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Probabilistic elastic-plastic analysis of repaired cracks with bonded composite patch

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Abstract. The objective of this work was to evaluate the ductile cracked structures with bonded composite patch used in probabilistic elastic plastic fracture mechanics subjected to tensile load. The finite element method is used to analyze the stress intensity factors for elastic case, the effect of cracks and the thickness of the patch (e_r) are presented for calculating the stress intensity factors. For elastic-plastic the Monte Carlo method is used to predict the distribution function of the mechanical response. According to the obtained results, we note that the stress variations are important factors influencing on the distribution function of (J/Je).

Keywords: composite; finite element method; fracture mechanics; elastic-plastic; probabilistic analysis

1. Introduction

The bonded composite repair has been recognized as an efficient and economical method to extend the service life of cracked aircraft structures. The scientific approach to designing and assessing repairs has probably started in the early 1970s. The work was pioneered by Baker (1984) at the Aeronautical and Maritime Research Laboratory (AMRL) for the Royal Australian Air force (RAAF) and later in the USA in the early 1980s.

Bonded composite repairs can be regarded as a versatile cost-effective method of repairing, strengthening, or upgrading inadequate metallic structures (da Costa Mattos *et al*. 2012, Cetisli and Kaman 2014). Oudad *et al*. (2009) investigated the influence of the patch parameters on the size of the plastic zone at the tip of repairing cracks. Albedah *et al*. (2011) studied the elastic–plastic behaviour of repairing cracks with bonded composite patch, their results show that the plastic zone sizes around the crack tip decrease significantly when the patch is bonded.

Stress-intensity factors are important in the prediction of crack growth rates and fracture strengths.

Thus, in the field of fracture mechanics, one of the major research activities is the development of new techniques to obtain accurate stress-intensity factors for arbitrarily shaped plates with cracks (Ibrahim *et al*. 2013, Noda and Lan 2012, Ayhan and Yücel 2011, Sripichai and Pan 2012,

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Itou 2009).

A realistic evaluation of structural performance can be conducted only if the uncertainty in structural loads, flaw sizes and material properties, and hence responses, is taken into consideration (Miglis *et al*. 2013, Carter *et al*. 2012, Yu and Guo 2012, Cao *et al*. 2012 , Leonel *et al*. 2011).

Probabilistic fracture mechanics are a means of quantifying the failure probability resulting from uncertainties in the values of the parameters used to perform a failure assessment of cracked structures through probabilistic analysis techniques (Feng *et al*. 2012, Leonel *et al*. 2012, Su and Zheng 2012, Mechab *et al*. 2014). Some common techniques include the Monte Carlo simulation (MCS) and the first/ second-order reliability methods (FORM/SORM).

A probabilistic methodology for elastic-Plastic fracture mechanics analysis of cracked structures (Rahman 2009, Fujioka 2013), which is capable of predicting accurate deterministic and probabilistic characteristics of J integral.

In this work, the finite element method is used to analyse the stress intensity factors for elastic case, the effect of cracks, the thickness of the patch (e_r) and form of patch are presented. For elastic-plastic the Monte Carlo method is used to predict the distribution function of the mechanical response.

2. Geometrical model

The basic geometry of the cracked structure considered in this study is shown in Fig. 1. Consider a plate with the following dimensions: height H_p = 600 mm, width w_p = 300 mm, thickness e_p = 4 mm. The plate is subjected to uniaxial tensile load giving a remote stress state of σ = 50 MPa for elastic analysis. A central crack of length 2a perpendicular to the loading axis is supposed to exist in the plate. This crack is repaired with unidirectional Boron/Epoxy composites patches. The ply orientation is parallel to the loading axis. The initial dimensions of the patch are: height H_r = 100 mm, width w_r = 100 mm and thickness e_r = 2 mm, 3 mm. The mechanical properties of the different materials are given in Table 1, The stress–strain curve of aluminum 2024-T3 is presented in Fig. 2.

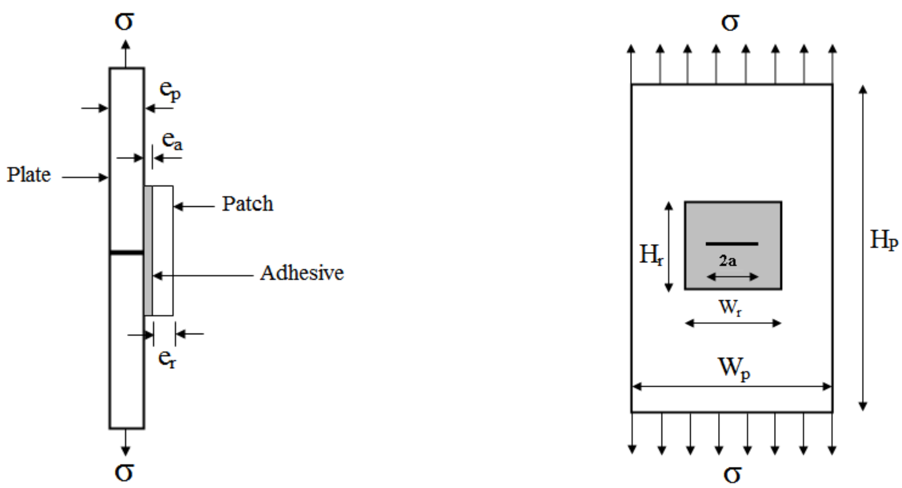

Fig. 1 Geometrical model

Table 1 Elastic properties of different materials

	Aluminium alloy T3	Boron/époxy	Adhésive (FM-73)
E_1 (GPa)	72	200	2.55
E_2 (GPa)		19.6	
E_3 (GPa)		19.6	
υ_{12}	0.3	0.3	0.32
υ_{13}		0.28	
υ_{23}		0.28	
G_{12} (GPa)		7.2	
G_{13} (GPa)		5.5	
G_{23} (GPa)		5.5	

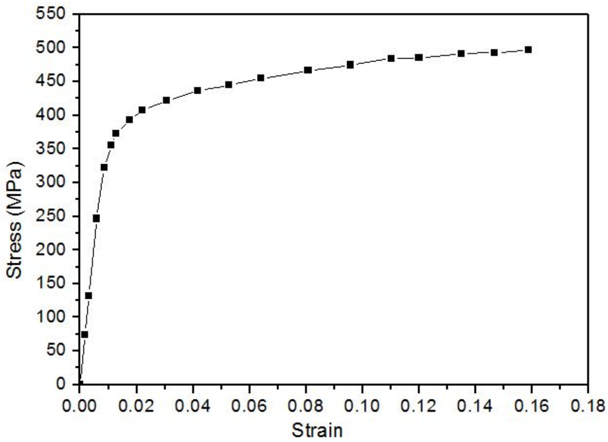

Fig. 2 Stress–strain curves for aluminum 2024-T3

3. Finite element modelling

The three-dimensional finite element analysis is carried out using the commercial finite element code ABAQUS (2007) The finite element model consists of three subsections to model the cracked plate, the adhesive, and the composite patch. Due to loading symmetry, only the quarter of the repaired plate was considered. The J integral values were extracted using a domain integral method within ABAQUS. This method provides high accuracy with rather coarse models in three-dimensions. The J integral values were extracted using a domain integral method within ABAQUS. This method provides high accuracy with rather coarse models in three-dimensions. To generate crack front some brick elements are replaced by “crack block”. These crack-blocks are meshes of brick elements which are mapped into the original element space and merged with surrounding mesh. Boundary conditions and loads are transferred to the crack-block elements. The mesh was refined near the crack-tip area with an element dimension of 0.067 mm using at least 15 such fine elements in the front and back of the crack tip. The finite element mesh was generated using brick elements with 20 nodes. The number of element used in this analysis is 49351 and number of degrees of freedom DOF is 322016. Fig. 3 shows the overall mesh of the specimen and mesh refinement in the crack-tip region.

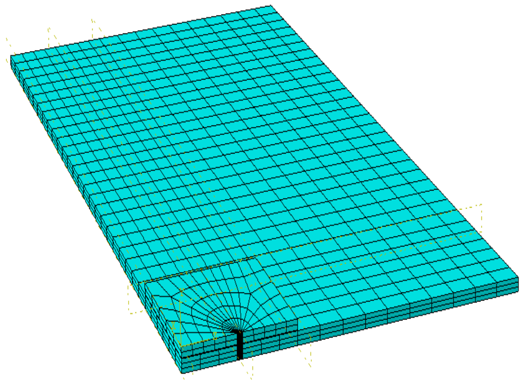

Fig. 3 Typical mesh model of the quarter of the structure

4. Results and discussion

4.1 Elastic analyses

4.1.1 Effect of the patch

The stress intensity factor (SIF), can be found

$$K_I = \sqrt{E.J_e} \tag{1}$$

J_e : The elastic of the J-integral
E : Young's modulus

Fig. 4 presents the variation of the stress intensity factor (SIF), according to the crack length for the cases with and without the patch, the value of applied stress is σ = 50 MPa for elastic analysis. The adhesive used for calculation is the FM 73 and the thicknesses of the different materials are: plate (e_p = 4 mm), patch (e_r = 3 mm), and adhesive (e_a = 0.2 mm). It can be seen, according to

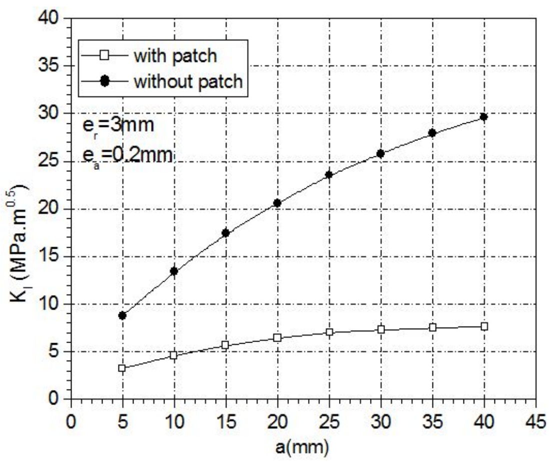

Fig. 4 Variation of the SIF according to the crack length for the cases with and without Patch

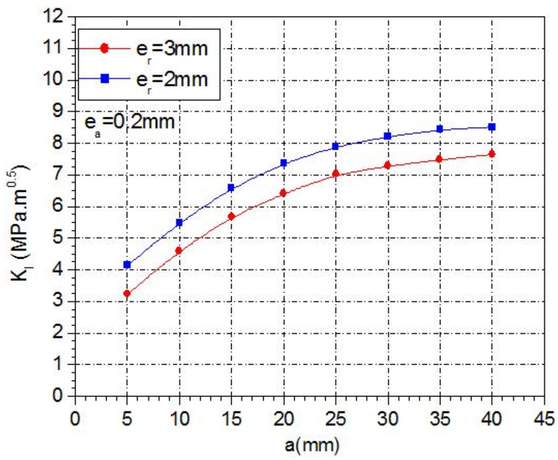

Fig. 5 Variation of the stress intensity factor (SIF) according to the crack length for different patch thicknesses

Fig. 4 that the presence of the patch has a considerable effect on the stress intensity factor (SIF) variation at the crack tip. It shows that the patch repair highly decreases the stress intensity factor, the maximum reduction of stress intensity factor (K_I) is about to70% of the crack length $a = 40$ mm.

4.1.2 Effect of the patch thickness

Fig. 6 shows the variation of the stress intensity factors according to the crack length for two values of the patch thickness. we note that the increase of the patch thickness reduces the stress intensity factor at the crack tip in a proportional way. Several authors (Bachir Bouiadjra *et al*. 2012) showed the importance of the effect of the patch thickness on the repair performance in damaged aircraft structures showed that, under pure mechanical loading, the increase of the patch thickness to 50% decreases the SIF at the crack tip in the same order. They concluded that it is useful to use a patch with multiple layers for repairing aircraft structures.

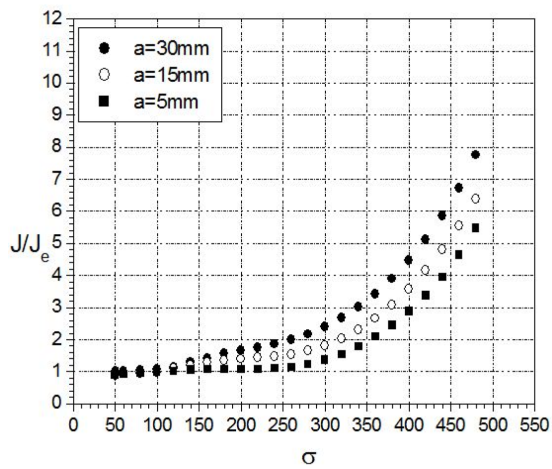

Fig. 6 Variation of the integral J according to the stress for different crack length

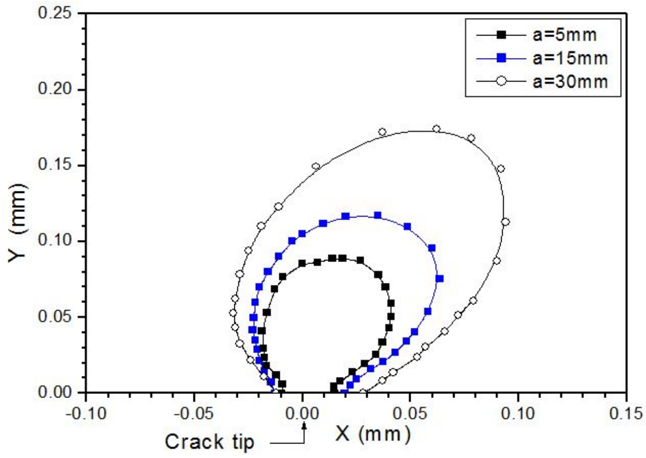

Fig. 7 Contour of the plastic zone for different crack length

4.2 Elastic-plastic analyses

$$J = J_e + J_P \tag{2}$$

The results of Fig. 6 show that the increase in the size of the crack leads to the increase in the integral J.

Fig. 7 presents the contour of the plastic zone ahead of repairing cracks for different crack lengths. It can be seen that the size of the plastic zone is slightly affected by the variation of the crack length. There is a weak reduction of the plastic zone size as the crack length decreases. This is due to the fact that the absorption of the stresses by the patch does not permit a great extension of the plastic zone as the crack length increases.

5. Probabilistic elastic-plastic analysis

5.1 Random parameters and fracture response

The uncertainties are related to load estimation, geometrical fluctuations and scatter of material properties; these parameters are modeled by random variables, described by distribution type and parameters (i.e., mean and coefficient of variation COV). For design purpose, the system uncertainties should be controlled in order to avoid unsafe situations. eight random variables are considered to model the plate uncertainties related to geometry (Thickness (t), height ($H_p/2$) and width ($w_p/2$) of plate), applied stress (σ), material properties (Young Modulus (E), Strain hardening index in the (R-O) ramberg-osgood (n) , Coefficient for the (R-O) ramberg-osgood (α)), and Crack length (a). Table 2 indicates the mean values and coefficients of variation for the six selected random variables.

Hence, any relevant fracture response, such as the (J/Je) (x_i), should be evaluated by the probability.

$$F_{(J/Je)}(j_o) \overset{def}{=} \Pr\left|(J/Je)(x_i) < j_o\right| \overset{def}{=} \int_{(J/Je)(x_i)<j_o} f_x(x)dx \tag{3}$$

Table 2 Random variables and corresponding parameters

Variable	Mean	Coefficient of variation (COV)
Young modulus (*E*)	72 GPa	1%
Ramberg-osgood exponent (*n*)	8	1%
Ramberg-osgood coefficient (*α*)	0.2	1%
Crack length (*a*/*t*)	30 mm	1%
Height ($H_p/2$)	300 mm	1%
Width ($w_p/2$)	150 mm	1%
Applied stress (*σ*)	200 MPa	4%

or the probability density function (PDF), where $F_{(J/Je)}(j_o)$ is the cumulative distribution function of (J/Je) and $f_x(x)$ is the known joint probability density function of x_i.

Under the plate uncertainties, the mechanical behaviour becomes random and the J integral (J/Je) is defined by a probability density. In reliability-based design, the uncertainties are taken into account by considering the safety margin (J/Je) (x_i) described in terms of the random variables

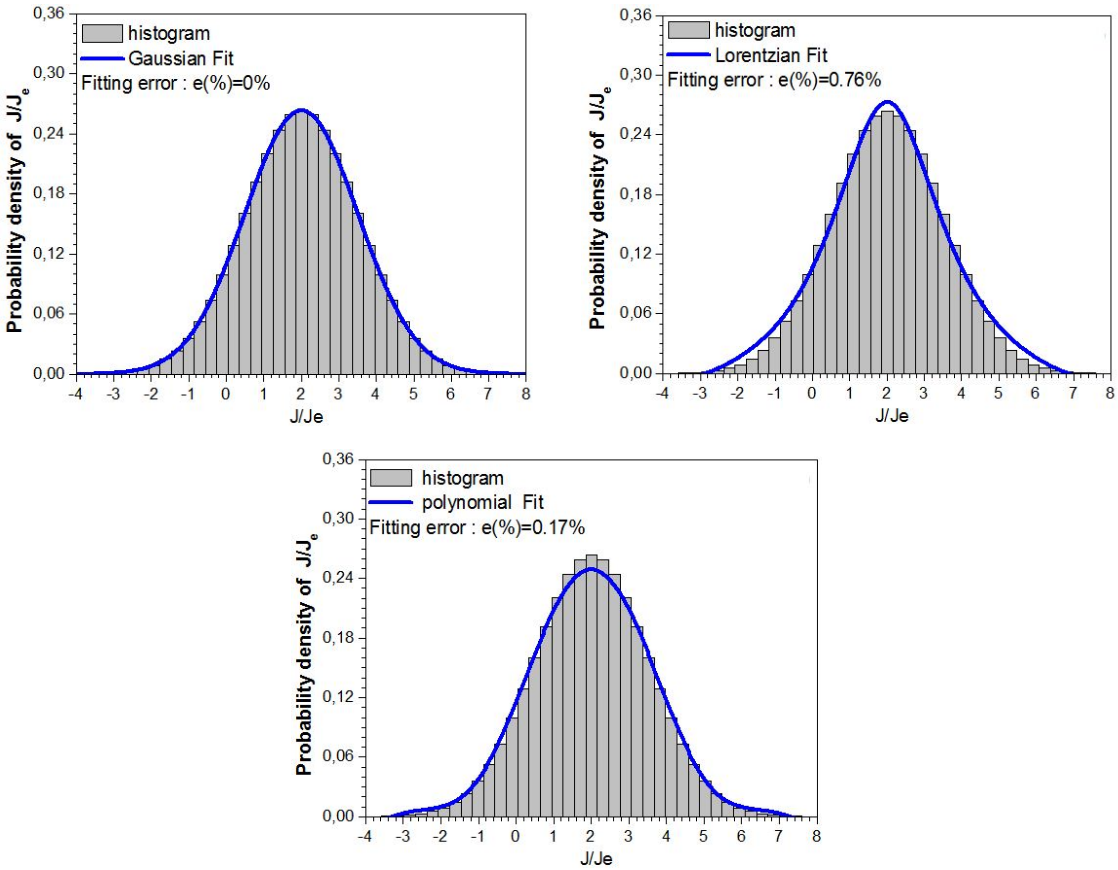

Fig. 8 Histogram and Probability density function of (J/J_e) a=30mm,σ=200MPa

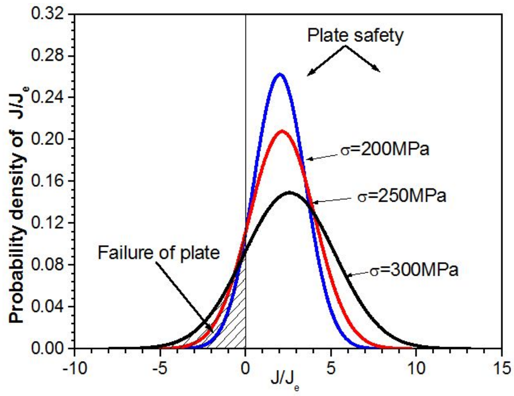

Fig. 9 Probability density of (J/J_e) for different values of stress a=30mm

x_i which define the uncertainties. The safety margin (J/Je) (x_i) is the probabilistic design rule, which defines the plate safety by the condition (J/Je) (x_i) > 0 and the plate failure by (J/Je) (x_i) ≤ 0. In our study, the density function is evaluated by using Monte Carlo method. The basic idea is to draw random samples for the input parameters, then to compute the mechanical response for each sample. When a large number of Monte Carlo samples are achieved, it becomes possible to make statistical analysis of the response sets in order to provide the probability density functions of the (J/Je), The failure probability can be obtained by computing the ratio between the number of failed samples and the total number of drawn samples. The sensitivity measures can be also obtained by computing the dispersion of the mechanical response in terms of the scatter of the input parameters. In order to analyze the ductile cracked structures with bonded composite patch by the FORTRAN program, which are developed by the authors: the first program provides the mechanical response by calculating the (J/Je) distribution and the second program computes the probabilistic response by using Monte Carlo simulations . To achieve a high accuracy of the results, we have carried out 10^5 simulations.

5.2 Probabilistic results

Fig. 8 plots the histograms of the (J/Je) obtained by Monte Carlo simulations. The probability density function (pdf) is obtained by fitting the histogram with theoretical models. Three distribution laws are investigated: Lorentizian (Cauchy), Gaussian (Normal law) and Polynomial (9th order); from Fig. 8, it can be clearly observed that the three distributions give more or less good approximation of the (J/Je). The polynomial distribution gives a lower mean value than for Gaussian and Lorentizian distributions. By comparing these three distributions, we can conclude that the Gaussian law offers an acceptable approximation of the (J/Je) probability density function, with good estimation of the average (see Fig. 8). Fig. 9 presents the probability density of (J/Je) for different values of the stress. We noted that when the stress is large the value of the probability density of (J/Je) is small. It can be seen that the margin increases significantly with the uncertainties related to the load applied, leading to larger failure probability, finally, the failure probabilities depend on the load applied.

6. Conclusions

This paper present, the finite element method used to analyse the stress intensity factors for elastic case, the effect of cracks and the thickness of the patch (e_r) are presented. The obtained results in is slightly affected by the variation of the crack length. The Monte Carlo method is used to pre this study allow us to deduce the presence of a patch reduces considerably the stress intensity at the crack tip witch increase the fatigue life of the structure. The reduction of the stress intensity factors (SIF) at the crack tip depends on the thickness of patch. For elastic-plastic analysis, the contour of the plastic zone ahead of repaired crack for different crack lengths. It can be seen that the size of the plastic zone dict the distribution function of the mechanical response. According to the obtained results, we note that the stress variations are important factors influencing on the distribution function of (J/Je).

References

ABAQUS (2007), ABAQUS Sandard/User's Manual; version 6.5, Hibbit Karlsson & Sorensen, Inc., Pawtucket, RI, USA.

Albedah, A., Bachir Bouiadjra, B., Ouddad, W., Es-Sahe, M. and Benyahia, F. (2011), "Elastic plastic analysis of bonded composite repair in cracked aircraft structures", *J. Reinf. Plast. Compos.*, **30**(1), 66-72.

Ayhan, A.O. and Yücel, U. (2011), "Stress intensity factor equations for mixed-mode surface and corner cracks in finite-thickness plates subjected to tension loads", *Int. J. Pres. Ves. Pip.*, **88**(5-7), 181-188.

Bachir Bouiadjra, B., Oudad, W., Albedah, A., Benyahia, F. and Belhouari, M. (2012), "Effects of the adhesive disband on the performances of bonded composite repairs in aircraft structures", *Mater. Des.*, **37**, 89-95.

Baker, A. (1984), "Repair of cracked or defective metallic aircraft components with advanced fiber composites — An overview of Australian work", *J. Compos. Struct*,, **2**(2), 153-181.

Carter, B.J., Schenck, E.C., Wawrzynek, P.A., Ingraffea, A.R. and Barlow, K.W. (2012), "Three-dimensional simulation of fretting crack nucleation and growth", *Eng. Fract. Mech.*, **96**, 447-460.

Cao, Z. and Liu, Y. (2012), "A new numerical modelling for evaluating the stress intensity factors in 3-D fracture analysis", *Struct. Eng. Mech., Int. J.*, **43**(3), 321-336.

Cetisli, F. and Kaman, M.O. (2014), "Numerical analysis of interface crack problem in composite plates jointed with composite patch", *Steel Compos. Struct., Int. J*,, **16**(2), 203-220.

da Costa Mattos, H.S., Monteiro, A.H. and Palazzetti, R. (2012), "Failure analysis of adhesively bonded joints in composite materials", *Mater. Des.*, **33**, 242-247.

Feng, G.Q., Garbatov, Y. and Guedes Soares, C. (2012), "Probabilistic model of the growth of correlated cracks in a stiffened panel", *Eng. Fract. Mech.*, **84**, 83-95.

Fujioka, T. (2013), "Simplified estimate of elastic–plastic *J*-integral of cracked components subjected to secondary stresses by the enhanced reference stress method and elastic follow-up factors", *Int. J. Pres. Ves. Pip.*, **109**, 28-39.

Ibrahim, A.M., Ozturk, H. and Sabuncu, M. (2013), "Vibration analysis of cracked frame structures Vibration analysis of cracked frame structures", *Struct. Eng. Mech., Int. J.*, **45**(1), 33-52.

Itou, S. (2009), "Dynamic stress intensity factors for two parallel cracks in an infinite orthotropic plate subject to an impact load", *Struct. Eng. Mech., Int. J.*, **33**(6), 697-708.

Leonel, E.D., Beck, A.T. and Venturini, W.S. (2011), "On the performance of response surface and direct coupling approaches in solution of random crack propagation problems", *Struct. Saf.*, **33**(4-5), 261-274.

Leonel, E.D., Chateauneuf, A. and Venturini, W.S. (2012), "Probabilistic crack growth analyses using a boundary element model: Applications in linear elastic fracture and fatigue problems", *Eng. Anal. Bound. Elem.*, **36**(6), 944-959.

Noda, N.A. and Lan, X. (2012), "Stress intensity factors for an edge interface crack in a bonded semiinfinite plate for arbitrary material combination", *Int. J. Solid. Struct.*, **49**(10), 1241-1251.

Mechab, B., Serier, B., Kaddouri, K. and Bachir Bouiadjra, B. (2014), "Probabilistic elastic-plastic analysis of cracked pipes subjected to internal pressure load", *Nucl. Eng. Des.*, **275**, 281-286.

Miglis, Y., Elishakoff, I. and Presuel-Moreno, F. (2013), "Analysis of a cracked bar under a tensile load in a corrosive environment", *Ocean Syst. Eng., Int. J.*, **3**(1), 1-8.

Oudad, W., Bachir Bouiadjra, B., Belhouari, M., Touzain, S. and Feaugas, X. (2009), "Analysis of the plastic zone size ahead of repaired cracks with bonded composite patch of metallic aircraft structures", *Comput. Mater. Sci.*, **46**(4), 950-954.

Rahman, S. (2009), "A new computational method for probabilistic elastic-plastic fracture analysis", *J. Press. Vess. Tech.*, **131**(6), 1-8.

Sripichai, K. and Pan, J. (2012), "Closed-form structural stress and stress intensity factor solutions for spot welds in square overlap parts of cross-tension specimens", *Eng. Fract. Mech.*, **96**, 307-327.

Su, C. and Zheng, C. (2012), "Probabilistic fracture mechanics analysis of linear-elastic cracked structures by spline fictitious bounda"ry element method", *Eng. Anal. Bound. Elem.*, **36**(12), 1828-1837.

Yu, P. and Guo, W. (2012), "An equivalent thickness conception for prediction of surface fatigue crack growth life and shape evolution" , *Eng. Fract. Mech.*, **93**, 65-74.

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