Steel and Composite Structures, *Vol. 22, No. 6 (2016) 1391-1415*
DOI: http://dx.doi.org/10.12989/scs.2016.22.6.1391

Evaluation of limit load analysis for pressure vessels – Part I: Linear and nonlinear methods

Xiaohui Chen [*1,2], Bingjun Gao [3a] and Xingang Wang [**1b]

[1] *School of Control Engineering, Northeastern University, Qinhuangdao, 066004, China*
[2] *College of Mechanical Engineering, Yanshan University, Qinhuangdao, 066004, China*
[2] *School of Chemical Engineering and Technology, Hebei University of Technology, Tianjin, 300000, China*

(*Received May 20, 2016, Revised November 02, 2016, Accepted November 29, 2016*)

Abstract. Limit load of pressure bearing structures was reviewed in this article. By means of the finite element analysis, limit load of pressurized cylinder with nozzle was taken as an example. Stress classification method and Elastic-plastic finite element analysis combining with limit load determination methods were used to determine limit load of cylinder with nozzle. Comparison of limit load determined by different methods, the results indicated that limit load determined by linearization method was the smallest. Limit load determined by twice elastic slope criterion was the nearest than experimental results. Elastic-plastic finite element analysis had comparably computational precision, but required time consuming. And then the requirements of computer processing and storage capacity by power system became higher and higher. Most of criteria for limit load estimation included any human factors based on a certain substantive characteristics of experimental results. The reasonable criterion should be objective and operational.

Keywords: pressure vessel; limit load; estimation criteria; ANSYS

1. Introduction

Structural and mechanical integrity assessment played an important role in many industry efforts relating to fitness-for-service evaluation of components or structures. Fitness-for-service assessments evaluated the structural integrity of components and their suitability for continuous service. The fitness of components or structures for service can be evaluated, and safety margins determined at operating conditions by having an understanding of these parameters. In the past decades, the integrity assessment had been carried out by some of the available standards such as ASME (1986), R5 (1990), EN13445 (2002), API 579/ASME FFS1 fitness for service code, and so on. Standards used to carry out these assessments provided guidelines which can be used to make run-repair-replace decisions, assisting plant management in identifying appropriate mitigation actions to ensure that the component can be operated safely. A comprehensive overview of existing assessment methods and comparison of various acceptance criteria was presented in (Brighenti 2001, Cosham *et al*. 2007). These studies highlighted the use of plastic instability and buckling as

∗Corresponding author, Ph.D., E-mail: chenxh@neuq.edu.cn
** Professor, E-mail: xgwang@neuq.edu.cn
[a] Ph.D., E-mail: bjgao@hebut.edu.cn

Copyright © 2016 Techno-Press, Ltd.
http://www.techno-press.org/?journal=scs&subpage=6 ISSN: 1229-9367 (Print), 1598-6233 (Online)

governing failure criteria to evaluate the burst pressure for thin piping with metal loss defects. Furthermore, they outlined the role of assumed flow stress on the assessment results. Simplified assessment rules commonly employed limit load analysis to evaluate the burst pressure assuming elastic perfectly-plastic material models BS 7910 2005. In contrast, plastic collapse load analysis using nonlinear stress–strain material data and advanced numerical techniques such as FEA yields more realistic and less conservative results (Chiodo and Ruggieri 2009, Kamaya *et al*. 2008, Khyabani and Sadrnejad 2009). This can be explained due to the non-zero post-yield stiffness, which allowed an entirely plastic region to sustain some of the increase in post yield load. Similar to limit load evaluation technique, the plastic collapse load was evaluated as the highest convergent load increment in a well-configured finite element analysis with sufficiently small load increments.

Limit analysis can provide an assessment of the integrity of a mechanical component during operation by applying the appropriate boundary conditions and geometrical behavior. Limit load analysis, which determined plastic collapse load of mechanical structures or components, was an important tool during design process of mechanical structures or components to ensure their functionality within operating conditions. Moreover, limit load analysis had contributed to provide an assessment of structures or components behaviors for other failure modes.

Limit load of components or structures was mainly determined using experimental methods, analytical techniques and elastic-plastic finite element analysis. Analytical approaches can determine the exact limit load for simple loading cases and geometrical configurations. For complex problems, some assumptions were made to obtain an approximate analytical solution. Such procedures were categorized into lower bound and upper-bound solutions. For more complex problems, numerical methods such as the elastic-plastic analysis were used to solve limit load of components or structures by means of iterative calculation. The ASME code had some guidelines for calculating the limit load from elastic-plastic analysis results. The elastic-plastic analysis can yield a relatively accurate solution, but huge amount of time and advanced computing resources were required. Moreover, considerable input and experience were required in defining the convergence criteria of the solution and the limit load conditions. In addition, the ASME code, EN13445 code, and so on, had set some guidelines to interpret a linear elastic analysis results and categorized the stresses into primary, secondary and peak stresses. These categories were used to find a design load to avoid most of the failure modes.

The purpose of this research was to summarize and compare these limit load methods. Experimental methods and analytical techniques were firstly reviewed. Limit pressures of a cylindrical pressure vessel with nozzle were then estimated and results of different criteria were compared and assessed.

2. Limit load determination methods

In this study, the available methods for limit load analysis were reviewed in detail including analytical techniques, numerical procedures and criteria for limit load estimation.

2.1 Analytical techniques

The analytical techniques for limit analysis, which were used to determine component load carrying capacity with mathematical formulations from plasticity theory, included variational tlmethods and slip lines methods.

2.1.1 Variational methods

Variational principles were proposed by Mura *et al*. (Mura *et al*. 1964, Mura and Lee 1965), with one of the most important applications in plasticity theory to be the limit load analysis. In plasticity theory, variational principles stated that the material of a body was elastic-perfectly plastic.

Limit load was determined by the variational principles according to plasticity theory, which was used to determine the limit load multipliers m at the impending plastic limit state of a body. However, variational principles were only feasible for some simple geometries and boundary conditions. For complicated problems it may be very difficult to find the exact limit load. In order to solve these problems, the classical upper bound and lower bound theories (Calladine 2000) were employed to estimate the limit load directly without considering the entire loading history.

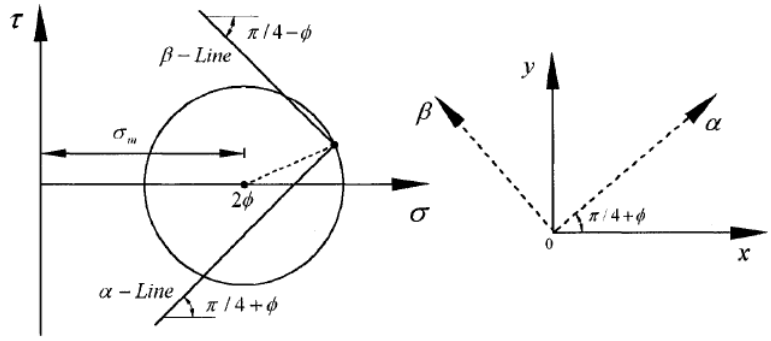
2.1.2 Slip lines methods

Slip line method which was widely used in metal forming and soil mechanics was proposed by Hencky (1923), Prandtl (1923) and Caratheodory and Schmidt (1923). The method estimated limit load in a component or structure in plane condition. Detailed studies of the slip line method had been done by Hill (1950) and Prager and Hodge (1951).

Davis and Selvadurai (2002) thought that the aim of slip line method was to define a coordinate system that lied on a potential failure surfaces in a given component. The axis of the new coordinate system, said α and β, needed to be defined in such a way that at each point they aligned on the potential failure surfaces. In other words, the direction of the new axis was on the direction of maximum shear stress when plastic flow occurred. The definition of the parameters α and β can be illustrated with Mohr diagram, as shown in Fig. 1. The α-line and β-line were located at two maximum shear stress failure lines (potential failure surface). Therefore, these two lines were orthogonal. In a two-dimensional system the potential failure surfaces became slip lines, and combinations of these lines form a network that covered the failing regions.

2.2 Linear and nonlinear numerical methods

For simple structures or components and boundary conditions, the analytical techniques can obtain the exact limit load. For more complicated problems, numerical methods were proposed to



(a) (b)

Fig. 1 Definition of slip line on Mohr diagram determine the limit load. The numerical procedures included programming method and robust methods with elastic modulus adjustment procedure. Moreover, stress classification method and elastic-plastic finite element analysis were developed to determine limit load of components or structures. These methods were reviewed in detail as follows.

2.2.1 Programming method

Charnes and Greenberg (1951) proposed programming methods to determine limit load. The programming methods can be divided into two approaches, linear programming and nonlinear programming. The nonlinear programming approach was more accurate and difficult than the linear programming approach.

The limit load analysis using programming method was realized by minimizing or maximizing a function subjected to certain constrains. Limit load was determined by finite element method combined with programming methods. Therefore, limit load analysis was a standard optimization problem. The objective function was the maximization or minimization of lower or upper bound limit load multiplier subjected to set of equality and inequality constraints.

So far, the programming method had only been used for two-dimensional components and some simple three-dimensional components. For three-dimensional components using finite element method, only simple element type was used by Lyamin and Sloan (2002a, b) and Zouain *et al*. (1993). Moreover, the programming method required large computer memory and a considerable amount of computational time, which was attributed to the huge number of constraint equations and the degree of freedom. Thus, the programming method was not easily implemented into available commercial finite element software.

2.2.2 Stress classification method

Stress classification method was mainly the equivalent linearization method, including stress classification points (He 1995), lines (Kroenke 1973) and planes (Hechmer and Hollinger 1989). So far, stress classification lines method which had been widely applied was used to determine limit load of structures or components. Thus only stress classification lines were employed in this study.

2.2.2.1 Linearization method

According to ASME Section III, the stress fields were obtained by linear elastic finite element analysis based on the combined applied load and the reaction forces. The basic stress classification procedure based on the location, origin and type was given in ASME Section III. The stresses were divided into several parts according to ASME Section III, including general primary membrane stress P_m, local primary membrane stress P_L, primary membrane plus primary bending stress $P_L + P_b$, primary plus secondary stress $P_L + P_b + Q$ and total stress $P_L + P_b + Q + F$. Each type of stress was compared to an allowable stress limit S_m or S_a.

$$P_m \leq S_m \tag{1}$$

$$P_L \leq 1.5S_m \tag{2}$$

$$P_L + P_b \leq 1.5S_m \tag{3}$$

$$P_L + P_b + Q \leq 3S_m \tag{4}$$

where, S_m was the basic allowable stress calculated according to the material.

The generated stresses during stress linearization procedure must be interpreted according to ASME Section III. In many pressure vessel structures, it was difficult to obtain $P_L + P_b$ due to the lack of information about primary bending stress. Based on the superposition principle, Gao *et al.* (2010) proposed a method of multistep applied load for primary bending stress and corresponding stress intensity $P_L + P_b$ for a wide range of axisymmetric problems in the pressure vessel. This would provide a reasonable alternative for some typical pressure vessel component design.

According to the stress categories, limit load of pressure vessel component was determined as follows. The stress intensities were proportional to internal pressure within the elasticity range. Therefore, the maximum allowable load could be deduced from Eq. (12).

$$P : P_{\max}^{I} \leq P_m : S_m \tag{5}$$

Namely

$$P_{\max}^{I} = \frac{PS_m}{P_m} \tag{6}$$

Similarly, the maximum allowable load could also be deduced from Eqs. (13)-(15) as follows

$$P_{\max}^{II} = \frac{1.5PS_m}{P_L} \tag{7}$$

$$P_{\max}^{III} = \frac{1.5PS_m}{P_L + P_b} \tag{8}$$

$$P_{\max}^{IV} = \frac{3PS_m}{P_L + P_b + Q} \tag{9}$$

The maximum allowable load based on stress categories could be expressed as follows

$$[P] = min\left\{P_{\max}^{I}, P_{\max}^{II}, P_{\max}^{III}, P_{\max}^{IV}\right\} \tag{10}$$

ASME B&PV Code VIII-2 (2010) Appendix 4–136.3 stated that the limits of the general primary membrane stress intensity, local primary membrane stress intensity, and primary membrane plus primary bending stress intensity did not necessarily to be satisfied at a specific location if the specified loadings did not exceed two-thirds of the collapse load P_1. If the above requirement was met, the allowable load was expressed as follows, corresponding to the limit load requirement.

$$[P] = \frac{2}{3} P_l \tag{11}$$

2.2.2.2 Gloss R-node method

Seshadri and Marriott (1993), Fanous and Seshadri (2007) extended the application of R-Node method to stress classification. The analysis procedure was proposed as follows to illustrate the R-node method.

Firstly, a linear elastic finite element analysis was carried out to find out the load corresponding to initial yielding.

Secondly, the results of the linear analysis was carried out in conjunction with the stress linearization tool in finite element software ANSYS or ABAQUS to find out P_m, P_b and F.

Thirdly, another linear elastic analysis was carried out in conjunction with elastic modulus adjustment procedure to locate R-node in the pressure component. The maximum R-node equivalent stress was identified and need to be less than allowable stress S_m. This ensured the satisfaction of both P_m and P_{eq}.

Lastly, a complete elastic-plastic analysis was carried out in order to compare the results obtained by the foregoing methods.

2.2.2.3 m_α^T method

Adibi-Asl and Seshadri (2007) proposed the m_α^T method for stress categorization which was applicable to both mechanical and thermal loads. Similar to stress linearization method, linear elastic finite element analysis was performed, and the results of finite element analysis were used to evaluate classical lower bounder m_L and upper bound multiplier m^0. The multiplier m_α^T was calculated using Eq. (18).

$$m_\alpha^T = \frac{m^0}{1+\left(1-\frac{1}{\sqrt{2}}\right)\left(\zeta_f - 1\right)} \tag{12}$$

where, ζ_f was obtained from $\zeta_f = (1 + C) \pm \sqrt{(1+C)^2 - 1}$, $C = 0.2929(\zeta_i - 1)$ and $\zeta_i = \frac{m^0}{m_L}$.

If m^0/m_L was less than $1+\sqrt{2}$, only primary and secondary stresses were considered in the structure or component. If m^0/m_L was greater than $1+\sqrt{2}$, all three categories of stresses were expected to be present in the structure or component. The magnitude of peak stresses was either zero or negligible for the two cases.

2.2.3 Elastic-plastic finite element method

Although experimental methods determined limit load based on actual behavior of a structure or component, limit load analysis using experimental methods was expensive and time consuming. With the development of finite element method, the assumptions of ideal conditions, strain hardening or larger deformations were considered. In order to determine the limit load of a structure or component, bilinear or multilinear kinematic hardening rule which was available in finite element software ANSYS or ABAQUS was used to determine limit load combined with the assumption of larger displacement formulation. Loading-strain curve was obtained by means of nonlinear finite element analysis, and then limit load determination criteria in Section 2.3 were used to determine limit load of a structure or component. Compared to experimental methods, finite element method was low cost and can realize all kinds of loading or load combinations. A lot of researches indicated that finite element analysis determining limit load can obtain reliable approximate solution. Therefore, finite element analysis was till a common analytical tool.

2.3 Criteria for limit load estimation

Criteria for limit load estimation which represented the assumption of ideal conditions were used to determine limit load based on actual behavior of a component (i.e., real material like strain hardening, and large deformation). The loading versus strain or displacement curve was firstly measured by experimental techniques and finite element analysis. And then limit load was determined by the following criteria based on the loading versus strain or displacement curve. Experimental techniques were the best estimate, but its cost was not particularly economical. Therefore, with only few experiments carried out, inelastic finite element analysis with strain hardening or large deformations were employed based on loading versus strain or displacement curve and combined with the following criterions.

(l) Tangent intersection criterion

Tangent intersection criterion was proposed by Save (1972), as shown in Fig. 2. The tangent of elastic and plastic parts of loading-deformation curve was drawn, and the tangent point corresponding to loading value was defined as limit load P_n.

(2) l% plastic strain criterion

Townley *et al*. (1971) put forward l% plastic strain criterion. The load corresponding to l% plastic strain on loading–strain curve was called limit loading.

(3) Twice elastic deformation criterion

ASME (1974) proposed twice elastic deformation criterion where the location of elastic and inelastic behavior of a component was determined on load-deformation curve (say δ on deformation axis). Elastic deformation δ was assigned to initial yield loading. Then a vertical line at the distance of 2δ was drawn as depicted in Fig. 4. The load corresponded to the intersection of this line and load-deformation curve was considered as limit load. In 1974, twice elastic deformation criterion was employed by ASME.

(4) Twice elastic slope criterion

Since 1975, ASME code had been used to determine limit load of pressurized equipment with

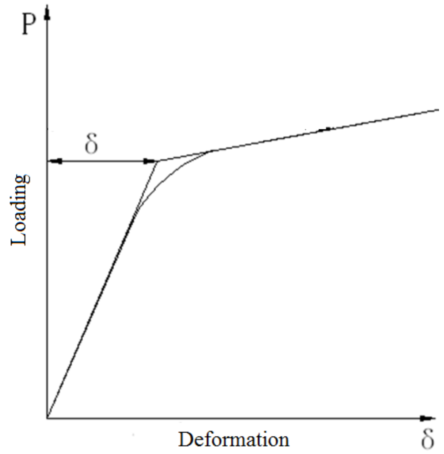


Fig. 2 Tangent intersection criterion

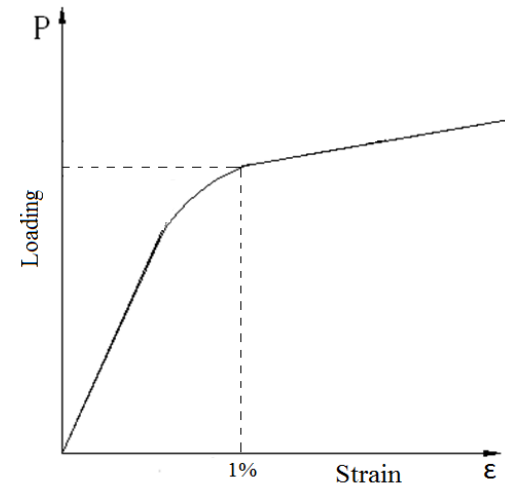


Fig. 3 l% plastic strain criterion

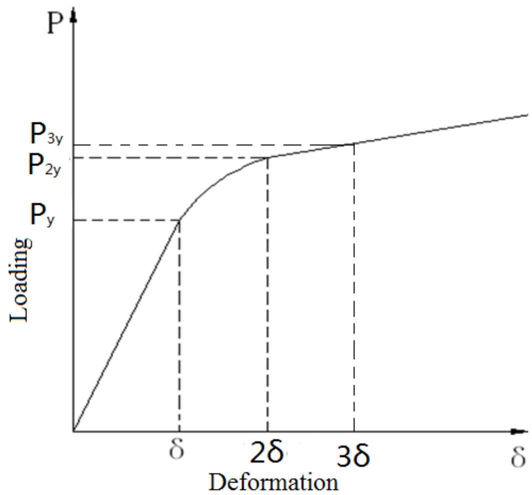


Fig. 4 Schematic of twice elastic deformation criterion

twice elastic slope criterion. Twice elastic slope criterion (ASME 1986) which was based on an empirical procedure was used for calculating collapse loads in experimental stress analysis of pressure vessels, as illustrated in Fig. 5. The plastic load P_φ corresponded to the intersection of the load-deformation curve and a straight line called the collapse limit line which emanated from the origin of the load deformation curve at angle $\Phi = \tan^{-1}(2\tan\theta)$. In this procedure stresses were allowed to exceed yield limits. Hence thinner materials could be used, resulting in economical design. The twice elastic slope criterion of plastic collapse was shown in Fig. 5.

(5) 0.2% offset strain criterion (ASME 1971)

In 1971, ASME code defined limit load which resulted from 0.2% offset strain, as shown in Fig. 6. This method was influenced by the definition of yield stress $\sigma_{0.2}$.

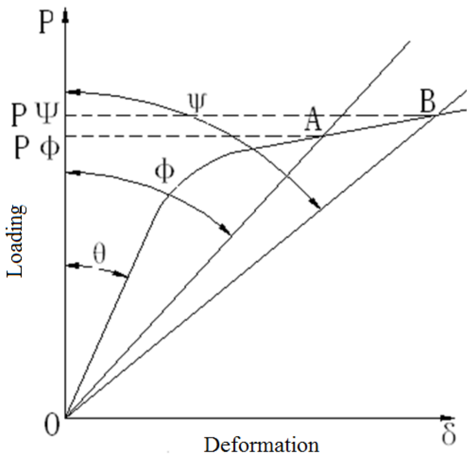


Fig. 5 Twice elastic slope criterion of plastic collapse

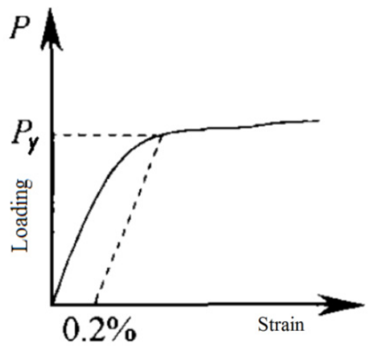


Fig. 6 0.2% residual strain criterion

(6) Demir and Drueker criterion

Demir and Drucker (1963) proposed that the intersection of a vertical line from the distance of 3δ and load-deformation curve was considered as limit load, as drawn in Fig. 4. This method was called as thrice elastic deformation criterion, the same as twice elastic deformation criterion, namely $\tan \psi = 3 \tan \theta$.

(7) Thrice δ criterion

Schroeder (1985) proposed that the vertical axis value on loading-deformation curve where the horizontal axis was 3 δ was defined as limit load, as drawn in Fig. 2.

(8) Plastic work criterion

In 1979, Gerdeen (1979) proposed the suitable ratio of elastic work W_e and plastic work W_P whose corresponding vertical axis value was called limit load. The elastic work W_e and plastic work W_p was defined by the shaded area, respectively.

It was difficult to find the separation point, namely the suitable ratio of elastic work W_e and plastic work W_p. Muscat *et al*. (1979) introduced a loading coefficient λ ($0 \le \lambda \le 1$) as vertical axis and the plastic work as horizontal axis. Like the tangent intersection criterion, the intersection λ_p of

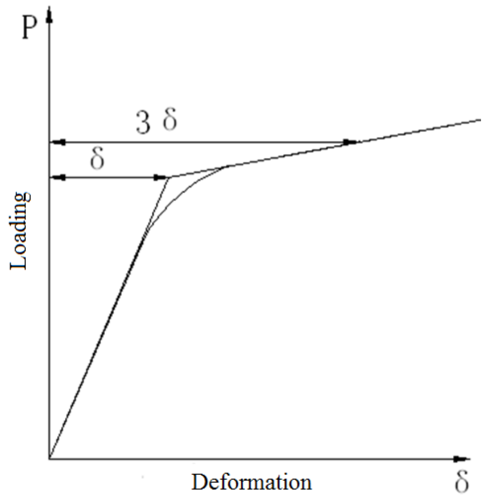
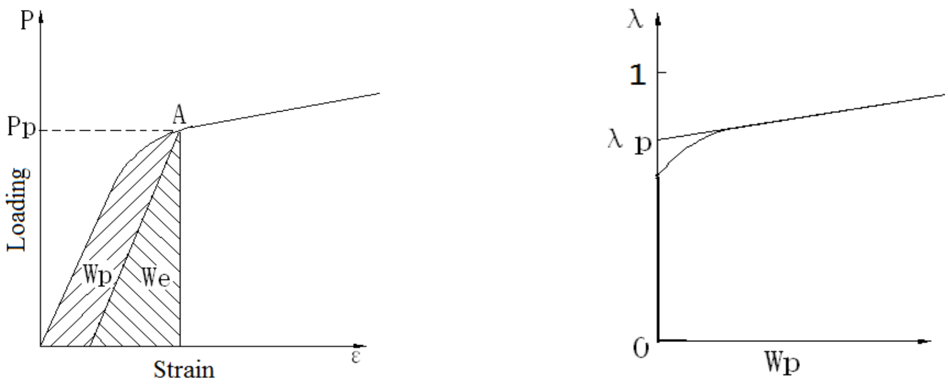


Fig. 7 Thrice δ criterion



(a) (b)

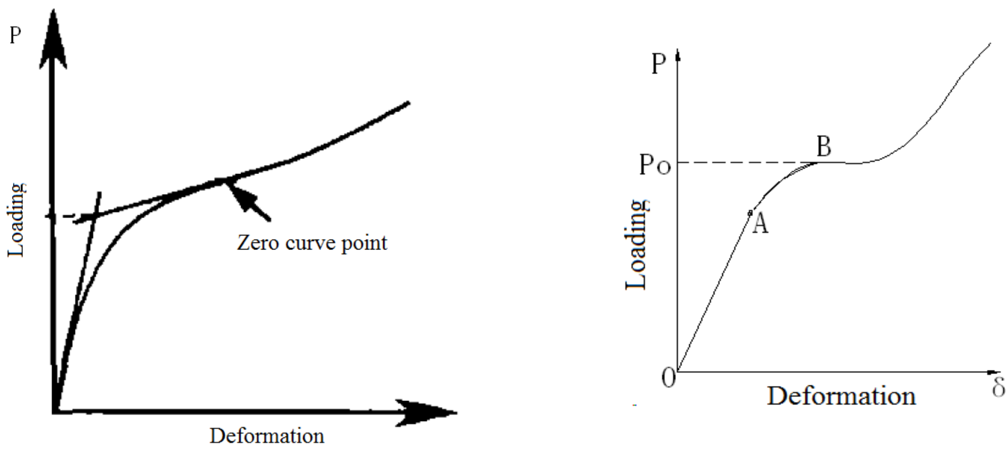
Fig. 8 Plastic work criterion

the tangent of plastic part of λ_W_p curve and the vertical axis was called limit load, as given in Fig. 8(b). The limit load was expressed as

$$P_L = \lambda^p P \tag{13}$$

(9) Zero curvature criterion

In 1989, Zhang *et al.* (1989) modified tangent intersection criterion and presented zero curvature criterion which determined component limit load similar as tangent intersection criterion, as shown in Fig. 9(a). Zero curvature point corresponded to the point where the tangent of plastic part on the loading-deformation curve started to deviate. Moreover, if the loading-deformation curve had yielded plateau as given in Fig. 9(b), zero curvature point corresponded to the point where the tangent of yield plateau started to deviate. The vertical axis value corresponding to zero curvature point was seen as limit load P_o.



(a) (b)

Fig. 9 Zero curvature criterion

(10) Five times elastic slope criterion

According to ideal material and small deformation assumption, Kirkwood (1986, 1989) stated that the intersection of five times elastic slope line and load-deformation curve was considered as limit load point, as drawn in Fig. 5.

(11) Fifteen times elastic slope criterion

Like five times elastic slope criterion, according to ideal material and small deformation assumption, in 2000 Lynch and Moffat (2000) proposed that the intersection of fifteen times elastic slope line and load-deformation curve was considered as limit load point, as drawn in Fig. 5.

(12) Plastic modulus criterion

Schroeder (1980) proposed the next equation which was used to determine plastic slope.

$$k_p = \frac{E_p}{E} k \tag{14}$$

where, k and k_p were the elastic and plastic slope of the loading-deformation curve of the structure, respectively. E and E_p were the elastic and plastic modulus of the loading-deformation curve, respectively. Limit load point was determined as the intersection of the slope of elastic and plastic part of the loading-deformation curve.

(13) Linear limit criterion (Miklus and Kosel 1991)

The load corresponding to the end of the linear part of loading-displacement curve loading was called limit load. Load determined from this criterion was the smallest among all the criteria.

(14) Ultimate strength criterion (Miklus and Kosel 1991)

Based on loading-displacement curve, the load corresponding to asymptote position was defined as limit load. The loading-displacement curve after the point was approximate to linear, but the slope of the loading-displacement curve was far smaller than that of initial linear elastic part.

(15) Maximum principal strain criterion

Su *et al*. (2005) proposed that the load corresponding to the maximum principal strain 5% on loading-maximum principal strain curve was seen as limit load.

2.4 Brief summary

The analytical techniques were only applicable for some simple geometries and boundary conditions. However, it can lead to good estimation of limit load or even the exact solution. For more complicated conditions, numerical procedures including linear and nonlinear methods were developed to determine limit load of structures or components. Inelastic finite element method were often required for engineering designs.

Experimental methods were used to determine the limit load of structures or components. However, it was expensive and time-consuming. Li *et al*. (2008) identified the plastic limit load of cylindrical vessels with different lateral angles under increasing internal loadings by means of experimental testing. Moreover, a three-dimensional nonlinear finite element numerical simulation was also performed. The limit load of cylindrical vessels with nozzles was obtained using a twice-elastic-slope criterion. It was found that the limit loads determined by experiment and numerical simulation methods were in good agreement. Patel and Kumat (2014) obtained limit load of pressure vessel with different inlet and outlet openings by means of experiment methods such as twice elastic slope method, tangent intersection method and nonlinear finite element method. The tangent intersection method used to estimate the lower value of limit pressure was more effective for higher elastic slope of limit pressure vs strain.

3. Application and discussion

Taking a cylinder with nozzle (Chen 2005) as an example in this paper, the limit load of a cylinder with nozzle was identified by stress classification method, elastic-plastic finite element

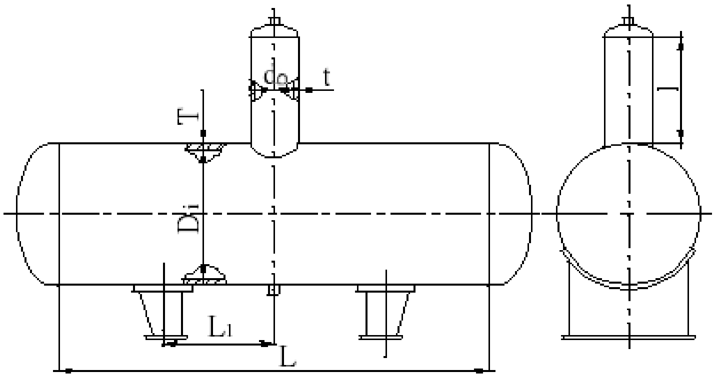


Fig. 10 Schematic diagram of cylinder with nozzle

Table 1 Structure parameters

Di/mm	*L*/mm	*L*1/mm	*T*/mm	*d*o/mm	*l*/mm	*t*/mm
400	800	240	5.4	133	300	4.3

Table 2 Material properties parameters

Structure	Material	Elastic modulus E / GPa	Poisson ratio v	Yield stress σ_s / MPa	Ultimate stress σ_b / MPa
Nozzle	20	212	0.300	332	472
Cylinder	Q235-A	201	0.300	320	485

analysis and elastic finite element method in conjunction with robust methods by mean of finite element software ANSYS.

3.1 Geometrical model

Fig. 10 depicted the structure of a cylinder with nozzle. The dimensions of cylinder with nozzle considered under analysis were shown in Table 1. The cylinder and nozzle material used for analysis was Q235-A. Since, the main purpose of this work was to find the limit pressure of the shell intersection, the yield stress, ultimate stress of the material were important parameters. The material properties were shown in Table 2.

The true stress - strain curves of Q235-A and 20 steel were shown in Figs. 11 and 12, respectively.

Chen (2005) measured the strains of a cylinder with nozzle by mean of experimental method. Locations of strain gages for testing models were given in Fig. 13. Experimental results indicated that dangerous sections were horizontal and longitudinal plane of symmetry of the cylinder with nozzle.

Twice elastic slope criterion and tangent intersection criterion were used to determine limit inner pressure of the cylinder with nozzle, as shown in Fig. 14. Limit inner pressure of test point 7 determined by twice elastic slope criterion and tangent intersection criterion was respectively $P_{L\phi}$ = 4.72 MPa andP_{LT} = 5.79 MPa. Limit inner pressure of test point 8 was respectively $P_{L\phi}$ = 5.05 MPa and P_{LT} = 5.58 MPa.

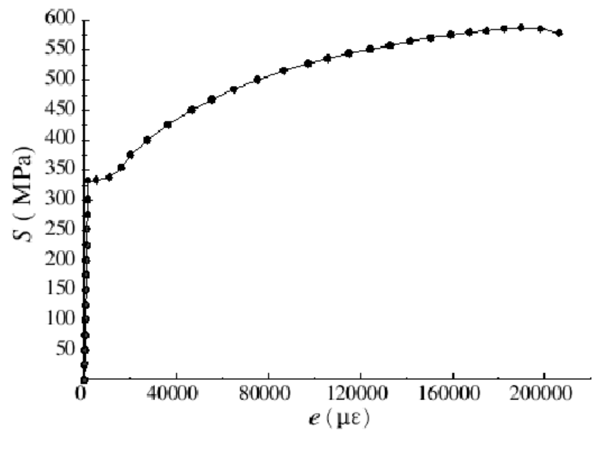


Fig. 11 The curve of true stress-strain for Q235-A

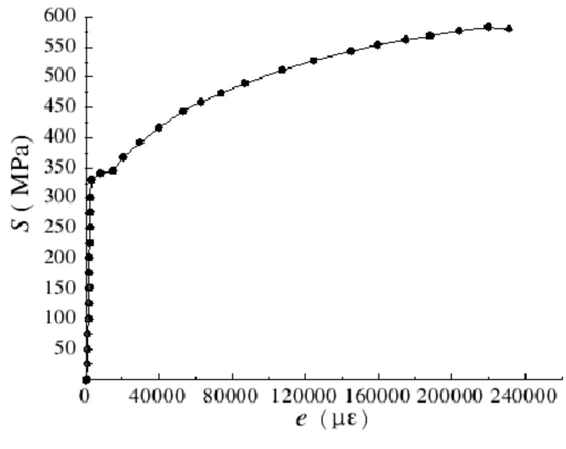


Fig. 12 The curve of true stress-strain for 20

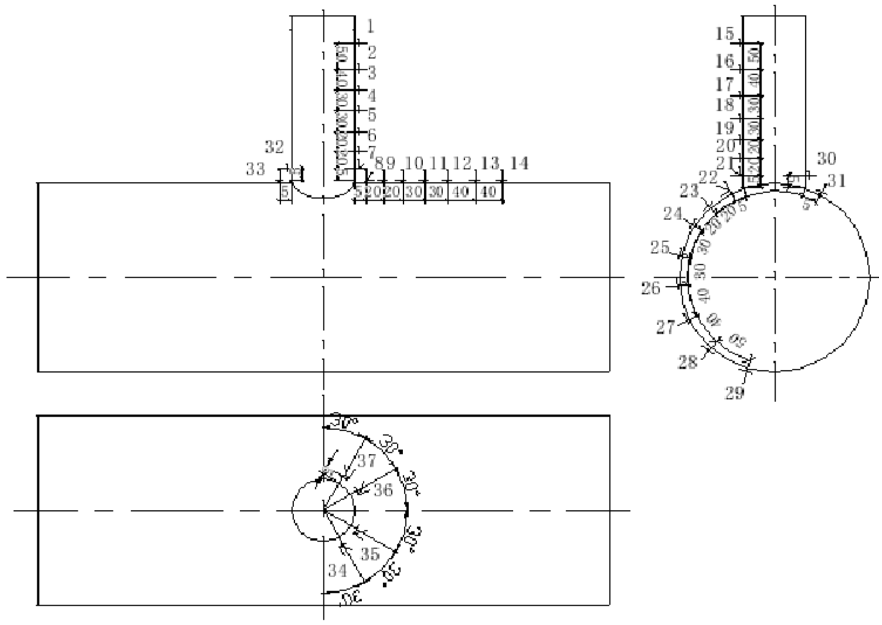
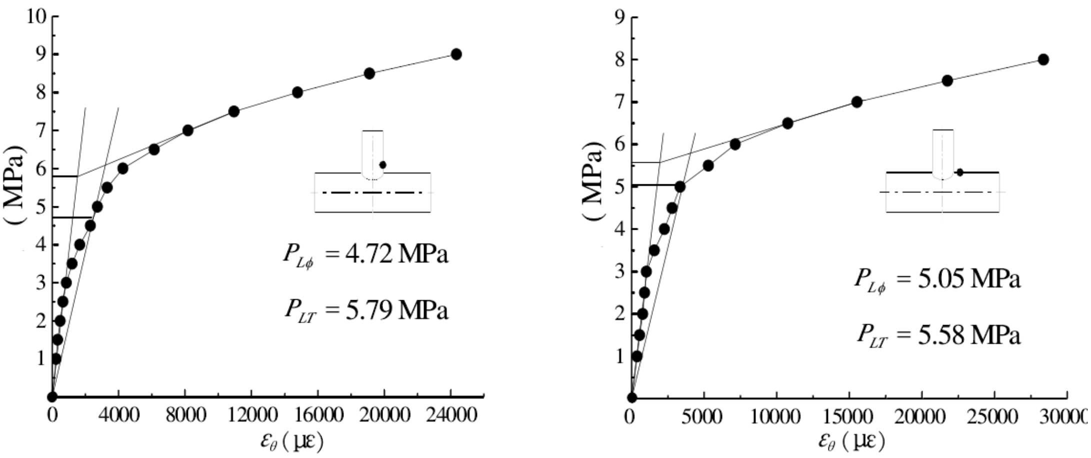


Fig. 13 Locations of strain gages for testing models (Chen 2005)



(a) Test point 7 (b) Test point 8

Fig. 14 Limit inner pressure from test (Chen 2005)

3.2 Finite element simulation

Three dimensional isoparametric element solid45 which was defined by eight nodal points were used to generate the finite element method mesh of the cylinder with nozzle. Due to the symmetry of the structure, a quarter of the cylinder with nozzle was modeled as shown in Fig. 15. Four elements were modeled throughout the thickness. Fig. 16 indicated the mesh for the cylinder with nozzle. The total number of elements was 40960. The intersection of the cylinder and nozzle had a

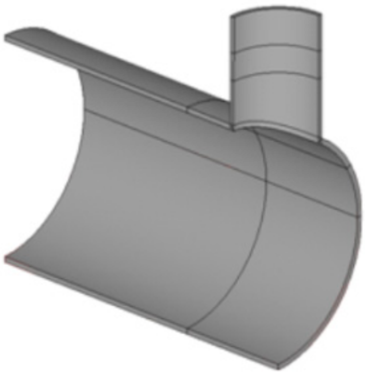


Fig. 15 Finite element model

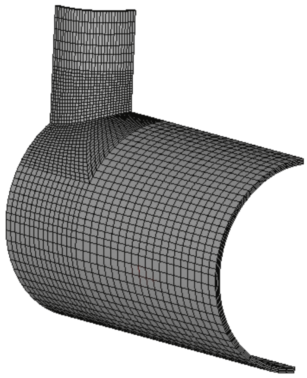
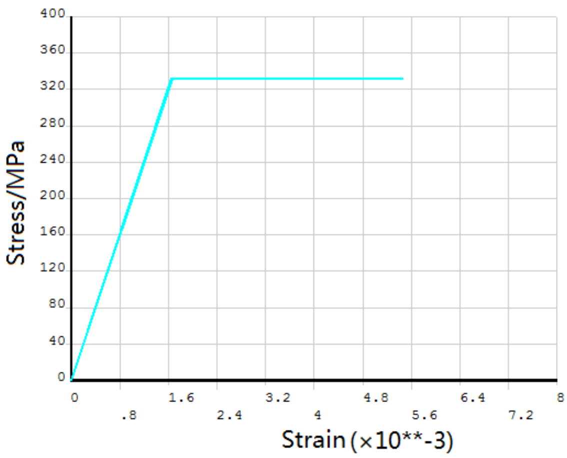


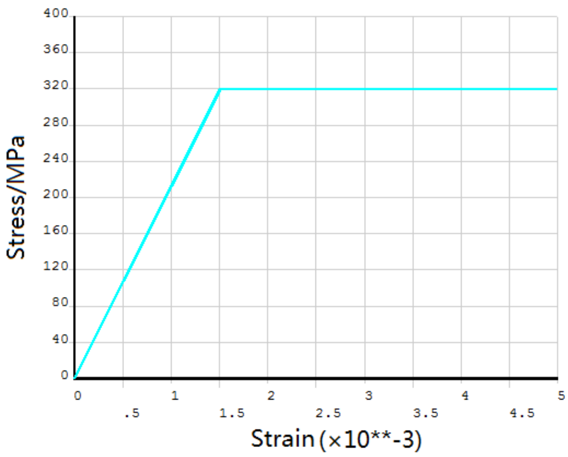
Fig. 16 Finite element mesh

very fine mesh with large number of elements. The accuracy of results was also studied by means of double meshes (Lu and Xu 2006). Comparison of the results of twice finite element analysis, the error of both results was 3%-5%. The results indicated that the first element number was suitable.

Ideal elastic-plastic model, which was relatively safe for the actual engineering, thus was usually used to determine limit load of structures or components in the actual engineering. The bilinear (BISO) elastic-plastic material model in ANSYS software was employed in this paper, as given in Fig. 17. Elastic slope was elastic modulus, plastic modulus was zero. BISO in ANSYS need to input respectively elastic modulus and yield stress of the material by means of the command 'MP' and 'TB'. Material yielding was based on the von Mises yield criterion. The large displacement theory was used during numerical simulation. The boundary conditions used in the finite element numerical simulation were set as follows: all nodes on the symmetric section (longitudinal plane and transverse plane of the vessel) were constrained against deformation in the direction normal to the symmetric plane. The node located at the center of one saddle support was



(a) 20 steel



(b) Q235-A

Fig. 17 The simplified curve of true stress-strain for 20 steel and Q235-A in ANSYS

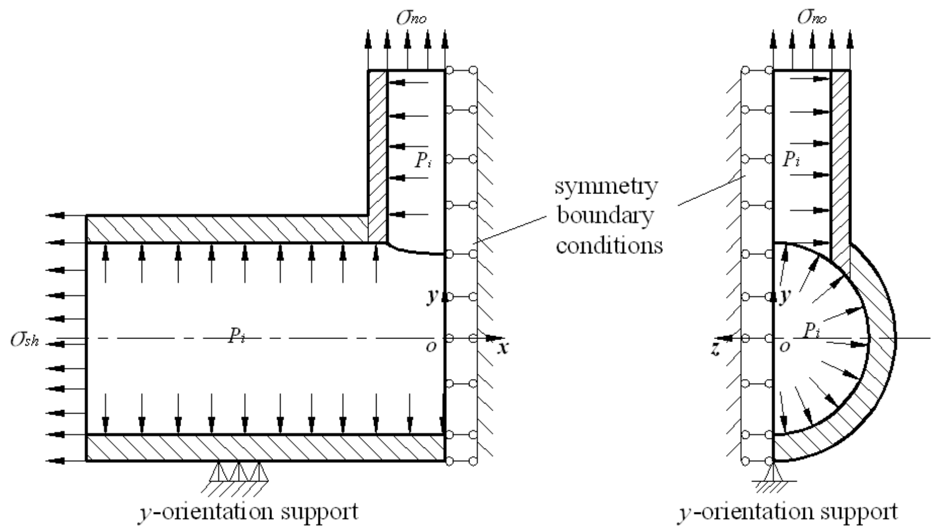


Fig. 18 Load and boundary condition of finite element model

restricted in the vertical direction only. The internal pressure loading of the analysis model was loaded incrementally inside the surface of the cylinder and nozzle. The axial equivalent tensile forces of the ending of the cylinder and nozzle were applied according to Eq. (15).

$$P_c = \frac{P_i D_i^2}{\left(D_i + 2t\right)^2 - D_i^2} \tag{15}$$

4. Finite element results

4.1 Limit load of stress classification

4.1.1 Numerical example

According to Section 2.3.3, three stress categorization methods were described. Linearization method was widely used for stress categorization. Stress contour and stress classification lines were shown in Fig. 19. Table 3 listed stress classification and verification results. According to reference (Gao *et al*. 2010), limit load was determined based on stress classification results.

Mackenzie and his co-worker (1994) observed that Gloss R-node method can be valid to extract primary stress and secondary stress of simple structural elements. But for complex structures such as axisymmetric cylinder and flat head, the results of Gloss R-node method were inconsistent with those of ASME code. Therefore, linearization method was so far widely applied for stress categorization. For the cylinder with nozzle in this study, Gloss R-node method was not used.

For m_α^T method, m^0/m_L was greater than $1+\sqrt{2}$ in this study, primary stress, secondary stress and peak stress were listed in Table 4.

For stress classification method, it can be concluded that when loading condition changed, the maximum stress location might shift from its previous location. Therefore, for components having

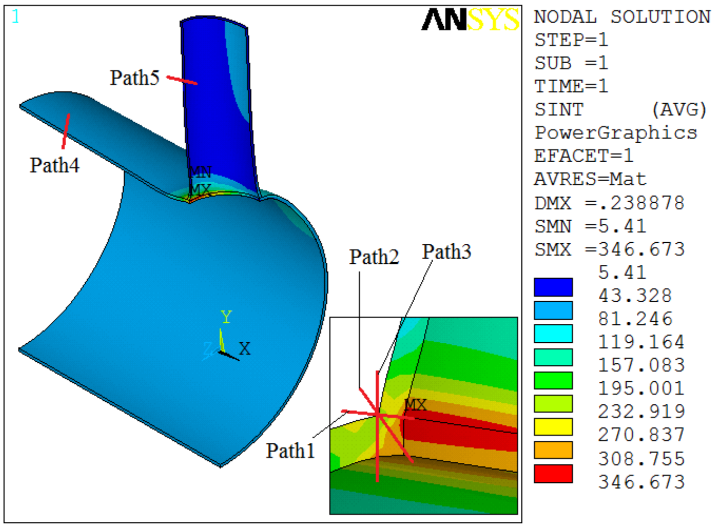


Fig. 19 Stress contour and stress classification line

Table 3 Stress classification and verification results

Position	$P_m\ (P_L)$	$P_m\ (P_L) + P_b + Q$	$P_{\max}^{II}$	$P_{\max}^{IV}$	$[P]$	P_1
Path1	222.9	286.6	2.98	3.48	2.98	3.50
Path2	241.7	298.7				
Path3	215.2	265.5				
Path4	64.78	65.40				
Path5	25.48	43.12				

Table 4 Stress classification of m_α^T method

Stress	P	$P+Q$	F	$P_{\max}^{II}$	$P_{\max}^{IV}$	$[P]$	P_l
	122.4/127.7	172.8/244.8	35	3.68	3. 84	3.68	5.52

non-uniform stress distribution for combined loading might lead to an improper selection of class lines during the stress linearization process. Whereas the proposed m_α^T - tangent method gave one value of the stress category irrespective of the loading condition. This was a very important advantage of using the m_α^T - tangent method for combined loading cases.

4.1.2 Brief summary

The categorization of stresses by elastic FEA was a challenging task even with the finest computing facilities and advanced numerical techniques. The categorization of elastic stresses in complex pressure vessels was involved and demanded substantial skill. The purpose of the present work was to introduce a new stress categorization method based only on linear elastic FEA and demonstrate its application. The proposed method used limit load multiplier estimated to decompose the elastic stresses into appropriate categories, using a single linear elastic FEA. The proposed method was able to identify the primary (*P*), primary plus secondary (*P*+*Q*) and peak (*F*) stress components for mechanical and thermal loads within reasonable accuracy. The results were directly comparable with the ASME B&PV Code (2007) limits. Notably, there was only a single primary stress to be evaluated against S_m, as opposed to separate limits for membrane and bending stress. Since the method delivered directly only a single bounding value for each stress category, its application was very convenient and straightforward. Several example problems were worked out to demonstrate the method, including typical two and three dimensional pressure vessel components. The primary stresses obtained from the proposed method were in reasonably good agreement with the elastic-plastic FEA results. For simple axisymmetric pressure vessel (cylindrical vessel and torispherical head), the primary plus secondary stresses obtained from the proposed method were compared with those obtained from stress linearization method. The results were again in reasonably good agreement. The same approach was expected to work well for more complicated structures e.g., oblique nozzle on a cylindrical vessel. The proposed method had several potential benefits over conventional stress categorization approaches. This method made

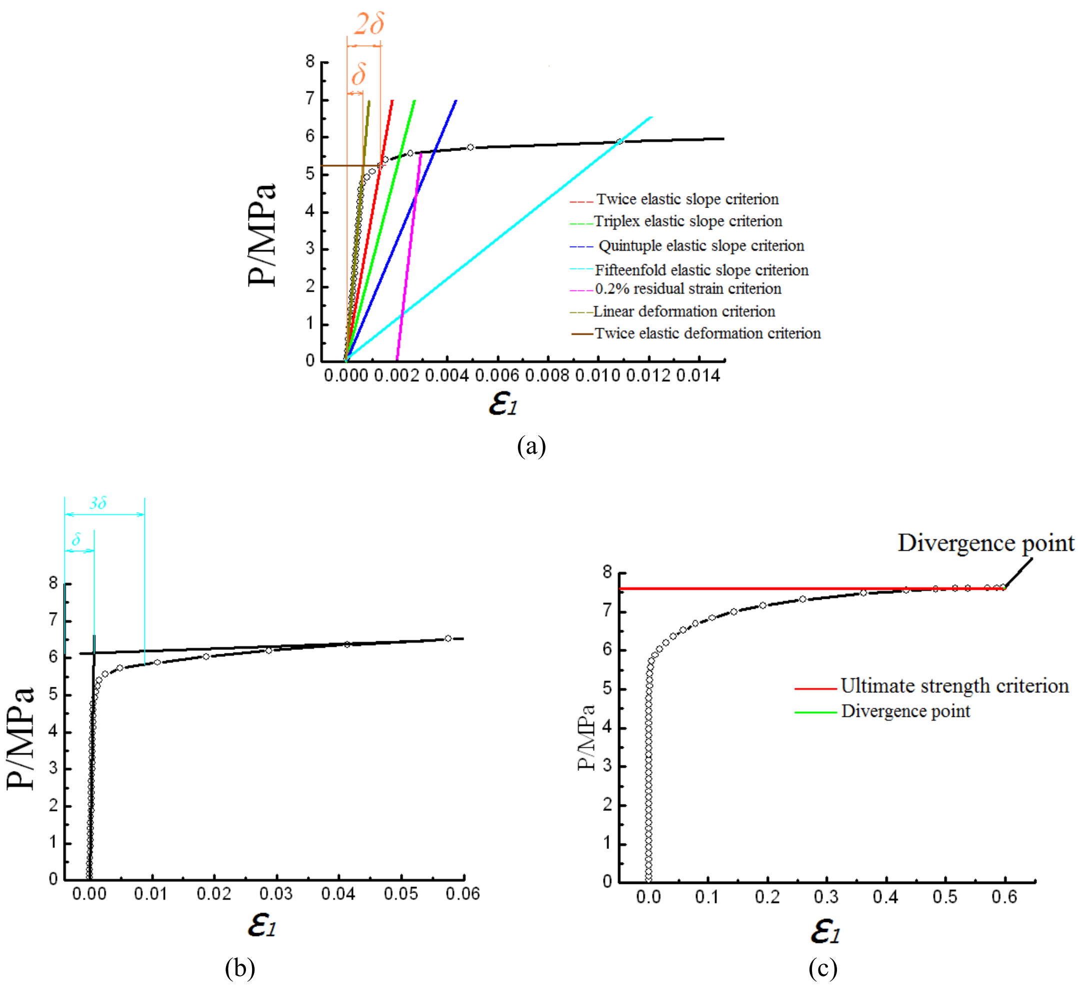


Fig. 20 Criterions of limit load determination

Table 5 Numerical solution of limit load

Methods	Limit load/MPa
Experimental result	4.72/5.05
Stress classification	3.50
m_{α}^{T} Stress classification	5.52
Twice elastic slope criterion	5.26
Triplex elastic slope criterion	5.49
Quintuple elastic slope criterion	5.63
Fifteenfold elastic slope criterion	5.89
Tangent intersection criterion	6.23
Triplex δ criterion	5.56
Zero curvature criterion	6.68
1% equivalent plastic strain criterion	5.85
0.2% residual strain criterion	5.57
5% maximum principal strain criterion	6.45
Linear deformation criterion	5.24
Twice elastic deformation criterion	5.72
ultimate strength criterion	7.10
Plastic work criterion	6.10
Plastic modulus criterion	6.34
Divergence point	7.62

use of available FEA codes and currently required a moderate amount of post-processing by the user, which could be automated. As a result, the method gave three numbers, namely the primary stress, primary plus secondary stress, and peak stress. The method directly delivered the bounding values for the analyzed component. This sidestepped the potential difficulties encountered in justifying the appropriate location and orientation of the SCLs. The method was applicable to a wide range of pressure vessels including three dimensional vessel with complex geometry as shown in Example 7.4 in this paper. The proposed method was able to categorize the stresses for combined loading (pressure and thermal) without requiring two separate analyses. Therefore, the proposed method could be used as a tool for simplified stress categorization of pressure vessels with minimum computational effort.

4.2 Limit load determined by elastic-plastic finite element method

4.2.1 Numerical example

Variational principles were only feasible for some simple geometries and boundary conditions. The slip lines method estimated limit load in a component or structure in plane condition. Therefore, limit load of the cylinder with nozzle was determined by finite element analysis combining with those criteria in Section 2.2, as listed in Table 5. Fig. 20(a) illustrated the crossing point of twice elstic slope criterion, three times elastic slope criterion, five times elastic slope criterion, fifteen times elastic slope criterion, 0.2% residual strain criterion, linear limit criterion and twice elastic strain criterion and loading - strain curve. Fig. 20(b) showed the crossing point of tangent intersection criterion and Demir & Drueker criterion and loading-deformation curve. Fig. 20(c) indicated the crossing point of ultimate strength criterion and divergent point and loading-strain curve.

4.2.2 Brief summary

The advantages and disadvantages of those criterions in Table 5 were described as follows:

(1) These criteria in Table 5 which were inequivalent with theoretical limit load were used to determine limit load of structures that were called as actual limit load or engineering limit load.

(2) The definition of plastic collapse of actual structures was included in these criteria. These criteria had human factors. The plastic collapse of actual structures was a definition of engineering significance. The engineering backgrounds for these criterions were different. Therefore, the maximum plastic limit load determined by one criterion cannot be seen as the reasonable index.

(3) The reasonable criterion should be objective and operational. The objective criterion can not include any human factors based on a certain substantive characteristics of experimental results. The operability criterion which should be convenient for engineering application can obtain limit load of the conservatism that could satisfy the engineering requirements. For the same relationship of loading versus deformation, different operator may obtain the same limit load. Tangent intersection criterion among these criterions was an objective criterion. If the plastic section of loading–deformation curve did not show obvious plastic platform or steady section, the tangent point position of plastic section would not be accurately determined. Therefore, EN13445 code (2002) suggested that if the plastic section was larger than 5%, the corresponding maximum principal strain 5% point was the tangent point of the plastic section. It was found that the maximum principal strain can represent the eigenvalue of total plastic deformation of pressurized equipment. Therefore, Su *et al*. (2005) proposed a 5% maximum principal strain criterion. Meanwhile, 5% maximum principal strain criterion was used to determine limit load of structures which was easy to be realized in commercial finite element software ANSYS.

(4) The other criteria were based on human factors except for tangent intersection criterion, as shown below

$$P_{2y} < P_{\varphi} < P_{\psi} \text{ and } P_{3\delta} \approx P_{\psi} \tag{16}$$

The plastic work criterion had difficulty in determining limit load of structures, which was attributed to the shape of plastic work and loading P-deformation ω curve. For example, twice elastic slope criterion cannot determine the split point between elastic work and plastic work. Therefore, Muscat *et al*. (2003) presented loading parameter - plastic work curve. According to EN13445 code (2002), the maximum principal strain 5% point was tangent point of plastic section. The intersection between tangent line and vertical axis λ_p was obtained, thus limit load was determined.

(5) Most of the criteria for limit load estimation in Section 2.2 determined limit load of structures through loading P-deformation δ curve. Sometimes loading P- angle of rotation θ curve or loading P-strain ε curve was also used. The loading-strain curve was usually applied for pressurized vessels and its components. For 0.2% residual strain criterion including bending effect resulted from surface strain, if loading-maximum strain curve

was determined by experiments, the maximum strain was not easy to be measured, and residual strain was not directly measured. Therefore, the practicality of 0.2% residual strain criterion was bad. But if finite element method was used such as ANSYS combined with APDL parametric design language, it was easy to determine the limit load by 0.2% residual strain criterion.

(6) Zero curvature criterion was also an objective criterion and did not contain any human factors. This criterion reflected the characteristic of the overall plastic flow of structures, and sustained almost the effect of local geometrical shape. Thus, compared to experimental results, the dispersibility of limit load determined by zero curvature criterion was very small. Like tangent intersection criterion, because the transition part of loading – displacement curve and tangent line of plastic flow zone was tangential and did not intersect, zero curvature point was not easy to obtain through plotting contours. But by means of nonlinear finite element method, the difference between back and forth sub-step slope of loading-displacement curve was smaller than 10^{-3}: 10^{-4}, the corresponding point was zero curvature point.

In general, the results of zero curvature criterion was larger than those of theoretical calculation, which indicated hardening effect of materials and the influence of large deformation. Compared to tangent intersection criterion, the zero curvature criterion result was even more approximate to actual limit load.

(7) Twice elastic slope criterion had a strong operability. For the same loading-deformation curve, the error of limit load determined by different people was very small. The result was certified in the ASME code. Twice elastic slope criterion had reasonable conservatism and was suitable for the engineering application. Quintuple and fifteenfold elastic slope criterions were proposed based on ideal material and small deformation assumption. These two criterions were suitable for material with longer yield platform from stress-strain curve. If they were used for large deformation conditions, unreliable results might be obtained.

(8) 1% equivalent plastic strain criterion was affected by the material and geometrical shape of pressurized equipment. Strain at plastic hinge of pressurized equipment was larger than those of other positions. Therefore, when the relationship between loading and deformation was determined by experiments, limit load determined by 1% equivalent plastic strain criterion had a strong relationship with measurement point of pressurized equipment.

(9) Twice elastic deformation criterion was combined with finite element method, and initial yield loading can be predefined. Elastic deformation was obtained, and then the limit load was determined. But the initial yield loading was difficult to be determined from the loading-deformation curve. Therefore, the deformation corresponding to proportional limit was defined as elastic deformation. The limit load error determined by twice elastic deformation criterion was rather large.

(10) Limit load determined by linear deformation criterion which was less than other criterions was widely recognized by the counterparts.

(11) Those criterions in Table 5 aimed at structure under simple loading conditions, *i.e.*, limit load of the structure was determined by simplify tension, compression, bending and internal pressure, and so on. Gerden (1979) proposed plastic work criterion which employed plastic work reflecting action effects of applied load. Gerden (1979) thought that the plastic work criterion could be employed for complex loading conditions, but it was

difficult during actual application. On one hand, it was difficult to determine the boundary between elastic work and plastic work. On the other hand, plastic work criterion need be several variant parameters that were corresponded to applied load and needed to reflect the work of applied load which was not arbitrarily determined.

(12) It was feasible for single loading condition, the loading - displacement curve or loading - angle of rotation curve was applied by these criteria in Table 5. It was difficult for complex loading conditions, which was attributed to a coordinate system which cannot be used for different loading type. Different variant parameters and measurement points affected the accuracy of limit load.

5. Conclusions

The purpose of this research was to summarize and compare the application of these limit load methods. The objectives were listed as follows. Analytical techniques, elastic and elastic-plastic numerical methods were firstly reviewed. Then taking a cylinder with nozzle as an example, the limit loads of a cylinder with nozzle were calculated and compared. The results indicated that limit load determined by linearization method was the smallest. Limit load determined by twice elastic slope criterion was the nearest than experimental results. Elastic-plastic finite element analysis had comparably computational precision. But Elastic-plastic finite element analysis required considerable amount of computation time and higher processing and storage capacity of the computer. Moreover, most of criteria for limit load estimation included any human factors based on a certain substantive characteristics of experimental results. The reasonable criterion should be objective and operational. Therefore, the scholars had been trying to develop an objective and operational determined limit load method.

Acknowledgments

The authors gratefully acknowledge financial support for this work from the Doctoral Scientific Research Foundation of Liaoning Province (No.201601017), Youth Foundation of Hebei Educational committee (No. QN2015336), Fundamental Research Funds for the Central Universities (XNB2015001) and National Natural Science Foundation of China (No. 51475086). The authors acknowledge China Scholarship Council (CSC) which has supplied funding for us to undertake collaborative research. Dr. Haofeng Chen at University of Strathclyde is acknowledged for the assistance of the thought of this paper.

References

Adibi-Asl, R. and Seshadri, R. (2007), “Simplified Limit Load Determination of Cracked Components Using the Reference Two-Bar Structure”, ASME PVP, Paper No. PVP2007-26747, San Antonio, TX, USA.

ASME (1971), ASME Boiler and Pressure Vessel Code; Section III. Division 1, American Society Mechanical Engineers, New York, NY, USA.

ASME (1974), ASME Boiler and Pressure Vessel Code; Section III. Division 1 Par. 1430, American Society Mechanical Engineers, New York, NY, USA.

ASME (1975), ASME Boiler and Pressure Vessel Code; Section III. Division 1 Par. 1430, American Society

Mechanical Engineers, New York, NY, USA.
ASME (1986), ASME Boiler and Pressure Vessel Code; Section XI. Division 2, American Society Mechanical Engineers, New York, NY, USA.
ASME (2010), ASME Boiler and Pressure Vessel Code; Section VIII. Division 2, American Society Mechanical Engineers, New York, NY, USA.
Brighenti, R. (2001), "Influence of a central straight crack on the buckling behavior of thin plates under tension, compression, or shear loading", *Int. J. Mech. Mater. Des.*, **6**(1), 73-87.
Calladine, C.R. (2000), *Plasticity for Engineers*, Horwood Publisheing, Chichester, UK.
Caratheodory, C. and Schmidt, E. (1923), "Uber Die Hencky-Prandtlschen Kurven", *Z. angew. Math. Mech.*, **3**, 468-475.
Charnes, A. and Greenberg, H.J. (1951), "Plastic collapse and linear programming", *Bull. Am. Math. Soc.*, **57**(6), 480 p.
Chen, F. (2005), "Investigation on the limit and burst pressures for the vessels with nozzle", Ph.D. Dissertation; Nanjing University of Technology, Nanjing, China.
Chen, X.H., Chen, X., Chen, G. and Li, D.M. (2015), "Ratcheting behavior of pressurized Z2CND18.12N stainless steel pipe under different control modes", *Steel Compos. Struct., Int. J.*, **18**(1), 29-50.
Chen, X.H. and Chen, X. (2016), "Effect of local wall thinning on ratcheting behavior of pressurized 90° elbow pipe under reversed bending using finite element analysis", *Steel Compos. Struct., Int. J.*, **20**(4), 931-950.
Chiodo, M.S.G. and Ruggieri, C. (2009), "Failure assessments of corroded pipelines with axial defects using stress-based criteria: Numerical studies and verification analyses", *Int. J. Press. Vessel. Pip.*, **86**(2-3), 164-176.
Cosham, A., Hopkins, P. and Macdonald, K.A. (2007), "Best practice for the assessment of defects in pipelines – Corrosion", *Eng. Fail. Anal.*, **14**(7), 1245-1265.
Davis, R.O. and Selvadurai, A.P. (2002), *Plasticity and Geomechanics*, Cambridge University Press, UK.
Demir, H.H. and Drueker, D.C. (1963), "An Experimental Study of Cylindrieal Shells under Ring Loads", In: *Progress in Applied Mechanics*, (Prager Anniversary Volume), MaeMillan, New York, pp. 205-220.
EN13445-3 (2002), European Committee for Standardization (CEN); European Standard for unfired pressure vessel—Part 3: design.
Fanous, Ihab F.Z.R. and Seshadri, R. (2007), "Stress classification using the R-node method", **129**(4), 676-682.
Gao, B.J., Chen, X.H., Shi, X.P. and Dong, J.H. (2010), "An approach to derive primary bending stress from finite element analysis for pressure vessels and applications structural design", *ASME J. Press. Vessel Technol.*, **132**(6), 061101-1-8.
Gerdeen, J.C. (1979), "A critical evaluation of plastic behavior data and a united definition of plastic loads for pressure components", WRC Bulletin 254.
Hencky, H. (1923), "Über Einige Statisch Bestimmte Fälle des Gleichgewichts in Plastischen Körpern", *Z. Angew. Math. Mech.*, **3**, 241-251.
Hill, R. (1950), *The Mathematical Theory of Plasticity*, Oxford at the Clarendon Press.
He, K.G. (1995), *Design Basis for Pressure Vessel*, China Machine Press, Beijing, China.
Hechmer, J.L. and Hollinger, G.L. (1989), "Code evaluation of 3D stresses on a plane", *Am. Soc. Mech. Eng., Press. Vessels Pip. Div. (Publication) PVP*, **161**, 33-46.
Hechmer, J.L. and Hollinger, G.L. (1991), "The ASME code and 3D stress evaluation", *ASME J. Press. Vessel Technol.*, **113**(4), 481-487.
Hechmer, J.L. and Hollinger, G.L. (1998), "3D stress criteria guidelines for application", WRC Bulletin 429.
Hodge, P.G. (1964), "Yield-point load determination by non-linear programming", *Proceedings of the 11th International Congress of Applied Mechanics*, Munich, Germany, pp. 554-561.
Hollinger, G.L. and Hechmer, J.L. (2000), "Three-dimensional stress criteria-summary of the PVRC project", *ASME J. Press. Vessel Technol.*, **122**(1), 105-109.
Hsu, K.H. and Mckinley, D.A. (1990), "SOAP-A computer program for classification of three dimension finite element stresses on a plane", *PVP ASME*, **185**, 11-19.

Kamaya, M., Suzuki, T. and Meshii, T. (2008), “Failure pressure of straight pipe with wall thinning under internal pressure”, *Int. J. Press. Vessel Pip.*, **85**(9), 628-634.
Khyabani, A. and Sadrnejad, S.A. (2009), “Finite element evaluation of residual stresses in thick plates, *Int. J. Mech. Mat. Des.*, **5**(3), 253-261.
Kirkood, M.G. (1986), “Finite element stress analysis of an equal diameter branch pipe intersection subjected to out-of-plane and twisting moments”, *Strain Anal.*, **21**(1), 171-183.
Kirkood, M.G. (1989), “Techniques for measuring plastic loads in branch pipe connections loads by pressure and in-plane moments”, *Pipework Engineering and Operation*, London, UK, pp. 197-208.
Kroenke, W.C. (1973), “Classification of finite element stress according to ASME section III”, *Stress Categories the Winter Annual Meeting of the American Society Engineering*, November.
Li, N., Sang, Z.F. and Widera, G.E.O. (2008), “Study of plastic limit load on pressurized cylinders with lateral nozzle”, *ASME J. Press. Vessel Technol.*, **130**(4), 041210-1-7.
Lu, M.W. and Xu, H. (2006), “Discussion on some important problems of design by analysis (1)”, *J. Pres. Ves.* (China), **23**(1), 15-19.
Lyamin, A.V. and Sloan, S.W. (2002a), “Upper bound limit analysis using linear finite elements and nonlinear programming”, *Int. J. Numer. Anal. Meth. Geomech.*, **26**(2), 181-216.
Lyamin, A.V. and Sloan, S.W. (2002b), “Lower bound limit analysis using nonlinear programming”, *Int. J. Numer. Meth. Eng.*, **55**(5), 573-611.
Lynch, M.A. and Moffat, D.G. (2000), “Limit load for cracked piping branch junctions under pressure and branch out-of-plane bending”, *Int. J. Press. Vessels Pip.*, **77**(5), 185-194.
Mackenzie, D., Boyle, J.T. and Spence, J. (1994), “Some recent developments in pressure vessel design by analysis”, *J. Pro. Mech. Eng.*, **208**(15), 23-29.
Miklus, S. and Kosel, F. (1991), “Plastic collapse of pipe bi-furcation”, *Int. J. Press. Vessels Pip.*, 79-92.
Mura, T. and Lee, S.L. (1965), “Application of variational principles to limit analysis”, *Q. Appl. Math.*, **21**(3), 243-248.
Mura, T., Kao, J.S. and Lee, S.L. (1964), “Limit analysis of circular orthotropic plates”, *J. Eng. Mech. Div. ASCE*, **90**(5), 375-395.
Mura, T., Rimawi, W.H. and Lee, S.L. (1965), “Extended theorems of limit analysis”, *Q. Appl. Math.*, **23**(2), 171-179.
Muscat, M., Mackenzie, D. and Hamilton, R. (2003), “A work criterion plastic collapse”, *Int. J. Press. Vessels Pip.*, **80**(1), 49-58.
Patel, D. M. and Kumar, D.B. (2014), “Pressure vessel limit load estimation by FEM and experimental method”, *Int. J. Innov. Res. Adv. Eng.*, **1**(9), 109-114.
Plancq, D. and Berton, M.N. (1998), “Limit analysis based on elastic compensation method of branch pipe tee connection under internal pressure and out-of-plane moment loading”, *Int. J. Press. Vessels Pip.*, **75**(11), 819-825.
Prager, W. and Hodge, P.G. (1951), *Theory of Perfectly Plastic Solids*, Wiley, New York, NY, USA.
Prandtl, L. (1923), “Anwendungsbeispiele Henckyschen Satz uber das Plastische Glei Zeitschr”, *Angkew. Mathematik u. Mechanik*, **407**.
R5 (1990), Assessment Procedure for the High Temperature Response of Structures; Nuclear Electric plc, **2**.
Samo, M. and Franc, K. (1991), “Plastic collapse of pipe bifurcation”, *Int. J. Press. Vessels Pip.*, **48**(1), 79-92.
Save, M. (1972), “Experimental verification of plastic limit analysis of torispherical and toriconical heads”, *Press. Vessel Pip. Des. Anal. I*, *ASME*, **1**, 382-416.
Schroeder, J. (1980), “A plastic modulus approach to experimental limit loads”, ASME Paper No. 80 - C2/PVP -1.
Schroeder, J. (1985), “Experimental limit couples for branch moment loads on 4 in. ANSI B16.9 Tees”, WRC Bulletin 304.
Seshadri, R. and Marriott, D.L. (1993), “On Relating the Reference Stress, Limit Load and the ASME Stress Classification Concepts,” *Int. J. Press. Vessels Pip.*, **56**, 387-408.
Su, W.X., Zheng, J.Y. and Kai, F.M. (2005), “A 5% maximum principal strain criterion to prevent gross

plastic deformation", *Press. Vessel Technol.*, **22**(12), 17-21.
Townley, C.H.A., Findlay, G.E., Goodman, A.M. and Stanley, P. (1971), "Elastic-plastic computations a basic for design charts for torispherical pressure vessel ends", *Proceedings of the Institution of Mechanical Engineers. Part E: Journal of Process Mechanical Engineering*, **185**(63), 869-877.
Zhang, W.M., Lu, M.W. and Zhang, R.Y. (1989), "A zero-curvature criterion to determine the practical collapse load", *Acta Mech. Solida Sin.*, **10**(2), 152-159.
Zouain, N., Herskovits, J., Borges, L.A. and Feijoo, R.A. (1993), "An iterative algorithm for limit analysis with nonlinear yield function", *Int. J. Solids Struct.*, **30**(10), 1397-1417.

CC