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A novel and simple higher order shear deformation theory for stability and vibration of functionally graded sandwich plate

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Abstract. In this work, a new higher shear deformation theory (HSDT) is developed for the free vibration and buckling of functionally graded (FG) sandwich plates. The proposed theory presents a new displacement field by using undetermined integral terms. Only four unknowns are employed in this theory, which is less than the classical first shear deformation theory (FSDT) and others HSDTs. Equations of motion are obtained via Hamilton's principle. The analytical solutions of FG sandwich plates are determined by employing the Navier method. A good agreement between the computed results and the available solutions of existing HSDTs is found to prove the accuracy of the developed theory.

Keywords: buckling; vibration; sandwich plate; functionally graded materials; plate theory

1. Introduction

It is noted long that sandwich structures are widely employed in areas of aerospace, marine, construction, transportation, and wind energy systems due to their outstanding mechanical characteristics (Vinson 2001, 2005, Tian *et al.* 2016). Although sandwich structures provides the benefits over other kinds of structures, the abrupt variation in material characteristics within the interfaces between the face sheets and the core can lead in high interlaminar stresses, often resulting to delamination, which is a great problem in classical sandwich structures. One way to overcome this problem is the use of functionally graded material (FGM). FGM presents non-homogenous composite material where the material characteristics are gradually changed from one surface of the structure to the other, which leads to eliminating the above indicated abrupt variations of mechanical properties (Koizumi 1997, Shaw 1998, Birman *et al*. 2013, Bouderba *et al.* 2013, Swaminathan *et al*. 2015, Akbaş 2015, Arefi 2015a, b, Arefi and Allam 2015, Zemri *et al*. 2015, Kar *et al*. 2015a, b, Bouguenina *et al*. 2015, Darabi and Vosoughi 2016, Celebi *et al.* 2016, Chikh *et al.* 2016, Trinh *et al*. 2016, Turan *et al*. 2016, Ebrahimi and Shafiei 2016, Bounouara *et al*. 2016, Barka *et al.* 2016, Mahapatra *et al*. 2017, El-Haina *et al*. 2017, Zidi *et al*. 2017).

A number of applications of functionally graded (FG) structures have led the development of various plate/beam models to examine accurately their bending, stability and vibration responses. These theories can be generally presented by: classical plate theory (CPT) neglecting the effects of transverse shear deformation (Feldman and Aboudi 1997, Javaheri and Eslami 2002, Mahdavian 2009, Chen *et al*. 2006), first-shear deformation theory (FSDT) with linear distribution of displacements (Chen *et al*. 2006, Praveen and Reddy 1998, Croce and Venini 2004, Efraim and Eisenberger 2007, Hosseini-Hashemi *et al.* 2011, Panda and Katariya 2015, Meksi *et al*. 2015, Adda Bedia *et al*. 2015, Bouderba *et al.* 2016, Bellifa *et al*. 2016, Ebrahimi and Jafari 2016, Hadji *et al*. 2016), higher-order shear deformation theory (HSDT) with nonlinear variations of displacements within the structure thickness such as third-order shear deformation plate theory (TSDT), sinusoidal shear deformation plate theory (SSDT), hyperbolic shear deformable plate theory (HDT) (Reddy 2000, Jha *et al.* 2013, Reddy 2011, Talha and Singh 2010, Matsunaga 2008, El Meiche *et al.* 2011, Bourada *et al*. 2012, Tounsi *et al*. 2013, Zidi *et al*. 2014, Ait Atmane *et al.* 2015, Mahi *et al.* 2015, Mahapatra and Panda 2015, Merazi *et al*. 2015, Belkorissat *et al* 2015, Bennai *et al.* 2015, Nguyen *et al.* 2015, Mahapatra *et al*. 2015, Bakora and Tounsi 2015, Bousahla *et al*. 2016, Barati and Shahverdi 2016, Mouaici *et al.* 2016, Beldjelili *et al*. 2016, Mahapatra *et al*. 2016a, b, c, d, Kar *et al.* 2016, Mahapatra and Panda 2016, Becheri *et al.* 2016, Baseri *et al.* 2016, Laoufi *et al*. 2016, Mohammadimehr *et al.* 2016, Ebrahimi and Habibi 2016, Houari *et al.* 2016, Ahouel *et al.* 2016, Raminnea *et al*. 2016, Saidi *et al*. 2016, Ghorbanpour Arani *et al.* 2016, El-Hassar *et al*. 2016, Benferhat *et al*. 2016, Javed *et al*. 2016, Mouffoki *et al.* 2017, Taibi *et al*. 2017, Bellifa *et al.* 2017, Kar *et al*. 2017, Tounsi *et al*. 2016, Klouche *et al.* 2017), quasi-3D theories taking into account the effect of normal

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stretching (Carrera *et al*. 2008, Wu and Chiu 2011, Chen *et al*. 2009, Hebali *et al*. 2014, Bourada *et al.* 2015, Akavci 2016). Moreover, owing to smooth distributions of material characteristics, FG sandwich plates have recently been proposed to overcome interface problems between faces and core found in conventional sandwich structures. Many plate models have been employed to predict behaviors of FG sandwich plates: static behaviors (Neves *et al*. 2013, Hamidi *et al.* 2015, Carrera *et al*. 2011), vibration and buckling behaviors (Neves *et al.* 2013, Carrera *et al*. 2011, Li *et al.* 2008, El Meiche *et al.* 2011, Sobhy 2013, Natarajan and Manickam 2012, Ait Amar Meziane *et al*. 2014, Bennoun *et al.* 2016).

The objective of this article is to propose a new hyperbolic shear deformation theory for vibration and buckling analyses of FG sandwich plates. The present theory differs from other HSDTs because, in proposed theory the displacement field which use undetermined integral terms and contains only four variables which is not considered by the other articles. Equations of motion derived here are solved for sandwich plates with FG faces. Closed-form solutions are obtained to predict the critical buckling loads and natural frequencies of simply supported FG sandwich plates. Comparison studies are performed to check the validity of the present results.

2. Problem formulation

Consider a rectangular FG sandwich plate with uniform thickness h, length a and width b. The Cartesian coordinate system xyz is considered such that the xy plane ($z=0$) coincides with the mid-surface of the sandwich plate. Layer 1, 2, 3 denote the bottom, middle and top layer, respectively. The sandwich core is isotropic (fully ceramic) and face sheets are made of a FGM through the thickness. The bottom face sheet varies from a metal-rich surface ($z=h_0$) to a ceramic-rich surface while the top face sheet changes from a ceramic-rich surface to a metal-rich surface ($z=h_3$), as presented in Fig. 1.

There are no interfaces between core and face sheets. The volume fraction of sandwich plate is expressed as

$$\begin{cases} V^{(1)}(z)=\left(\dfrac{z-h_0}{h_1-h_0}\right)^p & \text{for } z\in[h_0,h_1] \\ V^{(2)}(z)=1 & \text{for } z\in[h_1,h_2] \\ V^{(3)}(z)=\left(\dfrac{z-h_3}{h_2-h_3}\right)^p & \text{for } z\in[h_2,h_3] \end{cases} \tag{1}$$

where $V^{(n)}$ is the volume fraction of *n*-th layer and p is a parameter that denotes the power index and takes values greater than or equal to zero.

The effective material properties for *n*-th layer, like the Young's modulus $E^{(n)}$, the Poisson's ratio $\nu^{(n)}$ and the mass density $\rho^{(n)}$ at a point can be obtained by the linear rule of mixture (Marur 1999, Attia *et al*. 2015) as

$$P^{(n)}(z)=\left(P_1-P_2\right)V^{(n)}(z)+P_2 \tag{2}$$

where $P^{(n)}$ is the effective material property of FGM of layer *n*. P_1 and P_2 are the properties of the top an d bottom faces of layer 1, respectively, and vice versa for layer 3 depending on the volume fraction $V^{(n)}$, ($n=1,2,3$).

2.1 Kinematics and strains

In this study, further simplifying consideration are taken to the classical HSDTs so that the number of variables is reduced. The displacement field of the classical HSDTs is written by

$$u(x,y,z,t)=u_0(x,y,t)-z\frac{\partial w_0}{\partial x}+f(z)\varphi_x(x,y,t) \tag{3a}$$

$$v(x,y,z,t)=v_0(x,y,t)-z\frac{\partial w_0}{\partial y}+f(z)\varphi_y(x,y,t) \tag{3b}$$

$$w(x,y,z,t)=w_0(x,y,t) \tag{3c}$$

where u_0 ; v_0 ; w_0 , φ_x , φ_y are five unknown displacements of the mid-plane of the plate, $f(z)$ denotes shape function representing the variation of the transverse shear strains and stresses within the thickness. By considering that $\varphi_x=\int\theta(x,y)dx$ and $\varphi_y=\int\theta(x,y)dy$, the displacement field of the present theory can be written in a simpler form as (Bourada *et al.* 2016, Hebali *et al*. 2016, Merdaci *et al.* 2016, Chikh *et al.* 2017, Besseghier *et al*. 2017, Fahsi *et al.* 2017, Khetir *et al.* 2017, Meksi *et al.* 2017)

$$u(x,y,z,t)=u_0(x,y,t)-z\frac{\partial w_0}{\partial x}+k_1\,f(z)\int\theta(x,y,t)\,dx \tag{4a}$$

$$v(x,y,z,t)=v_0(x,y,t)-z\frac{\partial w_0}{\partial y}+k_2\,f(z)\int\theta(x,y,t)\,dy \tag{4b}$$

$$w(x,y,z,t)=w_0(x,y,t) \tag{4c}$$

In this study, the present HSDT is obtained by setting

$$f(z)=z-z\left[1+\frac{3\pi}{2}\operatorname{sech}^2\left(\frac{1}{2}\right)\right]-\frac{3\pi}{2}h\tanh\left(\frac{z}{h}\right) \tag{5}$$

It can be observed that the displacement field in equation (4) uses only four variables (u_0, v_0, w_0 and θ). The nonzero strains associated with the displacement field in Eq. (4) are

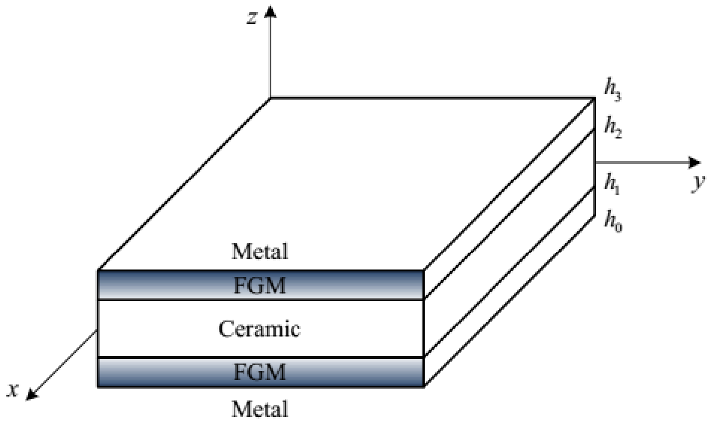

Fig. 1 FGM face sheets and homogeneous core.

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}+z\begin{Bmatrix}k_x^0\\ k_y^0\\ k_{xy}^0\end{Bmatrix}+f(z)\begin{Bmatrix}L_x^0\\ L_y^0\\ L_{xy}^0\end{Bmatrix},\qquad \begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=g(z)\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix} \tag{6a}$$

where

$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial u_0}{\partial x}\\ \dfrac{\partial v_0}{\partial x}\\ \dfrac{\partial u_0}{\partial y}+\dfrac{\partial v_0}{\partial x}\end{Bmatrix},\quad \begin{Bmatrix}k_x^0\\ k_y^0\\ k_{xy}^0\end{Bmatrix}=\begin{Bmatrix}-\dfrac{\partial^2 w_0}{\partial x^2}\\ -\dfrac{\partial^2 w_0}{\partial y^2}\\ -2\dfrac{\partial^2 w_0}{\partial x\partial y}\end{Bmatrix}, \tag{6b}$$

$$\begin{Bmatrix}L_x^0\\ L_y^0\\ L_{xy}^0\end{Bmatrix}=\begin{Bmatrix}k_1\theta\\ k_2\theta\\ k_1\dfrac{\partial}{\partial y}\int\theta\,dx+k_2\dfrac{\partial}{\partial x}\int\theta\,dy\end{Bmatrix},\quad \begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix}=\begin{Bmatrix}k_2\int\theta\,dy\\ k_1\int\theta\,dx\end{Bmatrix}$$

and

$$g(z)=\frac{df(z)}{dz} \tag{6c}$$

The integrals used in the above equations shall be resolved by a Navier type procedure and can be expressed as follows

$$\frac{\partial}{\partial y}\int\theta\,dx=A'\frac{\partial^2\theta}{\partial x\partial y},\quad \frac{\partial}{\partial x}\int\theta\,dy=B'\frac{\partial^2\theta}{\partial x\partial y},\qquad \int\theta\,dx=A'\frac{\partial\theta}{\partial x},\quad \int\theta\,dy=B'\frac{\partial\theta}{\partial y} \tag{7}$$

where the coefficients A' and B' are expressed according to the type of solution used, in this case via Navier method. Therefore, A', B', k_1 and k_2 are expressed as follows

$$A'=-\frac{1}{\alpha^2},\quad B'=-\frac{1}{\beta^2},k_1=\alpha^2,k_2=\beta^2 \tag{8}$$

where α and β are defined in expression (24).

For elastic and isotropic FGMs, the constitutive relations can be written as

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\\ \tau_{yz}\\ \tau_{xz}\end{Bmatrix}^{(n)}=\begin{bmatrix}C_{11}&C_{12}&0&0&0\\ C_{12}&C_{22}&0&0&0\\ 0&0&C_{66}&0&0\\ 0&0&0&C_{44}&0\\ 0&0&0&0&C_{55}\end{bmatrix}^{(n)}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\\ \gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}^{(n)} \tag{9}$$

where (σ_x, σ_y, τ_{xy}, τ_{yz}, τ_{xz}) and (ε_x, ε_y, γ_{xy}, γ_{yz}, γ_{xz}) are the stress and strain components, respectively. Using the material properties defined in Eq. (2), stiffness coefficients, C_{ij}, can be given as

$$C_{11}=C_{22}=\frac{E(z)}{1-\nu^2},\; C_{12}=\frac{\nu\,E(z)}{1-\nu^2},\qquad C_{44}=C_{55}=C_{66}=\frac{E(z)}{2(1+\nu)}, \tag{10}$$

2.2 Equations of motion

Hamilton’s principle is herein employed to deduce the equations of motion (Ait Yahia *et al.* 2015)

$$0=\int_0^t(\delta U+\delta V-\delta K)\,dt \tag{11}$$

where δU is the variation of strain energy; δV is the variation of the external work done by external load applied to the plate; and δK is the variation of kinetic energy.

The variation of strain energy of the plate is expressed by

$$\begin{aligned}\delta U &= \int_V \left[\sigma_x \delta\,\varepsilon_x + \sigma_y \delta\,\varepsilon_y + \tau_{xy}\delta\,\gamma_{xy} + \tau_{yz}\delta\,\gamma_{yz} + \tau_{xz}\delta\,\gamma_{xz}\right] dV \\ &= \int_A \left[N_x \delta\,\varepsilon_x^0 + N_y \delta\,\varepsilon_y^0 + N_{xy}\delta\,\gamma_{xy}^0 + M_x \delta\,k_x^0 + M_y \delta\,k_y^0 + M_{xy}\delta\,k_{xy}^0\right. \\ &\quad \left. + S_x \delta\,L_x^0 + S_y \delta\,L_y^0 + S_{xy}\delta\,L_{xy}^0 + R_{yz}\delta\,\gamma_{yz}^0 + R_{xz}\delta\,\gamma_{xz}^0\right] dA = 0\end{aligned} \tag{12}$$

where A is the top surface and the stress resultants N, M, S, and R are defined by

$$\left(N_i, M_i, S_i\right) = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} \left(1, z, f\right)\sigma_i dz\,, \quad \left(i = x, y, xy\right) \text{ and}$$
$$\left(R_{xz}, R_{yz}\right) = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} g\left(\tau_{xz}, \tau_{yz}\right) dz \tag{13}$$

where h_n and h_{n-1} are the top and bottom z-coordinates of the nth layer

The variation of the external work can be written as

$$\delta V = -\int_A \overline{N}\delta w_0 dA \tag{14a}$$

with

$$\overline{N} = \left[N_x^0 \frac{\partial^2 w_0}{\partial x^2} + 2N_{xy}^0 \frac{\partial^2 w_0}{\partial x \partial y} + N_y^0 \frac{\partial^2 w_0}{\partial y^2}\right] \tag{14b}$$

where (N_x^0, N_y^0, N_{xy}^0) are in-plane applied loads.

The variation of kinetic energy of the plate can be written as

$$\begin{aligned}\delta K &= \int_V \left[\dot{u}\,\delta\,\dot{u} + \dot{v}\,\delta\,\dot{v} + \dot{w}\,\delta\,\dot{w}\right] \rho(z)\, dV \\ &= \int_A \{I_0\left[\dot{u}_0\delta\dot{u}_0 + \dot{v}_0\delta\dot{v}_0 + \dot{w}_0\delta\dot{w}_0\right] \\ &- I_1\left(\dot{u}_0 \frac{\partial\delta\dot{w}_0}{\partial x} + \frac{\partial\dot{w}_0}{\partial x}\delta\,\dot{u}_0 + \dot{v}_0 \frac{\partial\delta\dot{w}_0}{\partial y} + \frac{\partial\dot{w}_0}{\partial y}\delta\,\dot{v}_0\right) \\ &+ J_1\left(\left(k_1\, A'\right)\left(\dot{u}_0 \frac{\partial\delta\,\dot{\theta}}{\partial x} + \frac{\partial\dot{\theta}}{\partial x}\delta\,\dot{u}_0\right) + \left(k_2\, B'\right)\left(\dot{v}_0 \frac{\partial\delta\dot{\theta}}{\partial y} + \frac{\partial\dot{\theta}}{\partial y}\delta\,\dot{v}_0\right)\right) \\ &+ I_2\left(\frac{\partial\dot{w}_0}{\partial x}\frac{\partial\delta\,\dot{w}_0}{\partial x} + \frac{\partial\dot{w}_0}{\partial y}\frac{\partial\delta\,\dot{w}_0}{\partial y}\right) + K_2\left(\left(k_1\, A'\right)^2\left(\frac{\partial\dot{\theta}}{\partial x}\frac{\partial\delta\,\dot{\theta}}{\partial x}\right) + \left(k_2\, B'\right)^2\left(\frac{\partial\dot{\theta}}{\partial y}\frac{\partial\delta\,\dot{\theta}}{\partial y}\right)\right) \\ &- J_2\left(\left(k_1\, A'\right)\left(\frac{\partial\dot{w}_0}{\partial x}\frac{\partial\delta\,\dot{\theta}}{\partial x} + \frac{\partial\dot{\theta}}{\partial x}\frac{\partial\delta\,\dot{w}_0}{\partial x}\right) + \left(k_2\, B'\right)\left(\frac{\partial\dot{w}_0}{\partial y}\frac{\partial\delta\,\dot{\theta}}{\partial y} + \frac{\partial\dot{\theta}}{\partial y}\frac{\partial\delta\,\dot{w}_0}{\partial y}\right)\right)\Bigg\} dA\end{aligned} \tag{15}$$

where dot-superscript convention indicates the differentiation with respect to the time variable t; $\rho(z)$ is the mass density given by Eq. (1); and (I_i, J_i, K_i) are mass inertias expressed by

$$\left(I_0, I_1, I_2\right) = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} \left(1, z, z^2\right)\rho(z) dz \tag{16a}$$

$$\left(J_1, J_2, K_2\right) = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} \left(f, z\,f, f^2\right)\rho(z) dz \tag{16b}$$

By substituting Eqs. (12), (14) and (15) into Eq. (11), the following can be derived:

$$\begin{aligned}&\delta\, u_0:\ \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0\ddot{u}_0 - I_1\frac{\partial\ddot{w}_0}{\partial x} + k_1 A' J_1 \frac{\partial\ddot{\theta}}{\partial x} \\ &\delta\, v_0:\ \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_0\ddot{v}_0 - I_1\frac{\partial\ddot{w}_0}{\partial y} + k_2\, B'\, J_1 \frac{\partial\ddot{\theta}}{\partial y} \\ &\delta\, w_0:\ \frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x\partial y} + \frac{\partial^2 M_y}{\partial y^2} + N_x^0\frac{\partial^2 w_0}{\partial x^2} + 2\,N_{xy}^0\frac{\partial^2 w_0}{\partial x\partial y} + N_y^0\frac{\partial^2 w_0}{\partial y^2} = I_0\ddot{w}_0 \\ &\quad + I_1\left(\frac{\partial\ddot{u}_0}{\partial x} + \frac{\partial\ddot{v}_0}{\partial y}\right) - I_2\nabla^2\ddot{w}_0 + J_2\left(k_1\, A'\frac{\partial^2\ddot{\theta}}{\partial x^2} + k_2\, B'\frac{\partial^2\ddot{\theta}}{\partial y^2}\right) \\ &\delta\,\theta:\ -\mathrm{k}_1 A'\frac{\partial^2 S_x}{\partial x^2} - -\mathrm{k}_2 B'\frac{\partial^2 S_y}{\partial y^2} - \left(k_1 A' + k_2\, B'\right)\frac{\partial^2 S_{xy}}{\partial x\partial y} + k_1\, A'\frac{\partial R_{xz}}{\partial x} + k_2\, B'\frac{\partial R_{yz}}{\partial y} = -J_1\left(k_1\, A'\frac{\partial\ddot{u}_0}{\partial x} + k_2\, B'\,\frac{\partial\ddot{v}_0}{\partial y}\right) \\ &\quad - K_2\left(\left(k_1\, A'\right)^2\,\frac{\partial^2\ddot{\theta}}{\partial x^2} + \left(k_2\, B'\right)^2\,\frac{\partial^2\ddot{\theta}}{\partial y^2}\right) + J_2\left(k_1\, A'\frac{\partial^2\ddot{w}_0}{\partial x^2} + k_2\, B'\frac{\partial^2\ddot{w}_0}{\partial y^2}\right)\end{aligned} \tag{17}$$

Substituting Eq. (5) into Eq. (9) and the subsequent results into Eqs. (13), the stress resultants are obtained in terms of strains as following compact form

$$\begin{Bmatrix} N \\ M \\ S \end{Bmatrix} = \begin{bmatrix} A & B & B^s \\ B & D & D^s \\ B^s & D^s & H^s \end{bmatrix} \begin{Bmatrix} \varepsilon \\ k^0 \\ L^0 \end{Bmatrix}, \quad R = A^s\gamma \tag{18}$$

in which

$$N = \left\{N_x, N_y, N_{xy}\right\}^t, \quad M = \left\{M_x, M_y, M_{xy}\right\}^t, \quad S = \left\{S_x, S_y, S_{xy}\right\}^t \tag{19a}$$

$$\varepsilon = \left\{\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0\right\}^t, \quad k^0 = \left\{k_x^0, k_y^0, k_{xy}^0\right\}^t, \quad L^s = \left\{L_x^0, L_y^0, L_{xy}^0\right\}^t \tag{19b}$$

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}, \quad B = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix}, \quad D = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \tag{19c}$$

$$B^s = \begin{bmatrix} B_{11}^s & B_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & B_{66}^s \end{bmatrix}, \quad D^s = \begin{bmatrix} D_{11}^s & D_{12}^s & 0 \\ D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & D_{66}^s \end{bmatrix}, \quad H^s = \begin{bmatrix} H_{11}^s & H_{12}^s & 0 \\ H_{12}^s & H_{22}^s & 0 \\ 0 & 0 & H_{66}^s \end{bmatrix} \tag{19d}$$

$$R=\{R_{xz},R_{yz}\}^t,\quad \gamma=\{\gamma^0_{xz},\gamma^0_{yz}\}^t,$$
$$A^s=\begin{bmatrix} A^s_{44} & 0 \\ 0 & A^s_{55}\end{bmatrix} \tag{19e}$$

and stiffness components are defined as

$$\begin{Bmatrix} A_{11} & B_{11} & D_{11} & B^s_{11} & D^s_{11} & H^s_{11} \\ A_{12} & B_{12} & D_{12} & B^s_{12} & D^s_{12} & H^s_{12} \\ A_{66} & B_{66} & D_{66} & B^s_{66} & D^s_{66} & H^s_{66}\end{Bmatrix}=\sum_{n=1}^{3}\int_{h_{n-1}}^{h_n} C^{(n)}_{11}\left(1,z,z^2,f(z),z\,f(z),f^2(z)\right)\begin{Bmatrix}1\\ \nu^{(n)} \\ \dfrac{1-\nu^{(n)}}{2}\end{Bmatrix}dz \tag{20a}$$

$$\left(A_{22},B_{22},D_{22},B^s_{22},D^s_{22},H^s_{22}\right)=\left(A_{11},B_{11},D_{11},B^s_{11},D^s_{11},H^s_{11}\right),$$
$$C^{(n)}_{11}=\frac{E(z)}{1-\nu^2}, \tag{20b}$$

$$A^s_{44}=A^s_{55}=\sum_{n=1}^{3}\int_{h_{n-1}}^{h_n}\frac{E(z)}{2(1+\nu)}\left[g(z)\right]^2dz, \tag{20c}$$

By substituting Eq. (18) into Eq. (17), the equations of motion can be expressed in terms of displacements (u_0, v_0, w_0, θ) and the appropriate equations take the form

$$\begin{aligned}&A_{11}d_{11}u_0+A_{66}d_{22}u_0+\left(A_{12}+A_{66}\right)d_{12}v_0-B_{11}d_{111}w_0-\left(B_{12}+2B_{66}\right)d_{122}w_0\\&+\left(B^s_{66}\left(k_1A'+k_2B'\right)+B^s_{12}k_2B'\right)d_{122}\theta+B^s_{11}k_1A'\,d_{111}\theta=I_0\ddot{u}_0-I_1d_1\ddot{w}_0+J_1A'k_1d_1\ddot{\theta},\end{aligned} \tag{21a}$$

$$\begin{aligned}&A_{22}d_{22}v_0+A_{66}d_{11}v_0+\left(A_{12}+A_{66}\right)d_{12}u_0-B_{22}d_{222}w_0-\left(B_{12}+2B_{66}\right)d_{112}w_0\\&+\left(B^s_{66}\left(k_1A'+k_2B'\right)+B^s_{12}k_1A'\right)d_{112}\theta+B^s_{22}k_2B'd_{222}\theta=I_0\ddot{v}_0-I_1d_2\ddot{w}_0+J_1B'k_2d_2\ddot{\theta},\end{aligned} \tag{21b}$$

$$\begin{aligned}&B_{11}d_{111}u_0+\left(B_{12}+2B_{66}\right)d_{122}u_0+\left(B_{12}+2B_{66}\right)d_{112}v_0+B_{22}d_{222}v_0-D_{11}d_{1111}w_0-2\left(D_{12}+2D_{66}\right)d_{1122}w_0\\&-D_{22}d_{2222}w_0+D^s_{11}k_1A'd_{1111}\theta+\left((D^s_{12}+2D^s_{66})\left(k_1A'+k_2B'\right)\right)d_{1122}\theta+D^s_{22}k_2B'd_{2222}\theta+N^0_xd_{11}w_0\\&+2N^0_{xy}d_{12}w_0+N^0_yd_{22}w_0=I_0\ddot{w}_0+I_1\left(d_1\ddot{u}_0+d_2\ddot{v}_0\right)-I_2\left(d_{11}\ddot{w}_0+d_{22}\ddot{w}_0\right)+J_2\left(k_1A'd_{11}\ddot{\theta}+k_2B'd_{22}\ddot{\theta}\right)\end{aligned} \tag{21c}$$

$$\begin{aligned}&-k_1A'B^s_{11}d_{111}u_0-\left(B^s_{12}k_2B'+B^s_{66}\left(k_1A'+k_2B'\right)\right)d_{122}u_0-\left(B^s_{22}k_1A'+B^s_{66}\left(k_1A'+k_2B'\right)\right)d_{112}v_0-B^s_{22}k_2B'd_{222}v_0\\&+D^s_{11}k_1A'd_{1111}w_0+\left((D^s_{12}+2D^s_{66})\left(k_1A'+k_2B'\right)\right)d_{1122}w_0+D^s_{22}k_2B'd_{2222}w_0-H^s_{11}(k_1A')^2d_{1111}\theta\\&-H^s_{22}(k_2B')^2d_{2222}\theta-\left(2H^s_{12}k_1k_2A'B'+\left(k_1A'+k_2B'\right)^2H^s_{66}\right)d_{1122}\theta+A^s_{44}\left(k_1A'\right)^2d_{11}\theta+A^s_{55}\left(k_2B'\right)^2d_{22}\theta=\\&-J_1\left(k_1A'd_1\ddot{u}_0+k_2B'd_2\ddot{v}_0\right)+J_2\left(k_1A'd_{11}\ddot{w}_0+k_2B'd_{22}\ddot{w}_0\right)-K_2\left(\left(k_1A'\right)^2d_{11}\ddot{\theta}+\left(k_2B'\right)^2d_{22}\ddot{\theta}\right)\end{aligned} \tag{21d}$$

where d_{ij}, d_{ijl} and d_{ijlm} are the following differential operators

$$d_{ij}=\frac{\partial^2}{\partial x_i\partial x_j},\quad d_{ijl}=\frac{\partial^3}{\partial x_i\partial x_j\partial x_l},$$
$$d_{ijlm}=\frac{\partial^4}{\partial x_i\partial x_j\partial x_l\partial x_m},\quad d_i=\frac{\partial}{\partial x_i}, \tag{22}$$
$$(i,j,l,m=1,2).$$

3. Close-form solutions

The Navier solution procedure is employed to determine the analytical solutions for which the displacement variables are expressed as product of arbitrary parameters and known trigonometric functions to respect the equations of motion and boundary conditions.

$$\begin{Bmatrix}u_0\\ v_0\\ w_0\\ \theta\end{Bmatrix}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\begin{Bmatrix}U_{mn}e^{i\omega t}\cos(\alpha x)\sin(\beta y)\\ V_{mn}e^{i\omega t}\sin(\alpha x)\cos(\beta y)\\ W_{mn}e^{i\omega t}\sin(\alpha x)\sin(\beta y)\\ X_{mn}e^{i\omega t}\sin(\alpha x)\sin(\beta y)\end{Bmatrix} \tag{23}$$

where ω is the frequency of free vibration of the plate, $\sqrt{i}=-1$ the imaginary unit.
with

$$\alpha=m\pi/a \text{ and } \beta=n\pi/b \tag{24}$$

Considering that the plate is subjected to in-plane compressive forces of form: $N^0_x=-N_0$, $N^0_y=-\gamma N_0$, $N^0_{xy}=0$, $\gamma=N^0_y/N^0_x$ (here γ are non-dimensional load parameter). Substituting Eq. (23) into Eq. (21), the following problem is obtained

$$\left(\begin{bmatrix}S_{11} & S_{12} & S_{13} & S_{14}\\ S_{12} & S_{22} & S_{23} & S_{24}\\ S_{13} & S_{23} & S_{33}+k & S_{34}\\ S_{14} & S_{24} & S_{34} & S_{44}\end{bmatrix}-\omega^2\begin{bmatrix}m_{11} & 0 & m_{13} & m_{14}\\ 0 & m_{22} & m_{23} & m_{24}\\ m_{13} & m_{23} & m_{33} & m_{34}\\ m_{14} & m_{24} & m_{34} & m_{44}\end{bmatrix}\right)\begin{Bmatrix}U_{mn}\\ V_{mn}\\ W_{mn}\\ X_{mn}\end{Bmatrix}=\begin{Bmatrix}0\\0\\0\\0\end{Bmatrix} \tag{25}$$

where

$$S_{11}=A_{11}\alpha^2+A_{66}\beta^2,\ S_{12}=\alpha\beta\left(A_{12}+A_{66}\right),$$

$$S_{13}=-\alpha\left(B_{11}\alpha^2+(B_{12}+2B_{66})\beta^2\right),$$

$$S_{14}=\alpha\left((k_2B'B^s_{12}+\left(k_1A'+k_2B'\right)B^s_{66})\beta^2+k_1A'B^s_{11}\alpha^2\right),$$

$$S_{22}=A_{66}\alpha^2+A_{22}\beta^2,$$

$$S_{23}=-\beta\left(B_{22}\beta^2+(B_{12}+2B_{66})\alpha^2\right),$$

$$S_{24}=\beta\left((k_1A'B^s_{12}+\left(k_1A'+k_2B'\right)B^s_{66})\alpha^2+k_2B'B^s_{22}\beta^2\right),$$

$$S_{33}=D_{11}\alpha^4+2(D_{12}+2D_{66})\alpha^2\beta^2+D_{22}\beta^4,$$

$$S_{34}=-k_1A'D^s_{11}\alpha^4-((D^s_{12}+2D^s_{66})(k_1A'+k_2B'))\alpha^2\beta^2-k_2B'D^s_{22}\beta^4,$$

$$\begin{aligned}S_{44}=&(k_1A')^2H^5_{11}\alpha^4+(2k_1k_2A'B'H^s_{12}+\\&(k_1A'+k_2B')^2H^s_{66})\alpha^2\beta^2+(k_2B')^2H^s_{22}\beta^4+\\&(k_2B')^2A^s_{55}\beta^2+(k_1A')^2A^s_{44}\alpha^2\end{aligned} \tag{26}$$

$$k=-N_0\left(\alpha^2+\gamma\beta^2\right)$$

$$m_{11}=I_0,\quad m_{13}=-\alpha I_1,\quad m_{14}=J_1k_1A'\alpha,$$

$$m_{22}=I_0,\quad m_{23}=-\beta I_1$$

$$m_{24}=k_2B'\beta J_1,\ m_{33}=I_0+I_2(\alpha^2+\beta^2),$$
$$m_{34}=-J_2\left(k_1A'\alpha^2+k_2B'\beta^2\right),$$

$$m_{44}=K_2\left(\left(k_1A'\right)^2\alpha^2+\left(k_2B'\right)^2\beta^2\right)$$

Table 1 Displacement models

Model	Theory	Unknown function
CPT	Classical plate theory	3
FSDT	First shear deformation plate theory (Mindlin 1951)	5
PSDT	Parabolic shear deformation plate theory (Reddy 2011)	5
SSDT	Sinusoidal shear deformation plate theory (Touratier 1991)	5
HySDT	Hyperbolic shear deformation plate theory (El Meiche *et al*. 2011)	4
Present	New HSDT	4

Eq. (25) is a general form for stability and free vibration analysis of FG sandwich plates under in-plane loads. The critical buckling loads (N_{cr}) can be determined from the stability problem $\left|S_{ij}\right|=0$ while the free vibration problem is achieved by omitting in-plane loads.

4. Numerical results and discussion

In this section, natural frequencies and critical buckling loads of simply supported FG sandwich plates are presented and compared with existing solutions to check the accuracy of the proposed new HSDT. The FG plate is considered to be made of aluminum and alumina with the following material characteristics:

- Ceramic (P_1 : Alumina, Al_2O_3): $E_1=380$ GPa, $\nu=0.3$, $\rho_1=3800$ kg/m^3.
- Metal (P_2 : Aluminum, Al): $E_2=70$ GPa, $\nu=0.3$, $\rho_2=2707$ kg/m^3.

For convenience, the following non-dimensional parameters are employed:

$$\bar{\omega}=\frac{\omega b^2}{h}\sqrt{\frac{\rho_0}{E_0}}, \quad \bar{N}=\frac{Na^2}{100h^2E_0} \tag{27}$$

Where $\rho_0=1$ kg/m^3 and $E_0=1$ GPa.

4.1 Results of free vibration analysis

The natural frequencies of the structure are computed using Eq. (25) as eigenvalue problem by omitting in-plane loads. The non-dimensional fundamental frequencies of FG sandwich plates are presented here to estimate the accuracy of the presented new HSDT.

First, for the verification purpose, the results determined by the present HSDT are compared with other theories existing in the literature such as CPT, FSDT, PSDT, SSDT, HySDT and three-dimensional linear theory of elasticity by Li *et al.* (2008). The description of various displacement models is presented in Table 1. We also take the shear correction factor $K=5/6$ in FSDT.

The results of the FG sandwich plates with five material distributions are compared in Table 2 with the results based on CPT, FSDT, PSDT, SSDT, HySDT and 3D elasticity. Young's modulus E and mass density ρ are based on the power-law distribution (Eq. (2)). Table 2 demonstrates a good agreement by comparisons of FG plates of five different gradient index with other HSDTs. Hence, the proposed theory (with only four unknown functions) gives comparable results to those determined with higher order theories with five variables. Compared to the 3D linear theory of elasticity (Li *et al*. 2008), the proposed theory gives more accurate results than the other theories such as PSDT and SSDT and especially for the case of non-symmetric (2–1–1) and (2–2–1).

The second comparison is presented in Table 3 for both the symmetric (1–2–1) and non-symmetric (2–2–1) types of square FG sandwich plates. It can be seen that increasing the mode number lead to an increase of frequencies. Again, we found that the natural frequencies obtained by the present HSDT are in a good agreement with other shear deformation theories.

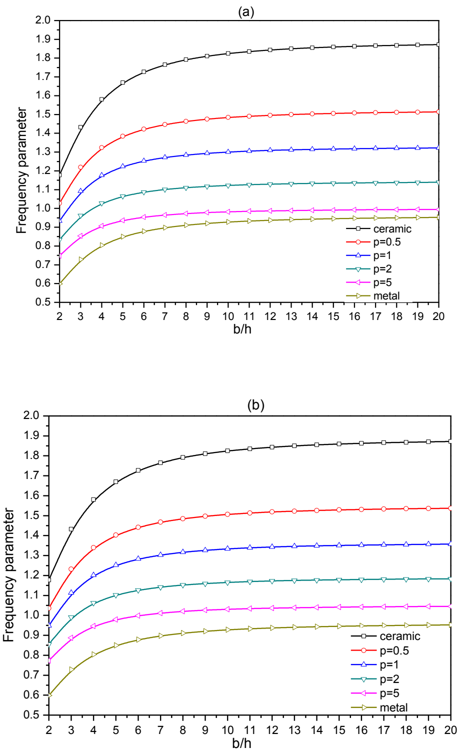

Fig. 2 Fundamental frequency ($\bar{\omega}$) versus the side-to-thickness ratio (b/h) of symmetric and non-symmetric square FG sandwich plates for various values of p: (a) The (2–1–2) FG sandwich plate and (b) the (2–1–1) FG sandwich plate

Table 2 Comparisons of natural fundamental frequency parameters $\bar{\omega}$ simply supported square FG plates with other theories ($h/b = 0.1$)

	Theory	$\bar{\omega}$					
		1–0–1	2–1–2	2–1–1	1–1–1	2–2–1	1–2–1
0	CPT	1.87359	1.87359	1.87359	1.87359	1.87359	1.87359
	FSDT	1.82442	1.82442	1.82442	1.82442	1.82442	1.82442
	PSDT	1.82445	1.82445	1.82445	1.82445	1.82445	1.82445
	SSDT	1.82452	1.82452	1.82452	1.82452	1.82452	1.82452
	Elasticity	–	–	–	–	–	–
	HySDT	1.82449	1.82449	1.82449	1.82449	1.82449	1.82449
	Present	1.82449	1.82449	1.82449	1.82449	1.82449	1.82449
0.5	CPT	1.47157	1.51242	1.54264	1.54903	1.58374	1.60722
	FSDT	1.44168	1.48159	1.51035	1.51695	1.55001	1.57274
	PSDT	1.44424	1.48408	1.51253	1.51922	1.55199	1.57451
	SSDT	1.44436	1.48418	1.51258	1.51927	1.55202	1.57450
	Elasticity	1.44614	1.48608	1.50841	1.52131	1.54926	1.57668
	HySDT	1.44419	1.48405	1.50636	1.51922	1.54714	1.57458
	Present	1.44432	1.48415	1.50644	1.51925	1.54717	1.5745
1	CPT	1.26238	1.32023	1.37150	1.37521	1.43245	1.46497
	FSDT	1.24031	1.29729	1.34637	1.35072	1.40555	1.43722
	PSDT	1.24320	1.30011	1.34888	1.35333	1.40789	1.43934
	SSDT	1.24335	1.30023	1.34894	1.35339	1.40792	1.43931
	Elasticity	1.24470	1.30181	1.33511	1.35523	1.39763	1.44137
	HySDT	1.24310	1.30004	1.33328	1.35331	1.39559	1.43940
	Present	1.2433	1.30019	1.33344	1.35337	1.39567	1.43931
5	CPT	0.95844	0.99190	1.08797	1.05565	1.16195	1.18867
	FSDT	0.94259	0.97870	1.07156	1.04183	1.14467	1.17159
	PSDT	0.94598	0.98184	1.07432	1.04466	1.14731	1.17397
	SSDT	0.94630	0.98207	1.07445	1.04481	1.14741	1.17399
	Elasticity	0.94476	0.98130	1.02942	1.04532	1.10983	1.17567
	HySDT	0.94574	0.98166	1.03033	1.04455	1.10875	1.17397
	Present	0.94621	0.98201	1.03069	1.04477	1.10904	1.17398
10	CPT	0.94321	0.95244	1.05185	1.00524	1.11883	1.13614
	FSDT	0.92508	0.93962	1.03580	0.99256	1.10261	1.12067
	PSDT	0.92839	0.94297	1.03862	0.99551	1.10533	1.12314
	SSDT	0.92875	0.94232	1.04558	0.99519	1.04154	1.13460
	Elasticity	0.92727	0.94078	0.98929	0.99523	1.06104	1.12466
	HySDT	0.92811	0.94275	0.99184	0.99536	1.06081	1.12311
	Present	0.92864	0.94453	0.99222	0.99564	1.06115	1.12317

Table 3 Comparisons of natural frequency parameters $\bar{\omega}$ simply supported square FG sandwich plates with other theories ($p = 2$, $h/b = 0.1$)

m	n	1–2–1					
		CPT	FSDT	PSDT	SSDT	HySDT	Present
1	1	1.32200	1.30020	1.30246	1.30244	1.30250	1.30244
1	2	3.26976	3.14452	3.15698	3.15686	3.15726	3.15692
2	2	5.17700	4.88021	4.90879	4.90849	4.90978	4.90895
1	3	6.42690	5.98487	6.02667	6.02622	6.02866	6.02741
2	3	8.27066	7.57215	7.63674	7.63601	7.64151	7.63953
1	4	10.67355	9.57284	9.67233	9.67121	9.68465	9.68143
3	3	11.26475	10.05424	10.16314	10.16193	10.17821	10.17467
2	4	12.43611	10.99612	11.12461	11.12321	11.14644	11.1422
3	4	15.30248	13.23801	13.41936	13.41755	13.46652	13.46038
4	4	19.17579	16.13722	16.40035	16.39820	16.50693	16.49783

m	n	2–2–1					
		CPT	FSDT	PSDT	SSDT	HySDT	Present
1	1	1.28650	1.26524	1.26775	1.26780	1.24375	1.24392
1	2	3.18172	3.05968	3.07353	3.07382	3.01698	3.01796
2	2	5.03724	4.74815	4.77998	4.78065	4.69456	4.69689
1	3	6.25311	5.82264	5.86924	5.87022	5.76658	5.76981
2	3	8.04649	7.36640	7.43850	7.44002	7.31319	7.31812
1	4	10.38339	9.31198	9.42315	9.42552	9.27437	9.28178
3	3	10.95830	9.78007	9.90179	9.90439	9.74847	9.75652
2	4	12.09731	10.69588	10.83951	10.84261	10.67885	10.68823
3	4	14.88418	12.87543	13.07809	13.08260	12.91005	12.92283
4	4	18.64932	15.69346	15.98701	15.99393	15.83764	15.85526

In general, the frequencies given by the CPT are much higher than those calculated from the shear deformation theories. This implies the well-known fact that the results computed by the CPT are grossly in error for a thick plate and/or for higher mode numbers. From these results (Tables 2 and 3), the frequencies decrease as the gradient index p increases and as the core thickness, with respect to the total thickness of the plate, decreases.

The variations in non-dimensional fundamental frequencies of FG sandwich plates for different gradient index p as a function of the side-to-thickness ratio is presented in Fig. 2 by employing the proposed theory. It can be deduced from this figure that the fundamental frequency is reduced with increasing the gradient index p. The maximum values are found for the ceramic plates while the minimum ones are for the metal plates.

4.2 Results of buckling analysis

The critical stability loads of the structure are computed using Eq. (25) as an eigenvalue problem by omitting mass matrix. The critical buckling forces of FG sandwich plates are illustrated here to demonstrate the accuracy of the proposed HSDT.

Tables 4 and 5 give critical buckling forces of various types of FG sandwich plates by employing different plate models and different values of the gradient index p. It can be deduced from these two tables that the results of the proposed model are in an excellent agreement with those reported by other shear deformation theories. Hence, the proposed model (with only four unknown functions) provides comparable results to those determined with higher order theories with five unknown functions. From these results, it can be seen that the critical buckling loads reduce with increasing the gradient parameter decrease p. In general, the fully ceramic plates produce the higher critical buckling loads. The uniaxial buckling load may be twice the biaxial one and this irrespective of the used value of p and the type of the FG sandwich plate.

Fig. 3 and 4 present the variation of the critical buckling loads of the symmetric (1–2–1) and non-symmetric (2–2–1) types of square FG sandwich plates versus side-to-thickness ratio by employing the proposed new theory. It can be seen from these figures that the critical buckling load decreases with increasing the gradient index p. Indeed, the maximum values are found for the ceramic plates while the minimum ones are for the metal plates.

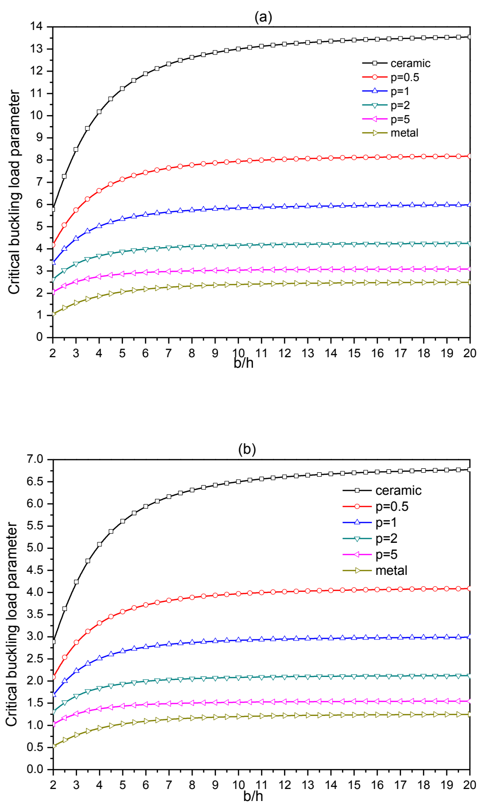

Fig. 3 Non-dimensional critical buckling load ($\overline{N}$) versus the side-to-thickness ratio (b/h) of (2–1–2) FG sandwich plates for various values of p: (a) Plate subjected to uniaxial compressive load ($\gamma = 0$) and (b) Plate subjected to biaxial compressive load ($\gamma = 1$)

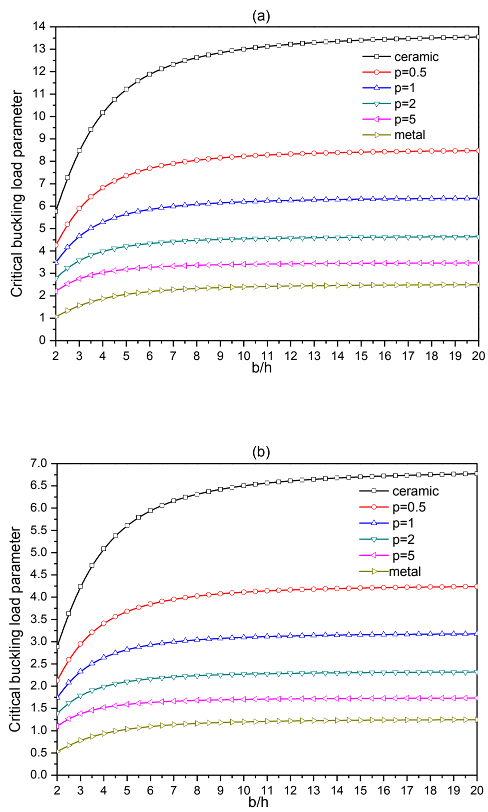

Fig. 4 Non-dimensional critical buckling load ($\overline{N}$) versus the side-to-thickness ratio (b/h) of (2–1–1) FG sandwich plates for various values of p: (a) Plate subjected to uniaxial compressive load ($\gamma = 0$) and (b) Plate subjected to biaxial compressive load ($\gamma = 1$)

Table 4 Comparisons of non-dimensional critical buckling load $\overline{N}$ of square FG plates subjected to uniaxial compressive load ($\gamma = 0$, $h/b = 0.1$)

p	Theory	$\overline{N}$					
		1–0–1	2–1–2	2–1–1	1–1–1	2–2–1	1–2–1
0	CPT	13.73791	13.73791	13.73791	13.73791	13.73791	13.73791
	FSDT	13.00449	13.00449	13.00449	13.00449	13.00449	13.00449
	PSDT	13.00495	13.00495	13.00495	13.00495	13.00495	13.00495
	SSDT	13.00606	13.00606	13.00606	13.00606	13.00606	13.00606
	HySDT	13.00552	13.00552	13.00552	13.00552	13.00552	13.00552
	Present	13.00552	13.00552	13.00552	13.00552	13.00552	13.00552
0.5	CPT	7.65398	8.25597	8.56223	8.78063	9.18254	9.61525
	FSDT	7.33732	7.91320	8.20015	8.41034	8.78673	9.19517
	PSDT	7.36437	7.94084	8.22470	8.43645	8.80997	9.21681
	SSDT	7.36568	7.94195	8.22538	8.43712	8.81037	9.21670
	HySDT	7.36380	7.94046	8.22471	8.43647	8.81029	9.21757
	Present	7.36523	7.94155	8.2251	8.43684	8.81016	9.21663
1	CPT	5.33248	6.02733	6.40391	6.68150	7.19663	7.78406
	FSDT	5.14236	5.81379	6.17020	6.43892	6.92571	7.48365
	PSDT	5.16713	5.84006	6.19394	6.46474	6.94944	7.50656
	SSDT	5.16846	5.84119	6.19461	6.46539	6.94980	7.50629
	HySDT	5.16629	5.83941	6.19371	6.46450	6.94952	7.50719
	Present	5.16804	5.84083	6.19437	6.46515	6.94964	7.5063
5	CPT	2.73080	3.10704	3.48418	3.65732	4.21238	4.85717
	FSDT	2.63842	3.02252	3.38538	3.55958	4.09285	4.71475
	PSDT	2.65821	3.04257	3.40351	3.57956	4.11209	4.73467
	SSDT	2.66006	3.04406	3.40449	3.58063	4.11288	4.73488
	HySDT	2.65679	3.04141	3.40280	3.57874	4.11157	4.73463
	Present	2.65951	3.04362	3.40419	3.58031	4.11263	4.7348
10	CPT	2.56985	2.80340	3.16427	3.27924	3.79238	4.38221
	FSDT	2.46904	2.72626	3.07428	3.27521	3.68890	4.26040
	PSDT	2.48727	2.74632	3.09190	3.19471	3.70752	4.27991
	SSDT	2.48928	2.74844	3.13443	3.19456	3.14574	4.38175
	HySDT	2.48574	2.74498	3.09111	3.19373	3.70686	4.27964
	Present	2.48966	2.74256	3.09263	3.19556	3.70812	4.28014

Table 5 Comparisons of non-dimensional critical buckling load $\overline{N}$ of square FG plates subjected to biaxial compressive load ($\gamma = 1$, $h/b = 0.1$)

p	Theory	$\overline{N}$					
		1–0–1	2–1–2	2–1–1	1–1–1	2–2–1	1–2–1
0	CPT	6.86896	6.86896	6.86896	6.86896	6.86896	6.86896
	FSDT	6.50224	6.50224	6.50224	6.50224	6.50224	6.50224
	PSDT	6.50248	6.50248	6.50248	6.50248	6.50248	6.50248
	SSDT	6.50303	6.50303	6.50303	6.50303	6.50303	6.50303
	HySDT	6.50276	6.50276	6.50276	6.50276	6.50276	6.50276
	Present	6.50276	6.50276	6.50276	6.50276	6.50276	6.50276
0.5	CPT	3.82699	4.12798	4.28112	4.39032	4.59127	4.80762
	FSDT	3.66866	3.95660	4.10117	4.20517	4.39336	4.59758
	PSDT	3.68219	3.95660	4.11235	4.21823	4.40519	4.60878
	SSDT	3.68248	3.97097	4.112269	4.21856	4.40519	4.60835
	HySDT	3.68190	3.97023	4.11236	4.21823	4.40514	4.60878
	Present	3.68261	3.97078	4.11255	4.21842	4.40508	4.60832
1	CPT	2.66624	3.01366	3.20195	3.34075	3.59831	3.89203
	FSDT	2.57118	2.90690	3.08510	3.21946	3.46286	3.74182
	PSDT	2.58357	2.92003	3.09697	3.23237	3.47472	3.75328
	SSDT	2.58423	2.92060	3.090731	3.23270	3.47490	3.75314
	HySDT	2.58315	2.91970	3.09686	3.23225	3.47476	3.75359
	Present	2.58402	2.92041	3.09719	3.23258	3.47482	3.75315
5	CPT	1.36540	1.55352	1.74209	1.82866	2.10619	2.42859
	FSDT	1.31921	1.51126	1.69269	1.77979	2.04642	2.35737
	PSDT	1.32910	1.52129	1.70176	1.78978	2.05605	2.36734
	SSDT	1.33003	1.52203	1.70224	1.79032	2.05644	2.36744
	HySDT	1.32839	1.52071	1.70140	1.78937	2.05578	2.36731
	Present	1.32976	1.52181	1.7021	1.79015	2.05632	2.3674
10	CPT	1.28493	1.40170	1.58214	1.62962	1.89619	2.19111
	FSDT	1.23452	1.36313	1.53714	1.58760	1.84445	2.13020
	PSDT	1.24363	1.37316	1.54595	1.59736	1.85376	2.13995
	SSDT	1.24475	1.37422	1.56721	1.59728	1.57287	2.19089
	HySDT	1.24287	1.37249	1.54556	1.59687	1.85343	2.13982
	Present	1.24435	1.37374	1.54631	1.59779	1.85406	2.14007

5. Conclusions

A new HSDT has been proposed for the buckling and free vibration analyses of FG sandwich plates. By assuming further simplifying considerations to the conventional HSDTs, with introducing undetermined integral term, the number of variables and equations of motion of the proposed theory are reduced by one, and thus, make this formulation simple and efficient to use. Analytical solutions are found for simply-supported sandwich plates to examine the critical buckling load and natural frequencies for various gradient index and side-to-thickness and skin-core-skin thickness ratios. A good agreement between the computed results and those reported by existing shear deformation theories is demonstrated within several numerical examples which shows the accuracy of the proposed theory in predicting the stability and vibration responses of FG sandwich plates. An improvement of present formulation will be considered in the future work to consider the thickness stretching effect by using quasi-3D shear deformation models (Bessaim *et al.* 2013, Bousahla *et al.* 2014, Belabed *et al.* 2014, Fekrar *et al.* 2014, Hebali *et al.* 2014, Meradjah *et al.* 2015, Larbi Chaht *et al*. 2015, Hamidi *et al.* 2015, Bourada *et al.* 2015, Bennoun *et al*. 2016, Draiche *et al*. 2016, Benbakhti *et al*. 2016, Benahmed *et al.* 2017, Ait Atmane *et al*. 2017, Benchohra *et al*. 2017, Bouafia *et al.* 2017) and the wave propagation problem (Mahmoud *et al*. 2015, Ait Yahia *et al.* 2015, Boukhari *et al.* 2016).

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