

## Optimum design of braced steel frames via teaching learning based optimization

Musa Artar\*

*Department of Civil Engineering, Bayburt University, Bayburt 69000, Turkey*

*(Received July 16, 2016, Revised October 17, 2016, Accepted October 18, 2016)*

**Abstract.** In this study, optimum structural designs of braced (non-swaying) planar steel frames are investigated by using one of the recent meta-heuristic search techniques, teaching-learning based optimization. Optimum design problems are performed according to American Institute of Steel Construction- Allowable Stress Design (AISC-ASD) specifications. A computer program is developed in MATLAB interacting with SAP2000 OAPI (Open Application Programming Interface) to conduct optimization procedures. Optimum cross sections are selected from a specified list of 128W profiles taken from AISC. Two different braced planar frames taken from literature are carried out for stress, geometric size, displacement and inter-storey drift constraints. It is concluded that teaching-learning based optimization presents robust and applicable optimum solutions in multi-element structural problems.

**Keywords:** teaching-learning based optimization; optimum design; planar steel frames; MATLAB-SAP2000 OAPI

### 1. Introduction

In the last 30 years, several algorithm techniques based on meta-heuristic methods have been developed for optimum structural designs. In literature, several studies exist on discrete design problems using different methods such as genetic algorithm (GA), harmony search (HS) algorithm, ant colony optimization (ACO), artificial bee colony (ABC) algorithm, bat-inspired (BI) search algorithm, big bang-big crunch (BB-BC) algorithm, tabu search (TS) algorithm, simulated annealing (SA) algorithm and teaching-learning-based optimization (TLBO).

Daloglu and Armutcu (1998) studied optimum design of plane steel frames according to TS 648 (Turkish Building Code for Steel Structures) using genetic algorithm. Togan and Daloglu (2006) used this basic algorithm method for space trusses. Lee and Geem (2004) investigated harmony search algorithm. Degertekin (2007) researched optimum design of nonlinear steel space frames using simulated annealing and genetic algorithm methods. Saka (2009) used harmony search algorithm for optimum design of steel sway frames according to BS5950. Degertekin and Hayalioglu (2009) used tabu search for optimum design of steel space frames. Degertekin and Hayalioglu (2010) studied optimum design of steel frames with semi-rigid connections and column bases using harmony search algorithm. Hasancebi *et al.* (2010a) used simulated annealing

---

\*Corresponding author, Assistant Professor, E-mail: [martar@bayburt.edu.tr](mailto:martar@bayburt.edu.tr)

algorithm for frame structures. Hasancebi *et al.* (2010b) studied structural optimization problems using adaptive harmony search method. Hasancebi *et al.* (2011) studied optimum design of high-rise steel buildings using an evolution strategy integrated parallel algorithm. Aydogdu and Saka (2012) researched optimization of irregular steel frames using ant colony optimization. Degertekin (2012) focused on optimum design of geometrically non-linear steel frames using artificial bee colony algorithm. Togan (2012) used teaching–learning based optimization for optimum design of planar steel frames. Dede (2013) used teaching-learning based optimization for optimum design of grillage structures. Also, Dede and Ayvaz (2013) used this new algorithm method for structural optimizations. Hasancebi *et al.* (2013) studied a bat-inspired algorithm for structural optimization. Rafiee *et al.* (2013) used big bang-big crunch method for optimum design of steel frames with semi-rigid connections. Azad *et al.* (2014) focused on guided stochastic search technique for discrete sizing optimization of steel trusses: A design-driven heuristic approach. Artar and Daloglu (2015) used genetic algorithm method for optimum design of steel space frames with composite beams. Artar (2016) studied optimum design of steel space frames under earthquake effect using harmony search algorithm.

Teaching-learning-based optimization (TLBO), one of the recent efficient optimization methods, was developed by Rao *et al.* (2011). This innovative algorithm method is selected to solve different braced (non-swaying) planar problems in the present study. These design examples taken from literature are a 162-member X-braced planar steel frame and a 304-member K-braced planar steel frame. To obtain optimum feasible profiles, a program was coded in MATLAB to incorporate with SAP2000-OAPI. The results show the robustness and applicability of TLBO method for structural problems.

## 2. Optimum design problem

The optimum design problem of steel frames is defined as

$$\min W = \sum_{k=1}^{ng} A_k \sum_{i=1}^{nk} \rho_i L_i \quad (1)$$

where  $W$  is the weight of the frame,  $A_k$  is cross-sectional area of group  $k$ ,  $\rho_i$  and  $L_i$  are density and length of member  $i$ ,  $ng$  is total number of groups,  $nk$  is the total number of members in group  $k$ .

-The stress constraints taken from AISC–ASD (1989) are expressed as

$$g_i(x) = \left[ \frac{f_a}{F_a} + \frac{C_{mx} f_{bx}}{\left(1 - \frac{f_a}{F'_{ex}}\right)_{bx}} \right] - 1.0 \leq 0 \quad i = 1, \dots, nc \quad (2)$$

$$g_i(x) = \left[ \frac{f_a}{0.60 F_a} + \frac{f_{bx}}{F_{bx}} \right] - 1.0 \leq 0 \quad i = 1, \dots, nc \quad (3)$$

if  $\frac{f_a}{F_a} \leq 0.15$ , Eq.(4) is determined instead of Eqs.(2) and (3)

$$g_i(x) = \left[ \frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} \right] - 1.0 \leq 0 \quad i = 1, \dots, nc \quad (4)$$

where  $nc$  is total number of members subjected to both axial compression and bending stresses,  $f_a$  is computed axial stress,  $F_a$  is allowable axial stress under axial compression force alone,  $f_{bx}$  is computed bending stresses due to bending of the member about its major (x),  $F_{bx}$  is allowable compressive bending stresses about major,  $F_{ex}$  is Euler stresses,  $F_y$  is yield stress of steel,  $C_{mx}$  is a factor. It is calculated from  $C_{mx} = 0.6 - 0.4(M_1 / M_2)$  for braced frame member without transverse loading between the ends and  $C_{mx} = 1 + \psi (f_a / F_e)$  for braced frame member with transverse loading.

The effective length factors  $K$  for braced member are determined as below (Dumonteil 1992)

$$K = \frac{3G_A G_B + 1.4(G_A + G_B) + 0.64}{3G_A G_B + 2.0(G_A + G_B) + 1.28} \quad (5)$$

where  $G_A$  and  $G_B$  are the relative stiffness factors at  $A^{\text{th}}$  and  $B^{\text{th}}$  ends of columns.

-The displacement constraints are presented as

$$g_{jl}(x) = \frac{\delta_{jl}}{\delta_{ju}} - 1 \leq 0 \quad \begin{matrix} i = 1, \dots, m \\ l = 1, \dots, nl \end{matrix} \quad (6)$$

where  $\delta_{jl}$  is the displacement of  $j^{\text{th}}$  degree of freedom under load case  $l$ ,  $\delta_{ju}$  is the upper bound,  $m$  is the number of restricted displacements,  $nl$  is the total number of loading cases.

-The column-to-column geometric constraints are expressed as

$$g_n(x) = \frac{D_{un}}{D_{ln}} - 1 \leq 0 \quad n = 2, \dots, ns \quad (7)$$

$$g_{na}(x) = \frac{A_{un}}{A_{ln}} - 1 \leq 0 \quad na = 2, \dots, ns \quad (8)$$

where  $D_{un}$  is the depth of upper floor column,  $D_{ln}$  is the depth of lower floor column,  $A_{un}$  is the section area of upper floor column and  $A_{ln}$  is the section area of lower floor column.

-The beam-to-column geometric constraints are defined as

$$g_n(x) = \frac{b_{fbk,i}}{b_{bck,i}} - 1 \leq 0 \quad i = 2, \dots, n_{bf} \quad (9)$$

where  $n_{bf}$  is the number of joints where beams are connected to flange of column,  $b_{fbk,i}$  and  $b_{fck,i}$  are the flange widths of beam and column, respectively.

### 3. Teaching-Learning Based Optimization (TLBO)

Teaching-learning based optimization technique uses the teaching-learning procedures which are between teacher and students in a class. The teacher is considered as a person having the highest information and shares his/her information with the students in class for better results. This innovative optimization method was developed by Rao *et al.* (2011) and includes two basic phases such as teaching and learning. These two processes provide practical solutions for optimization in structural problems. TLBO method is successfully used for various structural problems in the literature studies (Togan 2012, Dede 2013, Dede and Ayvaz 2013).

All steps of TLBO method are expressed as,

- Class including several students shows population as below

$$class(population) = \begin{bmatrix} x_1^1 & x_2^1 & \dots & x_{n-1}^1 & x_n^1 \\ x_1^2 & x_2^2 & \dots & x_{n-1}^2 & x_n^2 \\ \dots & \dots & \dots & \dots & \dots \\ x_1^{S-1} & x_2^{S-1} & \dots & x_{n-1}^{S-1} & x_n^{S-1} \\ x_1^S & x_2^S & \dots & x_{n-1}^S & x_n^S \end{bmatrix} \begin{matrix} \rightarrow f(x^1) \\ \rightarrow f(x^2) \\ \rightarrow \dots \\ \rightarrow f(x^{S-1}) \\ \rightarrow f(x^S) \end{matrix} \quad (10)$$

where each row represents a student and introduce a design solution,  $S$  is population size (the number of students),  $n$  is the number of design variables,  $f(x^{1,2,\dots,S})$  is unconstrained objective function value of each student in the class. Initial class is randomly created.

-Teaching phase: The best solution in the class has minimum objective value and it is called as “teacher”. The other students in the class are modified by following formula

$$x^{new,i} = x^i + r(x_{teacher} - T_F x_{mean}) \quad (11)$$

where  $x^{new,i}$  is the new student,  $x^i$  is the current student,  $r$  is a random number in the range  $[0,1]$ ,  $T_F$ , a teaching factor, is either 1 or 2.  $x_{mean}$  is the mean of the class is determined as

$$x_{mean} = (mean(x_1) \dots mean(x_S)) \quad (12)$$

If the new student  $x^{new,i}$  gives a better solution ( $f(x^{new,i})$ ) than the current solution ( $f(x^i)$ ), the new student is replaced with the current student.

- Learning phase: The procedures in this phase are very similar to these mentioned in teaching phases. In this phase, a better solution is tried to obtain from the learning between student  $i$  and  $j$  by the modification formula Eq. (13)

$$\begin{aligned} \text{if } f(x^i) < f(x^j) & \Rightarrow x^{new,i} = x^i + r(x^j - x^i) \\ \text{if } f(x^i) > f(x^j) & \Rightarrow x^{new,i} = x^i + r(x^i - x^j) \end{aligned} \quad (13)$$

As mentioned in teaching phase, if the new student  $x^{new,i}$  presents a better solution ( $f(x^{new,i})$ ) than the current solution ( $f(x^i)$ ), the new student is replaced with the current student.

#### 4. Design examples

Two different braced planar frames taken from literature are carried out by using teaching-learning based optimization (TLBO). First example is a 162-member X-braced planar steel frame and second example is a 304-member K-braced planar steel frame. Optimum profiles are selected from a specified list including 128W taken from American Institute of Steel Construction. The stress constraints obeying AISC-ASD (1989), geometric size (column-column and column-beam), displacement and inter-storey drift constraints. The material properties used in both design examples are modulus of elasticity ( $E$ ) = 29000 ksi and yield stress ( $F_y$ ) = 36 ksi.

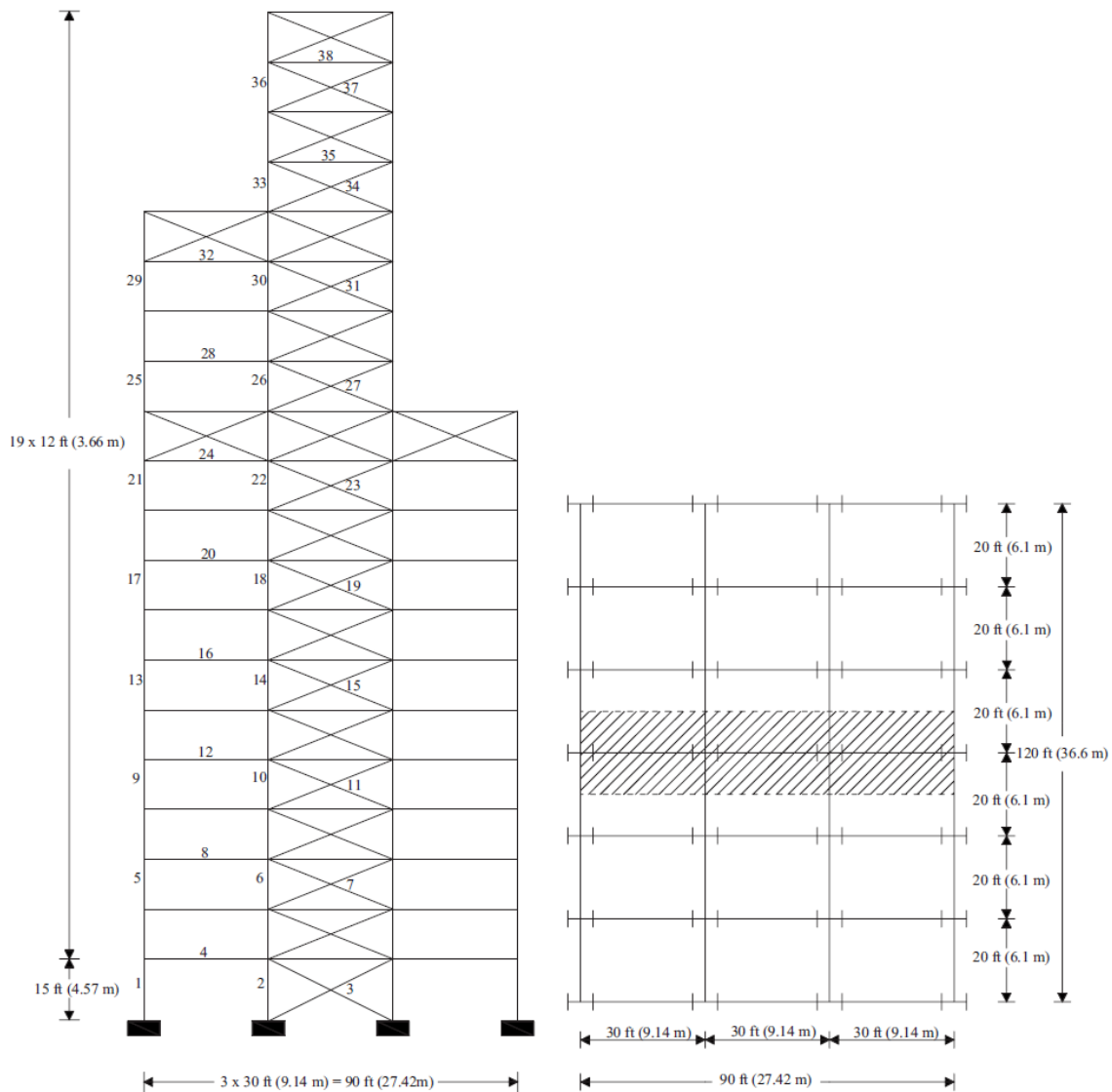


Fig. 1 162-member X-braced planar steel frame

#### 4.1 162-member X-braced planar steel frame

The plan and elevation views of a 162-member X-braced planar steel frame previously studied by Hasancebi *et al.* (2010a) are presented in Fig. 1. The members collected in 38 groups are also shown in Fig. 1.

The braced frame is designed for a single loading case including dead, live, snow and wind loads according to ASCE (2005). The vertical (dead, live and snow) loads are defined as 2.88 kN/m<sup>2</sup>, 2.39 kN/m<sup>2</sup> and 1.20 kN/m<sup>2</sup>, respectively. According to the vertical loads, uniformly distributed gravity loads calculated as 18.87 kN/m and 25.29 kN/m are applied on top story beams and other story beams. The wind speed is 105 mph and the wind (lateral) loads are applied on each floor level as presented in Table 1. The maximum lateral displacement and inter-storey drift are restricted to height/400. The design results are compared with literature results in Table 2. Fig. 2 shows the variation of minimum steel weight with iteration steps (design history).

In literature study (Hasancebi *et al.* 2010a), the braced planar frame was designed by using different meta-heuristic techniques (HS, GA, ACO) according to stress, geometric size (column-beam), top and inter story drift constraints. In the present study, column-column geometric size constraints (Eqs. (7) and (8)) are also imposed on the design problem. It is observed from Table 2 that the design cross sections obtained in the present study are similar to literature results. However, minimum steel weight 1082.90 kN is about 12%, 7% and 0.2% lighter than the minimum weight results of the other basic methods (HS, GA and ACO) used in reference study

Table 1 Wind loads on 162-member X-braced planar steel frame (kN)

Floor number	Windward	Leeward
1	10.01	13.81
2	11.45	13.81
3	12.85	13.81
4	13.95	13.81
5	14.87	13.81
6	15.67	13.81
7	16.37	13.81
8	17.01	13.81
9	17.59	13.81
10	18.13	13.81
11	18.63	13.81
12	19.10	13.81
13	19.54	13.81
14	19.96	13.81
15	20.36	13.81
16	20.74	13.81
17	21.10	13.81
18	21.45	13.81
19	21.78	13.81
20	11.05	6.91

(Hasancebi *et al.* 2010a), respectively. It shows the robustness and applicability of TLBO method for structural designs. Moreover, Hasancebi *et al.* (2010a) used maximum number of design samples = 50000 in their study to get optimum solutions. On the other hand, this number for this structural problem in this study is 32000.

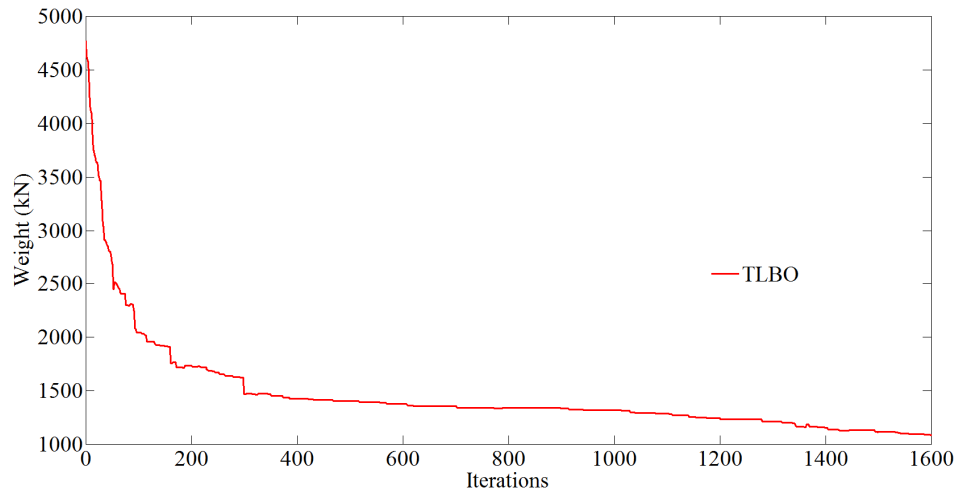


Fig. 2 Design history of the braced frame

Table 2 Design sections of 162-member x-braced planar steel frame

Size variables	Reference study (Hasancebi <i>et al.</i> 2010a)			This study
	Standard harmony search (HS)	Simple genetic algorithm (GA)	Ant colony optimization (CO)	Teaching-learning based optimization (TLBO)
1	W14×109	W12×96	W16×100	W44×224
2	W30×235	W36×230	W30×235	W40×298
3	W14×74	W10×49	W10×54	W12×14
4	W24×68	W24×68	W24×68	W14×68
5	W12×96	W12×87	W27×84	W44×224
6	W18×192	W30×173	W27×178	W36×194
7	W8×40	W10×33	W8×35	W12×14
8	W24×76	W44×198	W24×68	W18×86
9	W24×104	W10×88	W16×77	W40×149
10	W21×182	W12×152	W27×146	W30×148
11	W8×40	W8×35	W8×40	W14×30
12	W24×76	W24×68	W21×73	W14×74
13	W27×114	W16×67	W24×68	W40×149
14	W27×146	W14×109	W14×109	W30×108
15	W10×39	W8×31	W8×31	W12×14
16	W24×68	W24×68	W21×73	W18×86

Table 2 Continued

Size variables	Reference study (Hasancebi <i>et al.</i> 2010a)			This study
	Standard harmony search (HS)	Simple genetic algorithm (GA)	Ant colony optimization (CO)	Teaching-learning based optimization (TLBO)
17	W14×99	W12×65	W10×88	W30×108
18	W33×152	W18×86	W14×90	W18×97
19	W10×45	W10×33	W8×31	W12×14
20	W21×73	W24×68	W24×76	W16×89
21	W12×79	W27×84	W24×104	W30×108
22	W18×86	W14×74	W14×74	W18×97
23	W10×49	W10×49	W10×49	W12×22
24	W27×102	W24×103	W24×94	W14×74
25	W10×77	W12×53	W14×74	W21×101
26	W21×166	W14×68	W14×68	W18×71
27	W8×35	W10×33	W8×35	W8×21
28	W24×68	W24×68	W24×68	W12×58
29	W14×61	W12×53	W14×74	W21×68
30	W12×79	W12×53	W10×49	W16×50
31	W8×40	W8×31	W8×31	W8×15
32	W24×68	W21×73	W24×68	W18×71
33	W16×40	W14×43	W12×58	W10×49
34	W10×33	W10×33	W8×31	W12×14
35	W21×44	W16×45	W16×45	W14×61
36	W21×101	W12×53	W12×79	W10×33
37	W10×33	W8×31	W10×33	W10×15
38	W24×68	W21×73	W21×73	W18×60
Weight (kN)	1206.36	1155.06	1084.49	1082.90

Table 3 Gravity loading on 304-member K-braced planar steel frame (kN)

Beams	Outer span beams	Inner span beams
Roof beams (dead+snow loads)	14.77	17.42
Floor beams (dead+live loads)	21.49	25.29

Table 4 Minimum steel weights of 304-member K-braced planar steel frame

Minimum weight (kN)	Reference study (Hasancebi <i>et al.</i> 2010b)		This study
	Tabu search (TS)	Harmony search (HS)	Teaching-learning based optimization (TLBO)
	1052.76	1197.24	1093.91



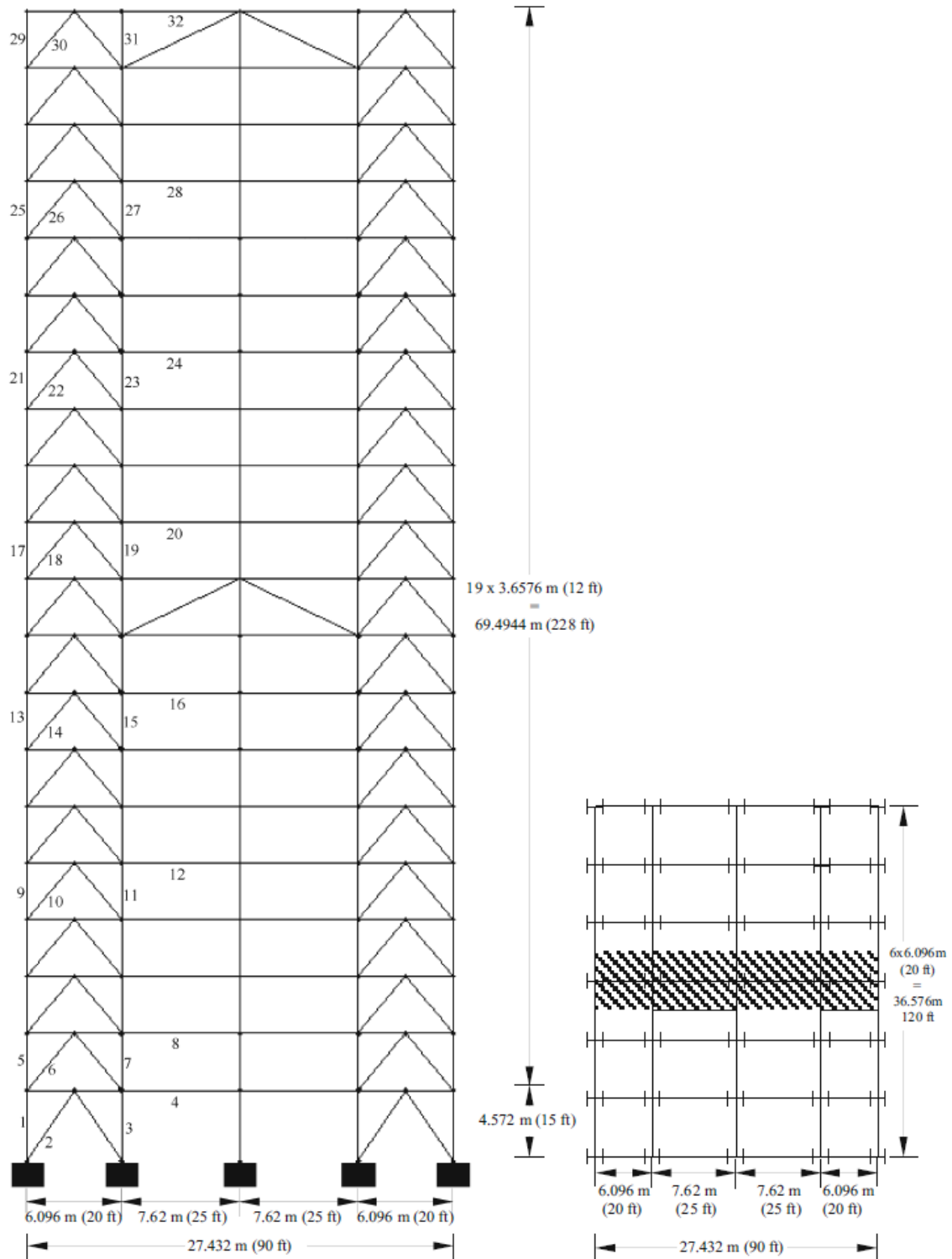


Fig. 3 304-member K-braced planar steel frame

#### 4.2 304-member K-braced planar steel frame

Fig. 3 shows the plan and elevation views of a 304-member K-braced planar steel frame previously studied by Hasancebi *et al.* (2010b). All members are collected in 32 groups. 304-member K-braced planar steel frame is solved for a single loading condition including dead, live, snow and wind loads. The design loads according to ASCE (2005) are dead load (D) = 2.88 kN/m<sup>2</sup>, live load (L) = 2.39 kN/m<sup>2</sup>, snow load (S) = 1.20 kN/m<sup>2</sup> and wind speed =105 mph. The uniformly

Table 5 The design cross sections of 304 K-braced planar frame

Size variables	Teaching-learning based optimization (TLBO)	Size variables	Teaching-learning based optimization (TLBO)
1	W30×148	17	W8×40
2	W12×14	18	W12×16
3	W44×224	19	W21×83
4	W14×53	20	W18×71
5	W18×130	21	W8×31
6	W12×14	22	W12×14
7	W44×224	23	W21×62
8	W12×53	24	W18×71
9	W14×90	25	W8×31
10	W12×14	26	W12×14
11	W40×149	27	W12×40
12	W12×53	28	W16×57
13	W10×60	29	W8×31
14	W12×16	30	W16×26
15	W24×117	31	W8×31
16	W14×61	32	W14×43

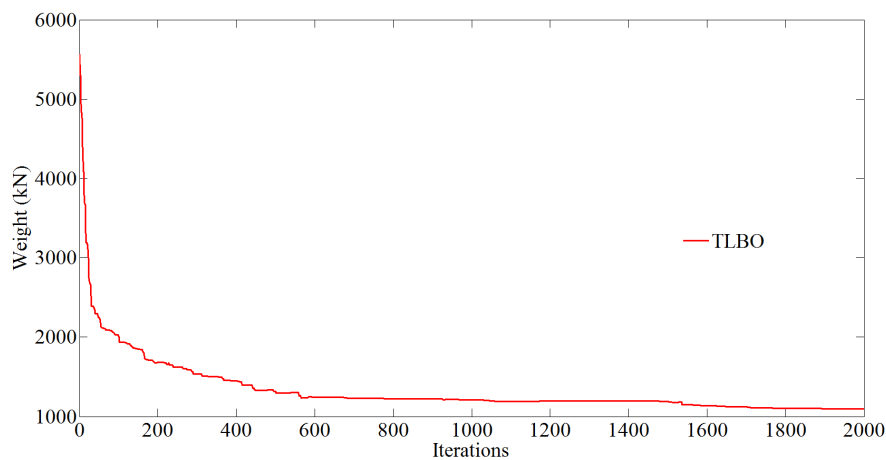


Fig. 4 Design history of the braced frame

distributed gravity loads calculated according to dead, live and snow loads are presented in Table 3. The wind (lateral) loads are applied on each floor level as given in previous design problem (Table 1). The top and inter story drifts are restricted to height/400. Minimum design weight of this study is compared with the ones of reference study in Table 4. The design cross sections of TLBO are presented in Table 5 and its design history is seen in Fig. 4.

It is obviously seen in Table 4 that minimum steel weight obtained by TLBO is 1093.91 kN which is very close to the minimum weights of reference study. This value is nearly 9% lighter than the minimum weight of HS although it is 3.7% heavier than the result of TS. However, in the present study, column-column geometric size constraints (Eqs. (7) and (8)) in addition to the other constraints (stress, displacement and column-beam geometric size constraints) used in the reference study (Hasancebi *et al.* 2010b) are imposed on the braced frame. As regards Fig. 3, TLBO method reduces the design weight of the braced frame successfully. Also, Hasancebi *et al.* (2010b) used 50000 structural analyses in their solutions to obtain optimum profiles. On the other hand, this number for this structural problem in this study is 40000.

## 5. Conclusions

Teaching-learning-based optimization (TLBO) is an innovative algorithm technique developed in recent years (Rao *et al.* 2011). In the present study, its applicability and robustness on optimum structural designs of braced (non-swaying) planar steel frames is investigated by using MATLAB-SAP2000 OAPI. For this purpose, a program was developed in MATLAB. Two different frame problems such as a 162-member X-braced planar steel frame and a 304-member K-braced planar steel frame are designed by selecting from a specified list including 128W profiles taken from AISC. In the first example, the minimum steel weight (1082.90 kN) of the design solution is about 12%, 7% and 0.2% lighter than the minimum weights of the other basic methods (HS, GA and ACO) used in reference study (Hasancebi *et al.* 2010a). In the second example, the minimum weight of the brame is calculated as 1093.91 kN which is about 9% lighter than the minimum weight of HS although it is 3.7% heavier than the one of TS. The results obtained from analyses prove that teaching-learning based optimization presents robust and applicable optimum solutions in multi-element structural problems.

## References

- AISC – ASD (1989), Manual of Steel Construction: Allowable Stress Design, American Institute of Steel Construction, Chicago, IL, USA.
- Artar, M. (2016), “Optimum design of steel space frames under earthquake effect using harmony search”, *Struct. Eng. Mech., Int. J.*, **58**(3), 597-612.
- Artar, M. and Daloglu, A.T. (2015), “Optimum design of steel space frames with composite beams using genetic algorithm”, *Steel Compos. Struct., Int. J.*, **19**(2), 503-519.
- ASCE (2005), Minimum design loads for building and other structures, ASCE7-05, New York, NY, USA.
- Aydogdu, I. and Saka, M.P. (2012), “Ant colony optimization of irregular steel frames including elemental warping effect”, *Adv. Eng. Softw.*, **44**(1), 150-169.
- Azad, S.K., Hasancebi, O. and Saka, M.P. (2014), “Guided stochastic search technique for discrete sizing optimization of steel trusses: A design-driven heuristic approach”, *Comput. Struct.*, **134**, 62-74.
- Daloglu, A. and Armutcu, M. (1998), “Optimum design of plane steel frames using genetic algorithm”, *Teknik Dergi*, **116**, 1601-1615.

- Dede, T. (2013), "Optimum design of grillage structures to LRFD-AISC with teaching-learning based optimization", *Struct. Multidisc. Optim.*, **48**(5), 955-964.
- Dede, T. and Ayvaz, Y. (2013), "Structural optimization with teaching-learning-based optimization algorithm", *Struct. Eng. Mech., Int. J.*, **47**(4), 495-511.
- Degertekin, S.O. (2007), "A comparison of simulated annealing and genetic algorithm for optimum design of nonlinear steel space frames", *Struct. Multidisc. Optim.*, **34**(4), 347-359.
- Degertekin, S.O. (2012), "Optimum design of geometrically non-linear steel frames using artificial bee colony algorithm", *Steel Compos. Struct., Int. J.*, **12**(6), 505-522.
- Degertekin, S.O. and Hayalioglu, M.S. (2009), "Optimum design of steel space frames: Tabu search vs. simulated annealing and genetic algorithms", *Int. J. Eng. Appl. Sci. (IJEAS)*, **1**(2), 34-45.
- Degertekin, S.O. and Hayalioglu, M.S. (2010), "Harmony search algorithm for minimum cost design of steel frames with semi-rigid connections and column bases", *Struct. Multidisc. Optim.*, **42**(5), 755-768.
- Dumonteil, P. (1992), "Simple equations for effective length factors", *Eng. J. AISC*, **29**(3), 111-115.
- Hasançebi, O., Erdal, F. and Saka, M.P. (2010a), "Adaptive harmony search method for structural optimization", *J. Struct. Eng. ASCE*, **136**(4), 419-431.
- Hasançebi, O., Çarbaş, S. and Saka, M.P. (2010b), "Improving the performance of simulated annealing in structural optimization", *Struct. Multidisc. Optim.*, **41**, 189-203.
- Hasançebi, O., Bahçecioğlu, T., Kuş, Ö. and Saka, M.P. (2011), "Optimum design of high-rise steel buildings using an evolution strategy integrated parallel algorithm", *Comput. Struct.*, **89**, 2037-2051.
- Hasançebi, O., Teke, T. and Pekcan, O. (2013), "A bat-inspired algorithm for structural optimization", *Comput. Struct.*, **128**, 77-90.
- Lee, K.S. and Geem, Z.W. (2004), "A new structural optimization method based on the harmony search algorithm", *Comput. Struct.*, **82**, 781-798.
- MATLAB (2009), The Language of Technical Computing, The Mathworks Inc., Natick, MA, USA.
- Rafiee, A., Talatahari, S. and Hadidi, A. (2013), "Optimum design of steel frames with semi-rigid connections using Big Bang-Big Crunch method", *Steel Compos. Struct., Int. J.*, **14**(5), 431-451.
- Rao, R.V., Savsani, V.J. and Vakharia, D.P. (2011), "Teaching-learning-based optimization: A novel method for constrained mechanical design optimization problems", *Computer-Aided Design*, **43**(3), 303-315.
- Saka, M.P. (2009), "Optimum design of steel sway frames to BS5950 using harmony search algorithm", *J. Constr. Steel Res.*, **65**(1), 36-43.
- SAP2000 (2008), Integrated Finite Elements Analysis and Design of Structures, Computers and Structures, Inc, Berkeley, CA, USA.
- Togan, V. (2012), "Design of planar steel frames using teaching-learning based optimization", *Eng. Struct.*, **34**, 225-232.
- Togan, V. and Daloglu, A.T. (2006), "Optimization of 3d trusses with adaptive approach in genetic algorithms", *Eng. Struct.*, **28**(7), 1019-1027.