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Thermal post-buckling behavior of imperfect temperature-dependent sandwich FGM plates resting on Pasternak elastic foundation

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Abstract. In this paper, post-buckling behavior of sandwich plates with functionally graded (FG) face sheets under uniform temperature rise loading is examined based on both sinusoidal shear deformation theory and stress function. It is supposed that the sandwich plate is in contact with an elastic foundation during deformation, which acts in both compression and tension. Thermo-elastic non-homogeneous properties of FG layers change smoothly by the variation of power law within the thickness, and temperature dependency of material constituents is considered in the formulation. In the present development, Von Karman nonlinearity and initial geometrical imperfection of sandwich plate are also taken into account. By employing Galerkin method, analytical solutions of thermal buckling and post-buckling equilibrium paths for simply supported plates are determined. Numerical examples presented in the present study discuss the effects of gradient index, sandwich plate geometry, geometrical imperfection, temperature dependency, and the elastic foundation parameters.

Keywords: functionally graded materials; thermal post-buckling; sinusoidal shear deformation theory; elastic foundation; imperfection

1. Introduction

Buckling and post-buckling behaviors of functionally graded (FG) plates subjected to different types of loading are important for practical uses and have taken considerable interest. Wu (2004) employed the first order shear deformation theory (FSDT) to determine the analytical expressions of critical buckling temperatures for simply supported FG plates. Thermo-mechanical post-buckling behavior of FG plates based on an analytical approach is examined by Woo *et al.* (2005). Liew *et al.* (2003, 2004) utilized the higher order shear deformation theory in conjunction with differential quadrature method to study the post-buckling of pure and hybrid FG plates with and without imperfection on the point of view that buckling only occurs for fully clamped FG plates. The post-buckling response of pure and hybrid FG plates subjected to the combination of different loading types were also examined by Shen (2007, 2009) by employing higher order shear

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deformation theory and two-step perturbation technique taking temperature dependence of material characteristics into consideration. Zhao *et al*. (2009) investigated the mechanical and thermal stability of FG plates by employing element- free Ritz method. Lee *et al*. (2010) have employed element-free Ritz technique to study the post-buckling of FG plates under to compressive and thermal loads. Tung and Duc (2010) proposed a simple accurate analytical solution to investigate the buckling and post-buckling behavior of thin FG plates. By considering the initial imperfection for an FG plate, they demonstrated that imperfect plates do not follow bifurcation-type buckling and commence to deflect by initiation of compression. They investigated possible combinations of movable and immovable simply supported edges for each case of thermo-mechanical loading. Tounsi *et al*. (2013) proposed a refined trigonometric shear deformation theory for thermo-elastic bending of FG sandwich plates. Bachir Bouiadjra *et al*. (2013) presented a nonlinear thermal buckling analysis for FG plates using an efficient sinusoidal shear deformation theory. Ahmed (2014) studied the post-buckling behavior of FG sandwich beams using a consistent higher order theory. Swaminathan and Naveenkumar (2014) developed an analytical approach for the buckling analysis of simply supported FG sandwich plates based on two higher-order refined computational models. Based on an efficient and simple trigonometric shear deformation theory, Tebboune *et al*. (2015) presented a thermal buckling analysis of FG plates resting on elastic foundation. Akbaş (2015) discussed the wave propagation of a FG beam in thermal environments. Bouchafa *et al*. (2015) analyzed thermal stresses and deflections of FG sandwich plates using a new refined hyperbolic shear deformation theory. Bouguenina *et al*. (2015) investigated the thermal stability of FGM plates with variable thickness using a finite differential method. Laoufi *et al*. (2016) analyzed the mechanical and hygrothermal behavior of FG plates using a hyperbolic shear deformation theory. Bourada *et al*. (2016) presented a new displacement field to analyze the buckling behavior of isotropic and orthotropic plates. Additional works on buckling and post-buckling analysis of laminated composite and FG structures under thermo-mechanical load are presented in the literature by Panda and his co-workers (Kar and Panda 2015a, b, 2016a, b, Katariya and Panda 2016, Bouderba *et al*. 2016, Panda and Katariya 2015, Panda and Singh 2013a, b, c, 2011, 2010a, b, 2009). The reason for the increasing use of FGMs in a variety of aerospace, automotive, civil, and mechanical engineering structures is that their material properties can be tailored to different applications and working environments (Arefi 2015a, b, Hamidi *et al*. 2015, Darılmaz 2015, Arefi and Allamm 2015, Meksi *et al*. 2015, Ebrahimi and Dashti 2015, Pradhan and Chakraverty 2015, Kar and Panda 2015a, b, Boukhari *et al*. 2016, Bounouara *et al*. 2016, Ebrahimi and Habibi 2016, Hadji *et al*. 2016, Moradi-Dastjerdi 2016, Bousahla *et al*. 2016, Ebrahimi and Salari 2016, Trinh *et al*. 2016).

The influence of the Pasternak elastic foundation on mechanical post-buckling of moderately thick FG plates is treated by Yang *et al*. (2005). Their work covers plates with all four edges clamped, and formulation is based on the FSDT. They determined the post-buckling equilibrium paths based on a 2D differential quadrature method combined with the perturbation technique. Librescu and Lin (1997) and Lin and Librescu (1998) have extended previous studies (Librescu and Stein 1991, 1992) to discuss the post-buckling response of flat and curved laminated composite panels resting on Winkler elastic foundations. Duc and Tung (2011) studied mechanical and thermal post-buckling of FG plate on elastic foundation by employing third order shear deformation plate theory and simple power law variation of the volume fraction for metal and ceramic. Bouderba *et al*. (2013) discussed the thermo-mechanical bending behavior of FG thick plates resting on Winkler-Pasternak elastic foundations. Zidi *et al*. (2014) studied the bending response of FG plates on elastic foundation under hygro-thermo-mechanical loading using a four variable refined plate theory. Ait Amar Meziane *et al*. (2014) presented and efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions and resting on Winkler-Pasternak elastic foundations. Khalfi *et al*. (2014) developed a refined and simple shear deformation theory for thermal stability of solar FG plates on elastic foundation. Bakora and Tounsi (2015) investigated the thermo-mechanical post-buckling behavior of thick P-FGM plates resting on elastic foundations. Recently, Chikh *et al*. (2016) examined the thermo-mechanical post-buckling of symmetric S-FGM plates resting on Pasternak elastic foundations using hyperbolic shear deformation theory. Also, many paper are published concerning with analysis of FGM structures based on higher order shear deformation theories (Benachour *et al*. 2011, Bourada *et al*. 2012, Ould Larbi *et al*. 2013, Belabed *et al*. 2014, Hebali *et al*. 2014, Bousahla *et al*. 2014, Ait Yahia *et al*. 2015, Larbi Chaht *et al*. 2015, Mahi *et al*. 2015, Meradjah *et al*. 2015, Merazi *et al*. 2015, Belkorissat *et al*. 2015, Nguyen *et al*. 2016, Al-Basyouni *et al*. 2015, Bourada *et al*. 2015, Attia *et al*. 2015, Bennai *et al*. 2015, Sallai *et al*. 2015, Tagrara *et al*. 2015, Zemri *et al*. 2015, Ait Atmane *et al*. 2015, Mouaici *et al*. 2016, Beldjelili *et al*. 2016, Bennoun *et al*. 2016, Saidi *et al*. 2016, Tounsi *et al*. 2016).

This work presents a simple analytical formulation to examine the post-buckling behavior of sandwich plates with FGM face sheets under uniform temperature rise loading. Present model is easily applied after some modifications for any types of loading with constant pre-buckling loads which lead to bifurcation-type buckling of simply supported plates. Material characteristics of the FGM layers follow power law variation within the thickness, and for all three layers, temperature dependency of thermo-mechanical characteristics is considered. A two-parameter Pasternak-type elastic foundation is supposed to be in contact during deformation, which acts in both tension and compression. Finally, analytical expressions are presented, which properly gives the temperature-deflection path and critical buckling temperature of symmetric sandwich FG plates.

2. Sandwich FGM plates

In this paper, a symmetrically mid-plane rectangular plate with a three-layered sandwich plate configuration made of two similar FG face sheets and a homogeneous core (Fig. 1) is considered (Liew *et al*. 2004, Houari *et al*. 2011, Li *et al*. 2008). Total height, width, and length of the plate are mentioned as h, b, and a, respectively. Considering a simple power law variation in the thickness direction, the volume fraction of metal constituent of the structure V_m may be expressed in the form

$$V_m = \begin{cases} \left(\dfrac{2z+h}{2h_f}\right)^k & -\dfrac{1}{2}h \le z \le \dfrac{1}{2}h_H \\ 1 & -\dfrac{1}{2}h_H \le z \le \dfrac{1}{2}h_H \\ \left(\dfrac{-2z+h}{2h_f}\right)^k & \dfrac{1}{2}h_H \le z \le \dfrac{1}{2}h \end{cases} \tag{1}$$

where h_H and h_f present the thickness of homogeneous core and each of face sheets, respectively.

Material characteristics of a sandwich FGM plate can be determined by means of the Voigt rule

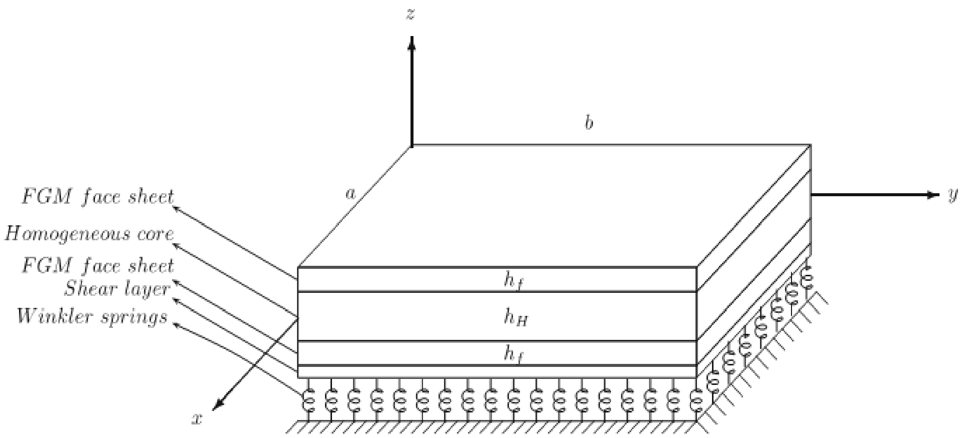

Fig. 1 Coordinate system and geometry of three-layered sandwich FG plates over an elastic foundation

of mixture (Suresh and Mortensen 1998). Hence, by employing Eq. (1), each non-homogeneous characteristic of sandwich plate *P* versus the thickness coordinate becomes

$$P(z)=\begin{cases} P_c+P_{mc}\left(\dfrac{2z+h}{2h_f}\right)^k & -\dfrac{1}{2}h\le z\le \dfrac{1}{2}h_H \\ P_{\mathrm{m}} & -\dfrac{1}{2}h_H\le z\le \dfrac{1}{2}h_H \\ P_c+P_{mc}\left(\dfrac{-2z+h}{2h_f}\right)^k & \dfrac{1}{2}h_H\le z\le \dfrac{1}{2}h \end{cases} \tag{2}$$

Where, $P_{mc} = P_m - P_c$ and P_m and P_c are the corresponding properties of the metal and ceramic, respectively, and k is the gradient index that takes the values greater or equal to zero. In the present study, we consider that the Young modulus E and thermal expansion coefficient α are defined by Eq. (2), while Poisson's ratio v is assumed to be constant within the thickness (Tung and Duc 2010, Bakora and Tounsi 2015, Akavci 2015, Hadji and Adda Bedia 2015, Kar and Panda 2015a, Bellifa *et al.* 2016).

3. Mathematical formulations

In this work, the sinusoidal shear deformation plate theory is employed with the following kinematic

$$u(x,y,z)=u_0(x,y)-z\frac{\partial w(x,y)}{\partial x}+\Psi(z)\ \phi_x(x,y) \tag{3a}$$

$$v(x,y,z)=v_0(x,y)-z\frac{\partial w(x,y)}{\partial y}+\Psi(z)\ \phi_y(x,y) \tag{3b}$$

$$w(x,y,z) = w_0(x,z) \tag{3c}$$

with

$$\Psi(z) = \frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right). \tag{3d}$$

here u_0, v_0, w_0, ϕ_x, ϕ_y are five unknown displacements of the mid-plane of the plate.

The non-linear von Karman strain–displacement equations are as follows

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} + \Psi(z)\begin{Bmatrix} \eta_x \\ \eta_y \\ \eta_{xy} \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \Psi'(z)\begin{Bmatrix} \gamma_{xz}^0 \\ \gamma_{yz}^0 \end{Bmatrix}, \tag{4}$$

where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} u_{0,x} + (w_{0,x})^2/2 \\ v_{0,x} + (w_{0,y})^2/2 \\ u_{0,y} + v_{0,x} + w_{0,x}w_{0,y} \end{Bmatrix}, \quad \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} = \begin{Bmatrix} -w_{0,xx} \\ -w_{0,yy} \\ -2w_{0,xy} \end{Bmatrix}, \quad \begin{Bmatrix} \eta_x \\ \eta_y \\ \eta_{xy} \end{Bmatrix} = \begin{Bmatrix} \phi_{x,x} \\ \phi_{y,y} \\ \phi_{x,y} + \phi_{y,x} \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{xz}^0 \\ \gamma_{yz}^0 \end{Bmatrix} = \begin{Bmatrix} \phi_x \\ \phi_y \end{Bmatrix}, \tag{5}$$

The linear constitutive relations of the sandwich FG plate can be written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 & 0 & 0 \\ \nu & 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1-\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1-\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}\begin{Bmatrix} \varepsilon_x - \alpha\,\Delta T \\ \varepsilon_y - \alpha\,\Delta T \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{6}$$

where ΔT is temperature rise from stress free initial state or temperature difference between two surfaces of the sandwich FG plate.

By employing the virtual work principle to minimize the functional of total potential energy function result in the expressions for the nonlinear equilibrium equations of a perfect plate resting on two parameters elastic foundation as

$$N_{x,x} + N_{xy,y} = 0 \tag{7a}$$

$$N_{xy,x} + N_{y,y} = 0 \tag{7b}$$

$$(M_{x,xx} + 2M_{xy,xy} + M_{y,yy}) + N_x w_{,xx} + 2\,N_{xy} w_{,xy} + N_y w_{,yy} - k_w w + k_g \nabla^2 w = 0 \tag{7c}$$

$$S_{x,x} + S_{xy,y} - Q_x = 0 \tag{7d}$$

$$S_{xy,x} + S_{y,y} - Q_y = 0 \tag{7e}$$

where the force and moment resultants (N, Q, S and M) of the sandwich FG plate are expressed by

$$(N_i, M_i, S_i) = \int_{-h/2}^{h/2} \sigma_i (1, z, \Psi(z))\, dz, \quad (i = x, y, xy) \tag{8a}$$

$$Q_i = \int_{-h/2}^{h/2} \sigma_j\, \Psi'(z)\, dz, \quad (i = x, y); \; (j = xz, yz) \tag{8b}$$

where the force and moment resultants (N, Q, S and M) of the sandwich FG plate are expressed by

$$\begin{aligned}(N_x, M_x, S_x) = \frac{1}{1-\nu^2}[&(E_1, E_2, E_3)(\varepsilon_x^0 + \nu\, \varepsilon_y^0) + (E_2, E_4, E_5)(k_x + \nu\, k_y) \\ &+ (E_3, E_5, E_7)(\eta_x + \nu\, \eta_y) - (1+\nu)(\Phi_1, \Phi_2, \Phi_3)]\end{aligned} \tag{9a}$$

$$\begin{aligned}(N_y, M_y, S_y) = \frac{1}{1-\nu^2}[&(E_1, E_2, E_3)(\varepsilon_y^o + \nu\, \varepsilon_x^0) + (E_2, E_4, E_5)(k_y + \nu\, k_x) \\ &+ (E_3, E_5, E_7)(\eta_y + \nu\, \eta_x) - (1+\nu)(\Phi_1, \Phi_2, \Phi_3)]\end{aligned} \tag{9b}$$

$$\begin{aligned}(N_{xy}, M_{xy}, S_{xy}) = \frac{1}{2(1+\nu)}[&(E_1, E_2, E_3)\gamma_{xy}^0 + (E_2, E_4, E_5) k_{xy} \\ &+ (E_3, E_5, E_7)\eta_{xy}]\end{aligned} \tag{9c}$$

$$(Q_x, Q_y) = \frac{1}{2(1+\nu)} E_8 \left(\gamma_{xz}^0, \gamma_{yz}^0\right) \tag{9d}$$

where

$$\begin{aligned}\left(E_1, E_4, E_5, E_7\right) = \int_{-h/2}^{h/2} \left(1, z^2, z\,\Psi(z), \Psi(z)^2\right) E\left(z\right) dz, \quad \left(E_2, E_3\right) = \int_{-h/2}^{h/2} \left(z, \Psi(z)\right) E\left(z\right) dz = \left(0, 0\right), \\ E_8 = \int_{-h/2}^{h/2} \left(\Psi'\left(z\right)\right)^2 E(z)\, dz\end{aligned} \tag{10a}$$

$$(\Phi_1, \Phi_2, \Phi_3) = \int_{-h/2}^{h/2} (1, z, \Psi) E(z)\, \alpha(z)\, \Delta T(z)\, dz \tag{10b}$$

The last three equations of Eq. (7) can be expressed into two equations in terms of variables w_0 and $\phi_{x,x} + \phi_{y,y}$ by substituting Eqs. (5) and (9) into Eqs. (7c)-(7e). Subsequently, elimination of the variable $\phi_{x,x}$ + $\phi_{y,y}$ from two the resulting equations, conducts to the following system of equilibrium equations

$$N_{x,x} + N_{xy,y} = 0 \tag{11a}$$

$$N_{xy,x} + N_{y,y} = 0 \tag{11b}$$

$$\begin{aligned}&\left(D_2^2 - D_1 D_3\right)\nabla^6 w + D_1 D_4\ \nabla^4 w + D_3 \nabla^2\left(N_x w_{,xx} + 2N_{xy} w_{,xy} + N_y w_{,yy} - k_w w + k_g \nabla^2 w\right)\\ &- D_4\left(N_x w_{,xx} + 2N_{xy} w_{,xy} + N_y w_{,yy} - k_w w + k_g \nabla^2 w\right) = 0\end{aligned} \tag{11c}$$

where

$$D_1 = \frac{E_4}{\left(1-\nu^2\right)},\quad D_2 = \frac{E_5}{\left(1-\nu^2\right)},\quad D_3 = \frac{E_7}{\left(1-\nu^2\right)},\quad D_4 = \frac{E_8}{2\left(1+\nu\right)}. \tag{12}$$

For an imperfect sandwich FG plate, Eq. (11) are modified into form as

$$\begin{aligned}&\left(D_2^2 - D_1 D_3\right)\nabla^6 w + D_1 D_4 \nabla^4 w + D_3 \nabla^2\begin{bmatrix} f_{,yy}\left(w_{,xx} + w^*_{,xx}\right) - 2 f_{,xy}\left(w_{,xy} + w^*_{xy}\right) \\ + f_{,xx}\left(w_{,yy} + w^*_{,yy}\right) - k_w w + k_g \nabla^2 w\end{bmatrix}\\ &- D_4\left[f_{,yy}(w_{,xx} + w^*_{,xx}) - 2 f_{,xy}(w_{,xy} + w^*_{xy}) + f_{,xx}(w_{,yy} + w^*_{,yy}) - k_w w + k_g \nabla^2 w\right] = 0\end{aligned} \tag{13}$$

In which $w^*(x, y)$ is a known function denoting initial small imperfection of the plate. Note that equation (13) gets a complicated form under the sinusoidal shear deformation theory which includes the 6th-order partial differential term $\nabla^6 w_0$. Also, $f(x, y)$ is stress function defined by

$$N_x = f_{,yy}\ ,\ N_y = f_{,xx}\ ,\ N_{xy} = -f_{,xy} \tag{14}$$

The geometrical compatibility equation for an imperfect plate is written as

$$\varepsilon^0_{x,yy} + \varepsilon^0_{y,xx} - \gamma^0_{xy,xy} = w^2_{0,xy} - w_{0,xx} w_{0,yy} + 2 w_{0,xy} w^*_{0,xy} - w_{0,xx} w^*_{0,yy} - w_{0,yy} w^*_{0,xx} \tag{15}$$

From the constitutive relations Eqs. (9) and (14) one can write

$$\begin{aligned}&(\varepsilon^0_x, \varepsilon^0_y) = \frac{1}{E_1}[(f_{,yy}, f_{,xx}) - \nu\ (f_{,xx}, f_{,yy}) + \Phi_1\,(1,1)]\\ &\gamma^0_{xy} = -\frac{1}{E_1}[2\ (1+\nu)\, f_{,xy}]\end{aligned} \tag{16}$$

Substituting Eq. (16) into Eq. (15), the compatibility equation of an imperfect sandwich plate becomes

$$\nabla^4 f - E_1\,(w^2_{0,xy} - w_{0,xx}\, w_{0,yy} + 2\, w_{0,xy}\, w^*_{0,xy} - w_{0,xx}\, w^*_{0,yy} - w_{0,yy}\, w^*_{0,xx}) = 0 \tag{17}$$

In this work, plate is considered to be simply supported in all edges where normal to edge displacement is prevented at boundaries. This type of edge conditions is also known as immovable simply supported conditions (Shen 2007). Mathematical expression for this class of edge supports may be written as (Shen 2007)

$$w_0 = u_0 = \phi_y = M_x = S_x = 0, \quad N_x = N_{x0} \text{ at } x = 0, a \tag{18a}$$

$$w_0 = v_0 = \phi_x = M_y = S_y = 0, \quad N_y = N_{y0} \text{ at } y = 0, b \tag{18b}$$

where N_{x0}, N_{y0} are fictitious compressive edge loads at immovable edges.

The proposed solutions of w and f respecting boundary conditions Eq. (18) are considered to be (Librescu and Lin 1997, Lin and Librescu 1998)

$$\left(w, w^*\right) = (W, \mu h)\sin(\lambda_m x)\sin(\delta_n y) \tag{19a}$$

$$\begin{aligned} f = A_1 \cos(2\lambda_m x) + A_2 \cos(2\delta_n y) + A_3 \sin(\lambda_m x)\sin(\delta_n y) \\ + \frac{1}{2}N_{x0} y^2 + \frac{1}{2}N_{y0} x^2 \end{aligned} \tag{19b}$$

where $\lambda_m = m\pi / a$, $\delta_n = n\pi / b$, m, n are odd numbers, W is amplitude of the deflection and μ is imperfection parameter. The coefficients A_i (i = 1, 2, 3) are obtained by substitution of Eqs. (19a), (19b) into Eq. (17) as

$$A_1 = \frac{E_1 \delta_n^2}{32\lambda_m^2} W(W + 2\mu h), \quad A_2 = \frac{E_1 \lambda_m^2}{32\delta_n^2} W(W + 2\mu h), \quad A_3 = 0 \tag{20}$$

Then, setting Eqs. (19a), (19b) into Eq. (13) and using the Galerkin method for the resulting equation yield

$$\begin{aligned} &((D_1 D_3 - D_2^2)(\lambda_m^2 + \delta_n^2)^3 + D_1 D_4 (\lambda_m^2 + \delta_n^2)^2 + \left[k_w + k_g (\lambda_m^2 + \delta_n^2)\right][(D_3 (\lambda_m^2 + \delta_n^2) + D_4]) W \\ &+ \frac{E_1}{16}\left(D_3 \left(5\left(\lambda_m^4 \delta_n^2 + \lambda_m^2 \delta_n^4\right) + \lambda_m^6 + \delta_n^6\right) + D_4\left(\lambda_m^4 + \delta_n^4\right)\right) \times W\left(W + \mu h\right)\left(W + 2\mu h\right) \\ &+ \left(D_3 (\lambda_m^2 + \delta_n^2) + D_4\right) \times \left(N_{x0}\, \lambda_m^2 + N_{y0}\, \delta_n^2\right)\left(W + \mu h\right) = 0 \end{aligned} \tag{21}$$

This equation will be used to examine the buckling and post-buckling responses of thick sandwich FG plates under thermal loads.

4. Solving equations

The in-plane condition on immovability at all edges, i.e., $u_0 = 0$ at $x = 0$, a and $v_0 = 0$ at $y = 0$, b, is given in an average sense as (Tung and Duc 2010)

$$\int_0^b\int_0^a \frac{\partial u_0}{\partial x}\,dxdy = 0\,, \qquad \int_0^a\int_0^b \frac{\partial v_0}{\partial y}\,dydx = 0 \tag{22}$$

From Eqs. (5) and (9) one can obtain the following expressions in which Eq. (14) and imperfection have been included

$$\frac{\partial u_0}{\partial x} = \frac{1}{E_1}(f_{,yy} - \nu f_{,xx}) - \frac{1}{2}w_{,x}^2 - w_{,x}w_{,x}^* + \frac{\Phi_1}{E_1} \tag{23a}$$

$$\frac{\partial v_0}{\partial y} = \frac{1}{E_1}(f_{,xx} - \nu f_{,yy}) - \frac{1}{2}w_{,y}^2 - w_{,y}w_{,y}^* + \frac{\Phi_1}{E_1} \tag{23b}$$

Substituting Eq. (19) into Eq. (23) and then the result into Eq. (22) give

$$N_{x0} = -\frac{\Phi_1}{1-\nu} + \frac{E_1}{8(1-\nu^2)}\left(\lambda_m^2 + \nu\,\delta_n^2\right) W\left(W + 2\mu h\right). \tag{24a}$$

$$N_{y0} = -\frac{\Phi_1}{1-\nu} + \frac{E_1}{8(1-\nu^2)}(\nu\,\lambda_m^2 + \delta_n^2)\, W\,(W + 2\mu h). \tag{24b}$$

When the deflection dependence of fictitious edge loads is ignored, i.e., $W = 0$, Eq. (25) becomes

$$N_{x0} = N_{y0} = -\frac{\Phi_1}{1-\nu} \tag{25}$$

Substituting Eq. (24) into Eq. (21) yields the expression of thermal parameter as

$$\begin{aligned}\frac{\Phi_1}{1-\nu} &= \left[\frac{\left(D_1D_3 - D_2^2\right)\left(\lambda_m^2 + \delta_n^2\right)^2 + D_1D_4\left(\lambda_m^2 + \delta_n^2\right)}{D_3\left(\lambda_m^2 + \delta_n^2\right) + D_4} + \frac{k_w + k_g\left(\lambda_m^2 + \delta_n^2\right)}{(\lambda_m^2 + \delta_n^2)}\right]\frac{W}{W + \mu h} \\ &+ \left[\frac{E_1\left[D_3\left(5\left(\lambda_m^4\delta_n^2 + \lambda_m^2\delta_n^4\right) + \lambda_m^6 + \delta_n^6\right) + D_4\left(\lambda_m^4 + \delta_n^4\right)\right]}{16\left[D_3(\lambda_m^2 + \delta_n^2) + D_4\right](\lambda_m^2 + \delta_n^2)} + \frac{E_1\left[\left(\lambda_m^4 + 2\,\upsilon\,\lambda_m^2\delta_n^2 + \delta_n^4\right)\right]}{8\left(1-\nu^2\right)\left(\lambda_m^2 + \delta_n^2\right)}\right] W\left(W + 2\,\mu\,h\right)\end{aligned} \tag{26}$$

The sandwich FG plate is exposed to temperature environments uniformly raised from stress free initial state T_i to final value T_f, and temperature change $\Delta T = T_f - T_i$ is assumed to be independent from thickness variable. The thermal parameter Φ_1 is obtained from Eq. (10b), and substitution of the result into Eq. (26) yields

$$\Delta\,T = e_1^2\,\frac{W}{W + \mu\,h} + e_2^2\,W\left(W + 2\mu\,h\right) \tag{27}$$

where

$$e_1^2 = \frac{(1-\nu)}{L\,[D_3(\lambda_m^2+\delta_n^2)+D_4]} \times \left[(D_1 D_3 - D_2^2)(\lambda_m^2+\delta_n^2)^2 + D_1 D_4(\lambda_m^2+\delta_n^2)\right] + \frac{\left[K_w + K_g\, a^2(\lambda_m^2+\delta_n^2)\right](1-\nu)\,D_0}{a^4 L\,(\lambda_m^2+\delta_n^2)}, \tag{28a}$$

$$e_2^2 = \frac{E_1(1-\nu)}{16\,L(\lambda_m^2+\delta_n^2)[D_3(\lambda_m^2+\delta_n^2)+D_4]} \times \left[D_3\left(5(\lambda_m^4\delta_n^2+\lambda_m^2\delta_n^4)+\lambda_m^6+\delta_n^6\right)+D_4(\lambda_m^4+\delta_n^4)\right] + \frac{E_1(\lambda_m^4+2\,\nu\,\lambda_m^2\,\delta_n^2+\delta_n^4)}{8\,L\,(1+\nu)(\lambda_m^2+\delta_n^2)} \tag{28b}$$

in which

$$L = \int_{-h/2}^{h/2} E(z)\alpha(z)\,dz \tag{29}$$

5. Results and discussion

To check the proposed formulation, a sandwich plate with metallic core and FGM face sheets is examined. The FGM layers are graded within the thickness. The combination of materials for FGM consists of ZrO_2 and Ti6Al4V. Reference temperature $T_0$is considered to be 300 K (Shen 2007, Liew *et al*. 2004, Kiani and Eslami 2012). Temperature-dependent coefficients for these materials are presented in Table 1, and thus, each property may be calculated as follow (Kiani and Eslami 2012)

$$P = P_0\left(1+\frac{P_{-1}}{T}+P_1T+P_2T^2+P_3T^3\right) \tag{30}$$

For simplicity, the following non-dimensional parameters are used

$$K_w = \frac{k_w a^4}{D_0}, \qquad K_g = \frac{k_g a^2}{D_0}, \qquad D_0 = \frac{E_m^0 h^3}{12(1-\nu^2)} \tag{31}$$

Table 1 Temperature-dependent coefficients for ZrO2 and Ti6Al4V (Kiani and Eslami, 2012)

Material	P_0	P_{-1}	P_1	P_2	P_3
			ZrO_2		
E (Pa)	244.27e+9	0	−1.371e−3	1.214e−6	−3.681e−10
α (1/°K)	12.766e−6	0	−1.491e−3	1.006e−5	−6.778e−11
			Ti6Al4V		
E (Pa)	122.56e+9	0	−4.586e−4	0	0
α (1/°K)	7.5788e−6	0	6.638e−4	−0.3147e−6	0

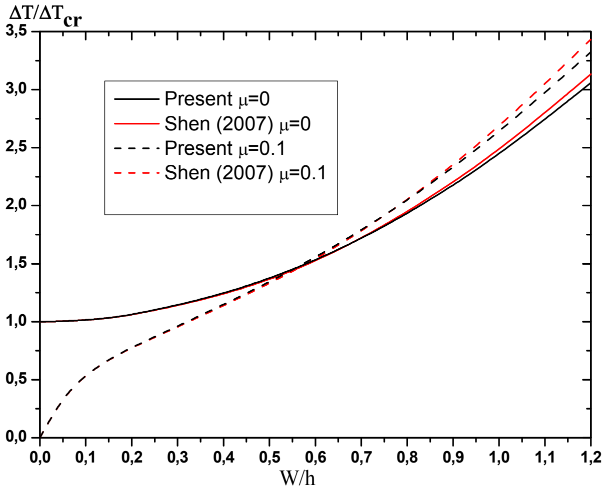

Fig. 2 A comparison on post-buckling responses of initially perfect and imperfect contact-less homogeneous square plate with those of given by Shen (2007)

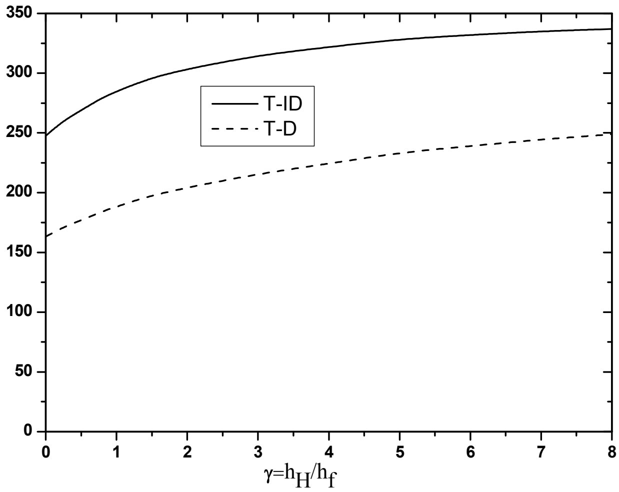

Fig. 3 Effect of temperature dependency of the material constituents on ΔT_{Cr} (k = 1, a/h = 20)

For (ZrO_2/Ti6Al4V) sandwich plate, Poisson's ratio is assumed to be constant and chosen as v = 0.29 (Shen 2007, Liew *et al*. 2004, Kiani and Eslami 2012). The plate is supposed to be simply supported on all four edges with expansion prevention capability of edge supports.

5.1 Comparative studies

For checking of the buckling and post-buckling solutions determined from the proposed approach, four comparative studies are examined in Tables 2, 3, 4 and Fig. 3.

Table 2 shows a comparative study on critical buckling temperature difference of isotropic homogeneous plate determined by the present method and the available data in the literature (k = 0).

Table 2 Critical bucking temperature difference ΔT_{Cr} for a simply-supported square plate in contact with the Winkler elastic foundation and subjected to uniform temperature rise

(K_w, K_g)	$h/b = 0.01$	$h/b = 0.02$	$h/b = 0.05$
		(0,0)	
Present	14.36	57.35	354.34
Kiani and Eslami (2012)	14.36	57.35	354.27
Shen (1997)	14.37	57.48	359.26
Raju and Rao (1988)	14.26	57.04	356.21
		$(\pi^4, 0)$	
Present	17.86	71.72	444.16
Kiani and Eslami (2012)	17.95	71.72	444.09
Shen (1997)	17.96	71.85	449.07
Raju and Rao (1988)	17.86	71.45	446.56
		$(2\pi^4, 0)$	
Present	21.55	86.10	533.97
Kiani and Eslami (2012)	21.55	86.09	533.90
Shen (1997)	21.56	86.22	538.89
Raju and Rao (1988)	21.47	85.86	536.64
		$(5\pi^4, 0)$	
Present	32.33	129.21	803.42
Kiani and Eslami (2012)	32.33	129.20	803.34
Shen (1997)	32.33	129.33	808.33
Raju and Rao (1988)	32.27	129.08	806.77

Table 3 Effect of temperature dependency on vfor two-layered square FGM plate

Theory	$k = 0$	$k = 0.2$	$k = 0.5$	$k = 1$	$k = 2$	$k = 5$
Present (T−ID)	354.3428	315.9042	279.5835	247.6850	219.2483	193.0968
Kiani and Eslami (2012) (T−ID)	354.2707	315.9903	279.7846	247.9336	219.4674	193.2106
Shen (1997) (T−ID)	354.3356	315.9033	279.5919	247.7017	219.2681	193.1101
Present (T−D)	321.3564	226.7279	187.6934	163.1828	144.9072	129.5516
Kiani and Eslami (2012) (T−D)	321.3050	226.8111	187.6975	163.1947	144.9294	129.6938
Shen (1997) (T−D)	321.3503	226.7268	187.6960	163.1888	144.9149	129.5569

Solution to thermal post-buckling problem in works of Shen (1997) and Raju and Rao (1988) are determined based on regular perturbation and iterative non-linear finite elements method, respectively, and the solution in work of Kiani and Eslami (2012) is based on FSDT. However, the present solution is based on sinusoidal shear deformation theory and stress function. As observed, in this case, comparison is well-demonstrated.

Table 3 presents the buckling temperature difference for a two-layered FGM plate, and results are compared with those given by Shen (2007) based on an iterative two-step perturbation method. Both temperature-dependent material characteristics and non-dependent material characteristics are considered into account. Here, $T-D$ shows that the material characteristics are temperature dependent and $T-ID$ indicates the temperature independency of the material characteristics.

Table 4 demonstrates the thermal post-buckling behavior of an isotropic homogeneous square plate which is in contact with the Winkler elastic foundation and a comparison with the available data in the literature is carried out. Results give the non-dimensional thermal parameter defined by $\lambda_T = \dfrac{12(1+\nu)\alpha\Delta T b^2}{h^2\pi^2}$. This example demonstrates the accuracy and efficiency of the present formulation.

In Fig. 2, to confirm the accuracy of the present formulation in the case of imperfect plate (without elastic foundation), results of the present work are shown against those given in (Shen 2007) for a moderately thick homogeneous square plate ($h/b = 0.1$), when materials are considered to be temperature independent. As observed from Tables 2, 3, 4 and Fig. 2, comparisons are well-demonstrated.

5.2 Parametric studies

Fig. 3 shows the effect of temperature dependency of the material constituents on critical buckling temperature difference of the square plate without elastic foundation ($K_w = K_g = 0$). Linear

Table 4 Comparison on thermal deflection response of a thin perfect square homogeneous plate ($h/b = 0.01$, $\nu = 0.3$) in contact with the Winkler elastic foundation

K_w	Theory	W/h					
		0	0.2	0.4	0.6	0.8	1
0	Present	1.9989	2.1042	2.4202	2.9469	3.6842	4.6322
	Kiani and Eslami (2012)	2.0000	2.1053	2.4212	2.9477	3.6848	4.6325
	Shen (1997)	2.0000	2.1054	2.4231	2.9571	3.7144	4.7049
	Raju and Rao (1988)	1.9847	2.1058	2.4170	2.9528	3.7136	4.6990
π^4	Present	2.4989	2.6042	2.9202	3.4469	4.1842	5.1322
	Kiani and Eslami (2012)	2.5000	2.6053	2.9212	3.4477	4.1848	5.1325
	Shen (1997)	2.5000	2.6054	2.9232	3.4576	4.2160	5.2088
	Raju and Rao (1988)	2.4860	2.5897	2.9181	3.4540	4.2322	5.2174
$2\pi^4$	Present	2.9989	3.1042	3.4202	3.9469	4.6842	5.6322
	Kiani and Eslami (2012)	3.0000	3.1053	3.4212	3.9477	4.6848	5.6325
	Shen (1997)	3.0000	3.1054	3.4233	3.9581	4.7177	5.7129
	Raju and Rao (1988)	2.9874	3.0911	3.4197	3.9556	4.7335	5.7018

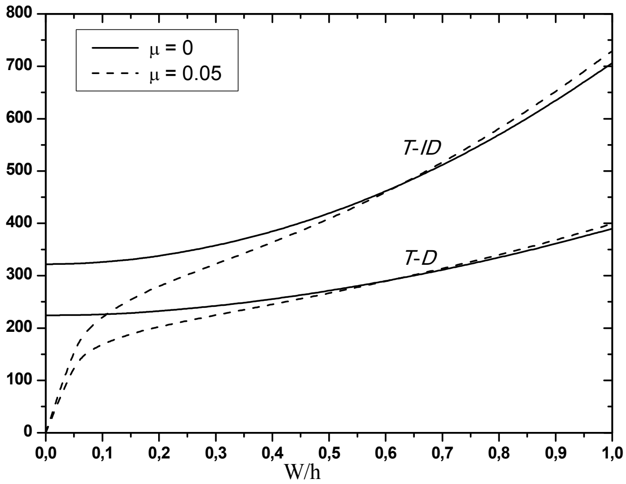

Fig. 4 Effect of temperature dependency on post-buckling response of perfect and imperfect sandwich square plates. Plates with all edges immovable simply-supported are pre-assumed (k = 1, a/h = 20, $K_w = K_g = 0$)

composition of material constituents is supposed for face sheets, and the other parameters are $b / a = 20h / b = 1$. As observed, the effect of temperature-dependent material characteristics is significant on ΔT_{Cr}. Therefore, when temperature dependency is not considered, the critical buckling temperatures become considerable. The critical buckling temperature difference of sandwich plates increases permanently when the thickness of metal core increases, because the thermal expansion coefficient of ceramic constituent is much more than that of metal.

The effect of considering temperature dependency of the material constituents on post-buckling response of sandwich plates is presented in Fig. 4. As can be observed, for perfect plate we found a bifurcation point in which buckling occurs, while for imperfect plates, there is no buckling point and plate commence to lateral deflection by initiation of thermal loading. Also, the impact of temperature dependency is significant, where the post-buckling curves for both perfect and imperfect plates become lower. Note that when W / h becomes larger, the effect of temperature dependency is more revealable. As plate deforms more and more, curves are highly descended when temperature dependency is considered.

Fig. 5 demonstrates the influence of elastic foundation on critical buckling temperature difference of perfect sandwich plates. As can be observed, the Winkler parameter of elastic foundation postpones the bifurcation point of plates in comparison with a foundationless plate. For plate without elastic foundation, both $T - D$ and $T - I\,D$ curves are completely smooth, which means that sandwich plate buckles in first modes for all values of a/b. For a plate resting on elastic foundation, some local extrema are found in the curves which demonstrate the alternation in buckling modes. Thus, the Winkler parameter of elastic foundation directly changes the buckling modes of the plate. As observed, for plates with/without elastic foundation, the critical buckling temperature is almost constant when a/b >2. However, these constant values are obtained under different buckled shapes of the plate.

Fig. 6 shows the elastic foundation influence on post-buckling response of square sandwich plate. Both $T - D$ and $T - I\,D$ cases are presented to assure the importance of temperature

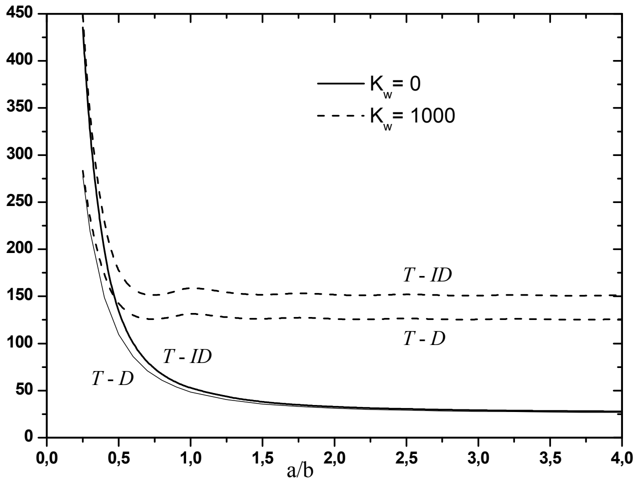

Fig. 5 Effects of elastic foundation and aspect ratio on ΔT_{Cr}. All edges are prevented from thermal expansion ($k = 1$, $h/b = 0.02$, $\gamma = h_H / h_f = 4$, $K_g = 0$)

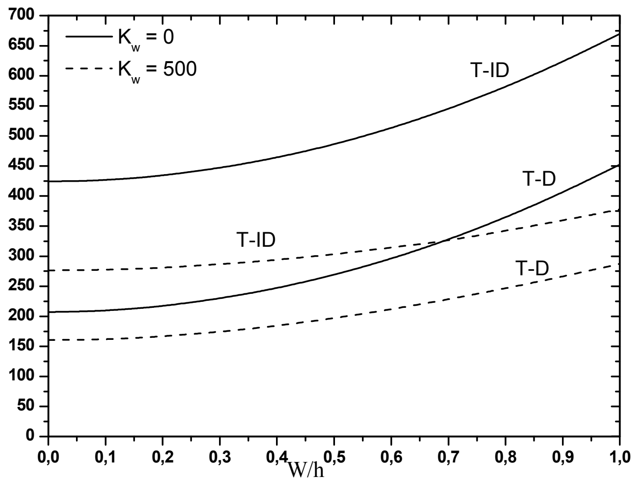

Fig. 6 Effects of temperature dependency and elastic foundation on temperature-deflection curves of perfect sandwich FG square sandwich plate ($k = 1$, $h/b = 0.04$, $\gamma = h_H / h_f = 4$, $K_g = 0$). All edges are assumed to be immovable

dependency influence. As expected, plates on elastic foundation have highly raised post-buckling curves due to the opposition of the elastic foundation against the plate deformation. The influence of temperature dependency is presented again, and it is remarked that for sandwich plates on elastic foundation, the effect of dependency of the material constituent to temperature is more significant.

Fig. 7 presents the load-deflection curves of both perfect and imperfect sandwich plates with various types of FG face sheets ($k = 0, 1, 10$). Here, an elastic foundation with Winkler coefficient $K_w = 0$ and Pasternak coefficient $K_g = 20$ resists against the deflection of the plate. As indicated in

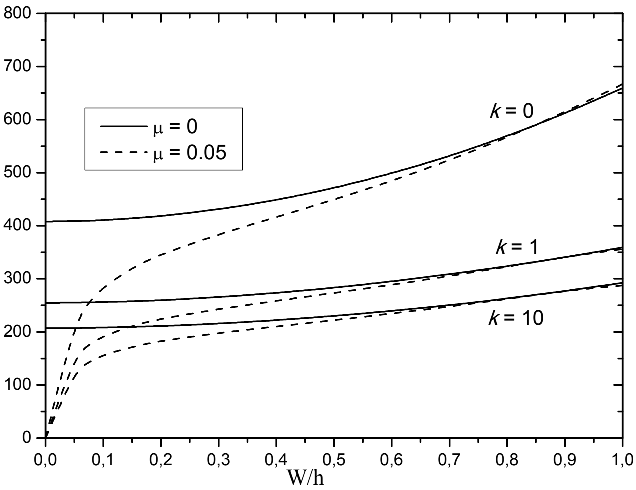

Fig. 7 Effects of geometrical imperfection and power law index on post-buckling response of sandwich FG square sandwich plate with all edges simply-supported (k = 1, h/b = 0.04, $\gamma = h_H / h_f = 4$, $K_k = 0$, $K_g = 20$)

Figs. 3, 4, 5, 6, to gain accurate load-deflection curves, the temperature dependency of the material constituents should be considered, and therefore, in Fig. 7, only $T - D$ is examined. Note that, for the imperfect plates, there is no bifurcation response and the curves are completely smooth. No sudden change is remarked in the temperature-deflection curve. This means that geometrically imperfect plates present bending when they are subjected to uniform thermal loading, while perfect plates follow bifurcation-type buckling. As observed, due to symmetrically mid-plane configuration of the structure and immovability of the boundary conditions, plate remains un-deformed in pre-buckling state, while a non-linear equilibrium path exists in post-buckling regime. As the power law index of FG layers increases, the temperature-deflection curves descend. Note that, however, the initial imperfection has significant influences on the primary response of the plate; this effect vanishes if someone follows the post-buckling path of the plate. As plate bends more and more, both imperfect and its associated perfect curves present the same response.

6. Conclusions

In the present work, an analytical approach to investigate the post-buckling behavior of sandwich plates with FGM face sheets supported by elastic foundations and subjected to uniform temperature rise loading. The derivation is based on the sinusoidal shear deformation plate theory and the stress function concept, with the assumption of power law composition for the constituent materials of FGM layers. The boundary conditions of plate on all edges are supposed to be simply supported with thermal expansion prevention. Temperature dependency of the core and FGM layers and initial geometrical imperfection of the plate are also considered in this work. It is concluded that:

- Temperature dependency of the material constituents has a considerable effect on the thermal buckling and post-buckling path. The critical temperatures are over-evaluated when materials are considered to be temperature independent. Also, temperature-deflection curves are over-predicted when independence of material characteristics to the temperature is carried out.

- Geometrical imperfection of the plate has a considerable influence on equilibrium path of the plate. Symmetrically mid-plane perfect plates follow bifurcation-type buckling, and hence, post-buckling paths exist, while imperfect plates exhibit bending with the onset of in-plane thermal loading.
- For plates with all edges immovable and without elastic foundation, thermal buckling occurs in first modes, while an elastic foundation may increase the buckling modes of the plate. Increasing each of the elastic foundation parameters increases the critical temperature. The Winkler parameter of elastic foundation has an important influence on the buckling modes, while the buckling modes of plates are independent of the Pasternak parameter of elastic foundation.

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