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Free vibrations of laminated composite plates using a novel four variable refined plate theory

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Abstract. In this research, the free vibration response of laminated composite plates is investigated using a novel and simple higher order shear deformation plate theory. The model considers a non-linear distribution of the transverse shear strains, and verifies the zero traction boundary conditions on the surfaces of the plate without introducing shear correction coefficient. The developed kinematic uses undetermined integral terms with only four unknowns. Equations of motion are obtained from the Hamilton's principle and the Navier method is used to determine the closed-form solutions of antisymmetric cross-ply and angle-ply laminates. Numerical examples studied using the present formulation is compared with three-dimensional elasticity solutions and those calculated using the first-order and the other higher-order theories. It can be concluded that the present model is not only accurate but also efficient and simple in studying the free vibration response of laminated composite plates.

Keywords: laminated composite plates; free vibration; refined plate theory; navier solution

1. Introduction

Laminated plate structures are widely employed in the aerospace, automotive, civil, marine and other structural applications due to advantageous features such as high ratio of stiffness and strength to weight and low maintenance cost (Khandan *et al*. 2012, Grover *et al*. 2013, Guo *et al*. 2014, Mahapatra *et al*. 2016). However, shear deformation influences become more important in such structures because of the low transverse shear moduli as compared to high in-plane tensile moduli, when subjected to transverse loads. This requires the accurate structural investigation of composite plates.

The classical laminated plate theory (CLPT), which does not introduce the transverse shear deformation effects, gives reasonable results for thin plates. However, it under-predicts transverse displacements and over-predicts frequencies as well as buckling loads with moderately thick plates (Reddy 1997). In order to overcome the problem encountered in CLPT, shear deformation theories considering the transverse shear deformation effect, have been proposed. The first-order shear deformation theory (FSDT) considers linear distribution of axial displacements within the thickness. Many investigations of the free vibration of laminated plates have presented by employing FSDT (Whitney and Pagano 1970, Noor and Burton 1989, Khdeir 1989a). Since FSDT does not verify equilibrium conditions at the top and bottom faces of the plate, shear correction factors are necessitated to correct the unrealistic distribution of the shear strain/stress across the thickness (Bouderba *et al*. 2016). The value of shear correction coefficient is influenced not only on the lamination and geometric parameters, but also on the loading and boundary conditions. To avoid the employ of shear correction coefficients, the higher-order shear deformation theories (HSDT) have been recommended. The HSDT has been widely utilized to study the behavior of composite structures (Ren 1986, Khdeir 1989b, Matsunaga 2001, Singh *et al*. 2001, Aagaah *et al*. 2006, Swaminathan and Patil 2008, Tounsi *et al*. 2013, Bouderba *et al*. 2013, Ait Amar Meziane *et al*. 2014, Draiche *et al*. 2014, Nedri *et al*. 2014, Fekrar *et al*. 2014, Zidi *et al*. 2014, Ait Yahia *et al*. 2015, Mahi *et al*. 2015, Ait Atmane *et al*. 2015, Attia *et al*. 2015, Belkorissat *et al*. 2015, Taibi *et al*. 2015, Akavci and Tanrikulu 2015, Bousahla *et al*. 2016, Houari *et al*. 2016, Bellifa *et al*. 2016, Beldjelili *et al*. 2016, Boukhari *et al*. 2016, Bounouara *et al*. 2016, Chikh *et al*. 2017, Mouffoki *et al*. 2017, Klouche *et al*. 2017, Besseghier *et al*. 2017, Bellifa *et al*. 2017, El-Haina *et al*. 2017). A review of various shear deformation models for the investigation of laminated composite plates is found in Refs (Noor 1989, Reddy 1990, Mallikarjuna and Kant 1993, Dahsin and Xiaoyu 1996, Sayyad and Ghugal 2015). Recently, a new and simple FSDT is developed by Mantari and Ore (2015) for laminated composite and sandwich plates. In addition, it can be found in some studies the thickness stretching effect in structures behaviors such as (Bessaim *et al*. 2013, Bousahla *et al*. 2014, Belabed *et al*. 2014, Hebali *et al*. 2014, Larbi Chaht *et al*. 2015, Meradjah *et al*. 2015, Bourada *et al*. 2015, Hamidi *et al*. 2015, Draiche *et al*.

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2016, Bennoun *et al*. 2016).

The purpose of this work is to propose a new displacement field for the free vibration of laminated composite plates. The proposed kinematic employs undetermined integral terms with only four variables. Equations of motion are deduced from the Hamilton's principle. The analytical solutions for simply supported antisymmetric cross-ply and angle-ply laminates are determined by utilizing Navier procedure. The influences of parameters such as the aspect ratio, thickness ratio, modulus ratio and number of layers on the natural frequencies of the laminates are examined. Numerical results are discussed to demonstrate the accuracy and efficiency of the present model in studying the vibration behavior of laminated plates by comparing the obtained results with those calculated via various theories and the exact solutions of three-dimensional elasticity theory.

2. Mathematical formulation

Consider a rectangular plate of total thickness h having n orthotropic layers with the coordinate system as plotted in Fig. 1.

2.1 Kinematics

In this investigation, some simplifying suppositions are adopted to the existing HSDT so that the number of unknowns is reduced. The displacement field of the existing HSDT is expressed by

$$u(x,y,z,t) = u_0(x,y,t) - z\frac{\partial w_0}{\partial x} + f(z)\varphi_x(x,y,t) \tag{1a}$$

$$v(x,y,z,t) = v_0(x,y,t) - z\frac{\partial w_0}{\partial y} + f(z)\varphi_y(x,y,t) \tag{1b}$$

$$w(x,y,z,t) = w_0(x,y,t) \tag{1c}$$

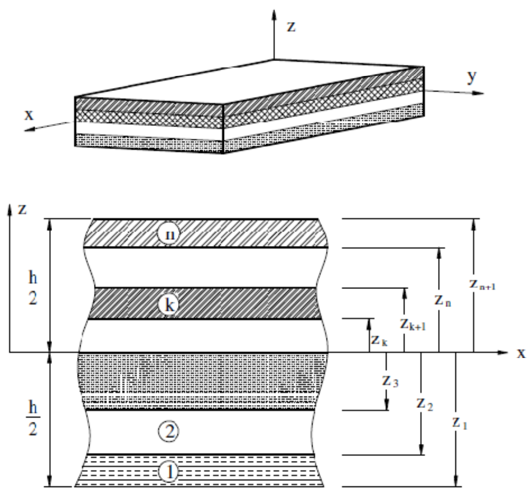

Fig. 1 Coordinate system and layer numbering used for a typical laminated plate

where u_0, v_0, w_0, φ_x and φ_y are five generalized displacements, $f(z)$ is the shape function representing the variation of the transverse shear strains and stresses within the thickness. By adopting that D and $\varphi_y = \int \theta(x,y)dy$, the kinematic of the proposed theory can be expressed in a simpler form as (Bourada *et al*. 2016, Hebali *et al*. 2016, Merdaci *et al*. 2016, Meksi *et al*. 2017, Fahsi *et al*. 2017)

$$u(x,y,z,t) = u_0(x,y,t) - z\frac{\partial w_0}{\partial x} + k_1\, f(z)\int \theta(x,y,t)\,dx \tag{2a}$$

$$v(x,y,z,t) = v_0(x,y,t) - z\frac{\partial w_0}{\partial y} + k_2\, f(z)\int \theta(x,y,t)\,dy \tag{2b}$$

$$w(x,y,z,t) = w_0(x,y,t) \tag{2c}$$

where $u_0(x, y)$, $v_0(x, y)$, $w_0(x, y)$ and $\theta(x, y)$ are the four unknown displacement functions of middle surface of the laminate. The constants k_1 and k_2 depends on the geometry. The integrals utilized are undetermined.

In this work, the present HSDT is obtained by putting

$$f(z) = z\left(\frac{5}{4} - \frac{5z^2}{3h^2}\right) \tag{3}$$

The strains associated with the displacements in Eq. (2) are

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + f(z)\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix}, \qquad \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g(z)\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix}, \tag{4}$$

where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial x} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial^2 w_0}{\partial y^2} \\ -2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix},$$

$$\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} k_1\theta \\ k_2\theta \\ k_1\dfrac{\partial}{\partial y}\displaystyle\int \theta\, dx + k_2\dfrac{\partial}{\partial x}\displaystyle\int \theta\, dy \end{Bmatrix}, \tag{5a}$$

$$\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} k_1\displaystyle\int \theta\, dy \\ k_2\displaystyle\int \theta\, dx \end{Bmatrix},$$

and

$$g(z) = \frac{df(z)}{dz} \tag{5b}$$

The integrals employed in the above relations shall be resolved by a Navier solution and can be expressed by

$$\frac{\partial}{\partial y}\int \theta\, dx = A'\frac{\partial^2 \theta}{\partial x \partial y}, \qquad \frac{\partial}{\partial x}\int \theta\, dy = B'\frac{\partial^2 \theta}{\partial x \partial y},$$
$$\int \theta\, dx = A'\frac{\partial \theta}{\partial x}, \qquad \int \theta\, dy = B'\frac{\partial \theta}{\partial y} \tag{6}$$

where the parameters A' and B' are defined according to the type of solution utilized, in this case via Navier. Hence, A' and B' are expressed by

$$A' = -\frac{1}{\alpha^2}, \quad B' = -\frac{1}{\beta^2}, \quad k_1 = \alpha^2, \quad k_2 = \beta^2 \tag{7}$$

where α and β are defined in Eq. (23).

2.2 Constitutive equations

With the consideration that each layer posses a plane of elastic symmetry parallel to the x–y plane, the constitutive equations for a layer can be expressed as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{8}$$

where Q_{ij} are the plane stress-reduced stiffnesses, and are expressed in terms of the engineering constants in the material axes of the layer Q

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}; \; Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}}; \; Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}};$$
$$Q_{66} = G_{12}; \; Q_{44} = G_{23}; \; Q_{55} = G_{13} \tag{9}$$

The constitutive equations of each lamina must be transformed to the laminate coordinates (x, y, z). The stress-strain relations in the laminate coordinates of the kth layer are expressed as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix}^{(k)} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} & 0 & 0 \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} & 0 & 0 \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \overline{Q}_{44} & \overline{Q}_{45} \\ 0 & 0 & 0 & \overline{Q}_{45} & \overline{Q}_{55} \end{bmatrix}^{(k)} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}^{(k)} \tag{10}$$

where $\overline{Q}_{ij}^k$ are the transformed material constants given as

$$\begin{aligned}
\overline{Q}_{11}^k &= Q_{11}\cos^4\theta_k + 2(Q_{12}+2Q_{66})\sin^2\theta_k\cos^2\theta_k + Q_{22}\sin^4\theta_k \\
\overline{Q}_{12}^k &= (Q_{11}+Q_{22}-4Q_{66})\sin^2\theta_k\cos^2\theta_k + Q_{12}(\sin^4\theta_k+\cos^4\theta_k) \\
\overline{Q}_{16}^k &= Q_{11}\cos^3\theta_k\sin\theta_k + Q_{12}(\cos\theta_k\sin^3\theta_k - \cos^3\theta_k\sin\theta_k) - Q_{22}\cos^3\theta_k\sin\theta_k \\
&\quad - 2Q_{66}\cos\theta_k\sin\theta_k(\cos^2\theta_k - \sin^2\theta_k) \\
\overline{Q}_{26}^k &= Q_{11}\cos\theta_k\sin^3\theta_k + Q_{12}(\cos^3\theta_k\sin\theta_k - \cos\theta_k\sin^3\theta_k) - Q_{22}\cos\theta_k\sin^3\theta_k \\
&\quad + 2Q_{66}\cos\theta_k\sin\theta_k(\cos^2\theta_k - \sin^2\theta_k) \\
\overline{Q}_{22}^k &= Q_{11}\sin^4\theta_k + 2(Q_{12}+2Q_{66})\sin^2\theta_k\cos^2\theta_k + Q_{22}\cos^4\theta_k \\
\overline{Q}_{44}^k &= Q_{44}\cos^2\theta_k + Q_{55}\sin^2\theta_k \\
\overline{Q}_{45}^k &= (Q_{55}-Q_{44})\cos\theta_k\sin\theta_k \\
\overline{Q}_{55}^k &= Q_{55}\cos^2\theta_k + Q_{44}\sin^2\theta_k \\
\overline{Q}_{66}^k &= (Q_{11}+Q_{22}-2Q_{12}-2Q_{66})\sin^2\theta_k\cos^2\theta_k + Q_{66}(\sin^4\theta_k+\cos^4\theta_k)
\end{aligned} \tag{11}$$

where θ_k is the angle of material axes with the reference coordinate axes of each layer.

2.3 Equation of motions

Hamilton's principle is employed to deduce the equations of motion

$$0 = \int_0^t (\delta U - \delta K)\, dt \tag{12}$$

where δU is the variation of strain energy; and δK is the variation of kinetic energy.

The variation of strain energy of the plate is computed by

$$\begin{aligned}
\delta U &= \int_V \left[\sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \tau_{xy}\delta\gamma_{xy} + \tau_{yz}\delta\gamma_{yz} + \tau_{xz}\delta\gamma_{xz}\right] dV \\
&= \int_A \left[N_x \delta\varepsilon_x^0 + N_y \delta\varepsilon_y^0 + N_{xy}\delta\gamma_{xy}^0 + M_x^b \delta k_x^b + M_y^b \delta k_y^b + M_{xy}^b \delta k_{xy}^b \right. \\
&\quad \left. + M_x^s \delta k_x^s + M_y^s \delta k_y^s + M_{xy}^s \delta k_{xy}^s + S_{yz}^s \delta\gamma_{yz}^s + S_{xz}^s \delta\gamma_{xz}^0\right] dA = 0
\end{aligned} \tag{13}$$

where A is the top surface and the stress resultants N, M, and S are expressed by

$$\left(N_i, M_i^b, M_i^s\right) = \int_{-h/2}^{h/2} (1, z, f)\sigma_i\, dz, \quad (i = x, y, xy)$$
$$\text{and} \quad \left(S_{xz}^s, S_{yz}^s\right) = \int_{-h/2}^{h/2} g\left(\tau_{xz}, \tau_{yz}\right) dz \tag{14}$$

The variation of kinetic energy of the plate can be calculated by

$$\begin{aligned}
\delta K &= \int_V \left[\dot{u}\,\delta\dot{u} + \dot{v}\,\delta\dot{v} + \dot{w}\,\delta\dot{w}\right] \rho\, dV \\
&= \int_A \Big\{ I_0\left[\dot{u}_0\delta\dot{u}_0 + \dot{v}_0\delta\dot{v}_0 + \dot{w}_0\delta\dot{w}_0\right] \\
&\quad - I_1\left(\dot{u}_0\frac{\partial\delta\dot{w}_0}{\partial x} + \frac{\partial\dot{w}_0}{\partial x}\delta\dot{u}_0 + \dot{v}_0\frac{\partial\delta\dot{w}_0}{\partial y} + \frac{\partial\dot{w}_0}{\partial y}\delta\dot{v}_0\right) \\
&\quad + J_1\left((k_1 A')\left(\dot{u}_0\frac{\partial\delta\dot{\theta}}{\partial x} + \frac{\partial\dot{\theta}}{\partial x}\delta\dot{u}_0\right) + (k_2 B')\left(\dot{v}_0\frac{\partial\delta\dot{\theta}}{\partial y} + \frac{\partial\dot{\theta}}{\partial y}\delta\dot{v}_0\right)\right) \\
&\quad + I_2\left(\frac{\partial\dot{w}_0}{\partial x}\frac{\partial\delta\dot{w}_0}{\partial x} + \frac{\partial\dot{w}_0}{\partial y}\frac{\partial\delta\dot{w}_0}{\partial y}\right) + K_2\left((k_1 A')^2\left(\frac{\partial\dot{\theta}}{\partial x}\frac{\partial\delta\dot{\theta}}{\partial x}\right) + (k_2 B')^2\left(\frac{\partial\dot{\theta}}{\partial y}\frac{\partial\delta\dot{\theta}}{\partial y}\right)\right) \\
&\quad - J_2\left((k_1 A')\left(\frac{\partial\dot{w}_0}{\partial x}\frac{\partial\delta\dot{\theta}}{\partial x} + \frac{\partial\dot{\theta}}{\partial x}\frac{\partial\delta\dot{w}_0}{\partial x}\right) + (k_2 B')\left(\frac{\partial\dot{w}_0}{\partial y}\frac{\partial\delta\dot{\theta}}{\partial y} + \frac{\partial\dot{\theta}}{\partial y}\frac{\partial\delta\dot{w}_0}{\partial y}\right)\right)\Big\} dA
\end{aligned} \tag{15}$$

where dot-superscript convention indicates the differentiation with respect to the time variable t; ρ is the mass density of the material; and (I_i, J_i, K_i) are mass inertias calculated by

$$
\left(I_0, I_1, I_2\right) = \sum_{k=1}^{n} \int_{z_k}^{z_{k+1}} \rho^{(k)}\left(1, z, z^2\right) dz \quad \text{and}
$$
$$
\left(J_1, J_2, K_2\right) = \sum_{k=1}^{n} \int_{z_k}^{z_{k+1}} \rho^{(k)}\left(f, z\,f, f^2\right) dz \tag{16}
$$

Substituting the relations for δU, and δK from Eqs. (13) and (15) into Eq. (12) and integrating by parts, and collecting the coefficients of δu_0, δv_0, δw_0 and $\delta\theta$, the following equations of motion for the laminated plate are deduced as follows

$$
\begin{aligned}
&\delta u_0: \ \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0 \ddot{u}_0 - I_1 \frac{\partial \ddot{w}_0}{\partial x} + k_1 A' J_1 \frac{\partial \ddot{\theta}}{\partial x} \\
&\delta v_0: \ \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_0 \ddot{v}_0 - I_1 \frac{\partial \ddot{w}_0}{\partial y} + k_2 B' J_1 \frac{\partial \ddot{\theta}}{\partial y} \\
&\delta w_0: \ \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} = I_0 \ddot{w}_0 + I_1\left(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y}\right) \\
&\qquad - I_2 \nabla^2 \ddot{w}_0 + J_2\left(k_1 A' \frac{\partial^2 \ddot{\theta}}{\partial x^2} + k_2 B' \frac{\partial^2 \ddot{\theta}}{\partial y^2}\right) \\
&\delta \theta: -\mathrm{k}_1 M_x^s - \mathrm{k}_2 M_y^s - (k_1 A' + k_2 B')\frac{\partial^2 M_{xy}^s}{\partial x \partial y} + k_1 A' \frac{\partial S_{xz}^s}{\partial x} + k_2 B' \frac{\partial S_{yz}^s}{\partial y} = -J_1\left(k_1 A' \frac{\partial \ddot{u}_0}{\partial x} + k_2 B' \frac{\partial \ddot{v}_0}{\partial y}\right) \\
&\qquad - K_2\left((k_1 A')^2 \frac{\partial^2 \ddot{\theta}}{\partial x^2} + (k_2 B')^2 \frac{\partial^2 \ddot{\theta}}{\partial y^2}\right) + J_2\left(k_1 A' \frac{\partial^2 \ddot{w}_0}{\partial x^2} + k_2 B' \frac{\partial^2 \ddot{w}_0}{\partial y^2}\right)
\end{aligned} \tag{17}
$$

Substituting Eq. (10) into Eq. (14) and integrating across the thickness of the laminated plate, the stress resultants are expressed as

$$
\left\{\begin{matrix} \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} \\ \begin{Bmatrix} M_x^b \\ M_y^b \\ M_{xy}^b \end{Bmatrix} \\ \begin{Bmatrix} M_x^s \\ M_y^s \\ M_{xy}^s \end{Bmatrix} \end{matrix}\right\} = \begin{bmatrix} \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} & \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} & \begin{bmatrix} B_{11}^s & B_{12}^s & B_{16}^s \\ B_{12}^s & B_{22}^s & B_{26}^s \\ B_{16}^s & B_{26}^s & B_{66}^s \end{bmatrix} \\ \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} & \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} & \begin{bmatrix} D_{11}^s & D_{12}^s & D_{16}^s \\ D_{12}^s & D_{22}^s & D_{26}^s \\ D_{16}^s & D_{26}^s & D_{66}^s \end{bmatrix} \\ \begin{bmatrix} B_{11}^s & B_{12}^s & B_{16}^s \\ B_{12}^s & B_{22}^s & B_{26}^s \\ B_{16}^s & B_{26}^s & B_{66}^s \end{bmatrix} & \begin{bmatrix} D_{11}^s & D_{12}^s & D_{16}^s \\ D_{12}^s & D_{22}^s & D_{26}^s \\ D_{16}^s & D_{26}^s & D_{66}^s \end{bmatrix} & \begin{bmatrix} H_{11}^s & H_{12}^s & H_{16}^s \\ H_{12}^s & H_{22}^s & H_{26}^s \\ H_{16}^s & H_{26}^s & H_{66}^s \end{bmatrix} \end{bmatrix} \left\{\begin{matrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} \\ \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} \\ \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} \end{matrix}\right\} \tag{18a}
$$

$$
\begin{Bmatrix} S_{yz}^s \\ S_{xz}^s \end{Bmatrix} = \begin{bmatrix} A_{44}^s & A_{45}^s \\ A_{45}^s & A_{55}^s \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} \tag{18b}
$$

and stiffness components are calculated by

$$
(A_{ij}, B_{ij}, B_{ij}^s, D_{ij}, D_{ij}^s, H_{ij}^s) = \sum_{k=1}^{n} \int_{z_k}^{z_{k+1}} \overline{Q}_{ij}^{(k)} (1, z, f(z), z^2, z\,f(z), f(z)^2)\, dz\,, \ (i, j = 1,2,6), \tag{19a}
$$

$$
A_{ij}^s = \sum_{k=1}^{n} \int_{z_k}^{z_{k+1}} \overline{Q}_{ij}^{(k)} \,(g(z))^2\, dz, \ (i, j = 4,5) \tag{19b}
$$

Eq. (17) can be expressed in terms of displacements (u_0, v_0, w_0, θ) by substituting for the stress resultants from Eq. (18). For homogeneous laminates, the equations of motion (17) take the form

$$
\begin{aligned}
&A_{11} d_{11} u_0 + 2 A_{16} d_{12} u_0 + A_{66} d_{22} u_0 + (A_{12} + A_{66}) d_{12} v_0 + A_{26} d_{22} v_0 + A_{16} d_{11} v_0 \\
&- (B_{11} d_{111} w_0 + 3 B_{16} d_{112} w_0 + (B_{12} + 2 B_{66}) d_{122} w_0 + B_{26} d_{222} w_0) \\
&+ (k_1 A' + k_2 B') B_{66}^s d_{122} \theta + (k_1 B_{11}^s + k_2 B_{12}^s) d_1 \theta + (k_1 A' + k_2 B') B_{16}^s d_{112} \theta \\
&\qquad - (k_1 B_{16}^s + k_2 B_{26}^s) d_2 \theta = I_0 \ddot{u}_0 - I_1 d_1 \ddot{w}_0 + A' J_1 k_1 d_1 \ddot{\theta},
\end{aligned} \tag{20a}
$$

$$
\begin{aligned}
&A_{11} d_{16} u_0 + (A_{12} + A_{66}) d_{12} u_0 + A_{26} d_{22} u_0 + A_{66} d_{22} v_0 + 2 A_{26} d_{12} v_0 + A_{22} d_{22} v_0 \\
&- (B_{16} d_{111} w_0 + 3 B_{26} d_{122} w_0 + (B_{12} + 2 B_{66}) d_{112} w_0 + B_{22} d_{222} w_0) \\
&+ (k_1 A' + k_2 B') B_{66}^s d_{112} \theta + (k_2 B_{22}^s + k_1 B_{12}^s) d_2 \theta + (k_1 A' + k_2 B') B_{26}^s d_{122} \theta \\
&\qquad + (k_1 B_{16}^s + k_2 B_{26}^s) d_1 \theta = I_0 \ddot{v}_0 - I_1 d_2 \ddot{w}_0 + B' J_1 k_2 d_2 \ddot{\theta},
\end{aligned} \tag{20b}
$$

$$
\begin{aligned}
&(B_{11} d_{111} u_0 + 3 B_{16} d_{112} u_0 + (B_{12} + 2 B_{66}) d_{122} u_0 + B_{26} d_{222} u_0) \\
&+ (B_{16} d_{111} v_0 + 3 B_{26} d_{122} v_0 + (B_{12} + 2 B_{66}) d_{112} v_0 + B_{22} d_{222} v_0) \\
&- D_{11} d_{1111} w_0 - 2(D_{12} + 2 D_{66}) d_{1122} w_0 - D_{22} d_{2222} w_0 - 4 D_{16} d_{1112} w_0 - 4 D_{26} d_{1222} w_0 \\
&+ (k_1 D_{11}^s + k_2 D_{12}^s) d_{11} \theta + 2(k_1 A' + k_2 B') D_{66}^s d_{1122} \theta + (k_1 D_{12}^s + k_2 D_{22}^s) d_{22} \theta + 2(k_1 D_{16}^s + k_2 D_{26}^s) d_{12} \theta \\
&+ (k_1 A' + k_2 B') D_{16}^s d_{1112} \theta + (k_1 A' + k_2 B') D_{26}^s d_{1222} \theta = I_0 \ddot{w}_0 + I_1 (d_1 \ddot{u}_0 + d_2 \ddot{v}_0) \\
&\qquad - I_2 \nabla^2 \ddot{w}_0 + J_2 (A' k_1 d_{11} \ddot{\theta} + B' k_2 d_{22} \ddot{\theta})
\end{aligned} \tag{20c}
$$

$$
\begin{aligned}
&-((k_1 A' + k_2 B') B_{66}^s d_{122} u_0 + (k_1 B_{11}^s + k_2 B_{12}^s) d_1 u_0 + (k_1 A' + k_2 B') B_{16}^s d_{112} u_0 - (k_1 B_{16}^s + k_2 B_{26}^s) d_2 u_0) \\
&-((k_1 A' + k_2 B') B_{66}^s d_{112} v_0 + (k_2 B_{22}^s + k_1 B_{12}^s) d_2 v_0 + (k_1 A' + k_2 B') B_{26}^s d_{122} v_0 + (k_1 B_{16}^s + k_2 B_{26}^s) d_1 v_0) \\
&+ (k_1 D_{11}^s + k_2 D_{12}^s) d_{11} w_0 + 2(k_1 A' + k_2 B') D_{66}^s d_{1122} w_0 + (k_1 D_{12}^s + k_2 D_{22}^s) d_{22} w_0 + 2(k_1 D_{16}^s + k_2 D_{26}^s) d_{12} w_0 \\
&+ (k_1 A' + k_2 B') D_{16}^s d_{1112} w_0 + (k_1 A' + k_2 B') D_{26}^s d_{1222} w_0 \\
&- H_{11}^s k_1 \theta - H_{22}^s k_2 \theta - 2 H_{12}^s k_1 k_2 \theta - (k_1 A' + k_2 B')^2 H_{66}^s d_{1122} \theta - 2(k_1 A' + k_2 B')(H_{16}^s + H_{26}^s) d_{12} \theta \\
&+ A_{44}^s (k_2 B')^2 d_{22} \theta + A_{55}^s (k_1 A')^2 d_{11} \theta + 2 A_{45}^s k_1 k_2 A' B' d_{12} \theta = -J_1 (k_1 A' d_1 \ddot{u}_0 + k_2 B' d_2 \ddot{v}_0) \\
&\qquad - K_2 ((k_1 A')^2 d_{11} \ddot{\theta} + (k_2 B')^2 d_{22} \ddot{\theta}) + J_2 (k_1 A' d_{11} \ddot{w}_0 + k_2 B' d_{22} \ddot{w}_0)
\end{aligned} \tag{20d}
$$

where d_{ij}, d_{ijl} and d_{ijlm} are the following differential operators

$$
d_{ij} = \frac{\partial^2}{\partial x_i \partial x_j}, \quad d_{ijl} = \frac{\partial^3}{\partial x_i \partial x_j \partial x_l},
$$
$$
d_{ijlm} = \frac{\partial^4}{\partial x_i \partial x_j \partial x_l \partial x_m}, \quad d_i = \frac{\partial}{\partial x_i}, \quad (i, j, l, m = 1,2). \tag{21}
$$

3. Analytical solutions for anti-symmetric cross-ply laminates

The Navier method is considered to present the analytical solutions of the partial differential equations in Eq. (19) for simply supported rectangular plates. For anti-symmetric cross-ply laminated plates, the following stiffness components are identically zero

$$
\begin{aligned}
&A_{16} = A_{26} = D_{16} = D_{26} = B_{16}^s = B_{26}^s = D_{16} = D_{26} = D_{16}^s = D_{26}^s = H_{16}^s = H_{26}^s = A_{45}^s = 0 \\
&B_{12} = B_{66} = B_{12}^s = B_{66}^s = 0
\end{aligned} \tag{22}
$$

Based on the Navier procedure, the following solutions of displacements are employed to automatically respect the simply supported boundary conditions of plate

$$
\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta \end{Bmatrix} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn} \cos(\alpha\, x) \sin(\beta\, y)\, e^{i\omega t} \\ V_{mn} \sin(\alpha\, x) \cos(\beta\, y)\, e^{i\omega t} \\ W_{mn} \sin(\alpha\, x) \sin(\beta\, y)\, e^{i\omega t} \\ X_{mn} \sin(\alpha\, x) \sin(\beta\, y)\, e^{i\omega t} \end{Bmatrix} \tag{23}
$$

where U_{mn}, V_{mn}, W_{mn} and X_{mn} are coefficients; ω is the natural frequency of the system; and α and β are expressed as

$$
\alpha = m\pi / a \quad \text{and} \quad \beta = n\pi / b \tag{24}
$$

Substituting Eqs. (22) into Eq. (19), the Navier solution of anti-symmetric cross-ply laminates can be deduced from equations

$$
\left(\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & 0 & m_{13} & m_{14} \\ 0 & m_{22} & m_{23} & m_{24} \\ m_{13} & m_{23} & m_{33} & m_{34} \\ m_{14} & m_{24} & m_{34} & m_{44} \end{bmatrix} \right) \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ X_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{25}
$$

4. Numerical results and discussion

In this section the accuracy of the developed HSDT which contain a displacement field with four variables, is

Table 1 Displacement models

Model	Theory	Unknown variables
CLPT	Classical laminate plate theory	3
FSDT	First-order shear deformation theory (Whitney and Pagano 1970)	5
TSDT	Third-order shear deformation theory (Reddy 1997)	5
HSDT	Higher-order shear deformation (Swaminathan and Patil 2008)	12
Present	Present higher-order shear deformation theory	4

assessed in studying the dynamic response of simply supported anti-symmetric cross-ply and angle-ply laminated plates. The influences of aspect ratio, thickness ratio, modulus ratio and number of layers on the free vibration behavior of plates are examined. The results calculated by employing the present model are compared with those given via various plate models and exact solutions of 3D elasticity theory. The different plate theories used in this study are described in Table 1.

In the proposed examples, a shear correction coefficient of 5/6 is considered for FSDT. All layers are supposed to have the same thickness, mass of density and orthotropic material characteristics in the material principle axes. The following lamina properties are utilized

- Material 1 (Noor 1973)

$$E_1 / E_2 = \text{open} \,,\; G_{12} = G_{13} = 0.6\, E_2 \,, \quad G_{23} = 0.5\, E_2 \,,\; \nu_{12} = 0.25 \tag{26a}$$

- Material 2 (Noor and Burton 1990)

$$E_1 = 15\, E_2 \,,\; G_{12} = G_{13} = 0.5\, E_2 \,, \quad G_{23} = 0.35\, E_2 \,,\; \nu_{12} = 0.3 \tag{26b}$$

Table 2 Non-dimensional natural frequencies of anti-symmetric cross-ply square laminates with $a/h = 5$

No. of layers	Theory	E_1/E_2				
		3	10	20	30	40
$(0/90)_1$	Exact(*)	6.2578	6.9845	7.6745	8.1763	8.5625
	TSDT	6.2169	6.9887	7.8210	8.5050	9.0871
	FSDT	6.2085	6.9392	7.7060	8.3211	8.8333
	Present	6.2169	6.9887	7.8210	8.5050	9.0871
$(0/90)_2$	Exact(*)	6.5455	8.1445	9.4055	10.1650	10.6789
	TSDT	6.5008	8.1954	9.6265	10.5348	11.1716
	FSDT	6.5043	8.2246	9.6885	10.6198	11.2708
	Present	6.5008	8.1954	9.6265	10.5348	11.1716
$(0/90)_3$	Exact(*)	6.6100	8.4143	9.8398	10.6958	11.2728
	TSDT	6.5558	8.4052	9.9181	10.8547	11.5012
	FSDT	6.5569	8.4183	9.9427	10.8828	11.5264
	Present	6.5558	8.4052	9.9181	10.8547	11.5012
$(0/90)_5$	Exact(*)	6.6458	8.5625	10.0843	11.0027	11.6245
	TSDT	6.5842	8.5126	10.0674	11.0197	11.6730
	FSDT	6.5837	8.5132	10.0638	11.0058	11.6444
	Present	6.5842	8.5126	10.0674	11.0197	11.6730

(*) Values taken from Noor (1973)

For convenience, the following dimensionless natural frequency is employed in investigating the numerical results

$$\bar{\omega} = \omega^2 (b^2 / h)\sqrt{\rho / E_2} \tag{27}$$

Example 1: A simply supported anti-symmetric cross-

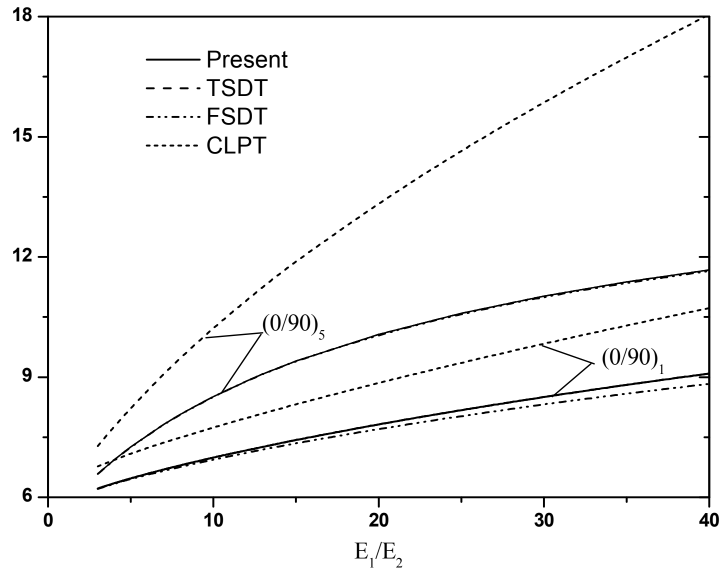

Fig. 2 The effect of modulus ratio on nondimensionalized natural frequencies of antisymmetric cross-ply $(0/90)_n$ square laminates with $a/h = 5$

ply square plates with thickness ratio a/h = 5 is examined using Material 1 and by considering the effects of the modulus ratio and the number of layers on the non-dimensional natural frequency. Numerical results are presented in Table 2 where a good agreement between the shear deformations theories is demonstrated. It can be observed that the results calculated by TSDT and the present theory are identical. The influence of modulus ratio on natural frequencies of two-layer (0/90) and ten-layer $(0/90)_5$ square plates with a/h = 5 is demonstrated in Fig. 2.

Example 2: A simply supported anti-symmetric cross-ply square plates with modulus ratio E_1/E_2 = 40 is investigated using Material 1 and by considering the influences the thickness ratio and the number of layers on dimensionless natural frequency. The natural frequencies calculated via various plate theories for the considered anti-symmetric cross-ply square laminates are presented in Table 3. It is remarked that there is an excellent agreement between the results predicted by present model, and TSDT.

The effect of thickness ratios on dimensionless fundamental frequencies of two-layer (0/90) and six-layer $(0/90)_3$ square plates, is shown in Fig. 3. It can be observed that this influence is considerable for thick plates where the trans-

Table 3 Non-dimensional natural frequencies of anti-symmetric cross-ply square laminates with $E_1/E_2 = 40$

No. of layers	Theory	a/h					
		2	4	10	20	50	100
$(0/90)_1$	TSDT	5.7170	8.3546	10.5680	11.1052	11.2751	11.3002
	FSDT	5.21047	8.0349	10.4731	11.0779	11.2705	11.2990
	CLPT	8.6067	10.4244	11.1537	11.2693	11.3023	11.3070
	Present	5.7170	8.3546	10.5680	11.1052	11.2751	11.3002
$(0/90)_2$	TSDT	5.7546	9.7357	14.8463	16.5733	17.1849	17.2784
	FSDT	5.6656	9.8148	14.9214	16.6008	17.1899	17.2796
	CLPT	14.1036	16.3395	17.1448	17.2682	17.3032	17.3082
	Present	5.7546	9.7357	14.8463	16.5733	17.1849	17.2784
$(0/90)_3$	TSDT	5.8741	9.9878	15.4632	17.3772	18.0644	18.1698
	FSDT	5.5992	9.9852	15.5010	17.3926	18.0673	18.1706
	CLPT	15.0895	17.2676	18.0461	18.1652	18.1990	18.2038
	Present	5.8741	9.9878	15.4632	17.3772	18.0644	18.1698
$(0/90)_5$	TSDT	5.9524	10.1241	15.7700	17.7743	18.4984	18.6097
	FSDT	5.7140	10.0628	15.7790	17.7800	18.4995	18.6100
	CLPT	15.6064	17.7314	18.4916	18.6080	18.6410	18.6457
	Present	5.9524	10.1241	15.7700	17.7743	18.4984	18.6097

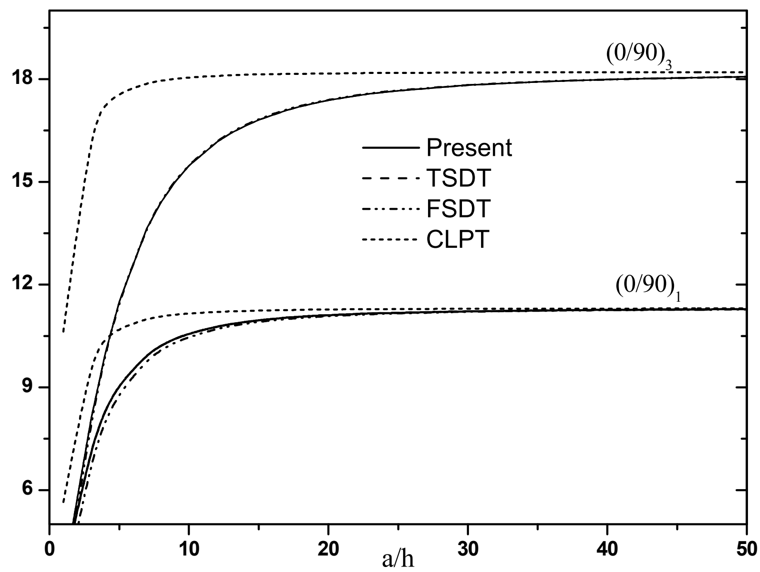

Fig. 3 The effect of thickness ratio on nondimensionalized natural frequencies of antisymmetric cross-ply $(0/90)_n$ square laminates with $E_1/E_2 = 40$

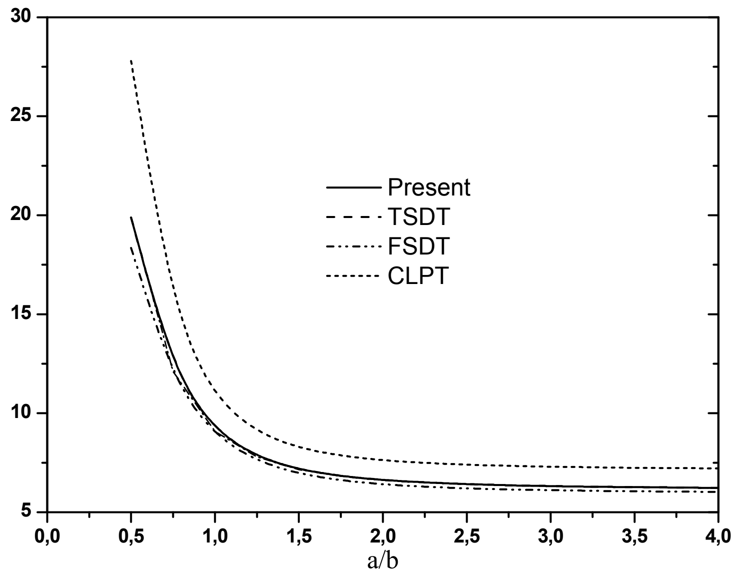

Fig. 4 The effect of aspect ratio on nondimensionalized natural frequencies of two-layer (0/90) rectangular laminates with $b/h = 5$ and $E_1/E_2 = 40$

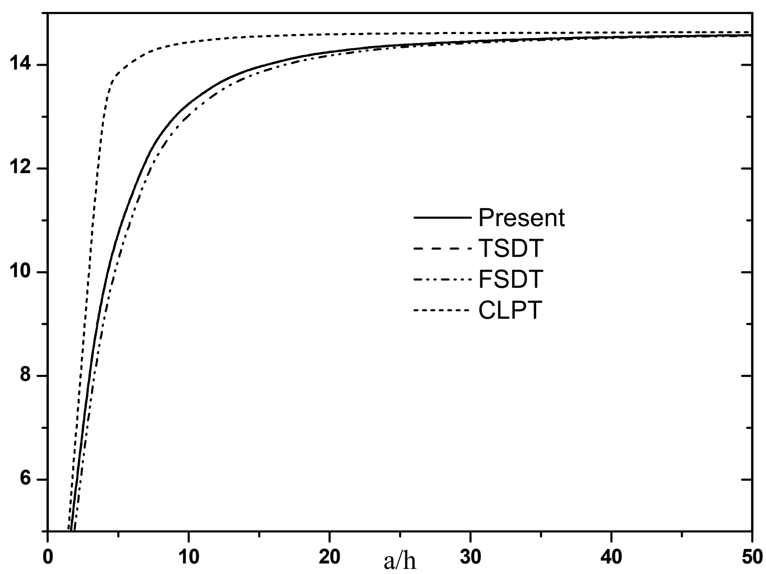

Fig. 5 The effect of thickness ratio on nondimensionalized natural frequencies of two-layer (45/-45) square laminates with $E_1/E_2 = 40$

Table 4 Non-dimensional natural frequencies of ten-layer anti-symmetric angle-ply $(45/\text{-}45)_5$ square laminates

a/h	Theory	$\bar{\omega}$
5	TSDT	10.1537
	FSDT	10.1288
	CLPT	15.4661
	Present	10.1537
10	TSDT	13.6078
	FSDT	13.614
	CLPT	15.846
	Present	13.6078

Table 4 Continued

a/h	Theory	$\bar{\omega}$
100	TSDT	15.9482
	FSDT	15.9484
	CLPT	15.9775
	Present	15.9482

verse shear deformation effect is more important. As the number of layers increases, the difference between the present theory and TSDT decreases significantly.

The influence of aspect ratio on fundamental frequen-

Table 5 Non-dimensional natural frequencies of anti-symmetric angle-ply (45/-45) square laminates

E_1/E_2	a/h	Theory				
		Present	HSDT [(a)]	TSDT	FSDT	CLPT
3	4	6.0861	6.1223	6.0861	6.0665	6.9251
	10	7.0739	7.1056	7.0739	7.0700	7.2699
	20	7.2705	7.3001	7.2705	7.2694	7.3228
	50	7.3293	7.3583	7.3293	7.3291	7.3378
	100	7.3378	7.3666	7.3378	7.3378	7.3400
10	4	7.3470	7.2647	7.3470	7.2169	8.7950
	10	8.9660	8.9893	8.9660	8.9324	9.3444
	20	9.3266	9.3265	9.3266	9.3173	9.4304
	50	9.4377	9.4377	9.4377	9.4362	9.4548
	100	9.4540	9.5123	9.4540	9.4537	9.4583
20	4	8.4152	8.0490	8.4152	8.1185	10.6314
	10	10.7151	10.6412	10.7151	10.6265	11.3406
	20	11.2772	11.2975	11.2772	11.2517	11.4525
	50	11.4553	11.5074	11.4553	11.4511	11.4844
	100	11.4816	11.5385	11.4816	11.4806	11.4889
30	4	9.1752	8.5212	9.1752	8.7213	12.1586
	10	12.0971	11.8926	12.0971	11.9456	12.9888
	20	12.8659	12.8422	12.8659	12.8208	13.1203
	50	13.1153	13.1566.	13.1153	13.1077	13.1577
	100	13.1524	12.2035	13.1524	13.1505	13.1631
40	4	9.7594	8.8426	9.7594	9.1609	13.5059
	10	13.2631	12.9115	13.2631	13.0439	14.4392
	20	14.2463	14.1705	14.2463	14.1790	14.5873
	50	14.5724	14.6012	14.5724	14.5608	14.6295
	100	14.6212	14.6668	14.6212	14.6183	14.6356

[(*)] Values taken from Noor (1973)

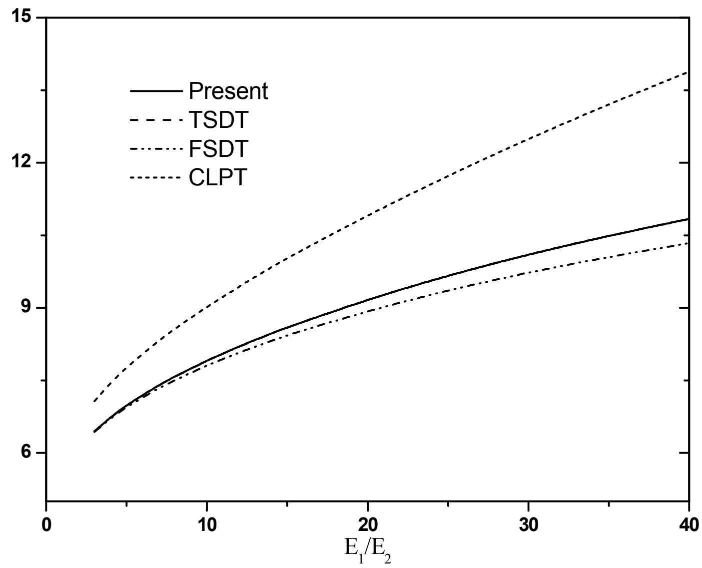

Fig. 6 The effect of modulus ratio on nondimensionalized natural frequencies of two-layer (45/-45) square laminates with $a/h = 5$

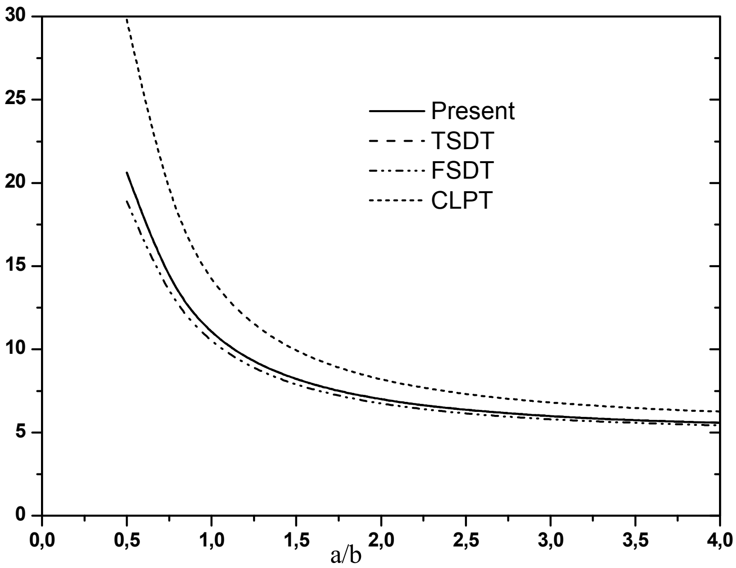

Fig. 7 The effect of aspect ratio on nondimensionalized natural frequencies of two-layer (45/45) rectangular laminates with $b/h = 5$ and $E_1/E_2 = 40$

cies of two-layer (0/90) rectangular plates with $b/h = 5$ and $E_1/E_2 = 40$ is also demonstrated in Fig. 4.

Example 3: A simply supported anti-symmetric angle-ply $(45/–45)_5$ square plates is studied using Material 2 for different values of thickness ratio. The fundamental frequencies predicted by employing various plate models are reported in Table 4. It can be shown from Table 4, that both TSDT and the present theory provide identical results. The influence of thickness ratio on fundamental frequencies of (45/–45) square plates with $E_1/E_2 = 40$ is demonstrated in Fig. 5 by considering various plate models.

Example 4: A simply supported anti-symmetric angle-ply (45/–45) square composite plates is examined using Material 1 for different values of thickness ratio and modulus ratio. The fundamental frequencies calculated by employing various plate models are presented in Table 5. Compared to HSDT solutions (Swaminathan and Patil 2008), the results predicted by the proposed theory for thick laminated plates with higher values of modulus ratio are considerably different to those of HSDT (Swaminathan and Patil 2008). This might be due to the thickness stretching influence in very thick plates which is neglected in the proposed theory and TSDT. It should be mentioned that the proposed model contains four variables as against five in the case of TSDT and twelve in the case of HSDT (Swaminathan and Patil 2008).

The influences of modulus ratio on fundamental frequencies of (45/–45) square laminates are illustrated in Fig. 6.

The influence of aspect ratio on fundamental frequencies of (45/–45) rectangular plates with $b/h = 5$ and $E_1/E_2 = 40$ is also demonstrated in Fig. 7.

5. Conclusions

In this work, a novel version of HSDT is proposed for free vibration of laminated composite plates. By considering some additional simplifying suppositions to the existing HSDT, with introducing an undetermined integral term, the number of unknowns and equations of motion of the developed HSDT are diminished by one, and hence, make this theory simple and efficient to use. The model provides parabolic distribution of the transverse shear strains, and respects the zero traction boundary conditions on the surfaces of the laminated plate without utilizing shear correction coefficients. The validity and efficiency of the present model has been proved for dynamic responses of simply supported anti-symmetric cross-ply and angle-ply plates. In conclusion, the proposed model can improve the numerical computational cost because of their reduced degrees of freedom.

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