Steel and Composite Structures, *Vol. 25, No. 2 (2017) 157-175*
DOI: https://doi.org/10.12989/scs.2017.25.2.157

A new and simple HSDT for thermal stability analysis of FG sandwich plates

Abderrahmane Menasria [1,2], Abdelhakim Bouhadra [1,2], Abdelouahed Tounsi [*1,3,4], Abdelmoumen Anis Bousahla [1,4,5] and S.R. Mahmoud [6]

[1] *Materials and Hydrology Laboratory, University of Sidi Bel Abbes, Faculty of Technology, Civil Engineering Department, Algeria*
[2] *Université de Abbés Laghrour Khenchela, Faculté de Sciences & Technologie, Département de Génie Civil, Algeria*
[3] *Laboratoire des Structures et Matériaux Avancés dans le Génie Civil et Travaux Publics, Université de Sidi Bel Abbes, Faculté de Technologie, Département de Génie Civil, Algeria*
[4] *Laboratoire de Modélisation et Simulation Multi-échelle, Université de Sidi Bel Abbés, Algeria*
[5] *Centre Universitaire de Relizane, Algérie*
[6] *Department of Mathematics, Faculty of Science, King Abdulaziz University, Saudi Arabia*

(*Received March 29, 2017, Revised June 21, 2017, Accepted June 28, 2017*)

Abstract. The novelty of this work is the use of a new displacement field that includes undetermined integral terms for analyzing thermal buckling response of functionally graded (FG) sandwich plates. The proposed kinematic uses only four variables, which is even less than the first shear deformation theory (FSDT) and the conventional higher shear deformation theories (HSDTs). The theory considers a trigonometric variation of transverse shear stress and verifies the traction free boundary conditions without employing the shear correction factors. Material properties of the sandwich plate faces are considered to be graded in the thickness direction according to a simple power-law variation in terms of the volume fractions of the constituents. The core layer is still homogeneous and made of an isotropic material. The thermal loads are assumed as uniform, linear and non-linear temperature rises within the thickness direction. An energy based variational principle is employed to derive the governing equations as an eigenvalue problem. The validation of the present work is checked by comparing the obtained results the available ones in the literature. The influences of aspect and thickness ratios, material index, loading type, and sandwich plate type on the critical buckling are all discussed.

Keywords: thermal buckling; sandwich plate; functionally graded materials; plate theory

1. Introduction

Sandwich structures, because of their outstanding characteristics, such as high stiffness and low weight, have been widely employed in fields of aircraft, aerospace, naval/marine, construction, transportation, and wind energy systems (Vinson 2001, 2005). However, the abrupt variation in material characteristics within the interfaces between the face sheets and the core can result in large interlaminar stresses inducing delamination, which is an important problem in classical sandwich structures. Furthermore, the difference in the values of thermal coefficients of the materials may induce residual stresses. In order to overcome these problems, the concept of functionally graded material (FGM) is introduced in sandwich plates design. FGM are heterogeneous advanced composite material where the mechanical properties are changed continuously and gradually from one surface of the structure to the other. The use of such materials helps us to eliminate mechanically and thermally the induced stresses because of the material property mismatch and to improve the bonding strength (Birman *et al*. 2013, Bouderba *et al*. 2013, Zidi *et al*. 2014, Merazi *et al*. 2015, Swaminathan *et al*. 2015, Akbaş 2015, Arefi 2015a, b, Arefi and Allam 2015, Zemri *et al*. 2015, Belkorissat *et al*. 2015, Boukhari *et al*. 2016, Bounouara *et al*. 2016, Mouaici *et al*. 2016, Hadji *et al*. 2016, Saidi *et al*. 2016, Abdelbari *et al*. 2016, Ebrahimi and Shafiei 2016, Bousahla *et al*. 2016, Turan *et al*. 2016, Celebi *et al*. 2016, Akbarov *et al*. 2016, Raminnea *et al*. 2016, Aizikovich *et al*. 2016, Darabi and Vosoughi 2016, Ebrahimi and Jafari 2016, Benferhat *et al*. 2016, Houari *et al*. 2016, Mouffoki *et al*. 2017, Zidi *et al*. 2017).

Buckling and bending of FGM plates under thermal or thermo-mechanical loads has been investigated by many researchers. Cheng and Batra (2000) examined the thermo-mechanical deformations of a linear elastic FG elliptic plate with rigidly clamped edges. Reddy and Cheng (2001) studied 3D thermo-mechanical deformations of simply supported FG rectangular plates by employing an asymptotic approach. Vel and Batra (2002) proposed an exact solution for 3D deformations of a simply supported FG thick plate subjected to mechanical and thermal loads. Feldman and Aboudi (1997) investigated the elastic bifurcation of FG plates under in-plane compressive loading based on a combination of micromechanical and structural approaches. Matsunaga (1997) showed the buckling instabilities of a simply supported thick elastic plate subjected to in-plane stresses. Najafizadeh and Eslami (2002) analyzed the buckling response of radially loaded solid circular plate made of functionally graded material.

*Corresponding author, Professor,
E-mail: tou_abdel@yahoo.com

ISSN: 1229-9367 (Print), 1598-6233 (Online)

By employing the HSDT, Najafizadeh and Heydari (2008) also presented an exact solution for buckling of FG circular plates under uniform radial compression. Thermal stability of a simply supported FG skew plate by employing the FSDT in conjunction with the finite element method was studied by Ganapathi and Prakash (2006). Based on the classical plate theory (CPT), Ma and Wang (2003) examined the axisymmetric large deflection bending of a FG circular plate is investigated under mechanical, thermal loadings. Bui *et al*. (2013) investigated the transient responses and natural frequencies of sandwich beams with inhomogeneous functionally graded core by proposing a novel truly meshfree method in which the displacement field is approximated by the radial point interpolation method (RPIM) regardless of predefined mesh, and the domain integrals are evaluated by the so-called Cartesian transformation method (CTM) to obviate the need for a background cell. Kar and Panda (2015a) analyzed the large deformation bending behavior of FG spherical shell using finite element method. Kar and Panda (2015b) also studied the nonlinear flexural vibration of shear deformable FG spherical shell panel. Morimoto *et al*. (2006) discussed the thermal buckling behavior of FG rectangular plates subjected to partial heating in a plane and uniform temperature rise within its thickness. Na and Kim (2006) investigated the FG composite structures that composed of ceramic, functionally graded material (FGM), and metal layers. By employing finite element method, Na and Kim (2004) also studied the 3D thermal buckling of FG plates. Lee *et al*. (2010) studied the post-buckling response of FG plates under edge compression and temperature field conditions using the element-free kp-Ritz method. Based on neutral surface of structures, Lee *et al*. (2016) also examined the thermal stability behavior of FG plates. The post-buckling of piezoelectric FG plates under thermo-electro-mechanical loading is studied by Liew *et al*. (2003) by employing the HSDT. Bourada *et al*. (2012) proposed a new four-variable refined plate theory for thermal buckling analysis of FG sandwich plates. Tran *et al*. (2013) proposed an isogeometric finite element approach (IGA) in combination with the third-order deformation plate theory for thermal stability analysis of FG plates. Yaghoobi *et al*. (2014) presented and analytical study on postbuckling and nonlinear free vibration analysis of FG beams resting on nonlinear elastic foundation under thermo-mechanical loading. By utilizing a local Kriging meshless method Zhang *et al*. (2014) examined the mechanical and thermal buckling behaviors of FG plates. Tung (2015) investigated the nonlinear bending and post-buckling response of FG sandwich plates resting on elastic foundations and subjected to uniform external pressure. By employing the exact 3D theory of elasticity, instead of the approximate plate models, Asemi and Shariyat (2016) the influences of heterogeneous auxetic of the materials in uniaxial and biaxial post-buckling responses of the FG plates. Yu *et al*. (2016a) studied the numerical results of thermal stability of FG plates with internal defects (e.g., crack or cutout) using the extended isogeometric investigation (XIGA). Based on the FSDT, Yaghoobi and Yaghoobi (2013) investigated the stability response of sandwich plates with FG face sheets resting on elastic foundation. Sobhy (2013) examined the stability and free vibration of exponentially graded sandwich plates resting on elastic foundations under various boundary conditions. Ait Amar Meziane *et al*. (2014) discussed the buckling and dynamic behavior of FG sandwich plates using an efficient and simple refined shear deformation theory. Swaminathan and Naveenkumar (2014) presented analytical solutions for the stability analysis of FG plates using higher order refined computational models. Adda Bedia *et al*. (2015) examined the thermal buckling characteristics of armchair single-walled carbon nanotube embedded in an elastic medium based on nonlocal continuum elasticity. Nguyen *et al*. (2015) proposed a refined higher-order shear deformation theory for bending, vibration and buckling analysis of FG sandwich plates. Bouguenina *et al*. (2015) presented a numerical analysis of FGM plates with variable thickness subjected to thermal buckling. Bakora and Tounsi (2015) investigated the thermo-mechanical post-buckling behavior of thick FG plates resting on elastic foundations. Barati and Shahverdi (2016) presented a four-variable plate theory for thermal vibration of embedded FG nanoplates under non-uniform temperature distributions with different boundary conditions. Beldjelili *et al*. (2016) studied the hygro-thermo-mechanical bending of S-FGM plates resting on variable elastic foundations using a four-variable trigonometric plate theory. Ahouel *et al*. (2016) discussed the Size-dependent mechanical behavior of functionally graded trigonometric shear deformable nanobeams including neutral surface position concept. Abdelhak *et al*. (2016) examined the thermal buckling response of functionally graded sandwich plates with clamped boundary conditions. Barka *et al*. (2016) discussed the thermal post-buckling behavior of imperfect temperature-dependent sandwich FGM plates resting on Pasternak elastic foundation. El-Hassar *et al*. (2016) analyzed the thermal stability of solar functionally graded plates on elastic foundation using an efficient hyperbolic shear deformation theory. Ghorbanpour Arani *et al*. (2016) studied dynamic buckling of FGM viscoelastic nano-plates resting on orthotropic elastic medium based on sinusoidal shear deformation theory. Trinh *et al*. (2016) analyzed post-buckling responses of elastoplastic FGM beams on nonlinear elastic foundation. Chikh *et al*. (2016) investigated the thermo-mechanical postbuckling of symmetric S-FGM plates resting on Pasternak elastic foundations using hyperbolic shear deformation theory. Bouderba *et al*. (2016) analyzed the thermal buckling response of FG sandwich plates using a simple shear deformation theory. Laoufi *et al*. (2016) analyzed the mechanical and hygrothermal behaviour of FG plates using a hyperbolic shear deformation theory. Khetir *et al*. (2017) proposed a new nonlocal trigonometric shear deformation theory for thermal buckling analysis of embedded nanosize FG plates. Klouche *et al*. (2017) presented an original single variable shear deformation theory for buckling analysis of thick isotropic plates. Meksi *et al*. (2017) presented and analytical solution for bending, buckling and vibration responses of FGM sandwich plates. Liu *et al*. (2017) analyzed FG plates by a simple locking-free quasi-

3D hyperbolic plate isogeometric method. Yu *et al*. (2017) employed isogeometric analysis (IGA) based on the FSDT without shear-locking effect to present new numerical results of thermal-mechanical buckling of FG rectangular and skew plates under combined thermal and mechanical loads. El-Haina *et al*. (2017) proposed a simple analytical approach for thermal buckling of thick FG sandwich plates. Bellifa *et al*. (2017) proposed a nonlocal zeroth-order shear deformation theory for nonlinear postbuckling of nanobeams. Yin *et al*. (2016) presented a numerical study of buckling and free vibration of functionally graded plates considering in-plane material inhomogeneity using a new and effective approach based on isogeometric analysis (IGA) and HSDT. Bui *et al*. (2016) presented a finite element formulation for heated FG plate taking the advantages of a novel simple third-order shear deformation plate theory. The advantage of this novel theory is it substantially provides more accuracy than other higher-order shear deformation plate theories. It may be due to the fact that the kinematic of displacements is derived from an elasticity formulation rather than the hypothesis of displacements as is described by Shi (2007). Although the theory presented by Bui *et al*. (2016) has only five variables, the development of other theories with fewer variables is interesting for the present work.

This work presents an analytical solution to investigate the thermal stability behavior of FG sandwich plates under uniform, linear and non-linear temperature rise using a new trigonometric shear deformation theory. The new plate theory is constructed by including the integral term in the kinematics leading to a reduction in the number of unknowns and governing equations. Thus, it can be noted that the novelty of this theory is the use of the integral term in in-plane displacements contrary to other similar simple theories with four variables (Vu *et al*. 2017, Yu *et al*. 2015, 2015, 2016b) where we this term is not employed. The mathematical formulations include the influences of thermal loads, and von-Karman-type nonlinearity. Effects of gradient index, geometric parameters, and temperature distributions on the response of FG sandwich FG plate are discussed in details. The results of this work are compared with the known data in the literature.

2. Statement of the problem

The geometry and dimensions of the sandwich FG plate made are represented in Fig. 1. Rectangular Cartesian coordinates (x, y, z) are employed to describe infinitesimal deformations of a three-layer sandwich elastic plate occupying the region $[0, a] \times [0, b] \times [-h/2, h/2]$ in the unstressed reference configuration, and the axes are parallel to the edges of the plate. The plate has length a, width b, and uniform thickness h. The mid-plane of the sandwich FG plate is defined by $z = 0$ and its external surfaces being defined by $z = \pm h/2$. The vertical positions of the lower surface, the two interfaces between the core and faces layers, and the upper surface are denoted, respectively, by $h_0 = -h/2$, h_1, h_2 and $h_3 = h/2$.

The effective material properties for each layer, such as Young's modulus, Poisson's ratio, and thermal expansion

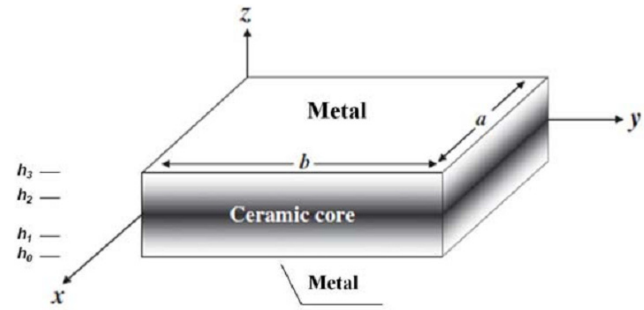

Fig. 1 Geometry of the FGM sandwich plate

coefficient, can be defined as (Ait Atmane *et al*. 2015, Mahi *et al*. 2015, Meksi *et al*. 2015, Ait Yahia *et al*. 2015, Attia *et al*. 2015, Tounsi *et al*. 2016)

$$P^{(j)}(z) = (P_c - P_m)V^{(j)}(z) + P_m \tag{1}$$

where $P^{(j)}$ is the effective material property of FGM of layer j. P_m and P_c are the Young's modulus (E), Poisson's ratio (v) of the lower and upper faces of layer 1 ($h_0 \le z \le h_1$), respectively, and vice versa for layer 3 ($h_2 \le z \le h_3$) depending on the volume fraction $V^{(j)}$ ($j = 1, 2, 3$). Note that P_m and P_c are, respectively, the corresponding properties of the metal and ceramic of the FG sandwich plate. The volume fraction $V^{(j)}$ of the FGMs is assumed to obey a power-law function along the thickness direction (Bourada *et al*. 2012, Tounsi *et al*. 2013, Taibi *et al*. 2015, Bennoun *et al*. 2016)

$$\begin{cases} V^{(1)}(z) = \left(\dfrac{z-h_0}{h_1-h_0}\right)^k & \text{for } z \in [h_0, h_1] \\ V^{(2)}(z) = 1 & \text{for } z \in [h_1, h_2] \\ V^{(3)}(z) = \left(\dfrac{z-h_3}{h_2-h_3}\right)^k & \text{for } z \in [h_2, h_3] \end{cases} \tag{2}$$

where k is the gradient index, which takes values greater than or equals to zero. The core layer is independent of the value of k, which is a fully ceramic layer. However, the value of k equal to zero represents a fully ceramic plate. Thus, this parameter permits us to control the mixture ceramic - metal.

2.1 Kinematics and strains

In this work, the classical HSDT is modified by taking into account some simplifying assumptions so that the number of unknowns is reduced. The kinematic of the classical HSDT is defined by

$$u(x, y, z) = u_0(x, y) - z\frac{\partial w_0}{\partial x} + f(z)\varphi_x(x, y) \tag{3a}$$

$$v(x, y, z) = v_0(x, y) - z\frac{\partial w_0}{\partial y} + f(z)\varphi_y(x, y) \tag{3b}$$

$$w(x,y,z) = w_0(x,y) \tag{3c}$$

where u_0; v_0; w_0, φ_x, φ_y are five unknown displacements of the mid-plane of the plate, $f(z)$ represents shape function defining the variation of the transverse shear strains and stresses within the thickness. By considering that $\varphi_x = \int \theta(x,y)dx$ and $\varphi_y = \int \theta(x,y)dy$, the displacement field of the proposed HSDT can be written in a simpler form as (Merdaci *et al*. 2016, Bourada *et al*. 2016, Hebali *et al*. 2016, Besseghier *et al*. 2017, Chikh *et al*. 2017, Fahsi *et al*. 2017)

$$u(x,y,z) = u_0(x,y) - z\frac{\partial w_0}{\partial x} + k_1 f(z)\int \theta(x,y)\,dx \tag{4a}$$

$$v(x,y,z) = v_0(x,y) - z\frac{\partial w_0}{\partial y} + k_2 f(z)\int \theta(x,y)dy \tag{4b}$$

$$w(x,y,z) = w_0(x,y) \tag{4c}$$

In this work, the shape function is defined by:

$$f(z) = \frac{z\left(\pi + 2\cos\left(\frac{\pi}{h}z\right)\right)}{(2+\pi)} \tag{5}$$

It can be seen that the kinematic in Eq. (4) uses only four unknowns (u_0, v_0, w_0 and θ). The non-linear von Karman strain–displacement equations are as follows

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + f(z)\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g(z)\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix}, \tag{6}$$

where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2 \\ \frac{\partial v_0}{\partial y} + \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} + \left(\frac{\partial w_0}{\partial x}\right)\left(\frac{\partial w_0}{\partial y}\right) \end{Bmatrix}, \quad \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\frac{\partial^2 w_0}{\partial x^2} \\ -\frac{\partial^2 w_0}{\partial y^2} \\ -2\frac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}, \tag{7a}$$

$$\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} k_1\theta \\ k_2\theta \\ k_1\frac{\partial}{\partial y}\int \theta\,dx + k_2\frac{\partial}{\partial x}\int \theta\,dy \end{Bmatrix}, \tag{7b}$$

$$\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} k_1\int \theta\,dy \\ k_2\int \theta\,dx \end{Bmatrix}, \tag{7b}$$

and

$$g(z) = \frac{df(z)}{dz} \tag{7c}$$

The integrals used in the above equations shall be resolved by a Navier type method and can be given as follows

$$\frac{\partial}{\partial y}\int \theta\,dx = A'\frac{\partial^2\theta}{\partial x\partial y}, \quad \frac{\partial}{\partial x}\int \theta\,dy = B'\frac{\partial^2\theta}{\partial x\partial y}, \quad \int \theta\,dx = A'\frac{\partial\theta}{\partial x}, \quad \int \theta\,dy = B'\frac{\partial\theta}{\partial y} \tag{8}$$

where the coefficients A' and B' are expressed according to the type of solution used, in this case via Navier. Therefore, A', B', k_1 and k_2 are expressed as follows

$$A' = -\frac{1}{\mu^2}, \quad B' = -\frac{1}{\beta^2}, \quad k_1 = \mu^2, \quad k_2 = \beta^2 \tag{9}$$

where μ and β are used in expression (20).

It should be noted that unlike the FSDT, this model does not require shear correction coefficients.

The constitutive relations of a FG sandwich plate can be expressed as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix}^{(j)} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}^{(j)} \begin{Bmatrix} \varepsilon_x - \alpha T \\ \varepsilon_y - \alpha T \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}^{(j)} \tag{10}$$

where C_{ij} (i, j = 1, 2, 4, 5, 6) are the elastic stiffness of the FG sandwich plate defined by

$$C_{11}^{(j)} = C_{22}^{(j)} = \frac{E^{(j)}(z)}{1-\nu^2}, \quad C_{12}^{(j)} = \frac{\nu E^{(j)}(z)}{1-\nu^2}, \quad C_{44}^{(j)} = C_{55}^{(j)} = C_{66}^{(j)} = \frac{E^{(j)}(z)}{2(1+\nu)}, \tag{11}$$

and $T(x, y, z)$ is the temperature rise through-the-thickness.

2.2 Stability equations

The governing equations of FG sandwich plates subjected to thermal loads may be determined on the basis of the stationary potential energy (Reddy 1984, Bellifa *et al*. 2016, Draiche *et al*. 2016). The equilibrium equations are deduced as

$$\delta u_0: \ \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0, \qquad \delta v_0: \ \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \tag{12}$$

$$\delta w_0 : \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} + N_x^0 \frac{\partial^2 w_0}{\partial x^2} + 2N_{xy}^0 \frac{\partial^2 w_0}{\partial x \partial y} + N_y^0 \frac{\partial^2 w_0}{\partial y^2} = 0$$

$$\delta \theta : -k_1 M_x^s - k_2 M_y^s - (k_1 A' + k_2 B')\frac{\partial^2 M_{xy}^s}{\partial x \partial y} + k_1 A' \frac{\partial S_{xz}^s}{\partial x} + k_2 B' \frac{\partial S_{yz}^s}{\partial y} = 0 \tag{12}$$

Using constitutive relations, the stress and moment resultants are defined by

$$\left(N_i, M_i^b, M_i^s\right) = \sum_{j=1}^{3} \int_{h_{j-1}}^{h_j} (1, z, f)\sigma_i^{(j)} dz, \qquad (i = x, y, xy)$$

and $$\left(S_{xz}^s, S_{yz}^s\right) = \sum_{j=1}^{3} \int_{h_{j-1}}^{h_j} g\left(\tau_{xz}, \tau_{yz}\right)^{(j)} dz \tag{13}$$

Upon substitution of Eq. (6) into Eq. (10) and the subsequent results into Eq. (13) the stress resultants are obtained in the matrix form as

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x^b \\ M_y^b \\ M_{xy}^b \\ M_x^s \\ M_y^s \\ M_{xy}^s \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 & B_{11}^s & B_{12}^s & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 & B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} & 0 & 0 & B_{66}^s \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 & D_{11}^s & D_{12}^s & 0 \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 & D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} & 0 & 0 & D_{66}^s \\ B_{11}^s & B_{12}^s & 0 & D_{11}^s & D_{12}^s & 0 & H_{11}^s & H_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 & D_{12}^s & D_{22}^s & 0 & H_{12}^s & H_{22}^s & 0 \\ 0 & 0 & B_{66}^s & 0 & 0 & D_{66}^s & 0 & 0 & H_{66}^s \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ k_x^b \\ k_y^b \\ k_{xy}^b \\ k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} - \begin{Bmatrix} N_x^T \\ N_y^T \\ 0 \\ M_x^{bT} \\ M_y^{bT} \\ 0 \\ M_x^{sT} \\ M_y^{sT} \\ 0 \end{Bmatrix} \tag{14a}$$

$$\begin{Bmatrix} S_{yz}^s \\ S_{xz}^s \end{Bmatrix} = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} \tag{14b}$$

where stiffness components are expressed as

$$\begin{Bmatrix} A_{11} & B_{11} & D_{11} & B_{11}^s & D_{11}^s & H_{11}^s \\ A_{12} & B_{12} & D_{12} & B_{12}^s & D_{12}^s & H_{12}^s \\ A_{66} & B_{66} & D_{66} & B_{66}^s & D_{66}^s & H_{66}^s \end{Bmatrix} = \sum_{j=1}^{3} \int_{h_{j-1}}^{h_j} C_{11}^{(j)} \left(1, z, z^2, f(z), z\, f(z), f^2(z)\right) \begin{Bmatrix} 1 \\ \nu \\ \dfrac{1-\nu}{2} \end{Bmatrix} dz \tag{15a}$$

$$\left(A_{22}, B_{22}, D_{22}, B_{22}^s, D_{22}^s, H_{22}^s\right) = \left(A_{11}, B_{11}, D_{11}, B_{11}^s, D_{11}^s, H_{11}^s\right), \tag{15b}$$

$$A_{44}^s = A_{55}^s = \sum_{j=1}^{3} \int_{h_{j-1}}^{h_j} C_{44}^{(j)} [g(z)]^2 dz, \tag{15c}$$

The stress and moment resultants, $N_x^T = N_y^T$; $M_x^{bT} = M_y^{bT}$ and $M_x^{sT} = M_y^{sT}$; to thermal loading are defined by

$$\begin{Bmatrix} N_x^T \\ M_x^{bT} \\ M_y^{bT} \end{Bmatrix} = \sum_{j=1}^{3} \int_{h_{j-1}}^{h_j} \frac{E^{(j)}(z)}{1-\nu} \alpha^{(j)} T \begin{Bmatrix} 1 \\ z \\ f(z) \end{Bmatrix} dz \tag{16}$$

In order to determine the stability equations and study the thermal stability response of the FG sandwich plate, the adjacent equilibrium criterion is used (Brush and Almroth 1975). By employing this approach, the stability equations become

$$\frac{\partial N_x^1}{\partial x} + \frac{\partial N_{xy}^1}{\partial y} = 0$$

$$\frac{\partial N_{xy}^1}{\partial x} + \frac{\partial N_y^1}{\partial y} = 0$$

$$\frac{\partial^2 M_x^{b1}}{\partial x^2} + 2\frac{\partial^2 M_{xy}^{b1}}{\partial x \partial y} + \frac{\partial^2 M_y^{b1}}{\partial y^2} + N_x^0 \frac{\partial^2 w_0^1}{\partial x^2} + 2N_{xy}^0 \frac{\partial^2 w_0^1}{\partial x \partial y} + N_y^0 \frac{\partial^2 w_0^1}{\partial y^2} = 0 \tag{17}$$

$$-k_1 M_x^{s1} - k_2 M_y^{s1} - (k_1 A' + k_2 B')\frac{\partial^2 M_{xy}^{s1}}{\partial x \partial y} + k_1 A' \frac{\partial S_{xz}^{s1}}{\partial x} + k_2 B' \frac{\partial S_{yz}^{s1}}{\partial y} = 0$$

where N_x^0, N_{xy}^0 and N_y^0 are the pre-buckling forces. Eq. (17) can be expressed in terms of displacements (u_0^1, v_0^1, w_0^1, θ^1) by substituting for the stress resultants from Eq. (14). For FG sandwich plate, the governing equations Eq. (17) take the form

$$A_{11}\frac{\partial^2 u_0^1}{\partial x^2} + A_{12}\frac{\partial^2 v_0^1}{\partial x \partial y} + A_{66}\left(\frac{\partial^2 u_0^1}{\partial y^2} + \frac{\partial^2 v_0^1}{\partial x \partial y}\right) - B_{11}\frac{\partial^3 w_0^1}{\partial x^3} - B_{12}\frac{\partial^3 w_0^1}{\partial x \partial y^2} - 2B_{66}\frac{\partial^3 w_0^1}{\partial x \partial y^2} + B_{11}^s A' k_1 \frac{\partial^3 \theta^1}{\partial x^3}$$
$$B_{12}^s B' k_2 \frac{\partial^3 \theta^1}{\partial x \partial y^2} + B_{66}^s (A' k_1 + B' k_2)\frac{\partial^3 \theta^1}{\partial x \partial y^2} = 0 \tag{18a}$$

$$A_{12}\frac{\partial^2 u_0^1}{\partial x \partial y} + A_{22}\frac{\partial^2 v_0^1}{\partial y^2} + A_{66}\left(\frac{\partial^2 u_0^1}{\partial x \partial y} + \frac{\partial^2 v_0^1}{\partial x^2}\right) - B_{12}\frac{\partial^3 w_0^1}{\partial x^2 \partial y} - B_{22}\frac{\partial^3 w_0^1}{\partial y^3} - 2B_{66}\frac{\partial^3 w_0^1}{\partial x^2 \partial y} + B_{12}^s A' k_1 \frac{\partial^3 \theta^1}{\partial x^2 \partial y}$$
$$B_{22}^s B' k_2 \frac{\partial^3 \theta^1}{\partial y^3} + B_{66}^s (A' k_1 + B' k_2)\frac{\partial^3 \theta^1}{\partial x^2 \partial y} = 0 \tag{18b}$$

$$B_{11}\frac{\partial^3 u_0^1}{\partial x^3} + B_{12}\left(\frac{\partial^3 u_0^1}{\partial x \partial y^2} + \frac{\partial^3 v_0^1}{\partial x^2 \partial y}\right) + B_{22}\frac{\partial^3 v_0^1}{\partial y^3} + 2B_{66}\left(\frac{\partial^3 u_0^1}{\partial x \partial y^2} + \frac{\partial^3 v_0^1}{\partial x^2 \partial y}\right) - D_{11}\frac{\partial^4 w_0^1}{\partial x^4} \tag{18c}$$

$$\begin{aligned}
&-2D_{12}\frac{\partial^4 w_0^1}{\partial x^2\partial y^2}-D_{22}\frac{\partial^4 w_0^1}{\partial y^4}-4D_{66}\frac{\partial^4 w_0^1}{\partial x^2\partial y^2}\\
&+D_{11}^s A'k_1\frac{\partial^4\theta^1}{\partial x^4}+D_{12}^s\left(A'k_1+B'k_2\right)\frac{\partial^4\theta^1}{\partial x^2\partial y^2}\\
&+D_{22}^s B'k_2\frac{\partial^4\theta^1}{\partial y^4}+2D_{66}^s\left(A'k_1+B'k_2\right)\frac{\partial^4\theta^1}{\partial x^2\partial y^2}\\
&+N_x^0\frac{\partial^2 w_0^1}{\partial x^2}+N_y^0\frac{\partial^2 w_0^1}{\partial y^2}+2N_{xy}^0\frac{\partial^2 w_0^1}{\partial x\partial y}=0
\end{aligned} \tag{18c}$$

$$\begin{aligned}
&-B_{11}^s A'k_1\frac{\partial^3 u_0^1}{\partial x^3}-B_{12}^s\left(A'k_1\frac{\partial^3 v_0^1}{\partial x^2\partial y}+B'k_2\frac{\partial^3 u_0^1}{\partial x\partial y^2}\right)\\
&-B_{22}^s B'k_2\frac{\partial^3 v_0^1}{\partial y^3}\\
&-D_{66}\left((A'k_1+B'k_2)\frac{\partial^3 u_0^1}{\partial x\partial y^2}+(A'k_1+B'k_2)\frac{\partial^3 v_0^1}{\partial x^2\partial y}\right)\\
&+D_{11}^s A'k_1\frac{\partial^4 w_0^1}{\partial x^4}+D_{12}^s(A'k_1+B'k_2)\frac{\partial^4 w_0^1}{\partial x^2\partial y^2}\\
&+D_{22}^s B'k_2\frac{\partial^4 w_0^1}{\partial y^4}+2D_{66}^s(A'k_1+B'k_2)\frac{\partial^4 w_0^1}{\partial x^2\partial y^2}\\
&-H_{11}^s(A'k_1)^2\frac{\partial^4\theta^1}{\partial x^4}-2H_{12}^s A'k_1B'k_2\frac{\partial^4\theta^1}{\partial x^2\partial y^2}\\
&-H_{22}^s(B'k_2)^2\frac{\partial^4\theta^1}{\partial y^4}-H_{66}^s(A'k_1+B'k_2)^2\frac{\partial^4\theta^1}{\partial x^2\partial y^2}\\
&+A_{55}^s(A'k_1)^2\frac{\partial^2\theta^1}{\partial x^2}+A_{44}^s(B'k_2)^2\frac{\partial^2\theta^1}{\partial y^2}=0
\end{aligned} \tag{18d}$$

3. Analytical solution

Rectangular sandwich plates are generally classified according to the type of support used. Here, we are concerned with the exact solutions of Eq. (18) for a simply supported FG sandwich plate.

Based on the Navier procedure, the following expansions of displacements u_0^1; v_0^1; w_0^1 and θ^1 are chosen to automatically satisfy the boundary conditions.

$$\begin{Bmatrix} u_0^1 \\ v_0^1 \\ w_0^1 \\ \theta^1 \end{Bmatrix}=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\begin{Bmatrix} U_{mn}\cos(\mu\, x)\sin(\beta\, y) \\ V_{mn}\sin(\mu\, x)\cos(\beta\, y) \\ W_{mn}\sin(\mu\, x)\sin(\beta\, y) \\ X_{mn}\sin(\mu\, x)\sin(\beta\, y) \end{Bmatrix} \tag{19}$$

where U_{mn}, V_{mn}, W_{mn}, X_{mn} are arbitrary parameters to be determined. μ and β are defined as

$$\mu=m\pi/a \quad \text{and} \quad \beta=n\pi/b \tag{20}$$

Substituting Eq. (19) into Eq. (18), the closed-form solution of buckling load can be obtained from

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{12} & a_{22} & a_{23} & a_{24} \\ a_{13} & a_{23} & a_{33}+N_x^0\mu^2+N_y^0\beta^2 & a_{34} \\ a_{14} & a_{24} & a_{34} & a_{44} \end{bmatrix}\begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ X_{mn} \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{21}$$

where

$$\begin{aligned}
a_{11}&=-(A_{11}\mu^2+A_{66}\beta^2)\\
a_{12}&=-\mu\,\beta\,(A_{12}+A_{66})\\
a_{13}&=\mu(B_{11}\mu^2+(B_{12}+2B_{66})\beta^2)\\
a_{14}&=-\mu(B_{11}^s A'k_1\mu^2+B_{12}^s B'k_2\beta^2+B_{66}^s(A'k_1+B'k_2)\beta^2)\\
a_{22}&=-\mu^2A_{66}-\beta^2A_{22}\\
a_{23}&=\beta(B_{22}\beta^2+(B_{12}+2B_{66})\mu^2)\\
a_{24}&=-\beta(B_{22}^s B'k_2\beta^2+\mu^2(B_{12}^s A'k_1+B_{66}^s(A'k_1+B'k_2))\\
a_{33}&=-\mu^2(D_{11}\mu^2+(2D_{12}+4D_{66})\beta^2)-D_{22}\beta^4\\
a_{34}&=D_{11}^s A'k_1\mu^4+D_{12}^s(A'k_1+B'k_2)\beta^2\mu^2\\
&\quad+D_{22}^s B'k_2\beta^4+2D_{66}^s(A'k_1+B'k_2)\beta^2\mu^2\\
a_{44}&=-(H_{11}^s\mu^2k_1+2k_1\beta^2H_{66}^s+2H_{66}^s\mu^2k_2+H_{12}^s\mu^2k_2\\
&\quad+k_1\beta^2H_{12}^s+k_2\beta^2H_{22}^s+A_s^{44}k_1+A_s^{55}k_2)
\end{aligned} \tag{22}$$

By using the condensation technique to eliminate the axial displacements U_{mn} and V_{mn}, Eq. (21) can be rewritten as

$$\begin{bmatrix} \bar{a}_{33}+N_x^0\mu^2+N_y^0\beta^2 & \bar{a}_{34} \\ \bar{a}_{43} & \bar{a}_{44} \end{bmatrix}\begin{Bmatrix} W_{mn} \\ X_{mn} \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{23}$$

where

$$\begin{aligned}
\bar{a}_{33}&=a_{33}-\frac{a_{13}\left(a_{13}a_{22}-a_{12}a_{23}\right)-a_{23}\left(a_{11}a_{23}-a_{12}a_{13}\right)}{a_{11}a_{22}-a_{12}^2}\\
\bar{a}_{34}&=a_{34}-\frac{a_{14}\left(a_{13}a_{22}-a_{12}a_{23}\right)-a_{24}\left(a_{11}a_{23}-a_{12}a_{13}\right)}{a_{11}a_{22}-a_{12}^2}\\
\bar{a}_{43}&=a_{34}-\frac{a_{13}\left(a_{14}a_{22}-a_{12}a_{24}\right)-a_{23}\left(a_{11}a_{24}-a_{12}a_{14}\right)}{a_{11}a_{22}-a_{12}^2}\\
\bar{a}_{44}&=a_{44}-\frac{a_{14}\left(a_{14}a_{22}-a_{12}a_{24}\right)-a_{24}\left(a_{11}a_{24}-a_{12}a_{14}\right)}{a_{11}a_{22}-a_{12}^2}
\end{aligned} \tag{24}$$

The system of homogeneous Eq. (23) has a nontrivial solution only for discrete values of the buckling load. For a nontrivial solution, the determinant of the coefficients (W_{mn}, X_{mn}) must equal zero

$$\begin{vmatrix} \bar{a}_{33}+N_x^0\mu^2+N_y^0\beta^2 & \bar{a}_{34} \\ \bar{a}_{43} & \bar{a}_{44} \end{vmatrix}=0 \tag{25}$$

The obtained equation may be solved for the buckling load. This gives the following relation for buckling load

$$N_x^0\mu^2 + N_y^0\beta^2 = \frac{\bar{a}_{34}\bar{a}_{43} - \bar{a}_{33}\bar{a}_{44}}{\bar{a}_{44}} \tag{26}$$

In this case, a rectangular sandwich plate subjected to thermal loads is examined. To obtain the critical buckling temperature, the pre-buckling thermal loads should be determined. Hence, solving the membrane form of the equilibrium equations and by using the technique proposed by Meyers and Hyer (1991), the pre-buckling load resultants of FG sandwich plate exposed to the temperature variation within the thickness are found to be

$$\begin{aligned} N_x^0 &= -\sum_{j=1}^{3}\int_{h_{j-1}}^{h_j}\frac{E^{(j)}(z)}{1-\nu}\alpha^{(j)}(z)T(z)dz, \\ N_y^0 &= -\sum_{j=1}^{3}\int_{h_{j-1}}^{h_j}\frac{E^{(j)}(z)}{1-\nu}\alpha^{(j)}(z)T(z)dz, \qquad N_{xy}^0 = 0 \end{aligned} \tag{27}$$

In this work, to investigate the influence of the considered type of temperature variation within the thickness on thermal buckling response of FG sandwich plate, three types of thermal loading within the plate thickness are considered.

3.1 Uniform temperature rise (*UTR*)

It is considered that the initial uniform temperature of the FG sandwich plate is T_i, and the temperature is uniformly elevated to a final value T_f such that the plate buckles. Thus, the temperature change is

$$T(z) = T_f - T_i = \Delta T \tag{28}$$

By considering the equations (26), (27), and (28) the following equation for thermal stability load is obtained:

$$\Delta T(m,n) = \frac{1}{\mu^2+\beta^2}\frac{\bar{a}_{33}\bar{a}_{44} - \bar{a}_{34}\bar{a}_{43}}{\bar{a}_{44}}\frac{1}{\int_{-h/2}^{h/2}\frac{E(z)\,\alpha(z)}{1-\nu}dz} \tag{29}$$

3.2 Linear temperature distribution through the thickness (*LTD*)

The following linear temperature distribution within the thickness of the FG sandwich plate is considered

$$T(z) = \Delta T\left(\frac{z}{h}+\frac{1}{2}\right) + T_m, \qquad \Delta T = T_c - T_m \tag{30}$$

Identically to the UTR procedure, the following expression for thermal buckling load is deduced

$$\Delta T(m,n) = \frac{1}{\int_{-h/2}^{h/2}\frac{E(z)\,\alpha(z)\left(\frac{z}{h}+\frac{1}{2}\right)}{1-\nu}dz} \left(\frac{1}{\mu^2+\beta^2}\frac{\bar{a}_{33}\bar{a}_{44} - \bar{a}_{34}\bar{a}_{43}}{\bar{a}_{44}} - \int_{-h/2}^{h/2}\frac{E(z)\,\alpha(z)T_M}{1-\nu}dz\right) \tag{31}$$

3.3 Non-linear temperature distribution through the thickness (*NTD*)

The following non-linear temperature distribution within the thickness of the FG sandwich plate is considered

$$T(z) = \Delta T\left(\frac{z}{h}+\frac{1}{2}\right)^{\gamma} + T_m, \qquad \Delta T = T_c - T_m \tag{32}$$

Identically to the UTR procedure, the following expression for thermal buckling load is deduced

$$\Delta T(m,n) = \frac{1}{\int_{-h/2}^{h/2}\frac{E(z)\,\alpha(z)\left(\frac{z}{h}+\frac{1}{2}\right)^{\gamma}}{1-\nu}dz} \left(\frac{1}{\mu^2+\beta^2}\frac{\bar{a}_{33}\bar{a}_{44} - \bar{a}_{34}\bar{a}_{43}}{\bar{a}_{44}} - \int_{-h/2}^{h/2}\frac{E(z)\,\alpha(z)T_M}{1-\nu}dz\right) \tag{33}$$

where γ is the temperature exponent ($0 < \gamma < \infty$). Note that the value of γ equal to unity represents a linear temperature change across the thickness. While the value of γ excluding unity represents a non-linear temperature change through-the-thickness.

4. Results and discussion

To check the proposed formulation, a ceramic-metal functionally graded sandwich plate is examined. For this purpose, two different functionally graded plate materials are used for the present work. These are Titanium alloy (Ti–6A1–4V)-Zirconia (ZrO_2) and Aluminum (Al)-Alumina (Al_2O_3). The material properties of the constituents of these FGMs are reported in Table 1.

Table 1 Material properties used in the FG sandwich plate

Properties	Ti–6A1–4V	ZrO_2	Al	Al_2O_3
E (GPa)	66.2	244.27	70	380
ν	0.3	0.3	0.3	0.3
α (10^{-6}/K)	10.3	12.766	23	7.4

Table 2 Minimum critical temperature parameter αT_{cr} of the simply supported isotropic square plate ($\alpha_0 = 1.0\times10^{-6}$/K, $E = 1.0\times10^{-6}$N/m^2, $\nu = 0.3$)

a/h	Present	Matsunaga (2005)
10	0.1198×10^{-1}	0.1183×10^{-1}
20	0.3119×10^{-2}	0.3109×10^{-2}
100	0.1265×10^{-3}	0.1264×10^{-3}

Table 3 Critical buckling temperature T_{cr} of FG sandwich square plates under uniform temperature rise (a/h = 5)

k	Theory	T_{cr}			
		(1-0-1)	(1-1-1)	(2-1-2)	(3-1-3)
0	Present	3,24034	3,24034	3,24034	3,24034
	Bourada *et al*. (2012)	3.23652	3.23652	3.23652	3.23652
	SSDT	3.23775	3.23775	3.23775	3.23775
	TSDT	3.23652	3.23652	3.23652	3.23652
	FSDT	3.23493	3.23493	3.23493	3.23493
	CPT	3.96470	3.96470	3.96470	3.96470
0.2	Present	3,07450	3,05802	3,05834	3,06119
	Bourada *et al*. (2012)	3.07042	3.05484	3.05461	3.05729
	SSDT	3.07198	3.05591	3.05598	3.05875
	TSDT	3.07042	3.05484	3.05461	3.05729
	FSDT	3.04858	3.03637	3.03394	3.03603
	CPT	3.66606	3.65640	3.64978	3.65144
0.5	Present	2,87541	2,83512	2,83423	2,84099
	Bourada *et al*. (2012)	2.87074	2.83224	2.83030	2.83673
	SSDT	2.87277	2.83331	2.83194	2.83855
	TSDT	2.87074	2.83224	2.83030	2.83673
	FSDT	2.83507	2.80230	2.79675	2.80218
	CPT	3.34559	3.31343	3.30066	3.30593
1	Present	2,69376	2,59191	2,59707	2,61374
	Bourada *et al*. (2012)	2.68781	2.58882	2.59241	2.60856
	SSDT	2.69065	2.59015	2.59458	2.61100
	TSDT	2.68781	2.58882	2.59241	2.60856
	FSDT	2.64222	2.55161	2.55053	2.56519
	CPT	3.06734	2.96299	2.95538	2.97216
2	Present	2,63896	2,36407	2,39953	2,44692
	Bourada *et al*. (2012)	2.63018	2.36000	2.39637	2.43977
	SSDT	2.63460	2.36196	2.39953	2.44337
	TSDT	2.63018	2.36000	2.39637	2.43977
	FSDT	2.57355	2.31737	2.34734	2.38823
	CPT	2.96200	2.64806	2.68016	2.72994
5	Present	2,94934	2,21632	2,35871	2,47451
	Bourada *et al*. (2012)	2.93446	2.21009	2.34898	2.46321
	SSDT	2.94205	2.21327	2.35401	2.46905
	TSDT	2.93446	2.21009	2.34898	2.46321
	FSDT	2.86226	2.16069	2.28926	2.39882
	CPT	3.32950	2.44274	2.59922	2.73600
10	Present	3,32102	2,20243	2,43404	2,60902
	Bourada *et al*. (2012)	3.30340	2.19469	2.42186	2.59474
	SSDT	3.31230	2.20150	2.42733	2.60199
	TSDT	3.30340	2.19469	2.42186	2.59474
	FSDT	3.23289	2.14099	2.35529	2.52271
	CPT	3.82441	2.41650	2.68184	2.89384
∞	Present	4,01613	4,01613	4,01613	4,01613
	Bourada *et al*. (2012)	4.01293	4.01293	4.01293	4.01293
	SSDT	4.01141	4.01141	4.01141	4.01141
	TSDT	4.01293	4.01293	4.01293	4.01293
	FSDT	4.00943	4.00943	4.00943	4.00943
	CPT	4.91392	4.91392	4.91392	4.91392

Table 4 Critical buckling temperature T_{cr} of FG sandwich square plates under linear temperature rise (a/h = 5)

k	Theory	T_{cr}			
		(1-0-1)	(1-1-1)	(2-1-2)	(3-1-3)
0	Present	6,43068	6,43068	6,43068	6,43068
	Bourada *et al*. (2012)	6.42305	6.42305	6.42305	6.42305
	SSDT	6.42550	6.42550	6.42550	6.42550
	TSDT	6.42305	6.42305	6.42305	6.42305
	FSDT	6.41986	6.41986	6.41986	6.41986
	CPT	7.87940	7.87940	7.87940	7.87940
0.2	Present	6,09901	6,06604	6,06668	6,07237
	Bourada *et al*. (2012)	6.09084	6.05968	6.05922	6.06459
	SSDT	6.09396	6.06183	6.06197	6.06751
	TSDT	6.09084	6.05968	6.05922	6.06459
	FSDT	6.04716	6.02273	6.01789	6.02207
	CPT	7.28211	7.26279	7.24955	7.25287
0.5	Present	5,70082	5,62023	5,61846	5,63198
	Bourada *et al*. (2012)	5.69148	5.61449	5.61059	5.62346
	SSDT	5.69554	5.61663	5.61389	5.62710
	TSDT	5.69148	5.61449	5.61059	5.62346
	FSDT	5.62014	5.55460	5.54350	5.55435
	CPT	6.64118	6.57686	6.55131	6.56187
1	Present	5,33752	5,13382	5,14414	5,17747
	Bourada *et al*. (2012)	5.32562	5.12765	5.13482	5.16711
	SSDT	5.33130	5.13030	5.13918	5.17201
	TSDT	5.32562	5.12765	5.13482	5.16711
	FSDT	5.23443	5.05323	5.05105	5.08038
	CPT	6.08468	5.87599	5.86076	5.89431
2	Present	5,22793	4,67814	4,75538	4,84385
	Bourada *et al*. (2012)	5.21036	4.66999	4.74275	4.82954
	SSDT	5.21920	4.67392	4.74908	4.83673
	TSDT	5.21036	4.66999	4.74275	4.82954
	FSDT	5.09711	4.58475	4.64468	4.72645
	CPT	5.87400	5.24612	5.31032	5.40989
5	Present	5,84868	4,38263	4,66742	4,89903
	Bourada *et al*. (2012)	5.81891	4.37017	4.64797	4.87641
	SSDT	5.83411	4.37654	4.65805	4.88811
	TSDT	5.81891	4.37017	4.64797	4.87641
	FSDT	5.67452	4.27139	4.52851	4.74763
	CPT	6.60901	4.83549	5.14843	5.42200
10	Present	6,59203	4,35486	4,80636	5,16805
	Bourada *et al*. (2012)	6.55680	4.33937	4,81805	5.13948
	SSDT	6.57459	4.35224	4.80638	5.15396
	TSDT	6.55680	4.33937	4.79372	5.13948
	FSDT	6.41578	4.23198	4.66058	4.99542
	CPT	7.59882	4.78299	5.31369	5.73769
∞	Present	7,98226	7,98226	7,98226	7,98226
	Bourada *et al*. (2012)	7.97281	7.97281	7.97281	7.97281
	SSDT	7.97585	7.97585	7.97585	7.97585
	TSDT	7.97281	7.97281	7.97281	7.97281
	FSDT	7.96885	7.96885	7.96885	7.96885
	CPT	9.77784	9.77784	9.77784	9.77784

Table 5 Critical buckling temperature T_{cr} of FG sandwich square plates under non-linear temperature rise (γ = 5 and a/h = 5)

k	Theory	T_{ar}			
		(1-0-1)	(1-1-1)	(2-1-2)	(3-1-3)
0	Present	19,29203	19,29203	19,29203	19,29203
	Bourada *et al*. (2012)	19.26915	19.26915	19.26915	19.26915
	SSDT	19.27651	19.27651	19.27651	19.27651
	TSDT	19.26915	19.26915	19.26915	19.26915
	FSDT	19.25957	19.25957	19.25957	19.25957
	CPT	23.63820	23.63820	23.63820	23.63820
0.2	Present	20,59236	20,33007	20,44976	20,49461
	Bourada *et al*. (2012)	20.56479	20.30876	20.42463	20.46833
	SSDT	20.57531	20.31595	20.43388	20.47819
	TSDT	20.56479	20.30876	20.42463	20.46833
	FSDT	20.41729	20.18492	20.28528	20.32483
	CPT	24.58692	24.34093	24.43703	24.47887
0.5	Present	21,64883	21,14600	21,36814	21,45395
	Bourada *et al*. (2012)	21.61337	21.12438	21.33822	21.42148
	SSDT	21.62878	21.13244	21.35073	21.43534
	TSDT	21.61337	21.12438	21.33822	21.42148
	FSDT	21.34246	20.89907	21.08307	21.15824
	CPT	25.21986	24.74530	24.91598	24.99617
1	Present	22,46081	21,71806	22,02269	22,12553
	Bourada *et al*. (2012)	22.41074	21.69196	21.98279	22,14890
	SSDT	22.43462	21.70318	22.00140	22.12553
	TSDT	22.41074	21.69196	21.98279	22.10459
	FSDT	22.02700	21.37713	21.62417	21.73355
	CPT	25.60494	24.85771	25.09061	25.21549
2	Present	23,10689	22,02137	22,41227	22,59731
	Bourada *et al*. (2012)	23.02926	21.98304	22.35275	22.53055
	SSDT	23.06831	22.00152	22.38252	22.56412
	TSDT	23.02926	21.98304	22.35275	22.53055
	FSDT	22.52869	21.58175	21.89055	22.04964
	CPT	25.96247	24.69501	25.02775	25.23797
5	Present	23,83092	22,12608	22,70953	23,01642
	Bourada *et al*. (2012)	23.70963	22.06317	22.61489	22.91015
	SSDT	23.77153	22.09533	22.66384	22.96510
	TSDT	23.70963	22.06317	22.61489	22.91015
	FSDT	23.12129	21.56445	22.03367	22.30513
	CPT	26.92893	24.41235	25.04991	25.47341
10	Present	24,14028	22,18585	22,92909	23,30857
	Bourada *et al*. (2012)	24.01127	22.10708	22.81317	23.17972
	SSDT	24.07633	22.17208	22.86373	23.24502
	TSDT	24.01127	22.10708	22.81317	23.17972
	FSDT	23.49484	21.55996	22.17958	22.52996
	CPT	27.82720	24.36712	25.28770	25.87769
∞	Present	23,94679	23,94679	23,94679	23,94679
	Bourada *et al*. (2012)	23.91843	23.91843	23.91843	23.91843
	SSDT	23.92755	23.92755	23.92755	23.92755
	TSDT	23.91843	23.91843	23.91843	23.91843
	FSDT	23.90656	23.90656	23.90656	23.90656
	CPT	29.33351	29.33351	29.33351	29.33351

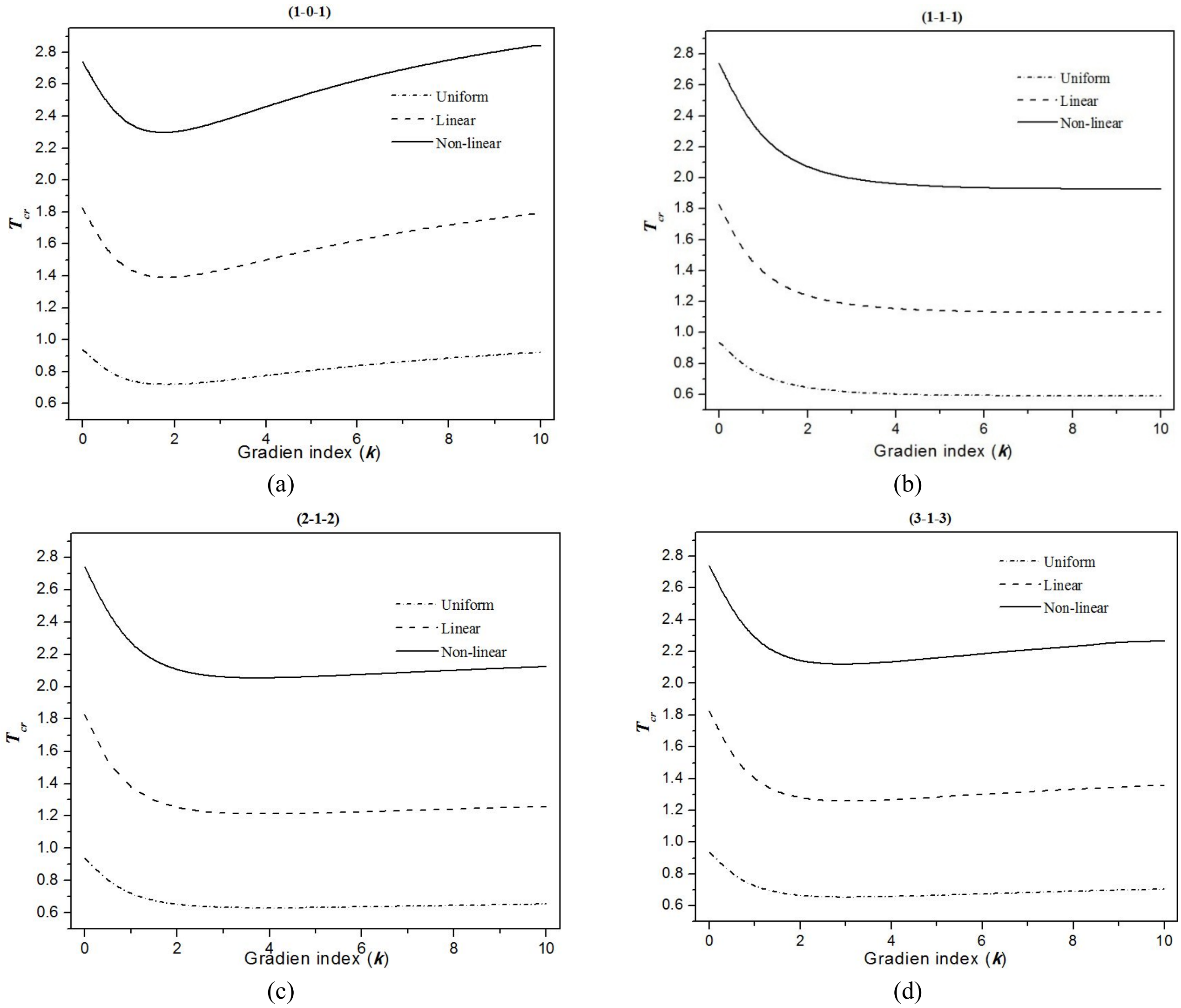

Fig. 2 Critical stability temperature difference T_{cr} vs. the gradient index k for various types of FG sandwich square plates (a/h = 10): (a) the (1–0–1) FG sandwich plate; (b) the (1–1–1) FG sandwich plate; (c) the (2–1–2) FG sandwich plate, and (d) the (3–1–3) FG sandwich plate. For non-linear temperature $\gamma = 2$

In the following, we note that several types of sandwich plates are employed:

- The (1-0-1) FG sandwich plate: The plate is symmetric and made of only two equal-thickness FGM layers, that is, there is no core layer. Thus, $h_1 = h_2 = 0$.
- The (1-1-1) FG sandwich plate: Here the plate is symmetric and made of three equal thickness layers. In this case, we have, $h_1 = -h/6$, $h_2 = h/6$.
- The (2-1-2) FG sandwich plate: The plate is symmetric and we have: $h_1 = -h/10$, $h_2 = h/10$.
- The (3-1-3) FG sandwich plate: Here, the plate is also symmetric and the thickness of the core is half the face thickness. In this case, we have, $h_1 = -h/14$, $h_2 = h/14$.

The shear correction factor for FSDT is set equal to 5/6. For the linear and nonlinear temperature rises through the thickness, $T_t = 25°C$.

To check the accuracy of the proposed HSDT, comparisons are carried out between the thermal stability results computed from the present theory and those calculated by Matsunaga (2005) as shown in Table 2. It can be observed that the obtained results are in agreement with the published results for simply supported isotropic plates.

In order to check also the validity of the proposed theory, results were determined for FG sandwich plates subjected to uniform, linear, and nonlinear thermal loading according to all plate theories. The critical stability temperature difference ($T_{cr} = 10^{-3}\Delta T_{cr}$) are determined for k = 0, 1, 2, 5, 10, and ∞ (metal) and for different types of FG sandwich plates as is shown in Tables 3-5. It can be seen from Tables 3-5 that there is a very good agreement between the proposed new plate theory and other HSDTs. It is observed that the thermal stability temperature increases with increasing thickness of the FG layers and especially for $k \geq 1$. For different values of gradient index k, the thermal stability temperature values are between those of plates made of ceramic (ZrO_2) and metal (Ti–6A1–4V). As the plate becomes more and more metallic, the thermal

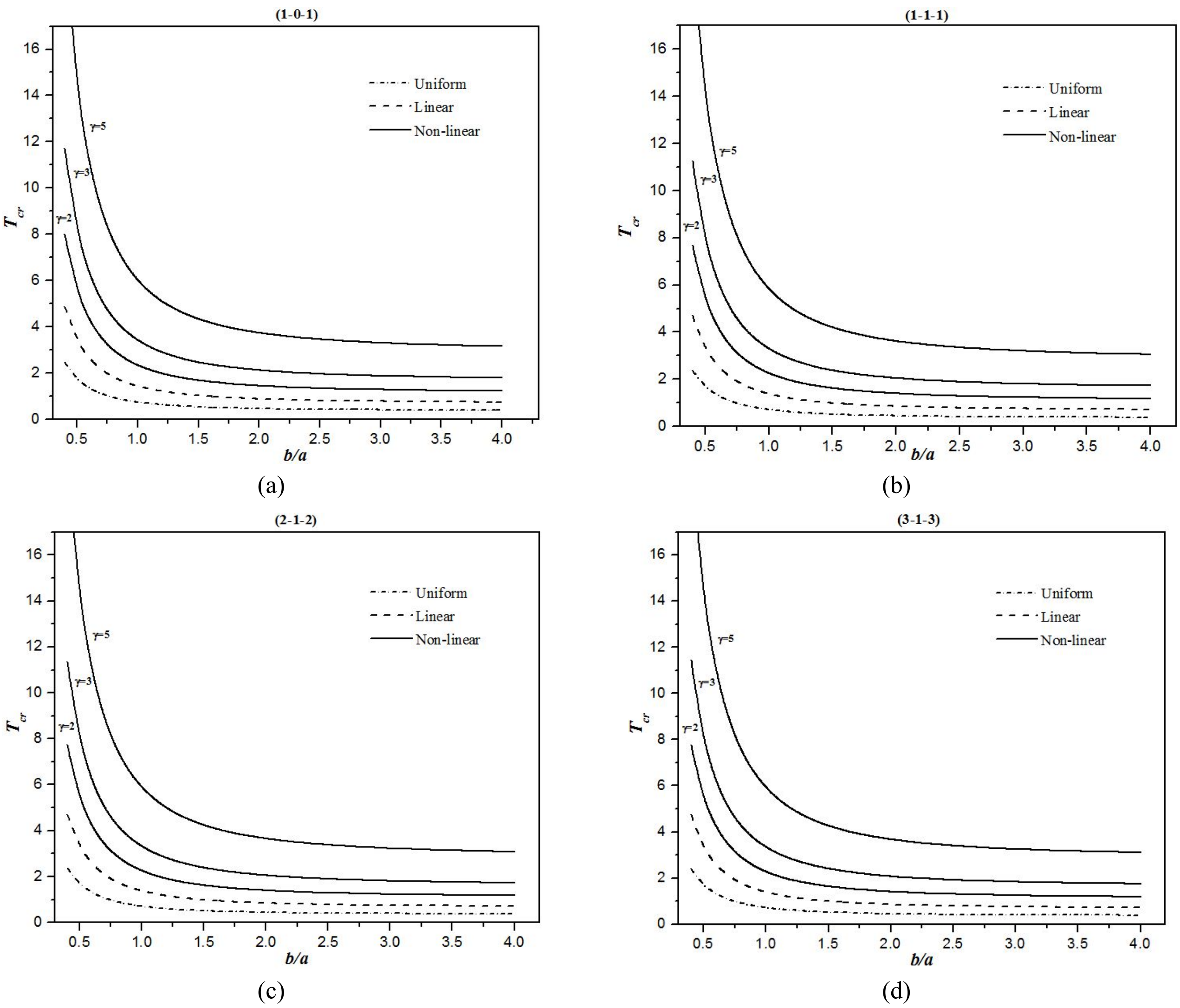

Fig. 3 Critical stability temperature difference T_{cr} vs. the plate aspect ratio b/a for various types of FG sandwich square plates ($k = 1$, $a/h = 10$): (a) the (1–0–1) FG sandwich plate; (b) the (1–1–1) FG sandwich plate; (c) the (2–1–2) FG sandwich plate; and (d) the (3–1–3) FG sandwich plate

stability temperature increases. It can be remarked from the results reported in Tables 3-5 that, the thermal stability temperature of fully metallic plate is higher to ceramic rich Ti–6A1–4V/ZrO_2 FG plate. This is due to the Young's modulus of ceramic (ZrO_2; 244.27 GPa) which is higher to that of metal (Ti–6A1–4V; 66.2 GPa). It is interesting to note that the critical stability temperatures determined based on CPT are noticeably greater than values determined based on HSDT.

Fig. 2 presents the influences of the gradient index k and the types of thermal loads on the critical stability temperature T_{cr} by using the proposed theory. It is clear that the critical stability temperature T_{cr} for the plates under a nonlinear thermal load is higher than that for the plates under uniform thermal load. While T_{cr} for the plates subjected to linear thermal load is intermediate to the two previous thermal loading cases. It is also observed that, for the plate without core, the critical stability T_{cr} diminishes rapidly to reach minimum values and then increases gradually as the gradient index k increases as indicated in Fig. 2(a). However, for the other sandwich FG plates (see Figs. 2(b), (c), and (d)), T_{cr} diminishes smoothly as the gradient index k increases.

Fig. 3 presents the influences of the aspect ratio b/a on the critical stability temperature change T_{cr} of FG sandwich plates subjected to various thermal loading types. It is noticed that, regardless of the sandwich plate types, the critical stability T_{cr} diminishes gradually with the increase of the plate aspect ratio b/a wherever the loading type is. It is also observed from Fig. 3 that the v increases with the increase of the nonlinearity parameter v.

Fig. 4 shows the variation of critical temperatures T_{cr} of FG sandwich square plates subjected to various thermal loading types with respect to the side-to-thickness ratio a/h. It is observed that the critical temperature difference diminishes monotonically as the side-to-thickness ratio a/h increases. It is also noticed from Fig. 4 that the critical temperatures v of the FG plate under uniform thermal load rise is smaller than that of the plate under linear thermal load and the latter is smaller than that of the plate under

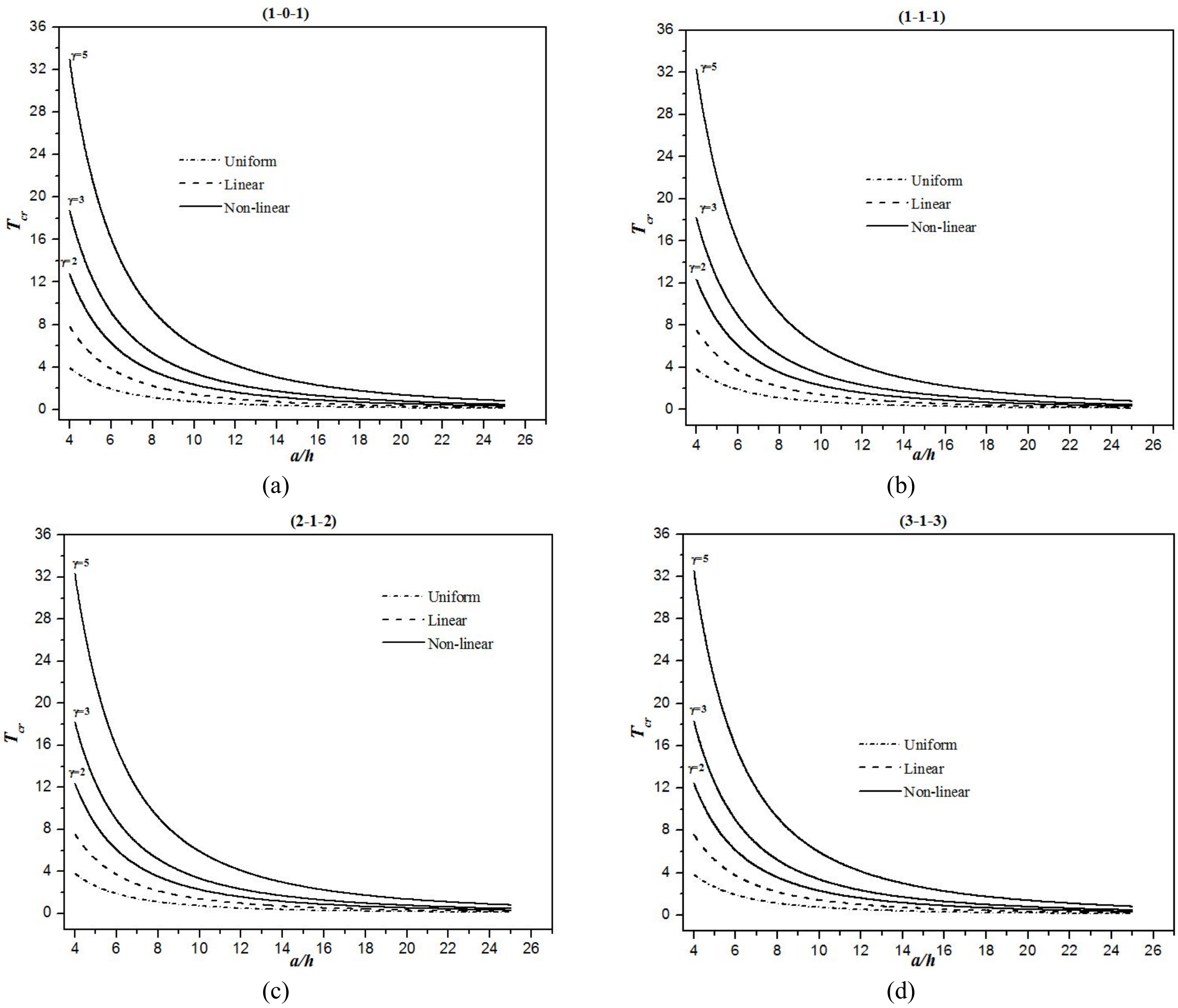

Fig. 4 Critical buckling temperature difference T_{cr} vs. the side-to-thickness ratio a/h for various types of FG sandwich square plates (k = 1): (a) the (1–0–1) FG sandwich plate; (b) the (1–1–1) FG sandwich plate; (c) the (2–1–2) FG sandwich plate; and (d) the (3–1–3) FG sandwich plate

nonlinear thermal loading. Also, it is observed that T_{cr} increases as the nonlinearity parameter γ increases.

The variation of the critical temperature versus the aspect ratio b/a is plotted in Figs. 5(a)-(d) for both Ti–6A1–4V/ZrO_2 and Al/Al_2O_3 FG sandwich plates exposed to nonlinear thermal loads by considering different types of sandwich plates. The Ti–6A1–4V/ZrO_2 plates present high critical temperatures T_{cr} because of the bending stiffness of these plates.

The critical buckling temperature versus the side-to-thickness ratio a/h plots in non-dimensional $T_{cr} - a/h$ plane is presented in Figs. 6(a)-(d). In each of the figures, thermal buckling behavior is shown for Ti–6A1–4V/ZrO_2 and Al/Al_2O_3 FG sandwich plates. In general, the critical buckling temperature is shown to be decreasing with increased the side-to-thickness ratio v level as a result of enhanced shear effect. Again, in all considered types of sandwich plates, the stiffness exhibited by Ti–6A1–4V/ZrO_2 sandwich plate is the highest.

5. Conclusions

In this work, a novel HSDT is proposed to investigate the thermal buckling responses of FG sandwich plates exposed to uniform, linear and nonlinear temperature distributions within the thickness. By considering further simplifying suppositions to the conventional HSDTs and with the incorporation of an undetermined integral term, the number of unknowns and governing equations of the proposed HSDT are reduced by one, and thus, make this model simple and efficient to utilize. The governing equations are obtained by employing the energy based variational principle and then are solved via Navier's method. The results determined by the proposed theory can be summarized as follows:

- The critical stability temperature computed using the proposed theory (with four variables) and other HSDTs (with five variables) are almost identical.

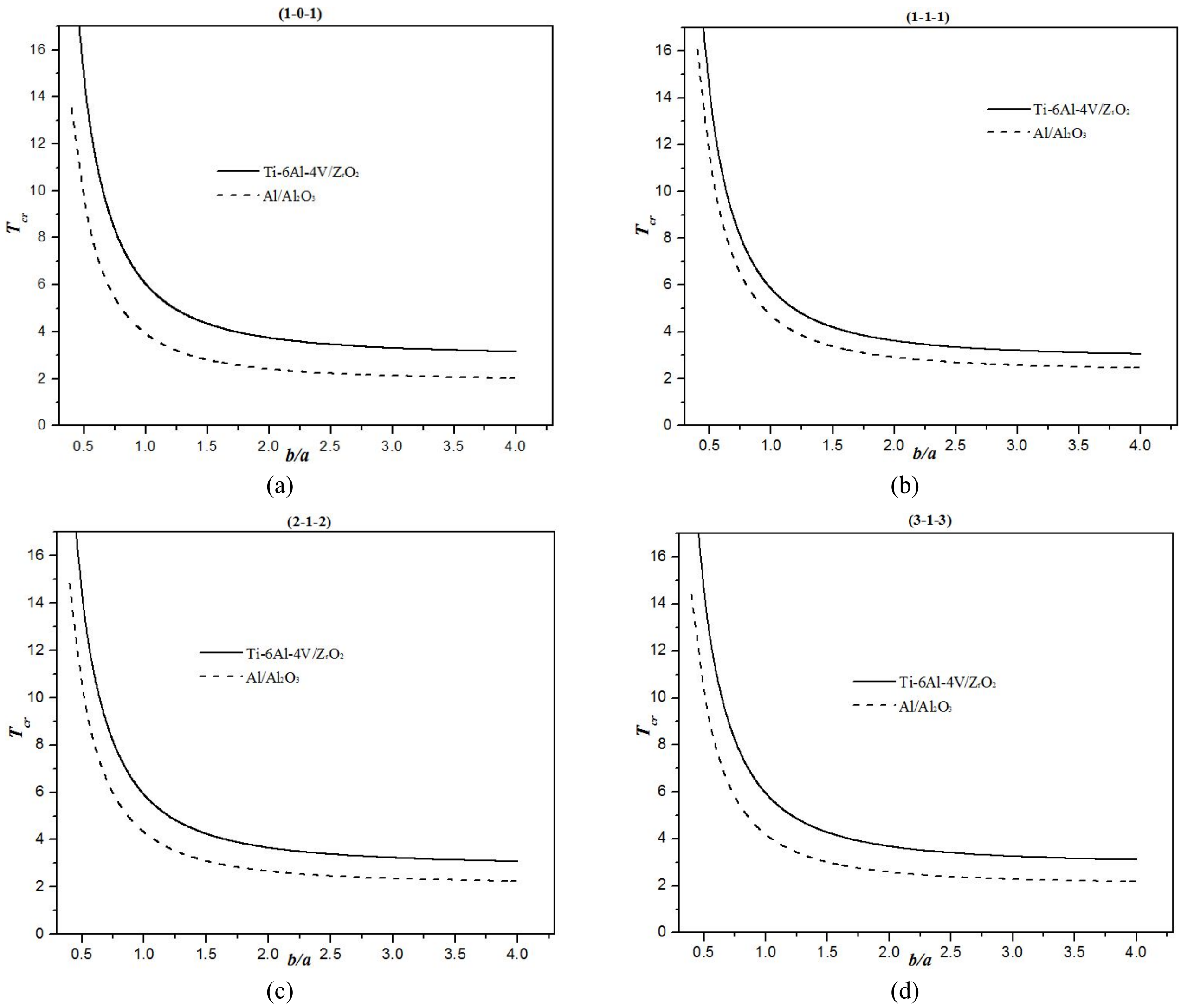

Fig. 5 Critical stability temperature difference T_{cr} of Ti–6A1–4V/ZrO_2 and Al/Al_2O_3 FG sandwich plates versus b/a with k = 1, a/h = 10 and γ = 5: (a) the (1–0–1) FG sandwich plate; (b) the (1–1–1) FG sandwich plate; (c) the (2–1–2) FG sandwich plate; and (d) the (3–1–3) FG sandwich plate

- The critical stability temperature difference diminishes as the side-to-thickness ratio and the plate aspect ratio increases.
- The critical stability temperature differences of FG sandwich plates are generally lower than the corresponding values for homogeneous ceramic plates.
- The critical stability temperature of FG sandwich plate under nonlinear temperature rise within the thickness increases as the temperature exponent γ increases.

Finally, an improvement of present formulation will be considered in the future work to account for the thickness stretching effect by using quasi-3D shear deformation models (Bessaim *et al*. 2013, Bousahla *et al*. 2014, Belabed *et al*. 2014, Fekrar *et al*. 2014, Hebali *et al*. 2014, Larbi Chaht *et al*. 2015, Meradjah *et al*. 2015, Hamidi *et al*. 2015, Bourada *et al*. 2015, Bennai *et al*. 2015, Bennoun *et al*. 2016, Draiche *et al*. 2016, Benbakhti *et al*. 2016, Benahmed *et al*. 2017, Bouafia *et al*. 2017, Benchohra *et al*. 2017) and the wave propagation problem (Mahmoud *et al*. 2015, Ait Yahia *et al*. 2015).

References

Abdelbari, S., Fekrar, A., Heireche, H., Saidi, H., Tounsi, A. and Adda Bedia, E.A. (2016), "An efficient and simple shear deformation theory for free vibration of functionally graded rectangular plates on Winkler–Pasternak elastic foundations", *Wind Struct*., *Int*. *J*., **22**(3), 329-348.

Abdelhak, Z., Hadji, L., Hassaine Daouadji, T. and Adda Bedia, E.A. (2016), "Thermal buckling response of functionally graded sandwich plates with clamped boundary conditions", *Smart Struct*. *Syst*., *Int*. *J*., **18**(2), 267-291.

Adda Bedia, W., Benzair, A., Semmah, A., Tounsi, A. and Mahmoud, S.R. (2015), "On the thermal buckling characteristics of armchair single-walled carbon nanotube embedded in an elastic medium based on nonlocal continuum elasticity", *Brazil*. *J*. *Phys*., **45**(2), 225-233.

Ahouel, M., Houari, M.S.A., Adda Bedia, E.A. and Tounsi, A.

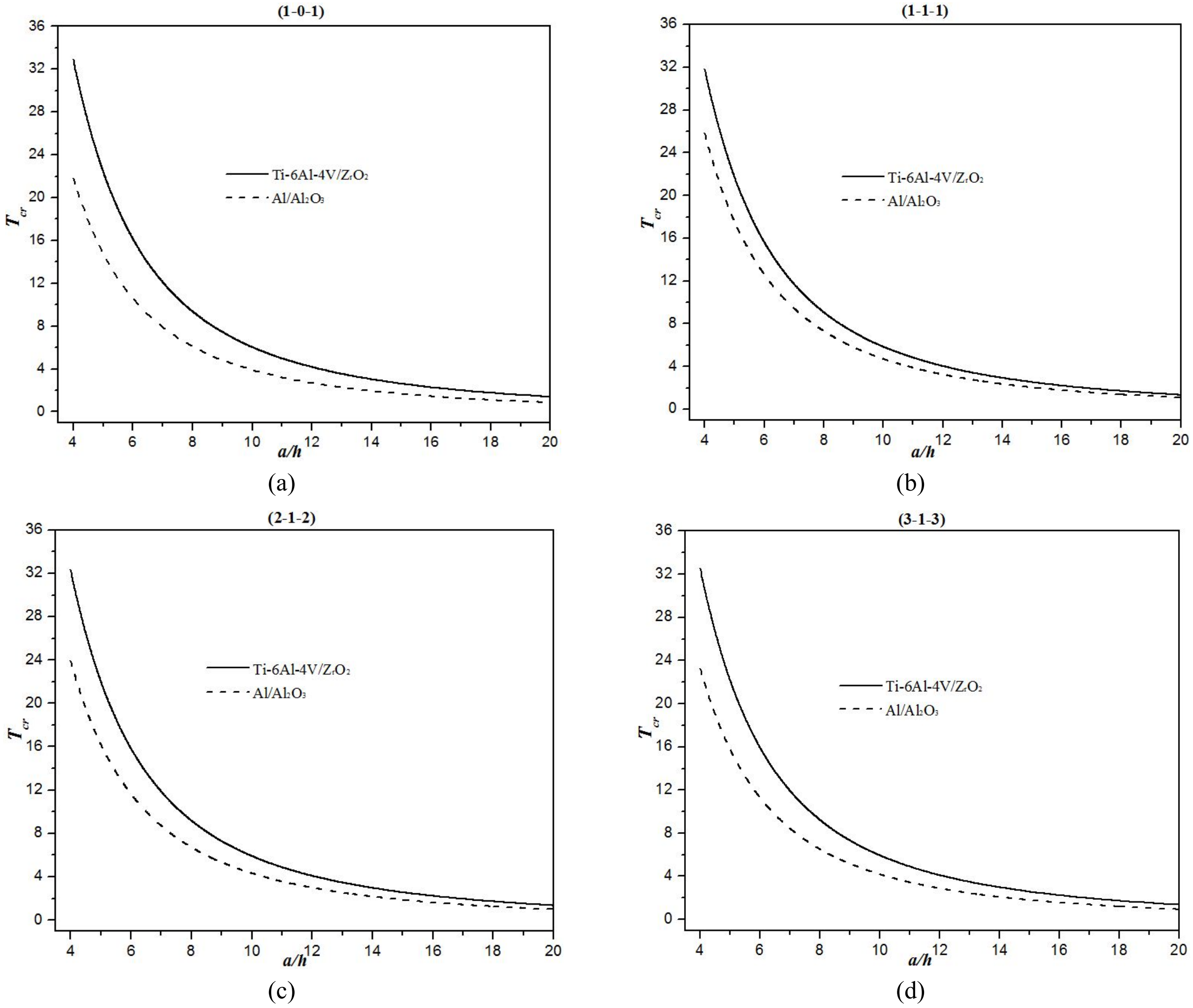

Fig. 6 Critical stability temperature difference T_{cr} of Ti–6A1–4V/ZrO_2 and Al/Al_2O_3 FG sandwich square plates versus a/h with $k = 1$ and $\gamma = 5$: (a) the (1–0–1) FG sandwich plate; (b) the (1–1–1) FG sandwich plate; (c) the (2–1–2) FG sandwich plate; and (d) the (3–1–3) FG sandwich plate

(2016), "Size-dependent mechanical behavior of functionally graded trigonometric shear deformable nanobeams including neutral surface position concept", *Steel Compos. Struct., Int. J.*, **20**(5), 963-981.

Ait Amar Meziane, M., Abdelaziz, H.H. and Tounsi, A. (2014), "An efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions", *J. Sandw. Struct. Mater.*, **16**(3), 293-318.

Ait Atmane, H., Tounsi, A., Bernard, F. and Mahmoud, S.R. (2015), "A computational shear displacement model for vibrational analysis of functionally graded beams with porosities", *Steel Compos. Struct., Int. J.*, **19**(2), 369-384.

Ait Yahia, S., Ait Atmane, H., Houari, M.S.A. and Tounsi, A. (2015), "Wave propagation in functionally graded plates with porosities using various higher-order shear deformation plate theories", *Struct. Eng. Mech., Int. J.*, **53**(6), 1143-1165.

Aizikovich, S.M., Mitrin, B.I., Seleznev, N.M., Wang, Y.C. and Volkov, S.S. (2016), "Influence of a soft FGM interlayer on contact stresses under a beam on an elastic foundation", *Struct. Eng. Mech., Int. J.*, **58**(4), 613-625.

Akbarov, S.D., Guliyev, H.H. and Yahnioglu, N. (2016), "Natural vibration of the three-layered solid sphere with middle layer made of FGM: Three-dimensional approach", *Struct. Eng Mech., Int. J.*, **57**(2), 239-264.

Akbaş, Ş.D. (2015), "Wave propagation of a functionally graded beam in thermal environments", *Steel Compos. Struct., Int. J.*, **19**(6), 1421-1447.

Arefi, M. (2015a), "Elastic solution of a curved beam made of functionally graded materials with different cross sections", *Steel Compos. Struct., Int. J.*, **18**(3), 659-672.

Arefi, M. (2015b), "Nonlinear electromechanical analysis of a functionally graded square plate integrated with smart layers resting on Winkler-Pasternak foundation", *Smart Struct. Syst., Int. J.*, **16**(1), 195-211.

Arefi, M. and Allam, M.N.M. (2015), "Nonlinear responses of an arbitrary FGP circular plate resting on the Winkler-Pasternak foundation", *Smart Struct. Syst., Int. J.*, **16**(1), 81-100.

Asemi, K. and Shariyat, M. (2016), "Three-dimensional biaxial post-buckling analysis of heterogeneous auxetic rectangular plates on elastic foundations by new criteria", *Comput. Methods Appl. Mech. Eng.*, **302**, 1-26.

Attia, A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2015), "Free vibration analysis of functionally graded plates with temperature-dependent properties using various four variable refined plate theories", *Steel Compos. Struct., Int. J.*, **18**(1), 187-212.

Bakora, A. and Tounsi, A. (2015), "Thermo-mechanical post-buckling behavior of thick functionally graded plates resting on elastic foundations", *Struct. Eng. Mech.*, **56**(1), 85-106.

Barati, M.R. and Shahverdi, H. (2016), "A four-variable plate theory for thermal vibration of embedded FG nanoplates under non-uniform temperature distributions with different boundary conditions", *Struct. Eng. Mech., Int. J.*, **60**(4), 707-727.

Barka, M., Benrahou, K.H., Bakora, A. and Tounsi, A. (2016), "Thermal post-buckling behavior of imperfect temperature-dependent sandwich FGM plates resting on Pasternak elastic foundation", *Steel Compos. Struct., Int. J.*, **22**(1), 91-112.

Belabed, Z., Houari, M.S.A., Tounsi, A., Mahmoud, S.R. and Anwar Bég, O. (2014), "An efficient and simple higher order shear and normal deformation theory for functionally graded material (FGM) plates", *Compos.: Part B*, **60**, 274-283.

Beldjelili, Y., Tounsi, A. and Mahmoud, S.R. (2016), "Hygro-thermo-mechanical bending of S-FGM plates resting on variable elastic foundations using a four-variable trigonometric plate theory", *Smart Struct. Syst., Int. J.*, **18**(4), 755-786.

Belkorissat, I., Houari, M.S.A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2015), "On vibration properties of functionally graded nano-plate using a new nonlocal refined four variable model", *Steel Compos. Struct., Int. J.*, **18**(4), 1063-1081.

Bellifa, H., Benrahou, K.H., Hadji, L., Houari, M.S.A. and Tounsi, A. (2016), "Bending and free vibration analysis of functionally graded plates using a simple shear deformation theory and the concept the neutral surface position", *J. Brazil. Soc. Mech. Sci. Eng.*, **38**(1), 265-275.

Bellifa, H., Benrahou, K.H., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2017), "A nonlocal zeroth-order shear deformation theory for nonlinear postbuckling of nanobeams", *Struct. Eng. Mech., Int. J.*, **62**(6), 695-702.

Benahmed, A., Houari, M.S.A., Benyoucef, S., Belakhdar, K. and Tounsi, A. (2017), "A novel quasi-3D hyperbolic shear deformation theory for functionally graded thick rectangular plates on elastic foundation", *Geomech. Eng., Int. J.*, **12**(1), 9-34.

Benbakhti, A., Bachir Bouiadjra, M., Retiel, N. and Tounsi, A. (2016), "A new five unknown quasi-3D type HSDT for thermomechanical bending analysis of FGM sandwich plates", *Steel Compos. Struct., Int. J.*, **22**(5), 975-999.

Benchohra, M., Driz, H., Bakora, A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2017), "A new quasi-3D sinusoidal shear deformation theory for functionally graded plates", *Struct. Eng. Mech., Int. J.* [Accepted]

Benferhat, R., Hassaine Daouadji, T., Hadji, L. and Said Mansour, M. (2016), "Static analysis of the FGM plate with porosities", *Steel Compos. Struct., Int. J.*, **21**(1), 123-136.

Bennai, R., Ait Atmane, H. and Tounsi, A. (2015), "A new higher-order shear and normal deformation theory for functionally graded sandwich beams", *Steel Compos. Struct., Int. J.*, **19**(3), 521-546.

Bennoun, M., Houari, M.S.A. and Tounsi, A. (2016), "A novel five variable refined plate theory for vibration analysis of functionally graded sandwich plates", *Mech. Adv. Mater. Struct.*, **23**(4), 423-431.

Bessaim, A., Houari, M.S.A., Tounsi, A., Mahmoud, S.R. and Adda Bedia, E.A. (2013), "A new higher order shear and normal deformation theory for the static and free vibration analysis of sandwich plates with functionally graded isotropic face sheets", *J. Sandw. Struct. Mater.*, **15**(6), 671-703.

Besseghier, A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2017), "Free vibration analysis of embedded nanosize FG plates using a new nonlocal trigonometric shear deformation theory", *Smart Struct. Syst., Int. J.*, **19**(6), 601-614.

Birman, V., Keil, T. and Hosder, S. (2013), "Functionally Graded Materials in Engineering", In: *Structural Interfaces and Attachments in Biology*, (S. Thomopoulus, V. Birman, G.M. Genin Eds.), Chapter 2, Springer, New York, NY, USA.

Bouafia, K., Kaci, A., Houari, M.S.A., Benzair, A. and Tounsi, A. (2017), "A nonlocal quasi-3D theory for bending and free flexural vibration behaviors of functionally graded nanobeams", *Smart Struct. Syst., Int. J.*, **19**(2), 115-126.

Bouderba, B., Houari, M.S.A. and Tounsi, A. (2013), "Thermo-mechanical bending response of FGM thick plates resting on Winkler-Pasternak elastic foundations", *Steel Compos. Struct., Int. J.*, **14**(1), 85-104.

Bouderba, B., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2016), "Thermal stability of functionally graded sandwich plates using a simple shear deformation theory", *Struct. Eng. Mech., Int. J*, **58**(3), 397-422.

Bouguenina, O., Belakhdar, K., Tounsi, A. and Adda Bedia, E.A. (2015), "Numerical analysis of FGM plates with variable thickness subjected to thermal buckling", *Steel Compos. Struct., Int. J.*, **19**(3), 679-695.

Boukhari, A., Ait Atmane, H., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2016), "An efficient shear deformation theory for wave propagation of functionally graded material plates", *Struct. Eng. Mech., Int. J.*, **57**(5), 837-859.

Bounouara, F., Benrahou, K.H., Belkorissat, I. and Tounsi, A. (2016), "A nonlocal zeroth-order shear deformation theory for free vibration of functionally graded nanoscale plates resting on elastic foundation", *Steel Compos. Struct., Int. J.*, **20**(2), 227-249.

Bourada, M., Tounsi, A., Houari, M.S.A. and Adda Bedia, E.A. (2012), "A new four-variable refined plate theory for thermal buckling analysis of functionally graded sandwich plates", *J. Sandw. Struct. Mater.*, **14**(1), 5-33.

Bourada, M., Kaci, A., Houari, M.S.A. and Tounsi, A. (2015), "Anew simple shear and normal deformations theory for functionally graded beams", *Steel Compos. Struct., Int. J.*, **18**(2), 409-423.

Bourada, F., Amara, K. and Tounsi, A. (2016), "Buckling analysis of isotropic and orthotropic plates using a novel four variable refined plate theory", *Steel Compos. Struct., Int. J.*, **21**(6), 1287-1306.

Bousahla, A.A., Houari, M.S.A., Tounsi, A. and Adda Bedia, E.A. (2014), "A novel higher order shear and normal deformation theory based on neutral surface position for bending analysis of advanced composite plates", *Int. J. Comput. Meth.*, **11**(6), 1350082.

Bousahla, A.A., Benyoucef, S., Tounsi, A. and Mahmoud, S.R. (2016), "On thermal stability of plates with functionally graded coefficient of thermal expansion", *Struct. Eng. Mech., Int. J.*, **60**(2), 313-335.

Brush, D.O. and Almroth, B.O. (1975), *Buckling of Bars, Plates, and Shells*, McGraw-Hill, New York, NY, USA.

Bui, T.Q., Khosravifard, A. Zhang, Ch., Hematiyan, M.R. and Golub, M.V. (2013), "Dynamic analysis of sandwich beams with functionally graded core using a truly meshfree radial point interpolation method", *Eng. Struct.*, **47**, 90-104.

Bui, T.Q., Do, T.V., Ton, L.H.T., Doan, D.H., Tanaka, S., Pham, D.T., Nguyen-Van, T.A., Yu, T. and Hirose, S. (2016), "On the high temperature mechanical behaviors analysis of heated functionally graded plates using FEM and a new third-order shear deformation plate theory", *Compos. Part B*, **92**, 218-241.

Celebi, K., Yarimpabuc, D. and Keles, I. (2016), "A unified method for stresses in FGM sphere with exponentially-varying properties", *Struct. Eng. Mech., Int. J.*, **57**(5), 823-835.

Cheng, Z.Q. and Batra, R.C. (2000), "Three-dimensional thermo-elastic deformations of a functionally graded elliptic plate", *Compos. Pt. B-Eng.*, **31**(2), 97-106.

Chikh, A., Bakora, A., Heireche, H., Houari, M.S.A., Tounsi, A. and Adda Bedia, E.A. (2016), "Thermo-mechanical postbuckling of symmetric S-FGM plates resting on Pasternak elastic foundations using hyperbolic shear deformation theory", *Struct. Eng. Mech., Int. J.*, **57**(4), 617-639.

Chikh, A., Tounsi, A., Hebali, H. and Mahmoud, S.R. (2017), "Thermal buckling analysis of cross-ply laminated plates using a simplified HSDT", *Smart Struct. Syst., Int. J.*, **19**(3), 289-297.

Darabi, A. and Vosoughi, A.R. (2016), "Hybrid inverse method for small scale parameter estimation of FG nanobeams", *Steel Compos. Struct., Int. J.*, **20**(5), 1119-1131.

Draiche, K., Tounsi, A. and Mahmoud, S.R. (2016), "A refined theory with stretching effect for the flexure analysis of laminated composite plates", *Geomech. Eng., Int. J.*, **11**(5), 671-690.

Ebrahimi, F. and Jafari, A. (2016), "Thermo-mechanical vibration analysis of temperature-dependent porous FG beams based on Timoshenko beam theory", *Struct. Eng. Mech., Int. J.*, **59**(2), 343-371.

Ebrahimi, F. and Shafiei, N. (2016), "Application of Eringen\'s nonlocal elasticity theory for vibration analysis of rotating functionally graded nanobeams", *Smart Struct. Syst., Int. J.*, **17**(5), 837-857.

El-Haina, F., Bakora, A., Bousahla, A.A., Tounsi, A. and Mahmoud, S.R. (2017), "A simple analytical approach for thermal buckling of thick functionally graded sandwich plates", *Struct. Eng. Mech., Int. J.*, **63**(5), 585-595.

El-Hassar, S.M., Benyoucef, S., Heireche, H. and Tounsi, A. (2016), "Thermal stability analysis of solar functionally graded plates on elastic foundation using an efficient hyperbolic shear deformation theory", *Geomech. Eng., Int. J.*, **10**(3), 357-386.

Fahsi, A., Tounsi, A., Hebali, H., Chikh, A., Adda Bedia, E.A. and Mahmoud, S.R. (2017), "A four variable refined *n*th-order shear deformation theory for mechanical and thermal buckling analysis of functionally graded plates", *Geomech. Eng., Int. J.* [In press]

Fekrar, A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2014), "A new five-unknown refined theory based on neutral surface position for bending analysis of exponential graded plates", *Meccanica*, **49**(4), 795-810.

Feldman, E. and Aboudi, J. (1997), "Buckling analysis of functionally graded plates subjected to uniaxial loading", *Compos. Struct.*, **38**, 29-36.

Ganapathi, M. and Prakash, T. (2006), "Thermal buckling of simply supported functionally graded skew plates", *Compos. Struct.*, **74**(2), 247-250.

Ghorbanpour Arani, A., Cheraghbak, A. and Kolahchi, R. (2016), "Dynamic buckling of FGM viscoelastic nano-plates resting on orthotropic elastic medium based on sinusoidal shear deformation theory", *Struct. Eng. Mech., Int. J.*, **60**(3), 489-505.

Hadji, L., Hassaine Daouadji, T., Ait Amar Meziane, M., Tlidji, Y. and Adda Bedia, E.A. (2016), "Analysis of functionally graded beam using a new first-order shear deformation theory", *Struct. Eng. Mech., Int. J.*, **57**(2), 315-325.

Hamidi, A., Houari, M.S.A., Mahmoud, S.R. and Tounsi, A. (2015), "A sinusoidal plate theory with 5-unknowns and stretching effect for thermomechanical bending of functionally graded sandwich plates", *Steel Compos. Struct., Int. J.*, **18**(1), 235-253.

Hebali, H., Tounsi, A., Houari, M.S.A., Bessaim, A. and Adda Bedia, E.A. (2014), "New quasi-3D hyperbolic shear deformation theory for the static and free vibration analysis of functionally graded plates", *J. Eng. Mech.*, ASCE, **140**(2), 374-383.

Hebali, H., Bakora, A., Tounsi, A. and Kaci, A. (2016), "A novel four variable refined plate theory for bending, buckling, and vibration of functionally graded plates", *Steel Compos. Struct., Int. J.*, **22**(3), 473-495.

Houari, M.S.A., Tounsi, A., Bessaim, A. and Mahmoud, S.R. (2016), "A new simple three-unknown sinusoidal shear deformation theory for functionally graded plates", *Steel Compos. Struct., Int. J.*, **22**(2), 257-276.

Kar, V.R. and Panda, S.K. (2015a), "Large deformation bending analysis of functionally graded spherical shell using FEM", *Struct. Eng. Mech., Int. J.*, **53**(4), 661-679.

Kar, V.R. and Panda, S.K. (2015b), "Nonlinear flexural vibration of shear deformable functionally graded spherical shell panel", *Steel Compos. Struct., Int. J.*, **18**(3), 693-709.

Khetir, H., Bachir Bouiadjra, M., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2017), "A new nonlocal trigonometric shear deformation theory for thermal buckling analysis of embedded nanosize FG plates", *Struct. Eng. Mech., Int. J.* [In press]

Klouche, F., Darcherif, L., Sekkal, M., Tounsi, A. and Mahmoud, S.R. (2017), "An original single variable shear deformation theory for buckling analysis of thick isotropic plates", *Struct. Eng. Mech., Int. J.* [In press]

Laoufi, I., Ameur, M., Zidi, M., Adda Bedia, E.A. and Bousahla, A.A. (2016), "Mechanical and hygrothermal behaviour of functionally graded plates using a hyperbolic shear deformation theory", *Steel Compos. Struct., Int. J.*, **20**(4), 889-911.

Larbi Chaht, F., Kaci, A., Houari, M.S.A., Tounsi, A., Anwar Bég, O. and Mahmoud, S.R. (2015), "Bending and buckling analyses of functionally graded material (FGM) size-dependent nanoscale beams including the thickness stretching effect", *Steel Compos. Struct., Int. J.*, **18**(2), 425-442.

Lee, Y.Y., Zhao, X. and Reddy, J.N. (2010), "Postbuckling analysis of functionally graded plates subject to compressive and thermal loads", *Comput. Methods Appl. Mech. Eng.*, **199**(25), 1645-1653.

Lee, Y.-H., Bae, S.-I. and Kim, J.-H. (2016), "Thermal buckling behavior of functionally graded plates based on neutral surface", *Compos. Struct.*, **137**, 208-214.

Liew, K.M., Yang, J. and Kitipornchai, S. (2003), "Postbuckling of piezoelectric FGM plates subject to thermo-electro-mechanical loading", *Int. J. Solids Struct.*, **40**(15), 3869-3892.

Liu, S., Yu, T., Bui, T.Q., Yin, S., Thai, D.K. and Tanaka, S. (2017), "Analysis of functionally graded plates by simple locking-free quasi-3D hyperbolic plate isogeometric method", *Compos. Part B: Eng.*, **120**, 182-196.

Ma, L.S. and Wang, T.J. (2003), "Nonlinear bending and post-buckling of a functionally graded circular plate under mechanical and thermal loadings", *Int. J. Solids Struct.*, **40**(13), 3311-3330.

Mahi, A., Adda Bedia, E.A. and Tounsi, A. (2015), "A new hyperbolic shear deformation theory for bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates", *Appl. Math. Model.*, **39**(9), 2489-2508.

Mahmoud, S.R., Abd-Alla, A.M., Tounsi, A. and Marin, M. (2015), "The problem of wave propagation in magneto-rotating orthotropic non-homogeneous medium", *J. Vib. Control*, **21**(16), 3281-3291.

Matsunaga, H. (1997), "Buckling instabilities of thick elastic plates subjected to in-plane stresses", *Comput. Struct.*, **62**(1), 205-214.

Matsunaga, H. (2005), "Thermal buckling of cross-ply laminated composite and sandwich plates according to a global higher-order deformation theory", *Compos. Struct.*, **68**(4), 439-454.

Meksi, A., Benyoucef, S., Houari, M.S.A. and Tounsi, A. (2015), "A simple shear deformation theory based on neutral surface position for functionally graded plates resting on Pasternak elastic foundations", *Struct. Eng. Mech., Int. J.*, **53**(6), 1215-1240.

Meksi, R, Benyoucef, S., Mahmoudi, A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2017), "An analytical solution for bending, buckling and vibration responses of FGM sandwich plates", *J. Sandw. Struct. Mater.*, 1099636217698443.

Meradjah, M., Kaci, A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2015), "A new higher order shear and normal deformation theory for functionally graded beams", *Steel Compos. Struct., Int. J.*, **18**(3), 793-809.

Merazi, M., Hadji, L., Daouadji, T.H., Tounsi, A. and Adda Bedia, E.A. (2015), "A new hyperbolic shear deformation plate theory for static analysis of FGM plate based on neutral surface position", *Geomech. Eng., Int. J.*, **8**(3), 305-321.

Merdaci, S., Tounsi, A. and Bakora, A. (2016), "A novel four variable refined plate theory for laminated composite plates", *Steel Compos. Struct., Int. J.*, **22**(4), 713-732.

Meyers, C.A. and Hyer, M.W. (1991), "Thermal buckling and postbuckling of symmetrically laminated composite plates", *J. Therm. Stress.*, **14**(4), 519-540.

Mouaici, F., Benyoucef, S., Ait Atmane, H. and Tounsi, A. (2016), "Effect of porosity on vibrational characteristics of non-homogeneous plates using hyperbolic shear deformation theory", *Wind Struct., Int. J.*, **22**(4), 429-454.

Mouffoki, A., Adda Bedia, E.A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2017), "Vibration analysis of nonlocal advanced nanobeams in hygro-thermal environment using a new two-unknown trigonometric shear deformation beam theory", *Smart Struct. Syst., Int. J.* [In press]

Morimoto, T., Tanigawa, Y. and Kawamura, R. (2006), "Thermal buckling of functionally graded rectangular plates subjected to partial heating", *Int. J. Mech. Sci.*, **48**(9), 926-937.

Na, K.-S. and Kim, J.-H. (2004), "Three-dimensional thermal buckling analysis of functionally graded materials", *Compos., Part B, Eng.*, **35**(5), 429-437.

Na, K.-S. and Kim, J.-H. (2006), "Three-dimensional thermo-mechanical buckling analysis for functionally graded composite plates", *Compos. Struct.*, **73**(4), 413-422.

Najafizadeh, M.M. and Eslami, M.R. (2002), "Buckling analysis of circular plates of functionally graded materials under uniform radial compression", *Int. J. Mech. Sci.*, **44**(12), 2479-2493.

Najafizadeh, M.M. and Heydari, H.R. (2008), "An exact solution for buckling of functionally graded circular plates based on higher order shear deformation plate theory under uniform radial compression", *Int. J. Mech. Sci.*, **50**(3), 603-612.

Nguyen, K.T., Thai, T.H. and Vo, T.P. (2015), "A refined higher-order shear deformation theory for bending, vibration and buckling analysis of functionally graded sandwich plates", *Steel Compos. Struct., Int. J.*, **18**(1), 91-120.

Raminnea, M., Biglari, H. and Vakili Tahami, F. (2016), "Nonlinear higher order Reddy theory for temperature-dependent vibration and instability of embedded functionally graded pipes conveying fluid-nanoparticle mixture", *Struct. Eng. Mech., Int. J.*, **59**(1), 153-186.

Reddy, J.N. (1984), *Energy Principles and Variational Methods in Applied Mechanics*, John Wiley, New York, NY, USA.

Reddy, J.N. and Cheng, Z.Q. (2001), "Three-dimensional thermo-mechanical deformations of functionally graded rectangular plates", *Eur. J. Mech. A-Solids*, **20**(5), 841-855.

Saidi, H., Tounsi, A. and Bousahla, A.A. (2016), "A simple hyperbolic shear deformation theory for vibration analysis of thick functionally graded rectangular plates resting on elastic foundations", *Geomech. Eng., Int. J.*, **11**(2), 289-307.

Shi, G. (2007), "A new simple third-order shear deformation theory of plates", *Int. J. Solids Struct.*, **44**(13), 4399-4417.

Sobhy, M. (2013), "Buckling and free vibration of exponentially graded sandwich plates resting on elastic foundations under various boundary conditions", *Compos. Struct.*, **99**, 76-87.

Swaminathan, K. and Naveenkumar, D.T. (2014), "Higher order refined computational models for the stability analysis of FGM plates: Analytical solutions", *Eur. J. Mech. A/Solid.*, **47**, 349-361.

Swaminathan, K., Naveenkumar, D.T., Zenkour, A.M. and Carrera, E. (2015), "Stress, vibration and buckling analyses of FGM plates—a state-of-the-art review", *Compos. Struct.*, **120**, 10-31.

Taibi, F.Z., Benyoucef, S., Tounsi, A., Bachir Bouiadjra, R., Adda Bedia, E.A. and Mahmoud, S.R. (2015), "A simple shear deformation theory for thermo-mechanical behaviour of functionally graded sandwich plates on elastic foundations", *J. Sandw. Struct. Mater.*, **17**(2), 99-129.

Tounsi, A., Houari, M.S.A., Benyoucef, S. and Adda Bedia, E.A. (2013), "A refined trigonometric shear deformation theory for thermoelastic bending of functionally graded sandwich plates", *Aerosp. Sci. Technol.*, **24**(1), 209-220.

Tounsi, A., Houari, M.S.A. and Bessaim, A. (2016), "A new 3-unknowns non-polynomial plate theory for buckling and vibration of functionally graded sandwich plate", *Struct. Eng. Mech., Int. J.*, **60**(4), 547-565.

Tran, L.V., Thai, C.H. and Nguyen-Xuan, H. (2013), "An isogeometric finite element formulation for thermal buckling analysis of functionally graded plates", *Finite Elem. Anal. Des.*, **73**, 65-76.

Trinh, T.H., Nguyen, D.K., Gan, B.S. and Alexandrov, S. (2016), "Post-buckling responses of elastoplastic FGM beams on nonlinear elastic foundation", *Struct. Eng. Mech., Int. J.*, **58**(3), 515-532.

Tung, H.V. (2015), "Thermal and thermomechanical postbuckling of FGM sandwich plates resting on elastic foundations with tangential edge constraints and temperature dependent properties", *Compos. Struct.*, **131**, 1028-1039.

Turan, M., Adiyaman, G., Kahya, V. and Birinci, A. (2016), "Axisymmetric analysis of a functionally graded layer resting on elastic substrate", *Struct. Eng. Mech., Int. J.*, **58**(3), 423-442.

Vel, S.S. and Batra, R.C. (2002), "Exact solution for thermoelastic deformations of functionally graded thick rectangular plates", *AIAA J.*, **40**(7), 1421-1433.

Vinson, J.R. (2001), "Sandwich structures", *Appl. Mech. Rev.*, **54**(3), 201-214.

Vinson, J.R. (2005), "Sandwich Structures: Past, Present, and Future", In: *Sandwich Structures 7: Advancing with Sandwich Structures and Materials*, (O.T. Thomsen, E. Bozhevolnaya, A. Lyckegaard Eds.), Chapter 1, Springer, Netherlands.

Vu, T.V., Nguyen, N.H., Khosravifard, A., Hematiyan, M.R., Tanaka, S. and Bui, T.Q. (2017), "A simple FSDT-based meshfree method for analysis of functionally graded plates", *Eng. Anal. Bound. Elem.*, **79**, 1-12.

Yaghoobi, H. and Yaghoobi, P. (2013), "Buckling analysis of sandwich plates with FGM face sheets resting on elastic foundation with various boundary conditions: An analytical approach", *Meccanica*, **48**, 2019-2035.

Yaghoobi, H., Valipour, M.S., Fereidoon, A. and Khoshnevisrad, P. (2014), "Analytical study on postbuckling and nonlinear free vibration analysis of FG beams resting on nonlinear elastic foundation under thermo-mechanical loading using VIM", *Steel Compos. Struct., Int. J.*, **17**(5), 753-776.

Yin, S., Hale, J.S., Yu, T., Bui, T.Q. and Bordas, S.P.A. (2014), "Isogeometric locking-free plate element: A simple first order shear deformation theory for functionally graded plates", *Compos. Struct.*, **118**, 121-138.

Yin, S., Yu, T., Bui, T.Q., Zheng, X. and Tanaka, S. (2016), "In-plane material inhomogeneity of functionally graded plates: A higher-order shear deformation plate isogeometric analysis", *Compos. Part B*, **106**, 273-284.

Yu, T., Yin, S., Bui, T.Q. and Hirose, S. (2015), "A simple FSDT-based isogeometric analysis for geometrically nonlinear analysis of functionally graded plates", *Finite Elem. Anal. Des.*, **96**, 1-10.

Yu, T., Bui, T.Q., Yin, S., Doan, D.H., Wu, C.T., Van Do, T. and Tanaka, S. (2016a), "On the thermal buckling analysis of functionally graded plates with internal defects using extended isogeometric analysis", *Compos*. *Struct*., **136**, 684-695.

Yu, T., Yin, S., Bui, T.Q., Xia, S., Tanaka, S. and Hirose, S. (2016b), "NURBS-based isogeometric analysis of buckling and free vibration problems for laminated composites plates with complicated cutouts using a new simple FSDT theory and level set method", *Thin-Wall*. *Struct*., **101**, 141-156.

Yu, T., Yin, S., Bui, T.Q., Liu, C. and Wattanasakulpong, N. (2017), "Buckling isogeometric analysis of functionally graded plates under combined thermal and mechanical loads", *Compos*. *Struct*., **162**, 54-69.

Zemri, A., Houari, M.S.A., Bousahla, A.A. and Tounsi, A. (2015), "A mechanical response of functionally graded nanoscale beam: an assessment of a refined nonlocal shear deformation theory beam theory", *Struct*. *Eng*. *Mech*., *Int*. *J*., **54**(4), 693-710.

Zhang, L.W., Zhu, P. and Liew, K.M. (2014), "Thermal buckling of functionally graded plates using a local Kriging meshless method", *Compos*. *Struct*., **108**, 472-492.

Zidi, M., Tounsi, A., Houari, M.S.A., Adda Bedia, E.A. and Anwar Bég, O. (2014), "Bending analysis of FGM plates under hygro-thermo-mechanical loading using a four variable refined plate theory", *Aerosp*. *Sci*. *Tech*., **34**, 24-34.

Zidi, M., Houari, M.S.A., Tounsi, A., Bessaim, A. and Mahmoud, S.R. (2017), "A novel simple two-unknown hyperbolic shear deformation theory for functionally graded beams", *Struct*. *Eng*. *Mech*., *Int*. *J*. [In press]

CC