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Progressive collapse analysis of steel frame structure based on the energy principle

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Abstract. The progressive collapse potential of steel moment framed structures due to abrupt removal of a column is investigated based on the energy principle. Based on the changes of component's internal energy, this paper analyzes structural member's sensitivity to abrupt removal of a column to determine a sub-structure resisting progressive collapse. An energy-based structural damage index is defined to judge whether progressive collapse occurs in a structure. Then, a simplified beam damage model is proposed to analyze the energies absorbed and dissipated by structural beams at large deflections, and a simplified modified plastic hinges model is developed to consider catenary action in beams. In addition, the correlation between bending moment and axial force in a beam during the whole deformation development process is analyzed and modified, which shows good agreement with the experimental results.

Keywords: progressive collapse; energy; sensitivity; modified plastic hinges model; steel frame

1. Introduction

Progressive collapse is a catastrophic structural failure that is caused by local structural damage that cannot be prevented by the inherent continuity and ductility of the structural system. (Ellingwood and Dusenberry 2005, Rezvani and Asgarian 2014, Mirtaheri and Abbi Zoghi 2016). The local damage or failure initiates a chain reaction of failures that propagates through the structural system, leading to an extensive partial or total collapse. The resulting damage is disproportionate to the local damage caused by the initiating event. Such local initiating failures can be caused by abnormal loads not considered in standard structural design. Abnormal loads include gas explosions, vehicular collisions, sabotage, severe fires, extreme environmental effects and human errors in design and construction etc. (Porcari *et al*. 2015, Agarwal and Varma 2014).

Past research on progressive collapse has proceeded in waves initiated in the aftermath of high profile failures, particularly the 1968 Ronan Point incident, 1995 Murrah Federal Building bombing and the 2001 World Trade Center collapse. The alternate path method is one of the most

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commonly used approaches to systematically investigate the load redistribution behavior of a building structure in resisting progressive collapse. With this method, critical load-bearing structural elements are notionally removed and then the damaged structure is analyzed to assess its ability to bridge over the removed elements (DoD 2009). In practical application of this method, effects such as the collapsing of structural elements from upper floors onto the remaining structure are typically not considered. Nonlinear dynamic analysis is one of the most accurate techniques for alternate path analysis. However, the associated computational expense is very high and may even be prohibitive for progressive collapse analysis of large scale, complicated buildings. Besides, skilled analysts are generally needed to perform the nonlinear dynamic analysis and interpret the computer results (Min Liu, 2013). Investigation of the load redistribution behavior of a damaged building using amplified gravity loads is often termed pushdown analysis (Kim *et al*. 2009, Khandelwal and El-Tawil 2011) and has been adopted in the current progressive collapse design guidelines (DoD 2009).

This paper was aimed to understanding the characteristics of progressive collapse in moment resisting steel framed buildings. The initial structural responses to abnormal loadings (e.g., a plane crash, an impact, an explosion) were not considered. Damage caused by the conditions cited above was assumed to result in abrupt removal of columns at critical locations. The main objective of current study was to enable the development of a rational energy-based analysis of progressive collapse of moment resisting steel framed buildings.

2. The sensitivity analysis of structure

The sensitivity of structural components to abrupt removal of a column is performed based on energy principle by the SAP2000 software to get the elastic strain energy of structure elements. The coefficient for importance of a member in progressive collapse analysis is given by

$$R_{j,importance} = \frac{\left|U_j^d - U_0\right|}{n \cdot U_0} \tag{1}$$

where: j is the number of a removed component and n is numbers of spans directly affected by the removed column. U_j^d and U_0 are the total elastic strain energy of damaged structure and the total elastic strain energy of undamaged structure, respectively. The most critical analysis case can be determined based on the $R_{j,\mathrm{importance}}$.

The component's internal energy (elastic deformation work) is defined as

$$E_{i,\mathrm{int}} = \iiint_{V_i} u_i \, dV \tag{2}$$

where: i is the number of the undamaged component; u_i and V_i are elastic strain energy density per unit volume and the volume of a member, respectively. If the component is formed by single material and has uniform section, then

$$u_i = \frac{E_{i,\mathrm{int}}}{V_i} \tag{3}$$

The change of the elastic strain energy density per unit volume is given by

$$\Delta u_i = \frac{\left|\Delta E_{i,\text{int}}^d\right|}{V_i} = \frac{\left|E_{i,\text{int}}^d - E_{i,\text{int}}^0\right|}{V_i} \tag{4}$$

where: $E_{i,\text{int}}^0$ is the internal energy (elastic deformation work) of a member before damaged, $E_{i,\text{int}}^d$ is the internal energy (elastic deformation work) of a member after damaged, and $\Delta E_{i,\text{int}}^d$ is the change in internal energy of a member before and after removal of a column.

Furthermore, the relative change in the elastic strain energy density per unit volume of a member is

$$\Delta u_{i,rel} = \frac{\Delta u_i}{\max\limits_i \{\ \Delta u_i\ \}} \tag{5}$$

The sensitivity of all structural undamaged components is defined as

$$R_{i,j,sensitivity} = \Delta u_{i,rel} \tag{6}$$

where: *i* is the number of a undamaged component, *j* is the number of a removed component.

According to the sensitivity of a structural component, we can get a sub-structure resisting progressive collapse. Then, the proposed sub-structure becomes an analysis target instead of a single beam or column in traditional analysis process. The whole structural performance resisting progressive collapse is characterized by analyzing the behavior of the sub-structure in a column removal scenario. Thus, the analysis process proposed in current study is relatively more reasonable.

3. The failure energy of structure

Energy-based method is focused on providing structures with energy dissipating capacities that are larger or equal than their expected energy demands (Bojórquez *et al*. 2010, Szyniszewski 2009). The design requirements of a steel moment frame structure can be formulated as

$$E_{EC} \geq E_{ED} \tag{7}$$

where: E_{EC} the structural energy dissipating capability after damaged (Energy Capacity) and E_{ED} is the energy demand to support upper structure after removal of a column.

Eq. (7) can be expressed as an energy-based structural damage index

$$I_{DE} = \frac{E_{ED}}{E_{EC}} \tag{8}$$

If $I_{DE} > 1$, then progressive collapse occurs in a steel moment frame structure, and the structure should be redesigned and strengthened. If $I_{DE} \leq 1$, the structure is safe.

Structural details, such as walls and partitions, can affect the response of the structure. Including these secondary elements in analysis would increase the complexity of the responses.

Therefore, the effects of secondary elements were not considered because the response of the major load carrying members should be clarified. Among all the energies absorbed and dissipated by a structure, the total failure energy of a column, E_{column}, and the total failure energy of a beams considering catenary action, E_{beam}, are crucial.

$$E_{EC} = E_{Beam} + E_{Column} = \sum_i R_{i,j,sensitivity} \cdot E_{i,beam} + \sum_k R_{k,j,sensitivity} \cdot E_{k,column} \tag{9}$$

The inherent potential energy of the upper structure is

$$E_{ED} = \sum_i R_{i,j,sensitivity} \cdot E_{i,potential,beam} + \sum_k R_{k,j,sensitivity} \cdot E_{k,potential,column} \tag{10}$$

where $E_{i,beam}$ is the flexural failure energy considering catenary action of a beam and $E_{k,column}$ is the failure energy of a column. $E_{i,potential,beam}$ and $E_{k,potential,column}$ are the external force work by applied forces due to deformation of a beam and a column, respectively. It should be noted that steel frames is regular, and is designed according to the strong column-weak beam approach, and exhibited fairly stiff beam-column connections in engineering practice. Thus, the paper assumes that the column and connecting joint are enough strong and cannot be destroyed in progressive collapse, namely, $E_{k,column} = 0$ and $E_{k,potential,column} = 0$. Thus, Eqs. (9) and (10) can be simplified as Eqs. (11) and (12), respectively.

$$E_{EC} = E_{Beam} = \sum_i R_{i,j,sensitivity} \cdot E_{i,beam} \tag{11}$$

$$E_{ED} = \sum_i R_{i,j,sensitivity} \cdot E_{i,potential,beam} \tag{12}$$

3.1 The failure energy of beam ($E_{i,beam}$)

A steel frame beam under normal load is mainly subjected to the combined effect of moment and shear force, and the axial force in steel beam is usually ignored. With the increase of load, bending moments at midspan and ends of the steel frame beam increase accordingly, which develop plastic hinges at these locations and large deflection at the beam midspan. In this case, the steel beam endures considerable axial force, which cannot be neglected when calculating the response of the beam. As the cross section yields fully, the axial force in the steel beam can further increase to resist the additional external loading, which results in decreasing of the bending moment at plastic hinge section caused by the plastic interaction between axial forces and bending moments. Finally, the steel beam mainly depends on the axial force to resist external loading and the steel beam fails at midspan or ends. This process is commonly referred as tensile catenary action (Wang *et al*. 2009, McConnell *et al*. 2015, Yin and Yang 2005, Izzuddin *et al*. 2008).

A simplified beam damage model is proposed in this paper to analyze beam failure energy incorporating catenary action, as shown in Fig. 1(a), satisfying the inelastic axial load–bending moment interaction. Four moment plastic hinges and two force plastic hinges are identified and the rest of the beam is treated as a rigid rod. Furthermore, in consideration of catenary action in beams, a modified plastic hinge model is developed in current study on base of the Federal Emergency Management Agency (FEMA 356 2000), as show in Figs. 1(b) and (c). The moment begins to decrease gradually after reaching flexural strength and the beam axial force begins to increase until rotational deformation of the beam achieves the maximal displacement value $\theta_{\max}$, and then the beam is broken.

Beam motion can be decoupled into rotation (producing bending in the moment hinges) and extending (producing extending in the force hinges) of beams. The key variable for the quantitative analysis is total vertical displacement (δ) by rotation and extending

$$\delta = v_\theta + v_\Delta = L_b^0 \cdot \sin\theta + \Delta \cdot \sin\varphi \tag{13}$$

$$\varphi = \cos^{-1}\left(\frac{L_b^0}{L_b^0 + \Delta}\right) \tag{14}$$

where: v_θ and v_Δ are vertical displacement due to rotation and extending, respectively. Δ and φ are the extending deflection and the corresponding rotation (McConnell *et al*. 2015). L_b^0 is the distance between the two moment plastic hinges, as shown in Fig. 1(a). $L_b^0 = (1-d)L_b$, and d is a length factor of a moment plastic hinge, equal to 0.1 in current study.

The plastic deformations (b_P and b_M) and the residual strength (c_P and c_M), as shown in Figs. 1(b) and (c), are determined based on the FEMA 356 (2000). Then, Beam's expected flexural strength and axial yield force of the member are calculated by

$$M_y = Zf_{ye} \tag{15}$$

$$P_y = (2A_f + A_w) \cdot f_{ye} \tag{16}$$

And the rotation at yield is

$$\theta_y = Zf_{ye}L_b / 6EI_b \tag{17}$$

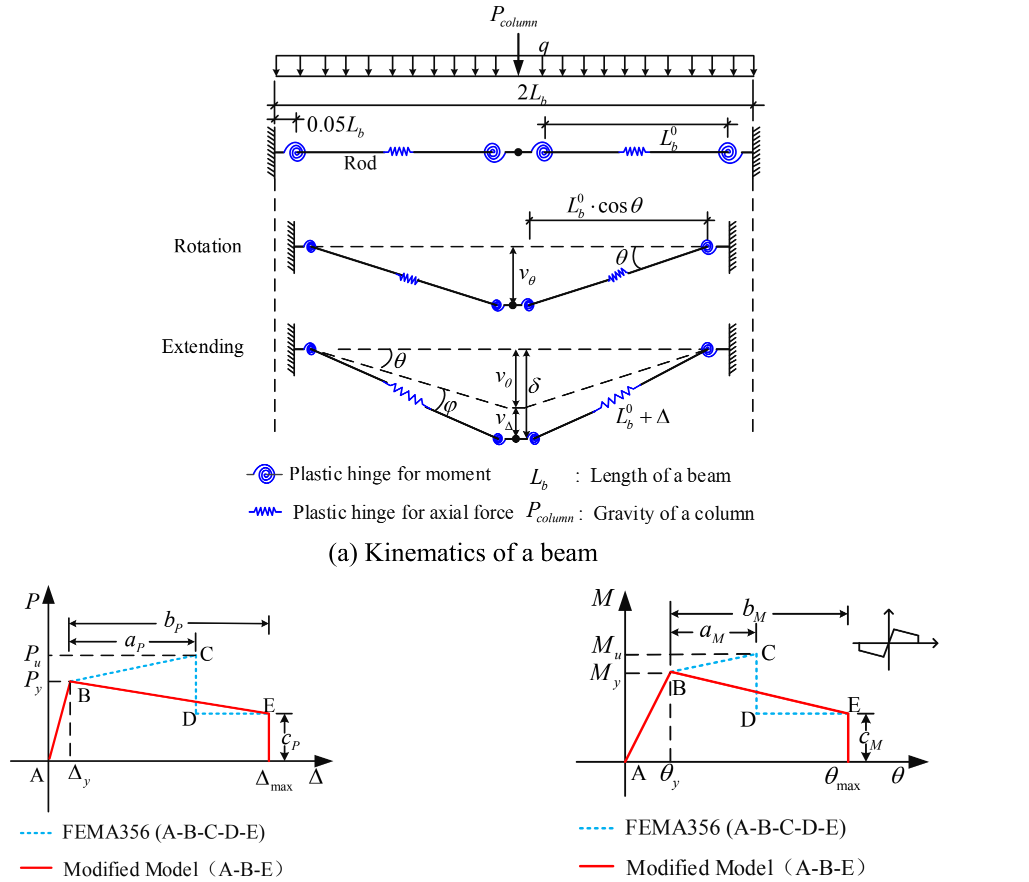
where: f_{ye} is expected yield strength of the material; L_b is beam length; Z is plastic section modulus; I_b is moment of inertia.

Catenary action is considered in current study, when the whole cross-section yields, the axial force and moment correlation equation is given by Yin and Yang (2005). If the plastic neutral axis is in the web, then

$$\frac{M}{M_y} + \frac{(1+\alpha)^2}{\alpha[2(1+\beta)+\alpha]}\left(\frac{P}{P_y}\right)^2 = 1 \tag{18}$$

where: $\alpha = A_w = (2A_f)$, A_w is web plate area and A_f is flange area ; $\beta = t / h_0$, t is flange thickness and h_0 is web plate height. M and P are bending moment and axial force, respectively; M_y and P_y are expected flexural strength and axial yield force of the member, respectively. If the plastic neutral axis is in the flange, then

$$\frac{1-\gamma}{1-\dfrac{(1+\alpha)^2\gamma^2}{\alpha[2(1+\beta)+\alpha]}}\frac{M}{M_y} + \frac{P}{P_y} = 1 \tag{19}$$



(a) Kinematics of a beam

(b) Modified axial force deformation relation for plastic force hinges

(c) Modified moment rotation relation for a beam plastic moment hinges

Fig. 1 The simplified mechanical damage model of a beam

where: $\gamma = A_w / (2A_f + A_w)$.

However, flange thickness t is far smaller than web plate height h_0 in engineering practice, the ratio of flange thickness to web height is approximately equal to zero, namely, $\beta \approx 0$. For simplicity, the curve from Eq. (18) can be replaced by a straight line (Wang *et al*. 2009). Thus the Eqs. (18) and (19) can be rewritten as Eqs. (20) and (21), respectively

$$\frac{M}{M_y} + \zeta\frac{P}{P_y} = 1 \tag{20}$$

$$\lambda\frac{M}{M_y} + \frac{P}{P_y} = 1 \tag{21}$$

where $\zeta = (1 + \alpha)^2 / [\alpha(2 + \alpha)]$; $\lambda = (1 - \gamma) / (1 - \zeta\gamma^2)$.

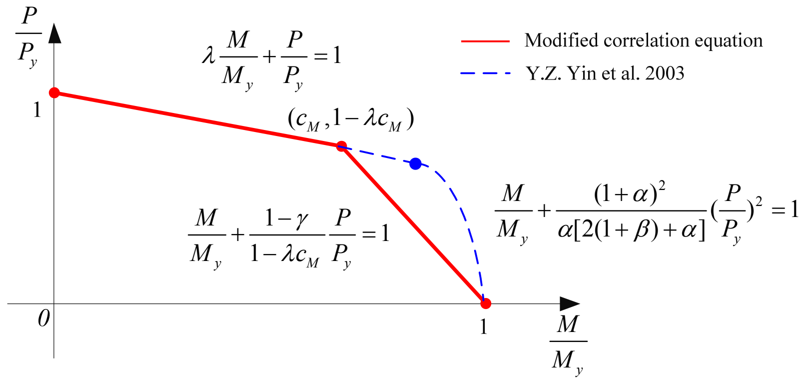


Fig. 2 The simplified modified relationship between bending moment and axial force

Based on the experimental results (Sadek *et al*. 2009, 2013, Yang and Tan 2013a, b, c), However, it is obviously found that axial force of a beam cannot reach the axial yield force P_y before the beam is broken. So, it is assumed in current study that if the plastic neutral axis is up to critical point of the web and flange in section, meanwhile the moment and rotational deformation of the beam are $c_M \cdot M_y$ and $\theta_{\max}$, respectively, and then the beam fails. Hence, combining with the proposed modified plastic hinge model shown in Fig. 1(c), the Eq. (20) can be modified as the Eq. (22), and the modified axial force and moment correlation curve is shown in Fig. 2.

$$\frac{M}{M_y}+\frac{1-\gamma}{1-\lambda c_M}\frac{P}{P_y}=1 \tag{22}$$

Different failure phases in a structural beam are given in Fig. 3. The relationship of beam axial force and vertical displacement is shown in Fig. 3(a). The failure of a beam has two different failure mechanisms, as shown in Fig. 3(b). The two primary mechanisms are beam mechanism (Phase ①-②) and catenary mechanism (Phase ②-⑥), respectively.

According to the Figs. 2 and 3, the analyses indicate that in the early stages of the behavior was dominated by flexure and the beam axial force is zero, as shown in Fig. 3. With increased vertical displacement, axial tension develops in the beams, and the behavior of catenary action will occur. The axial force in the beams increases with increased downward displacement, until the beam can no longer bear the combined axial and flexural stresses, resulting in the failure of the beam. In other words, if moment and rotational deformation of the beam reach $c_M \cdot M_y$ and $\theta_{\max}$ respectively, meanwhile, the corresponding axial force of the beam and tensile deformation reach $(1-\lambda c_M)P_y$ and Δ_3, respectively, and then the beam will fail and not sustain the combined axial and flexural stresses.

Hence, the plastic deformation work of a beam in the limit state is given by

$$E_{i,beam}=2\int_0^{\theta_{\max}} M(\theta)\,\mathrm{d}\theta+\int_0^{\Delta_3} P(\Delta)\,\mathrm{d}\Delta \tag{23}$$

where: Δ_3 and $\theta_{\max}$ are the axial tensile length and the maximum rotation, respectively.

Meanwhile, the maximum vertical displacement at the beam midspan is calculated by

$$\delta_{\max} = v_\theta + v_\Delta = L_b^0 \cdot \sin\theta_{\max} + \Delta_3 \cdot \sin\varphi \tag{24}$$

And the external force work due to deformation of a beam in the limit state is

$$E_{i,potential,beam} = q_i \cdot \left[\frac{1}{2}\delta_{\max}(dL_b + L_b^0 \cdot \cos\theta_{\max})\right] + P_{i,column} \cdot \delta_{\max} \tag{25}$$

where: $P_{i,column}$ is the gravity of a column over a beam, q_i is the distributed load applied on a beam.

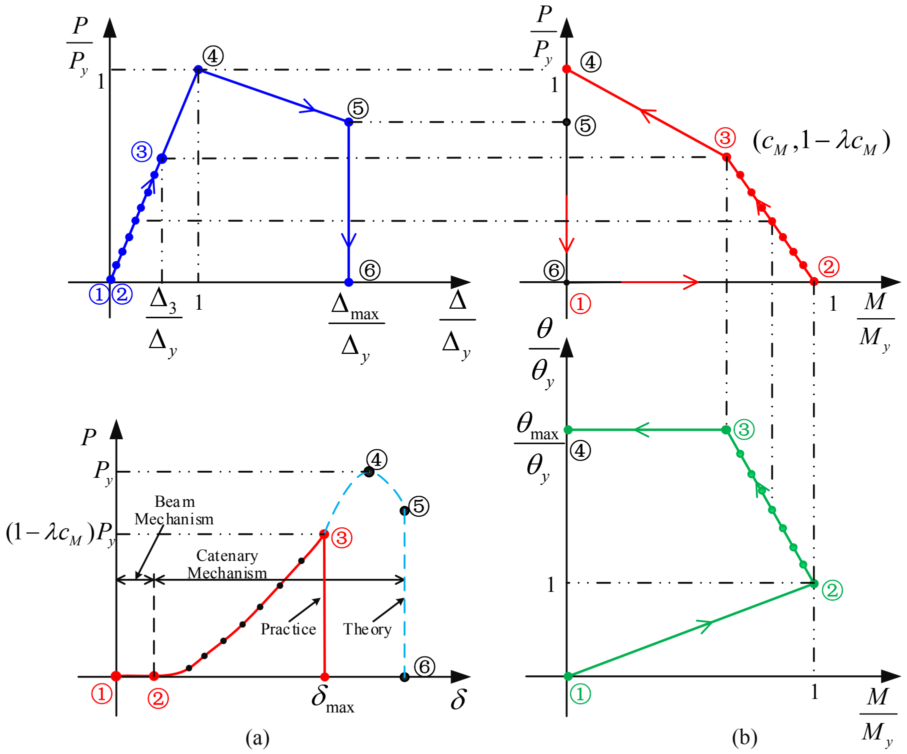


Fig. 3 The damaged process of a beam incorporating catenary action. (a) Beam axial force versus vertical displacement of center column; (b) Different phases in process of failure

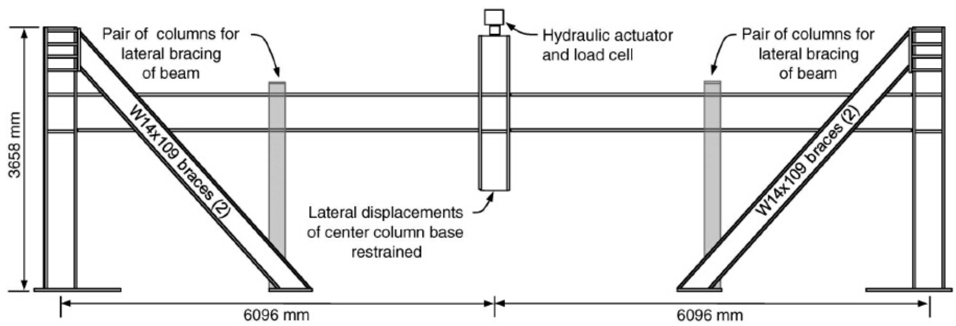


Fig. 4 Fahim's test set-up and instrumentation layout (Sadek *et al*. 2009, 2013)

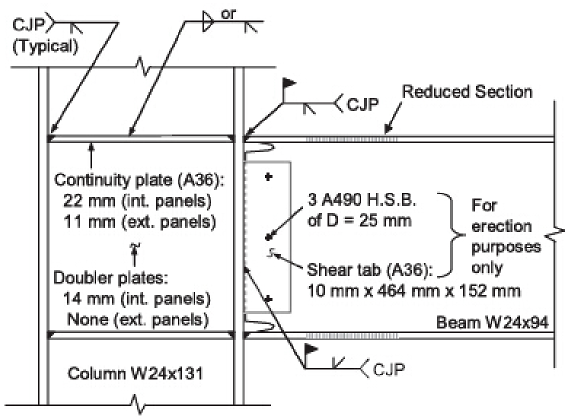


Fig. 5 RBS connection details

Fig. 6 Failure mode of steel SMF assembly (Sadek *et al*. 2009, 2013)

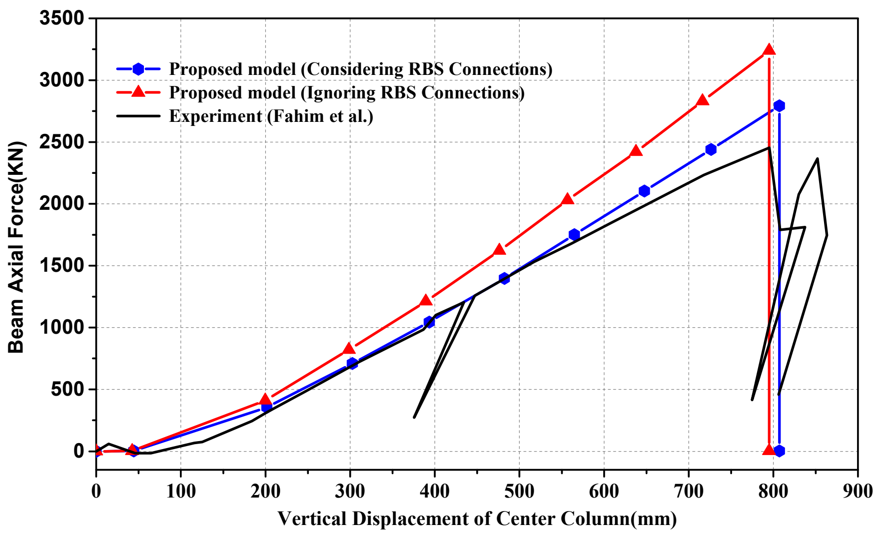


Fig. 7 Beam axial force versus vertical displacement of center column

3.2 Model validation

Fahim *et al*. (2009, 2013) conducted an experimental assessment of the performance of steel beam-column assemblies with two types of moment-resisting connections under a middle-column-removal scenario, which are Bolted Web Connection (WUF-B) and Reduced Beam-Section Connection (RBS), respectively. The test set-up and instrumentation layout are shown in Fig. 4.

A validation analysis is performed on action of a beam connected to columns by RBS connections, which accords with the assumptions that the structures are designed according to the strong column-weak beam approach, and stiff beam-column connections. The RBS connection is created by cutting away a portion of the top and bottom flanges of the beam at a distance from the beam-column interface so that yielding is concentrated in this reduced area. The reduced section thus acts as a fuse to protect the connection against premature fracture. The radius-cut section was found to yield the most reliable performance (FEMA 2000). The RBS connection has been commonly used for seismic design since the 1994 Northridge earthquake in California. FEMA 355D (FEMA 2000) provides extensive information on the testing and performance of the RBS connections under seismic loading. The RBS connection details and the failure modes are given in Figs. 5 and 6, respectively.

The experiment results of the axial force in the beams versus the vertical displacement and the results of the proposed model in current study are presented in Fig. 7. It can be observed that in the

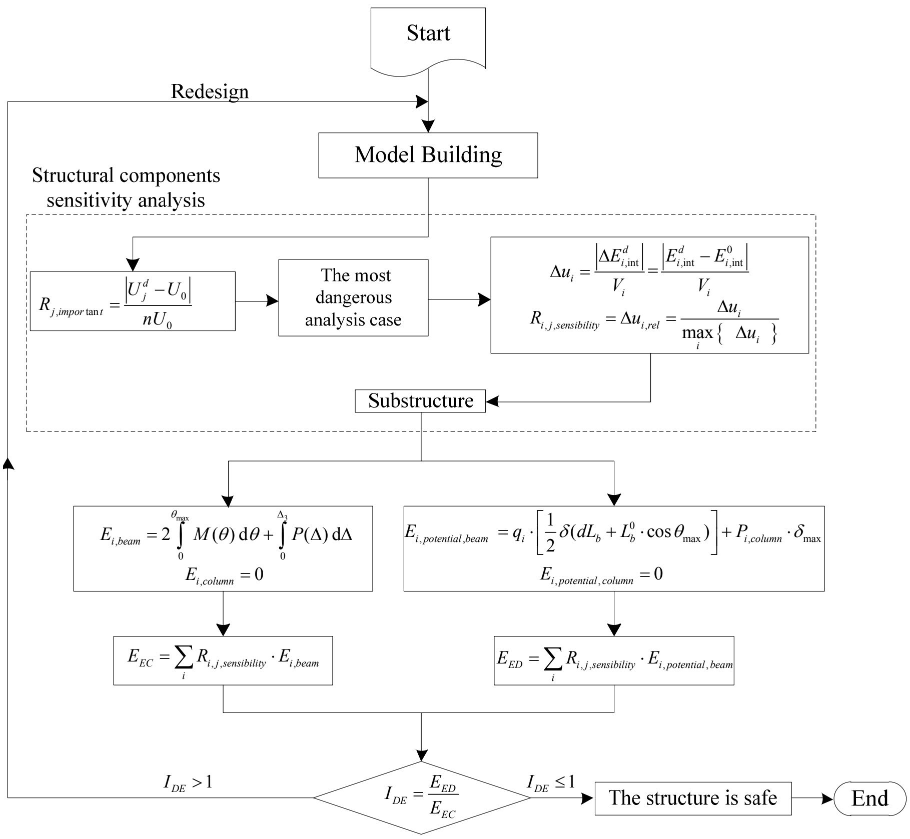


Fig. 8 General flow chart of simplified analysis of progressive collapse based on energy principle

early phases of the response, the behavior was dominated by flexure with less axial forces in the beams. With the increase of vertical displacement, axial forces developed in the beams and the behavior was dominated by catenary action. The axial force in the beams increased with increased downward displacement, until the beam could no longer carry the combined axial and flexural stresses, resulting in the failure of the beam and the fracture propagated from bottom flange to web. At the time of failure, the axial force in the beams was about 2450 kN and the vertical displacement was approximately 850 mm. The failure was characterized by fracture of the bottom flange in the middle of the reduced section of one of the connections near the center column. The modified model (considering RBS connections by reducing the flange width) proposed in current study shows good agreement with the experiment results. The proposed model considering RBS connections can be used for analysis of complete structural systems connected by RBS connections between beams and columns to assess their vulnerability to disproportionate collapse.

3.3 The frame of calculating program

General flow chart of the simplified analysis of progressive collapse of steel frame structure based on energy principle proposed in current study is shown in Fig. 8.

4. Analysis example

4.1 The analysis model

The building in our example is a nine-story steel frame structure, with six bays in the longitudinal direction and three in the transverse direction in accordance with the AISC Load Resistance Factor Design (2003). The longitudinal direction has a uniform column spacing of 8.25 m (27 ft), while on the three-bay side columns are spaced every 9.75 m (32 ft). Main girders are W21×57. Floor-to-floor height for every story is 4.3 m (14 ft). W14×159 columns span from the ground to the fifth floor and W14×90 columns span from the sixth floor to the roof. A three-

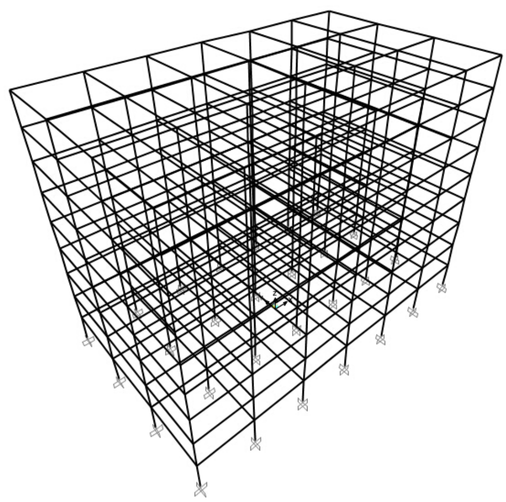


Fig. 9 Three-dimensional model of example building

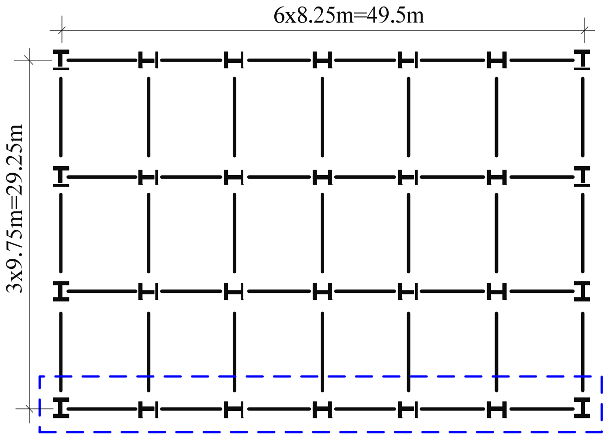


Fig. 10 Structural plan of the model structure

Table 1 Girder Section and Material Properties

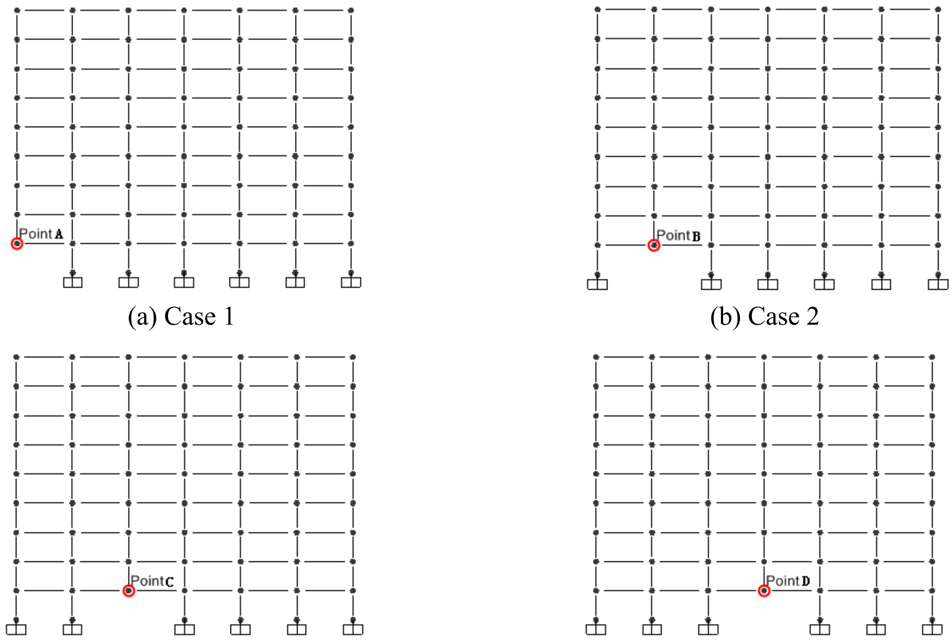
Girder section	W21×57 (AISC-LRFD 2003)
Expected yield strength	f_{ye} = 348 MPa (55 ksi)
Modulus of elasticity	E=2×10^5 MPa (29,000 ksi)
Moment of inertia	I = 4.84×10^{-4} m^4 (1170 in.^{4})
Plastic modulus	Z = 2.11×10^{-3} m^3 (129 in.^{3})
Plastic moment	$M_p = f_{ye}Z$ = 803.3 kN·m (592.8 kips·ft)
Flange compactness	$b_t / 2t_f = 5.04 < 52 / \sqrt{55} = 7.01$
Web compactness	$h / t_w = 46.3 < 418 / \sqrt{55} = 56.36$

dimensional finite-element model of the structure is shown in Fig. 9 and the structural plan is shown in Fig. 10. The two-dimensional frame enclosed in the dotted rectangle in Fig. 10 is taken out for analysis objects. Material and girder section properties used in this study are shown in Table 1.

For checking the capacity of a structure to withstand the effects of an extraordinary event, according to the recommendations of General Services Administration (GSA, 2003), the following load combination was used in the subsequent analysis

$$\text{Load} = 1.0\,\text{DL} + 0.5\,\text{LL} \tag{26}$$

where DL = dead load, which is generated based on element volume and material density; and LL = live load and is assumed to be 3.92 N/mm distributed uniformly across the entire beam span including roof. To estimate dead load, we have assumed uniform concrete slab thickness of 90 mm (3.5 in), with normal weight concrete density of 23.6 kN/m^3 (150 pcf), and the equivalent linear load is 4.28 N/mm. Additionally, dead load includes perimeter wall weight of 19.7 N/mm (1,350 plf) at every floor, except roof level.



(a) Case 1 (b) Case 2

(c) Case 3 (d) Case 4

Fig. 11 Four types of different analysis cases

4.2 Progressive collapse analysis based on proposed energy method

4.2.1 The sensitivity analysis

In this paper we attempt to simplify, conceptually explain and determine the most important column in first story so as to analyze progressive collapse by performing study of four different analysis cases, as shown in Figs. 11(a)-(d).

In case1, the corner column A in the first story is removed to initiate progressive collapse; The column B closing to the corner column in the first story is removed in case 2; The column C closing to the central column in the first story is removed in case 3 and the central column D in the first story is removed in case 4, as shown in Figs. 11 (a)-(d), respectively.

The coefficients of importance of four columns removed in different cases are listed in Table 2 based on the Eq. (1). In addition, a comparative finite element analysis by SAP2000 software between two-dimensional structure and three-dimensional structure is performed. The analysis results obtained from three-dimensional structure are listed in Table 3.

Table 2 The coefficients in two-dimensional structure

Case / Analysis results	Case 1	Case 2	Case 3	Case 4
U_i^d (kN·m)	165.43	212.49	201.39	201.21
U_0 (kN·m)	74.89	74.89	74.89	74.89
$R_{j,importance}$	1.21	0.92	0.84	0.84

Table 3 The coefficients in three-dimensional structure

Analysis results \ Case	Case 1	Case 2	Case 3	Case 4
U_j^d (kN·m)	1992.51	2070.98	2049.94	2049.57
U_0 (kN·m)	1726.67	1726.67	1726.67	1726.67
$R_{j,importance}$	0.15	0.10	0.09	0.09

According to the results of the coefficients of importance of structural components in Table 2 and Table 3, it can be found that case 1 is the most critical analysis case. Thus, the corner column A in the first story is removed in case 1 for analysis, as shown in Fig. 11(a). The structural component's number is show in Fig. 12.

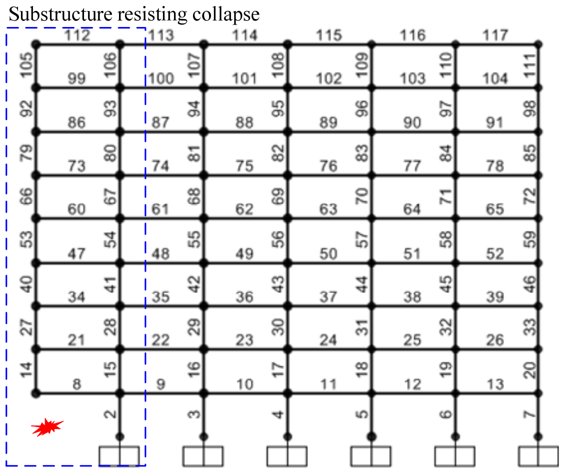


Fig. 12 The structural component's number

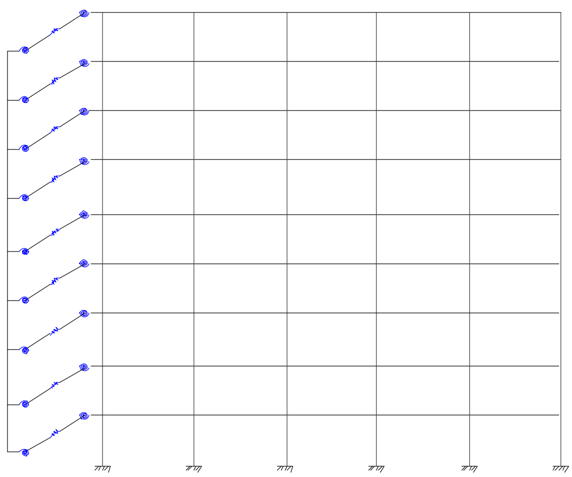


Fig. 13 The failure state of structural progressive collapse

Moreover, based on Eq. (6), the sensitivity of structural components to abrupt removal of the corner column shown in Fig. 11(a) can be determined. Analysis results are listed in Table 4. Then, a sub-structure enclosed in the dotted rectangle in Fig. 12 to resisting progressive collapse can be proposed based on the sensitivity of structural components. In accordance with the values in shadow region of the Table 4, it is noted that energies absorbed and dissipated by structure mainly are contributed by the failure of beams in the sub-structure, so the failure state of structural progressive collapse is shown in Fig. 13.

4.2.2 Failure energy of the structure

Based on the modified plastic hinges model, basic parameters are calculated and listed in Table 5.

The results of progressive collapse analysis based on energy method are listed in Table 6.

Table 4 The sensitivity of structural components to abrupt removal of the corner column

No. (i)	$R_{i,A,sensibility}$
2	0.402
8	0.828
9	0.052
13	0.023
14	0.076
15	0.307
20	0.021
21	1.000
22	0.067
23	0.031
24	0.030
25	0.029
26	0.036
28	0.242
34	0.926
35	0.057
36	0.031
37	0.031
38	0.032
39	0.037
40	0.027
41	0.176
47	0.900
48	0.054
49	0.033
50	0.031
51	0.032
52	0.034
53	0.038
54	0.128
60	0.780
61	0.056
62	0.027
63	0.026
64	0.026
65	0.030
66	0.058
67	0.234
72	0.024
73	0.638
74	0.054
78	0.027
79	0.079
80	0.156
85	0.023
86	0.626
87	0.050
90	0.020
91	0.025
92	0.073
93	0.102
98	0.022
99	0.647
100	0.051
101	0.022
104	0.026
105	0.203
106	0.084
107	0.032
108	0.022
109	0.021
112	0.309
113	0.073

* the "No." is the components' number. And the values are not listed in Table 4 if $R_{i,A,sensibility}$ <0.02.

Table 5 Basic parameters to calculate energy-based structural damage index

θ_y (rad)	$\theta_{\max}$ (rad)	M_y (kN·m)	P_y (kN)	Δ_y (mm)	α	ζ	γ	λ	c_M	Δ_3 (mm)
0.0113	0.1356	801.65	4085.7	15.65	0.940	1.362	0.485	0.758	0.6	8.53

Changings of structural internal energy (plastic deformation work) and potential energy (external force work) in the different phases are shown in Fig. 14. The relationship of beam axial force and vertical displacement is shown in Fig. 15.

According to the results listed in Table 6, the structural energy-based damage index I_{DE} is 0.88. Hence, progressive collapse does not occur and our example building is safe after removal of a corner column in the first story. It is can be observed that the energies absorbed and dissipated by

Table 6 The results of progressive collapse analysis based on energy method

Failure energy of a beam ($E_{i,beam}$):	$E_{i,beam} = 2\int_0^{\theta_{\max}} M(\theta)\,\mathrm{d}\theta + \int_0^{\Delta_3} P(\Delta)\,\mathrm{d}\Delta = 178.0\,kN\cdot m$
Energies absorbed and dissipated by a structure E_{EC}:	$E_{EC} = \sum R_{i,A,sensibility}\cdot E_{i,beam} = 1184.4 kN\cdot m$
External force work by applied forces due to deformation of a beam except roof level $E_{i,potential,beam}$:	Except roof level: $E_{i,potential,beam} = q_i\cdot\left[\frac{1}{2}\delta_{\max}(dL_b + L_b^0\cdot\cos\theta_{\max})\right] + P_{i,column}\cdot\delta_{\max} = 162.5\,kN\cdot m$ Roof level: $E_{i,potential,beam} = q_i\cdot\left[\frac{1}{2}\delta_{\max}(dL_b + L_b^0\cdot\cos\theta_{\max})\right] = 39.3\,kN\cdot m$
Inherent potential energy of the upper structure E_{ED}:	$E_{ED} = \sum_i R_{i,A,sensibility}\cdot E_{i,potential,beam} = 1043\,kN\cdot m$
Energy-based structural damage index I_{DE}:	$I_{DE} = \frac{E_{ED}}{E_{EC}} = \frac{1043.1}{1184.4} = 0.88 < 1$

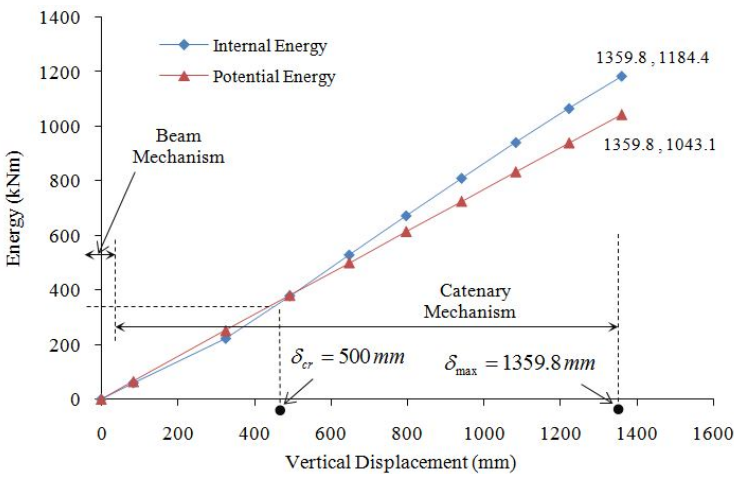


Fig. 14 Energy (internal and potential) versus vertical displacement

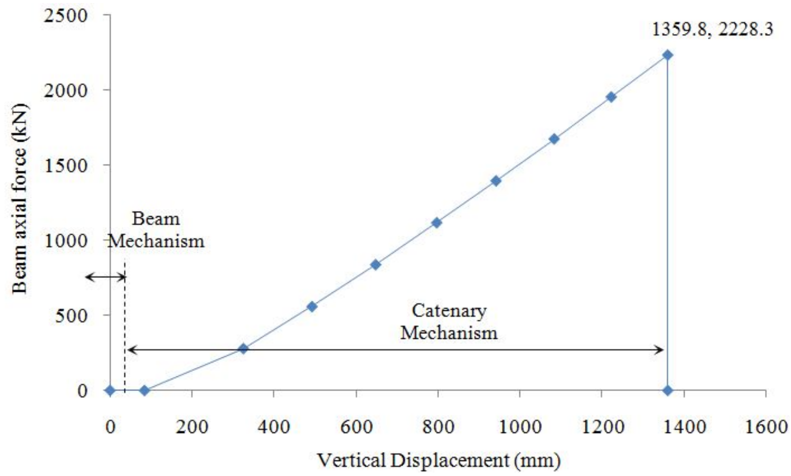
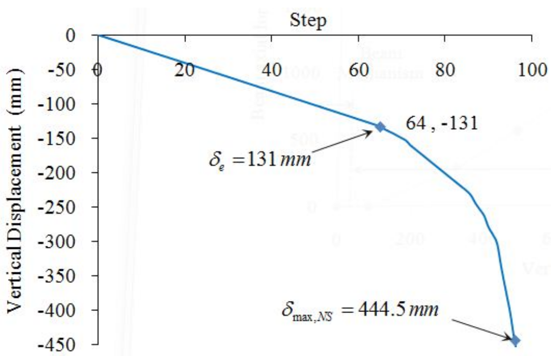


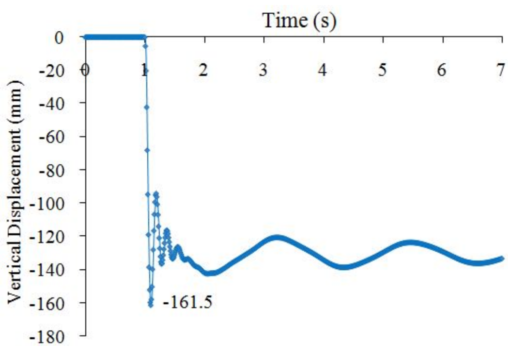
Fig. 15 Beam axial force versus vertical displacement

the catenary mechanism are greater than by the beam mechanism based on the Fig. 14, and the catenary mechanism is the last defense against progressive collapse based on Fig. 15. In addition, after the critical point where energy-based structural damage index I_{DE} is equal to 1.0 and the corresponding vertical displacement (defined as δ_{cr}) is approximately 500 mm, the internal energy is always greater than the potential energy until the vertical displacement reaching the maximum ($\delta_{\max}$ = 1359.8 mm). Thus, the structural safety reserve (SSR) of the analyzed example building for progressive collapse obtained by the proposed based-energy method can be defined as SSR_BE = $\delta_{\max} / \delta_{cr}$ = 2.72.

Moreover, Nonlinear Static (NS) and Nonlinear Dynamic (ND) analysis are performed by SAP2000 software as shown in Figs. 16(a) and (b), respectively. The loading combination Load = 2(DL + 0.25LL) is applied to NS analysis, and Load = DL + 0.25LL is applied to ND analysis (US GSA 2003). The maximum vertical displacement is 161.5 mm by ND analysis, which is lower than the limited value $\delta_{\text{limit}} = \min\{20\delta_e, 0.21L_b\}$ = 1732.5 mm obtained based on the acceptance criteria for nonlinear analysis recommended in the US GSA 2003, where δ_e = 131



(a) Nonlinear Static (NS) analysis



(b) Nonlinear Dynamic (ND) analysis

Fig. 16 Structural progressive collapse analysis based on traditional method recommended in GSA2003 mmis obtained by NS analysis shown in Fig. 16(a).The SSR_NS by NS analysis and the SSR_ND by ND analysis are 3.9 and 10.7, respectively, which are greater than the SSR_BE by proposed based-energy method, namely, SSR_BE < SSR_NS < SSR_ND. Thus, analysis results obtained by the proposed based-energy method are credible.

5. Conclusions

An energy-based method has been developed to analyze the behavior of steel moment frame structures subject to progressive collapse. The major conclusions are as following:

(1) Based on the changes of component's internal energy, sensitivity of structural members to abrupt removal of a column is proposed to determine a sub-structure resisting progressive collapse. An energy-based structural damage index is defined to judge whether progressive collapse occurs in structures with a removed column.
(2) A simplified beam damage model is proposed to analyze beam failure energy incorporating catenary action at large deflections, and a modified plastic hinge model is developed, the modified correlation between bending moment and axial force during the whole deformation development process shows good agreement with the experimental results.
(3) The proposed energy-based analysis method is a new and credible method to analyze progressive collapse in steel frame structure without considering the Dynamic Increase Factor (DIF).

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