Steel and Composite Structures, *Vol. 20, No. 5 (2016) 1103-1117*
DOI: http://dx.doi.org/10.12989/scs.2016.20.5.1103

2D deformation in initially stressed thermoelastic half-space with voids

Ibrahim A. Abbas[*1,2,3] and Rajneesh Kumar [4a]

[1] *Department of Mathematics, Faculty of Science and Arts - Khulais, University Of Jeddah, Saudi Arabia*
[2] *Nonlinear Analysis and Applied Mathematics Research Group (NAAM), Department of Mathematics, King Abdulaziz University, Jeddah, Saudi Arabia*
[3] *Department of mathematics, Faculty of Science, Sohag University, Sohag, Egypt*
[4] *Department of Mathematics, Kurukshetra University, Kurukshetra-136 119, India*

(*Received May 09, 2015, Revised December 29, 2015, Accepted January 12, 2016*)

Abstract. The present investigation is to study the plane problem in initially stressed thermoelastic half-space with voids due to thermal source. Lord-Shulman (Lord and Shulman 1967) theory of thermoelasticity with one relaxation time has been used to investigate the problem. A particular type of thermal source has been taken as an application of the approach. Finite element technique has been used to solve the problem. The components of displacement, stress, temperature change and volume fraction field are computed numerically. The resulting quantities are depicted graphically for different values of initial stress parameter. The relaxation time and the initial stress parameter have a significant effect on all distributions.

Keywords: thermoelastic half-space; voids; initially stressed; thermal sources finite element

1. Introduction

Biot (1956) introduced the theory of coupled thermoelasticity to overcome the first shortcoming in the classical uncoupled theory of thermoelasticity where it predicts two phenomena not compatible with physical observations. The theory of couple thermoelasticity was extended by Lord and Shulman (1967) and Green and Lindsay (1972) by including the thermal relaxation time in constitutive relations. In the decade of the 1990's Green and Naghdi (1991, 1992, 1993) proposed three new thermoelastic theories based on an entropy equality rather than the usual entropy inequality. During the second half of twentieth century, non-isothermal problems of the theory of elasticity become increasingly important. This is due to their many applications in widely diverse fields. First, the high velocities of modern aircraft give rise to aerodynamic heating, which produces intense thermal stresses that reduce the strength of the aircraft structure. Second, in the nuclear field, the extremely high temperature and temperature gradients originating inside nuclear reactors influence their design and operations.

The linear theory of elastic materials with voids is one of the generalization of the classical

*Corresponding author, Professor, E-mail: ibrabbas7@yahoo.com
[a] Professor, E-mail: rajneesh_kuk@rediffmail.com

Copyright © 2016 Techno-Press, Ltd.
http://www.techno-press.org/?journal=scs&subpage=6 ISSN: 1229-9367 (Print), 1598-6233 (Online)

theory of elasticity. This theory has practical utility to investigate various types of geological, biological and synthetic porous materials for which the elastic theory is inadequate. This theory is concerned with elastic materials consisting of a distribution of small pores (voids), in which the voids volume is included among the kinematics variables, and in the limiting case of volume tending to zero, the theory reduces to the classical theory of elasticity. The presence of voids is known to affect the estimation of the physical-mechanical properties of the composite and weaken the bond as these pores are spread over a wide area. The intended applications are to the materials like rock, soil and to manufactured porous materials.

A nonlinear theory of elastic materials with voids was developed by Nunziato and Cowin (1979). Later, Cowin and Nunziato (1983) developed a theory of linear elastic materials with voids for the mathematical study of the mechanical behavior of porous solids. They considered several applications of the linear theory by investigating the response of the materials to homogeneous deformations, pure bending of beams, and small amplitudes of acoustic waves. Puri and Cowin (1985) studied the behavior of plane waves in a linear elastic material with voids. Iesan (1986) developed a linear theory of thermoelastic material with voids. Rusu (1987) studied the existence and uniqueness in thermoelastic materials with voids. Saccomandi (1992) presented some remarks about the thermoelastic theory of materials with voids. Ciarletta and Scalia (1993b) investigated the uniqueness and reciprocity theorems in linear thermoelasticity of material with voids. Ciarletta and Scalia (1993a) discussed the nonlinear theory of nonsimple thermoelastic materials with voids. The domain of influence theorem in the linear theory of elastic materials with voids was discussed by Dhaliwal and Wang (1994), Scarpetta (1995) studied well-posedness theorems for linear elastic materials with voids. Ciarletta and Scarpetta (1995) discussed some results on thermoelasticity for dielectric materials with voids. Dhaliwal and Wang (1995) developed a heat-flux dependent theory of thermoelasticity with voids. Marin (1997a, b) studied uniqueness and domain-of-influence results in thermoelastic bodies with voids. Marin (1998) also presented the contributions on uniqueness in thermoelastodynamics for bodies with voids. Marin and Salca (1998) obtained the relation of Knopoff-de Hoop type in thermoelasticity of dipolar bodies with voids. Birsan (2000) established existence and uniqueness of a weak solution in the linear theory of elastic shells with voids. Chirita and Scalia (2001) studied the spatial and temporal behavior in linear thermoelasticity of materials with voids. Pompei and Scalia (2002) studied the asymptotic spatial behavior in linear thermoelasticity of materials with voids. Ciarletta *et al*. (2003) studied stress analysis for cracks in elastic materials with voids. Iesan (1987) developed a theory of initially stressed thermoelastic material with voids. Singh *et al*. (2006) discussed the reflection of generalized thermoelastic waves from a solid half-space under hydrostatic initial stress. Fahmy and El-Shahat (2008) studied the effect of initial stress and inhomogeneity on the thermoelastic stresses in a rotating anisotropic solid. Effect of magnetic field and initial stress on the propagation of interface waves in transversely isotropic perfectly conducting media was investigated by Acharya *et al*. (2009).

The finite element method is a powerful technique originally developed for numerical solution of complex problem in structural mechanics, and it remains the method of choice for complex system. A further benefit of this method is that it's allow physical effects to be visualized and quantified regardless of experimental limitations. The counterparts of our problem in the contexts of the thermoelasticity theories have been considered by using analytical and numerical methods (Zenkour and Abbas 2014, Abbas and Kumar 2014, Abbas 2013, 2014a, b, Abbas and Othman 2011, Tomar and Ogden 2014, Marin 1999, 2009, Abo-Dahab and Singh 2013, Sharma and Grover 2012, Bachher *et al*. 2014, 2015).

In the present paper, the components of displacement, stress, and temperature change and volume fraction field are obtained in an initially stressed thermoelastic half-space due to thermal source. The resulting quantities are computed numerically by finite element technique and depicted graphically for different values of initial stress parameter. The initial stress parameter has a significant effect on all distributions.

2. Basic equations

Following Lord and Shulman (1967), Iesan (2004), the basic equations for homogeneous initially stressed generalized thermoelastic with voids material are

The stress- strain relation in isotropic medium

$$t_{ij} = \lambda\delta_{ij}u_{l,l} + \mu(u_{i,j} + u_{j,i}) + t^0 u_{i,j} + \xi^* \delta_{ij}\,\varphi - \beta\,\delta_{ij}\,T \tag{1}$$

Equations of motion

$$(\mu + t^0)\Delta\boldsymbol{u} + (\lambda + \mu)grad\,div\boldsymbol{u} + \xi^*\,grad\,\varphi - \beta\ \ gard\,T = \rho\,\ddot{\boldsymbol{u}}\ . \tag{2}$$

Equations of equilibrated forces

$$(d\Delta - \zeta)\varphi - \omega_0\dot{\varphi} - \xi^*\,div\,\boldsymbol{u} + b_1^* T = \rho\chi\ddot{\varphi}. \tag{3}$$

Equation of heat conduction

$$(1 + \tau_0\frac{\partial}{\partial t})\left[\rho C^*\dot{T} + T_0(b_1^*\dot{\varphi} + \beta\ \ div\,\dot{\boldsymbol{u}})\right] = k\Delta T\ , \tag{4}$$

where λ, μ are Lame's constants, ρ is the density, $u = (u, v, w)$ is the displacement vector, σ_{ij} is the stress tensor, t_{jm}^0 is the initial stress parameter, ξ, ${}^*d$, b_1^*, ω_0, ζ are the constitutive constant of the medium C^* is the specific heat, $\beta = (3\lambda + 2\mu)\ \alpha_t$, α_t is the coefficient of linear thermal expansion. Δ is the Laplacian operator, ϕ $(= v - v_0)$ is the volume fraction field and v_0 is the matrix volume fraction at the reference state. T is the temperature change which is measured from the absolute temperature T_0 ($T_0 \neq 0$). We assume that T_0 and v_0 are constants. τ_0 is thermal relaxation time.

3. Formulation of the problem

We consider the medium of isotropic generalized thermoelastic with voids under initial stress. The origin of the Cartesian coordinate system (x, y, z) is taken at any point and z-axis taking vertically downward into the medium. For two dimensional problem, we have

$$\boldsymbol{u} = (\mathrm{u}, 0, \mathrm{w})\,. \tag{5}$$

We define the dimensionless quantities

$$x^{'}=\frac{\omega_1^{*}x}{c_1},z^{'}=\frac{\omega_1^{*}z}{c_1},\ t^{'}=\omega_1^{*}t,\ u^{'}=\frac{\omega_1^{*}u}{c_1},\ w^{'}=\frac{\omega_1^{*}w}{c_1},$$
$$\varphi^{'}=\frac{\omega_1^{*2}\chi\varphi}{c_1^2},T^{'}=\frac{\beta\ T}{\rho c_1^2},\quad t_{ij}^{'}=\frac{t_{ij}}{\beta T_0},\tau_0^{'}=\omega_1^{*}\tau_0,\tau_1^{'}=\omega_1^{*}\tau_1, \tag{6}$$

where $\omega_1^{*}=\dfrac{C^{*}(\lambda+2\mu)}{k}$ and $c_1^2=\dfrac{\lambda+2\mu}{\rho}$,

Eqs. (2)-(4) with the aid of Eqs. (5) and (6), yield after suppressing the primes

$$\delta_1\Delta u+\delta_2\frac{\partial e}{\partial x}+\delta_3\frac{\partial\varphi}{\partial x}-\frac{\partial T}{\partial x}=\ddot{u}\ , \tag{7}$$

$$\delta_1\Delta w+\delta_2\frac{\partial e}{\partial z}+\delta_3\frac{\partial\varphi}{\partial z}-\frac{\partial T}{\partial z}=\ddot{w}, \tag{8}$$

$$(\delta_4\Delta+\delta_5+\delta_6\frac{\partial}{\partial t})\varphi+\delta_7 e+\delta_8 T=\ddot{\varphi}, \tag{9}$$

$$(1+\tau_0\frac{\partial}{\partial t})\tau_{m0}\left[(\delta_9 e+\delta_{10}\varphi)+\delta_{11}T\right]=k\Delta T, \tag{10}$$

where

$$\delta_1=\frac{t^0+\mu}{\lambda+2\mu},\delta_2=\frac{\lambda+\mu}{\lambda+2\mu},\delta_3=\frac{\xi^{*}}{\chi\rho\omega_1^{*2}},\delta_4=\frac{d}{\chi(\lambda+2\mu)},$$
$$\delta_5=-\frac{\zeta}{\rho\chi\omega_1^{*2}},\delta_6=-\frac{\omega_0}{\rho\chi\omega_1^{*}},\delta_7=-\frac{\xi^{*}}{\lambda+2\mu},$$
$$\delta_8=\frac{b_1^{*}}{\beta},\ \delta_9=\frac{T_0\beta^2}{\rho\omega_1^{*}},\delta_{10}=\frac{T_0 b_1^{*}c_1^2\beta}{\rho\chi\omega_1^{*3}},\delta_{11}=\frac{\rho C^{*}c_1^2}{\omega_1^{*}}, \tag{11}$$
$$\delta_{12}=\frac{Bc_1^2}{\chi\omega_1^{*2}},e=\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z},\ \Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}.$$

From the Eq. (1) with the aid of Eq. (6), we obtain the stress components in dimensionless form as

$$t_{xx}=\frac{1}{\beta T_0}[(\lambda+2\mu+t^0)u_{,x}+\lambda w_{,z}+\delta_{12}\varphi-\rho c_1^2 T], \tag{12}$$

$$t_{zz}=\frac{1}{\beta T_0}[\lambda u_{,x}+(\lambda+2\mu+t^0)w_{,z}+\delta_{12}\varphi-\rho c_1^2 T], \tag{13}$$

$$t_{xz} = \frac{1}{\beta T_0}[(\mu + t^0)u_{,z} + \mu w_{,x}]. \tag{14}$$

4. Initial and boundary condition

The above Eqs. (7)-(10) are solved subjected to initial conditions

$$u = w = \varphi = T = 0, \quad \dot{u} = \dot{w} = \dot{\varphi} = \dot{T} = 0, \qquad t = 0. \tag{15}$$

The boundary condition for the problem may be taken a

$$T(0,z,t) = T_0 H(t)H(2l - |z|), \ \frac{\partial\varphi}{\partial x} = 0, \ \sigma_{xx}(0,z,t) = 0, \sigma_{xz}(0,z,t) = 0, \tag{16}$$

where $H()$ is the Heaviside unit step.

5. Finite element formulation

In this section, the governing equations of generalized thermoelastic with voids are summarized, followed by the corresponding finite element equations. In the finite element method, the displacement components u, w, volume fraction φ and temperature change T are related to the corresponding nodal values by

$$u = \sum_{i=1}^{m} N_i u_i(t), \ w = \sum_{i=1}^{m} N_i w_i(t), \ T = \sum_{i=1}^{m} N_i T_i(t), \ \varphi = \sum_{i=1}^{m} N_i \varphi_i(t) \tag{17}$$

where m denotes the number of nodes per element, and N_i are the shape functions. The eight-node isoparametric, quadrilateral element is used for displacement components, volume fractional field and temperature calculations. The weighting functions and the shape functions coincide. Thus

$$\delta u = \sum_{i=1}^{m} N_i \delta u_i, \ \delta w = \sum_{i=1}^{m} N_i \delta w_i, \ \delta\varphi = \sum_{i=1}^{m} N_i \delta\varphi_i, \ \delta T = \sum_{i=1}^{m} N_i \delta T_i \tag{18}$$

It should be noted that appropriate boundary conditions associated with the governing Eqs. (7)-(10) must be adopted in order to properly formulate a problem. Boundary conditions are either essential (or geometric) or natural (or traction) types. Essential conditions are prescribed displacements u, w, volume fraction φ and temperature change T while, the natural boundary conditions are prescribed tractions, heat flux and equilibrated stress which are expressed as

$$\sigma_{xx} n_x + \sigma_{xz} n_z = \bar{\tau}_x, \ \sigma_{xz} n_x + \sigma_{zz} n_z = \bar{\tau}_z, \ q_x n_x + q_z n_z = \bar{q}, \quad h_x n_x + h_z n_z = \bar{h}, \tag{19}$$

where n_x and n_z are direction cosines of the outward unit normal vector at the boundary, $\bar{\tau}_x$, $\bar{\tau}_z$ are the given tractions values, $\bar{q}$ is the given surface heat flux and $\bar{h}$ is the given equilibrated stress value. In the absence of body force, the governing equations are multiplied by weighting functions and then are integrated over the spatial domain Ω with the boundary Γ. Applying integration by parts and making use of the divergence theorem reduce the order of the spatial derivatives and allows for the application of the boundary conditions. Thus, the finite element equations corresponding to Eqs. (7)-(10) can be obtained as

$$\sum_{e=1}^{m}\left(\begin{bmatrix} M_{11} & 0 & 0 & 0 \\ 0 & M_{22} & 0 & 0 \\ 0 & 0 & M_{33} & 0 \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix}\begin{Bmatrix} \ddot{u} \\ \ddot{w} \\ \ddot{\varphi} \\ \ddot{T} \end{Bmatrix}+\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & R_{33} & 0 \\ R_{41} & R_{42} & R_{43} & R_{44} \end{bmatrix}\begin{Bmatrix} \dot{u} \\ \dot{w} \\ \dot{\varphi} \\ \dot{T} \end{Bmatrix} + \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \\ K_{31} & K_{32} & K_{33} & K_{34} \\ 0 & 0 & 0 & K_{44} \end{bmatrix}\begin{Bmatrix} u \\ w \\ \varphi \\ T \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix}\right) \tag{20}$$

where the coefficients in Eq. (20) are given below

$$M_{11}=\int_\Omega [N]^T[N]\mathrm{d}\Omega,\quad M_{22}=\int_\Omega [N]^T[N]\mathrm{d}\Omega,\quad M_{33}=\int_\Omega [N]^T[N]\mathrm{d}\Omega,$$

$$M_{41}=\int_\Omega \tau_0\delta_9[N]^T\left[\frac{\partial N}{\partial x}\right]\mathrm{d}\Omega,\quad M_{42}=\int_\Omega \tau_0\delta_9[N]^T\left[\frac{\partial N}{\partial z}\right]\mathrm{d}\Omega,$$

$$M_{43}=\int_\Omega \tau_0\delta_{10}[N]^T[N]\mathrm{d}\Omega,\quad M_{44}=\int_\Omega \delta_{12}\tau_0[N]^T[N]\mathrm{d}\Omega,$$

$$R_{33}=\int_\Omega \delta_6[N]^T[N]\mathrm{d}\Omega,\qquad R_{44}=\int_\Omega \delta_{11}[N]^T[N]\mathrm{d}\Omega, \tag{21}$$

$$R_{41}=\int_\Omega \delta_9[N]^T\left[\frac{\partial N}{\partial x}\right]\mathrm{d}\Omega, R_{42}=\int_\Omega \delta_9[N]^T\left[\frac{\partial N}{\partial z}\right]\mathrm{d}\Omega, R_{43}=\int_\Omega \delta_{10}[N]^T N\,\mathrm{d}\Omega,$$

$$K_{11}=\int_\Omega (\delta_1+\delta_2)\left[\frac{\partial N}{\partial x}\right]^T\left[\frac{\partial N}{\partial x}\right]+\delta_1\left[\frac{\partial N}{\partial z}\right]^T\left[\frac{\partial N}{\partial z}\right]\mathrm{d}\Omega,\qquad K_{12}=\int_\Omega \delta_2\left[\frac{\partial N}{\partial x}\right]^T\left[\frac{\partial N}{\partial z}\right]\mathrm{d}\Omega,$$

$$K_{13}=\int_\Omega \delta_3[N]^T\left[\frac{\partial N}{\partial x}\right]\mathrm{d}\Omega,\quad K_{14}=-\int_\Omega [N]^T\left[\frac{\partial N}{\partial x}\right]\mathrm{d}\Omega,$$

$$K_{21} = \int_\Omega \delta_2 \left[\frac{\partial N}{\partial z}\right]^T \left[\frac{\partial N}{\partial x}\right] \mathrm{d}\Omega, \qquad K_{22} = \int_\Omega (\delta_1+\delta_2)\left[\frac{\partial N}{\partial z}\right]^T \left[\frac{\partial N}{\partial z}\right] + \delta_1 \left[\frac{\partial N}{\partial x}\right]^T \left[\frac{\partial N}{\partial x}\right] \mathrm{d}\Omega,$$

$$K_{23} = \int_\Omega \delta_3 [N]^T \left[\frac{\partial N}{\partial z}\right] \mathrm{d}\Omega, \quad K_{24} = -\int_\Omega [N]^T \left[\frac{\partial N}{\partial z}\right] \mathrm{d}\Omega,$$

$$K_{31} = \int_\Omega \delta_7 [N]^T \left[\frac{\partial N}{\partial x}\right] \mathrm{d}\Omega, \quad K_{32} = \int_\Omega \delta_7 [N]^T \left[\frac{\partial N}{\partial z}\right] \mathrm{d}\Omega,$$

$$K_{33} = \int_\Omega \left(\delta_4 \left[\left[\frac{\partial N}{\partial x}\right]^T \left[\frac{\partial N}{\partial x}\right] + \left[\frac{\partial N}{\partial z}\right]^T \left[\frac{\partial N}{\partial z}\right] \right] + \delta_5 [N]^T N \right) \mathrm{d}\Omega, K_{34} = \int_\Omega \delta_8 [N]^T N \,\mathrm{d}\Omega, \tag{21}$$

$$K_{44} = \int_\Omega k \left(\left[\frac{\partial N}{\partial x}\right]^T \left[\frac{\partial N}{\partial x}\right] + \left[\frac{\partial N}{\partial z}\right]^T \left[\frac{\partial N}{\partial z}\right] \right) \mathrm{d}\Omega,$$

$$F_1 = \int_\Gamma [N]^T \overline{\tau}_x \,\mathrm{d}\Gamma, \quad F_2 = \int_\Gamma [N]^T \overline{\tau}_y \,\mathrm{d}\Gamma, \quad F_3 = \int_\Gamma [N]^T \overline{q} \,\mathrm{d}\Gamma, \quad F_4 = \int_\Gamma [N]^T \overline{h} \,\mathrm{d}\Gamma.$$

Symbolically, the discretized equations of Eq. (20) can be written as

$$M\ddot{d} + R\dot{d} + Kd = F^{ext}, \tag{22}$$

where M, R, K and F^{ext} represent the mass, damping, stiffness matrices and external force vectors, respectively; $d = [u\ w\ \varphi\ T]^T$. On the other hand, the time derivatives of the unknown variables have to be determined by Newmark time integration method (see Wriggers 2008).

6. Numerically results and discussion

With the view of illustrating theoretical results derived in the preceding sections, and compare these in the context of various theories of initially stressed thermoelastic with voids, we now present some numerical results for crystal-like material. The physical constants used are

$$\lambda = 7.76\times10^{10}\,Kg\,m^{-1}s^{-2}, \mu = 3.86\times10^{10}\,Kg\,m^{-1}s^{-2},$$
$$k = 386\,W\,m^{-1}K^{-1}, \rho = 8.954\times10^{3}\,Kg\,m^{-3} \tag{23}$$
$$\alpha_t = 1.78\times10^{-5}\,K^{-1}, C^* = 0.3831\times10^{3}\,J\,Kg^{-1}\,K^{-1}, T_0 = 0.293\times10^{3}\,Kg\,m^{-3}.$$

The voids parameter, initial stress parameters and relaxation times are

$$\omega_0 = 0.687\times10^{-5}\,\mathrm{N\,s\,m^{-2}}, \chi = 0.610\times10^{-15}\,\mathrm{m}^2, \mathrm{d} = 0.9798\times10^{-5}\,\mathrm{N},$$
$$\xi^* = 0.4\times10^{10}\,\mathrm{N\,m^{-2}}, \mathrm{t}^0 = 0.5\times10^{10}\,\mathrm{N\,m^{-2}}, \tau_0 = 0.02\,\mathrm{s},, \zeta = 0.196\times10^{4}\,\mathrm{N\,m^{-2}}. \tag{24}$$

The grid size has been refined until the values of u, w, volume fraction φ and temperature change T, stabilizes. Further refinement of mesh size over 500 × 500 elements does not change the values considerably. Thus, elements with $x \times y = 500 \times 500$ were used for this study.

Figs. 1-7 depict the variation of displacement, temperature variation and stress components for different values of initial stress parameters with fixed relaxations time (Case I). In these figure —— solid line, ------ dash line, …. Dot line, dash —·—· correspond to $t^0 = 0$, 5×10^{10}, 10×10^{10}, 15×10^{10} respectively.

Figs. 8-14 show the variation of these quantities for different values relaxations times with fixed initial stress parameters (Case II) and in Figs. 8-14 the above lines correspond to τ_0 = 0,0 .05, 0.1, 0.2 respectively.

Near the application of the source the values of u increase whereas the values of w decrease and away from the source, both converges to the boundary surface, these variations are shown in Figs. 1 and 2.

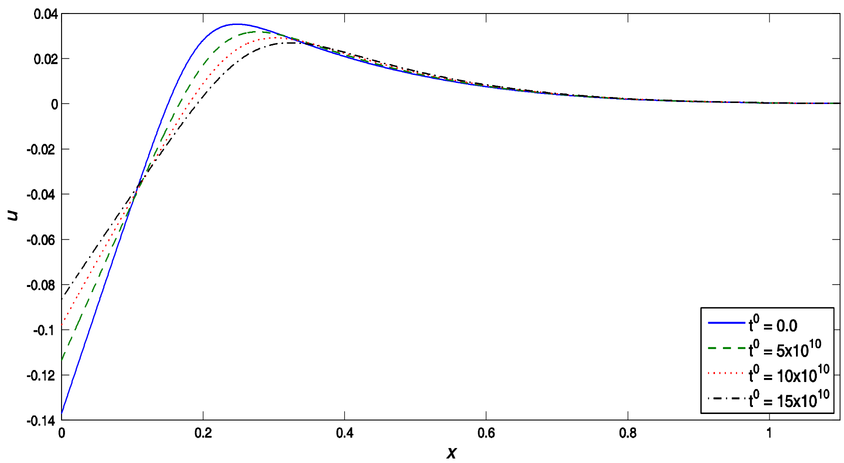

Fig. 1 Variation of normal displacement with distance

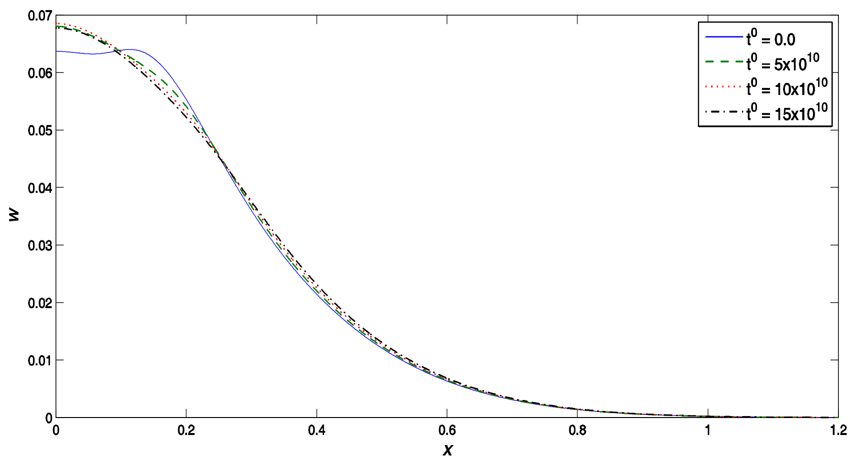

Fig. 2 Variation of Tangential displacement with distance

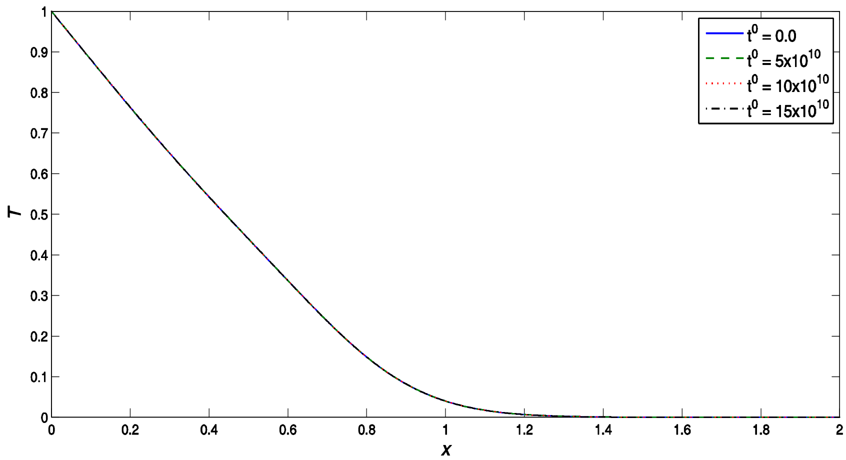

Fig. 3 Variation of Temperature change with distance

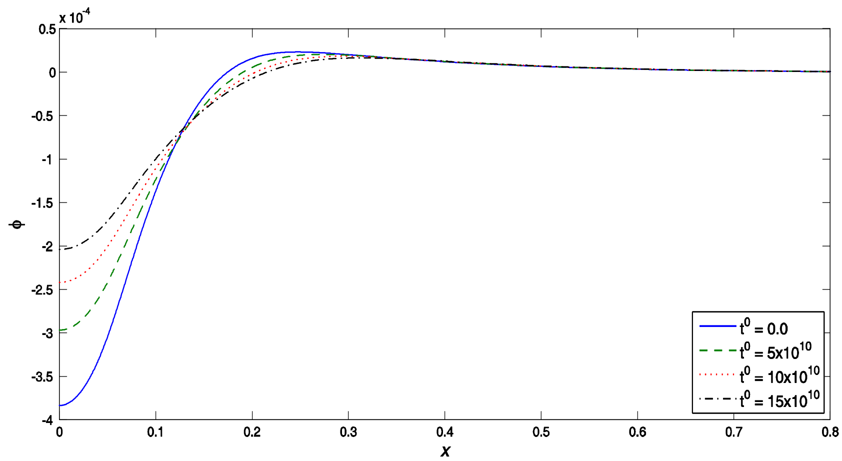

Fig. 4 Variation of volume fraction field with distance

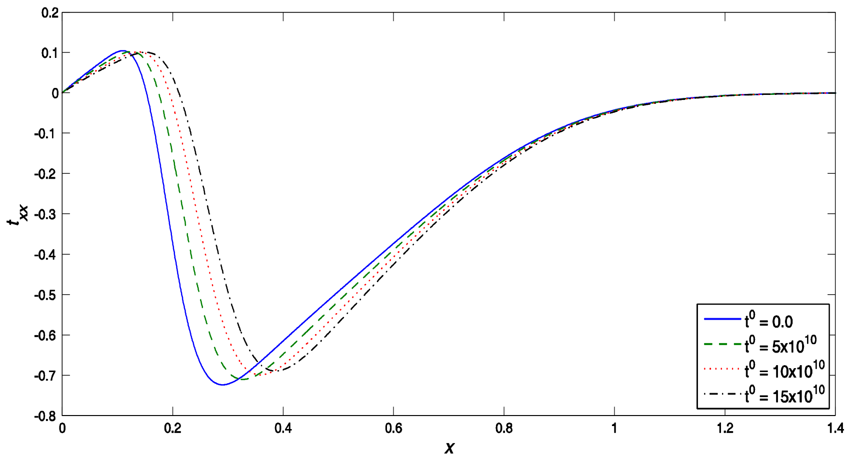

Fig. 5 Variation of stress component t_{xx} with distance

In Fig. 3, the values of temperature T decrease monotonically with distance x for all values of initial stress parameter with difference in their magnitude value.

The values of ϕ depict the oscillatory behavior for small value of x and shown dispersionless behavior at the boundary surface, these variations are shown in Fig. 4.

At the application of the source, the values of t_{xx}, t_{zz}, start with similar values, increase with small magnitude, decrease monotonically, again increase and take the stationary value as x increases, these variations are shown in Figs. 5 and 7.

The behaviour of variation of t_{xz} is of oscillatory when the source is applied for small values of x and as x increases, the variation is small and obtained the stationary values at the boundary surface and these variations are shown in Fig. 6.

The behaviour and trend of variation of u and w near the application of the source is opposite to each other and away from the source, both attained the similar trend. These variations are shown in Figs. 8 and 9.

Fig. 10 depicts the variation of T. The values of T are small for small relaxation time and more

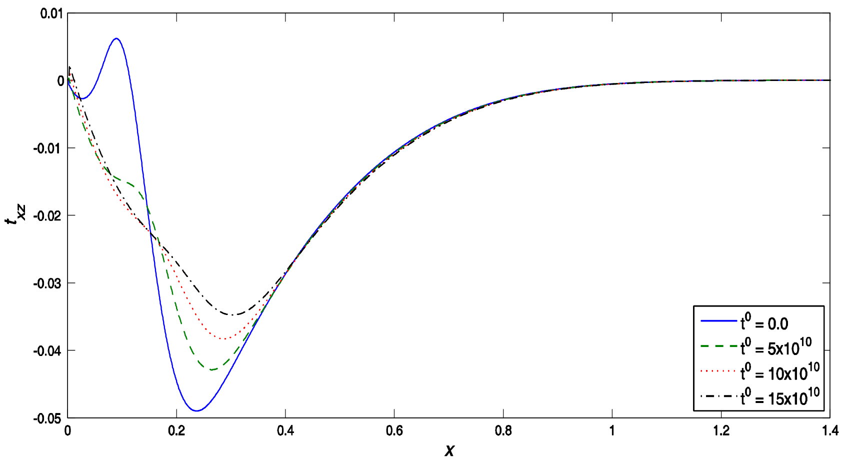

Fig. 6 Variation of stress component t_{xz} with distance

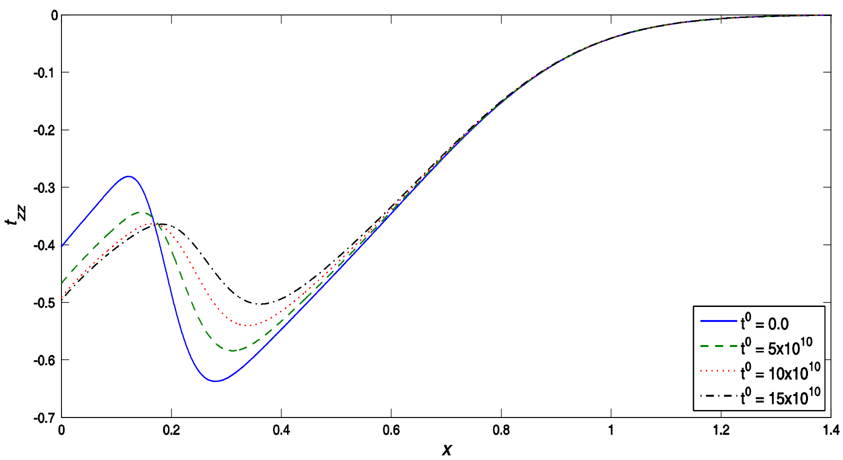

Fig. 7 Variation of stress component t_{zz} with distance

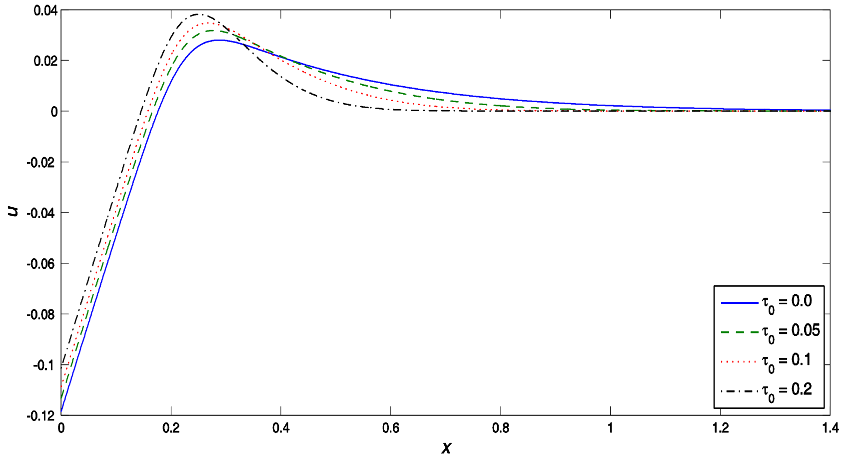

Fig. 8 Variation of normal displacement with distance

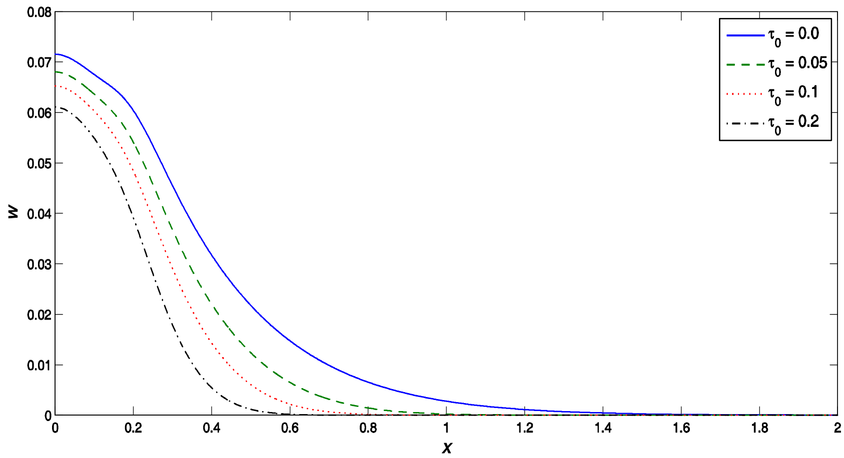

Fig. 9 Variation of tangential displacement with distance

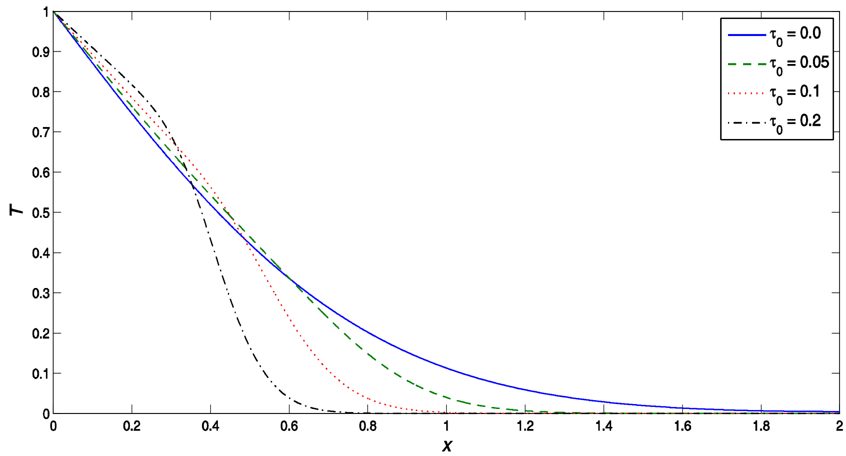

Fig. 10 Variation of Temperature change with distance

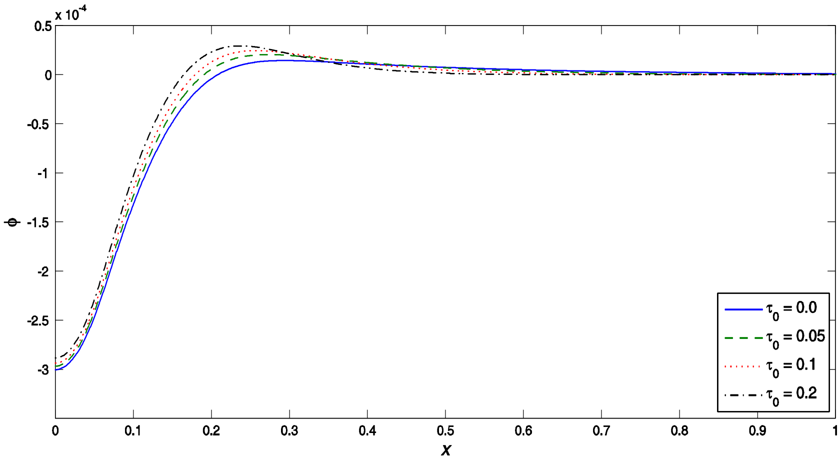

Fig. 11 Variation of volume fraction field with distance

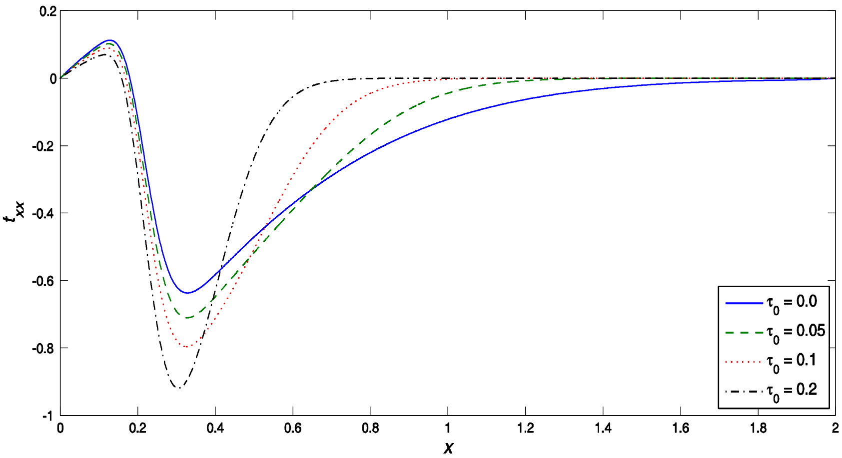

Fig. 12 Variation of stress component t_{xx} with distance

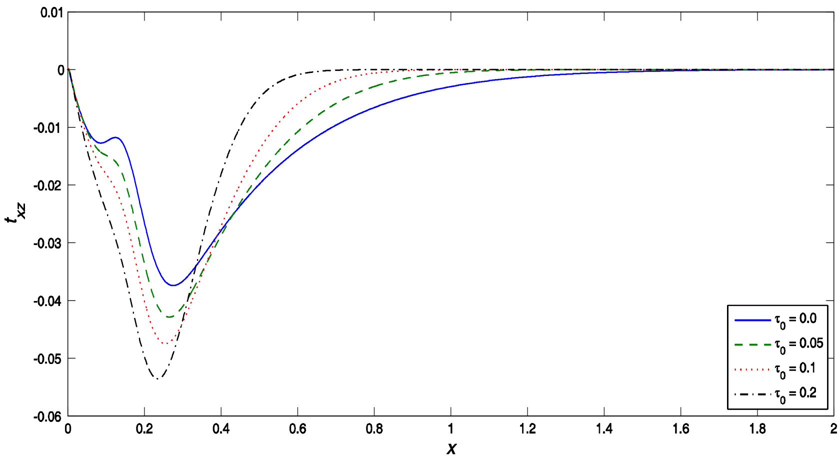

Fig. 13 Variation of stress component t_{xz} with distance

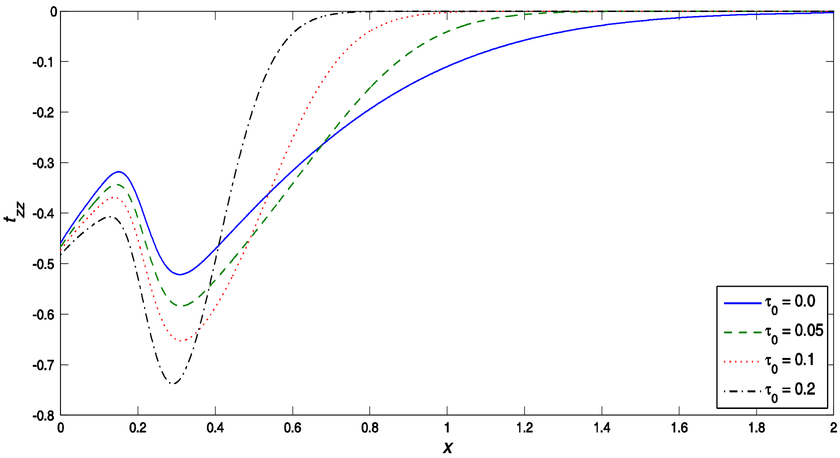

Fig. 14 Variation of stress component t_{zz} with distance

as relaxation time takes the higher value. The opposite trend is noticed as $x \geq 0.65$ whereas in the intermediate values there is oscillatory behavior.

The values of ϕ increase monotonically as x increases with different magnitude and as x increases further, the values remain stationary with difference in their magnitude values and these variations are shown in Fig. 11.

The behavior and variations of t_{xx}, t_{zz}, are similar with difference in their values, the values of these quantities are oscillatory for all values of relaxations times, and these variation are shown in Figs. 12 and 14.

The value of t_{xz} decrease monotonically near the source application for all values of relaxation times except at $\tau_0 = 0, 0.05$, whereas it has oscillatory behavior with small values of x. For higher values of x, it increase and then attain stationary values with difference in their magnitude values as $x \geq 1.4$.

7. Conclusions

The aim of the present study is to enhance our knowledge about the application of finite element method in initially stressed thermoelastic half-space with voids. All the curves converge to zero as distance from surface of medium increases, this satisfies the conditions for surface wave propagation. The effect of relaxation times and initial stress have significant effects on all the field quantities except the temperature respect to initial stress. Near the application of the source, the effect of initial stress and relaxation times are highly predominated on u, w, ϕ, t_{zz} whereas on other quantities effect are small predominated. Away from the source, they depict the stationary behavior. Also these effect decrease the value of u, w, T and increase the value of ϕ, t_{xx}, t_{zz}, t_{zz}, and converges to the boundary surface.

References

Abbas, I.A. (2013), "A GN model for thermoelastic interaction in an unbounded fiber-reinforced anisotropic medium with a circular hole", *Appl. Math. Lett.*, **26**(2), 232-239.

Abbas, I.A. (2014a), "Fractional order GN model on thermoelastic interaction in an infinite fibre-reinforced anisotropic plate containing a circular hole", *J. Comput. Theor. Nanosci.*, **11**(2), 380-384.

Abbas, I.A. (2014b), "Three-phase lag model on thermoelastic interaction in an unbounded fiber-reinforced anisotropic medium with a cylindrical cavity", *J. Comput. Theor. Nanosci.*, **11**(4), 987-992.

Abbas, I.A. and Kumar, R. (2014), "Deformation due to thermal source in micropolar generalized thermoelastic half-space by finite element method", *J. Comput. Theor. Nanosci.*, **11**(1), 185-190.

Abbas, I.A. and Othman, M.I. (2011), "Generalized thermoelastic interaction in a fiber-reinforced anisotropic half-space under hydrostatic initial stress", *J. Vib. Control*, **18**(2) 175-182.

Abo-Dahab, S.M. and Singh, B. (2013), "Rotational and voids effect on the reflection of P waves from stress-free surface of an elastic half-space under magnetic field and initial stress without energy dissipation", *Appl. Math. Model.*, **37**(20-21), 8999-9011**.**

Acharya, D.P., Roy, I. and Sengupta, S. (2009), "Effect of magnetic field and initial stress on the propagation of interface waves in transversely isotropic perfectly conducting media", *Acta Mechanics*, **202**(1), 35-45.

Bachher, M., Sarkar, N. and Lahiri, N.A. (2014), "Generalized thermoelastic infinite medium with voids subjected to a instantaneous heat sources with fractional derivative heat transfer", *Int. J. Mech. Sci.*, **89**, 84-91.

Bachher, M., Sarkar, N. and Lahiri, N.A. (2015), "Fractional order thermoelastic interactions in an infinite voids material due to distributed time-dependent heat sources", *Meccanica*, **50**(8), 2167-2178.

Biot, M.A. (1956), "Thermoelasticity and irreversible thermodynamics", *J. Appl. Phys.*, **27**(3), 240-253.

Birsan, M. (2000), "Existence and uniqueness of weak solution in the linear theory of elastic shells with voids", *Libertas Math.*, **20**, 95-105.

Chirita, S. and Scalia, A. (2001), "On the spatial and temporal behavior in linear thermoelasticity of materials with voids", *J. Therm. Stress.*, **24**(5), 433-455.

Ciarletta, M. and Scalia, A. (1993a), "On the nonlinear theory of non simple thermoelastic materials with voids", *Z. Angew. Math. Mech.*, **73**(2), 67-75.

Ciarletta, M. and Scalia, A. (1993b), "On uniqueness and reciprocity in linear thermoelasticity of material with voids", *Journal of Elasticity*, **32**(1), 1-17.

Ciarletta, M. and Scarpetta, E. (1995), "Some results on thermoelasticity for dielectric materials with voids", *Z. Angew. Math. Mech.*, **75**(9), 707-714.

Ciarletta, M., Iovane, G. and Sumbatyan, M.A. (2003), "On stress analysis for cracks in elastic materials with voids", *Int. J. Eng. Sci.*, **41**(20), 2447-2461.

Cowin, S.C. and Nunziato, J.W. (1983), "Linear elastic materials with voids", *J. Elasticity*, **13**(2), 25-147.

Dhaliwal, R.S. and Wang, J. (1994), "Domain of influence theorem in the theory of elastic materials with voids", *Int. J. Eng. Sci.*, **32**(11), 1823-1828.

Dhaliwal, R.S. and Wang, J. (1995), "A heat-flux dependent theory of thermoelasticity with voids", *Acta Mech.*, **110**(1), 33-39.

Fahmy, M.A. and El-Shahat, T.M. (2008), "The effect of initial stress and inhomogeneity on the thermoelastic stresses in a rotating anisotropic solid", *Arch. Appl. Mech.*, **78**(6), 431-442.

Green, A.E. and Lindsay, K.A. (1972), "Thermoelasticity", *J. Elast.*, **2**(1), 1-7.

Green, A. and Naghdi, P. (1991), "A re-examination of the basic postulates of thermomechanics", *Proceedings of the Royal Society of London. Series A: Mathematical and Physical Sciences*, **432**(1885), 171-194.

Green, A.E. and Naghdi, P.M. (1992), "On undamped heat waves in an elastic solid", *J. Therm. Stress.*, **15**(2), 253-264.

Green, A.E. and Naghdi, P.M. (1993), "Thermoelasticity without energy dissipation", *J. Elast.*, **31**(3), 189-208.

Iesan, D. (1986), "A theory of thermoelastic materials with voids", *Acta Mechanica*, **60**(1), 67-89.

Iesan, D. (1987), "A theory of initially stressed thermoelastic materials with voids", *An. St. Univ. Iasi, S. I-a Matematica*, **33**, 167-184.

Iesan, D. (2004), *Thermoelastic Models of Continua*, Springer, Berlin, Germany.

Lord, H.W. and Shulman, Y. (1967), "A generalized dynamical theory of thermoelasticity", *J. Mech. Phys. Solid.*, **15**(5), 299-309.

Marin, M. (1997a), "A uniqueness result for body with voids in linear thermoelasticity", *Rend. Mat. Appl.*, **17**(7), 103-113.

Marin, M. (1997b), "On the domain of influence in thermoelasticity of bodies with voids", *Arch. Math.* (*Brno*), **33**(4), 301-308.

Marin, M. (1998), "Contributions on uniqueness in thermoelastodynamics on bodies with voids", *Cienc. Mat.* (*Havana*), **16**(2), 101-109.

Marin, M. (1999), "An evolutionary equation in thermoelasticity of dipolar bodies", *J. Math. Phys.*, **40**(3), 1391-1399.

Marin, M. (2009), "On the minimum principle for dipolar materials with stretch", *Nonlinear Anal.: Real World Appl.*, **10**(3), 1572-1578.

Marin, M. and Salca, H. (1998), "A rrelation of Knopoff-de hoop type in thermoelasticity of Dipolar Bodies with voids", *Theor. Appl. Mech.*, **24**, 99-110.

Nunziato, J.W. and Cowin, S.C. (1979), "A nonlinear theory of elastic materials with voids", *Arch. Rat. Mech. Anal.*, **72**(2), 175-201.

Pompei, A. and Scalia, A. (2002), "On the asymptotic spatial behavior in linear thermoelasticity of materials with voids", *J. Therm. Stress.*, **25**(2), 183-193.

Puri, P. and Cowin, S.C. (1985), "Plane waves in linear elastic materials with voids", *J. Elasticity*, **15**(2), 167-183.

Rusu, G. (1987), "On existence and uniqueness in thermoelasticity of materials with voids", *B. Acad. Pol. Sci. Tech.*, **35**(7-8), pp. 339-346.

Saccomandi, G. (1992), "Some remarks about the thermoelastic theory of materials with voids", *Rend. Mat. Appl.*, **12**(7), 45-58.

Scarpetta, E. (1995), "Well posedness theorems for linear elastic materials with voids", *Int. J. Eng. Sci.*, **33**(2), 151-161.

Sharma, J.N. and Grover, D. (2012), "Thermoelastic vibration analysis of Mems/Nems plate resonators with voids", *Acta Mechanica*, **223**(1), 167-187.

Singh, B., Kumar, A. and Singh, J. (2006), "Reflection of generalized thermoelastic waves from a solid half-space under hydrostatic initial stress", *Appl. Math. Comput.*, **177**(1), 170-177.

Tomar, S.K. and Ogden, R.W. (2014), "Two-dimensional wave propagation in a rotating elastic solid with voids", *J. Sound Vib.*, **333**(7), 1945-1952.

Wriggers, P. (2008), *Nonlinear Finite Element Methods*, Berlin Heidelberg, Springer-Verlag.

Zenkour, A.M. and Abbas, I.A. (2014), "Nonlinear transient thermal stress analysis of temperature-dependent hollow cylinders using a finite element model", *Int. J. Struct. Stabil. Dyn.*, **14**(7), 1450025.

CC