	12.0887	10.4562
	HSDT (Touratier 1991)	12.1424	10.4567
	HSDT (Reddy and Liu 1985)	12.1580	10.4440
	FSDT (Reddy and Liu 1985)	12.3700	10.4460
Plate	Present	11.9895	10.6625
	HSDT (Mantari *et al*. 2012)	11.9895	10.6625
	HSDT (Mantari *et al*. 2011)	12.0914	10.6635
	HSDT (Touratier 1991)	12.1451	10.6641
	HSDT (Reddy and Liu 1985)	12.1610	10.6510
	FSDT (Reddy and Liu 1985)	12.3730	10.6530

Table 2 Normalized central deflections of cross-ply two layer (0/90) square laminated spherical shells under uniform load

R/a	Theory	a/h = 10	a/h = 100
5	Present	17.3545	1.7521
	HSDT (Mantari *et al*. 2012)	17.3697	1.7524
	HSDT (Mantari *et al*. 2011)	17.4886	1.7523
	HSDT (Touratier 1991)	17.5504	1.7523
	HSDT (Reddy and Liu 1985)	17.5660	1.7519
	FSDT (Reddy and Liu 1985)	19.9440	1.7535
10	Present	18.5181	5.5408
	HSDT (Mantari *et al*. 2012)	18.5170	5.5414
	HSDT (Mantari *et al*. 2011)	18.6543	5.5414
	HSDT (Touratier 1991)	18.7260	5.5415
	HSDT (Reddy and Liu 1985)	18.7440	5.5388
	FSDT (Reddy and Liu 1985)	19.0650	5.5428
20	Present	18.8336	11.2765
	HSDT (Mantari *et al*. 2012)	18.8274	11.2770
	HSDT (Mantari *et al*. 2011)	18.9699	11.2775
	HSDT (Touratier 1991)	19.0444	11.2778
	HSDT (Reddy and Liu 1985)	19.0640	11.2680
	FSDT (Reddy and Liu 1985)	19.3650	11.2730
50	Present	18.9234	15.7269
	HSDT (Mantari *et al*. 2012)	18.9161	15.7269
	HSDT (Mantari *et al*. 2011)	19.0601	15.7281
	HSDT (Touratier 1991)	19.1354	15.7287
	HSDT (Reddy and Liu 1985)	19.1550	15.7110
	FSDT (Reddy and Liu 1985)	19.4520	15.7140
100	Present	18.9368	16.6595
	HSDT (Mantari *et al*. 2012)	18.9289	16.6597
	HSDT (Mantari *et al*. 2011)	19.0731	16.6611
	HSDT (Touratier 1991)	19.1485	16.6618
	HSDT (Reddy and Liu 1985)	19.1680	16.6420
	FSDT (Reddy and Liu 1985)	19.4640	16.6450
Plate	Present	18.9409	16.9954
	HSDT (Mantari *et al*. 2012)	18.9331	16.9954
	HSDT (Mantari *et al*. 2011)	19.0774	16.9968
	HSDT (Touratier 1991)	19.1528	16.9975
	HSDT (Reddy and Liu 1985)	19.1720	16.9770
	FSDT (Reddy and Liu 1985)	19.4690	16.9800

Table 3 Normalized central deflections of cross-ply two layer (0/90) square laminated spherical shells under point load

R/a	Theory	a/h = 10	a/h = 100
5	Present	5.5391	0.8191
	HSDT (Mantari *et al*. 2012)	5.5548	0.8195
	HSDT (Mantari *et al*. 2011)	5.7174	0.8212
	HSDT (Touratier 1991)	5.8309	0.8222
	HSDT (Reddy and Liu 1985)	5.8953	–
	FSDT (Reddy and Liu 1985)	7.1015	–
10	Present	5.8307	1.8335
	HSDT (Mantari *et al*. 2012)	5.8422	1.8340
	HSDT (Mantari *et al*. 2011)	6.0098	1.8358
	HSDT (Touratier 1991)	6.1260	1.8369
	HSDT (Reddy and Liu 1985)	6.1913	–
	FSDT (Reddy and Liu 1985)	7.3836	–
20	Present	5.9096	3.2771
	HSDT (Mantari *et al*. 2012)	5.9198	3.2776
	HSDT (Mantari *et al*. 2011)	6.0888	3.2796
	HSDT (Touratier 1991)	6.2058	3.2808
	HSDT (Reddy and Liu 1985)	6.2714	–
	FSDT (Reddy and Liu 1985)	7.4692	–
50	Present	5.9322	4.3841
	HSDT (Mantari *et al*. 2012)	5.9420	4.3845
	HSDT (Mantari *et al*. 2011)	6.1115	4.3866
	HSDT (Touratier 1991)	6.2286	4.3879
	HSDT (Reddy and Liu 1985)	6.2943	–
	FSDT (Reddy and Liu 1985)	7.4909	–
100	Present	5.9355	4.6156
	HSDT (Mantari *et al*. 2012)	5.9452	4.6159
	HSDT (Mantari *et al*. 2011)	6.1147	4.6182
	HSDT (Touratier 1991)	6.2319	4.6195
	HSDT (Reddy and Liu 1985)	6.2976	–
	FSDT (Reddy and Liu 1985)	7.4940	–
Plate	Present	5.9366	4.6988
	HSDT (Mantari *et al*. 2012)	5.9463	4.6992
	HSDT (Mantari *et al*. 2011)	6.1158	4.7014
	HSDT (Touratier 1991)	6.2330	4.7028
	HSDT (Reddy and Liu 1985)	6.2987	–
	FSDT (Reddy and Liu 1985)	7.4853	–

by Mantari *et al*. (2011), LM3/4 (referential solution) and ED4D (both from CUF, see Giunta *et al*. (2011)), and the well-known trigonometric HSDT (Touratier 1991).

It can be seen from Tables 1 and 2 that results demonstrate a good agreement with other theories proposed for comparison. The difference between the solutions predicted by FSDT (Reddy and Liu 1985) and the rest of HSDTs diminishes with the increasing value of R/a ratio for thick shells. However, opposite occurs for thin shells. In fact, in the case of thin shells the FSDT appears to be enough.

Table 3 shows the results of cross-ply spherical shells under point load. The computed results are compared with the FSDT and HSDT obtained from Reddy and Liu (1985) for different values of R/a. The results are also compared with the HSDT developed by Mantari *et al*. (2011 and 2012) and the well-known trigonometric HSDT (Touratier 1991).

Again, it can be observed that the obtained results are in good agreement with those predicted by Mantari (Mantari *et al*. 2012). As is well-known, for this type of load at the center, the difference between the results provided by the FSDT and the HSDTs is more significant as pointed out by Reddy and Liu (1985).

Figs. 2-4 present the variation of non-dimensional center deflections under sinusoidal, uniform, and point loads for thick (a/h = -5) anti-symmetric cross-ply cylindrical shells with R/a. Comparisons between the proposed HSDT and both Mantari's HSDT (Mantari *et al*. 2012) and Touratier's HSDT (Touratier 1991) are demonstrated.

It can be noticed, from these figures that the proposed theory agree well with the theory developed by Mantari (Mantari *et al*. 2012). However, a visible difference in the prediction of the non-dimensional transverse center deflection is found when results are compared to Touratier's HSDT (Touratier 1991), which is the clear influence of the different HSDTs employed, i.e., the type of shear strain functions employed in the modeling of the kinematic of the theory.

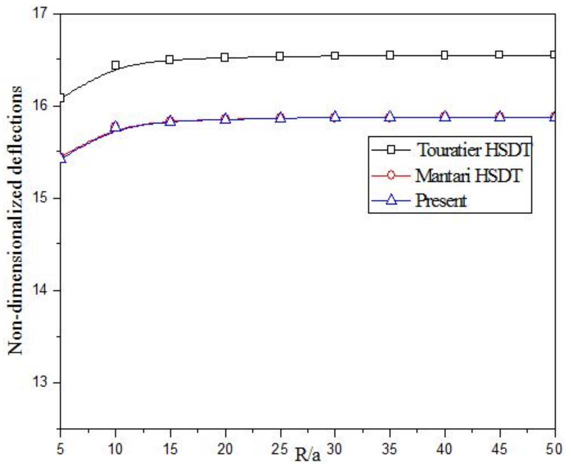

Fig. 2 Variation of normalized transverse center deflection with R/a ratio for different HSDTs of cross-ply laminated spherical shells [0/90] under sinusoidal load

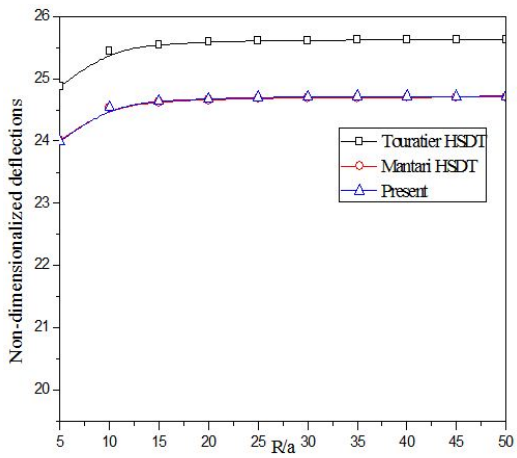

Fig. 3 Variation of normalized transverse center deflection with R/a ratio for different HSDTs of cross-ply laminated spherical shells [0/90] under uniform load

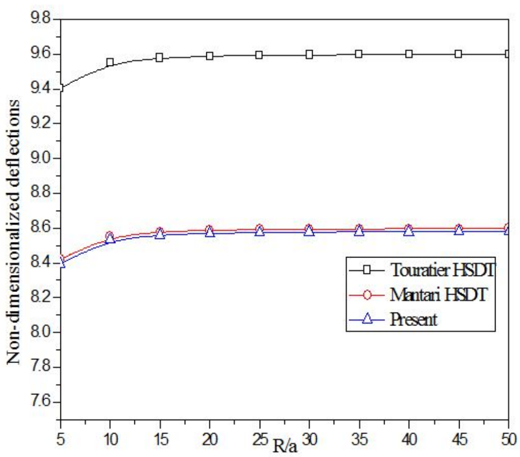

Fig. 4 Variation of normalized transverse center deflection with R/a ratio for different HSDTs of cross-ply laminated spherical shells [0/90] under point load

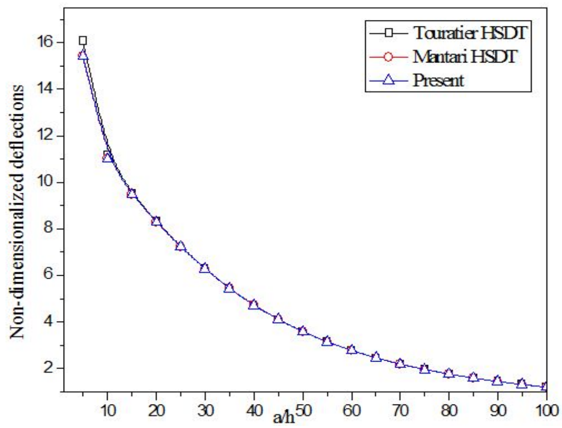

Fig. 5 Variation of normalized transverse center deflection with a/h ratio for different HSDTs of cross-ply laminated spherical shells [0/90] under sinusoidal load

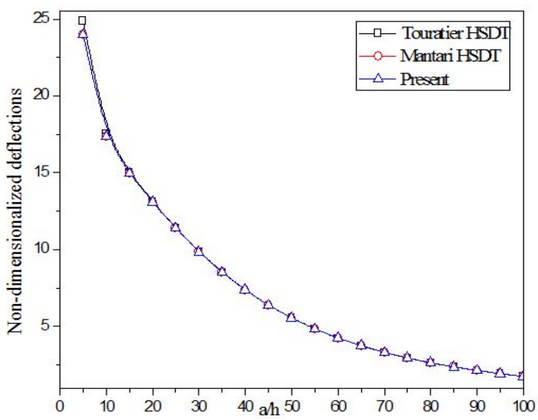

Fig. 6 Variation of normalized transverse center deflection with a/h ratio for different HSDTs of cross-ply laminated spherical shells [0/90] under uniform load

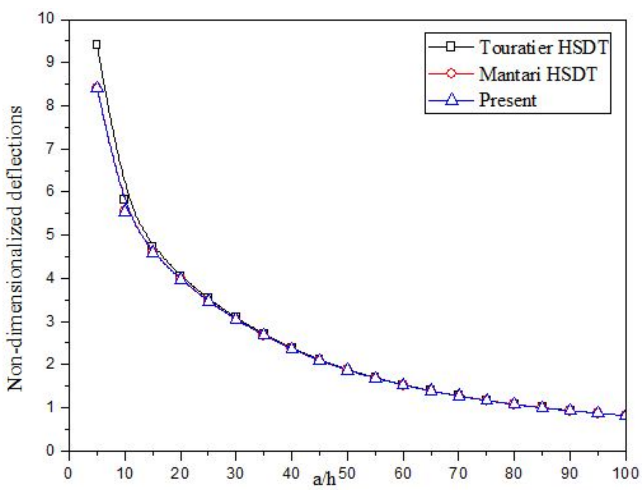

Fig. 7 Variation of normalized transverse center deflection with a/h ratio for different HSDTs of cross-ply laminated spherical shells [0/90] under point load

Table 4 Normalized fundamental frequencies of cross-ply two layer (0/90) square laminated cylindrical shells $\Omega = \omega(a^2/h)\sqrt{\rho / E_2}$

R/a	Theory	$a/h = 10$	$a/h = 100$
5	Present	9.1637	16.7042
	HSDT (Mantari *et al*. 2012)	9.1624	16.7033
	HSDT (Mantari *et al*. 2011)	9.1254	16.7030
	HSDT (Touratier 1991)	–	–
	HSDT (Reddy and Liu 1985)	9.0230	16.6900
	FSDT (Reddy and Liu 1985)	8.9082	16.6680
10	Present	9.0828	11.8447
	HSDT (Mantari *et al*. 2012)	9.0825	11.8444
	HSDT (Mantari *et al*. 2011)	9.0453	11.8440
	HSDT (Touratier 1991)	–	–
	HSDT (Reddy and Liu 1985)	8.9790	11.8400
	FSDT (Reddy and Liu 1985)	8.8879	11.8310
20	Present	9.0580	10.2712
	HSDT (Mantari *et al*. 2012)	9.0579	10.2712
	HSDT (Mantari *et al*. 2011)	9.0207	10.2707
	HSDT (Touratier 1991)	–	–
	HSDT (Reddy and Liu 1985)	8.9720	10.2700
	FSDT (Reddy and Liu 1985)	8.8900	10.2650
50	Present	9.0481	9.7848
	HSDT (Mantari *et al*. 2012)	9.0481	9.7848
	HSDT (Mantari *et al*. 2011)	9.0109	9.7843
	HSDT (Touratier 1991)	–	–
	HSDT (Reddy and Liu 1985)	8.9730	9.7830
	FSDT (Reddy and Liu 1985)	8.8951	9.7816
100	Present	9.0457	9.7132
	HSDT (Mantari *et al*. 2012)	9.0457	9.7132
	HSDT (Mantari *et al*. 2011)	9.0085	9.7127
	HSDT (Touratier 1991)	–	–
	HSDT (Reddy and Liu 1985)	8.9750	9.7120
	FSDT (Reddy and Liu 1985)	8.8974	9.7108
Plate	Present	9.0437	9.6890
	HSDT (Mantari *et al*. 2012)	9.0437	9.6890
	HSDT (Mantari *et al*. 2011)	9.0065	9.6886
	HSDT (Touratier 1991)	–	–
	HSDT (Reddy and Liu 1985)	8.9760	9.6880
	FSDT (Reddy and Liu 1985)	8.8998	9.6873

Figs. 5-7 present the variation of non-dimensional center deflections for cross-ply cylindrical shells under sinusoidal, uniform, and point loads for deep (R/a = 5) anti-symmetric cross-ply laminates with thickness a/h ratio. Again, comparisons between the well-known trigonometric HSDT (Touratier 1991), Mantari's HSDT (Mantari *et al*. 2012) and the proposed HSDT are demonstrated.

It can be seen from these results that there is no significant difference between the results produced by the three HSDTs.

4.2 Vibration analysis

The same mechanical characteristics of the laminate in the previous example problem are employed in this section. Table 4 presents the non-dimensional fundamental frequencies of cross-ply cylindrical shells. For thin anti-symmetric cross-ply shells, the FSDT (Reddy and Liu 1985) underpredicts the fundamental natural frequencies when compared with the rest HSDTs (Reddy and Liu 1985, Mantari *et al*. 2011, 2012).

From Table 4, it can be noticed that the proposed theory agree well with Mantari's HSDT (Mantari *et al*. 2012).

5. Conclusions

A new simplified HSDT is proposed for bending and dynamics analysis of composite plates and shells. By making further simplifying assumptions to the existing HSDTs, with the introduction of an undetermined integral term, the number of unknowns and equations of motion of the proposed HSDT are reduced by one, and hence, make the this theory simple and efficient to use. The model considers adequate variation of the transverse shear strains within the plate thickness and tangential stress-free boundary conditions on the shell boundary surface; therefore a shear correction factor is not needed. The equations of motion are obtained by using the Hamilton's principle. These equations are then solved via a Navier-type, closed form solution. Bending and dynamic results are provided for cylindrical and spherical shells. Shells and plates are subjected to sinusoidal, distributed and point loads. The accuracy of the proposed formulation is ascertained by comparing the obtained results with those of other HSDTs available in the literature. The results demonstrate that the proposed model performs better than all the HSDTs compared here for investigating the bending and vibration behavior of multilayered composite plates and shells.

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