Steel and Composite Structures, *Vol. 25, No. 6 (2017) 735-745*
DOI: https://doi.org/10.12989/scs.2017.25.6.735

An original HSDT for free vibration analysis of functionally graded plates

Imene Ait Sidhoum[*1], Djilali Boutchicha[1], Samir Benyoucef[2] and Abdelouahed Tounsi[2,3]

[1]*Laboratoire de Mécanique Appliqué, Département de Génie Mécanique, Université des Sciences et Technologie d'ORAN Mouhamed Boudiaf, BP 1505 Elmnouar 31000, ORAN, Algeria*
[2]*Material and Hydrology Laboratory, Civil Engineering Department, University of SidiBel Abbes, Faculty of Technology, Algeria*
[3]*Department of Civil and Environmental Engineering, King Fahd University of Petroleum & Minerals, 31261 Dhahran, Eastern Province, Saudi Arabia*

(*Received December 1, 2016, Revised September 17, 2017, Accepted September 21, 2017*)

Abstract. This work presents a free vibration analysis of functionally graded plates by employing an original high order shear deformation theory (HSDT). This theory use only four unknowns, which is even less than the classical HSDT. The equations of motion for the dynamic analysis are determined via the Hamilton's principle. The original kinematic allows obtaining interesting equations of motion. These equations are solved analytically via Navier procedure. The accuracy of the proposed solution is checked by comparing it with other closed form solutions available in the literature.

Keywords: vibration; functionally graded plate; plate theory

1. Introduction

Functionally graded materials (FGMs) are considered as a type of advanced composite material which was originally proposed in Japan (Bever and Duwez 1972, Koizumi 1993). The interesting advantage of this type of composites is the continuity over a desired direction through a structural component (shell, plate or beam) (Kar and Panda 2015, Bennai *et al*. 2015, Belkorissat *et al*. 2015, Ait Atmane *et al*. 2015, Bakora and Tounsi 2015, Barati and Shahverdi 2016, Ahouel *et al*. 2016, Beldjelili *et al*. 2016). Some kinds of conventional composites suffer in continuity within the thickness direction; such discontinuity can be attenuated by a gradual and smooth variation of mechanical characteristics across the thickness of the structural element as in FGMs. Moreover, FGMs allow us to have high thermal and toughness mechanical characteristics, via a mixing for example ceramic and metal.

For the structural applications generally, functionally graded (FG) plates are employed for which the properties change along the thickness. The FG plates find their applications in many engineering industries such as aerospace, nuclear and biomedical. This increase in engineering applications of FGMs has attracted the attention of many scientists. Several research works are found for the structural investigations of FG plates by employing elasticity solution as well as various HSDTs. The exact 3D solution by Kashtalyan (2004), Karami *et al*. (2017) give benchmark for the investigation. Proposing exact solution is a complex process and moreover it is possible only for a few special cases. Hence it is interesting to develop HSDTs which facilitates to propose solution in much simpler way and with reasonable accuracy. Most of the HSDTs are two dimensional and suppose plane stress condition which neglects transverse normal stress. The most fundamental deformation theory is classical plate theory (CPT) which is valid for thin plates, as it does not consider shear deformation. This may lead to inaccurate results for thick plates, which is thicker than 1/20 of its larger span (Chi and Chung 2006). This model has been implemented by several scientists to investigate the mechanical response of FG plates (Abrate 2008, Ghannadpour and Alinia 2006, Cheng and Batra 2000). The problem encountered in CPT was overcome by first order shear deformation theory (FSDT) developed by Reissner (1945), Mindlin (1951) which takes into consideration the influence of transverse shear deformation. According to FSDT, the transverse strains are considered to be uniform within the thickness of plate which is unrealistic and requires a shear correction factor to compensate transverse shear strains on the top and bottom of the plate (AddaBedia *et al*. 2015, Hadji *et al*. 2016, Bouderba *et al*. 2016). Praveen and Reddy (1998), Chinosi and Croce (2007), Singha *et al*. (2011), Alieldin *et al*. (2011), Wen and Aliabadi (2012), Castellazzi *et al*. (2013), Meksi *et al*. (2015), Bellifa *et al*. (2016) employed FSDT for the bending and vibration analysis of FG plates. Abdelbari *et al*. (2016) presented an efficient and simple shear deformation theory for free vibration of FG rectangular plates on Winkler-Pasternak elastic foundations. Abdelhak *et al*. (2016) studied thermal buckling response of FG sandwich plates with clamped boundary conditions. For avoiding the use of shear correction coefficients, several HSDT, such as, the third-order shear deformation theory (TSDT) (Reddy 2000, Hosseini-Hashemi *et al*. 2011, Qian *et al*. 2004, Akavci 2006, Tounsi *et al*. 2013, Zidi *et al*. 2014, Bourada *et al*. 2016, Merdaci *et al*. 2016, Raminnea *et al*. 2016, Fahsi *et al*. 2017), the sinusoidal shear deformation theory (SSDT) (Neves *et al*. 2012, Bouderba *et al*. 2013, Fekrar *et al*. 2014, Ait Amar Meziane *et al*. 2014,

∗Corresponding author, Ph.D.
E-mail: aitsidhoumimene@gmail.com

Copyright © 2017 Techno-Press, Ltd.
http://www.techno-press.org/?journal=scs&subpage=6 ISSN: 1229-9367 (Print), 1598-6233 (Online)

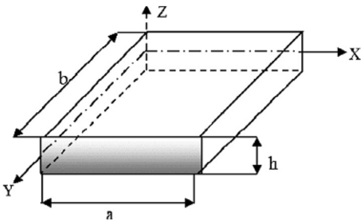

Fig. 1 Geometry of functionally graded plate

Attia *et al*. 2015, AitYahia *et al*. 2015, Hamidi *et al*. 2015, Houari *et al*. 2016, Draiche *et al*. 2016, El-Haina *et al*. 2017, Besseghier *et al*. 2017, Khetir *et al*. 2017), the hyperbolic shear deformation theory (Belabed *et al*. 2014, Hebali *et al*. 2014, Akavci and Tanrikulu 2015, Mahi *et al*. 2015, Saidi *et al*. 2016; Akavci 2016, El-Hassar *et al*. 2016, Mouaici *et al*. 2016, Bennoun *et al*. 2016, Saidi*et al*. 2016, Bousahla *et al*. 2014, 2016, Chikh *et al*. 2017, Abualnour *et al*. 2018), the exponential shear deformation theory (Klouche *et al*. 2017) and the zeroth-order shear deformation theory (Bounouara *et al*. 2016, Bellifa *et al*. 2017a) have been developed. It should be noted that many studies are presented in literature to present HSDTs for composite structures as well as graded CNT structures such as (Mehar *et al*. 2017a, b, c, d, Hirwani *et al*. 2017a, b, c; Mehar and Panda 2017, Kar *et al*. 2016, Sahoo *et al*. 2016a, b, Kar and Panda 2016a, b, Mahapatra and Panda 2016, Mehar and Panda 2016, Mehar *et al*. 2016, Mahapatra *et al*. 2016a, b, Katariya and Panda 2016, Sahoo *et al*. 2017, Singh and Panda 2017, Kar and Panda 2015, Singh and Panda 2015, Mahapatra and Panda 2015, Panda and Katariya 2015, Mahapatra *et al*. 2015, Panda and Mahapatra 2014).

In the present paper, the free vibration analysis of FG plates is investigated. The theory contains only four variables, which is less than others HSDTs. The mechanical properties of the plates are supposed to vary in the thickness direction according to a power law distribution in terms of the volume fractions of the constituents. The interesting plate equations of motion for the free vibration analysis are determined through the Hamilton's principle. These equations are then solved using Navier's procedure. The accuracy of the results of this theory is verified by comparing with other HSDTs available in the literature.

2. Analytical modeling

The geometry of a FG plate is as presented in Fig. 1. The dimensions of the plates are $a\times b\times h$, where 'a' is the length, 'b' is width and 'h' is thickness of the plate. The gradation of material characteristics is in the thickness direction with metal and ceramic being the typical constituents. Aluminum/Alumina (Al/Al_2O_3), Aluminum/ Zirconia (Al/ZrO_2) and Aluminum/ Silicon nitride (AL/Si_3N_4) are the examples of the FG plate.

2.1 Material variation laws

The material properties of FG plate such as the Young's modulus E and the mass density ρ are considered to vary continuously within the thickness of the plate according to the power law variation as follows (Bessaim *et al*. 2013, Meradjah *et al*. 2015, Benbakhti *et al*. 2016, Benchohra *et al*. 2017, Benahmed *et al*. 2017, Bouafia *et al*. 2017)

$$E(z) = E_m + \left(E_c - E_m\right)\left(\frac{1}{2}+\frac{z}{h}\right)^{p_1}, \qquad \rho(z) = \rho_m + \left(\rho_c - \rho_m\right)\left(\frac{1}{2}+\frac{z}{h}\right)^{p_2} \tag{1}$$

Where (E_c, ρ_c) and (E_m, ρ_m) are the corresponding properties of the ceramic and metal, respectively, and ρ_1, ρ_2 are constants. The Poisson's ratio v is considered to be constant and equal to 0.3 throughout the analyses (Bourada *et al*. 2012, Taibi *et al*. 2015, Zemri *et al*. 2015, Bourada *et al*. 2015, Larbi Chaht *et al*. 2015, Laoufi *et al*. 2016, Benferhat *et al*. 2016, Boukhari *et al*. 2016, Chikh *et al*. 2016, Hebali *et al*. 2016, Meksi *et al*. 2017, Mouffoki *et al*. 2017, Zidi *et al*. 2017, Menasria *et al*. 2017). The value of p (p_1 or p_2) equal to zero represents a fully ceramic plate and infinite p, a fully metallic plate. The distribution of the composition of ceramics and metal is linear for p=1. Typical values for metal and ceramics used in the FG plate are listed in Table 1.

Table 1 Material properties used in the FG plate

Properties	Metal	Ceramic		
	Aluminum (Al)	Alumina (Al_2O_3)	Zirconia (ZrO_2)	Silicon nitride (Si_3N_4)
E (GPa)	70	380	200	322.2
ρ (kg/m^3)	2702	3800	5700	2370

2.2 Displacement base field

In this work, further simplifying considerations are madeto the classical HSDT so that the number of variables is reduced. The displacement field of the classical HSDT is given by (Barka *et al*. 2016, Bellifa *et al*. 2017).

$$u(x,y,z,t) = u_0(x,y,t) - z\frac{\partial w_0}{\partial x} + f(z)\varphi_x(x,y,t) \tag{2a}$$

$$v(x,y,z,t) = v_0(x,y,t) - z\frac{\partial w_0}{\partial y} + f(z)\varphi_y(x,y,t) \tag{2b}$$

$$w(x,y,z,t) = w_0(x,y,t) \tag{2c}$$

Where u_0; v_0; w_0, φ_x, φ_y are five unknown displacements of the mid-plane of the plate, $f(z)$ denotes shape function representing the variation of the transverse shear strains and stresses within the thickness. By considering that $\varphi_x = \int \theta(x,y)dx$ and $\varphi_y = \int \theta(x,y)dy$, the displacement field of the present theory can be written in a simpler form as

$$u(x,y,z,t) = u_0(x,y,t) - z\frac{\partial w_0}{\partial x} + k_1\, f(z)\int \theta(x,y,t)\,dx \tag{3a}$$

$$v(x,y,z,t) = v_0(x,y,t) - z\frac{\partial w_0}{\partial y} + k_2\, f(z)\int \theta(x,y,t)\,dy \tag{3b}$$

$$w(x,y,z,t) = w_0(x,y,t) \tag{3c}$$

In this work, the present original HSDT is obtained by setting

$$f(z) = \frac{1}{2} h \tanh\left(\frac{2z}{h}\right) - \frac{4}{3}\frac{z^3}{h^2 \cosh(1)^2} \tag{4}$$

It can be seen that the displacement field in Eq. (3) Introduces only four unknowns (u_0, v_0, w_0 and θ).The constants k_1 and k_2 depends on the geometry.

In this work, the displacement field considers terms with integrals instead of derivatives, as often employed for example in the CPT or many HSDTs. Such strategy can be important since HSDTs with reduced number of variables can be proposed by utilizing the present foundation.

2.3 Kinematic relations and constitutive relations

In the derivation of the necessary equations, small strains are considered (i.e., displacements and rotations are small, and obey Hooke's law). The linear strain relations derived from the displacement model of Eq. (3) are as follows

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + f(z) \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g(z) \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix}, \tag{5}$$

Where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial x} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial^2 w_0}{\partial y^2} \\ -2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix},$$

$$\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} k_1\theta \\ k_2\theta \\ k_1 \dfrac{\partial}{\partial y}\int \theta\, dx + k_2 \dfrac{\partial}{\partial x}\int \theta\, dy \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} k_2 \int \theta\, dy \\ k_1 \int \theta\, dx \end{Bmatrix}, \tag{6a}$$

And

$$g(z) = \frac{df(z)}{dz} \tag{6b}$$

The analytical solution of this theory can be solved by a Navier type solution. The following relations can be obtained

$$\frac{\partial}{\partial y}\int \theta\, dx = A' \frac{\partial^2 \theta}{\partial x \partial y}, \qquad \frac{\partial}{\partial x}\int \theta\, dy = B' \frac{\partial^2 \theta}{\partial x \partial y},$$

$$\int \theta\, dx = A' \frac{\partial \theta}{\partial x}, \quad \int \theta\, dy = B' \frac{\partial \theta}{\partial y} \tag{7}$$

Where the coefficients A' and B' (defined according to the type of solution adopted), k_1 and k_2 are expressed as follows

$$A' = -\frac{1}{\alpha^2}, \quad B' = -\frac{1}{\beta^2}, \quad k_1 = \alpha^2, k_2 = \beta^2 \tag{8}$$

Note that α and β are terms related to the Navier type solution defined in Eq. (23).

For the FG plates, the stress-strain relation- ships can be written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{66} & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & C_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{9}$$

where (σ_x, σ_y, τ_{xy}, τ_{yz}, τ_{xz}) and (ε_x, ε_y, γ_{xy}, γ_{yz}, γ_{xz}) are the stress and strain components, respectively. Using the material properties defined in Eq. (1), stiffness coefficients, C_{ij}, can be given as

$$C_{11} = C_{22} = \frac{E(z)}{1-\nu^2}, \; C_{12} = \frac{\nu E(z)}{1-\nu^2}, \quad C_{44} = C_{55} = C_{66} = \frac{E(z)}{2(1+\nu)}, \tag{10}$$

2.3 Equations of motion

Hamilton's principle is employed for the free vibration problem and defined as follows (Hanifi Hachemi Amar *et al*. 2017)

$$0 = \int_0^t (\delta U - \delta K)\, dt \tag{11}$$

Where δU is the variation of strain energy; and δK is the variation of kinetic energy.

The variation of strain energy of the plate is given by

$$\begin{aligned} \delta U &= \int_V \left[\sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \tau_{xy}\delta\gamma_{xy} + \tau_{yz}\delta\gamma_{yz} + \tau_{xz}\delta\gamma_{xz}\right] dV \\ &= \int_A \left[N_x \delta\varepsilon_x^0 + N_y \delta\varepsilon_y^0 + N_{xy}\delta\gamma_{xy}^0 + M_x^b \delta k_x^b + M_y^b \delta k_y^b + M_{xy}^b \delta k_{xy}^b \right. \\ &\quad \left. + M_x^s \delta k_x^s + M_y^s \delta k_y^s + M_{xy}^s \delta k_{xy}^s + S_{yz}^s \delta\gamma_{yz}^s + S_{xz}^s \delta\gamma_{xz}^0\right] dA = 0 \end{aligned} \tag{12}$$

Where A is the top surface and the stress resultants N, M, and S are defined by

$$\left(N_i, M_i^b, M_i^s\right) = \int_{-h/2}^{h/2} (1, z, f)\sigma_i\, dz, \quad (i = x, y, xy) \quad \text{and} \quad \left(S_{xz}^s, S_{yz}^s\right) = \int_{-h/2}^{h/2} g\left(\tau_{xz}, \tau_{yz}\right) dz \tag{13}$$

The variation of kinetic energy of the plate can be written as

$$
\begin{aligned}
\delta K &= \int_V \left[\dot{u}\,\delta\dot{u} + \dot{v}\,\delta\dot{v} + \dot{w}\,\delta\dot{w}\right]\rho(z)\,dV \\
&= \int_A \Big\{ I_0\left[\dot{u}_0\delta\dot{u}_0 + \dot{v}_0\delta\dot{v}_0 + \dot{w}_0\delta\dot{w}_0\right] \\
&- I_1\left(\dot{u}_0\frac{\partial\delta\dot{w}_0}{\partial x} + \frac{\partial\dot{w}_0}{\partial x}\delta\dot{u}_0 + \dot{v}_0\frac{\partial\delta\dot{w}_0}{\partial y} + \frac{\partial\dot{w}_0}{\partial y}\delta\dot{v}_0\right) \\
&+ J_1\left((k_1 A')\left(\dot{u}_0\frac{\partial\delta\dot{\theta}}{\partial x} + \frac{\partial\dot{\theta}}{\partial x}\delta\dot{u}_0\right) + (k_2 B')\left(\dot{v}_0\frac{\partial\delta\dot{\theta}}{\partial y} + \frac{\partial\dot{\theta}}{\partial y}\delta\dot{v}_0\right)\right) \\
&+ I_2\left(\frac{\partial\dot{w}_0}{\partial x}\frac{\partial\delta\dot{w}_0}{\partial x} + \frac{\partial\dot{w}_0}{\partial y}\frac{\partial\delta\dot{w}_0}{\partial y}\right) \\
&+ K_2\left((k_1 A')^2\left(\frac{\partial\dot{\theta}}{\partial x}\frac{\partial\delta\dot{\theta}}{\partial x}\right) + (k_2 B')^2\left(\frac{\partial\dot{\theta}}{\partial y}\frac{\partial\delta\dot{\theta}}{\partial y}\right)\right) \\
&- J_2\left((k_1 A')\left(\frac{\partial\dot{w}_0}{\partial x}\frac{\partial\delta\dot{\theta}}{\partial x} + \frac{\partial\dot{\theta}}{\partial x}\frac{\partial\delta\dot{w}_0}{\partial x}\right) + (k_2 B')\left(\frac{\partial\dot{w}_0}{\partial y}\frac{\partial\delta\dot{\theta}}{\partial y} + \frac{\partial\dot{\theta}}{\partial y}\frac{\partial\delta\dot{w}_0}{\partial y}\right)\right)\Big\} dA
\end{aligned}
\tag{14}
$$

Where dot superscript convention indicates the differentiation with respect to the time variable t; and (I_i, J_i, K_i) are mass inertias expressed by

$$(I_0, I_1, I_2) = \int_{-h/2}^{h/2} (1, z, z^2)\rho(z)dz \tag{15a}$$

$$(J_1, J_2, K_2) = \int_{-h/2}^{h/2} (f, z\,f, f^2)\rho(z)dz \tag{15b}$$

Employing the displacement strain relations (Eqs. (5) and (6)) and stress strain relations (Eq. (9)); applying integrating by parts and the fundamental lemma of variational calculus; and collecting the coefficients of δu_0, δv_0, δw_0, $\delta\theta$ in Eq. (11), the equations of motion are obtained

$$
\begin{aligned}
&\delta u_0: \ \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0\ddot{u}_0 - I_1\frac{\partial\ddot{w}_0}{\partial x} + k_1 A' J_1\frac{\partial\ddot{\theta}}{\partial x} \\
&\delta v_0: \ \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_0\ddot{v}_0 - I_1\frac{\partial\ddot{w}_0}{\partial y} + k_2 B' J_1\frac{\partial\ddot{\theta}}{\partial y} \\
&\delta w_0: \ \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x\partial y} + \frac{\partial^2 M_y^b}{\partial y^2} = I_0\ddot{w}_0 + I_1\left(\frac{\partial\ddot{u}_0}{\partial x} + \frac{\partial\ddot{v}_0}{\partial y}\right) - I_2\nabla^2\ddot{w}_0 \\
&\qquad + J_2\left(k_1 A'\frac{\partial^2\ddot{\theta}}{\partial x^2} + k_2 B'\frac{\partial^2\ddot{\theta}}{\partial y^2}\right) \\
&\delta\theta: -\mathrm{k}_1 M_x^s - \mathrm{k}_2 M_y^s - (k_1 A' + k_2 B')\frac{\partial^2 M_{xy}^s}{\partial x\partial y} + k_1 A'\frac{\partial S_{xz}^s}{\partial x} + k_2 B'\frac{\partial S_{yz}^s}{\partial y} \\
&\quad = -J_1\left(k_1 A'\frac{\partial\ddot{u}_0}{\partial x} + k_2 B'\frac{\partial\ddot{v}_0}{\partial y}\right) - K_2\left((k_1 A')^2\frac{\partial^2\ddot{\theta}}{\partial x^2} + (k_2 B')^2\frac{\partial^2\ddot{\theta}}{\partial y^2}\right) \\
&\quad + J_2\left(k_1 A'\frac{\partial^2\ddot{w}_0}{\partial x^2} + k_2 B'\frac{\partial^2\ddot{w}_0}{\partial y^2}\right)
\end{aligned}
\tag{16}
$$

Substituting Eq. (5) into Eq. (9) and the subsequent results into Eqs. (13), the stress resultants are obtained in terms of strains as following compact form

$$\begin{Bmatrix} N \\ M^b \\ M^s \end{Bmatrix} = \begin{bmatrix} A & B & B^s \\ B & D & D^s \\ B^s & D^s & H^s \end{bmatrix} \begin{Bmatrix} \varepsilon \\ k^b \\ k^s \end{Bmatrix}, \quad S = A^s\gamma, \tag{17}$$

In which

$$N = \{N_x, N_y, N_{xy}\}^t,\ M^b = \{M_x^b, M_y^b, M_{xy}^b\}^t,\ M^s = \{M_x^s, M_y^s, M_{xy}^s\}^t, \tag{18a}$$

$$\varepsilon = \{\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0\}^t,\quad k^b = \{k_x^b, k_y^b, k_{xy}^b\}^t, \quad k^s = \{k_x^s, k_y^s, k_{xy}^s\}^t, \tag{18b}$$

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix},\quad B = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix},\quad D = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix}, \tag{18c}$$

$$B^s = \begin{bmatrix} B_{11}^s & B_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & B_{66}^s \end{bmatrix},\quad D^s = \begin{bmatrix} D_{11}^s & D_{12}^s & 0 \\ D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & D_{66}^s \end{bmatrix},\quad H^s = \begin{bmatrix} H_{11}^s & H_{12}^s & 0 \\ H_{12}^s & H_{22}^s & 0 \\ 0 & 0 & H_{66}^s \end{bmatrix}, \tag{18d}$$

$$S = \{S_{xz}^s, S_{yz}^s\}^t,\quad \gamma = \{\gamma_{xz}^0, \gamma_{yz}^0\}^t,\quad A^s = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix}, \tag{18e}$$

And stiffness components are expressed as

$$\begin{Bmatrix} A_{11} & B_{11} & D_{11} & B_{11}^s & D_{11}^s & H_{11}^s \\ A_{12} & B_{12} & D_{12} & B_{12}^s & D_{12}^s & H_{12}^s \\ A_{66} & B_{66} & D_{66} & B_{66}^s & D_{66}^s & H_{66}^s \end{Bmatrix} = \int_{-h/2}^{h/2} C_{11}(1, z, z^2, f(z), z\,f(z), f^2(z)) \begin{Bmatrix} 1 \\ \nu \\ \dfrac{1-\nu}{2} \end{Bmatrix} dz \tag{19a}$$

$$(A_{22}, B_{22}, D_{22}, B_{22}^s, D_{22}^s, H_{22}^s) = (A_{11}, B_{11}, D_{11}, B_{11}^s, D_{11}^s, H_{11}^s) \tag{19b}$$

$$A_{44}^s = A_{55}^s = \int_{-h/2}^{h/2} C_{44}[g(z)]^2\,dz, \tag{19c}$$

By substituting Eq. (17) into Eq. (16), the equations of motion can be expressed in terms of displacements (u_0, v_0, w_0, θ) and the appropriate equations take the form

$$
\begin{aligned}
&A_{11}d_{11}u_0 + A_{66}d_{22}u_0 + (A_{12} + A_{66})d_{12}v_0 - B_{11}d_{111}w_0 \\
&\quad - (B_{12} + 2B_{66})d_{122}w_0 + \left(B_{66}^s(k_1 A' + k_2 B')\right)d_{122}\theta \\
&\quad + \left(B_{11}^s k_1 + B_{12}^s k_2\right)d_1\theta = I_0\ddot{u}_0 - I_1 d_1\ddot{w}_0 + J_1 A' k_1 d_1\ddot{\theta},
\end{aligned}
\tag{20a}
$$

$$
\begin{aligned}
&A_{22}d_{22}v_0 + A_{66}d_{11}v_0 + (A_{12} + A_{66})d_{12}u_0 - B_{22}d_{222}w_0 \\
&\quad - (B_{12} + 2B_{66})d_{112}w_0 + \left(B_{66}^s(k_1 A' + k_2 B')\right)d_{112}\theta \\
&\quad + \left(B_{22}^s k_2 + B_{12}^s k_1\right)d_2\theta = I_0\ddot{v}_0 - I_1 d_2\ddot{w}_0 + J_1 B' k_2 d_2\ddot{\theta},
\end{aligned}
\tag{20b}
$$

$$
\begin{aligned}
&B_{11}d_{111}u_0 + (B_{12} + 2B_{66})d_{122}u_0 + (B_{12} + 2B_{66})d_{112}v_0 \\
&\quad + B_{22}d_{222}v_0 - D_{11}d_{1111}w_0 - 2(D_{12} + 2D_{66})d_{1122}w_0 \\
&\quad - D_{22}d_{2222}w_0 + \left(D_{11}^s k_1 + D_{12}^s k_2\right)d_{11}\theta \\
&\quad + 2\left(D_{66}^s(k_1 A' + k_2 B')\right)d_{1122}\theta + \left(D_{12}^s k_1 + D_{22}^s k_2\right)d_{22}\theta \\
&= I_0\ddot{w}_0 + I_1(d_1\ddot{u}_0 + d_2\ddot{v}_0) - I_2(d_{11}\ddot{w}_0 + d_{22}\ddot{w}_0) \\
&\quad + J_2\left(k_1 A' d_{11}\ddot{\theta} + k_2 B' d_{22}\ddot{\theta}\right)
\end{aligned}
\tag{20c}
$$

Table 2 Comparison of fundamental frequency parameter $\Omega = \omega h\sqrt{\rho_c / E_c}$ simply supported Al/Al$_2$O$_3$ square plates

a/h	Theories	Power indices (*p*)					
		0	0.5	1	4	10	∞
20	Benachour *et al*. (2011)	0.01480	0.01254	0.01130	0.00980	0.00940	-
	Hosseini-Hashemi *et al*. (2010)	0.01480	0.01281	0.01150	0.01013	0.00963	-
	Zhao *et al*. (2009)	0.01464	0.01241	0.01118	0.00970	0.00931	-
	Present	0.01480	0.01254	0.01130	0.00980	0.00940	-
10	Benachour *et al*. (2011)	0.05769	0.04900	0.04417	0.03804	0.03635	0.02936
	Matsunaga (2008)	0.05777	0.04917	0.04427	0.03811	0.03642	0.02933
	Hosseini-Hashemi *et al*. (2010)	0.05769	0.04920	0.04454	0.03825	0.03627	0.02936
	Zhao *et al*. (2009)	0.05673	0.04818	0.04346	0.03757	0.03591	-
	Matsunaga (2008)	0.06382	0.05429	0.04889	0.04230	0.04047	-
	Present	0.05769	0.04900	0.04417	0.03804	0.03635	0.02936
5	Benachour *et al*. (2011)	0.2112	0.1806	0.1628	0.1375	0.1300	0.1075
	Matsunaga (2008)	0.2121	0.1819	0.1640	0.1383	0.1306	0.1077
	Hosseini-Hashemi *et al*. (2010)	0.2112	0.1806	0.1650	0.1371	0.1304	0.1075
	Zhao *et al*. (2009)	0.2055	0.1757	0.1587	0.1356	0.1284	-
	Matsunaga (2008)	0.2334	0.1997	0.1802	0.1543	0.1462	-
	Present	0.2112	0.1806	0.1628	0.1375	0.1300	0.1075

$$\begin{aligned}
&-\left(B_{11}^s k_1 + B_{12}^s k_2\right)d_1 u_0 - \left(B_{66}^s\left(k_1 A' + k_2 B'\right)\right)d_{122}u_0 \\
&\quad -\left(B_{66}^s\left(k_1 A' + k_2 B'\right)\right)d_{112}v_0 - \left(B_{12}^s k_1 + B_{22}^s k_2\right)d_2 v_0 \\
&\quad +\left(D_{11}^s k_1 + D_{12}^s k_2\right)d_{11}w_0 + 2\left(D_{66}^s\left(k_1 A' + k_2 B'\right)\right)d_{1122}w_0 \\
&\quad +\left(D_{12}^s k_1 + D_{22}^s k_2\right)d_{22}w_0 - H_{11}^s k_1^2\,\theta - H_{22}^s\,k_2^2\,\theta \\
&\quad -2H_{12}^s k_1 k_2\theta - \left(\left(k_1 A' + k_2 B'\right)^2 H_{66}^s\right)d_{1122}\theta \\
&\quad +A_{44}^s\left(k_2 B'\right)^2 d_{22}\theta + A_{55}^s\left(k_1 A'\right)^2 d_{11}\theta \\
&= -J_1\left(k_1 A' d_1\ddot{u}_0 + k_2 B' d_2\ddot{v}_0\right) + J_2\left(k_1 A' d_{11}\ddot{w}_0 + k_2 B' d_{22}\ddot{w}_0\right) \\
&\quad -K_2\left(\left(k_1 A'\right)^2 d_{11}\ddot{\theta} + \left(k_2 B'\right)^2 d_{22}\ddot{\theta}\right)
\end{aligned} \tag{20d}$$

Where d_{ij}, d_{ijl} and d_{ijlm} are the following differential operators

$$d_{ij} = \frac{\partial^2}{\partial x_i \partial x_j}, \quad d_{ijl} = \frac{\partial^3}{\partial x_i \partial x_j \partial x_l},$$
$$d_{ijlm} = \frac{\partial^4}{\partial x_i \partial x_j \partial x_l \partial x_m}, \quad d_i = \frac{\partial}{\partial x_i}, \quad (i, j, l, m = 1,2). \tag{21}$$

3. Solution procedure

For the analytical solution of the partial differential equations (Eq. (20)), the Navier's procedure is used. Using this method, the solution of the displacement variables satisfying the boundary conditions can be written in the following Fourier series

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn} e^{i\omega t}\cos(\alpha\, x)\sin(\beta\, y) \\ V_{mn} e^{i\omega t}\sin(\alpha\, x)\cos(\beta\, y) \\ W_{mn} e^{i\omega t}\sin(\alpha\, x)\sin(\beta\, y) \\ X_{mn} e^{i\omega t}\sin(\alpha\, x)\sin(\beta\, y) \end{Bmatrix} \tag{22}$$

Where ω is the frequency of free vibration of the plate, $\sqrt{i} = -1$ the imaginary unit.

Where

$$\alpha = m\pi / a \quad \text{and} \quad \beta = n\pi / b \tag{23}$$

Substituting Eq. (22) in Eq. (20), the following equations are obtained

$$\left(\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & 0 & m_{13} & m_{14} \\ 0 & m_{22} & m_{23} & m_{24} \\ m_{13} & m_{23} & m_{33} & m_{34} \\ m_{14} & m_{24} & m_{34} & m_{44} \end{bmatrix} \right) \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ X_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{24}$$

Where

$$\begin{gathered}
S_{11} = -\left(A_{11}\alpha^2 + A_{66}\beta^2\right), \quad S_{12} = -\alpha\beta\left(A_{12} + A_{66}\right), \\
S_{13} = \alpha\left(B_{11}\alpha^2 + B_{12}\beta^2 + 2B_{66}\beta^2\right), \\
S_{14} = \alpha\left(k_1 B_{11}^s + k_2 B_{12}^s - \left(k_1 A' + k_2 B'\right)B_{66}^s \beta^2\right), \\
S_{22} = -\left(A_{66}\alpha^2 + A_{22}\beta^2\right), \quad S_{23} = \beta\left(B_{22}\beta^2 + B_{12}\alpha^2 + 2B_{66}\alpha^2\right), \\
S_{24} = \beta\left(k_2 B_{22}^s + k_1 B_{12}^s - \left(k_1 A' + k_2 B'\right)B_{66}^s \alpha^2\right) \\
S_{33} = -\left(D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4\right), \\
S_{34} = -k_1\left(D_{11}^s\alpha^2 + D_{12}^s\beta^2\right) + 2(k_1 A' + k_2 B')\,D_{66}^s\alpha^2\beta^2, \\
-k_2\left(D_{22}^s\beta^2 + D_{12}^s\alpha^2\right) \\
S_{44} = -k_1\left(H_{11}^s k_1 + H_{12}^s k_2\right) - (k_1 A' + k_2 B')^2 H_{66}^s\alpha^2\beta^2 \\
-k_2\left(H_{12}^s k_1 + H_{22}^s k_2\right) - \left(k_1 A'\right)^2 A_{55}^s\alpha^2 - \left(k_2 B'\right)^2 A_{44}^s\beta^2
\end{gathered}$$

$$\begin{gathered}
m_{11} = -I_0, \quad m_{13} = \alpha\, I_1, \quad m_{14} = -J_1 k_1 A'\alpha, \quad m_{22} = -I_0, \\
m_{23} = \beta\, I_1, m_{24} = -k_2 B'\,\beta\, J_1, m_{33} = -I_0 - I_2(\alpha^2 + \beta^2) \\
m_{34} = J_2\left(k_1 A'\alpha^2 + k_2 B'\beta^2\right), \\
m_{44} = -K_2\left(\left(k_1 A'\right)^2\alpha^2 + \left(k_2 B'\right)^2\beta^2\right)
\end{gathered} \tag{25}$$

4. Numerical results and discussions

In this section, various numerical examples are presented and discussed for checking the accuracy of the present HSDT in predicting the dynamic behaviors of simply supported FG plates. For the verification purpose,

Table 3 Comparison of fundamental frequency parameter ($\Omega = \omega a^2 \sqrt{\rho_c / E_c} / h$) for simply supported square FG plates when *h/a*=0.1

	FGMs Theories	Power indices (*p*)						
		0	0.5	1	2	5	8	10
Al/Al_2O_3	Benachour *et al*. (2011)	5.7694	4.9000	4.4166	4.0057	3.7660	3.6831	3.6357
	Hosseini-Hashemi *et al*. (2010)	5.7693	4.9207	4.4545	4.0063	3.7837	3.6830	3.6277
	Zhao *et al*. (2009)	5.6763	4.8209	4.3474	3.9474	3.7218	3.6410	3.5923
	Present	5.7694	4.9000	4.4166	4.0057	3.7660	3.6831	3.6357
Al/ZrO_2	Benachour *et al*. (2011)	5.7694	5.4380	5.3113	5.2923	5.3904	5.3950	5.3783
	Matsunaga (2008)	5.7769	-	5.3216	-	-	-	-
	Hosseini-Hashemi *et al*. (2010)	5.7693	5.3176	5.2532	5.3084	5.2940	5.2312	5.1893
	Zhao *et al*. (2009)	5.6763	5.1105	4.8713	4.6977	4.5549	4.4741	4.4323
	Present	5.7694	5.4380	5.3113	5.2923	5.3904	5.3950	5.3783

Table 4 Comparison of fundamental frequency parameter $\Omega = \omega h^2 \sqrt{\rho_m / E_m}$ for simply supported square FG plates

Theories	*p*=1			*h/a*=0.2		
	h/a=0.05	*h/a*=0.1	*h/a*=0.2	*p*=2	*p*=3	*p*=5
Benachour *et al*. (2011)	0.0158	0.0618	0.2270	0.2249	0.2255	0.2266
Matsunaga (2008)	0.0158	0.0618	0.2285	0.2264	0.2270	0.2281
Pradyumna and Bandyopadhyay (2008)	0.0157	0.0613	0.2257	0.2237	0.2243	0.2253
Hosseini-Hashemi *et al*. (2010)	0.0158	0.0611	0.2270	0.2249	0.2254	0.2265
Pradyumna and Bandyopadhyay (2008)	0.0162	0.0633	0.2323	0.2325	0.2334	0.2334
Present	0.0158	0.0618	0.2270	0.2249	0.2255	0.2267

the results obtained by the proposed HSDT are compared with the existing data in the literatureand discussed for checking the accuracy of the present HSDT in predicting the dynamic behaviors of simply supported FG plates. For the verification purpose, the results obtained by the proposed HSDT are compared with the existing data in the literature.

Table 2 presented the non-dimensional fundamental frequencies of the simply supported square Al/Al_2O_3 plates for different values of the thickness to length ratios (*h/a*=20; 10 and 5) with p_1=p_2=p=0, 0.5, 1, 4, 10, and ∞. The obtained results are compared with those of Matsunaga (2008), Zhao *et al*. (2009), Hosseini-Hashemi *et al*. (2010), Benachour *et al*. (2011). It should be noted that the results given by Matsunaga (2008) are based on the both FSDT and 2D HSDT; whereas Zhao *et al*. (2009) used the FSDT and employed different values of shear correction factors in their study work. However, the results reported by Hosseini-Hashemi *et al*. (2010) are based on FSDT with a new formula for the shear correction factors. The resultsgiven by Benachour *et al*. (2011) are based on refined plate theory (RPT). For convenience in comparison, a novel frequency parameter is defined as $\Omega = \omega h \sqrt{\rho_c / E_c}$. From Table 2, it can be seen that the present results are in excellent agreement with those given by the 2D HSDT (Matsunaga 2008), the RPT (Benachour *et al*. 2011) and the FSDT (Hosseini-Hashemi *et al*. 2010) which use a novel shear correction factors. It is worth noting that all results given on the basis of the FSDT (Matsunaga 2008) are inappropriate since the value of shear correction factor is considered to be constant (k^2=1) for any values of thickness to length ratios and power indices. In addition, the influence of truncated power series to approximate displacement, strain components, and in-plane stress (Matsunaga 2008) on these apparent discrepancies cannot be neglected. The results given by the FSDT (Hosseini-Hashemi *et al*. 2010) are also different from those reported by the proposed theory and by the 2D HSDT (Matsunaga 2008), the RPT (Benachour *et al*. 2011) and the FSDT (Hosseini-Hashemi *et al*. 2010). Another reason of this difference is due to the fact that Zhao *et al*. (2009) utilized a numerical solution (element-free *kp*-Ritz method) to determine the natural frequencies of the FG plates.

Table 3 shows the comparison of present non-dimensional fundamental frequencies with those obtained with FSDT by Zhao *et al*. (2009) and with higher order theories by Hosseini-Hashemi *et al*. (2010) and Benachour *et al*. (2011) for Al/Al_2O_3 and Al/ZrO_2 squareplates square plate with span to thickness ratio10.

From Table 3, it can be seen that the results of FSDT (Hosseini-Hashemi *et al*. 2010) are in good agreement with both the present theory and the theory presented by Benachour *et al*. (2011) comparatively to those obtained by FSDT Zhao *et al*. (2009). This is due to the factthat the shear correction factor is considered to be constant in the FSDT Zhao *et al*. (2009) (k^2=5/6) for any values of power indices, whereas in the FSDT (Hosseini-Hashemi *et al*. 2010) a novel formula for the shear correction factors is supposed taking into account the power indices and the thickness to length ratios. Also, it can be observed that the fundamental frequencies computed by the proposed theory

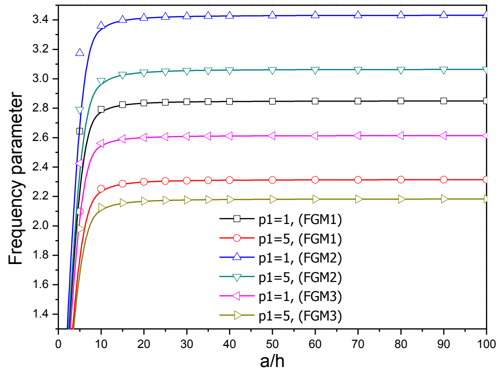

Fig. 2 Variation of frequency parameter with *a/h* ratio and p_1 index (a/b=0.5 and p_2=1)

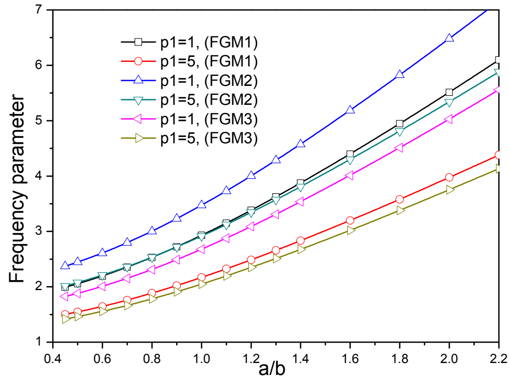

Fig. 4 Variation of frequency parameter with *a/b* ratio and p_1 index. (a/h=2and p_2=1)

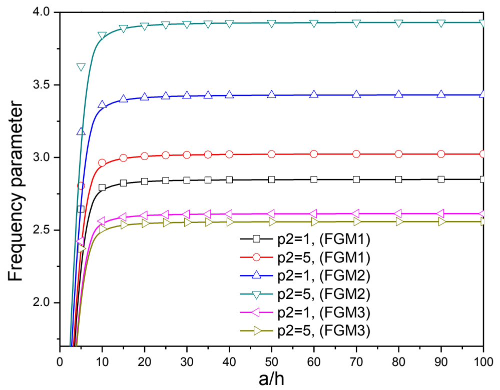

Fig. 3 Variation of frequency parameter with *a/h* ratio and p_2 index. (a/b=0.5 and p_1=1)

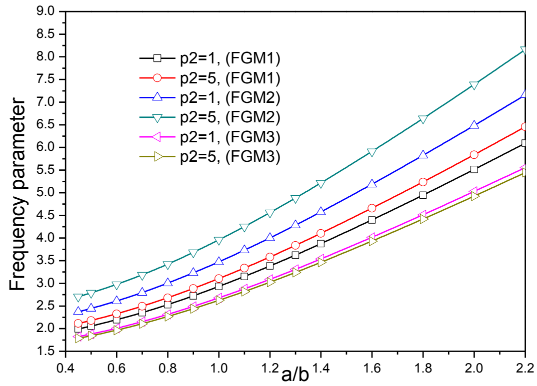

Fig. 5 Variation of frequency parameter with *a/h* ratio and p_1 index. (a/h=2 and p_1=1)

demonstrate also a satisfied agreement with 2D HSDT (Matsunaga 2008) for the Al/ZrO_2 square plate.

In Table 4, we present the non-dimensional fundamental frequencies of the Al/ZrO_2 square plate with simply-supported boundary conditions at four edges for h/a=0.1, 0.2, and $1/\sqrt{10}$ when p_1=p_2=p=0, 1, 2, 3 and 5. For convenience in comparison, the non-dimensional fundamental frequency is defined as $\Omega = \omega h^2 \sqrt{\rho_m / E_m}$. Comparing the obtained results with those reported by the 2D HSDT (Matsunaga 2008), FSDT (Hosseini-Hashemi *et al*. 2010) and HSDT of Pradyumna and Bandyopadhyay (2008) and of Benachour *et al*. (2011) demonstrates that all results are in excellent agreement with each other. It is also observed that the proposed analytical method provides the results lower than those reported by the2D HSDT (Matsunaga 2008) and greater than those given on the basis of the HSDT (Pradyumna and Bandyopadhyay 2008). However, the results computed by the present theory are almost identical to those reported by the FSDT (Hosseini-Hashemi *et al*. 2010) which employ a new shear correction factors. In addition, it should be indicated that the discrepancy between the FSDT (Pradyumna and Bandyopadhyay 2008) and five other theories (i.e., the proposed theory, 2D HSDT (Matsunaga 2008), FSDT (Hosseini-Hashemi *et al*. 2010), and HSDTs of Pradyumna and Bandyopadhyay (2008) and of Benachour *et al*. (2011)) is also considerable.

To make the effects of a/h ratio and power indices more apparent, Figs. 2 and 3 are shown for Aluminum/Alumina (FGM1), Aluminum/Zirconia (FGM2) and Aluminum/ Silicon nitride (FGM3) plates, to show the variation of the non-dimensional fundamental frequency with *a/h* ratio and p_i (i=1,2) power indices, respectively. According to these results the non-dimensional fundamental frequency increases with increasing *a/h* ratio when a/h<20. The non-dimensional frequency is found to be independent of the length-thickness ratio *a/h* for a/h>20. It is shown from Fig. 2 that the effect of p_1 is to make the plate stiffer when this gradient index is reduced. However, decreasing the second power index p_2, makes the plate soften as is presented in Fig. 3. In addition, it is observed that the non-dimensional fundamental frequency is approximately insensitive to p_2 for Aluminum/ Silicon nitride (FGM3) plate.

The dynamic behavior of plate in Ω−*a/b* plane is presented in Figs. 4 and 5 for FGM 1, FGM 2 and FGM 3. It can be seen that the non-dimensional fundamental frequency increases with increasing the aspect ratio *a/b*. It is observed from the results that non-dimensional frequency decreases with increasing p_1 however, it increases with increasing p_2. Again, it can be observed that the non-dimensional frequency is approximately insensitive to p_2 for Aluminum/ Silicon nitride (FGM3) plate.

5. Conclusions

This work presents a free vibration analysis for FG plates by employing an original HSDT with only 4 unknown variables. The nonlinear shear strain function of the theory ensures the accuracy to model the FG plate which depicts non-linear distribution of material characteristics within the thickness. The equations of motion are obtained through the Hamilton's principle. These equations are solved via Navier's procedure. The results were compared with the solutions of several theories. It is concluded that the results of the proposed original HSDT has an excellent agreement with the other theories used for comparison for free vibration problems.

References

Abdelbari, S., Fekrar, A., Heireche, H., Saidi, H., Tounsi, A. and AddaBedia, E.A. (2016), "An efficient and simple shear deformation theory for free vibration of functionally graded rectangular plates on Winkler-Pasternak elastic foundations", *Wind Struct.*, **22**(3), 329-348.

Abdelhak, Z., Hadji, L., HassaineDaouadji, T. and AddaBedia, E.A. (2016), "Thermal buckling response of functionally graded sandwich plates with clamped boundary conditions", *Smart Struct. Syst.*, **18**(2), 267-291.

Abrate, S. (2008), "Functionally graded plates behave like homogeneous plates", *Compos. Part B Eng*., **39**(1), 151-158.

Abualnour, M., Houari, M.S.A., Tounsi, A., AddaBedia, E.A. and Mahmoud, S.R. (2018), "A novel quasi-3D trigonometric plate theory for free vibration analysis of advanced composite plates", *Compos. Struct.*, **184**, 688-697.

AddaBedia, W., Benzair, A., Semmah, A., Tounsi, A. and Mahmoud, S.R. (2015), "On the thermal buckling characteristics of armchair single-walled carbon nanotube embedded in an elastic medium based on nonlocal continuum elasticity", *Brazil. J. Phys*., **45**(2), 225-233.

Ahouel, M., Houari, M.S.A., AddaBedia, E.A. and Tounsi, A. (2016) "Size-dependent mechanical behavior of functionally graded trigonometric shear deformable nanobeams including neutral surface position concept", *Steel Compos. Struct.*, **20**(5), 963-981.

Ait Amar Meziane, M., Abdelaziz, H.H. and Tounsi, A. (2014), "An efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions", *J. Sandw. Struct. Mater.*, **16**(3), 293-318.

AitAtmane, H., Tounsi, A., Bernard, F. and Mahmoud, S.R. (2015), "A computational shear displacement model for vibrational analysis of functionally graded beams with porosities", *Steel Compos. Struct.*, **19**(2), 369-384.

AitYahia, S., AitAtmane, H., Houari, M.S.A. and Tounsi, A. (2015), "Wave propagation in functionally graded plates with porosities using various higher-order shear deformation plate theories", *Struct. Eng. Mech*., **53**(6), 1143-1165.

Akavci, S.S. (2006), "Analysis of shear deformable symmetrically laminated composite plates on elastic foundation", *Sci. Eng. Compos. Mater.*, **13**, 79-91.

Akavci, S.S. (2016), "Mechanical behavior of functionally graded sandwich plates on elastic foundation", *Compos. Part B*, **96**, 136-152.

Akavci, S.S. and Tanrikulu, A.H. (2015), "Static and free vibration analysis of functionally graded plates based on a new quasi-3D and 2D shear deformation theories", *Compos. Part B Eng*., **83**, 203-215.

Alieldin, S.S., Alshorbagy, A.E. and Shaat, M. (2011), "A first-order shear deformation finite element model for elastostatic analysis of laminated composite plates and the equivalent functionally graded plates", *Ain Shams Eng. J.*, **2**, 53-62.

Attia, A., Tounsi, A., AddaBedia, E.A. and Mahmoud, S.R. (2015), "Free vibration analysis of functionally graded plates with temperature-dependent properties using various four variable refined plate theories", *Steel Compos. Struct.*, **18**(1), 187-212.

Bakora, A. and Tounsi, A. (2015), "Thermo-mechanical post-buckling behavior of thick functionally graded plates resting on elastic foundations", *Struct. Eng. Mech*., **56**(1), 85-106.

Barati, M.R. and Shahverdi, H. (2016), "A four-variable plate theory for thermal vibration of embedded FG nanoplates under non-uniform temperature distributions with different boundary conditions", *Struct. Eng. Mech*., **60**(4), 707-727.

Barka, M., Benrahou, K.H., Bakora, A. and Tounsi, A. (2016), "Thermal post-buckling behavior of imperfect temperature-dependent sandwich FGM plates resting on Pasternak elastic foundation", *Steel Compos. Struct.*, **22**(1), 91-112.

Belabed, Z., Houari, M.S.A., Tounsi, A., Mahmoud, S.R. and Anwar Bég, O. (2014), "An efficient and simple higher order shear and normal deformation theory for functionally graded material (FGM) plates", *Compos. Part B*, **60**, 274-283.

Beldjelili, Y., Tounsi, A. and Mahmoud, S.R. (2016), "Hygro-thermo-mechanical bending of S-FGM plates resting on variable elastic foundations using a four-variable trigonometric plate theory", *Smart Struct. Syst.*, **18**(4), 755-786.

Belkorissat, I., Houari, M.S.A., Tounsi, A., AddaBedia, E.A. and Mahmoud, S.R. (2015), "On vibration properties of functionally graded nano-plate using a new nonlocal refined four variable model", *Steel Compos. Struct.*, **18**(4), 1063-1081.

Bellifa, H., Bakora, A., Tounsi, A., Bousahla, A.A. and Mahmoud, S.R. (2017b), "An efficient and simple four variable refined plate theory for buckling analysis of functionally graded plates", *Steel Compos. Struct.*, **25**(3), 257-270.

Bellifa, H., Benrahou, K.H., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2017a), "A nonlocal zeroth-order shear deformation theory for nonlinear postbuckling of nanobeams", *Struct. Eng. Mech*., **62**(6), 695-702.

Bellifa, H., Benrahou, K.H., Hadji, L., Houari, M.S.A. and Tounsi, A. (2016), "Bending and free vibration analysis of functionally graded plates using a simple shear deformation theory and the concept the neutral surface position", *J. Braz. Soc. Mech. Sci. Eng*., **38**(1), 265-275.

Benachour, A., Daouadji, H.T., Ait Atmane, H., Tounsi, A. and Meftah, S.A. (2011), "A four variable refined plate theory for free vibrations of functionally graded plates with arbitrary gradient", *Compos. Part B*, **42**(6), 1386-1394.

Benahmed, A., Houari, M.S.A., Benyoucef, S., Belakhdar, K. and Tounsi, A. (2017), "A novel quasi-3D hyperbolic shear deformation theory for functionally graded thick rectangular plates on elastic foundation", *Geomech. Eng.*, **12**(1), 9-34.

Benbakhti, A., BachirBouiadjra, M., Retiel, N. and Tounsi, A. (2016), "A new five unknown quasi-3D type HSDT for thermomechanical bending analysis of FGM sandwich plates", *Steel Compos. Struct.*, **22**(5), 975-999.

Benchohra, M., Driz, H., Bakora, A., Tounsi, A., AddaBedia, E.A. and Mahmoud, S.R. (2017), "A new quasi-3D sinusoidal shear deformation theory for functionally graded plates", *Struct. Eng. Mech*., (Accepted)

Benferhat, R., Hassaine Daouadji, T., Hadji, L. and Said Mansour, M. (2016), "Static analysis of the FGM plate with porosities", *Steel Compos. Struct.*, **21**(1), 123-136.

Bennai, R., Ait Atmane, H. and Tounsi, A. (2015), "A new higher-order shear and normal deformation theory for functionally graded sandwich beams", *Steel Compos. Struct.*, **19**(3), 521-546.

Bennoun, M., Houari, M.S.A. and Tounsi, A. (2016), "A novel five variable refined plate theory for vibration analysis of functionally graded sandwich plates", *Mech. Adv. Mater. Struct.*, **23**(4), 423-431.

Bessaim, A., Houari, M.S.A., Tounsi, A., Mahmoud, S.R. and AddaBedia, E.A. (2013), "A new higher order shear and normal deformation theory for the static and free vibration analysis of sandwich plates with functionally graded isotropic face sheets", *J. Sandw. Struct. Mater*., **15**, 671-703.

Besseghier, A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2017), "Free vibration analysis of embedded nanosize FG plates using a new nonlocal trigonometric shear deformation theory", *Smart Struct. Syst*., **19**(6), 601-614.

Bever, M.B. and Duwez, P.E. (1972), "Gradients in composite materials", *Mater. Sci. Eng*., **10**, 1-8.

Bouafia, K., Kaci, A., Houari, M.S.A., Benzair, A. and Tounsi, A. (2017), "A nonlocal quasi-3D theory for bending and free flexural vibration behaviors of functionally graded nanobeams", *Smart Struct. Syst*., **19**(2), 115-126.

Bouderba, B., Houari, M.S.A. and Tounsi, A. (2013), "Thermomechanical bending response of FGM thick plates resting on Winkler-Pasternak elastic foundations", *Steel Compos. Struct*., **14**(1), 85-104.

Bouderba, B., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2016), "Thermal stability of functionally graded sandwich plates using a simple shear deformation theory", *Struct. Eng. Mech*., **58**(3), 397-422.

Boukhari, A., AitAtmane, H., Tounsi, A., AddaBedia, E.A. and Mahmoud, S.R. (2016), "An efficient shear deformation theory for wave propagation of functionally graded material plates", *Struct. Eng. Mech*., **57**(5), 837-859.

Bounouara, F., Benrahou, K.H., Belkorissat, I. and Tounsi, A. (2016), "A nonlocal zeroth-order shear deformation theory for free vibration of functionally graded nanoscale plates resting on elastic foundation", *Steel Compos. Struct.*, **20**(2), 227-249.

Bourada, F., Amara, K. and Tounsi, A. (2016), "Buckling analysis of isotropic and orthotropic plates using a novel four variable refined plate theory", *Steel Compos. Struct.*, **21**(6), 1287-1306.

Bourada, M., Kaci, A., Houari, M.S.A. and Tounsi, A. (2015), "Anew simple shear and normal deformations theory for functionally graded beams", *Steel Compos. Struct.*, **18**(2), 409-423.

Bourada, M., Tounsi, A., Houari, M.S.A. and AddaBedia, E.A. (2012), "A new four-variable refined plate theory for thermal buckling analysis of functionally graded sandwich plates", *J. Sandw. Struct. Mater*., **14**, 5-33.

Bousahla, A.A., Benyoucef, S., Tounsi, A. and Mahmoud, S.R. (2016), "On thermal stability of plates with functionally graded coefficient of thermal expansion", *Struct. Eng. Mech*., **60**(2), 313-335.

Bousahla, A.A., Houari, M.S.A., Tounsi, A. and AddaBedia, E.A. (2014), "A novel higher order shear and normal deformation theory based on neutral surface position for bending analysis of advanced composite plates", *Int. J. Comput. Meth*., **11**(6), 1350082.

Castellazzi, G., Gentilini, C., Krysl, P. and Elishakoff, I. (2013), "Static analysis of functionally graded plates using a nodal integrated finite element approach", *Compos. Struct*., **103**, 197-200.

Cheng, Z.Q. and Batra, R.C. (2000), "Deflection relationship between the homogenous Kirchhoff plate theory and different functionally graded plates theories", *Arch. Appl. Mech*., **52**(1), 143-158.

Chi, S.H. and Chung, Y.L. (2006), "Mechanical behavior of functionally graded material plates under transverse load-part I: analysis", *Int. J. Solid. Struct*., **43**(13), 3657-3674.

Chikh, A., Bakora, A., Heireche, H., Houari, M.S.A., Tounsi, A. and AddaBedia, E.A. (2016), "Thermo-mechanical postbuckling of symmetric S-FGM plates resting on Pasternak elastic foundations using hyperbolic shear deformation theory", *Struct. Eng. Mech*., **57**(4), 617-639.

Chikh, A., Tounsi, A., Hebali, H. and Mahmoud, S.R. (2017), "Thermal buckling analysis of cross-ply laminated plates using a simplified HSDT", *Smart Struct. Syst.*, **19**(3), 289-297.

Chinosi, C. and Croce, L.D. (2007), "Approximation of functionally graded plates with non-conforming finite elements", J. *Comput. Appl. Math*., **210**, 106-115.

Draiche, K., Tounsi, A. and Mahmoud, S.R. (2016), "A refined theory with stretching effect for the flexure analysis of laminated composite plates", *Geomech. Eng.*, **11**(5), 671-690.

El-Haina, F., Bakora, A., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2017), "A simple analytical approach for thermal buckling of thick functionally graded sandwich plates", *Struct. Eng. Mech*., **63**(5), 585-595.

El-Hassar, S.M., Benyoucef, S., Heireche, H. and Tounsi, A. (2016), "Thermal stability analysis of solar functionally graded plates on elastic foundation using an efficient hyperbolic shear deformation theory", *Geomech. Eng.*, **10**(3), 357-386.

Fahsi, A., Tounsi, A., Hebali, H.,Chikh, A., AddaBedia, E.A. and Mahmoud, S.R. (2017), "A four variable refined nth-order shear deformation theory for mechanical and thermal buckling analysis of functionally graded plates", *Geomech. Eng.*, **13**(3), 385-410.

Fekrar, A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2014), "A new five-unknown refined theory based on neutral surface position for bending analysis of exponential graded plates", *Meccanica*, **49**, 795-810.

Ghannadpour, S.A.M. and Alinia, M.M. (2006), "Large deflection behavior of functionally graded plates under pressure loads", *Compos. Struct*., **75**(1-4), 67-71.

Hadji, L., HassaineDaouadji, T., Ait Amar Meziane, M., Tlidji, Y. and AddaBedia, E.A. (2016), "Analysis of functionally graded beam using a new first-order shear deformation theory", *Struct. Eng. Mech.*, **57**(2), 315-325.

Hamidi, A., Houari, M.S.A., Mahmoud, S.R. and Tounsi, A. (2015), "A sinusoidal plate theory with 5-unknowns and stretching effect for thermomechanical bending of functionally graded sandwich plates", *Steel Compos. Struct*., **18**(1), 235-253.

HanifiHachemi Amar, L., Kaci, A. and Tounsi, A. (2017), "On the size-dependent behavior of functionally graded micro-beams with porosities", *Struct. Eng. Mech.*, **64**(5), 527-541.

Hebali, H., Bakora, A., Tounsi, A. and Kaci, A. (2016), "A novel four variable refined plate theory for bending, buckling, and vibration of functionally graded plates", *Steel Compos. Struct.*, **22**(3), 473-495.

Hebali, H., Tounsi, A., Houari, M.S.A., Bessaim, A. and AddaBedia, E.A. (2014), "A new quasi-3D hyperbolic shear deformation theory for the static and free vibration analysis of functionally graded plates", *ASCE J. Eng. Mech*., **140**(2), 374-383.

Hirwani, C.K., Mahapatra, T.R., Panda, S.K., Sahoo, S.S., Singh, V.K. and Patle, B.K. (2017b), "Nonlinear free vibration analysis of laminated carbon/epoxy curved panels", *Defen. Sci. J.*, **67**(2), 207-221.

Hirwani, C.K., Panda, S.K., Mahapatra, T.R. and Mahapatra, S.S. (2017a), "Numerical study and experimental validation of dynamic characteristics of delaminated composite flat and curved shallow shell structure", *ASCE J. Aerosp. Eng.*, **30**(5), 04017045.

Hirwani, C.K., Panda, S.K., Mahapatra, T.R. and Mahapatra, S.S. (2017c), "Nonlinear transient finite element analysis of delaminated composite shallow shell panels", *AIAA J.*, **55**(5), 1734-1748.

Hosseini-Hashemi, S., Fadaee, M. and Atashipour, S.R. (2011), "Study on the free vibration of thick functionally graded rectangular plates according to a new exact closed form procedure", *Compos. Struct.*, **93**(2), 722-735.

Hosseini-Hashemi, Sh., RokniDamavandiTaher, H., Akhavan, H. and Omidi, M. (2010), "Free vibration of functionally graded rectangular plates using first-order shear deformation plate theory", *Appl. Math. Model*., **34**(5), 1276-1291.

Houari, M.S.A., Tounsi, A., Bessaim, A. and Mahmoud, S.R. (2016), "A new simple three-unknown sinusoidal shear deformation theory for functionally graded plates", *Steel Compos. Struct.*, **22**(2), 257-276.

Kar, V.R. and Panda, S.K. (2015), "Free vibration responses of temperature dependent functionally graded curved panels under thermal environment", *Latin Am. J. Solid. Struct.*, **12**(11), 2006-2024.

Kar, V.R. and Panda, S.K. (2015), "Nonlinear flexural vibration of shear deformable functionally graded spherical shell panel", *Steel Compos. Struct*., **18**(3), 693-709.

Kar, V.R. and Panda, S.K. (2016a), "Geometrical nonlinear free vibration analysis of FGM spherical panel under nonlinear thermal loading with TD and TID properties", *J. Therm. Stress.*, **39**(8), 942-959.

Kar, V.R. and Panda, S.K. (2016b), "Nonlinear free vibration of functionally graded doubly curved shear deformable panels using finite element method", *J. Vib. Control*, **22**(7), 1935-1949.

Kar, V.R., Panda, S.K. and Mahapatra, T.R. (2016), "Thermal buckling behaviour of shear deformable functionally graded single/doubly curved shell panel with TD and TID properties", *Adv. Mater. Res.*, **5**(4), 205-221.

Karami, B., Janghorban, M. and Tounsi, A. (2017), "Effects of triaxial magnetic field on the anisotropic nanoplates", *Steel Compos. Struct*., **25**(3), 361-374.

Kashtalyan, M. (2004), "Three-dimensional elasticity solution for bending of functionally graded rectangular plates", *Eur. J. Mech. A/Solid*., **23**, 853-864.

Katariya, P.V. and Panda, S.K. (2016), "Thermal buckling and vibration analysis of laminated composite curved shell panel", *Aircraft Eng. Aerosp. Technol*., **88**(1), 97-107.

Khetir, H., BachirBouiadjra, M., Houari, M.S.A., Tounsi, A. and S.R. Mahmoud, (2017), "A new nonlocal trigonometric shear deformation theory for thermal buckling analysis of embedded nanosize FG plates", *Struct. Eng. Mech.*, **64**(4), 391-402.

Klouche, F., Darcherif, L., Sekkal, M., Tounsi, A. and Mahmoud, S.R. (2017), "An original single variable shear deformation theory for buckling analysis of thick isotropic plates", *Struct. Eng. Mech*., **63**(4), 439-446.

Koizumi, M. (1993), "The concept of FGM, ceramic transactions", *Funct. Grad. Mater*., **34**, 3-10.

Laoufi, I., Ameur, M., Zidi, M., AddaBedia, E.A. and Bousahla, A.A. (2016), "Mechanical and hygrothermalbehaviour of functionally graded plates using a hyperbolic shear deformation theory", *Steel Compos. Struct.*, **20**(4), 889-911.

LarbiChaht, F., Kaci, A., Houari, M.S.A., Tounsi, A., Anwar Bég, O. and Mahmoud, S.R. (2015), "Bending and buckling analyses of functionally graded material (FGM) size-dependent nanoscale beams including the thickness stretching effect", *Steel Compos. Struct.*, **18**(2), 425-442.

Mahapatra, T.R. and Panda, S.K. (2015), "Thermoelastic vibration analysis of laminated doubly curved shallow panels using non-linear FEM", *J. Therm. Stress.*, **38**(1), 39-68.

Mahapatra, T.R. and Panda, S.K. (2016), "Nonlinear free vibration analysis of laminated composite spherical shell panel under elevated hygrothermal environment: A micromechanical approach", *Aerosp. Sci. Technol.*, **49**, 276-288.

Mahapatra, T.R., Kar, V.R. and Panda, S.K. (2015), "Nonlinear free vibration analysis of laminated composite doubly curved shell panel in hygrothermal environment", *J. Sandw. Struct. Mater.*, **17**(5), 511-545.

Mahapatra, T.R., Kar, V.R. and Panda, S.K. (2016b), "Large amplitude vibration analysis of laminated composite spherical panels under hygrothermal environment", *Int. J. Struct. Stab. Dyn.*, **16**(3), 1450105.

Mahapatra, T.R., Panda, S.K. and Kar, V.R. (2016a), "Nonlinear hygro-thermo-elastic vibration analysis of doubly curved composite shell panel using finite element micromechanical model", *Mech. Adv. Mater. Struct.*, **23**(11), 1343-1359.

Mahi, A., AddaBedia, E.A. and Tounsi, A. (2015), "A new hyperbolic shear deformation theory for bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates", *Appl. Math. Model.*, **39**(9), 2489-2508.

Matsunaga, H. (2008), "Free vibration and stability of functionally graded plates according to a 2-D higher-order deformation theory", *Compos. Struct*., **82**, 499-512.

Mehar, K. and Panda, S.K. (2016), "Geometrical nonlinear free vibration analysis of FG-CNT reinforced composite flat panel under uniform thermal field", *Compos. Struct.*, **143**, 336-346.

Mehar, K. and Panda, S.K. (2017), "Thermal free vibration behavior of FG-CNT reinforced sandwich curved panel using finite element method", *Polym. Compos.*, DOI: 10.1002/pc.24266.

Mehar, K., Panda, S.K. and Mahapatra, T.R. (2017a), "Thermoelastic nonlinear frequency analysis of CNT reinforced functionally graded sandwich structure", *Eur. J. Mech. A/Solid.*, **65**, 384-396.

Mehar, K., Panda, S.K. and Patle, B.K. (2017b), "Stress, deflection, and frequency analysis of CNT reinforced graded sandwich plate under uniform and linear thermal environment: A finite element approach", *Polym. Compos.*, DOI: 10.1002/pc.24409.

Mehar, K., Panda, S.K. and Patle, B.K. (2017c), "Thermoelastic vibration and flexural behavior of FG-CNT reinforced composite curved panel", *Int. J. Appl. Mech.*, **9**(4), 1750046.

Mehar, K., Panda, S.K., Bui, T.Q. and Mahapatra, T.R. (2017d), "Nonlinear thermoelastic frequency analysis of functionally graded CNT-reinforced single/doubly curved shallow shell panels by FEM", *J. Therm. Stress.*, **40**(7), 899-916.

Mehar, K., Panda, S.K., Dehengia, A. and Kar, V.R. (2016), "Vibration analysis of functionally graded carbon nanotube reinforced composite plate in thermal environment", *J. Sandw. Struct. Mater.*, **18**(2), 151-173.

Meksi, A., Benyoucef, S., Houari, M.S.A. and Tounsi, A. (2015), "A simple shear deformation theory based on neutral surface position for functionally graded plates resting on Pasternak elastic foundations", *Struct. Eng. Mech.*, **53**(6), 1215-1240.

Meksi, R., Benyoucef, S., Mahmoudi, A., Tounsi, A., AddaBedia, E.A. and Mahmoud, SR. (2017), "An analytical solution for bending, buckling and vibration responses of FGM sandwich plates", *J. Sandw. Struct. Mater.*, 1099636217698443.

Menasria, A., Bouhadra, A., Tounsi, A., Bousahla, A.A. and Mahmoud, S.R. (2017), "A new and simple HSDT for thermal stability analysis of FG sandwich plates", *Steel Compos. Struct.*, **25**(2), 157-175.

Meradjah, M., Kaci, A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2015), "A new higher order shear and normal deformation theory for functionally graded beams", *Steel Compos. Struct.*, **18**(3), 793-809.

Merdaci, S., Tounsi, A. and Bakora, A. (2016), "A novel four variable refined plate theory for laminated composite plates", *Steel Compos. Struct*., **22**(4), 713-732.

Mindlin, R.D. (1951), "Influence of rotary inertia and shear on flexural motions of isotropic elastic plates", *J. Appl. Mech*., **18**, 31-38.

Mouaici, F., Benyoucef, S., AitAtmane, H. and Tounsi, A. (2016), "Effect of porosity on vibrational characteristics of non-homogeneous plates using hyperbolic shear deformation theory", *Wind Struct.*, **22**(4), 429-454.

Mouffoki, A., AddaBedia, E.A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2017), "Vibration analysis of nonlocal advanced nanobeams in hygro-thermal environment using a new two-unknown trigonometric shear deformation beam theory", *Smart Struct. Syst.*, **20**(3), 369-383.

Neves, AMA, Ferreira, AJM, Carrera, E, Roque, CMC, Cinefra, M, Jorge, RMN, *et al.* (2012), "A quasi-3D sinusoidal shear deformation theory for the static and free vibration analysis of functionally graded plates", *Compos. Part B Eng.*, **43**(2), 711-725.

Panda, S.K. and Katariya, P.V. (2015), "Stability and free vibration behaviour of laminated composite panels under thermo-mechanical loading", *Int. J. Appl. Comput. Math.*, **1**(3), 475-490.

Panda, S.K. and Mahapatra, T.R. (2014), "Nonlinear finite element analysis of laminated composite spherical shell vibration under uniform thermal loading", *Meccanica*, **49**(1), 191-213.

Pradyumna, S. and Bandyopadhyay, J.N. (2008), "Free vibration analysis of functionally graded curved panels using a higher-order finite element formulation", *J. Sound Vib*., **318**, 176-192.

Praveen, G.N. and Reddy, J.N. (1998), "Nonlinear transient thermoelastic analysis of functionally graded ceramic-metal plates", *Int. J. Solid. Struct*., **35**, 4457-4471.

Qian, L.F., Batra, R.C. and Chen, L.M. (2004), "Static and dynamic deformations of thick functionally graded elastic plates by using higher-order shear and normal deformable plate theory and meshless local PetroveGalerkin method", *Compos. Part B Eng.*, **35**(6-8), 685-697.

Raminnea, M., Biglari, H. and VakiliTahami, F. (2016), "Nonlinear higher order Reddy theory for temperature- dependent vibration and instability of embedded functionally graded pipes conveying fluid-nanoparticle mixture", *Struct. Eng. Mech*., **59**(1), 153-186.

Reddy, J. (2000), "Analysis of functionally graded plates", *Int. J. Numer. Meth. Eng*., **47**, 663-684.

Reissner, E. (1945), "The effect of transverse shear deformation on the bending of elastic plates", *J. Appl. Mech*., **12**(2), 69-77.

Sahoo, S.S., Panda, S.K. and Mahapatra, T.R. (2016a), "Static, free vibration and transient response of laminated composite curved shallow panel-An experimental approach", *Eur. J. Mech. A/Solid.*, **59**, 95-113.

Sahoo, S.S., Panda, S.K. and Sen, T.R. (2016b), "Effect of delamination on static and dynamic behavior of laminated composite plate", *AIAA J*., **54**(8), 2530-2544.

Sahoo, S.S., Panda, S.K. and Singh, V.K. (2017), "Experimental and numerical investigation of static and free vibration responses of woven glass/epoxy laminated composite plate", *Proc. IMechE Part L: J. Mater. Des. Appl.*, **231**(5), 463-478.

Saidi, H., Tounsi, A. and Bousahla, A.A. (2016), "A simple hyperbolic shear deformation theory for vibration analysis of thick functionally graded rectangular plates resting on elastic foundations", *Geomech. Eng.*, **11**(2), 289-307.

Singh, M.K. and Panda, S.K. (2017), "Geometrical nonlinear free vibration analysis of laminated composite doubly curved shell panels embedded with piezoelectric layers", *J. Vib. Control*, **23**(13), 2078-2093.

Singh, M.K., Mahapatra, T.R. and Panda, S.K. (2016), "Nonlinear transient analysis of smart laminated composite plate integrated with PVDF sensor and AFC actuator", *Compos. Struct.*, **157**, 121-130.

Singh, M.K., Prakash, T. and Ganapathi, M. (2011), "Finite element analysis of functionally graded plates under transverse load", *Finite Elem. Anal. Des*., **47**, 453-460.

Singh, V.K. and Panda, S.K. (2015), "Large amplitude free vibration analysis of laminated composite spherical shells embedded with piezoelectric layers", *Smart Struct. Syst.*, **16**(5), 853-872.

Taibi, F.Z., Benyoucef, S., Tounsi, A., BachirBouiadjra, R., AddaBedia, E.A. and Mahmoud, S.R. (2015), "A simple shear deformation theory for thermo-mechanical behaviour of functionally graded sandwich plates on elastic foundations", *J. Sandw. Struct. Mater.*, **17**(2), 99-129.

Tounsi, A., Houari, M.S.A., Benyoucef, S. and AddaBedia, E.A. (2013), "A refined trigonometric shear deformation theory for thermoelastic bending of functionally graded sandwich plates", *Aerosp. Sci. Technol*., **24**(1), 209-220.

Wen, P.H. and Aliabadi, M.H. (2012), "Analysis of functionally graded plates by meshless method: A purely analytical formulation", *Eng. Anal. Bound. Elem*., **36**, 639-650.

Zemri, A., Houari, M.S.A., Bousahla, A.A. and Tounsi, A. (2015), "A mechanical response of functionally graded nanoscale beam: an assessment of a refined nonlocal shear deformation theory beam theory", *Struct. Eng. Mech*., **54**(4), 693-710.

Zhao, X., Lee, Y.Y. and Liew, K.M. (2009), "Free vibration analysis of functionally graded plates using the element-free kp-Ritz method", *J. Sound Vib*., **319**, 918-939.

Zidi, M., Houari, M.S.A., Tounsi, A., Bessaim, A. and Mahmoud, S.R. (2017),"A novel simple two-unknown hyperbolic shear deformation theory for functionally graded beams", *Struct. Eng. Mech.*, **64**(2), 145-153.

Zidi, M., Tounsi, A., Houari M.S.A., AddaBedia, E.A. and Anwar Bég, O. (2014), "Bending analysis of FGM plates under hygro-thermo-mechanical loading using a four variable refined plate theory", *Aerosp. Sci. Technol*., **34**, 24-34.

CC