

## Numerical analysis of the Influence of the presence of disbond region in adhesive layer on the stress intensity factors (SIF) and crack opening displacement (COD) in plates repaired with a composite patch

Aicha Benchiha <sup>\*1</sup>, Kouider Madani <sup>1a</sup>, Sebastien Touzain <sup>2</sup>,  
Xavier Feaugas <sup>2</sup> and Mohan Ratwani <sup>3</sup>

<sup>1</sup> LMPM, Department of Mechanical Engineering,  
University of Sidi Bel Abbes, Cité Ben M'hidi, Sidi Bel Abbes, Algeria

<sup>2</sup> La Rochelle University, Laboratoire des sciences pour l'ingénieur  
pour l'environnement (LASIE), La Rochelle, France

<sup>3</sup> R-Tec, Rolling Hills Estates, CA 90274, USA

(Received September 27, 2015, Revised November 29, 2015, Accepted December 25, 2015)

**Abstract.** The determination of the stress intensity factor at the crack tip is one of the most widely used methods to predict the fatigue life of aircraft structures. This prediction is more complicated for repaired cracks with bonded composite patch. This study is used to compute the stress intensity factor (SIF) and crack opening displacement (COD) for cracks repaired with single and double-sided composite patches. The effect of the presence of disbond region in adhesive at the crack was taken into consideration. The results show that there is a considerable reduction in the asymptotic value of the stress-intensity factors and the crack opening displacement at the crack tip. The use of a double-sided patch suppresses the bending effect due to the eccentricity of the patch on one side only.

**Keywords:** stress intensity factor (SIF); crack opening displacement (COD); crack; bonded composite repair; disbond

---

### 1. Introduction

The use of adhesively bonding is of great interest to many industrial sectors including aerospace and automotive, as well as pack aging and domestic appliance industries, Da Silva *et al.* (2011). Adhesive bonded patching is one of the most widely used repair techniques to cracked or damaged metallic and composite structures. In this technique, a composite patch is bonded to the parent structure to reinforce the cracked zone and to try to restore the structure to its original design specifications, Ouinas *et al.* (2007). The adhesively bonded composite repair is an efficient and cost-effective method to extend the fatigue life of cracked components in advanced aerospace structures. The bonded patch repair prevents or retards the crack reinitiating and crack propagation.

---

\*Corresponding author, Professor, E-mail: [aicha.benchiha@yahoo.fr](mailto:aicha.benchiha@yahoo.fr)

<sup>a</sup> Ph.D., E-mail: [koumad10@yahoo.fr](mailto:koumad10@yahoo.fr)

The bonded composite repair carries a part of the load acting at the crack tip throughout the bonded region, thus reducing the stress-intensity factors more effectively compared to mechanically fastened repairs, Madani *et al.* (2009). The bonded repair reduces stresses in the cracked region and keeps the crack from opening, and therefore from growing. The determination of the stress intensity factors (SIFs, KI and KII) can be used to analyze the performance of bonded composite crack repairs, Rezgani *et al.* (2010). The first numerical model proposed by Ratwani used two dimensional finite elements to represent cracked panel and shear spring elements to represent adhesive layer by neglecting the influence of out-of-plane bending, which may lead to errors in single-sided repair situations. Several authors have used Mindlin plate finite elements to represent cracked panels in the numerical analyses, Madani *et al.* (2009). Lee and Lee (2004) considered out of-plane bending effects in analytical approach of single sided repairs. By both analytical and 3D linear FE approaches in their study, SIF followed a linear variation through-the-thickness, accordingly with the results by Sun *et al.* Also, the value of the root-mean-square of SIF along the front, K, obtained from the 3D FE analysis was in good agreement with those obtained from two-rmslayer model by Lee and Lee (2004). Seo and Lee (2002). Modeled the crack front as a skewed straight one in their 3D FE analysis, and showed the effect of crack front shape on the SIF variation through-the-thickness. The most of studies are limited to the case of mode I opening of the crack, Megueni *et al.* (2003) computed the stress intensity factor at the crack tip in mode II. found that the lead fatigue crack growth rates in cross-ply GLARE 3-3/2 laminates containing multiple open holes increased because of the presence of multi-site damage. Albedah *et al.* (2011) used the finite elements method to analyze the performances of the bonded composite repair. They showed that the presence of the patch repair highly reduced the stress intensity factor (SIF) at the crack tip, which can improve the fatigue life of the cracked structures. Albedah *et al.* (2011) have conducted finite element analysis to estimate SIF for single and double sided repairs having a circular patch shape. They have compared the mass gain for both cases.

The aim of the present study is to analyze the effects of different patch properties and adhesive thickness with and without presence of disbond region in adhesive, on stress intensity factors in a centrally cracked aluminum plate repaired with single and double-sided patches. Stress intensity factors and the crack opening displacement are calculated using a software program developed by Ratwani *et al.* (2006). The program computes stress intensity factors for cracked plate with repair patch bonded on one or both the sides of the plate. The program also computes crack opening displacement at the centerline of the crack. The software program, developed using mathematical analysis, is operational on a personal computer or a work station.

## 2. Mathematical formulation

The cracked aluminum plate configuration shown in Fig. 1 is considered for analysis. A crack length of  $(2a)$  is assumed in metallic plate and no crack in repair patch. A complex variables method is used to analyze the problem of the cracked plate with a bonded repair patch as shown in Fig. 1. In the generalized formulation the cracked plate is considered as an orthotropic material so that the materials such as titanium which exhibit some orthotropic can be analyzed by the present analysis method.

The solution to the Fig. 1 problem is obtained by considering the perturbation problem in which loads are applied to crack surfaces only. The following assumptions are made in the analysis:

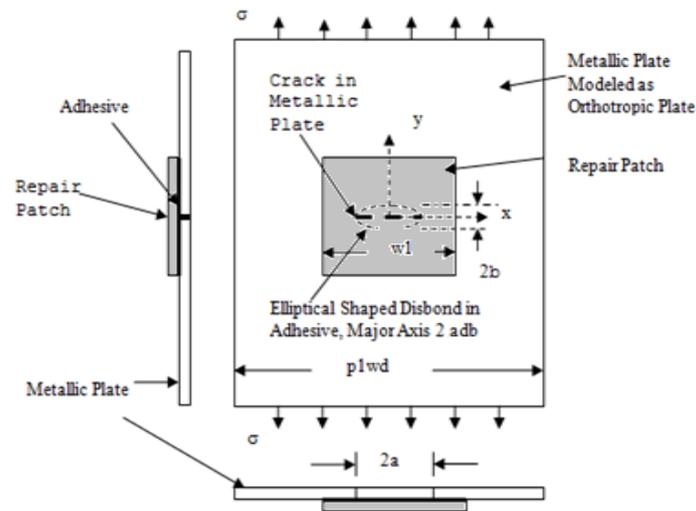


Fig. 1 Geometrical model of the patched structure

- (1) Plate and repair patch are considered as orthotropic materials with principal directions along the crack plane and at right angles.
- (2) Plate, patch and adhesive are linearly elastic materials.
- (3) Variations of stresses through the thickness of plate and repair patch are neglected.
- (4) Adhesive is modeled as shear spring to transfer load between plate and repair patch.
- (5) The surface shears between the plate and repair patch, transmitted through the adhesive, acts as body forces on the plate and repair patch.
- (6) Load transfer takes place between the plate and the repair patch in the bonded region only.
- (7) The plate and patch are sufficiently large in size so that the Green's functions for infinite plate can be used (Madani *et al.* 2008).

### 3. Development of the computer program

The software program solves integral equations numerically for shear stresses and from these stress intensity factors in the cracked plate, shear stresses in the adhesive, and stresses in the cracked plate and repair patch are computed. The program incorporates the effect of out-of-plane bending and finite width effects. The stresses in the cracked plate are computed in the neighborhood of the crack plane only. The program computes stress intensity factors for cracked plate with repair patch bonded on one or both sides of the plate. The program also computes crack opening displacement at the centerline of the crack. The software program is operational on a personal computer.

The present analysis is for infinitely wide plate. To account for the finite plate width effects in present analysis, the finite width correction for cracks in isotropic monolithic sheets given by Tada and Paris. Is applied to the SIFs obtained from the infinite width plate analysis. The finite width correction is given by

$$f\left(\frac{a}{W}\right) = \left[ 1 - 0.025\left(\frac{2a}{W}\right)^2 + 0.06\left(\frac{2a}{W}\right)^4 \right] \sqrt{\sec\left(\frac{\pi a}{W}\right)} \tag{1}$$

Where, ( $W$ ) is plate width and, ( $a$ ) is half crack length.

#### 4. Comparison between test and analysis

The analytical predictions of crack opening displacements were made for the panels tested using the software developed in the present studies. The material properties, discussed above in parametric studies were used. A comparison of analytical and test crack opening displacements at centreline of crack for panel with GLARE repair patch and applied remote stress of 120 MPa is shown in Table 1. The correlation between test and analytical predictions is very good.

SIFs were obtained experimentally for panel bonded with boron patch, using the strain gage data, obtained ahead of the crack tip. The normalized stress intensity factor for a crack length of 25 mm was found to be 0.55. The SIF from present analysis is 0.533. Thus, a good correlation (97%) is obtained between analytical prediction and experimental SIF. Other details for the software program used in this study are given elsewhere (Ratwani *et al.* 2006).

The elastic properties of the different materials, aluminum plate, adhesive layer and composite patch are given in Table 3. (Madani *et al.* 2008, Mokhtari *et al.* 2013).

Where, ( $E_1, E_2, E_3$ ) are elastic modulus, ( $G_{12}, G_{13}, G_{23}$ ) Shear modulus and, ( $\nu_{12}, \nu_{13}, \nu_{23}$ ) are Poisson ratio.

Table 1 Comparison of analytical and test crack opening displacements (COD) at centreline of crack for panel with glare repair patch (applied remote stress 120 MPa)

Crack length (mm)	COD analysis (mm)	COD test (mm)	Percentage of correlation
25.0	0.0535	0.06	89
37.5	0.0748	0.07	93
50.0	0.0965	0.085	86

Table 2 Dimensions of the plate, adhesive layer and patch

Dimensions	2024-T3 Aluminum	Adhesive	Composite patch
Length	$H_p = 100$ mm	$H_a = 50$ mm	$H_r = 50$ mm
Width	$W = 50$ mm	$W = 25$ mm	$W = 25$ mm
Thickness	$e_p = 2$ mm	$e_a = 0.1$ mm	$e_r = 1.60$ mm

Table 3 Material properties of aluminum plate, adhesive layer and composite patch

Materials	$E_1$ (GPa)	$E_2$ (GPa)	$E_3$ (GPa)	$G_{12}$ (GPa)	$G_{13}$ (GPa)	$G_{23}$ (GPa)	$\nu_{12}$	$\nu_{13}$	$\nu_{23}$
Al 2024-T3	68.88	-	-	25.90	-	-	0.33	-	-
Adekit A140	2.69	-	-	1.00	-	-	0.34	-	-
Carbon-epoxy	109.00	8.819	8.819	4.315	4.315	3.20	0.34	0.34	0.38

### 5. Analysis and results

To examine the effect of different patch properties and adhesive thicknesses on the stress intensity factors (SIFs) and crack opening displacement (COD), three parts was considered in the analysis. In the first part of our study, we consider no disbonding region in adhesive. In the second part we change only the patch properties. Then, in the third part of this study, we consider the disbonding region in adhesive (elliptical shape disbonding in adhesive), that the patch composite has the same material properties.

#### 5.1 SIF without presence of adhesive disbond

This section concerns the determination of SIF with crack length (2a) in an aluminum plate repaired with single and double-sided composite patch without presence of adhesive disbond.

##### 5.1.1 Influence of out-of-plane bending on stress intensity factors (SIFs)

The influence of out-of-plane bending on stress intensity factor at the crack according to the

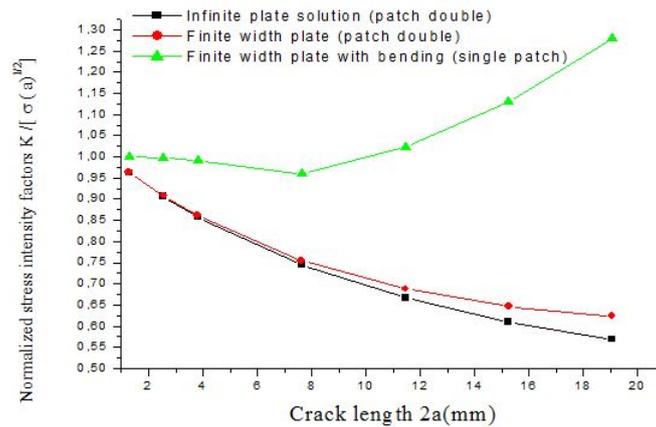


Fig. 2 Variation of stress intensity factor (SIF) with crack length, adhesive thickness = 0.1 mm

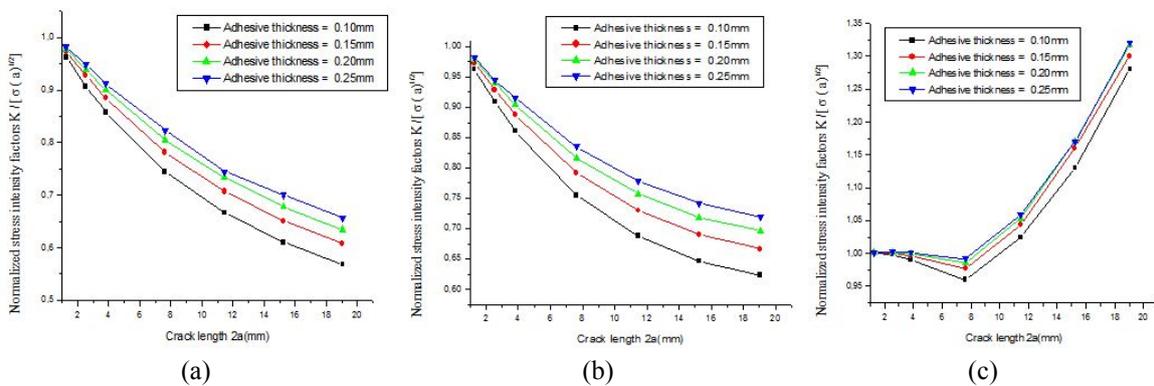


Fig. 3 Influence of adhesive thicknesses on stress intensity factors: (a) infinitely wide plate (double patch); (b) finite width plate (double patch); (c) finite width plate with bending(single patch)

crack length for single and double symmetric composite patch without presence of the adhesive disbond is shown in Fig. 2. For the same crack length, the values of normalized SIFs decrease considerably for single patch as well as double patch with an increase in crack length, but when the length of crack is rather significant (larger than about 15 mm), the values of SIFs increase for single patch. This is due to more load transfer to the repair patch from the cracked plate. A plate with repair on both sides shows more reduction in SIFs as compared to a plate with repair on one side only.

Comparing the values of the SIF between the single and double composite repair Fig. 2, it can be noted that the double patch reduces significantly the stress intensity factor at the crack compared to the single patch, due to the effect of out-of-plane bending caused by bonding of patch on one side only. Belhouari *et al.* (2004) have shown that there is an additional reduction of 10% in SIFs for a two-sided patch as compared to a single-sided patch. This indicates that the fatigue life of a structure can be improved considerably by using a two-sided patch.

### 5.1.2 Influence of adhesive thicknesses on stress intensity factors (SIFs)

Chukwujekwu Okafor and Bhogapurapu (2006) Adhesive properties influence the strength of the joint since the load is transferred from the skin to patch via the adhesive. The thickness of adhesive layer is an important issue in designing a patch.

The influence of adhesive thicknesses on stress intensity factors for infinitely wide plate (double patch), finite width plate (double patch), and finite width plate with bending (single patch) is shown in Fig. 3. It is seen that a reduction of adhesive thickness decreases the SIFs due to an increase in load transfer to the repair patch. This implies that lower adhesive thickness is desirable for repairing cracks. The lower thickness well supports the transfer loads towards the patch but increases the risk of adhesive failure. However, a certain minimum adhesive thickness is necessary to develop full bond strength and avoid adhesive failure. (Ratwani *et al.* 2006)

### 5.1.3 Influence of adhesive thickness on crack opening displacement (COD)

The influence of adhesive thicknesses on crack opening displacement is shown in Fig. 4.

It can be seen according to Fig. 4 that a thicker adhesive causes larger crack opening

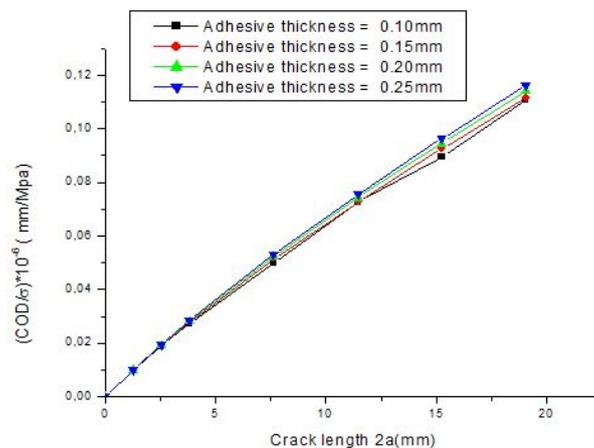


Fig. 4 Influence of adhesive thicknesses on COD, infinitely wide plate

Table 4 Properties of different patch used in the analysis (Mokhtari *et al.* 2013)

Materials	$E_1$ (GPa)	$E_2$ (GPa)	$E_3$ (GPa)	$G_{12}$ (GPa)	$G_{13}$ (GPa)	$G_{23}$ (GPa)	$\nu_{12}$	$\nu_{13}$	$\nu_{23}$
Boron-epoxy	193.06	18.617	18.617	5.516	5.516	7.757	0.21	0.21	0.2
Graphite/epoxy	127.5	9.00	4.80	4.80	4.80	2.55	0.28	0.28	0.41
Aramid-epoxy	76.0	5.5	5.5	2.3	2.3	2.3	0.34	0.34	0.34

displacement due to less load transfer to the repair patch. This is due to a reduction in the effective adhesive shear stiffness, which is inversely proportional to the adhesive thicknesses.

### 5.2 Influence of different patch properties

The parameters of fracture are influenced by the rigidity of patch, the size of the affected region and the adhesive resistance. The patch properties directly influence the variation of SIF, Ouinan *et al.* (2007). In this part of study, we use three different patch properties, the Boron-epoxy, Graphite-epoxy, and Aramid-epoxy with the same thickness of patch. Material properties of different patch used in the analysis are given in Table 4.

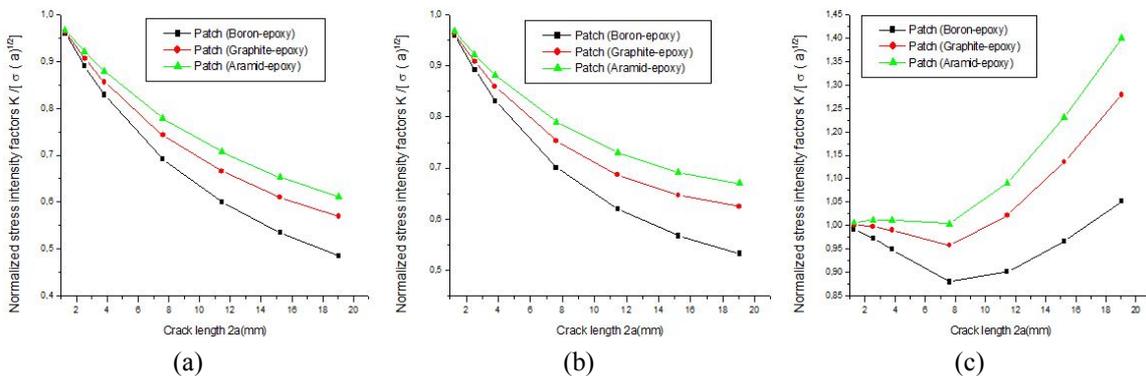


Fig. 5 Influence of patch properties on stress intensity factors: (a) infinitely wide plate (double patch); (b) finite width plate (double patch); (c) finite width plate with bending(single patch)

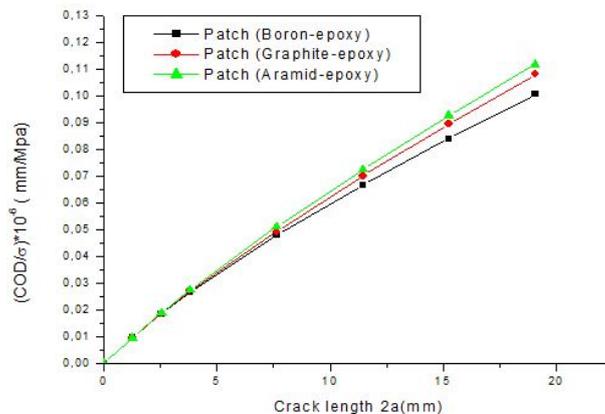


Fig. 6 Influence of patch properties on COD, finite width plate with bending

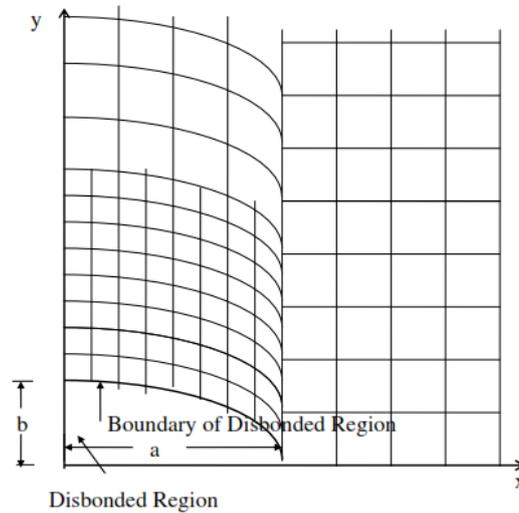


Fig. 7 Location of grid points for computing adhesive shear stresses (elliptical shape disbond in adhesive)

### 5.2.1 Influence of different patch properties on stress intensity factors (SIFs)

Reduction in the stress intensity factor (SIF) is the key issue in designing a patch. There is a considerable decrease of SIF just by applying one layer of patch.

The Fig. 5 shows the variation of the SIF according to the crack length for different patch properties without presence of disbond region in adhesive. It can be seen that the rate of decrease of the stress intensity factor is similar for a single patch to that for a double patch with the increase in the patch properties. The SIF increases exponentially as the patch properties decreases. The Boron/epoxy patch reduces the SIF more than other composite patch (Graphite-epoxy, and Aramid-epoxy). This is due to the rigidity of material.

### 5.2.2 Influence of different patch properties on crack opening displacement (COD)

The Fig. 6 shows the influence of patch properties on crack opening displacement.

### 5.3 Effect of the adhesive disbond (elliptical shape disbonding in adhesive)

Before analyzing the effect of the adhesive disbond on the performance of composite repair in cracked aluminum plate, we have found useful to compute the SIF for single and double symmetric composite patches with adhesive disbond. The disbond in adhesive layer is assumed to be elliptical in shape with the principal axes along and perpendicular to the crack plane. The loading on the panel, shown in Figure 1, can be uniaxial or biaxial. The location of grid for computing adhesive shear stresses with and without disbond region in adhesive layer (Belhouari *et al.* 2004) is shown in Fig. 7.

- Ratio of the principal axes of adhesive disbond (ratio of principal axis at right angles to crack plane to that along the crack plane,  $b/a$  ratio in Fig. 7). For no adhesive disbond  $b/a = 0$ . This quantity has no dimensions.
- Semi-axis of adhesive disbond along crack plane. For the case of no adhesive disbond semi-axis is zero.

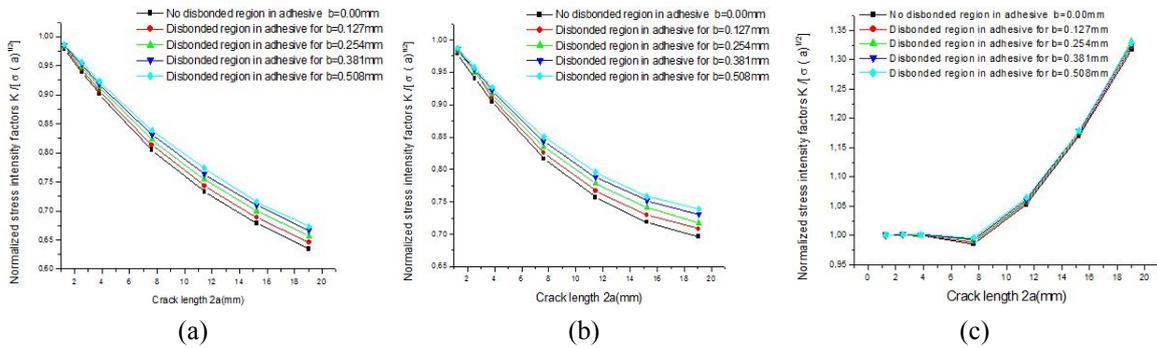


Fig. 8 Influence of disbond in adhesive on stress intensity factors: (a) infinitely wide plate (double patch); (b) finite width plate (double patch); (c) finite width plate with bending(single patch)

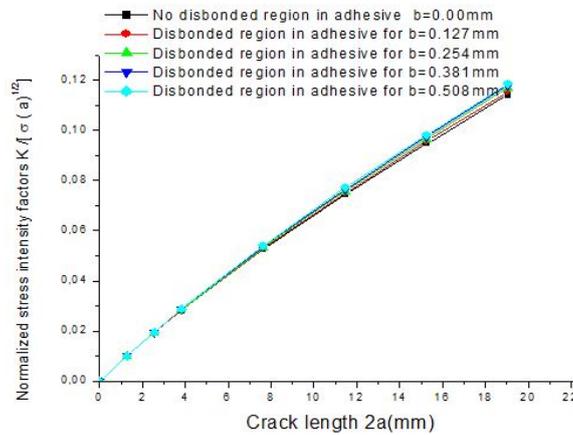


Fig. 9 Influence of disbonding in adhesive on COD, infinitely wide plate

**5.3.1 Influence of disbond region in adhesive on stress intensity factors for adhesive thickness  $e_a = 0.20$  mm**

The effect of presence of disbond in adhesive at the crack on the SIF variation for single patch and double symmetric patch are illustrated in Fig. 8.

In Fig. 8, it can observe that when the adhesive disbond increases the stress intensity factor increases. The presence of the adhesive disbond increases the stress intensity at the tip of repaired crack which can reduce the repair efficiency. A comparison of the SIF variation between single and double symmetric patches according to the presence of disband can be made. It can be seen that for weak values of disbond, the difference in the SIF between the two cases (single and double symmetric patch) is not significant. Beyond this weak value, the difference in the SIF becomes very significant as the disbond increases. In addition, this presence increases the risk of adhesion failure between the repaired structure and the composite patch.

**5.3.2 Influence of disbond region in adhesive on crack opening displacement (COD) for adhesive thickness  $e_a = 0.20$  mm**

The Fig. 9 shows the influence of disbond region in adhesive on crack opening displacement according to the crack length. It can be seen that the crack opening displacement increases as the

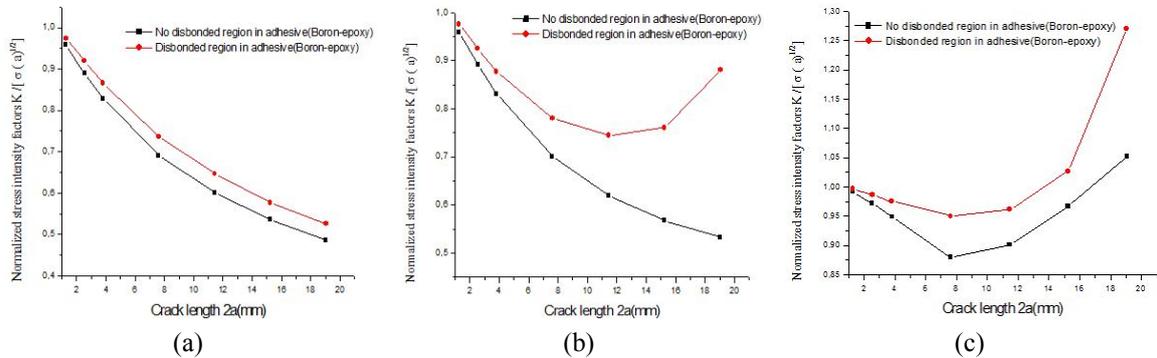


Fig. 10 Comparison between adhesive with and without disbond for Boron/epoxy patch: (a) infinitely wide plate (double patch); (b) finite width plate (double patch); (c) finite width plate with bending (single patch)

disbond region increases. It can be noted that the effect of the presence of adhesive disbond is no important on the crack opening displacement variation.

### 5.3.3 Comparison between adhesive with and without disbond for Boron/epoxy patch

When comparing the values of the SIFs between adhesive with and without disbond region for the same composite repair patch (Boron/epoxy), it can be noted that the patch without disbond reduces significantly the stress intensity factor at the crack tip compared to the adhesive with disbond region. These results confirm the effect of the presence of disbond region in adhesive at the crack. The stress intensity factor is influenced by a disbond propagation. This comparison is illustrated in Fig. 10.

### 5.4 Comparison of variation of stress intensity factor (SIF) for various thicknesses of adhesive with and without disbond

In this paragraph, the stress intensity factor is computed with and without the presence of disbond region in adhesive in order to estimate the effect of various thicknesses of adhesive on

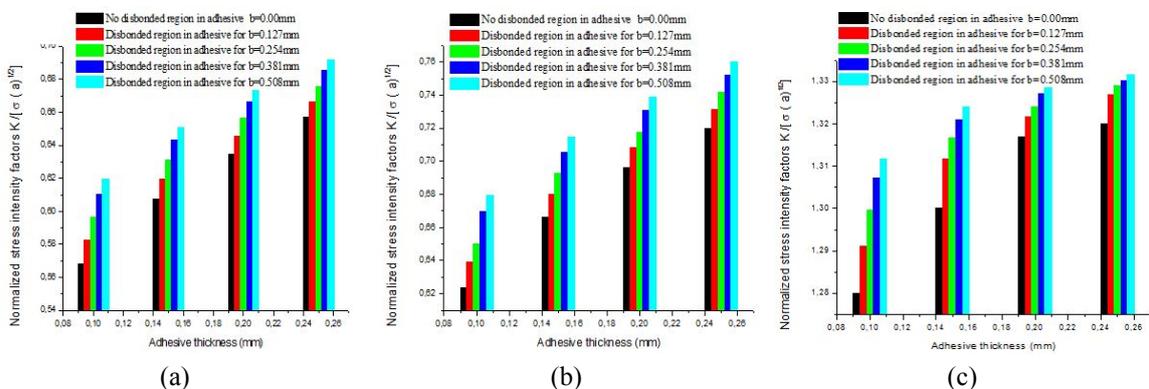


Fig. 11 Distribution of stress intensity factors; (a) infinitely wide plate; (b) finite width plate; (c) finite width plate with bending, for various thicknesses of adhesive with and without defects

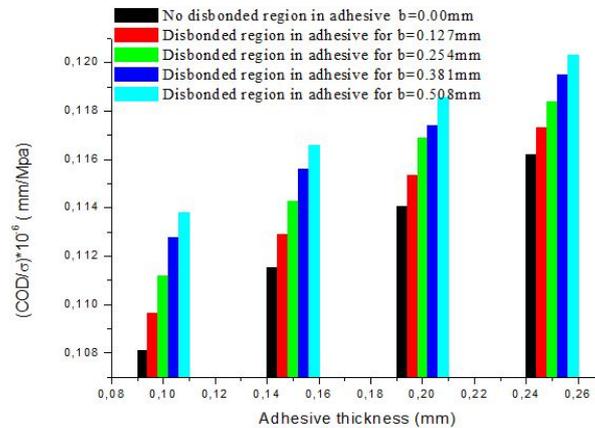


Fig. 12 Distribution of COD, infinitely wide plate, for various thicknesses of adhesive with and without disbond

these stresses. Fig. 11 presents the variation of the SIF variation according to the adhesive thicknesses for two cases with and without disbond.

It can be seen according to Fig. 11 that the presence of the disbond has a considerable effect on the SIF variation at the crack tip. Indeed, the values of the SIF are highly increased by the presence of the disbond. It is also shown, according to this figure, that for small adhesive thicknesses, the effect of the disbond on the SIF variation is less significant. The effect of the presence of the disbond on the SIF variation increases as the crack length increases.

### 5.5 Comparison of variation of crack opening displacement (COD) for a maximum crack length for various thicknesses of adhesive with and without disbond

Fig. 12 shows the variation of the crack opening displacement depending on the thickness of the adhesive for various value of the size of the debonded area. we clearly notice that the COD increases in the presence of debonded area in the adhesive. The charge transfer from the damaged plate to the patch will be minimal and thereafter, the stresses remain high at the crack tip and thus a High COD. If in addition, the thickness of the adhesive increases, the adhesive behaves as a third material and therefore no charge transfer to the patch, the COD increases.

## 6. Conclusions

The results obtained in this study yield the following conclusions:

- The double patch reduces significantly the stress intensity factor at the crack compared to the single patch, due to the effect of out-of-plane bending caused by bonding of patch on one side only.
- The stress intensity factor at the crack tip is highly reduced by the use of the double patch, what can improve the fatigue life of the aircraft structures and increases considerably the repair performances.
- The SIF increases when the crack approaches the limit external of the patch. This increase is due to the rise of the stress concentration at the level of the patch. There is a bad transfer of

the stress from the plate to the patch.

- The increase of the patch properties decreases the SIF value at the crack tip.
- The presence of the adhesive disbond increases the stress intensity at the tip of repaired cracks which can reduce the repair efficiency.
- The propagation of the adhesive disbond perpendicularly to the crack has a very negative effect on the repair efficiency.

## References

- Albedah, A., Bachir Bouiadjra, B., Mhamdia, R., Benyahia, F. and Es-Saheb, M. (2011), "Comparison between double and single sided bonded composite repair with circular shape", *Mater. Des.*, **32**(2), 996-1000.
- Belhouari, M., Bachir Bouiadjra, B., Megueni, A. and Kaddouri, K. (2004), "Comparison of double and single bonded repairs to symmetric composite structures: a numerical analysis", *Compos. Struct.*, **65**(1), 47-53.
- Chen, J.F., Evgeny, V., Morozov, K.S. (2014), "Progressive failure analysis of perforated aluminium/CFRP fibre metal laminates using a combined elastoplastic damage model and including delamination effects", *Compos. Struct.*, **114**, 64-79.
- Chukwujekwu Okafor, A. and Bhogapurapu, H. (2006), "Design and analysis of adhesively bonded thick composite patchrepair of corrosion grind-out and cracks on 2024 T3 clad aluminum aging aircraft structures", *Compos. Struct.*, **76**(1-2), 138-150.
- Da Silva, L.F.M., Öchsner, A. and Adams, R.D. (2011), *Handbook of Adhesion Technology*, Springer-Verlag, BerlinHeidelberg, Germany.
- Lee, W.Y. and Lee, J.J. (2004), "Successive 3D FE analysis technique for characterization offatigue crack growth behavior in composite-repaired aluminum plate", *Compos. Struct.*, **66**(1-4), 513-520.
- Madani, K., Touzain, S., Feugas, X., Benguediab, M. and Ratwani, M. (2008), "Numerical analysis for the determination of the stress intensity factors and crack opening displacements in plates repaired with single and double composite patches", *Comput. Mater. Sci.*, **42**(3), 385-393.
- Madani, K., Touzain, S., Feugas, X., Benguediab, M. and Ratwani, M. (2009), "Stress distribution in a 2024-T3 aluminum plate with a circular notch, repaired by a graphite/epoxy composite patch", *Int. J. Adhes. Adhes.*, **29**(3), 225-233.
- Megueni, A., Tounsi, A., Bachir Bouiadjra, B. and Serier, B. (2003), "The effect of a bonded hygrothermal aged composite patch on the stress intensity factor for repairing cracked metallic structures", *Compos. Struct.*, **62**(2), 171-176.
- Mokhtari, M., Madani, K., Belhouari, M., Touzain, S., Feugas, X. Ratwani, M. (2013), "Effects of composite adherend properties on stresses in double lap bonded joints", *Mater. Des.*, **44**(0), 633-639.
- Ouinias, D., Bouiadjra, B.B. and Serier, B. (2007), "The effects of disbonds on the stress intensity factor of aluminium panels repaired using composite materials", *Compos. Struct.*, **78**(2), 278-284.
- Ramji, M., Srilakshmi, R. and Bhanu Prakash, M. (2013), "Towards optimization of patch shape on the performance of bonded composite repair using FEM", *Compos.: Part B*, **45**(1), 710-720.
- Ratwani, M.M., Kieboom, O.T. and Heslehurst, R.B. (2006), "Crack opening displacement in plate with bonded repair patch", *Fatig. Fract. Eng. Mater. Struct.*, **29**(6), 425-430.
- Rezgani, L., Bachir Bouiadjra, B., Belhouari, M., Madani, K., Serier, B. and Feugas, X. (2010), "Effect of composite hygrothermal aging on the SIF variation in bonded composite repair of aircraft structures", *J. Reinf. Plast. Compos.*, **29**(24), 3631-3636.
- Seo, D.C. and Lee, J.J. (2002), "Fatigue crack growth behavior of cracked aluminum plate repaired with composite patch", *Compos. Struct.*, **57**, 323-330.