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An efficient method for reliable optimum design of trusses

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Abstract. This paper introduces a new and effective design amplification factor-based approach for reliable optimum design of trusses. This paper may be categorized as in the family of decoupled methods that aiming for a reliable optimum design based on a Design Amplification Factor (DAF). To reduce the computational expenses of reliability analysis, an improved version of Response Surface Method (RSM) was used. Having applied this approach to two planar and one spatial truss problems, it exhibited a satisfactory performance.

Keywords: design amplification factor; reliable; optimum; trusses

1. Introduction

Reliability-based design optimization (RBDO) is a concept that accounts for uncertainty all along the optimization process. The deterministic performance model is wrapped into a more realistic probabilistic constraint which is referred to as the failure probability. The general truss RBDO problem with both deterministic and probabilistic constraints can be formulated as (Zhou and Hong 2004)

$$\begin{aligned}
&\text{Min/Max} \quad f(\boldsymbol{d}) \\
&\text{Subject to:} \quad P(G_i(\boldsymbol{d},\boldsymbol{X}) \le 0) \le P_f^i, \quad i = 1, \dots, N_{PC} \\
&\qquad \text{and/or} \quad \sigma_j(\boldsymbol{d},\boldsymbol{X}) \le \sigma_{all.}, \quad j = 1, \dots, N_m \\
&\qquad \text{and/or} \quad u_k(\boldsymbol{d},\boldsymbol{X}) \le u_{all.}, \quad k = 1, \dots, N_{DOF} \\
&\qquad \text{and/or} \quad \boldsymbol{d}^L \le \boldsymbol{d} \le \boldsymbol{d}^U
\end{aligned} \tag{1}$$

Where $\boldsymbol{d} = [d_1, d_2, \dots, d_n]^T$ is a column vector of n deterministic design variables, $\boldsymbol{X} = [x_1,\ x_2, \dots, x_m]^T$ is the m-dimensional vector of random variables, $f(\boldsymbol{d})$ is the objective functio n, $P(G_i(\boldsymbol{d},\boldsymbol{X}) \le 0)$ denotes the failure probability for the i-th limit state function $G_i(\boldsymbol{d},\boldsymbol{X})$. P_f^i is the target failure probability of i-th constraint and N_{PC} is the number of probabilistic constraints. In Eq. (1), σ and u are the stress of jth member and the nodal displacement of k-th degrees of freedom, respectively. $\sigma_{all.}$, $u_{all.}$, $\boldsymbol{d}^L$, $\boldsymbol{d}^U$, N_m and N_{DOF} are allowable member stress, allowable nodal displacement, allowable lower and uper bounds of $\boldsymbol{d}$,

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total number of members and total number of degrees-of-freedom, respectively. The target failure probability could simply be expressed in terms of the target reliability index as $P_f^i = \Phi(-\beta_{ti})$, where $\Phi(.)$ is the standard normal cumulated distribution function (Lu *et al.* 2015).

The most common routines to solve Eq. (1) consists of 1- nesting a reliability analysis within a constrained optimization loop that are referred to as double or loop nested approaches. It is also known as the Reliability Index Approach (RIA), 2- decoupled approaches consist in decoupling the optimization loop from the reliability analyses. Such approaches are referred to as sequential approaches or decoupled approaches (Royset *et al.* 2001, Du and Chen 2004, Chen *et al.* 2013, Dizangian and Ghasemi 2015a), 3-Single-loop approaches (Kuschel and Rackwitz 1997, Kirjner-Neto *et al.* 1998, Kharmanda *et al.* 2002, Shan and Wang 2008, Mansour and Olsson 2016) attempt to fully reformulate the original RBDO problem into an equivalent DDO problem by means of classical optimization algorithms. As far as structural and mechanical RBDO is concerned, many attempts have been made through the last decades to reduce the computational expenses of reliability analysis using response-surface-based approaches (see e.g., Basaga *et al.* 2012, Bucher and Bourgund 1990, Guan and Melchers 2001, Kang *et al.* 2010, Lü *et al.* 2007, Zhao and Qiu 2013, Goswami *et al.* 2016, Zhao *et al.* 2016, Roussouly *et al.* 2013, Shu and Gong 2016). The present paper introduces an efficient framework for solving RBDO of trusses by implementing stress and/or displacement Design Amplification Factors (DAFs) into the design constraints in order to construct kind of hypothetical constraints surfaces after which it proceeds with the optimization procedure. Besides, in order to reduce the computational efforts, an improved version of RSM that was previously proposed by (Zhao and Qiu 2013) is employed for reliability analysis. To assess the performance of the proposed RBDO method, three truss problems were investigated.

2. Methodology

2.1 Description of conventional RSM

To improve the accuracy of the Response Surface Method (RSM), Bucher and Bourgund 1990, Roussouly *et al.* 2013, suggested an alternative process of selecting the experimental points. In the first step of this algorithm, the mean vector is selected as the center point. Then the RSF obtained is used to find an estimation of the design point $\boldsymbol{X}_D$. In the next step, the new center point $\boldsymbol{X}_M$ is chosen on a straight line from the mean vector $\overline{\boldsymbol{X}}$ to $\boldsymbol{X}_D$ so that $G(\boldsymbol{X}) = 0$ at the new center point $\boldsymbol{X}_M$ from linear interpolation, i.e.

$$\boldsymbol{X}_M = \overline{\boldsymbol{X}} + \left(\boldsymbol{X}_D - \overline{\boldsymbol{X}}\right)\frac{G\left(\overline{\boldsymbol{X}}\right)}{G\left(\overline{\boldsymbol{X}}\right) - G(\boldsymbol{X}_D)} \tag{2}$$

2.2 A brief description to the Zhao's improved RSM

For reliability analysis, at one stage, it is required to determine the reliability index (β). For this purpose, the improved RSM just as introduced and proposed by (Zhao and Qiu 2013) was utilized in the present study. For the sake of a better understanding of the modified RSM, a brief explanation is given here as follows:

(1) Select n+1 initial experimental points, $\overline{\boldsymbol{X}}$ and $\boldsymbol{X}_i = \overline{\boldsymbol{X}} - f\boldsymbol{\sigma}$, i = 1, 2, …, n. The value f = 3 has been recommended by several scholars (Kaymaz and Mcmahon 2005, Zhao and Qiu

2013, Kang *et al.* 2010). The value of f = 3 is also used in the present work.

(2) Calculate the values of $G(\overline{\boldsymbol{X}})$ and $G(\boldsymbol{X}_i)$ at the points selected in step 1.

(3) Calculate the differences between $G(\overline{\boldsymbol{X}})$ and $G(\boldsymbol{X}_i)$, as follows

$$F(\boldsymbol{X}_i) = \mathrm{G}(\overline{\boldsymbol{X}}) - \mathrm{G}(\boldsymbol{X}_i), \qquad i = 1, 2, \ldots, n \tag{3}$$

(4) Use the following expression to obtain the weight for each experiment point

$$w_i = \frac{F(\boldsymbol{X}_i)}{\sum_{j=1}^{n} |F(\boldsymbol{X}_j)|}, i = 1,2, \ldots, n \tag{4}$$

(5) Determine the control point in the standard normal space by employing from the following expression

$$\boldsymbol{U}_C = \sum_{i=1}^{n} w_i \boldsymbol{U}_i \,, i = 1,2, \ldots, n \tag{5}$$

where

$$\boldsymbol{U}_i = (\boldsymbol{X}_i - \overline{\boldsymbol{X}})./\boldsymbol{\sigma} \tag{6}$$

where $./$ represents the division of corresponding components between two vectors. Finally, the following equation is defined to express the control point of experiment points in the actual space

$$\boldsymbol{X}_C = \boldsymbol{U}_C . \times \boldsymbol{\sigma} + \overline{\boldsymbol{X}} \tag{7}$$

where $.\times$ represents the division of corresponding components between two vectors.

2.3 Design amplification factor-based design

2.3.1 Role of Design Amplification Factors (DAFs)

Most engineering optimization problems may be expressed as minimizing (or maximizing) a function subject to inequality and equality constraints and can be stated as the general form (Chen *et al.* 2013, Kaveh and Bakhshpoori 2015)

$$\begin{array}{lll} \text{Minimize/Maximize} & f(\boldsymbol{d}) & \\ \text{subject to} & \sigma_i(\boldsymbol{d}) \le \sigma_{all.} & i=1,\ldots,NM \\ \text{and/or} & u_j(\boldsymbol{d}) \le u_{all.} & j=1,\ldots,ND \\ \text{and/or} & \boldsymbol{d}^L \le \boldsymbol{d} \le \boldsymbol{d}^U & \end{array} \tag{8}$$

Where $\boldsymbol{d} = [d_1, d_2, \ldots, d_n]^T$ is a column vector of n deterministic design variables, *NM* stands for the number of members and *ND* is the number of degrees of freedom of the structure. In Eq. (8), f is the objective function, σ is the stress of i-th member and u represents the nodal displacement of j-th degrees of freedom. $\sigma_{all.}$, $u_{all.}$, $\boldsymbol{d}^L$ and $\boldsymbol{d}^U$ are the allowable values for member stress, nodal displacement and lower and upper bounds of $\boldsymbol{d}$, respectively. A design $\boldsymbol{d}$ that satisfies all inequality and equality constraints is referred to as feasible.

The normalized penalized objective function Z for Equation 8 related to the member stresses and nodal displacement may be defined as in Eq. (9) given (Belegundu and Chandrupatla 2011)

$$Z(\boldsymbol{d}) = \sum_{i=1}^{NM} [\max(0, \frac{\sigma_i(\boldsymbol{d})}{\sigma_{all.}} - 1)]^2 + \sum_{j=1}^{ND} [\max(0, \frac{u_j(\boldsymbol{d})}{u_{all.}} - 1)]^2 \tag{9}$$

Assuredly, the resulting optimal design will have an unqualified reliability. Instead of using probabilistic design to raise reliability, here design amplification factors, like the design codes, together with the mean values of random variables are entered into the constraint formulation of Eq. (10). The above equation can be modified as in Eq. (10) where the concept and significant role of DAF is introduced and applied (Dizangian and Ghasemi 2015a)

$$Z'(\boldsymbol{d}, \widehat{\boldsymbol{X}}) = \sum_{i=1}^{NM} [\max(0, |\frac{\sigma_i(\boldsymbol{d}, \widehat{\boldsymbol{X}})}{\gamma_s \sigma_{all.}}| - 1)]^2 + \sum_{j=1}^{ND} [\max(0, |\frac{u_j(\boldsymbol{d}, \widehat{\boldsymbol{X}})}{\gamma_d u_{all.}}| - 1)]^2 \tag{10}$$

Where Z' is called hypothetical constraint surface, γ_s and γ_d are defined here as Design Amplification Factors (DAF) corresponding to stress and displacement, respectively. In Eq. (10), $\widehat{\boldsymbol{X}}$ denotes vector of the mean values of random variables. Design amplification factors of γ_d and γ_s, similar to design safety factors, should be between 0 and 1. In the proposed approach, use of DAF in the objective function simply allows a combined reliability-based objective function for optimization. The new formulation of DAF-based RBDO is introduced as the following equation (Dizangian and Ghasemi 2015a)

$$\begin{gathered} \text{Find } \gamma^* \text{and corresponding } \boldsymbol{d}^* \\ \text{subject to:} \\ Z'(\boldsymbol{d}, \widehat{\boldsymbol{X}}) = 0 \\ P_f(\boldsymbol{d}^*, \widehat{\boldsymbol{X}}) = P_f^{Target} \end{gathered} \tag{11}$$

where in Eq. (11), in the current study the probability of failure P_f is then computed using an improved Response Surface Method (RSM).

2.4 Procedure of the proposed DAF-based RBDO of trusses

The essence of the algorithm is to reduce substantially the searching time for the RBDO point by allowing a confined number of design amplification factors for which the P_f should be computed. For such aim, the polynomial curve-fitting concept was utilized. Fig. 1 contains the flowchart of the proposed RBDO procedure.

The following steps details the routine. In all steps the optimum points are computed for the mean values of the random variables $\widehat{\boldsymbol{X}} = [\mu_{x_1}, \mu_{x_2}, \dots, \mu_{x_m}]^T$. P_f^a means the probability of failure corresponding to $\gamma = a$.

(1) First for the mean value of design amplification factor, $\gamma = 0.5$, first optimum design $\boldsymbol{d}^*$ will be determined with regard to the allotted value for γ embedded in Z' of Eq. (10). The $P_f^{0.5}$ will then be computed using an improved RSM.

(2) Compare P_f^{Target} with $P_f^{0.5}$ as a result of which the range of γ^* will be found according to Eq. (12)

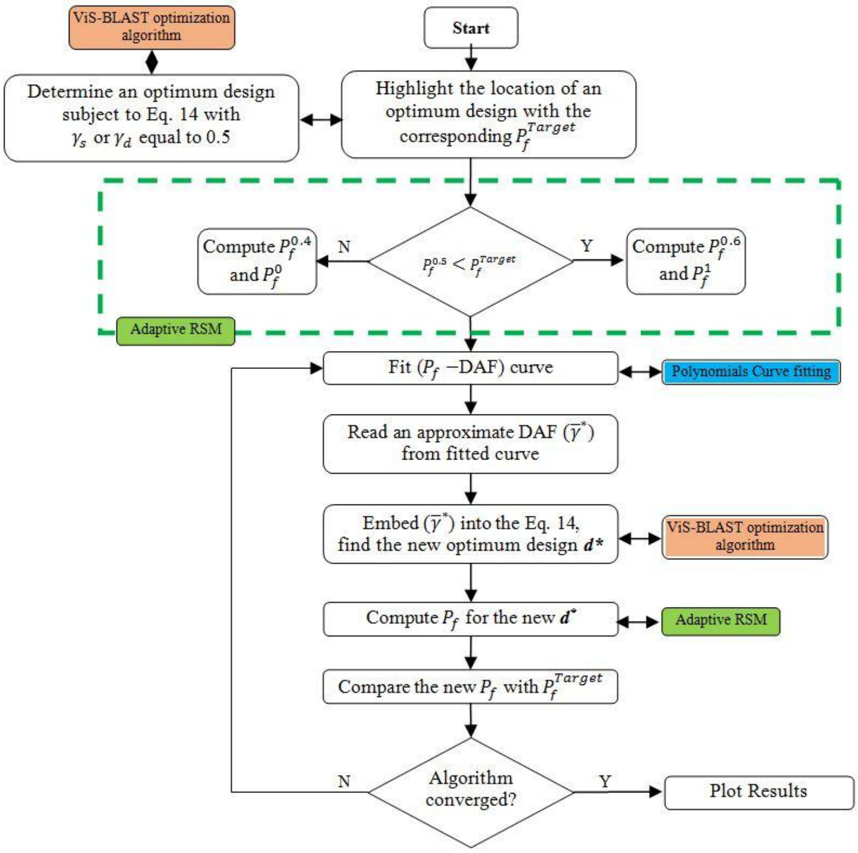

Fig. 1 Flowchart of the proposed RBDO of trusses

$$\begin{cases} (I) \quad \text{if} \quad P_f^{0.5} > P_f^{Target} \quad \text{search for} \quad \gamma^* \quad 0 < \gamma < 0.5 \\ (II) \quad \text{if} \quad P_f^{0.5} < P_f^{Target} \quad \text{search for} \quad \gamma^* \quad 0.5 < \gamma < 1 \end{cases} \tag{12}$$

In most structural problems, computing $P_f^{0.4}$ for region *I* and $P_f^{0.6}$ for region *II* could be very efficient. In case of (*I*), compute the values of P_f for the two auxiliary points $\gamma = 0$ and 0.4; in case of (*II*), compute the values of P_f for the two auxiliary points $\gamma = 0.6$ and 1. It should be noted that when $\gamma = 0$ that means P_f is obviously equal to zero. This is a case where the structure becomes fully over-designed, a case not being a part of the aim of the present work.

(3) Fit $(P_f \; vs. \; \gamma)$ curve by using the coordinates of the 3 points found.
(4) Extract the approximate DAF $\bar{\gamma}^*$corresponding to the value of P_f^{Target} given, from the fitted curve of step 3.
(5) Use DDO approach to find the optimum design point $\boldsymbol{d}^*$subject to Z' based on the extracted $\bar{\gamma}^*$using the mean values of random variables $\widehat{\boldsymbol{X}}$.
(6) Determine the P_f^{t+1} for the deterministic optimum design $\boldsymbol{d}^*$of step 6 using an improved RSM.
(7) Compute the relative distance $error^{P_f}$ between P_f^{t+1} and P_f^{Target} using Eq. (13)

$$error^{P_f} = \frac{P_f^{t+1} - P_f^{Target}}{P_f^{Target}} \tag{13}$$

(8) Check convergence. If relative distance $error^{P_f}$ is smaller than the desired value of tolerance δ, the P_f^{t+1} will be assigned as the P_f^{Target} and the reliability based optimum design is said to be found.

(9) If according to Eq. (13) the convergence has not occurred, the P_f^{t+1} will be considered as another auxiliary point to more accurately fit the curve. Steps 3 to 9 will be repeated until converged and the reliable optimum design point $\boldsymbol{d}^*$ will be recorded.

3. Examples

Three truss problems will be investigated here. First example is a ten-bar planar truss studied by Zhao and Qiu 2013 and Dizangian and Ghasemi 2015a. Second example is a seventeen-bar planar truss which has been studied by several researchers as a deterministic design optimization problem. However, in the present study it will be investigated from the RBDO point of view. The third example is the RBDO study of a twenty-five-bar space truss benchmarked for the first time by Ho-Huu *et al.* 2016. In all examples, ViS-BLAST method (Dizangian and Ghasemi 2015b and c) for optimization and adaptive RSM method (Zhao and Qiu 2013) for reliability analysis were utilized. The permissible error for P_f in all examples is $\delta = 1\%$.

3.1 A Ten-bar truss problem

3.1.1 Definition

A 10 bar truss structure is considered as illustrated in Fig. 2. Random variables include five basic variables reflecting different properties of structural components, the modulus of elasticity E, the length of bar L and loads P_1, P_2 and P_3 are basic random variables. All variables are assumed independent and normally distributed as listed in Table 1. The cross-sectional areas of 10 bars are design variables. The lower and upper bound of design variables are 0.0001 m^2 and 0.002 m^2, respectively.

The total area of bars is to be minimized. For this example, two types of limit states were considered such as:

(1) Serviceability limit state: Maximum vertical displacement of node 3, which should be less than 0.004 m as recorded by (Zhao and Qiu 2013) with the target reliability index of 2.5, i.e., the target failure probability 6.21×10^{-3}.

(2) Strength and Serviceability limit states: Limit state of Case (1) together with the stress constraints of 200 MPa equal to the yield stress F_y for all members. In order to show the impact of target reliability level on the optimization results for Case 2, two different target levels of reliability were also considered. These reliability levels were considered as the target reliability indexes of 2.5, and 3.5 with the corresponding probability of failure 6.21×10^{-3} and 2.326×10^{-4}, respectively. An implicit limit state function of Case (1) is expressed as in Eq. (14) for P_f calculation

$$G = 0.004 - u_{3,y}(\boldsymbol{d}^*, \boldsymbol{X}) \tag{14}$$

Table 1 Statistical properties of random variables, 10-Bar truss

Variable	Distribution	Unit	Mean value	Coefficient of variation(C.V.)
P_1	Normal	kN	60.00	0.2
P_2	Normal	kN	40.00	0.2
P_3	Normal	kN	10.00	0.2
E	Normal	Gpa	200.00	0.1
L	Normal	m	1.00	0.05

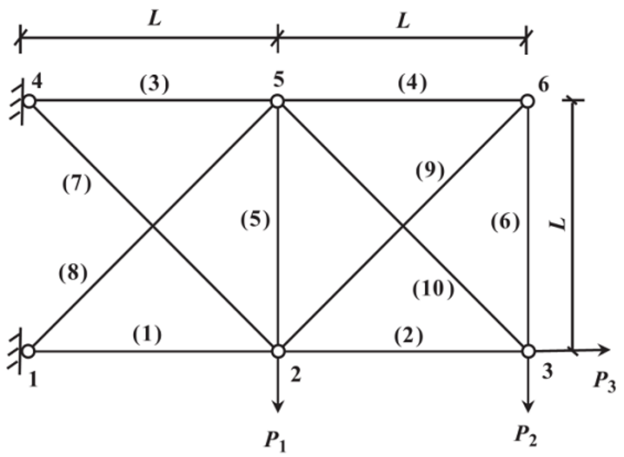

Fig. 2 10-Bar truss

Considering Eq. (10), the following hypothetical constraint formulation is introduced as utilizing the displacement DAF (γ_d) to obtain optimum deterministic points

$$Z'\left(\boldsymbol{d},\widehat{\boldsymbol{X}}\right) = [\max(0, \frac{u_{3,y}\left(\boldsymbol{d},\widehat{\boldsymbol{X}}\right)}{\gamma_d 0.004} - 1)]^2 \tag{15}$$

In a similar way, limit state function and hypothetical constraint formulation for Case (2) may respectively be regarded as Eqs. (16) and (17)

$$\begin{cases} G_1 = 0.004 - u_{3,y}(\boldsymbol{d}^*, \boldsymbol{X}) \\ G_2 = F_y - |\sigma_i(\boldsymbol{d}^*, \boldsymbol{X})| \end{cases} \tag{16}$$

and

$$Z'\left(\boldsymbol{d},\widehat{\boldsymbol{X}}\right) = [\max(0, \frac{u_{3,y}\left(\boldsymbol{d},\widehat{\boldsymbol{X}}\right)}{\gamma_d 0.004} - 1)]^2 + \sum_{i=1}^{10}[\max(0, \frac{\sigma_i\left(\boldsymbol{d},\widehat{\boldsymbol{X}}\right)}{\gamma_s F_y} - 1)]^2 \tag{17}$$

In Eqs. (16) and (17), the aim is to find an optimum design amplification factors of γ_d^* and γ_s^* that give an optimum solution $\boldsymbol{d}^*$, such that $P_f(\boldsymbol{d},^*\boldsymbol{X})$ is less than the P_f^{Target} considering the allowable probability of failure error δ.

Table 2 RBDO results of 10-Bar truss, Case 1

Variable group	Bar areas	Optimal cross sectional area (m^2) ×10^{-4}							
		Zhao and Qiu 2013	Dizangian and Ghasemi 2015a	Current work: Initial sample points			Current work: Auxiliary sample points (Iterations)		
				$\gamma_d = 0.5$	$\gamma_d = 0.6$	$\gamma_d = 1$	$\overline{\gamma}^*_{d,1} = 0.61$	$\overline{\gamma}^*_{d,2} = 0.626$	$\overline{\gamma}^*_{d,3} = 0.6264$
1	A_1	10.705	10.000	12.500	13.999	6.250	10.000	10.000	10.000
2	A_2	5.914	5.042	6.250	10.000	3.125	7.2	5.040	5.040
3	A_3	14.424	14.0280	17.562	10.000	8.693	13.832	13.9627	13.951
4	A_4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
5	A_5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
6	A_6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
7	A_7	5.531	9.000	12.500	10.000	6.250	9.000	9.000	9.000
8	A_8	11.853	10.000	12.500	10.000	6.250	10.000	10.000	10.000
9	A_9	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
10	A_{10}	11.223	10.000	12.500	10.000	6.250	10.000	10.000	10.000
Total area (×10^{-4})		**63.649**	**62.068**	**77.812**	**67.999**	**40.818**	**64.0322**	**62.0027**	**61.991**
β (RSM)		2.777	2.5017	3.0618	2.747	0.1	2.683	2.5050	2.5022
Exact P_f (MCS)		2.742 ×10^{-3}	6.15 ×10^{-3}	0.13 ×10^{-3}	2.6 ×10^{-3}	0.43	3.64 ×10^{-3}	6.122 ×10^{-3}	6.17 ×10^{-3}
$(error^{P_f}$×100)%		55.8	0.966				41.27	1.41	0.64

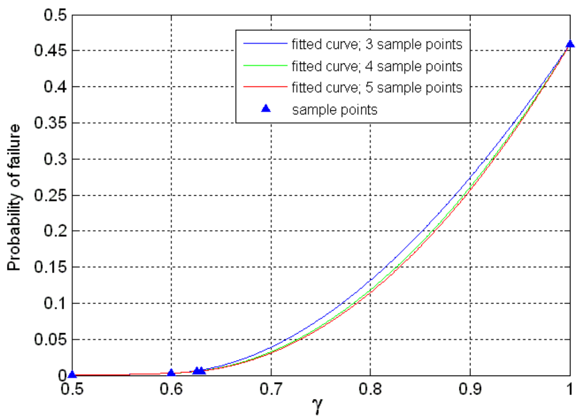

Fig. 3 Converged fit plots of P_f vs γ; 10-bar truss, Case 1

3.1.2 Results obtained

Case (1): Having found $P_f^{0.5}$ and compared the value with P_f^{Target}, the range $0.5 < \gamma^* < 1$ was selected. To first fit the curve, the two initial points corresponding to $\gamma = 0.6$ and 1 were computed for P_f.

The procedure then was repeated for three iteration until the curve was fitted for three auxiliary points as a result of which $error^{P_f}$ was found less than 1%. Fig. 3 illustrates the convergence leading to the optimum design corresponding to the P_f^{Target} . The results were alse listed in

Table 3 Properties of the initial sample points for two levels of reliability; 10-bar truss, Case 2

Variable group		Optimal cross sectional area (m^2) ×10^{-4}		
		Initial sample points		
		$\gamma_{d,s} = 0.5$	$\gamma_{d,s} = 0.6$	$\gamma_{d,s} = 1$
1	A_1	16.334	13.699	8.274
2	A_2	7.732	9.031	3.544
3	A_3	12.5	10	6.25
4	A_4	1.000	1.000	1.000
5	A_5	1.000	1.000	1.000
6	A_6	1.000	2.577	1.000
7	A_7	12.5	10	6.250
8	A_8	12.5	10	6.125
9	A_9	1.000	1.000	1.000
10	A_{10}	12.5	10.000	6.125
Total area (×10^{-4})		**78.067**	**68.309**	**40.818**
Computed P_f (MCS)		0.147×10^{-3}	3.394×10^{-3}	0.495

Table 4 Comparison of the RBDO results of 10-bar truss for two levels of reliability, Case 2

Variable group		Optimal cross sectional area (m^2) ×10^4						
		Auxiliary sample points ($\beta = 2.5$)			Auxiliary sample points ($\beta = 3.5$)			
		$\overline{\gamma}^*_{d,s,1} = 0.6218$	$\overline{\gamma}^*_{d,s,2} = 0.623$	$\overline{\gamma}^*_{d,s,3} = 0.6227$	$\overline{\gamma}^*_{d,s,1} = 0.5142$	$\overline{\gamma}^*_{d,s,2} = 0.5177$	$\overline{\gamma}^*_{d,s,3} = 0.5158$	$\overline{\gamma}^*_{d,s,4} = 0.5152$
1	A_1	13.226	13.171	13.201	16.152	15.991	16.330	16.236
2	A_2	6.443	6.185	6.443	6.956	6.102	6.121	6.121
3	A_3	10.000	10.000	10.000	12.500	12.500	12.500	12.500
4	A_4	1.000	1.0309	1.000	1.000	1.000	1.000	1.000
5	A_5	1.000	1.000	1.000	1.000	1.000	1.000	1.000
6	A_6	1.000	1.000	1.000	1.000	1.000	1.000	1.000
7	A_7	10.000	10.000	10.000	12.500	12.500	12.500	12.500
8	A_8	9.8	10.000	9.800	11.289	11.760	11.760	12.250
9	A_9	1.000	1.000	1.000	1.000	1.000	1.000	1.000
10	A_{10}	10.000	10.000	10.000	12.500	12.500	12.500	12.250
Total area ×10^4		**63.469**	**63.388**	**63.444**	**75.898**	**75.353**	**75.711**	**75.857**
β (RSM)		2.6	2.489	2.506	3.527	3.44	3.466	3.502
Exact P_f (MCS)		5.995×10^{-3}	6.281×10^{-3}	6.18×10^{-3}	2.053×10^{-4}	2.846×10^{-4}	2.52×10^{-4}	2.3×10^{-4}
($error^{P_f}$×100)%		3.46	1.14	0.483	11.74	22.3	8.32	1

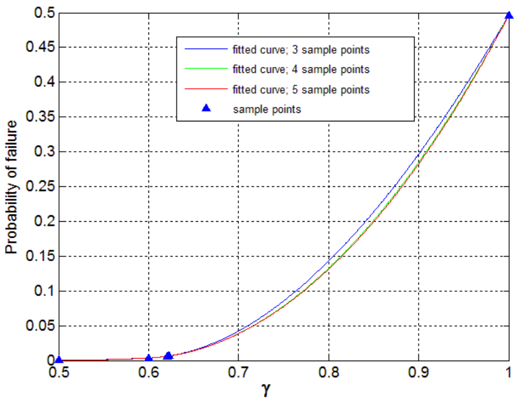

Fig. 4 Converged fit plots of P_f vs γ; 10-bar truss, Case 2 (Beta = 2.5)

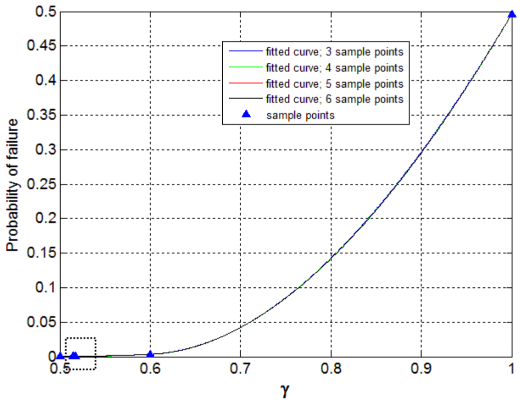

Fig. 5 Converged fit plots of P_f vs γ; 10-bar truss, Case 2 (Beta = 3.5)

Table 2. To compare the results, the exact P_f was also computed using Monte Carlo Simulation (MCS) (Ghorbani and Ghasemi 2011). It is worth to note that through the whole process of reliability based optimization with the proposed technique, the P_f was only computed for 6 point, where the converged optimum design possesses a P_f less than the target and the optimum weight was slightly lighter than that in the literature.

Case (2): Similar to Case (1), after computing $P_f^{0.5}$, it was found that the optimum DAFs are located in the range between 0.5 and 1. Table 3 contains the properties of the initial sample points that were generated for both reliability levels. The results of RBDO are presented in Table 4 for two different levels of target reliability.

As seen in Table 4, failure probability evaluated by adaptive RSM satisfies the target value with a relative error less than 1% for both reliability levels. The exact P_f was also determined with MCS to show the effectiveness of proposed DAF-based RBDO. In the case of $\beta = 3.5$, the reliable optimum solution was found with the objective function value of 75.857×10^{-4} m^2 that is about 20% greater than the objective value of 63.444×10^{-4} m^2 resulted by the reliability index equal to 2.5.

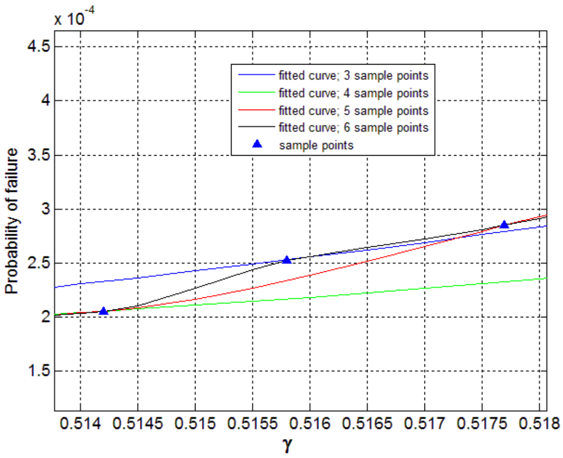

Fig. 6 Close-up of converged fit plots around an optimum DAFs; 10-bar truss, Case 2 (Beta = 3.5)

The converged fitted curves are illustrated in Figs. 4 and 5 for three and four sample points, as the reliability indexes of 2.5 and 3.5, respectively.

From Fig. 5, it is obvious that the optimum DAFs are located in finite range very close to each other; for this reason, Fig. 6 was also plotted for ease of comparison of these four fitted curves.

3.2 A Seventeen-Bar planar truss problem

3.2.1 Definition

Fig. 7 shows the schematic of 17-bar planar truss. In terms of deterministic design optimization, this truss has been studied by several researchers including Adeli and Kumar 1995 and Lee and Geem 2004. Here the RBDO of this truss is concerned. All members were assumed to be made of a material with an elastic modulus of E = 30,000 ksi (206.85 GPa) and a mass density of 0.268 lb/in^3 (7418.21446 kg/m^3). The members were subjected to stress limitations of ±50 ksi (344.7379 MPa), and displacement limitations of ±2.0 in. (5.08 cm) were imposed on all nodes in both directions (x and y) (Lee and Geem 2004). The loading consists of a single vertical downward load of P = 100 kips (444.822 kN) at node 9. In the current work, in order to get more practical design,

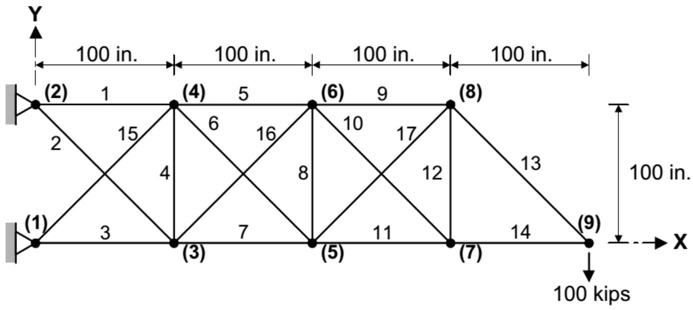

Fig. 7 17-Bar planar truss (Lee and Geem 2004)

Table 5 RBDO results of 17-Bar planar truss

Variable Group	Bar areas	Current work						
		Initial sample points			Auxiliary sample points (Iterations)			
		$\gamma_d = 0.5$	$\gamma_d = 0.6$	$\gamma_d = 1$	$\overline{\gamma}^*_{d,1} = 0.73$	$\overline{\gamma}^*_{d,2} = 0.77$	$\overline{\gamma}^*_{d,3} = 0.81$	$\overline{\gamma}^*_{d,4} = 0.798$
1	$A_{1,3}$	29.873	24.904	15.207	20.12	17.746	19.65	17.74
2	$A_{5,7}$	19.259	16.472	9.751	14.90	13.89	12.00	12.80
3	$A_{9,11}$	12.034	10.584	6.192	8.1	8.25	7.66	7.80
4	$A_{4,8,12}$	1.998	1.182	1.012	1.22	1.35	1.11	1.13
5	$A_{13,14}$	10.370	8.653	5.222	7.087	6.48	5.76	6.53
6	$A_{2,15}$	6.063	5.162	2.795	4.23	4.07	3.61	3.66
7	$A_{6,16}$	6.218	5.097	2.888	3.85	3.98	3.52	3.73
8	$A_{10,17}$	6.175	4.887	3.107	3.78	3.90	3.90	4.14
Weight (lb.)		**5509.394**	**4588.312**	**2755.342**	**3768.3**	**3574.434**	**3406.47**	**3443.712**
β (RSM)		4.99	4.53	-0.006	4.152	3.46	2.81	3.05
Exact P_f (MCS)		2×10^{-7}	3×10^{-6}	0.498	1.7×10^{-5}	0.000292	0.002381	0.001213

seventeen elements of truss were linked in eight groups illustrated in Table 5. The minimum cross-sectional area of the members was 0.1 in.^{2}. RBDO problem involves 10 random design variables including the cross-sectional area A_i of each members group, Young's modulus E and the external force P. All random variables were assumed statistically independent, normally distributed and have covariant of 5% of variable values. This RBDO problem was solved for the targeted reliability index β equal to 3 (P_f = 0.001349) only for displacement constraints.

3.2.2 Results obtained

The reliability-based design optimization results of 17-bar truss are given in Table 5. Table 5 also indicates that through the whole process of RBDO using the proposed DAF-based technique, the P_f was only computed for 7 point (3 initial and 4 auxiliary sample points), where the converged optimum design possesses a P_f less than that of the target. From these results, it is obvious that an adaptive RSM has acceptable performance since its results match those of the MCS.

According to Table 5, the deterministic optimum solution explored a global optimum weight of 2755.342 lb. with the γ_d equal to 1. For the RBDO, the converged γ_d was found equal to 0.798 corresponding to the global optimum weight of 3443.712 lb, a 20% heavier than that of the deterministic solution.

Figs. 8 and 9 show the converged fit plots at the end of four iterations. From these figures, with respect to the target P_f value set as 0.001349, one realizes that the zone through which the global optimum DAF is located, as the forth sample point with the value of 0.798, was inspected very fast.

3.3 A Twenty five-bar space truss problem

3.3.1 Definition

Fig. 10 shows a 25-bar space truss. This problem has been studied by many researchers including Togan *et al.* 2008, Dede *et al.* 2011, Kaveh *et al.* 2007 and Dizangian and Ghasemi 2015 b and c, as a deterministic truss optimization problem. Recently, Ho-Huu *et al.* 2016, solved this work as a RBDO problem by assuming probabilistic characteristics for some design variables and parameters. For this space truss, the material density was 0.1 lb./in.^{3} (2767.990 kg/m^3) and the modulus of elasticity was 10,000 ksi (68.950 GPa). This space truss was subjected to the loading condition as given in Table 6. Design constraints were considered as the maximum allowable

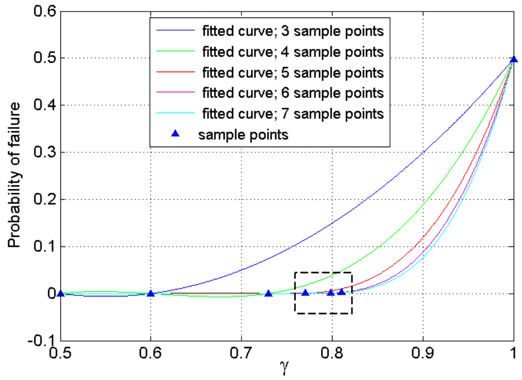

Fig. 8 Converged fit plots of P_f vs γ; 17-bar truss

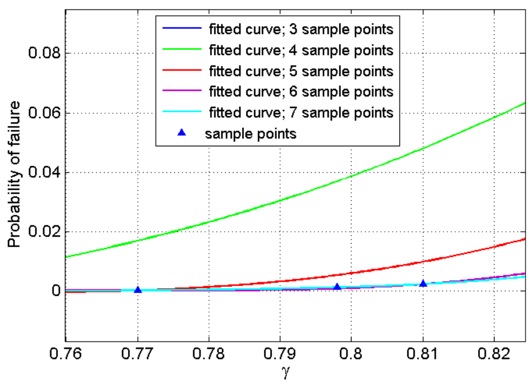

Fig. 9 Close-up of converged fit plots around an optimum DAFs; 17-bar truss

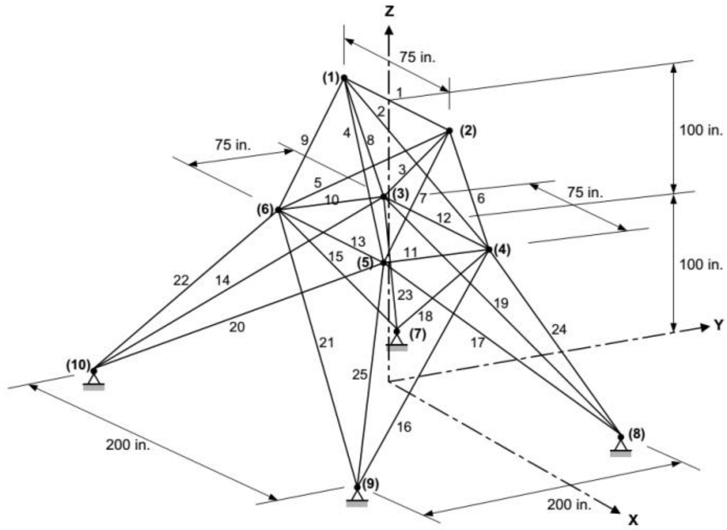

Fig. 10 25-Bar space truss

Table 6 Loading condition (kips) for 25-bar space truss (Ho-Huu *et al.* 2016)

Node	*x*	*y*	*z*
1	1.0	-10.0	-10.0
2	0	-10.0	-10.0
3	0.5	0	0
6	0.6	0	0

Note: 1 kips = 4.45 kN

displacement of ±0.35 in. (±8.89 mm) imposed on nodes 1, and 2 in *x* and *y* directions and the allowable stress for all members equals to ±40 ksi (±275.89 MPa). The minimum admissible cross-sectional areas of all members were set equally as 0.1 in^2 (6.45 mm^2). For consistency with the literature, all members were classified into eight groups as given in Table 7. For this problem, the random variables including the cross-sectional area A_i of each group, Young's modulus E and the external force P which are all considered to be statistically independent and follow normal distribution. The covariant of all random variables (C.O.V) is 5% of variable values. The targeted P_f for this problem was considered equal to 0.001349 (β = 3) for displacements of all nodes (Ho-Huu *et al.* 2016).

3.3.2 Results obtained

The reliable global optimum solution for the 25 bar truss example was converged through seven P_f computation, the results of which are presented in Table 7. The corresponding optimum weight using the proposed DAF-based RBDO was found equal to 660.804 lb. The reason for it being slightly heavier than that reported by Ho-Huu *et al.* 2016 is due to the fact that the solution obtained by Ho-Huu *et al.* (2016), deviates the target P_f value of 0.001349 (β = 3) by a small amount. This may be because SORA technique makes assumptions for solving RBDO problems.

Similar to Example 2, Figs. 11 and 12 show the converged fit plots again at the end of four

Table 7 RBDO results of 25-Bar space truss

Variable Group	Bar areas	Optimal cross sectional area (m^2) ×10^{-4}							
		Ho-Huu *et al.* 2016	Current work						
			Initial sample points			Auxiliary sample points (Iterations)			
			$\gamma_d = 0.5$	$\gamma_d = 0.6$	$\gamma_d = 1$	$\overline{\gamma}^*_{d,1} = 0.71$	$\overline{\gamma}^*_{d,2} = 0.76$	$\overline{\gamma}^*_{d,3} = 0.79$	$\overline{\gamma}^*_{d,4} = 0.793$
1	A_1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
2	$A_{2\text{-}5}$	2	0.131	0.18	1.25	0.365	0.845	0.1	0.1
3	$A_{6\text{-}9}$	3.4	7.632	6.82	2.5	5.305	4.56	5.27	4.71
4	$A_{10\text{-}11}$	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
5	$A_{12\text{-}13}$	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
6	$A_{14\text{-}17}$	1.2	1.820	1.45	1.25	1.404	1.08	1.02	0.86
7	$A_{18\text{-}21}$	1.9	4.44	3.44	2.5	2.4	1.94	2.28	2.71
8	$A_{22\text{-}25}$	3.4	4.87	4.05	2.5	3.98	4.19	3.53	3.57
Weight (lb.)		**659.527**	**1050.794**	**875.538**	**580.978**	**737.708**	**686.386**	**662.91**	**660.804**
β (RSM)			4.98	4.54	-0.1004	4.26	3.49	2.98	3.01
Exact P_f (MCS)		0.0023	1×10^{-7}	2×10^{-6}	0.5614	5×10^{-6}	0.00019	0.001195	0.00132

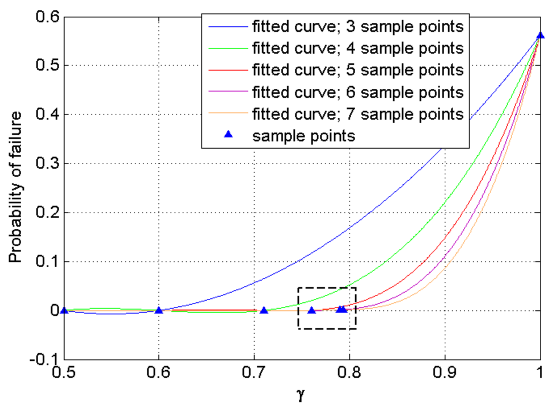

Fig. 11 Converged fit plots of P_f vs γ; 25-bar truss

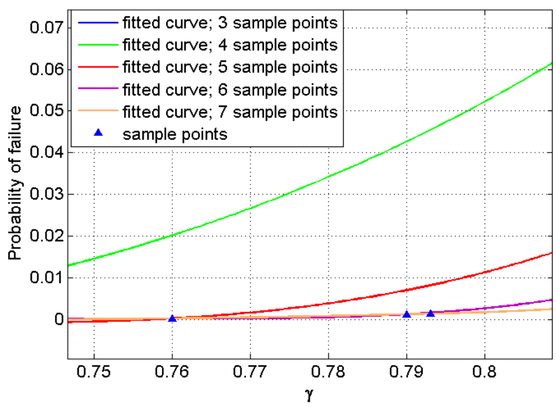

Fig. 12 Close-up of converged fit plots around an optimum DAFs; 25-bar truss

iterations. From these figures, one comprehends that the zone through which the global optimum DAF is located, being the forth sample point with the value of 0.793, was inspected very fast.

4. Conclusions

The proposed Design Amplification Factor-based RBDO approach was applied to three benchmark truss problems for obtaining reliability based optimum design. The features of the proposed technique may be summarized as follows: (1) The method introduces an objective function depending on a design amplification factor and allowing an inverse approach to find and lead to the targeted P_f with the aid of hypothetical constraint surfaces; (2) To find a design amplification factor corresponding to the targeted P_f, the interpolation curves of P_f versus DAF were modified each time with the new founded point; (3) An improve version of RSM shows a satisfactory performance, as its results compare to the MCS method, have an excellent accuracy.

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