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An efficient and simple four variable refined plate theory for buckling analysis of functionally graded plates

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Abstract. In this article, an efficient and simple refined theory is proposed for buckling analysis of functionally graded plates by using a new displacement field which includes undetermined integral variables. This theory contains only four unknowns, with is even less than the first shear deformation theory (FSDT). Governing equations are obtained from the principle of virtual works. The closed-form solutions of rectangular plates are determined. Comparison studies are carried out to check the validity of obtained results. The influences of loading conditions and variations of power of functionally graded material, modulus ratio, aspect ratio, and thickness ratio on the critical buckling load of functionally graded plates are examined and discussed.

Keywords: bending analysis; functionally graded plate; plate theory

1. Introduction

During last two decades, the need to design the high per Functionally graded materials (FGMs) are composite materials composed of two or more constituent phases with a continuously variable variation by gradually changing the volume fraction. These materials type have been proposed, developed and successfully employed in industrial application since 1980s (Koizumi 1993). FGMs were designed as a thermal barrier coating in aerospace application, such as ceramic-metal composite structure. Nowadays, FGMs are alternative materials widely employed in aerospace, civil, mechanical, nuclear, optical, electronic, chemical, shipbuilding, and biomechanical industries (Akavci 2016, Kar and Panda 2016, Kar and Panda 2015, Bourada *et al*. 2015, Eltaher *et al*. 2014, Belkorissat *et al*. 2015, Ait Atmane *et al*. 2015, Akbaş 2015, Arefi 2015a,b, Arefi and Allam 2015b, Zemri *et al*. 2015, Boukhari *et al*. 2016, Bounouara *et al*. 2016, Ahouel *et al*. 2016, Celebi *et al*. 2016, Darabi and Vosoughi 2016, Turan *et al*. 2016, Ebrahimi and Shafiei 2016, Mouaici *et al*. 2016, Mouffoki *et al*. 2017, Zidi *et al*. 2017).

In the past three decades, investigations on FG plates have received particular attention, and a variety of plate models has been proposed based on considering the transverse shear deformation influences. The classical plate theory (CPT), which ignores the transverse shear deformation influence, gives reasonable results for thin plate. This model was used for stability analysis of FG plate by Feldman and Aboudi (1997), Abrate (2008), Mahdavian (2009), and Mohammadi *et al*. (2010a). However, it under-predicts transverse displacements and over-predicts frequencies as well as buckling loads of moderately thick plate (Reddy 2004). To improve the limitation of CPT, many shear deformation plate models which consider the transverse shear deformation effect have been proposed. The Reissner (1945) and Mindlin (1951) theories are known as the first-order shear deformation plate theory (FSDT), and incorporate the transverse shear effect by the way of linear distribution of in-plane displacements across the thickness. Many works of the stability behavior of FG plate have been presented via FSDT (Yang *et al*. 2005, Zhao *et al*. 2009, Sepiani *et al*. 2010, Mohammadi *et al*. 2010b, Meksi *et al*. 2015, Adda Bedia *et al*. 2015, Ebrahimi and Jafari 2016, Bellifa *et al*. 2016, Hadji *et al*. 2016). Since FSDT does not respect the equilibrium conditions at the upper and lower surfaces of the plate, shear correction coefficients are needed to correct the unrealistic distribution of transverse shear stresses and shear strains within the thickness. These shear correction factors are sensitive not only to the geometric parameters of plate, but also to the boundary conditions and loading conditions. To avoid the employ of shear correction factors, a number of higher-order shear deformation plate theories (HSDT), which use the higher-order terms in Taylor's expansions of the displacements in the thickness coordinate, have been developed. Although the HSDTs have been utilized for stability and bending investigations of FG plates

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(Najafizadeh and Heydari 2007, 2008, Bourada *et al*. 2012, Bouderba *et al*. 2013, Tounsi *et al*. 2013, Swaminathan and Naveenkumar 2014, Bouguenina *et al*. 2015, Chikh *et al*. 2016, Barati and Shahverdi 2016, Becheri *et al*. 2016, El-Hassar *et al*. 2016, Fahsi *et al*. 2017, El-Haina *et al*. 2017), they are not convenience to use because of the higher-order terms included into the theory. Therefore, there is a scope to develop a HSDT which is simple to use. Recently, Mantari and Granados (2015) have developed a new simple FSDT with four variables in which integral terms in the plate kinematics are employed for the first time. However, in this theory the shear correction factors are required. Based on shear deformation theories, the bending, buckling and vibration of composite structures been presented by Mahi *et al.* (2015), Ait Yahia *et al.* (2015) and Ait Amar Meziane *et al.* (2014). More reports on the behavior of composite structures may be also found in the open literature (see, e.g., Panda and Singh 2009, 2010a,b, 2011, 2013a,b,c,d, Zidi *et al*. 2014, Taibi *et al.* 2015, Panda and Katariya 2015, Attia *et al*. 2015, Nguyen *et al*. 2015, Tounsi *et al*. 2016, Trinh *et al*. 2016, Raminnea *et al*. 2016, Saidi *et al*. 2016, Katariya and Panda 2016, Javed *et al.* 2016, Ebrahimi and Habibi 2016, Kar *et al.* 2016a,b, Houari *et al*. 2016, Beldjelili *et al*. 2016, Ghorbanpour Arani *et al.* 2016, Baseri *et al.* 2016, Laoufi *et al*. 2016, Benferhat *et al.* 2016, Barka *et al.* 2016, Kar and Panda 2016a,b 2017, Klouche *et al*. 2017, Bellifa *et al.* 2017, Meksi *et al*. 2017, Sekkal *et al*. 2017, Menasria *et al*. 2017). Bouderba *et al.* (2016) studied the thermal stability of FG sandwich plates using a simple shear deformation theory. Bousahla *et al*. (2016) analyzed also the thermal stability of plates with functionally graded coefficient of thermal expansion.

This work aims to develop a simple HSDT for stability behavior of FG plates. The addition of the integral term in the displacement field leads to a reduction in the number of unknowns and governing equations. Analytical solutions of rectangular plates are obtained. Comparison studies are performed to demonstrate the validity of the present results. The influences of loading conditions and variations of power of functionally graded material, modulus ratio, aspect ratio, and thickness ratio on the critical buckling load of FG plates are examined and discussed.

2. Refined plate theory for FG plates

2.1 Displacement base field

The displacement field of the novel theory is given as follows (Bourada *et al*. 2016, Hebali *et al*. 2016, Merdaci *et al*. 2016, Chikh *et al*. 2017, Besseghier *et al*. 2017, Khetir *et al*. 2017)

$$u(x,y,z) = u_0(x,y) - z\frac{\partial w_0}{\partial x} + k_1\, f(z)\int \theta(x,y)\,dx \tag{1a}$$

$$v(x,y,z) = v_0(x,y) - z\frac{\partial w_0}{\partial y} + k_2\, f(z)\int \theta(x,y)\,dy \tag{1b}$$

$$w(x,y,z) = w_0(x,y) \tag{1c}$$

where $u_0(x,y)$, $v_0(x,y)$, $w_0(x,y)$, and $\theta(x,y)$ are the four unknown displacement functions of middle surface of the plate. Note that the integrals do not have limits. In the present work is considered terms with integrals instead of terms with derivatives. The constants k_1 and k_2 depends on the geometry.

$$f(z) = z\left(\frac{5}{4} - \frac{5z^2}{3h^2}\right) \tag{2}$$

It should be noted that unlike the FSDT, this theory does not require shear correction factors. The kinematic relations can be obtained as follows

$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix} = \begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} + z\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix} + f(z)\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix},$$
$$\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix} = g(z)\begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix} \tag{3}$$

where

$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix} = \begin{Bmatrix}\dfrac{\partial u_0}{\partial x}\\ \dfrac{\partial v_0}{\partial x}\\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x}\end{Bmatrix}, \quad \begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix} = \begin{Bmatrix}-\dfrac{\partial^2 w_0}{\partial x^2}\\ -\dfrac{\partial^2 w_0}{\partial y^2}\\ -2\dfrac{\partial^2 w_0}{\partial x\partial y}\end{Bmatrix}, \tag{4a}$$

$$\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix} = \begin{Bmatrix}k_1\theta\\ k_2\theta\\ k_1\dfrac{\partial}{\partial y}\int\theta\,dx + k_2\dfrac{\partial}{\partial x}\int\theta\,dy\end{Bmatrix}, \quad \begin{Bmatrix}\gamma_{yz}^0\\ \gamma_{xz}^0\end{Bmatrix} = \begin{Bmatrix}k_1\int\theta\,dy\\ k_2\int\theta\,dx\end{Bmatrix}$$

and

$$g(z) = \frac{df(z)}{dz} \tag{4b}$$

The integrals defined in the above equations shall be resolved by a Navier type method and can be written as follows

$$\frac{\partial}{\partial y}\int\theta\,dx = A'\frac{\partial^2\theta}{\partial x\partial y}, \quad \frac{\partial}{\partial x}\int\theta\,dy = B'\frac{\partial^2\theta}{\partial x\partial y},$$
$$\int\theta\,dx = A'\frac{\partial\theta}{\partial x}, \quad \int\theta\,dy = B'\frac{\partial\theta}{\partial y} \tag{5}$$

where the coefficients A' and B' are expressed according to the type of solution used, in this case via Navier. Therefore, A' and B' are expressed as follows

$$A' = -\frac{1}{\alpha^2}, \quad B' = -\frac{1}{\beta^2}, k_1 = \alpha^2, k_2 = \beta^2 \tag{6}$$

where α and β are defined in expression (20).

2.2 Constitutive relations

Consider a FG plate formed from ceramic and metal, the material properties of FGM such as Young modulus E are assumed to vary through the plate thickness with a power law distribution of the volume fraction of the two materials as (Bakora and Tounsi 2015, Merazi *et al*. 2015)

$$E(z) = E_m + (E_c - E_m)\left(\frac{1}{2} + \frac{z}{h}\right)^p \tag{7}$$

where E_m and E_c are the properties of the metal and ceramic, respectively; and p is the volume fraction exponent. The value of p equal to zero represents a fully ceramic plate, whereas infinite p indicates a fully metallic plate. The distribution of the combination of ceramic and metal is linear for $p = 1$. The variation of Poisson's ratio ν is generally small and it is assumed to be a constant for convenience. The linear constitutive relations of a FG plate can be expressed as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{66} & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & C_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{8}$$

where

$$C_{11} = C_{22} = \frac{E(z)}{1-\nu^2}, \quad C_{12} = \frac{\nu\, E(z)}{1-\nu^2}, \quad C_{44} = C_{55} = C_{66} = \frac{E(z)}{2(1+\nu)}, \tag{9}$$

2.3 Governing equations

The principle of virtual works of the considered FG plates is expressed as

$$\delta U + \delta V = 0 \tag{10}$$

where δU is the variation of strain energy; and δV is the variation of the external work done by external load applied to the plate.

The variation of strain energy of the plate is given by

$$\begin{aligned} \delta U &= \int_V \left[\sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \tau_{xy}\delta\gamma_{xy} + \tau_{yz}\delta\gamma_{yz} + \tau_{xz}\delta\gamma_{xz}\right] dV \\ &= \int_A \left[N_x \delta\varepsilon_x^0 + N_y \delta\varepsilon_y^0 + N_{xy}\delta\gamma_{xy}^0 + M_x^b \delta k_x^b + M_y^b \delta k_y^b + M_{xy}^b \delta k_{xy}^b \right. \\ &\quad \left. + M_x^s \delta k_x^s + M_y^s \delta k_y^s + M_{xy}^s \delta k_{xy}^s + S_{yz}^s \delta\gamma_{yz}^s + S_{xz}^s \delta\gamma_{xz}^0 \right] dA = 0 \end{aligned} \tag{11}$$

where A is the top surface and the stress resultants N, M, and S are defined by

$$\left(N_i, M_i^b, M_i^s\right) = \int_{-h/2}^{h/2} (1, z, f)\,\sigma_i\, dz, \quad (i = x, y, xy)$$
$$\text{and} \quad \left(S_{xz}^s, S_{yz}^s\right) = \int_{-h/2}^{h/2} g\left(\tau_{xz}, \tau_{yz}\right) dz \tag{12}$$

Substituting Eq. (8) into Eq. (12) and integrating th rough the thickness of the plate, the stress resultants ar e given as

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x^b \\ M_y^b \\ M_{xy}^b \\ M_x^s \\ M_y^s \\ M_{xy}^s \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 & B_{11}^s & B_{12}^s & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 & B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} & 0 & 0 & B_{66}^s \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 & D_{11}^s & D_{12}^s & 0 \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 & D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} & 0 & 0 & D_{66}^s \\ B_{11}^s & B_{12}^s & 0 & D_{11}^s & D_{12}^s & 0 & H_{11}^s & H_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 & D_{12}^s & D_{22}^s & 0 & H_{12}^s & H_{22}^s & 0 \\ 0 & 0 & B_{66}^s & 0 & 0 & D_{66}^s & 0 & 0 & H_{66}^s \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ k_x^b \\ k_y^b \\ k_{xy}^b \\ k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} \tag{13a}$$

$$\begin{Bmatrix} S_{xz}^s \\ S_{yz}^s \end{Bmatrix} = \begin{bmatrix} A_{55}^s & 0 \\ 0 & A_{44}^s \end{bmatrix} \begin{Bmatrix} \gamma_{xz}^0 \\ \gamma_{yz}^0 \end{Bmatrix} \tag{13b}$$

where A_{ij}, B_{ij}, etc. are the plate stiffness, defined by

$$\left(A_{ij}, B_{ij}, D_{ij}, B_{ij}^s, D_{ij}^s, H_{ij}^s\right) = \int_{-h/2}^{h/2} C_{ij}\left(1, z, z^2, f(z), z\, f(z), f^2(z)\right) dz, \quad (i, j = 1,2,6) \tag{14a}$$

$$A_{ij}^s = \int_{-h/2}^{h/2} C_{ij}\left[g(z)\right]^2 dz, \quad (i, j = 4,5) \tag{14b}$$

The work done by applied forces can be expressed as

$$\delta V = -\int_A \left(N_x^0 \frac{\partial w_0}{\partial x}\frac{\partial \delta w_0}{\partial x} + 2N_{xy}^0 \frac{\partial w_0}{\partial x}\frac{\partial \delta w_0}{\partial y} + N_y^0 \frac{\partial w_0}{\partial y}\frac{\partial \delta w_0}{\partial y} \right) dA \tag{15}$$

where (N_x^0, N_y^0, N_{xy}^0) are transverse and in-plane applied loads.

Substituting Eqs. (11) and (15) into Eq. (10) and integrating the equation by parts, collecting the coefficients of δu_0, δv_0, δw_0, $\delta\theta$, the governing equations can be obtained as follows

$$\begin{aligned} &\delta u_0: \ \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \\ &\delta v_0: \ \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \\ &\delta w_0: \ \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} + N_x^0 \frac{\partial^2 w_0}{\partial x^2} + 2N_{xy}^0 \frac{\partial^2 w_0}{\partial x \partial y} + N_y^0 \frac{\partial^2 w_0}{\partial y^2} = 0 \\ &\delta\theta: \ -\mathrm{k}_1 M_x^s - \mathrm{k}_2 M_y^s - (k_1 A' + k_2 B')\frac{\partial^2 M_{xy}^s}{\partial x \partial y} + k_1 A' \frac{\partial S_{xz}^s}{\partial x} + k_2 B' \frac{\partial S_{yz}^s}{\partial y} = 0 \end{aligned} \tag{16}$$

Eq. (16) can be expressed in terms of displacements (u_0, v_0, w_0, θ) by substituting for the stress resultants from Eq. (13). For FG plate, the governing equations Eq. (16) take the form

$$A_{11}d_{11}u_0 + A_{66}\,d_{22}u_0 + (A_{12}+A_{66})d_{12}v_0 - B_{11}d_{111}w_0 - (B_{12}+2B_{66})d_{122}w_0 + \left(B_{66}^s(k_1A'+k_2\,B')\right)d_{122}\theta + \left(B_{11}^s k_1 + B_{12}^s\,k_2\right)d_1\theta = 0 \tag{17a}$$

$$A_{22}\,d_{22}v_0 + A_{66}\,d_{11}v_0 + (A_{12}+A_{66})\,d_{12}u_0 - B_{22}\,d_{222}w_0 - (B_{12}+2B_{66})\,d_{112}w_0 + \left(B_{66}^s(k_1A'+k_2\,B')\right)d_{112}\theta + \left(B_{22}^s k_2 + B_{12}^s k_1\right)d_2\theta = 0 \tag{17b}$$

$$B_{11}\,d_{111}u_0 + (B_{12}+2B_{66})d_{122}u_0 + (B_{12}+2B_{66})d_{112}v_0 + B_{22}\,d_{222}v_0 - D_{11}d_{1111}w_0 - 2(D_{12}+2D_{66})\,d_{1122}w_0 - D_{22}\,d_{2222}w_0 + \left(D_{11}^s\,k_1 + D_{12}^s\,k_2\right)d_{11}\theta + 2\left(D_{66}^s(k_1A'+k_2B')\right)d_{1122}\theta + \left(D_{12}^s k_1 + D_{22}^s k_2\right)d_{22}\theta + N_x^0\,d_{11}w_0 + 2\,N_{xy}^0\,d_{12}w_0 + N_y^0\,d_{22}w_0 = 0 \tag{17c}$$

$$-\left(B_{11}^s k_1 + B_{12}^s k_2\right)d_1u_0 - \left(B_{66}^s(k_1A'+k_2B')\right)d_{122}u_0 - \left(B_{66}^s(k_1A'+k_2B')\right)d_{112}v_0 - \left(B_{12}^s k_1 + B_{22}^s k_2\right)d_2v_0 + \left(D_{11}^s k_1 + D_{12}^s k_2\right)d_{11}w_0 + 2\left(D_{66}^s(k_1A'+k_2B')\right)d_{1122}w_0 + \left(D_{12}^s k_1 + D_{22}^s k_2\right)d_{22}w_0 - H_{11}^s k_1^2\,\theta - H_{22}^s\,k_2^2\,\theta - 2H_{12}^s\,k_1\,k_2\theta - \left((k_1A'+k_2B')^2 H_{66}^s\right)d_{1122}\theta + A_{44}^s(k_2B')^2\,d_{22}\theta + A_{55}^s\,(k_1\,A')^2\,d_{11}\theta = 0 \tag{17d}$$

where d_{ij}, d_{ijl} and d_{ijlm} are the following differential operators

$$d_{ij} = \frac{\partial^2}{\partial x_i \partial x_j}, \quad d_{ijl} = \frac{\partial^3}{\partial x_i \partial x_j \partial x_l},$$
$$d_{ijlm} = \frac{\partial^4}{\partial x_i \partial x_j \partial x_l \partial x_m}, \quad d_i = \frac{\partial}{\partial x_i}, \tag{18}$$
$$(i, j, l, m = 1,2).$$

3. Closed-form solution for rectangular plate

Consider a simply supported rectangular plate with l ength a and width b which is subjected to in-plane l oading in two directions ($N_x^0 = \gamma_1 N_{cr}$; $N_y^0 = \gamma_2 N_{cr}$; $N_{xy}^0 = 0$). Based on the Navier method, the following expansions of displacements (u_0, v_0, w_0, θ) are ado pted to automatically satisfy the boundary conditions.

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn}\cos(\alpha\,x)\sin(\beta\,y) \\ V_{mn}\sin(\alpha\,x)\cos(\beta\,y) \\ W_{mn}\sin(\alpha\,x)\sin(\beta\,y) \\ X_{mn}\sin(\alpha\,x)\sin(\beta\,y) \end{Bmatrix} \tag{19}$$

where ($U_{mn}, V_{mn}, W_{mn}, X_{mn}$) are unknown functions to be determined and (α, β) are expressed by

$$\alpha = m\pi / a \quad \text{and} \quad \beta = n\pi / b \tag{20}$$

Substituting Eq. (19) into Eq. (17), the closed-form solution of buckling load N_{cr} can be obtained from

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33}+k & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ X_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{21}$$

where

$$\begin{aligned}
&S_{11} = -\left(A_{11}\alpha^2 + A_{66}\beta^2\right), \quad S_{12} = -\alpha\beta\,(A_{12}+A_{66}), \\
&S_{13} = \alpha\left(B_{11}\alpha^2 + B_{12}\beta^2 + 2B_{66}\beta^2\right), \\
&S_{14} = \alpha\left(k_1B_{11}^s + k_2B_{12}^s - (k_1A'+k_2B')B_{66}^s\beta^2\right) \\
&S_{22} = -\left(A_{66}\alpha^2 + A_{22}\beta^2\right), \\
&S_{23} = \beta\left(B_{22}\beta^2 + B_{12}\alpha^2 + 2B_{66}\alpha^2\right), \\
&S_{24} = \beta\left(k_2B_{22}^s + k_1B_{12}^s - (k_1A'+k_2B')B_{66}^s\alpha^2\right) \\
&S_{33} = -\left(D_{11}\alpha^4 + 2(D_{12}+2D_{66})\alpha^2\beta^2 + D_{22}\beta^4\right), \\
&S_{34} = -k_1\left(D_{11}^s\alpha^2 + D_{12}^s\beta^2\right) + 2(k_1A' + k_2B') \\
&\quad D_{66}^s\alpha^2\beta^2 - k_2(D_{22}^s\beta^2 + D_{12}^s\alpha^2) \\
&S_{44} = -k_1\left(H_{11}^s k_1 + H_{12}^s k_2\right) - (k_1A' + k_2B')^2 \\
&\quad H_{66}^s\alpha^2\beta^2 - k_2(H_{12}^s k_1 + H_{22}^s k_2) \\
&\quad -(k_1A')^2A_{55}^s\alpha^2 - (k_2B')^2A_{44}^s\beta^2 \\
&\quad ,\; k = N_{cr}\left(\gamma_1\,\alpha^2 + \gamma_2\beta^2\right)
\end{aligned} \tag{22}$$

By applying the condensation approach to eliminate the in-plane displacements U_{mn} and V_{mn}, Eq. (21) can be rewritten as

$$\begin{bmatrix} \bar{S}_{33}+k & \bar{S}_{34} \\ \bar{S}_{43} & \bar{S}_{44} \end{bmatrix} \begin{Bmatrix} W_{mn} \\ X_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{23}$$

where

$$\begin{aligned}
\bar{S}_{33} &= S_{33} - \frac{S_{13}(S_{13}S_{22} - S_{12}S_{23}) - S_{23}(S_{11}S_{23} - S_{12}S_{13})}{S_{11}S_{22} - S_{12}^2} \\
\bar{S}_{34} &= S_{34} - \frac{S_{14}(S_{13}S_{22} - S_{12}S_{23}) - S_{24}(S_{11}S_{23} - S_{12}S_{13})}{S_{11}S_{22} - S_{12}^2} \\
\bar{S}_{43} &= S_{34} - \frac{S_{13}(S_{14}S_{22} - S_{12}S_{24}) - S_{23}(S_{11}S_{24} - S_{12}S_{14})}{S_{11}S_{22} - S_{12}^2} \\
\bar{S}_{44} &= S_{44} - \frac{S_{14}(S_{14}S_{22} - S_{12}S_{24}) - S_{24}(S_{11}S_{24} - S_{12}S_{14})}{S_{11}S_{22} - S_{12}^2}
\end{aligned} \tag{24}$$

The system of homogeneous Eq. (23) has a nontrivial solution only for discrete values of the buckling load. For a nontrivial solution, the determinant of the coefficients (W_{mn}, X_{mn}) must equal zero

$$\begin{vmatrix} \bar{S}_{33}+k & \bar{S}_{34} \\ \bar{S}_{43} & \bar{S}_{44} \end{vmatrix} = 0 \tag{25}$$

The resulting equation may be solved for the buckling load. This gives the following expression for buckling load:

$$k = \frac{\bar{S}_{34}\bar{S}_{43} - \bar{S}_{33}\bar{S}_{44}}{\bar{S}_{44}} \tag{26}$$

By employing the Eq. (25), the following expression for critical buckling load is determined

$$N_{cr}(m,n) = \frac{1}{\left(\gamma_1\,\alpha^2 + \gamma_2\beta^2\right)} \frac{\bar{S}_{34}\bar{S}_{43} - \bar{S}_{33}\bar{S}_{44}}{\bar{S}_{44}} \tag{27}$$

For the case of CPT, the expression of buckling load N_{cr} can be simplified by setting the shear component of transverse displacement to zero ($\theta = 0$) as

$$N_{cr}(m,n) = \frac{-\bar{S}_{33}}{\left(\gamma_1 \alpha^2 + \gamma_2 \beta^2\right)} \tag{28}$$

For each choice of m and n, there is a correspon sive unique value of N_{cr}. The critical buckling load i s the smallest value of $N_{cr}(m,n)$.

4. Results and discussion

In this section, numerical examples are proposed and discussed for checking the accuracy and simplicity of the developed theory in determining the critical buckling load of FG plates under in-plane loading. For the verification purpose, the results computed by present model are compared with those existing in the literature by employing CPT, FSDT and HSDT. The following material properties are employed:

- Material 1 (Al/Al_2O_3) $E_c = 380\,\text{GPa}$, $E_m = 70\,\text{GPa}$, $\nu = 0.3$
- Material 2 (Al/SiC) $E_c = 420\,\text{GPa}$, $E_m = 70\,\text{GPa}$, $\nu = 0.3$

4.1 Comparison studies

Example 1: The first example is performed for simply supported rectangular plate ($a/b = 0.5$) with linear distribution of the volume fractions of the constituents ($p = 1$). The structure is fabricated from a mixture of Aluminum (Al) and Alumina (Al_2O_3), and subjected to different types of axial loading. Table 1 presents the comparisons of the critical buckling loads computed by the present model with those reported by Javaheri and Eslami (2002) based on CPT, Shariat and Eslami (2005) based on FSDT, and Bodaghi and Saidi (2010) based on HSDT. It can be observed that the results of present model are in excellent agreement with those given by HSDT (Bodaghi and Saidi 2010) for all values of thickness ratio a/h. It should be noted that the proposed theory uses only four independent variables as against five in the case of HSDT (Bodaghi and Saidi 2010) and FSDT (Shariat and Eslami 2005). Also, the proposed theory does not required shear correction coefficients as in the case of FSDT. It can be confirmed that the proposed theory is not only accurate but also efficient and simple in determining critical buckling load of FG plates. It is also seen that the CPT overestimates the critical buckling force of FG plates. The difference between CPT and shear deformation theories is significant for thick plate and negligible for thin plate due to the effects of the transverse shear deformation.

Example 2: The next comparison is carried out for FG plates under various loading conditions. The plate is made from a mixture of Aluminum (Al) and SiliconCarbide (SiC). The critical buckling forces of simply supported plate for different values of thickness ratio b/h, aspect ratio a/b, and gradient index p are demonstrated in Table 2. It can be observed that the critical buckling force predicted by the proposed theor are almost identical with those given by (Bodaghi and Saidi 2010) based on HSDT, and the change of critical buckling mode of FG plate determined by the proposed model and HSDT are identical.

4.2 Parameter studies

Parameter investigations are presented to examine the influences of loading types and variations of gradientindex p, modulus ratio E_m/E_c, thickness ratio a/h, and aspect ratio b/a on the non-dimensional critical buckling load $\bar{N} = N_{cr}a^2/E_m h^3$ of Al/Al_2O_3 plates.

Fig. 1 illustrates the variation of non-dimensional critical buckling force of square FG plates with different loading types versus the gradient index p. The thickness ratio a/h is considered to be 10. It can be observed that with increasing the gradient index, the non-dimensional critical buckling force decreases, and the variation of the non-dimensional critical buckling force is considerable when the gradient index is small. This is due to the fact that higher values of gradient index correspond to high portion of metal in comparison with the ceramic part. Moreover, the non-dimensional critical buckling force of plate under uniaxial compression ($\gamma_1 = -1, \gamma_2 = 0$) is greater than that under biaxial compression ($\gamma_1 = \gamma_2 = -1$) and less than that under biaxial compression and tension ($\gamma_1 = -1, \gamma_2 = 1$).

Fig. 2 demonstrates the variation of non-dimensional critical buckling force of square plate versus the modulus ratio E_m/E_c for different values of gradient index. The thickness ratio a/h is considered to be 10. It can be observed that the non-dimensional critical buckling force increases as the modulus ratio E_m/E_c increases, and decreases as the gradient index increases.

The variation of non-dimensional critical buckling force of plate versus thickness ratio a/h is presented in Fig. 3 by employing the proposed theory and CPT. Since the transverse shear deformation influences of plate are neglected in the CPT, the values of non-dimensional critical buckling force computed by CPT are independent of thickness ratio. Whereas, the values of non-dimensional critical buckling force computed by the proposed theory, which considers the transverse shear deformation influences, are dependent of thickness ratio. It is demonstrated that the non-dimensional critical buckling force increases by increasing the thickness ratio a/h, while the CPT overestimates the non-dimensional critical buckling force of FG plate. The difference between two models is significant for thick plates ($a/h < 10$), and negligible for thin plates.

Table 1 Comparison of critical buckling load (MN) of simply supported Al/Al_2O_3 plate ($a/b = 0.5$, $p = 1$)

(γ_1, γ_2)	Method	a/h					
		5	10	20	30	40	50
(1.0)	CPT [(a)]	267.4800	33.4350	4.1794	1.2383	0.5224	0.2675
	FSDT [(b)]	243.4100	32.6280	4.1537	1.2349	0.5216	0.2672
	HSDT [(c)]	239.1500	32.4720	4.1486	1.2343	0.5215	0.2672
	Present	239.1450	32.4721	4.1486	1.2343	0.5215	0.2672
(1,1)	CPT [(a)]	213.9900	26.7480	3.4353	0.9907	0.4179	0.2140
	FSDT [(b)]	194.7300	26.1030	3.3230	0.9880	0.4173	0.2137
	HSDT [(c)]	191.3200	25.9780	3.3189	0.9879	0.4172	0.2137
	Present	191.3160	25.9777	3.3189	0.9879	0.4172	0.2137
(1.-1)	CPT [(a)]	356.6400	44.5800	5.5725	1.6511	0.6966	0.3566
	FSDT [(b)]	324.5400	43.5050	5.5383	1.6466	0.6955	0.3563
	HSDT [(c)]	318.8600	43.2960	5.5315	1.6457	0.6953	0.3562
	Present	318.8600	43.2961	5.5315	1.6457	0.6953	0.3562

[(a)] (Javaheri and Eslami 2002)
[(b)] (Shariat and Eslami 2005)
[(c)] (Bodaghi and Saidi 2010)

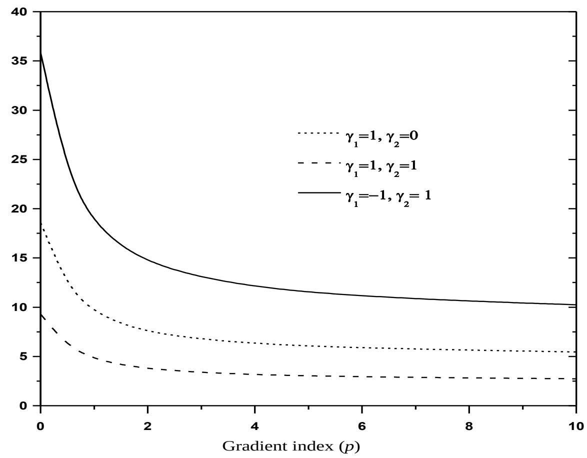

Fig. 1 The effect of the gradient index p on non-dimensional critical buckling load $\overline{N}$ of simply supported square plate ($a/h = 10$) under different loading conditions

The influences of aspect ratio b/a on non-dimensional critical buckling force of FG plate subjected to uniaxial compression and biaxial compression are illustrated in Figs. 4 and 5, respectively. The thickness ratio a/h is considered to be 10. It is observed that the non-dimensional critical buckling force generally decreases by the increase of b/a. In the case of uniaxial compression as demonstrated in Fig. 4, the graph is not smooth due to the change of critical buckling mode as the aspect ratio increases. Whereas, the graph in the case of biaxial compression as demonstrated in Fig. 5 is smooth because of the existence of a single critical buckling mode regardless of aspect ratio b/a.

Tables 3-5 provide the non-dimensional critical buckling loads for FG plates under uniaxial compression, biaxial compression, and biaxial compression and tension, respectively. It is demonstrated from Tables 3-5 that the non-dimensional critical buckling force increases by the decrease of gradient index and the increase of thickness ratio. Moreover, increasing not only increases the values of non-dimensional critical buckling force, but also induces the changes in critical buckling mode. For example, for the plate under uniaxial compression along x-axis with, the critical buckling mode varies from 3 to 2 as the value of thickness ratio increases from 5 to 10. In the case of plate subjected to biaxial compression (see Table 4 and Fig. 5), only one critical buckling mode exists regardless of aspect ratio, thickness ratio, and gradient index.

Table 2 Comparison of critical buckling load (MN/m) of simply supported Al/SiC plate

(γ_1,γ_2)	a/b	b/h	Method	p = 0	1	2
(-1.0)	0.5	10	HSDT (*)	2079.721	1028.412	780.097
			Present	2079.758	1028.449	780.023
		5	HSDT (*)	12162.119	6270.298	4692.542
			Present	12164.987	6272.425	4695.029
	1	10	HSDT (*)	1437.361	702.304	534.441
			Present	1437.389	702.251	534.835
		5	HSDT (*)	9915.620	4955.431	3746.054
			Present	9916.193	4955.484	3746.732
	1.5	10	HSDT (*)	1527.903[a]	748.920[a]	569.751[a]
			Present	1527.994[a]	748.988[a]	569.528[a]
		5	HSDT (*)	10044.721[a]	5067.219[a]	3819.109[a]
			Present	10044.962[a]	5068.084[a]	3820.079[a]
(-1.-1)	0.5	10	HSDT (*)	1663.777	822.738	624.158
			Present	1663.807	822.759	624.182
		5	HSDT (*)	9729.999	5016.384	3754.274
			Present	9731.990	5017.941	3756.023
	1	10	HSDT (*)	718.692	351.124	267.416
			Present	718.695	351.125	267.418
		5	HSDT (*)	4957.888	2477.589	1873.190
			Present	4958.097	2477.742	1873.366
	1.5	10	HSDT (*)	526.861	256.776	195.714
			Present	526.862	256.776	195.714
		5	HSDT (*)	3772.877	1871.038	1418.120
			Present	3772.964	1871.101	1418.193
(-1.1)	0.5	10	HSDT (*)	2772.980	1371.653	1040.519
			Present	2773.011	1371.265	1040.304
		5	HSDT (*)	16216.712	8360.541	6257.811
			Present	16219.983	8363.233	6260.038
	1	10	HSDT (*)	2772.980[a]	1371.653[a]	1040.519[a]
			Present	2773.011[a]	1371.265[a]	1040.304[a]
		5	HSDT (*)	16216.712[a]	8360.541[a]	6257.811[a]
			Present	16219.983[a]	8363.233[a]	6260.038[a]
	1.5	10	HSDT (*)	2772.980[b]	1371.653[b]	1040.519[b]
			Present	2773.011[b]	1371.265[b]	1040.304[b]
		5	HSDT (*)	16216.712[b]	8360.541[b]	6257.811[b]
			Present	16219.983[b]	8363.233[b]	6260.038[b]

(*) (Bodaghi and Saidi 2010)
[a] Mode for plate is (m, n) = (2, 1).
[b] Mode for plate is (m, n) = (3, 1)

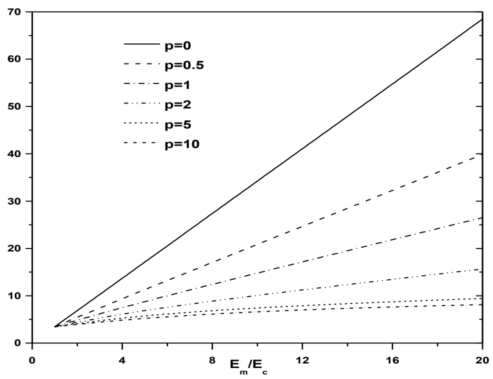

Fig. 2 The effect of modulus ratio on non-dimensional critical buckling load $\overline{N}$ of simply supported square plate ($a/h=10$) under uniaxial compression along the x-axis $(\gamma_1=-1,\gamma_2=0)$

Table 3 Non-dimensional critical buckling load $\overline{N}$ of simply supported Al/Al_2O_3 plate subjected to uniaxial compression along the *x*-axis ($\gamma_1 = -1$, $\gamma_2 = 0$)

a/b	a/h	p 0	0.5	1	2	5	10	20	100
0.5	5	6.7203	4.4235	3.4164	2.6451	2.1484	1.9213	1.7115	1.3737
	10	7.4053	4.8206	3.7111	2.8897	2.4165	2.1896	1.9387	1.5251
	20	7.5993	4.9315	3.7930	2.9582	2.4944	2.2690	2.0054	1.5683
	50	7.6555	4.9634	3.8166	2.9779	2.5172	2.2923	2.0250	1.5809
	100	7.6635	4.9680	3.8200	2.9808	2.5205	2.2957	2.0278	1.5827
1.0	5	16.0211	10.6254	8.2245	6.3432	5.0531	4.4807	4.0070	3.2586
	10	18.5785	12.1229	9.3391	7.2631	6.0353	5.4528	4.8346	3.8198
	20	19.3528	12.5668	9.6675	7.5371	6.3448	5.7668	5.0988	3.9923
	50	19.5914	12.6970	9.7636	7.6177	6.4373	5.8614	5.1782	4.0434
	100	19.6145	12.7158	9.7775	7.6293	6.4507	5.8752	5.1897	4.0508
1.5	5	28.1996[a]	19.2510[a]	19.2510[a]	11.4234[a]	8.4727[a]	7.2952[a]	6.6106[a]	5.6325[a]
	10	40.7476[a]	26.9091[a]	20.8024[a]	16.0793[a]	12.9501[a]	11.5379[a]	10.2958[a]	8.3112[a]
	20	45.8930[a]	29.9050[a]	23.0286[a]	17.9221[a]	14.9472[a]	13.5273[a]	11.9843[a]	9.4447[a]
	50	47.5784[a]	30.8691[a]	23.7414[a]	18.5177[a]	15.6238[a]	14.2156[a]	12.5629[a]	9.8207[a]
	100	47.8297[a]	31.0119[a]	23.8469[a]	18.6061[a]	15.7256[a]	14.3198[a]	12.6502[a]	9.8769[a]
2.0	5	37.7404[b]	26.3645[b]	20.7491[b]	15.5819[b]	10.9554[b]	9.1505[c]	8.3988[c]	7.4403[b]
	10	64.0842[a]	42.5015[a]	32.8980[a]	25.3727[a]	20.2123[a]	17.9227[a]	16.0280[a]	13.0345[a]
	20	74.3140[a]	48.4917[a]	37.3564[a]	29.0523[a]	24.1413[a]	21.8114[a]	19.3385[a]	15.2794[a]
	50	77.8004[a]	50.4890[a]	38.8338[a]	30.2858[a]	25.5363[a]	23.2278[a]	20.5301[a]	16.0561[a]
	100	78.3257[a]	50.7880[a]	39.0546[a]	30.4707[a]	25.7491[a]	23.4456[a]	20.7126[a]	16.1737[a]

[a] Mode for plate is (*m*, *n*) = (2, 1)
[b] Mode for plate is (*m*, *n*) = (3, 1)
[c] Mode for plate is (*m*, *n*) = (4, 1)

Table 4 Non-dimensional critical buckling load $\overline{N}$ of simply supported Al/Al_2O_3 plate subjected to biaxial compression ($\gamma_1 = -1$, $\gamma_2 = -1$)

a/b	a/h	p 0	0.5	1	2	5	10	20	100
0.5	5	5.3762	3.5388	2.7331	2.1161	1.7187	1.5370	1.3692	1.0990
	10	5.9243	3.8565	2.9689	2.3117	1.9332	1.7517	1.5510	1.2200
	20	6.0794	3.9452	3.0344	2.3665	1.9955	1.8152	1.6044	1.2547
	50	6.1244	3.9708	3.0533	2.3823	2.0137	1.8338	1.6200	1.2647
	100	6.1308	3.9744	3.0560	2.3846	2.0164	1.8365	1.6222	1.2662
1.0	5	8.0105	5.3127	4.1122	3.1716	2.5264	2.2403	2.0035	1.6293
	10	9.2893	6.0615	4.6696	3.6315	3.0177	2.7264	2.4173	1.9099
	20	9.6764	6.2834	4.8337	3.7686	3.1724	2.8834	2.5494	1.9961
	50	9.7907	6.3485	4.8818	3.8088	3.2186	2.9307	2.5891	2.0217
	100	9.8073	6.3579	4.8888	3.8147	3.2254	2.9376	2.5948	2.0254
1.5	5	11.6820	7.8299	6.0799	4.6637	3.6176	3.1718	2.8510	2.3600
	10	14.6084	9.5685	7.3793	5.7279	4.7124	4.2384	3.7657	2.9959
	20	15.7985	10.1332	7.7977	6.0761	5.1006	4.6300	4.0961	3.2135
	50	15.8875	10.3036	7.9236	6.1815	5.2212	4.7531	4.1995	3.2803
	100	15.9312	10.3284	7.9419	6.1969	5.2389	4.7712	4.2147	3.2900
2.0	5	15.7235	10.6622	8.3092	6.3353	4.7754	4.1382	3.7392	3.1534
	10	21.5050	14.1552	10.9323	8.4644	6.8750	6.1481	5.4769	4.3958
	20	23.6970	15.4260	11.8755	9.2469	7.7327	7.0067	6.2040	4.8802
	50	24.7985	15.8244	12.1700	9.4931	8.0132	7.2926	6.4440	5.0358
	100	24.4974	15.8830	12.2132	9.5294	8.0550	7.3353	6.4799	5.0589

Table 5 Non-dimensional critical buckling load $\overline{N}$ of simply supported Al/Al_2O_3 plate subjected to biaxial compression and tension ($\gamma_1 = -1, \gamma_2 = 1$)

a/b	a/h	p							
		0	0.5	1	2	5	10	20	100
0.5	5	8.9604	5.8980	4.5551	3.5268	2.8646	2.5617	2.2820	1.8316
	10	9.8738	6.4275	4.9481	3.8529	3.2219	2.9195	2.5850	2.0334
	20	10.1324	6.5753	5.0574	3.9442	3.3259	3.0253	2.6739	2.0911
	50	10.2073	6.6179	5.0888	3.9706	3.3562	3.0564	2.7000	2.1079
	100	10.2181	6.6241	5.0934	3.9744	3.3606	3.0609	2.7037	2.1103
1.0	5	26.2058[a]	17.7704[a]	13.8486[a]	10.5589[a]	7.9590[a]	6.8970[a]	6.2320[a]	5.2556[a]
	10	35.8416[a]	23.5920[b]	18.2206[a]	14.1073[a]	11.4583[a]	10.2468[a]	9.1281[a]	7.3263[a]
	20	39.4951[a]	25.7100[a]	19.7925[a]	15.4115[a]	12.8878[a]	11.6779[a]	10.3400[a]	8.1336[a]
	50	40.6574[a]	26.3740[a]	20.2833[a]	15.8219[a]	13.3554[a]	12.1543[a]	10.7401[a]	8.3931[b]
	100	40.8291[a]	26.4717[a]	20.3554[a]	15.8823[a]	13.4250[a]	12.2256[a]	10.7998[a]	8.4315[b]
1.5	5	29.0249[b]	20.1105[b]	15.7823[b]	11.9009[b]	8.5250[b]	7.2422[b]	6.6008[b]	5.7477[b]
	10	37.9819	24.8781	19.1863	14.8925	12.2523	11.0199	9.7909	7.7894
	20	40.5307	26.3463	20.2740	15.7980	13.2616	12.0379	10.6500	8.3551
	50	41.3076	26.7894	20.6013	16.0719	13.5752	12.3580	10.9186	8.5287
	100	41.4211	26.8539	20.6489	16.1118	13.6212	12.4052	10.9581	8.5541
2.0	5	26.2058	17.7704	13.8486	10.5589	7.9590	6.8970	6.2320	5.2556
	10	35.8416	23.5920	18.2206	14.1073	11.4583	10.2468	9.1281	7.3263
	20	39.4951	25.7100	19.7925	15.4115	12.8878	11.6779	10.3400	8.1336
	50	40.6574	26.3740	20.2833	15.8219	13.3554	12.1543	10.7401	8.3931
	100	40.8291	26.4717	20.3554	15.8823	13.4250	12.2256	10.7998	8.4315

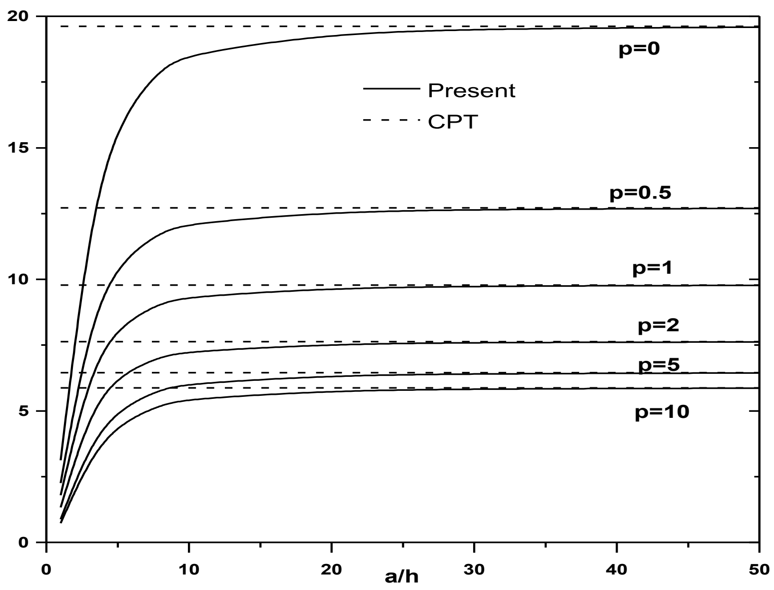

Fig. 3 The effect of thickness ratio on non-dimensional critical buckling load $\overline{N}$ of simply supported square plate under uniaxial compression along the *x*-axis $(\gamma_1 = -1, \gamma_2 = 0)$

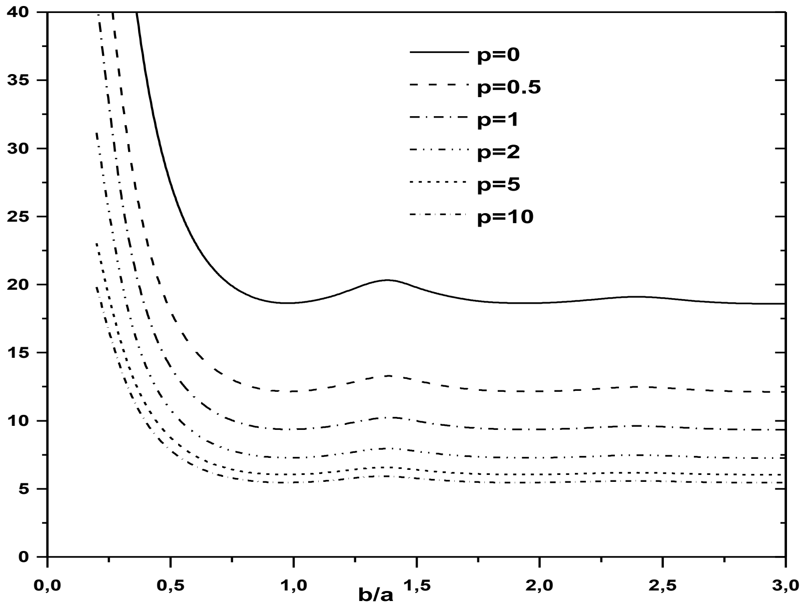

Fig. 4 The effect of aspect ratio on non-dimensional critical buckling load $\overline{N}$ of simply supported rectangular plate ($a/h=10$) under uniaxial compression along the *y*-axis $(\gamma_1 = 0, \gamma_2 = -1)$

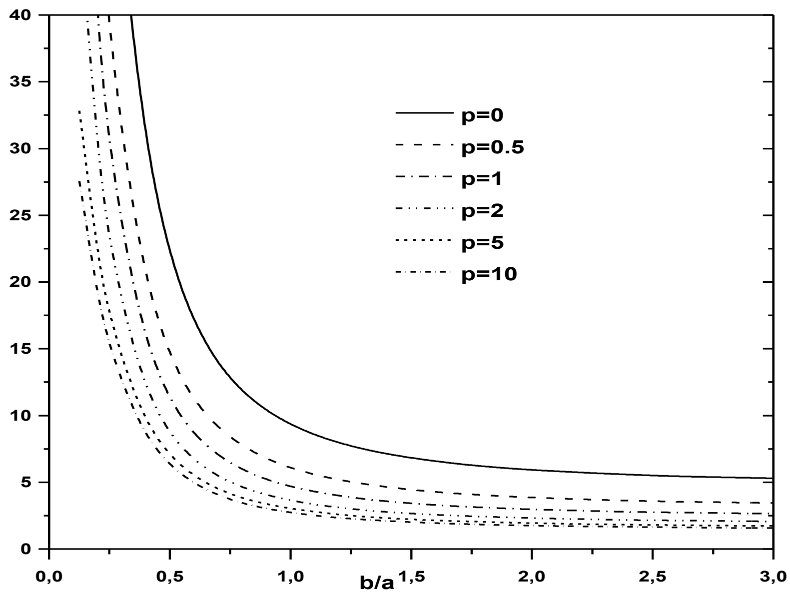

Fig. 5 The effect of aspect ratio on non-dimensional critical buckling load $\overline{N}$ of simply supported rectangular plate ($a/h=10$) under biaxial compression $(\gamma_1 = -1, \gamma_2 = -1)$

5. Conclusions

An efficient and simple refined plate theory is proposed for buckling behavior of FG plates. By making further simplifying assumptions to the existing HSDTs, with the inclusion of an undetermined integral term, the number of unknowns and governing equations of the proposed HSDT are reduced by one, and hence, make this theory simple and efficient to use. The accuracy and efficiency of the proposed model have been demonstrated for buckling investigation of simply supported FG plates. It can be concluded that the proposed theory is not only accurate but also efficient in determining the critical buckling forces of FG plate compared to other shear deformation plate theories such as FSDT and HSDT. An improvement of proposed formulation will be considered in the future work to consider the thickness stretching effect by using quasi-3D shear deformation models (Bessaim *et al.* 2013, Bousahla *et al.* 2014, Belabed *et al.* 2014, Fekrar *et al.* 2014, Hebali *et al.* 2014, Meradjah *et al.* 2015, Larbi Chaht *et al.* 2015, Bennai *et al.* 2015, Hamidi *et al.* 2015, Bourada *et al.* 2015, Bennoun *et al.* 2016, Draiche *et al*. 2016, Benbakhti *et al*. 2016, Benahmed *et al*. 2017, Ait Atmane *et al*. 2017, Benchohra *et al*. 2017, Bouafia *et al*. 2017) and the wave propagation problem (Mahmoud *et al*. 2015, Ait Yahia *et al*. 2015, Boukhari *et al*. 2016).

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