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Distortional buckling of I-steel concrete composite beams in negative moment area

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Abstract. The predominant type of buckling that I-steel concrete composite beams experience in the negative moment area is distortional buckling. The key factors that affect distortional buckling are the torsional and lateral restraints by the bottom flange. This study thoroughly investigates the equivalent lateral and torsional restraint stiffnesses of the bottom flange of an I-steel concrete composite beam under negative moments. The results show a coupling effect between the applied forces and the lateral and torsional restraint stiffnesses of the bottom flange. A formula is proposed to calculate the critical buckling stress of the I-steel concrete composite beams under negative moments by considering the lateral and torsional restraint stiffnesses of the bottom flange. The proposed method is shown to better predict the critical bending moment of the I-steel composite beams. This article introduces an improved method to calculate the elastic foundation beams, which takes into account the lateral and torsional restraint stiffnesses of the bottom flange and considers the coupling effect between them. The results show a close match in results from the calculation method proposed in this paper and the ANSYS finite element method, which validates the proposed calculation method. The proposed calculation method provides a theoretical basis for further research on distortional buckling and the ultimate resistance of I-steel concrete composite beams under a variable axial force.

Keywords: steel concrete composite beams; distortional buckling; rotational restraint stiffness; lateral restraint stiffness; elastic foundation beam method

1. Introduction

Steel concrete composite beams are important lateral-load-carrying composite elements in structures. Shear connections combine a concrete slab with a steel beam, so that the steel beam and the concrete slab can carry loads together. The inclusion of the concrete slab can improve both the local stability of the steel beam and the overall stability of the structure. In addition, owing to the effective use of recycled steel and the quick construction speed, steel-concrete composite beams are eco-friendly structures promising for the future construction market (Fu *et al.* 2013, Champenoy *et al.* 2014, Li *et al.* 2014).

The concrete slab restricts the extent of negative bending moment zone in the steel section in a

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steel concrete composite beam. Composite beams may suffer either from local buckling or from distortional buckling, the latter of which is characterized by lateral buckling in the compression flange accompanied by an out-of-plane distortion in the steel web. The Vlasov assumption that the cross-section remains undistorted does not apply to this mode of distortional buckling (Bradford 1992, Bradford and Gao 1992, Dekker *et al*. 1995). A significant amount of research has been done on the distortional buckling of composite beams under negative bending moment, which includes important studies by Bradford and Kemp (2000), Johnson and Chen (1993a, b), and Ronagh (2001). In 1983, Johnson and Bradford (1983) presented the first design rule for the complete buckling of composite beams, which was based on a rational buckling analysis. Johnson and Fan (1991) and Johnson and Chen (1993a, b) carried out a series of beam tests at Warwick University to investigate the distortional buckling failure of continuous composite beams. Their results verified that the modes of buckling may be classified as local and/or restrained distortional buckling, and showed that interaction between the local and the distortional buckling governs the ultimate resistance of the test specimens. These findings were further verified by Johnson and Fan (1991) and Johnson and Chen (1993a, b) for the ultimate resistance of a continuous composite beam.

Current research methods to determine the distortional buckling of the composite beam fall into two general categories: numerical analysis methods and simplified theoretical methods. Numerical analysis methods are most commonly used for inelastic buckling analyses. For example, Bradford and Johnson (1987) used inelastic finite element and finite strip analyses to develop design rules for both the local and the distortional buckling. Vrcelj and Bradford (2009) employed a spline finite strip method to investigate the inelastic buckling of simple and continuous composite beams. Simplified theoretical methods, such as the elastically supported bar method and the energy method, are used to study elastic distortional buckling. Although numerical methods can be used to solve elastic distortional buckling, the simplified theoretical methods can derive closed-form solutions, which have a strong ability to reveal the mechanism and to present a rational basis for practical design methods.

If the flexural stiffness of the web is sufficient, then the lateral restraint from the concrete slab should be able to stabilize the bottom flange (Weston *et al*. 1991, Dekker *et al*. 1995, Bradford 2000). In order to evaluate the critical stress of a composite beam at which lateral buckling occurs, Svensson (1985) assumed a totally rigid concrete slab and considered the bottom flanges of the beam as an elastically-supported strut. This method allowed obtaining numerical solutions for most of the common practical cases and boundary conditions. However, Williams (Williams and Jemah 1987) found that Svensson's method was not safe enough, and hence, suggested an increase in the involved area of the web. Goltermann (Goltermann and Svensson 1988) further developed Williams's models by solving the eigenvalue of a four-step differential equation in order to understand the buckling of steel concrete composite beams in the negative moment area. British Bridge Standard (BS5400) (1982) also employs this method for the design of continuous composite beams. Lawson and Rackham (1989) used an energy method to obtain a formula to calculate the critical buckling stress of steel concrete composite beams in the negative moment area. Jiang *et al*. (2013) presented a simplified calculation model of a steel concrete composite box beam. They put forward a formula to calculate the critical moments of local and distortional buckling by using energy method. However, due to the limited computational capacity at that time, the study did not perform a detailed analysis of the applicability of the elastic foundation beam method. Ye and Chen (Chen and Ye 2010, Ye and Chen 2013) examined the part of the web involved in to make an improvement, based on Svensson's elastic foundation strut model, to the lateral and torsional restraint stiffnesses of the elastic foundation strut, and discovered that the elastic foundation beam method was more practical than the energy method. However, the method proposed by Ye and Chen does not consider the coupling effect between the applied forces and the lateral and torsional restraint stiffnesses of the bottom flange.

This paper proposes analytical formulas for calculating the lateral and torsional restraint stiffnesses under negative bending moment with consideration to the coupling effect between the applied forces and the lateral and torsional restraint stiffnesses of the bottom flange. This study further develops expressions for the critical buckling stress and the critical bending moment of the I-steel concrete composite beam. Finally, this paper validates the proposed expressions by analyzing 24 composite beams and comparing the results with those from a finite element analysis.

2. Basic assumptions

Fig. 1 shows the cross-sectional dimensions of an I-steel concrete composite beam. The steel beam when in the composite beam has a different lateral buckling mode than as an unconstrained steel beam. In the composite beam, the top flange of the steel beam is embedded into the concrete slab, thus increasing the lateral and torsional restraint stiffnesses. Therefore, the concrete slab restrains both the lateral deformation and the torsional deformation of the steel beam. Fig. 2 shows the three types of buckling: torsional buckling, lateral flexural buckling, and lateral-torsional buckling. To simplify the calculation, the following assumptions are made (Dekker *et al*. 1995, Bradford 1998, Kalkan and Buyukkaragoz 2012, Zhou *et al*. 2014).

(1) The lateral and torsional stiffnesses of the concrete slab are relatively large. The concrete slab prevents the lateral and torsional distortions of the top flange of the steel beam.
(2) The tensile strength of the concrete slab is ignored.

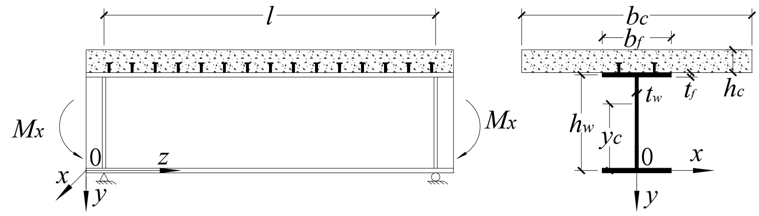
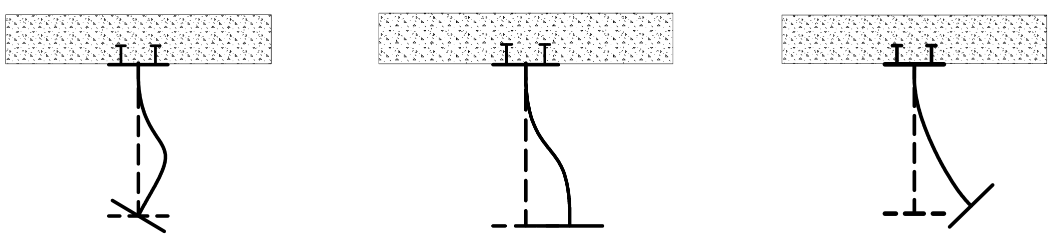


Fig. 1 Cross-sectional dimensions of I-steel concrete composite beams



(a) Torsional buckling (b) Lateral flexural buckling (c) Lateral-torsional buckling

Fig. 2 Distortional buckling modes of I-steel concrete composite beams under negative moment

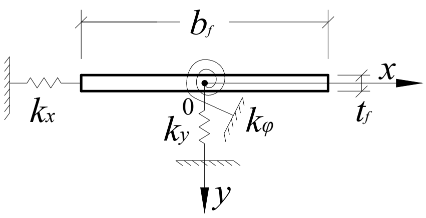


Fig. 3 Simplified calculation model of I-steel concrete composite beams

(3) Since no vertical deformation occurs when the distortional buckling takes place, the vertical restraint stiffness is deemed as infinity.

In accordance with the above mentioned assumptions, the buckling model of the I-steel concrete composite beams can be simplified as depicted in Fig. 3. Springs restrict the movements in the horizontal and torsional directions of the bottom flange, while the movement in the vertical direction is fixed by a rigid support.

3. Buckling analysis of the steel concrete composite beams

In accordance with the above assumptions and with consideration to the reinforcement within the concrete slabs, the compressive stress at the bottom of the web can be expressed as $\sigma_1 = M_c(z)y_c/I$. where $M_x(z)$ is the negative bending moment; I is the section moment of inertia; and y_c is the neutral axis coordinate of the equivalent cross section, which can be expressed as

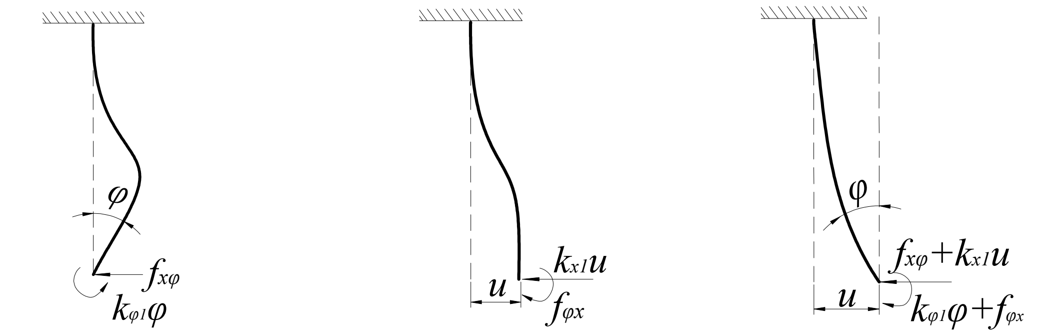
$$y_c = \frac{A_s y_s + A_t h_w + 0.5 A_w h_w}{A_s + A_t + A_w + A_f} \tag{1}$$

where A_f is the area of the bottom flange; A_t is the area of the top flange; A_w is the area of the web; A_s is the area of the reinforcements within the concrete slab; y_s is the distance between the gravity of the reinforcements and the bottom flange; and h_w is the height of the web as shown in Fig. 1.

3.1 The torsional restraint stiffness of the web

As shown in Figs. 2(a) and 4(a), the two transverse edges of the web are simply supported, and the top of the web is fixed (Zhou *et al*. 2014). Torsional buckling of the composite beam causes the bottom of the web, which bears lateral distributed force $f_{x\varphi}$ and the distributed bending moment $k_{\varphi 1}\varphi$, to rotate. Here $k_{\varphi 1}$ is the torsional restraint stiffness of the web. The boundary condition of the web can be expressed as

$$\left.\begin{aligned} &\left.\frac{\partial w}{\partial y}\right|_{y=-h_w} = 0, \quad w\big|_{z=0,\lambda} = 0, \quad w\big|_{y=0,-h_w} = 0 \\ &D_w\left(\frac{\partial^2 w}{\partial z^2} + \mu\frac{\partial^2 w}{\partial y^2}\right)\bigg|_{z=0,\lambda} = 0 \end{aligned}\right\} \tag{2}$$



(a) Torsional buckling (b) Lateral flexural buckling (c) Lateral-torsional buckling

Fig. 4 Simplified calculation model of web

where $D_w = Et_w^3/12(1-\mu^2)$; μ is the Poisson's ratio of steel; E is the elasticity modulus of steel; $w(y, z)$ is the buckling deformation function of the web; t_w is the thickness of the web; and λ is the length of the buckling half wave.

The buckling deformation function of the web can be expressed by using the boundary conditions as (Jiang *et al*. 2013)

$$w = c\left[\frac{y}{h_w} + 2\left(\frac{y}{h_w}\right)^2 + \left(\frac{y}{h_w}\right)^3\right]\sin\frac{\pi z}{\lambda} \tag{3}$$

According to the small deflection theory of thin plates, the bending strain energy of the web can be obtained as follows (Bradford 1988, Dekker *et al*. 1995)

$$U_1 = \frac{D_w}{2}\int_0^{\lambda}\int_{-h_w}^{0}\left[\left(\frac{\partial^2 w}{\partial y^2}\right)^2 + \left(\frac{\partial^2 w}{\partial z^2}\right)^2 + 2\mu\frac{\partial^2 w}{\partial y^2}\frac{\partial^2 w}{\partial z^2} + 2(1-\mu)\left(\frac{\partial^2 w}{\partial y\partial z}\right)^2\right]dydz \tag{4}$$

The strain energy of the spring restraint to the steel beam web (Bradford 1988, Dekker *et al*. 1995) is given as

$$U_2 = \frac{k_{\varphi 1}}{2}\int_0^{\lambda}\left(\frac{\partial w}{\partial y}\right)^2_{y=0} dz \tag{5}$$

The total external work is equal to (Bradford 1988, Dekker *et al*. 1995)

$$W = \frac{t_w}{2}\int_0^{\lambda}\int_{-h_w}^{0}\frac{\sigma_1(y_c+y)}{y_c}\left(\frac{\partial w}{\partial z}\right)^2 dydz \tag{6}$$

The sum of the strain energy and the external work is called the total potential energy and is given as

$$\Pi = U_1 + U_2 - W \tag{7}$$

According to the principle of stationary potential energy $\delta\Pi = 0$, the following expression can be obtained (Bradford 1988, Dekker *et al*. 1995)

$$\left(B_0 + k_{\varphi 1}T - \sigma_1 N_0\right)c = 0 \tag{8}$$

The general coordinate c cannot be equal to zero when buckling happens. Therefore, the generalized eigenvalue of the characteristic matrix shown below can be used to solve the buckling of the composite beam

$$\left|B_0 + k_{\varphi 1}T - \sigma_1 N_0\right| = 0 \tag{9}$$

By solving Eq. (9), $k_{\varphi 1}$ can be obtained as

$$k_{\varphi 1} = \left(\sigma_1 N_0 - B_0\right)/T \tag{10}$$

where $B_0 = \lambda D_w\left(\frac{2}{h_w^3} + \frac{h_w\beta^2}{210} + \frac{2\beta}{15h_w}\right)$, $T = \frac{\lambda}{2h_w^2}$, $N_0 = \frac{\pi^2}{\lambda}\left(\frac{t_w h_w}{210} - \frac{t_w h_w^2}{560 y_c}\right)$, $\beta = \frac{\pi^2}{\lambda^2}$, c is the general coordinate representing the buckling distortion.

Eq. (10) shows a coupling relationship between the external loads and the torsional restraint stiffness. This indicates that the torsional restraint stiffness of the bottom flange is not only determined by the cross sectional features of the composite beam but also dependent on the external loads. Therefore, it is no more appropriate to use the restraint stiffness as a constant cross sectional feature as practiced in the traditional elastic foundation beam method.

According to the elastic plate theory, the lateral distributed force produced by the bottom flange is exerted on the bottom of the web as follows (Atanackovic and Ardéshir 2012)

$$f_{x\varphi} = -D_w\left[\frac{\partial^3 w}{\partial y^3} + (2-\mu)\frac{\partial^3 w}{\partial z^2 \partial y}\right] \tag{11}$$

$$f_{x\varphi}\Big|_{y=0} = \left[\beta\frac{(2-\mu)}{h_w} - \frac{6}{h_w^3}\right] D_w c \sin\frac{\pi z}{\lambda} \tag{12}$$

3.2 The lateral restraint stiffness of the web

Figs. 2(b) and 4(b) demonstrate that the two transverse edges of the web are simply supported, while the top of the web is fixed (Zhou *et al*. 2014). The lateral flexural buckling of the composite beam causes the lateral movement of the bottom of the web, which bears the lateral distributed force $k_{x1}u$ and the distributed bending moment $f_{\varphi x}$. Here k_{x1} is the lateral restraint stiffness of the web. Then the boundary condition of the web can be expressed as

$$\left.\begin{aligned} &\left.\frac{\partial w}{\partial y}\right|_{y=0,-h_w} = 0, \quad w\big|_{z=0,\lambda} = 0, \quad w\big|_{y=-h_w} = 0 \\ &\left.\left(\frac{\partial^2 w}{\partial z^2} + \mu\frac{\partial^2 w}{\partial y^2}\right)\right|_{z=0,\lambda} = 0 \end{aligned}\right\} \tag{13}$$

The buckling deformation function of the steel web can be expressed by using the boundary condition as (Jiang *et al*. 2013)

$$w = d\left[1 - 3\left(\frac{y}{h_w}\right)^2 - 2\left(\frac{y}{h_w}\right)^3\right]\sin\frac{\pi z}{\lambda} \tag{14}$$

According to the principle of stationary potential energy (Bradford 1988, Dekker Kemp *et al*. 1995), the following expression can be obtained (Bradford 1988, Dekker *et al*. 1995)

$$\left(H_0 + k_{x1}R - \sigma_1 S_0\right)d = 0 \tag{15}$$

The general coordinate d cannot be equal to zero when buckling happens. Therefore, the generalized eigenvalue of the characteristic matrix shown below can be used to solve the buckling of the composite beam

$$\left|H_0 + k_{x1}R - \sigma_1 S_0\right| = 0 \tag{16}$$

By solving Eq. (16), k_{x1} can be obtained as

$$k_{x1} = \left(\sigma_1 S_0 - H_0\right)/R \tag{17}$$

where d is the general coordinate representing the buckling distortion of the bottom flange. Then, $S_0 = \dfrac{t_w h_w \pi^2}{\lambda}\left(\dfrac{13}{70} - \dfrac{3h_w}{70y_c}\right)$, $R = \dfrac{\lambda}{2}$, and $H_0 = \lambda D_w\left(\dfrac{6}{h_w^3} + \dfrac{13h_w}{70}\beta^2 + \dfrac{6\beta}{5h_w}\right)$.

Eq. (17) shows a coupling relationship between the external loads and the lateral restraint stiffness, which indicates that the lateral restraint stiffness of the bottom flange is determined by both the cross sectional features of the composite beam and the external loads. Therefore, it is no more appropriate to use the lateral restraint as a constant cross sectional feature as considered in the traditional elastic foundation beam method.

According to the elastic plate theory, the distributed bending moment produced by the bottom flange is exerted on the bottom of the web as follows (Atanackovic and Ardéshir 2012)

$$f_{\varphi x} = -D_w\left(\frac{\partial^2 w}{\partial y^2} + \mu\frac{\partial^2 w}{\partial z^2}\right) \tag{18}$$

$$f_{\varphi x}\Big|_{y=0} = \left(6/h_w^2 + \mu\beta\right)D_w d\sin\frac{\pi z}{\lambda} \tag{19}$$

3.3 Analysis of the buckling of the bottom flange

Figs. 5 and 4(c) show that the x-axis and the y-axis of the bottom flange are both symmetrical. The centroid of the plate is set to be the origin point. Under the assumption that the lateral displacement of the bottom flange is $u(z)$ and the torsional angle is $\varphi(z)$, the neutral equilibrium differential equation of an elastic thin-walled bar can be expressed as (Svensson 1985, Zhou *et al*. 2012)

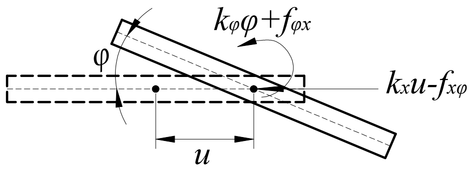


Fig. 5 Force method of analysis of the bottom flange

$$\left.\begin{aligned} &\left(Pr_0^2-GJ\right)\varphi''+k_\varphi\varphi+f_{\varphi x}=0 \\ &EI_y u^{IV}+Pu''+k_x u-f_{x\varphi}=0 \end{aligned}\right\} \tag{20}$$

where $I_y = t_f b_f^3/12$, $I_x = b_f t_f^3/12$, $J = b_f t_f^3/3$, $r_0^2 = (I_x + I_y)/A_f$; $k_\varphi = -k_{\varphi 1}$; $k_x = -k_{x1}$; G is the shear modulus of the steel; and P is the pressure of the bottom flange given as, $P = A_f\sigma_1$.

The torsional angle and the lateral displacement of the bottom flange can be obtained by combining Eqs. (3) and (14) as

$$\varphi(z)=\frac{c}{h_w}\sin\frac{\pi z}{\lambda} \tag{21}$$

$$u(z)=d\sin\frac{\pi z}{\lambda} \tag{22}$$

According to the Galerkin method (Bi and Ginting 2011, Tinh and Minh 2013, Wang and Peng 2013), the following expression can be obtained

$$\left.\begin{aligned} &\int_0^\lambda\left[\left(Pr_0^2-GJ\right)\varphi''+k_\varphi\varphi+f_{\varphi x}\right]\sin\frac{\pi z}{\lambda}dz=0 \\ &\int_0^\lambda\left(EI_y u^{IV}+Pu''+k_x u-f_{x\varphi}\right)\sin\frac{\pi z}{\lambda}dz=0 \end{aligned}\right\} \tag{23}$$

$$\begin{pmatrix} B_1+k_\varphi T-\sigma_1 N_1 & Q \\ M & H_1+k_x R-\sigma_1 S_1 \end{pmatrix}\begin{pmatrix} c \\ d \end{pmatrix}=0 \tag{24}$$

where $B_1=\dfrac{GJ\pi^2}{2\lambda h_w^2}$; $M=\dfrac{3D_w\lambda}{h_w^3}-D_w\dfrac{(2-\mu)\lambda\beta}{2h_w}$; $Q=3\dfrac{D_w\lambda}{h_w^3}+\dfrac{D_w\mu\lambda\beta}{2h_w}$; $H_1=EI_y\beta^2\dfrac{\lambda}{2}$; $N_1=\dfrac{A_f r_0^2\pi^2}{2\lambda h_w^2}$; and $S_1=\dfrac{A_f\pi^2}{2\lambda}$.

3.4 Formula for the calculation of buckling of the I-steel concrete composite beams

A combination of Eqs. (8), (15) and (24) leads to

$$\left[\begin{pmatrix} B & Q \\ M & H \end{pmatrix}-\sigma_1\begin{pmatrix} N & 0 \\ 0 & S \end{pmatrix}\right]\begin{pmatrix} c \\ d \end{pmatrix}=0 \tag{25}$$

where $B = B_0 + B_1$; $N = N_0 + N_1$; $H = H_0 + H_1$; and $S = S_0 + S_1$.

The deformation vector $\{c, d\}^T$ cannot be equal to 0 when buckling happens. Therefore, the generalized eigenvalue of the characteristic matrix shown below can be used to solve the buckling of the composite beam

$$\left| \begin{pmatrix} B & Q \\ M & H \end{pmatrix} - \sigma_1 \begin{pmatrix} N & 0 \\ 0 & S \end{pmatrix} \right| = 0 \tag{26}$$

By solving Eq. (26), two generalized eigenvalues σ_{ti} (i = 1, 2) can be obtained. Then letting σ_{cr} = min $\{\sigma_{ti}$ (i = 1, 2)$\}$, σ_{cr} is the critical buckling stress of the composite beam, as shown below

$$\sigma_{cr} = \frac{NH + SB - \sqrt{(NH - SB)^2 + 4NSMQ}}{2NS} \tag{27}$$

The following equation can calculate the critical half wavelength of a composite beam under negative moment (Ye and Chen 2013)

$$l_{cr} = 2.4h_w \left[b_f^3 t_f \left(1 - \mu^2\right) / \left(t_w^3 h_w\right) \right]^{0.25} \tag{28}$$

The half wave amount is

$$n = \frac{l}{\lambda} \tag{29}$$

where l is the length of the composite beam. If l/l_{cr} is an integer, then $\lambda = l_{cr}$. Otherwise, if l/l_{cr} is not an integer, then set the two integers adjacent to l/l_{cr} as n_1 and n_2, and plug $\lambda_1 = l/n_1$ and $\lambda_1 = l/n_2$ into Eq. (27) to obtain two corresponding critical buckling stresses σ_{cr1} and σ_{cr2}. The i^{th} lambda λ_i (i = 1, 2) that corresponds to the min$\{\sigma_{cr1}, \sigma_{cr2}\}$ is the critical half wavelength.

The following equation can calculate the critical buckling moment of the composite beam

$$M_{cr} = \sigma_{cr} I / y_c \tag{30}$$

4. Analysis of example composite beams

The critical buckling analysis of the composite beam under negative bending moment can be performed by two methods, namely, the calculation method proposed in this article and the finite element method. This study validated the introduced method by showing its superiority in predicting the critical bending moment by comparing the prediction results from Svensson's method, Williams' method, and Goltermann's method in the calculation. Table 1 lists the geometric dimensions of each example considered in this paper to validate the proposed analytical formulas. The finite element analysis was conducted with ANSYS commercial software, which has been generally used in the numerical calculation of composite structure. The steel beam was modeled with the element called Shell43. The concrete slab of the composite beam was replaced by the constraints in the numerical model. The motions in the x and y directions of the top edge of the flange were restrained to represent the lateral and torsional restrictions caused by the concrete slab in actual practice. The two ends of the model in the x and y directions had restricted degrees

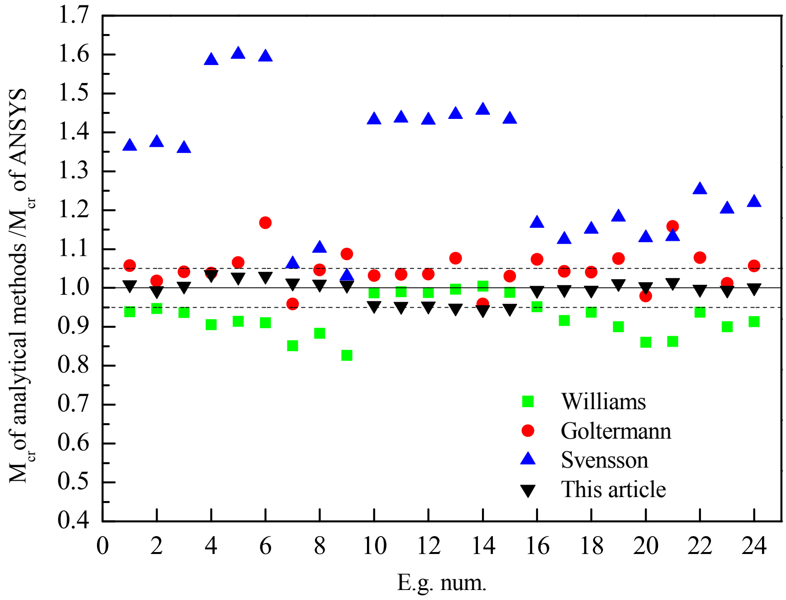


Fig. 5 Precision in predicting Mcr of different analytical methods

of freedom, and one node of the left end of the model in the z direction had a restricted degree of freedom in order to meet the static equilibrium. Table 2 lists the results of each analytical calculation method, and Fig. 6 shows the error analysis of each analytical method.

Based on the results presented in Table 2 and Fig. 6, the following conclusions can be drawn:

(1) Under negative bending moment, the length of the beam affected the critical bending buckling moment of the composite beam, because the critical half wavelength changed

Table 1 Basic geometric sizes of different composite beams cases

e.g. num.	Cross section Num.	h_w/mm	b_f/mm	t_f/mm	t_w/mm	l/mm
1						4500
2	1	600	200	12	12	7200
3						9600
4						3000
5	2	600	120	10	10	5400
6						8400
7						6000
8	3	719	268	25	16	10200
9						14400
10						3000
11	4	300	100	3	3	5400
12						8400

Table 1 Continued

e.g. num.	Cross section Num.	h_w/mm	b_f/mm	t_f/mm	t_w/mm	l/mm
13						4200
14	5	450	150	4	4	7200
15						10200
16						5500
17	6	700	300	20	13	7500
18						12000
19						3000
20	7	500	150	16	10	5000
21						8000
22						5000
23	8	800	250	20	14	8000
24						12000

Table 2 Distortional buckling critical moment of composite beams under negative uniform moment

e.g. num.	Distortional buckling critical moment M_{cr}/kN.m				
	ANSYS	Williams	Goltermann	Svensson	Eq. (30)
1	1129.1	1060.6	1193.6	1540.1	1138.3
2	1212.3	1148.0	1234.4	1664.7	1203.9
3	1149.7	1077.0	1197.1	1561.7	1155.0
4	390.98	354.02	405.88	619.6	404.45
5	410.19	375.12	436.86	656.6	421.65
6	401.97	366.03	469.19	640.6	414.12
7	3672.7	3126.4	3521.1	3899.1	3719.7
8	3836.5	3390.4	4014.3	4228.1	3874.5
9	3905.1	3226.8	4245.4	4024.2	3932.7
10	34.155	33.710	35.235	48.9	32.632
11	35.235	34.884	36.464	50.6	33.587
12	34.729	34.304	35.951	49.7	33.106
13	94.611	94.325	101.817	136.8	89.713
14	107.307	107.811	102.911	156.3	101.30
15	91.161	90.153	93.920	130.7	86.372
16	3095.4	2945.0	3322.2	3609.5	3077.0
17	2802.3	2567.2	2922.9	3151.7	2790.0
18	2916.2	2735.0	3033.9	3356.9	2900.0
19	808.47	727.99	869.44	955.61	817.60
20	857.85	737.69	839.86	968.22	860.87
21	752.03	648.61	870.74	851.33	762.63
22	2531.5	2372.6	2729.1	3169.7	2522.9
23	2472.5	2225.3	2501.7	2973.0	2458.2
24	2293.4	2094.1	2423.7	2797.2	2294.4

with the length of the composite beam.

(2) The results obtained, for the case of negative bending moment, from the proposed calculation method agreed closely with those from finite element analysis. The discrepancy was within 5%, which validates the accuracy and applicability of the proposed method.

(3) Predictions of the critical bending moment by traditional calculation methods, including Svensson's method, Williams's method, and Goltermann's method, deviate considerably from the predictions obtained by finite element method.

These findings indicate that the traditional elastic foundation beam methods need an improvement. This study also illustrates that the constant lateral and torsional restraint stiffness used in the traditional methods may lead to deviations.

5. Conclusions

This study proposed improvement to the traditional elastic foundation beam methods by considering the coupling relationship between the applied loads and the lateral and torsional restraint stiffnesses of the bottom flange. Based on this coupling effect, this paper developed a simplified method to calculate the critical buckling load of an I-steel concrete composite beam. This paper then compared the introduced method with various traditional methods, and the following conclusions were obtained:

- A linear coupling relationship exists between both the torsional and the lateral restraint stiffnesses and the applied loads. This relationship is different from the concept that traditional elastic foundation beam methods consider both the torsional and the lateral restraint rigidity as constants.
- The length affects the critical bending buckling moment of the I-steel concrete composite beam with same cross section, because the change in the critical half wavelength is dependent on the composite beam length.
- The results of the proposed calculation method matched well with the results from the finite element calculation method for the case of negative bending moment. The discrepancy between the two methods was within 5%, thus validating the applicability of the proposed method.

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