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The effect of magnetic field on a thermoelastic fiber-reinforced material under GN-III theory

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Abstract. In this work, the two-dimensional generalized magneto-thermoelastic problem of a fiber-reinforced anisotropic material is investigated under Green and Naghdi theory of type III. The solution will be obtained for a certain model when the half space subjected to ramp-type heating and traction free surface. Laplace and exponential Fourier transform techniques are used to obtain the analytical solutions in the transformed domain by the eigenvalue approach. The inverses of Fourier transforms are obtained analytically. The results have been verified numerically and are represented graphically. Comparisons are made with the results predicted by the presence and absence of reinforcement and magnetic field.

Keywords: analytical solution; magnetothermoelasticity; fiber-reinforced material; eigenvalue approach; Green and Naghdi theory

1. Introduction

The theory of generalized thermoelasticity has drawn attention of researchers due to its applications in various diverse fields such as nuclear reactor's design, earthquake engineering, high energy particle accelerators, etc. The first of such modeling is the extended thermoelasticity theory (L-S) of Lord and Shulman (1967), who established the generalized of thermoelasticity with one relaxation time by postulating a new law of heat conduction to replace the classical Fourier law. Green and Lindsay (1972) proposed the temperature rate dependent thermoelasticity (G-L) theory with two relaxation time. The theory was extended for anisotropic body by Dhaliwal and Sherief (1980). Green and Naghdi (1991, 1993) proposed a new generalized thermoelasticity theory by including the thermal-displacement gradient among the independent constitutive variables.

Fiber-reinforced thermoelastic materials are the composite materials which shows highly anisotropic elastic behavior such that the elastic parameters have an extension in the fiber direction

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which are of the order of 50 or more time larger than their parameter in the transverse direction. These composite materials are light weight, having high resistance, strength, stiffness at a high temperature. Due to theoretical and practical importance, many problems on waves and vibrations in fiber-thermoelastic materials have been investigated. (Singh 2002) showed that, for wave propagation in fiber-reinforced anisotropic media, this decoupling cannot be achieved by the introduction of the displacement potentials. (Sengupta and Nath 2001) discussed the problem of surface waves in fiber-reinforced anisotropic elastic media. Hashin and Rosen (1964) gave the elastic moduli for fiber-reinforced materials.

(Abbas 2012, 2013, 2015a, Abbas and Othman 2012, Zenkour and Abbas 2014b, Hussein *et al*. 2015, Said and Othman 2016) investigated different problems for the thermoelastic interaction in a fiber-reinforced materials using finite element method. Also, different researches have studied the source problem in fiber-reinforced thermoelastic media by using the different mathematical techniques as in (Kumar and Gupta 2010, Abbas *et al*. 2011, Othman and Said 2012, 2013, 2014, Gupta and Gupta 2013, Lotfy and Hassan 2013, Othman and Lotfy 2013, Sarkar and Lahiri 2013). Note that in most of the earlier studies mechanical or thermal loading on the boundary surface was considered to be in the form of a shock. (Abbas 2015b) studied the fractional order generalized magneto-thermoelastic medium due to moving heat source using eigenvalue approach. (Zenkour and Abbas 2014a) investigated finite element analyses in magneto-thermoelastic interaction in an infinite FG cylinder. (Othman *et al*. 2014) used the normal mode method to investigate the initial stress and gravitational effect on generalized magneto-thermo- microstretch elastic solid for the different theories. (Lotfy and Othman 2014) studied the effect of magnetic field for a mode-I crack on a two-dimensional problem fiber-reinforced in generalized thermoelasticity.

In this paper, the analytical expressions for displacement components, temperature and the components of stress are obtained in the physical domain by using the eigenvalue approach. The non-dimensional equations are handled by employing an analytical-numerical technique based on Fourier and Laplace transform and eigenvalues approach. The results have been verified numerically and are represented graphically.

2. Basic equation and formulation of the problem

We consider the problem of a thermoelastic half-space ($x \geq 0$). A magnetic field with constant intensity $\mathbf{H} = (0, 0, H_0)$, acting parallel to the boundary plane (taken as the direction of the z-axis). The surface of the half-space is subjected to a thermal shock which is a function of y and t. Thus, all the quantities considered will be functions of the time variable t, and of the coordinates x and y. We begin our consideration with linearized equations of electro-dynamics of slowly moving medium

$$\mathbf{J} = \mathrm{curl}\mathbf{h} - \varepsilon_0 \dot{\mathbf{E}}\mathbf{m}, \tag{1}$$

$$\mathrm{curl}\mathbf{E} = -\mu_0 \dot{\mathbf{h}}, \tag{2}$$

$$\nabla.\mathbf{h} = 0. \tag{3}$$

$$\mathbf{E} = -\mu_0 (\dot{\mathbf{u}} \times \mathbf{H}), \tag{4}$$

These equations are supplemented by the displacement equations of the theory of elasticity, taking into consideration the Lorentz force F_i to give

$$F_i = \mu_0 (J? \ )_i, \tag{5}$$

$$\sigma_{ij,j} + F_i = \rho \ddot{u}_i, \tag{6}$$

where **h** is the induced magnetic field vector; μ_0 is magnetic permeability; **E** is the induced electric field vector; ε_0 is the electric permeability; **J** is the current density vector and $\dot{\mathbf{u}}$ is the particle velocity of the medium. Following (Green and Naghdi 1991, 1993) and (Singh 2006), the equation of heat conduction for fiber-reinforced material in the absence of heat sources are considered as

$$K^* T_{,ij} + K_{ij} T_{,ij} = \rho c_e \ddot{T} + T_o \beta_{ij} \ddot{u}_{i,j}, \qquad i, j = 1, 2, 3. \tag{7}$$

The stress-strain relation for a fiber-reinforced linearly thermoelastic medium with respect to the reinforcement direction medium unit vector **a** will be as follows

$$\begin{aligned}\sigma_{ij} &= \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha(a_k a_m e_{km}\delta_{ij} + a_i a_j e_{kk}) + 2(\mu_L - \mu_T)(a_i a_k e_{kj} + a_j a_k e_{ki}) \\ &\quad + \beta a_k a_m e_{km} a_i a_j - \beta_{ij}(T - T_o)\delta_{ij}, \qquad i, j, k, m = 1, 2, 3,\end{aligned} \tag{8}$$

where u_i the displacement vector components; T the temperature change of a material particle; ρ is the mass density; e_{ij} the strain tensor; σ_{ij} the stress tensor; β_{ij} the thermal elastic coupling tensor; T_o the reference uniform temperature of the body; λ, μ_T are elastic parameters; c_e the specific heat at constant strain; K^* the material constant, characteristic of the theory; α, β, $(\mu_L - \mu_T)$ are the parameters of reinforced elastic; δ_{ij} is the Kronecker delta and K_{ij} the thermal conductivity. The comma notation is used for spatial derivatives and superimposed dot represents time differentiation and $\mathbf{a} \equiv (a_1, a_2, a_3)$, $a_1^2 + a_2^2 + a_3^2 = 1$. All the considered functions will be depend on the time t and the coordinates x and y as in Fig. 1. Thus, the displacement vector u_i will have the components

$$u = u_1 = u(x, y, t), \qquad v = u_2 = v(x, y, t), \qquad w = u_3 = 0. \tag{9}$$

We choose the fiber-direction as $\mathbf{a} \equiv (1, 0, 0)$ so that the preferred direction is the x-axes, Eqs. (5)-(8) simplifies, as given below

$$\sigma_{xx} = b_{11}\frac{\partial u}{\partial x} + (b_{12} - b_{13})\frac{\partial v}{\partial y} - \beta_{11}(T - T_o), \tag{10}$$

$$\sigma_{yy} = b_{22}\frac{\partial v}{\partial y} + (b_{12} - b_{13})\frac{\partial u}{\partial x} - \beta_{22}(T - T_o), \tag{11}$$

$$\sigma_{xy} = b_{13}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right), \tag{12}$$

$$F_x = \mu_0 H_0^2 \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} - \varepsilon_0 \mu_0 \frac{\partial^2 u}{\partial t^2}\right), \tag{13}$$

$$F_y = \mu_0 H_0^2 \left(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y^2} - \varepsilon_0 \mu_0 \frac{\partial^2 u}{\partial t^2} \right), \tag{14}$$

$$(b_{11} + \rho c_H^2) \frac{\partial^2 u}{\partial x^2} + (b_{12} + \rho c_H^2) \frac{\partial^2 v}{\partial x \partial y} + b_{13} \frac{\partial^2 u}{\partial y^2} - \beta_{11} \frac{\partial T}{\partial x} = \rho \left(1 + \frac{c_H^2}{c^2} \right) \frac{\partial^2 u}{\partial t^2}, \tag{15}$$

$$(b_{22} + \rho c_H^2) \frac{\partial^2 v}{\partial y^2} + (b_{12} + \rho c_H^2) \frac{\partial^2 u}{\partial x \partial y} + b_{13} \frac{\partial^2 v}{\partial x^2} - \beta_{22} \frac{\partial T}{\partial y} = \rho \left(1 + \frac{c_H^2}{c^2} \right) \frac{\partial^2 v}{\partial t^2}, \tag{16}$$

$$\left(K^* + K_{11} \frac{\partial}{\partial t} \right) \frac{\partial^2 T}{\partial x^2} + \left(K^* + K_{22} \frac{\partial}{\partial t} \right) \frac{\partial^2 T}{\partial y^2} = \frac{\partial^2}{\partial t^2} \left(\rho c_e T + T_0 \beta_{11} \frac{\partial u}{\partial x} + T_0 \beta_{22} \frac{\partial v}{\partial y} \right), \tag{17}$$

where $b_{11} = \lambda + 2\ (\alpha + \mu_T) + 4(\mu_L - \mu_T) + \beta$, $b_{12} = \alpha + \lambda + \mu_L$, $b_{13} = \mu_L$, $b_{22} = \lambda + 2\mu_T$, $c_H^2 = \frac{\mu_0 H_0^2}{\rho}$, $c^2 = \frac{1}{\varepsilon_0 \mu_0}$, $\beta_{11} = (2\lambda + 3\alpha + 4\mu_L - 2\mu_L + \beta)\ \alpha_{11} + (\lambda + \alpha)\ \alpha_{22}$, $\beta_{22} = (2\lambda + \alpha)\ \alpha_{11} + (\lambda + 2\mu_T)\ \alpha_{22}$ and α_{11}, α_{22} are the linear thermal expansion coefficients. It is convenient to change the preceding equations into the dimensionless forms. To do this, the dimensionless parameters are introduced as

$$(x', y', u', v') = \frac{c_1}{\chi}(x, y, u, v), \quad T' = \frac{T - T_0}{T_0}, \quad t' = \frac{c_1^2 t}{\chi}, \quad \sigma'_{ij} = \frac{\sigma_{ij}}{b_{11}}, \quad h' = \frac{h}{H_0} \tag{18}$$

where $c_1^2 = \frac{b_{11}}{\rho}$ and $\chi = \frac{K_{11}}{\rho c_e}$.

Upon introducing in Eqs. (15)-(17), and after suppressing the primes, we obtain

$$\gamma_1 \frac{\partial^2 u}{\partial x^2} + \gamma_2 \frac{\partial^2 v}{\partial x \partial y} + \gamma_3 \frac{\partial^2 u}{\partial y^2} - \gamma_4 \frac{\partial T}{\partial x} = \gamma_5 \frac{\partial^2 u}{\partial t^2}, \tag{19}$$

$$\gamma_6 \frac{\partial^2 v}{\partial y^2} + \gamma_2 \frac{\partial^2 u}{\partial x \partial y} + \gamma_3 \frac{\partial^2 v}{\partial x^2} - \gamma_7 \frac{\partial T}{\partial y} = \gamma_5 \frac{\partial^2 v}{\partial t^2}, \tag{20}$$

$$\gamma_8 \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \frac{\partial}{\partial t} \left(\frac{\partial^2 T}{\partial x^2} + \gamma_9 \frac{\partial^2 T}{\partial y^2} \right) = \frac{\partial^2}{\partial t^2} \left(T + \gamma_{10} \frac{\partial u}{\partial x} + \gamma_{11} \frac{\partial v}{\partial y} \right), \tag{21}$$

with

$$\sigma_{xx} = \frac{\partial u}{\partial x} + \gamma_{12} \frac{\partial v}{\partial y} - \gamma_4 T, \tag{22}$$

$$\sigma_{yy} = \gamma_{12}\frac{\partial u}{\partial x} + \gamma_{13}\frac{\partial v}{\partial y} - \gamma_7 T,$$
$$\sigma_{xy} = \gamma_3\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \tag{22}$$

where $\gamma_1 = 1 + R_H$, $\gamma_2 = \frac{b_{12}}{b_{11}} + R_H$, $\gamma_3 = \frac{b_{13}}{b_{11}}$, $\gamma_4 = \frac{\beta_{11}T_o}{b_{11}}$, $\gamma_5 = 1 + \frac{c_H^2}{c^2}$, $\gamma_6 = \frac{b_{22}}{b_{11}} + R_H$, $\gamma_7 = \frac{\beta_{22}T_o}{b_{11}}$, $\gamma_8 = \frac{K^*}{\rho c_e c_1^2}$, $\gamma_9 = \frac{K_{22}}{K_{11}}$, $\gamma_{10} = \frac{\beta_{11}}{\rho c_e}$ and $\gamma_{11} = \frac{\beta_{22}}{\rho c_e}$, $\gamma_{12} = \frac{b_{12} - b_{13}}{b_{11}}$, $\gamma_{13} = \frac{b_{22}}{b_{11}}$.

3. Initial and boundary conditions

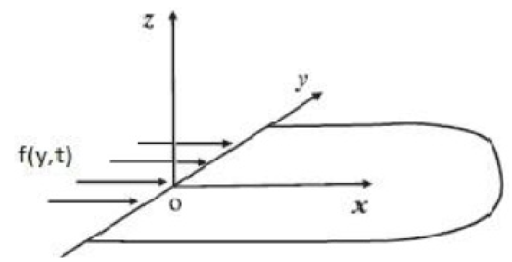
In order to solve the problem, both the initial and boundary conditions should be considered. The initial conditions of the problem are taken as

$$u(x,y,0) = \frac{\partial u(x,y,0)}{\partial t}, \quad v(x,y,0) = \frac{\partial v(x,y,0)}{\partial t} = 0, \quad T(x,y,0) = \frac{\partial T(x,y,0)}{\partial t} = 0. \tag{23}$$

The boundary conditions of the problem are taken as

$$\sigma_{xx}(0,y,t) = 0, \quad \sigma_{xy}(0,y,t) = 0, \quad T(0,y,t) = f(y,t), \tag{24}$$

with $f(y,t) = g(t)\ H(L - |y|)$ and $g(t) = T_1\begin{Bmatrix} 0 & t \le 0 \\ \frac{t}{t_o} & 0 < t < t_o \\ 1 & t_o \le t \end{Bmatrix}$, where H is the Heaviside unit step function and T_1 is a constant and t_0 is a constant and is called the ramping time parameter. This means that heat is applied on the surface of the half-space on a narrow band of width L surrounding the y-axis to keep it at temperature T_o, while the rest of the surface is kept at zero temperature.



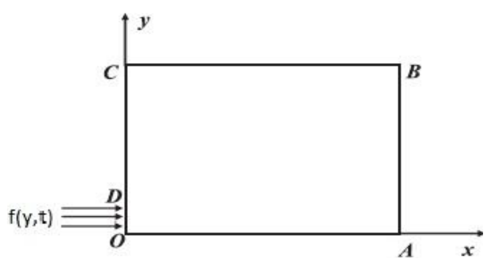


Fig. 1 Schematic of the half-space

4. Formulation in the laplace transform domain

We will apply Laplace transform defined as

$$\bar{f}(s) = L[f(t)] = \int_0^\infty f(t)e^{-st}\,dt, \tag{25}$$

Hence, the above equations will take the forms

$$\gamma_1 \frac{\partial^2 \bar{u}}{\partial x^2} + \gamma_2 \frac{\partial^2 \bar{v}}{\partial x \partial y} + \gamma_3 \frac{\partial^2 \bar{u}}{\partial y^2} - \gamma_4 \frac{\partial \bar{T}}{\partial x} = \gamma_5 s^2 \bar{u}, \tag{26}$$

$$\gamma_6 \frac{\partial^2 \bar{v}}{\partial y^2} + \gamma_2 \frac{\partial^2 \bar{u}}{\partial x \partial y} + \gamma_3 \frac{\partial^2 \bar{v}}{\partial x^2} - \gamma_7 \frac{\partial \bar{T}}{\partial y} = \gamma_5 s^2 \bar{v}, \tag{27}$$

$$\gamma_8\left(\frac{\partial^2 \bar{T}}{\partial x^2} + \frac{\partial^2 \bar{T}}{\partial y^2}\right) + s\left(\frac{\partial^2 \bar{T}}{\partial x^2} + \gamma_9 \frac{\partial^2 \bar{T}}{\partial y^2}\right) = s^2\left(\bar{T} + \gamma_{10}\frac{\partial \bar{u}}{\partial x} + \gamma_{11}\frac{\partial \bar{v}}{\partial y}\right). \tag{28}$$

$$\begin{aligned}
\bar{\sigma}_{xx} &= \frac{\partial \bar{u}}{\partial x} + \gamma_{12}\frac{\partial \bar{v}}{\partial y} - \gamma_4 \bar{T}, \\
\bar{\sigma}_{yy} &= \gamma_{12}\frac{\partial \bar{u}}{\partial x} + \gamma_{13}\frac{\partial \bar{v}}{\partial y} - \gamma_7 \bar{T}. \\
\bar{\sigma}_{xy} &= \gamma_3\left(\frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x}\right),
\end{aligned} \tag{29}$$

5. Formulation in the fourier transform domain

We will apply Fourier transform defined as

$$\bar{f}^*(q) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \bar{f}(t)e^{-iqy}\,dy, \tag{30}$$

where

$$\bar{f}(y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \bar{f}(q)e^{iqy}\,dq. \tag{31}$$

Hence, we obtain the following system of differential equations

$$\frac{d^2 \bar{u}^*}{\partial x^2} = m_{41}\bar{u}^* + m_{45}\frac{d\bar{v}^*}{dx} + m_{46}\frac{d\bar{T}^*}{dx}, \tag{32}$$

$$\frac{d^2\overline{v}^*}{\partial x^2} = m_{52}\overline{v}^* + m_{53}\overline{T}^* + m_{54}\frac{d\overline{u}^*}{dx}, \tag{33}$$

$$\frac{d^2\overline{T}^*}{\partial x^2} = m_{62}\overline{v}^* + m_{63}\overline{T}^* + m_{64}\frac{d\overline{u}^*}{dx},, \tag{34}$$

$$\begin{aligned}
\overline{\sigma}^*_{xx} &= \frac{d\overline{u}^*}{dx} + iq\gamma_{12}\overline{v}^* - \gamma_4\overline{T}^*,\\
\overline{\sigma}^*_{yy} &= \gamma_{12}\frac{d\overline{u}^*}{dx} + iq\gamma_{13}\overline{v}^* - \gamma_7\overline{T}^*,\\
\overline{\sigma}^*_{xy} &= \gamma_3\left(iq\overline{u}^* + \frac{\partial\overline{v}^*}{\partial x}\right),
\end{aligned} \tag{35}$$

with

$$\overline{\sigma}^*_{xx} = (0,q,s) = 0, \qquad \overline{\sigma}^*_{xy}(0,q,s) = 0, \qquad \overline{T}^*(0,q,s) = T_1\sqrt{\frac{2}{\pi}\frac{\sin(qL)}{sq}}, \tag{36}$$

where $m_{41} = \frac{1}{\gamma_1}(s^2\gamma_5 + q^2\gamma_3)$, $m_{45} = \frac{iq\gamma_2}{\gamma_1}$, $m_{46} = \frac{\gamma_4}{\gamma_1}$, $m_{52} = \frac{1}{\gamma_3}(s^2\gamma_5 + q^2\gamma_6)$, $m_{53} = \frac{iq\gamma_7}{\gamma_3}$, $m_{54} = \frac{iq\gamma_2}{\gamma_3}$, $m_{62} = \frac{s^2 + q^2(\gamma_8 + s\gamma_9)}{s + \gamma_8}$, $m_{63} = \frac{iqs^2\gamma_{11}}{s + \gamma_8}$ and $m_{64} = \frac{s^2\gamma_{10}}{s + \gamma_8}$.

Let us now proceed to solve the coupled differential Eqs. (32), (33) and (34) by the eigenvalue approach proposed by (Das *et al*. 1997, Abbas 2014). Eqs. (32-34) can be written in a vector-matrix differential equation as follows

$$\frac{d\vec{\Psi}}{dx} = M\vec{\Psi}, \tag{37}$$

where $\vec{\Psi} = \left[\overline{u}^* \ \overline{v}^* \ \overline{T}^* \ \frac{d\overline{u}^*}{dx} \ \frac{d\overline{v}^*}{dx} \ \frac{d\overline{T}^*}{dx}\right]^T$ and $M = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\\ m_{41} & 0 & 0 & 0 & m_{45} & m_{46}\\ 0 & m_{52} & m_{53} & m_{54} & 0 & 0\\ 0 & m_{62} & m_{63} & m_{64} & 0 & 0 \end{bmatrix}$.

The characteristic equation of the matrix M takes the form

$$\lambda^6 - F_1\lambda^4 + F_2\lambda^2 + F_3 = 0, \tag{38}$$

where

$$F_1 = m_{41} + m_{52} + m_{45}m_{54} + m_{63} + m_{46}m_{64},$$
$$F_2 = m_{41}m_{52} - m_{53}m_{62} - m_{46}m_{54}m_{62} + m_{41}m_{63} + m_{52}m_{63} + m_{45}m_{54}m_{63} + m_{46}m_{52}m_{64} - m_{45}m_{53}m_{64},$$
$$F_3 = m_{41}m_{53}m_{62} - m_{41}m_{52}m_{63}.$$

The roots of the characteristic Eq. (38) which are also the eigenvalues of matrix M are of the form $\pm\lambda_1$, $\pm\lambda_2$, $\pm\lambda_3$. The eigenvector $\vec{Y} = [Y_1,\ Y_2,\ Y_3,\ Y_4,\ Y_5,\ Y_6]^T$, corresponding to eigenvalue λ can be calculated as

$$\begin{gathered} Y_1 = -m_{45}m_{53}\lambda + m_{46}\lambda(m_{52} - \lambda^2), \\ Y_2 = -m_{46}m_{54}\lambda^2 + m_{53}\lambda(m_{41} - \lambda^2), \\ Y_3 = -m_{41}m_{52} + (m_{41} + m_{52} + m_{45}a_{54})\lambda^2 - \lambda^4, \\ Y_4 = \lambda Y_1, \quad Y_5 = \lambda Y_2, \quad Y_6 = \lambda Y_3. \end{gathered} \tag{39}$$

From Eq. (39), we can easily calculate the eigenvector $\vec{Y}_i$, corresponding to eigenvalue λ_j, j = 1, 2, 3, 4, 5, 6. For further reference, we shall use the following notations

$$\begin{gathered} \vec{Y}_1 = [\vec{Y}]_{\lambda=-\lambda_1},\ \vec{Y}_2 = [\vec{Y}]_{\lambda=-\lambda_2},\ \vec{Y}_3 = [\vec{Y}]_{\lambda=-\lambda_3}, \\ \vec{Y}_4 = [\vec{Y}]_{\lambda=-\lambda_1},\ \vec{Y}_5 = [\vec{Y}]_{\lambda=-\lambda_2},\ \vec{Y}_6 = [\vec{Y}]_{\lambda=-\lambda_3}, \end{gathered} \tag{40}$$

The solution of Eq. (37) can be written from as follows

$$\vec{\Psi}(x,q,s) = \sum_{j=1}^{3} B_j \vec{Y}_j e^{-\lambda_j x}, \tag{41}$$

where the terms containing exponentials of growing nature in the space variable x have been discarded due to the regularity condition of the solution at infinity, B_1, B_2 and B_3 are constants to be determined from the boundary condition of the problem.

6. Inversion of double transform

The expression for functions $\vec{\Psi}(x,q,s)$ in Fourier transform domain can be given as

$$\vec{\Psi}(x,y,s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \vec{\Phi}(x,y,s) e^{iqy} dq.. \tag{42}$$

Thus, the field variables can be written for x, y and s as

$$\bar{u}(x,y,s) = \frac{1}{\sqrt{2\pi}} \sum_{j=1}^{3} \int_{-\infty}^{\infty} B_j u_j e^{-\lambda_j x + iqy} dq, \tag{43}$$

$$\bar{v}(x,y,s)=\frac{1}{\sqrt{2\pi}}\sum_{j=1}^{3}\int_{-\infty}^{\infty}B_j v_j e^{-\lambda_j x+iqy}dq,, \tag{44}$$

$$\bar{T}(x,y,s)=\frac{1}{\sqrt{2\pi}}\sum_{j=1}^{3}\int_{-\infty}^{\infty}B_j T_j e^{-\lambda_j x+iqy}dq,, \tag{45}$$

$$\bar{\sigma}_{xx}(x,y,s)=\frac{1}{\sqrt{2\pi}}\sum_{j=1}^{3}\int_{-\infty}^{\infty}B_j(-\lambda_j u_j+iq\gamma_{12}v_j-\gamma_4 T_j)e^{-\lambda_j x+iqy}dq, \tag{46}$$

$$\bar{\sigma}_{yy}(x,y,s)=\frac{1}{\sqrt{2\pi}}\sum_{j=1}^{3}\int_{-\infty}^{\infty}B_j(-\lambda_j\gamma_{12}u_j+iq\gamma_{13}v_j-\gamma_7 T_j)e^{-\lambda_j x+iqy}dq, \tag{47}$$

$$\bar{\sigma}_{xy}(x,y,s)=\frac{\gamma_3}{\sqrt{2\pi}}\sum_{j=1}^{3}\int_{-\infty}^{\infty}B_j(iqu_j-v_j\lambda_j)e^{-\lambda_j x+iqy}dq, \tag{48}$$

where u_j, v_j, T_j, j = 1, 2, 3 are the corresponding eigenvectors components of variables.

In the time domain t and spaces x and y, for the final solution of displacement components, temperature and stress components distributions we adopt a numerical inversion method based on the Stehfest (1970). In this method, the inverse $f(t)$ of the Laplace transform $f(s)$ is approximated by the relation

$$f(t)=\frac{\ln 2}{t}\sum_{j=1}^{N}V_j F\left(\frac{\ln 2}{t}j\right), \tag{49}$$

Where V_j is given by the following equation

$$V_i=(-1)^{\left(\frac{N}{2}+1\right)}\sum_{k=\frac{i+1}{2}}^{\min\left(i,\frac{N}{2}\right)}\frac{k^{\left(\frac{N}{2}+1\right)}(2k)!}{\left(\frac{N}{2}-K\right)!k!(i-k)!(2k-1)!}. \tag{50}$$

The parameter N is the number of terms used in the summation in Eq. (49) and should be optimized by trial and error. Increasing N increases the accuracy of the result up to a point, and then the accuracy declines because of increasing round-off errors. An optimal choice of $10 \leq N \leq 14$ has been reported by Lee *et al*. for some problem of their interest (Lee *et al*. 1984).

7. Numerical results and discussion

We assume that the plate is made of fiber-reinforced material. The physical constants are listed below (Othman and Said 2015)

$$\rho = 2660\,\text{kg/m}^3,\ \lambda = 5.65\times10^{10}\ \text{N/m}^2,\ \mu_T = 2.46\times10^{10}\ \text{N/m}^2,\ \mu_L = 5.66\times10^{10}\ \text{N/m}^2,$$
$$\alpha_{22} = 0.015\times10^{-4}\ \text{deg}^{-1},\ c_e = 0.787\times10^3\ \text{J kg}^{-1}\text{deg}^{-1},\ T_0 = 293\,\text{k},\ L = 0.5,\ T_1 = 1,$$
$$\alpha = -1.28\times10^{10}\ \text{N/m}^2,\ \beta = 220.90\times10^{10}\ \text{N/m}^2,\ \alpha_{11} = 0.017\times10^{-4}\ \text{deg}^{-1},$$
$$K_{11} = 0.0921\times10^3\ \text{J m}^{-1}\text{s}^{-1}\text{deg}^{-1},\ K_{22} = 0.0963\times10^3\ \text{J m}^{-1}\text{s}^{-1}\text{deg}^{-1},\ t_0 = 0.5.$$

The computations were carried out for a value of time $t = 0.3$ and the ramping time parameter $t_0 = 0.5$. The variation of the temperature T, displacement components u, v and stress components σ_{xx}, σ_{xy} and σ_{yy} with distance x in the plane $y = 0.25$ for the problem under consideration based on Green and Naghdi theory. Calculated results for all the non-dimensional variables of the plate are shown in Figs. 2-19.

The first group (Figs. 2-7) represent the behavior of the field quantities under fiber-reinforced with magnetic field ($R_H = 0.2$) at different time ($t = 0.1$, $t = 0.2$, $t = 0.3$).

The second group (Figs. 8-13) shows the differences between the case of the presence of fiber-reinforced (with fiber), and the case of the absence of fiber-reinforced (without fiber) with magnetic field ($R_H = 0.2$) at ($t = 0.3$).

The last group (Figs. 14-19) demonstrate the variations of the physical quantities under fiber-reinforced respect to the distance x in the case of the absence of the magnetic field ($R_H = 0.0$), and the case of the presence of magnetic field ($R_H = 0.2$, $R_H = 0.4$). It is easy to see that, the presence of a magnetic field has very small effect on the temperature while, it has a significant effect on the other field quantities.

Fig. 2 shows the temperature variation with respect to x and it indicates that temperature field has maximum value at the boundary and then decreases to zero. Fig. 4 displays the variation of horizontal displacement with respect to x and it indicates that the magnitude of the displacement increases with time. It is apparent that when the surface of the half-space is taken to be traction free, and the ramp-type heating applied on the surface, the displacement at different values of time shows a negative value at the boundary of the half space and it attains a stationary maximum value after some distance and after that, it decreases to zero value. Fig. 5 shows the variation of vertical displacement with respect to x for different values of time in which we observed that, significant difference in the value of displacement is noticed for the different value of t. Figs. 5-7 display the stress distribution with distance x for different values of time. We observe that stress components

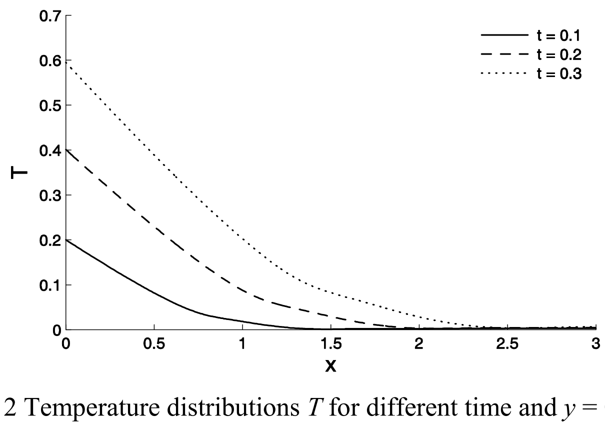


Fig. 2 Temperature distributions T for different time and $y = 0.25$

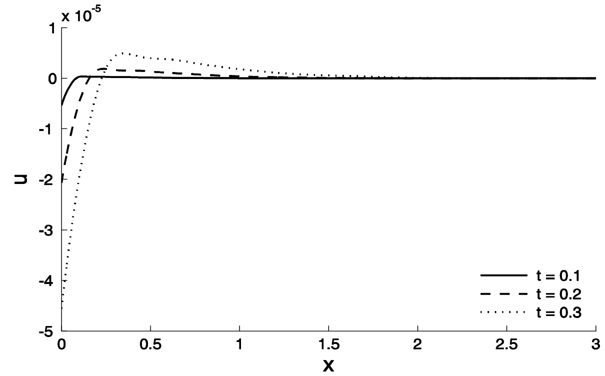


Fig. 3 Horizontal displacement distributions u for different time and $y = 0.25$

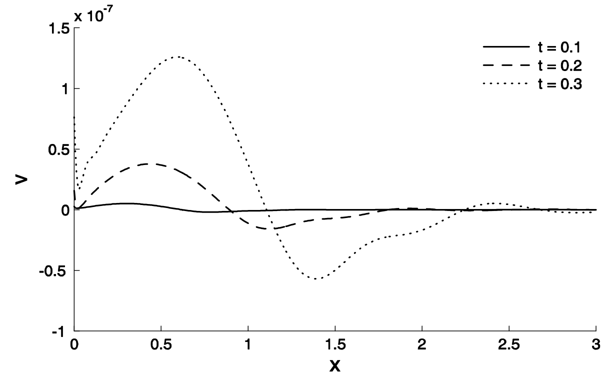


Fig. 4 Vertical displacement distributions V for different time and $y = 0.25$

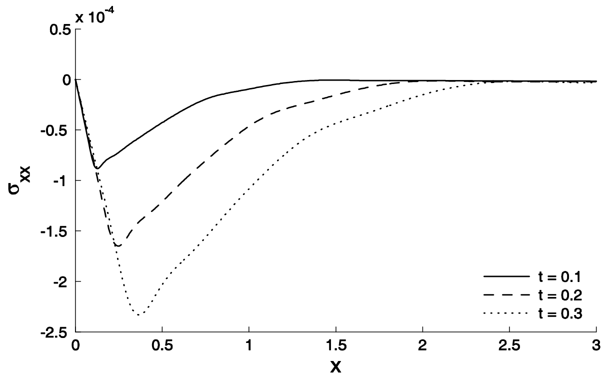


Fig. 5 The distribution of stress component σ_{xx} for different time and $y = 0.25$

σ_{xx} and σ_{xy}, always starts from the zero value and terminates at the zero value to obey the boundary conditions. In Figs. 2-7, the time parameter t has significant effects on all physical quantities distribution.

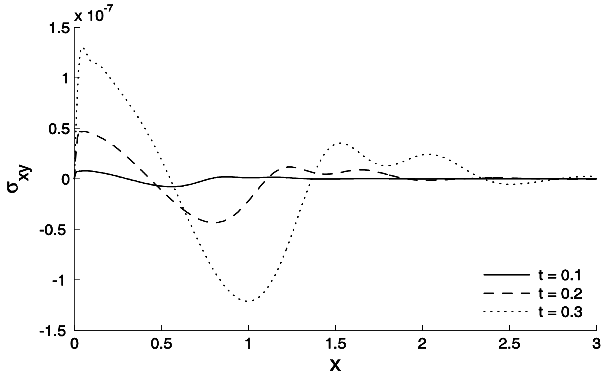


Fig. 6 The distribution of stress component σ_{xy} for different time and $y = 0.25$

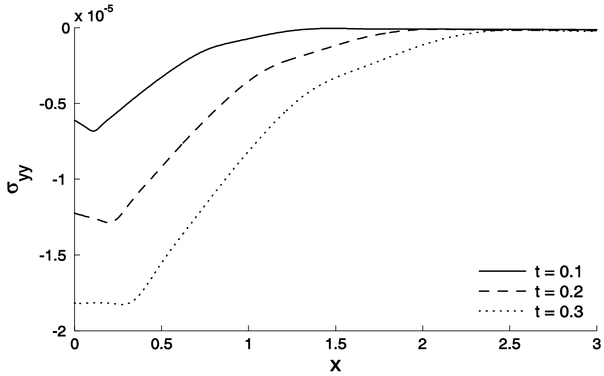


Fig. 7 The distribution of stress component σ_{yy} for different time and $y = 0.25$

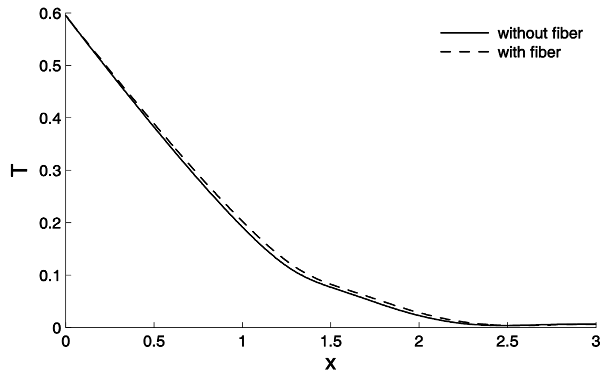


Fig. 8 The distribution of temperature T with and without fiber-reinforced for $y = 0.25$ and $t = 0.3$

The effect of fiber-reinforcement is given in Figs. 8-13, it is to be noted that the solid line (————) refer to the absence of fiber-reinforced (i.e., $\alpha = 0$, $\beta = 0$ and $\mu_L - \mu_T = 0$) while dashed line (---------) refer to the presence of fiber-reinforced. All variable quantities are very sensitive to the temperature-dependent on material properties.

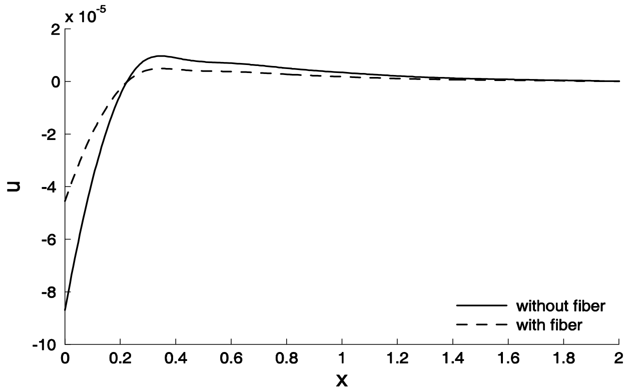


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Fig. 9 The distribution of Horizontal displacement u with and without fiber-reinforced for y = 0.25 and t = 0.3

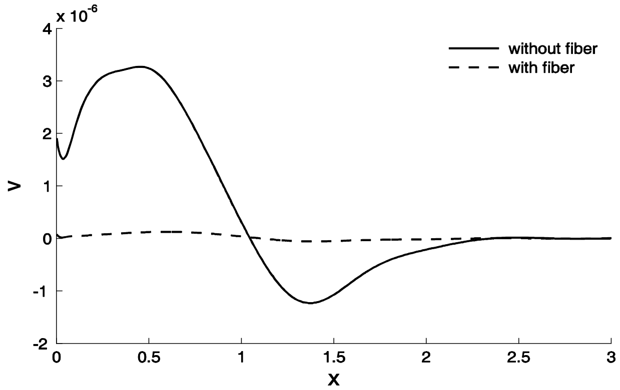

Fig. 10 The distribution of Horizontal displacement V with and without fiber-reinforced for y = 0.25 and t = 0.3

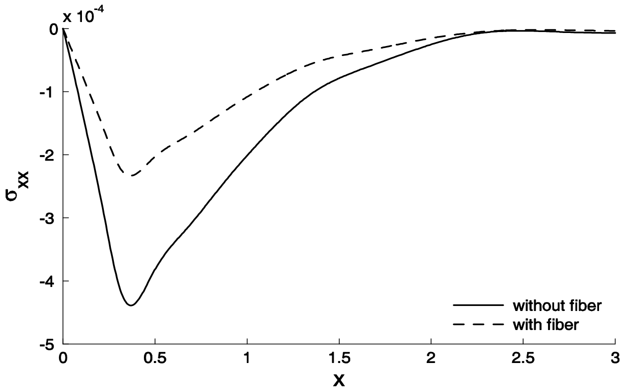

Fig. 11 The distribution of stress component σ_{xx} with and without fiber-reinforced for y = 0.25 and t = 0.3

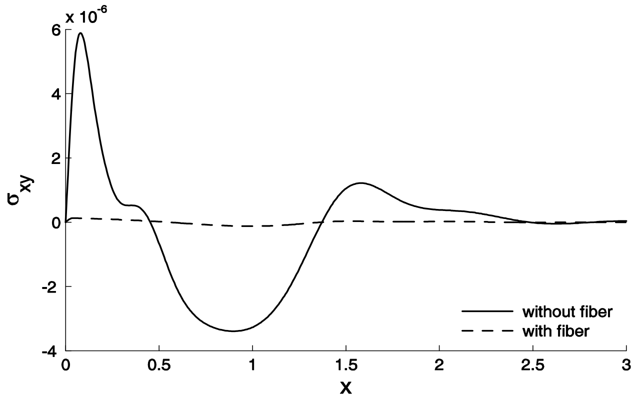

Fig. 12 The distribution of stress component σ_{xy} with and without fiber-reinforced for $y = 0.25$ and $t = 0.3$

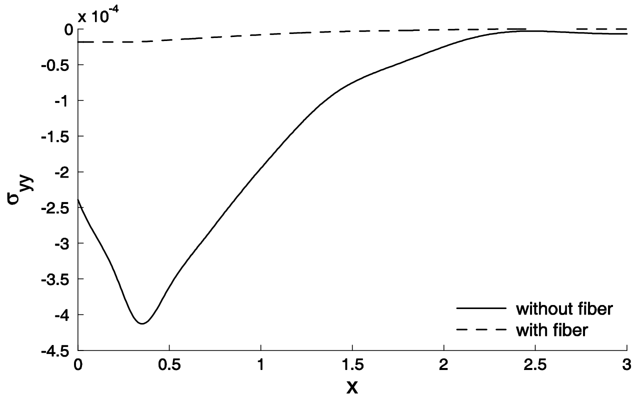

Fig. 13 The distribution of stress component σ_{yy} with and without fiber-reinforced for $y = 0.25$ and $t = 0.3$

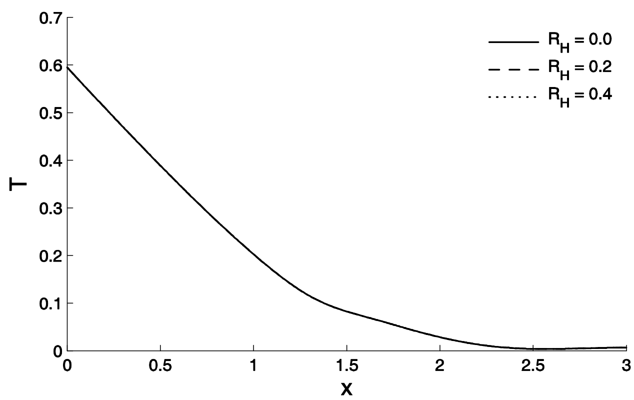

Fig. 14 Temperature distributions T for different values of R_H at $t = 0.3$ and $y = 0.25$

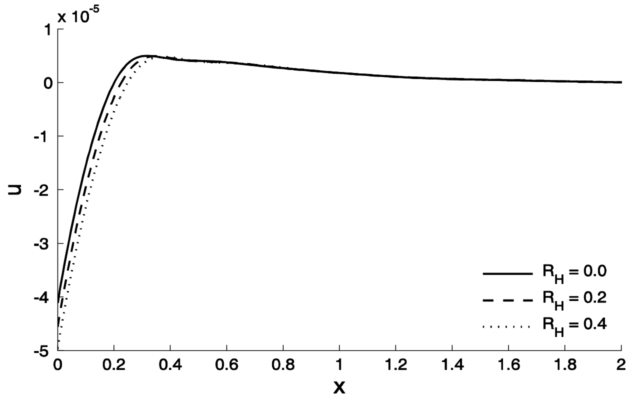

Fig. 15 Horizontal displacement distributions u for different values of R_H at $t = 0.3$ and $y = 0.25$

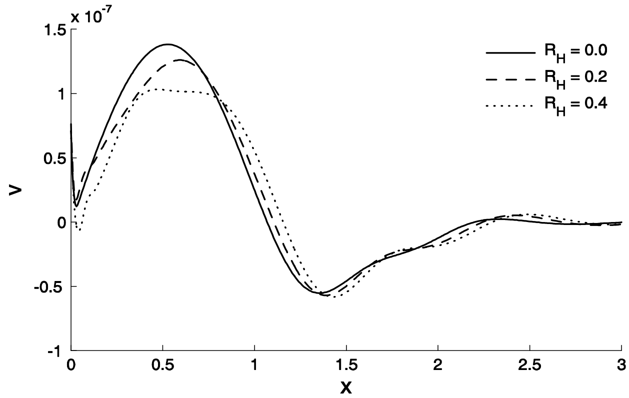

Fig. 16 Vertical displacement distributions V for different values of R_H at $t = 0.3$ and $y = 0.25$

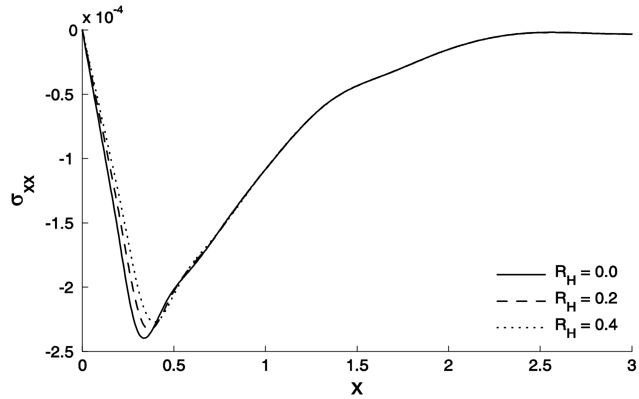

Fig. 17 The distributions of stress component σ_{xx} for different values of R_H at $t = 0.3$ and $y = 0.25$

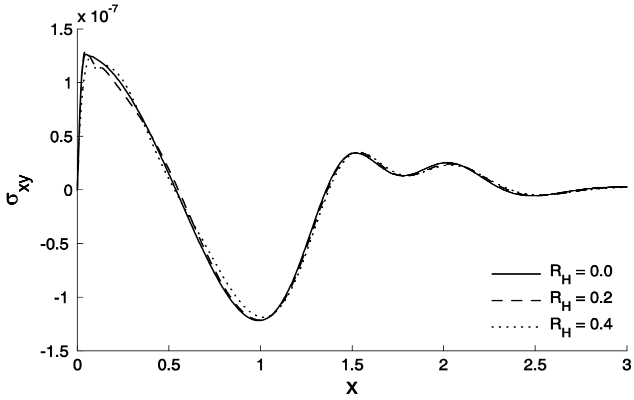

Fig. 18 The distributions of stress component σ_{xy} for different values of R_H at $t = 0.3$ and $y = 0.25$

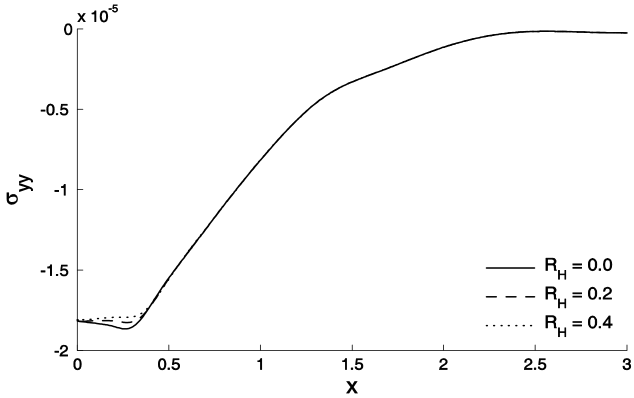

Fig. 19 The distributions of stress component σ_{yy} for different values of R_H at $t = 0.3$ and $y = 0.25$

8. Conclusions

In this work, the effect of magnetic field and reinforcement of the temperature, displacement components and the stress components have been studying for a two-dimensional problem of a anisotropic material is considered within the context of the GN-III theory. It is easy to see that, the reinforcement has a significant effect on all field quantities as expected. The fiber-reinforced material properties act to reduce the magnitudes of the considered variables, which may be significant in some practical applications, can easily be taken under consideration and accurately assessed i.e., the fiber-reinforced material has a high strength.

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