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A constitutive model for confined concrete in composite structures

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Abstract. The constitutive relation is an important factor in analysis of confined concrete in composite structures. In order to propose a constitutive model for nonlinear analysis of confined concrete, lateral restraint mechanism of confined concrete is firstly analyze to study the generalities. As the foundation of the constitutive model, peak stress and peak strain is the first step in research. According to the generalities and the Twin Shear Unified Strength Theory, a novel unified equation for peak stress and peak strain are established. It is well coincident with experimental results. Based on the general constitutive relations and the unified equation for peak stress and peak strain, we propose a unified and convenient constitutive model for confined concrete with fewer material parameters. Two examples involved with steel tube confined concrete and hoop-confined concrete are considered. The proposed constitutive model coincides well with the experimental results. This constitutive model can also be extended for nonlinear analysis to other types of confined concrete.

Keywords: confined concrete; lateral restraint force; the Twin Shear Unified Strength Theory; constitutive model

1. Introduction

The constraint of lateral pressure around the confined concrete can restrict the development of internal microcracks. The compressive features of the confined concrete are improved, and the compressive strength and ductility are increased (Tian *et al*. 2014, Han and Yang 2007, Soliman 2011, Lim and Ozbakkaloglu 2014, Samani and Attard 2012). This characteristic has been widely adopted in practice. For example, steel tube, hoops and FRP always are used to confine the concrete members, which are the main structural members. With lateral constraint, the filled concrete toughness is obviously improved, and the anti-collapse property of structure is increased (Cai 2003, Yu *et al*. 2010, Lee and Lee 2007, Ren *et al*. 2014, Toutanji and Saafi 2002). Therefore confined concrete has a wide practical engineering.

The constitutive relation for confined concrete is an important factor to study the nonlinear property of structure. Many scholars have proposed their own constitutive relation models (Shi *et al*. 2011, Mander *et al*. 1988, Scott *et al*. 1982, Daniel and Patrick 1995, Han and Yang 2007). Numerous studies have shown that the difference in the rising stage is less significant. Therefore, main research focuses on the declining stage of the models (Shi *et al*. 2011). Mander (1988) puts forward the effective restraint coefficient to describe the arching effect and proposes the unified constitutive model for the rising and declining stages of low-strength hoops confined concrete. But the computation values of both peak stress and peak strain of high-strength hoop specimen are generally higher, conversely, the computation values of peak strain with high constraint are lower. The model proposed by Scott (1982) assumes that the improvement of both peak stress and peak strain are just the same, and the declining stage of constitutive relation for the hoops confined concrete is simply fitted in a linear form. However, this model can be merely used to reasonably calculate the peak strain for the confined concrete with lower constraint degree and fit the constitutive relation for the concrete with weaker constraint effect. Afterwards, Daniel and Patrick (1995) put forward the constitutive model of high-strength hoops confined concrete. The declining stage of stress-strain curve is an exponential function form which is fitted by a series of parameters. To be more reasonable, the more parameters in this model and involve the iterative calculation method. Therefore, this model is unsuitable to the practical application. Han and Yang (2007) build the constitutive model of steel tube filled concrete, based on changing the control parameters of declining stage in the constitutive model of plain concrete. The confined concrete in square steel tube and circular steel tube has been discussed in the model. However, the declining stage curve of square steel tube filled concrete declines fast, while that of circular steel tube filled concrete declines slowly and even rises up.

The filled concrete has been investigated by many scholars through the experiments (Toutanji 2001, Shi *et al*. 2014). Their respective constitutive models of the particular confined concrete are put forward. However, there is not a unified model which is suitable for most types of confined concrete and could also be convenient for engineering application. Through the research, there are common characteristics of most types of confined concrete. Therefore, based on the Twin Shear Unified Strength

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Theory, this paper analyzes the confined concrete in the perspective of lateral compressive stress of concrete in the core area. Then the article builds unified computational equation of axial compressive mechanical properties. Furthermore, the unique constitutive model for most types of confined concrete is proposed.

2. Computation for stress and strain of confined concrete

2.1 Lateral restraint force of confined concrete

According to different section shapes of confined concrete, the common confined concretes in the practical engineering are circular steel tube filled concrete, square steel tube filled concrete and hoops confined concrete. These three kinds of concretes are studied in this paper. For the confined concrete, the main difference among various types is focused on lateral restraint force. The diagrammatic sketches of constraint forces generated by the lateral pressure on steel tube filled concrete and hoops confined concrete are shown in Fig. 1.

The direct interaction between steel tube and concrete for circular steel tube filled concrete is shown in Fig. 1(a). The forces exerting on steel tube are relatively definite, including longitudinal compressive stress, radial compressive stress and circumferential tensile stress. According to the computational results and the practical engineering, it can be obtained that when the circumferential tensile stress reached the yield stress, circumferential tensile stress is equal to the steel yield stress (f_y) directly. Therefore, the lateral compressive stress on the confined concrete in the core zone meet $p = 2tf_y / D$ by the mechanical analysis, D is the diameter, t is the thickness.

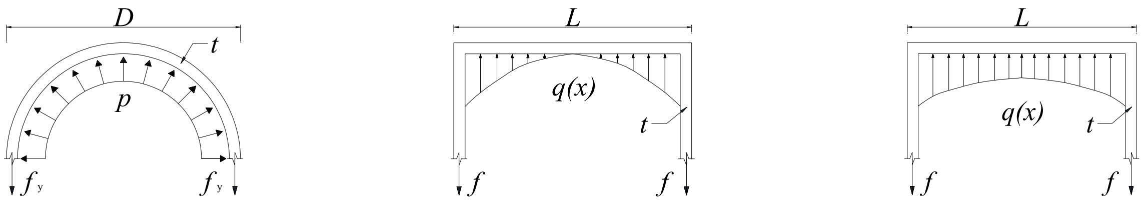
Compared with circular steel tube filled concrete, the internal constraint mechanism of square steel tube filled concrete is more complex. The damage of short square steel tube filled concrete column is caused by the yield at the corner of steel tube and longitudinal buckling of steel plate in the middle. Actually, the concrete in the core area is divided into effective constraint region and the non-effective constraint area under the lateral restraint force (Varma *et al*. 2005). But it is difficult to apply in the formula, this paper turns to determine the effective lateral restraint force. The normal constraint force between steel tube and concrete is the only factor to be considered, and the distribution curve along the side length changed with the width-to-thickness ratio L/t of tube wall, as shown in Fig. 1(b). The normal constraint force distributed along the side length following the law of quadratic curve, and then the equation is $q(x) = 4qx^2/L^2$. Where q is the maximum of lateral restraint force, and $q = 2tf_y/L$. According to the theory proposed by Mander (Mander *et al*. 1988), the homogeneous distribution of lateral pressure along the surface of core concrete, and it assumes that the average stress is equivalent to average value of the curve equation. Then the equation for the lateral compressive stress is as follows

$$p = \frac{2}{L}\int_0^{L/2} q(\mathrm{x}) = q / 3 \tag{1}$$

Internal constraint mechanism of hoops confined concrete is similar to that of square steel tube filled concrete. But the difference of hoops confined concrete is that there is larger lateral compressive stress on the midpoint of each side of hoops as the peak stress is reached. The hoops are mainly extruded in the lateral and vertical directions. The damage is caused by the yield at the corner of hoops. But there is no connection between each hoop, the hoops do not directly bear axial compressive stress, and the main stress is the lateral compressive stress from the inner concrete. Therefore, there is no relative vertical displacement between hoops and concrete. Compared with square steel tube, the hoops generated better confined effect in the cross sections of members. But because the constraint of hoops along the length direction of members is non-uniform, the “arch effect” would be along the length direction. According to the studies on a large number of the experimental data of hoops confined concrete (Mander *et al*. 1988, Shi *et al*. 2011, 2013), the reduced confinement effect of “arch effect” is mainly embodied on the constitutive model by the influence of volume-stirrup ratio, it would be studied in later section. Because of the similar restraint mechanism between hoops and square steel tube, the hoops could be considered as square steel tube with the same amount of steel in volume. The distribution curve of lateral compressive stress is shown in Fig. 1(c). Assume that the thickness of equivalent square steel tube is t and the side length of concrete is L, the volume-hoop ratio equation can be rewritten as $\rho_v = 4tL/L^2 = 4t/L$. The lateral restraint force from hoops is more homogeneous, and hoops are embedded inside the concrete. Hence the reduction effect of lateral restraint force can be ignored. Then $p = 2tf_y/L = \rho_v f_y/2$ is obtained by the mechanical analysis.

2.2 Peak stress computation of confined concrete

The confined concrete is compressed in three directions,



(a) Circular steel tube filled concrete (b) Square steel tube filled concrete (b) Hoops confined concrete

Fig. 1 Diagrammatic sketches of lateral restraint forces no matter what cross section form is, the peak stress of confined concrete can be analyzed by the same method. This paper computes ultimate bearing capacity of three-dimensional compressive concrete based on the Twin Shear Unified Strength Theory, which is suitable for various materials under complex stress states. The Twin Shear Unified Strength Theory (Yu *et al*. 2007) is that both the second principal shear stress and the corresponding principal stress are considered, the materials would be failed as the combination of them reaches the extremum. Its mathematical expression is:

$$F=\sigma_1-\frac{\alpha}{1+b}\left(b\sigma_2+\sigma_3\right)=\sigma_t,\ \ \sigma_2\le\frac{\sigma_1+\alpha\sigma_3}{1+\alpha} \tag{2a}$$

$$F'=\frac{1}{1+b}\left(\sigma_1+b\sigma_2\right)-\alpha\sigma_3=\sigma_t,\ \ \sigma_2>\frac{\sigma_1+\alpha\sigma_3}{1+\alpha} \tag{2b}$$

where σ_1, σ_2 and σ_3 are three principal stresses, $\alpha = \sigma_t / \sigma_c$ is the ratio of tensile strength to compressive strength of material, σ_t and σ_c are the uniaxial tensile strength limit and uniaxial compression strength limit of material, and b is a weighted parameter. In addition, the cohesive force c of concrete and internal friction angle φ are used to express the Eqs. (2a) and (2b), and then

$$F=\tau_{13}+b\tau_{12}+\sin\varphi\left(\sigma_{13}+b\sigma_{12}\right)=(1+\mathrm{b})c\cos\varphi,\ \ F\ge F' \tag{3a}$$

$$F'=\tau_{13}+b\tau_{23}+\sin\varphi\left(\sigma_{13}+b\sigma_{23}\right)=(1+\mathrm{b})c\cos\varphi,\ \ F<F' \tag{3b}$$

where τ_{12}, τ_{23} and τ_{13} are three principal shear stresses, $\tau_{12} = (\sigma_1 - \sigma_2)/2$, $\tau_{23} = (\sigma_2 - \sigma_3)/2$, $\tau_{13} = (\sigma_1 - \sigma_3)/2$, and σ_{12}, σ_{23} and σ_{13} are the principal stresses corresponded to three principal shear stresses, $\sigma_{12} = (\sigma_1 + \sigma_2)/2$, $\sigma_{23} = (\sigma_2 + \sigma_3)/2$, $\sigma_{13} = (\sigma_1 + \sigma_3)/2$.

Subtract Eq. (3a) from Eq. (3b), then

$$F'-F=b\left(1-\sin\varphi\right)\left(\sigma_1-\sigma_3\right)\ge 0$$

As shown in the above equations, the Eq. (3b) should be used to computation for the confined concrete, the peak stress state of confined concrete is $0 > \sigma_1 = \sigma_2 > \sigma_3$. Then the principal stress form is taken for Eq. (3b), and the Eq. (3b) is simplified as

$$-\sigma_3=\frac{2c\cos\varphi}{1-\sin\varphi}-\frac{1+\sin\varphi}{1-\sin\varphi}\sigma_1 \tag{4}$$

Since the cohesive force c and internal friction angle φ of concrete are difficult to measure, other parameters would be applied to replace. As unixial compression for concrete, $\sigma_3 = -f_c$, $\sigma_1 = \sigma_2 = 0$. According to Mohr strength criterion, $f_c = (2c\cos\varphi)/(1-\sin\varphi)$, $k = (1 + \sin\varphi)/(1 - \sin\varphi)$. For confined concrete, $\sigma_3 = -f_{cc}$, $\sigma_1 = \sigma_2 = -p$, then the Eq. (4) can be expressed as

$$f_{cc}=f_c+kp \tag{5}$$

It is observed that the peak stress equation is irrelevant to b. Because the second principal stress is equal to the third principal stress, $\tau_{12} + \mathrm{b}\tau_{23} = (1 + b)(\sigma_1 - \sigma_3)$ and $\sigma_{12} + b\,\sigma_{23} = (1 + b)(\sigma_1 + \sigma_3)$, then $1 + b$ can be removed, it mean that the influence of the second principal stress can be ignored. Therefore, b has no influence on the results based on the analysis of the Twin Shear Unified Strength Theory. Moreover, k is the lateral confinement coefficient. It is related to the internal friction angle. According to the triaxial compression state of concrete, the internal friction angle decreases with the increasing lateral compression. For steel tube filled concrete and hoops confined concrete, a large number of axial compression test is collected and analyzed in this paper (Cai 2003, Han and Yang 2007, Shi *et al*. 2011, 2013), 200 groups of representative experimental data are selected. According to analysis, it is found by that k is related to the lateral compression on the concrete and concrete strength. Furthermore, ω is defined as the relative lateral restraint force and the representative of the two influencing factors of k. It reflects the constraint effect of confined concrete ($\omega = p/f_c$, p is the lateral compressive stress on the confined concrete, f_c is the uniaxial compressive strength of concrete).

The relationship between k and ω is shown in Fig. 2. It is observed that k decreases with the increasing of ω, first trend of k values is sharply decline, then the trend is slowly, eventually they tend to be 1. Moreover, there is the functional relationship between k and ω. The computed equation for k is obtained, as shown in Eq. (6).

$$k=1+\frac{1}{3.5\omega^{0.6973}} \tag{6}$$

Simultaneously, when computing the peak stress, size effect should be considered. Therefore, the size reduction coefficient is introduced, γ_u (Sakino *et al*. 2004). As the lateral restraint force is applied to the confined concrete, the mechanism to increase the strength is the same. Therefore, the equations of confined concrete are similar, the peak stress equation of confined concrete is obtained by the combination of Eqs. (5) and (6).

$$f_{cc}=\gamma_u f_c+kp \tag{7}$$

Where γ_u is the strength reduction coefficient (Sakino *et al*. 2004), and $\gamma_u = 1.67D^{-0.112}$. The parameter D is the outer diameter of steel tube for the circular steel tube filled

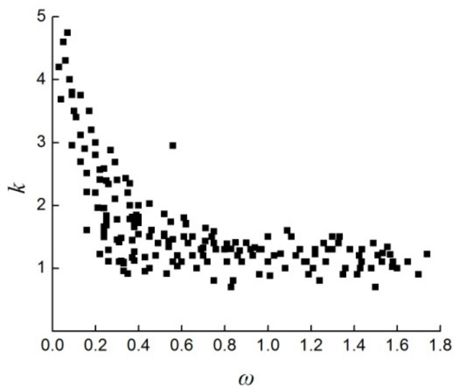
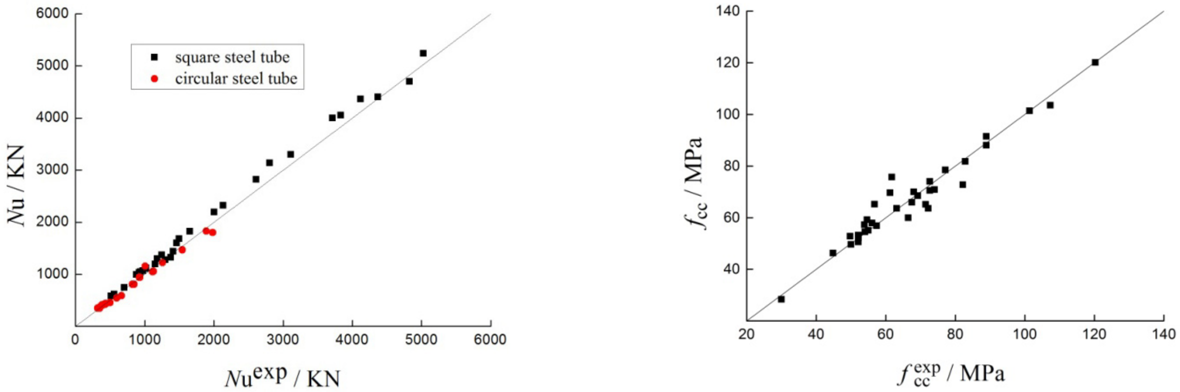


Fig. 2 Relation between k and ω



(a) Steel tube confined concrete (b) Concrete confined by hoops

Fig. 3 Comparison between experimental values and computational values for confined concrete peak stress

concrete, the range is from 50 mm to 800 mm. For square steel tube filled concrete and hoops confined concrete, the parameter D is the side length (Sakino *et al*. 2004) on outer surface of cross sections. Furthermore, f_c is the uniaxial compressive strength of concrete. In terms of square steel tube filled concrete and hoops confined concrete, f_c should take the standard compressive strength. But for circular steel tube filled concrete, the section of confined concrete is circular, f_c should be replaced by the uniaxial compressive strength of cylinder, $f_{cy}=mf_c$. In addition, m is transformation coefficient (Guo *et al*. 1982). And k is the restraint coefficient and can be obtained by Eq. (7).

To verify the computation equation for peak stress, this paper compares the experimental data (Chung *et al*. 2013, Li *et al*. 2015, Han and Yang 2007, Harries and Kharel 2003, Patel *et al*. 2012, Sakino *et al*. 2004, Zhang *et al*. 2007) of the three kinds of filled concrete with the computational data according to the equation. The comparisons for steel tube filled concrete and hoops confined concrete are shown in Fig. 3. According to coincidence between experimental results and computational results, this paper further compute the correlation coefficient R, residual mean Q and residual standard deviation S for the two groups of data as the evaluation parameters, the parameters can be established as shown in Eqs. (8a)-(8c). The evaluation parameters of steel tube filled concrete are R = 0.997, Q = 52.41 and S = 8.71, as well as the evaluation parameters of hoops confined concrete are R = 0.970, Q = 0.22 and S = 0.09. The values of Q and S of steel tube filled concrete are greater, due to the higher bearing capacity of steel tube filled concrete. To sum up, the computational results agree better with the experimental results in this paper.

$$R=\sum_1^z \frac{\exp_i - com_i}{\exp_i} \tag{8a}$$

$$Q=\frac{\sum_1^z \exp_i - com_i}{n} \tag{8b}$$

$$S=\sqrt{\frac{\sum_1^z (Q_i-Q)^2}{z-1}} \tag{8c}$$

where exp_i is the experimental data; com_i is the computation data; z is the quantity of the data.

2.3 Peak stress computation of confined concrete

Based on the experimental analysis and theoretical research, the axial compression of confined concrete can be divided into two parts. One part is equal to the lateral restraint force, which can be constituted to hydrostatic pressure with the lateral constraint force, called as hydrostatic pressure part. The other part is the stress without the hydrostatic pressure, which is higher than pressure of original plain concrete. Because the concrete performance is changed by the lateral restraint force, this part could be called as the well performance concrete part. Correspondingly, there are two parts in the peak strain, one part is the strain ε_c of concrete after performance improvement, and the other part is the strain ε_p generated by the hydrostatic pressure. According to the theoretical analysis and experimental data in the literatures (Cai 2003, Chung *et al*. 2002, Hong *et al*. 2006, Shi *et al*. 2014), the computation equation of peak strain ε_{cc} of confined concrete is obtained by fitting.

$$\varepsilon_{cc}=\varepsilon_c+\varepsilon_p \tag{9}$$

where $\varepsilon_c = 700+172\times f_c^{'0.5}\times 10^{-6}$, $f_c^{'}=f_c+(k-1)p$, $\varepsilon_p=800\times(2\omega)^{0.2}\times 10^{-6}$. The comparison between the experimental

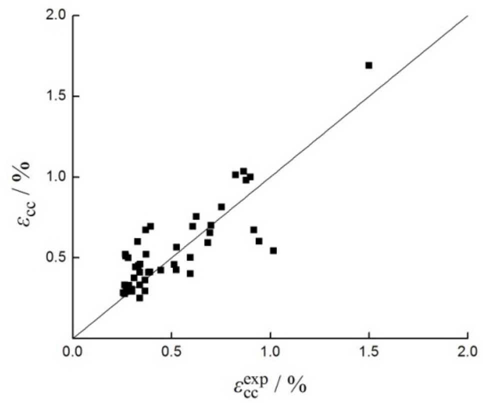


Fig. 4 Comparison between test values and computed values of peak strain

values and computed values (Cai 2003, Han and Yang 2007, Guo *et al*. 1982) of the peak strain of confined concrete is shown in Fig. 4. Simultaneously, the evaluation parameters for goodness of fit of data are R = 0.831, Q = 0.041 and S = 0.16. It suggests that the computed results from the equation agreed well with the experimental results.

3. Constitutive model

3.1 Design concept

By the compressive mechanical properties analysis for the confined concrete, there are few internal microcracks before the peak strain, and the influence of lateral compressive stress for the stress-strain development tendency is small. Therefore, the rising stage curve can be simplified as constitutive model of plain concrete. The lateral restraint force gradually increase with the lateral deformation of concrete, and both the ultimate bearing capacity and ultimate strain will increase due to the restriction for the internal microcracks of concrete.

The current uniaxial constitutive relation is sectional curve equation, which is proposed by Guo *et al*. (1982). The equation is based on the dimensionless coordinates and accordance with the characteristics of plain concrete. Moreover, the rising stage and the declining stage could be adjusted by some parameters. It can be the convenient method to fit constitutive relation of various confined concrete.

There are several limitations in the current models for some various confined concrete. There is no unified model available for all types of confined concrete. According to the convenient method and the compressive mechanical properties analysis, this paper proposes the unified constitutive relation model, which is based on lateral compressive stress and combined with the current constitutive relations (Guo *et al*. 1982). In consideration of the constraint effect, some parameters should be introduced and modified.

3.2 Constitutive model of confined concrete

Based on the above analysis, before the peak stress is reached, the lateral restraint force has a greater influence on the peak stress and strain, but has little influence on the change tendency of rising stage curve. Hence, the curve equation of rising stage is consistent with the constitutive relation model of plain concrete. Due to the constraint effect of the lateral restraint force, the internal microcracks of concrete developed slowly, and the declining stage of constitutive relation tended to be gentle. Because the declining stage to be gentle, the parameter α_d in the declining stage of confined concrete needs to be solved, as well as the slope coefficient n in the declining stage is introduced. For hoops confined concrete, the constraint effect on the concrete in the spacing area of hoops is less significant, namely, the constraint effect on the hoops confined concrete along the length direction is not homogeneous, and the overall constraint effect decreases. When the lateral restraint coefficient of hoops confined concrete is transformed into that of steel tube confined concrete and the volume-hoop ratio ρ_v of hoops confined concrete is smaller, the constraint of hoops along the length direction is non-uniform, and the internal microcrack development of concrete has not been effectively restricted. Therefore, the declining stage of constitutive relation model of hoops confined concrete will appear earlier. To meet the applicability of model in this paper, the effective constraint coefficient Ψ is proposed to consider the non-uniform constraint of hoops concrete confined along the length direction, which is referenced for the Mander model (Mander *et al*. 1988) and the declining stage parameters in experimental data for hoops confined concrete. The constitutive equation of confined concrete is as follows

$$\begin{cases} y = \alpha_a x + (3 - 2\alpha_a)x^2 + (\alpha_a - 2)x^3 & x \le 1 \\ y = \dfrac{x}{\psi\alpha_d (x-1)^n + x} & x > 1 \end{cases} \tag{10}$$

where $x = \varepsilon_c/\varepsilon_{cc}$, $y = f/f_{cc}$. The parameter (Guo *et al*. 1982) in the rising stage is α_d = 2.4 – 0.0124f_{cc}. In the declining stage, α_d is the important parameter which can be computed by Eq. (10); Ψ is the effective constraint coefficient (Mander *et al*. 1988), for the steel tube filled concrete Ψ = 1, for hoops confined concrete $\Psi = 0.021/\rho_v$; n is the declining slope coefficient (Shi *et al*. 2013), n = 1+ exp(-3ω).

According to arrangement and analysis for the experimental results of confined concrete (Han and Yang 2007, Mander *et al*. 1988), it is known that the characteristics in the declining stage of stress-strain curve of confined concrete are mainly related to the lateral restraint coefficient ω. This mainly shows that the greater the ω is, the stronger the constraint effect for confined concrete is. With the increase of deformation, the declining stage of constitutive relation model appears later, and even the declining stage does not appear. Based on the above analysis, the influence of increase in the ultimate compressive strain of confined concrete on the parameter of declining stage is used for reference, the influence of lateral constraint effect is fully considered. The computation equation of declining stage parameter α_d is proposed, based on a large number of experimental results and theories (Salim and Murat 1999, Shi *et al*. 2013, 2014).

$$\begin{cases} \alpha_d = \dfrac{3d}{17(d-1)^n} \\ d = \dfrac{0.0033}{\varepsilon_{cc}}\left[1 + 2.18\times10^5 \left(2\omega\sigma_c\varepsilon_{cc} / f_y\right)^{1.2}\right] \end{cases} \tag{11}$$

4. Constitutive models validation

The proposed model can be used for calculation to obtain the stress-strain curves of all kinds of confined concretes. There are three typical kinds of confined concrete have been analyzed. Therefore, we use two examples to verify the model, and three computation curves are compared with experimental curves. Finally, the results are shown in Figs. 5 and 6. Fig. 5(a) is for square steel tube confined concrete, the outer side length of steel tube is L = 200 mm, wall-thickness is t= 5.4 mm, compression strength of concrete is f_c = 44.5 MPa, yield strength of steel tube is f_y = 486 MPa. Fig. 5(b) is for circular steel tube confined concrete, the outer diameter of steel tube is D = 200 mm, wall-thickness is t = 2.5 mm, compression strength of concrete is f_c = 44.5 MPa, yield strength of steel tube is f_y = 486 MPa. Due to the complexity, figure is about hoops confined concrete. For Fig. 6(a), volumetric hoop ratios is ρ_v = 2.12%, compressive strength of concrete is f_c = 39.2 MPa, yield strength of hoops is f_y = 486 MPa. For Fig. 6(b), volumetric hoop ratios is ρ_v = 3.25%, compression strength of concrete is f_c= 45.5 MPa, yield strength of hoops is f_y = 486 MPa. or Fig. 6(c), volumetric hoop ratios is ρ_v = 1.49%, compressive strength of concrete is f_c = 44.5 MPa, yield strength of hoops is f_y= 1143 MPa.

To further illustrate the fitting of two curves, the measured strain in the experiments was taken as the known increment to be substituted into the proposed models in this paper, and then the computation results of stress of confined concrete are obtained. Afterwards, the computation results are compared with the stress values from the experiments to evaluate the fitting of curves. Finally, the evaluation parameters are shown in Table 1.

Table 1 Evaluation parameters of constitutive relation model

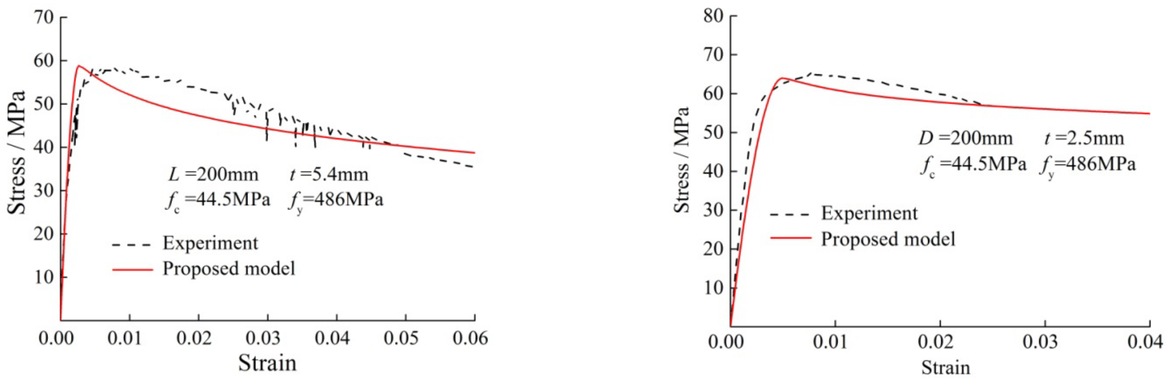
Serial number	R	Q	S
Fig. 5(a)	0.929	0.43	4.48
Fig. 5(b)	0.981	3.20	4.43
Fig. 6(a)	0.996	0.49	1.15
Fig. 6(b)	0.984	1.32	2.19
Fig. 6(c)	0.984	0.82	1.86

As shown in Fig. 5, the fitting degrees of curves in Figs. 5(a), (b), 6(a) are well, and the fitting degrees of curves in Figs. 6(b) and (c) are the best by the comparison for the computation results in Figs. 5, 6 and Table 1. Therefore, the proposed constitutive relation model in this paper can fit better with the stress-strain curve of confined concrete.

5. Conclusions

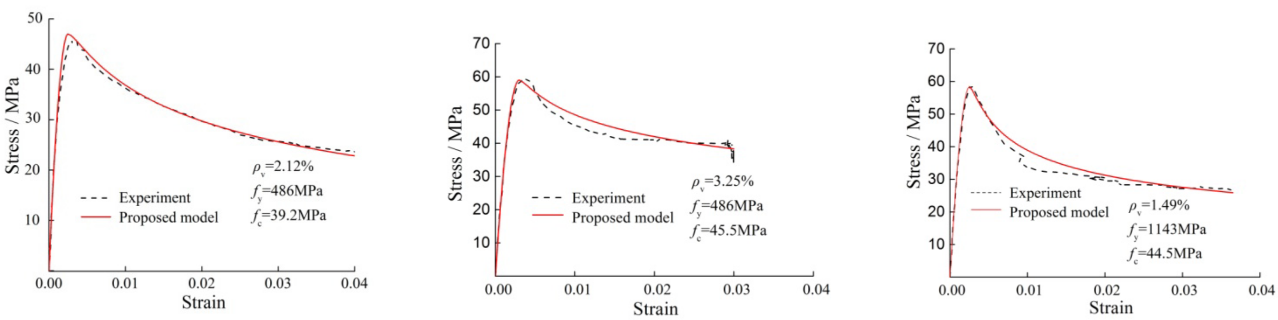
The lateral restraint force is an important factor to affect the confined concrete performance. The confined concrete has good compressive property and deformability. This paper obtains computation equations of lateral restraint forces for three kinds of confined concretes. As well as the lateral restraint coefficient ω is introduced to considerate the lateral restraint forces and performance of the concrete.

From the perspective of lateral compressive stress on the confined concrete, the computation equation of peak stress of confined concrete is proposed based on the Twin Shear Unified Strength Theory. Based on the computation



(a) Square section (a) Square section

Fig. 5 Comparison between the two curves of steel tube confined concrete



(a) (b) (c)

Fig. 6 Comparison between the two curves of hoops confined concrete equation of peak stress, the computation equation of peak strain is obtained. Moreover, since the equation is based on the lateral restraint force, this equation is suitable for various confined concretes.

Based on the constitutive relation model of plain concrete, by modifying main parameters and introducing some important parameters, the constitutive relation model of confined concrete is proposed. In addition, the model parameters can be obtained by the uniaxial stress performance of concrete. By means of comparison between computational curves and experimental curves, it shows good fit of model.

From the uniaxial compressive mechanical properties and lateral compressive stress, the constitutive relation model proposed in this paper is simple and convenient for use. Moreover, this solution method is suitable for most for the confined concrete, and this model belongs to a unified constitutive relation model of a variety of confined concrete. It is very useful for practical engineering.

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