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Natural frequencies and mode shapes of thin-walled members with shell type cross section

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Abstract. An analytical procedure based on the transfer matrix method to estimate not only the natural frequencies but also vibration mode shapes of the thin-walled members composed of interconnected cylindrical shell panels is presented. The transfer matrix is derived from the differential equations for the cylindrical shell panels. The point matrix relating the state vectors between consecutive shell panels are used to allow the transfer procedures over the cross section of the members. As a result, the interactions between the shell panels of the cross sections of the members can be considered. Although the transfer matrix method is naturally a solution procedure for the one-dimensional problems, this method is well applied to thin-walled members by introducing the trigonometric series into the governing equations of the problem. The natural frequencies and vibration mode shapes of the thin-walled members composed of number of interconnected cylindrical shell panels are observed in this analysis. In addition, the effects of the number of shell panels on the natural frequencies and vibration mode shapes are also examined.

Key words: natural frequency; mode shape; thin-walled member; cylindrical shell panel; transfer matrix method.

1. Introduction

Thin-walled members with shell type cross sections are widely used in a broad range of structural applications to reduce the material cost as well as the dead weight of a structure. Research works on vibration analysis of curved panels have been reported in many literatures. Sewall (1967) and Webster (1971) used a classical approach to solve some examples with simple boundary conditions. Many shell theories based on numerical approaches such as the Rayleigh-Ritz method, the Galerkin method, the finite element method and the finite strip method have been used to analyze the vibration of cylindrical shell structures (Tsui 1968, Petyt 1971, Petyt and Nath 1971, Maddox, Plumblee and King 1970, Bardell and Mead 1989, Sheinman and Reichman 1992, To and Wang 1991, Mizusawa 1988). However, to the authors knowledge there has been hardly any research work on thin-walled members composed of interconnected shell panels as illustrated in Fig. 1. Thin-walled members show both local

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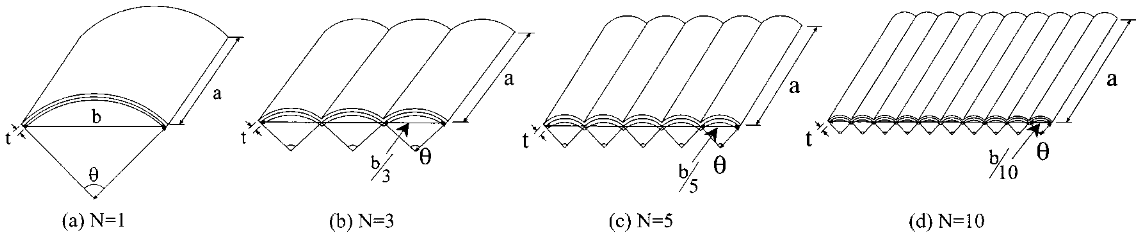


Fig. 1 Analytical models

and overall dynamic behaviors so that the investigation of the natural frequencies and vibration mode shapes of the members are very important to clarify the dynamic behaviors of these members.

In this paper, an analytical procedure to estimate not only the natural frequencies but also vibration mode shapes of the thin-walled members composed of interconnected cylindrical shell panels as given in Fig. 1 is presented. The natural frequencies and vibration mode shapes of many analytical models composed of different number of interconnected shell panels are examined. In addition, the effects of the number of shell panels on the natural frequencies and vibration mode shapes are also examined.

For this purpose transfer matrix method is used. The transfer matrix is derived from the differential equations for the cylindrical shell panels of the dynamic problem. The point matrix relating the state vectors between consecutive shell panels is used to allow the transfer procedures over the cross section of the members. As a result, the exact interactions between the shell panels can be considered. Although the transfer matrix method is naturally a solution procedure for the one-dimensional problems, this method is applied to thin-walled members by introducing the trigonometric series into the governing equations of the problem (Ohga 1995a, b, c, Tesar and Fillo 1988, Uhrig 1973).

2. Analytical procedure

2.1. Equilibrium equations for shell panels of dynamic problem

The equilibrium equations for the shell panel given in Fig. 2 are as follows (Uhrig 1973):

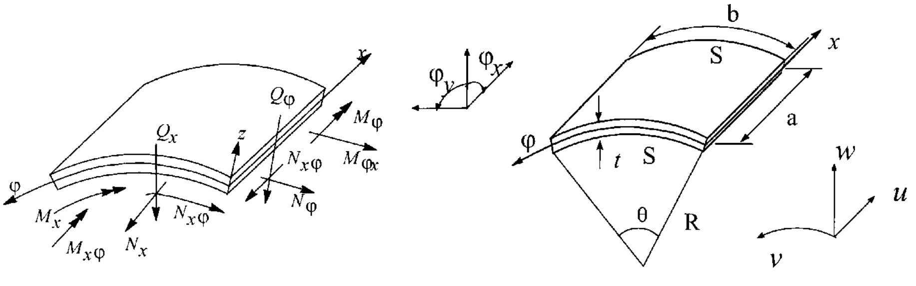
$$N_x' + N_{\varphi x}^{\bullet} + \omega^2\rho t u = 0,\ \ N_{x\varphi}' + N_{\varphi}^{\bullet} + \frac{Q_\varphi}{R} + \omega^2\rho t v = 0,\ \ -\frac{N_\varphi}{R} + Q_x' + Q_\varphi^{\bullet} + \frac{Q_\varphi}{R} + \omega^2\rho t w = 0$$

$$M_x' + M_{\varphi x}^{\bullet} - Q_x = 0,\ \ M_{\varphi x}' + M_{\varphi}^{\bullet} - Q_\varphi = 0,\ \ M_{x\varphi} - M_{\varphi x} - \frac{M_{\varphi x}}{R} = \tag{1}$$

where, N_x, N_φ, $N_{x\varphi}$, $N_{\varphi x}$: in-plane forces, M_x, M_φ, $M_{x\varphi}$, $M_{\varphi x}$: bending and twisting moments, Q_x, Q_φ: shear forces, ω: natural frequency, ρ : mass density, t: shell thickness, u, v, w: displacements in x, φ, z directions, R: radius of shell panel, $' \equiv \frac{\partial}{\partial x}$, $\bullet \equiv \frac{\partial}{R\partial\varphi}$.

2.2. Relation between strains and displacements

The relations between the strains and displacements according to the geometry and the sign



(a)forces (b)displacements

Fig. 2 Cylindrical shell panel

convention for shell panel given in Fig. 2(b) are as follows (Uhrig 1973):

$$\varepsilon_x = u', \quad \varepsilon_\varphi = v^\bullet + \frac{w}{R}, \quad \gamma_{x\varphi} = v' + u^\bullet, \quad \gamma_{xz} = w' + \varphi_x = 0_x, \quad \gamma_{\varphi z} = w^\bullet + \varphi_\varphi - \frac{v}{R} = 0$$

$$\kappa_x = \varphi_x', \quad \kappa_{x\varphi} = \varphi_x^\bullet + \frac{v'}{R}, \quad \kappa_{\varphi x} = \varphi'_\varphi, \quad \kappa_\varphi = \varphi_\varphi^\bullet + \frac{1}{R} = \left(v^\bullet + \frac{w}{R}\right) \tag{2}$$

where, ε_x, ε_φ : normal strains, $\gamma_{x\varphi}$, $\gamma_{\varphi z}$, γ_{xz} : shear strains, κ_x, κ_φ, $\kappa_{x\varphi}$, $\kappa_{\varphi x}$: curvatures of displacements.

2.3. Transfer matrix for shell panel

From Eqs. (1) and (2), the partial differential equations for the state vectors w, φ_φ, M_φ, V_φ, v, u, N_φ and $N_{\varphi x}$ are obtained as follows (Fig. 2):

$$w^\bullet = -\varphi_\varphi + \frac{v}{R}, \quad \varphi_\varphi^\bullet = \frac{K_{21}}{K_{22}}w'' + \frac{M_\varphi}{K_{22}} + \frac{I_{21}}{RI_{22}}u' - \frac{N_\varphi}{RI_{22}}, \quad M_\varphi^\bullet = V_\varphi - 4K_{33}\varphi_\varphi''$$

$$V_\varphi^\bullet = \left(K_{11} - \frac{K_{12}K_{21}}{K_{22}}\right)w'''' - \omega^2\rho t w - \frac{K_{12}}{K_{22}}M_\varphi'' + \frac{1}{R}N_\varphi, \quad v^\bullet = -\frac{w}{R} + \frac{N_\varphi}{I_{22}} - \frac{I_{21}}{I_{22}}u'$$

$$u^\bullet = \frac{1}{I_{33}}N_{\varphi x} + \frac{2K_{33}}{RI_{33}}\varphi_\varphi' - v', \quad N_\varphi^\bullet = -N_{\varphi x}' - \frac{V_\varphi}{R} - \omega^2\rho t v$$

$$N_{\varphi x}^\bullet = \left(\frac{I_{12}I_{21}}{I_{22}} - I_{11}\right)u'' - \frac{I_{12}}{I_{22}}N_\varphi' - \omega^2\rho t u \tag{3}$$

where

$$I_{11} = I_{22} = \frac{Et}{1-v^2}, \quad I_{21} = I_{12} = \frac{vEt}{1-v^2}, \quad I_{33} = \frac{Et}{2(1+v)}$$

$$K_{11} = K_{22} = \frac{Et^3}{12(1-v^2)}, \quad K_{21} = K_{12} = \frac{vEt^3}{12(1-v^2)}, \quad K_{33} = \frac{Et^3}{24(1+v)}$$

E: modulus of elasticity, v: Poisson's ratio

$V_\varphi \equiv Q_\varphi + M'_{\varphi x}$: equivalent shear force.

For a shell panel simply supported at x=0, a, and taking two lengthwise boundary conditions as arbitrary, the state variables can be assumed in the form given below (Fig. 2(b)):

$$
\begin{aligned}
w(x,\varphi) &= \bar{w}(\varphi)\sin\alpha x,\quad \varphi_\varphi(x,\varphi) = \bar{\varphi}_\varphi(\varphi)\sin\alpha x \\
M_\varphi(x,\varphi) &= \bar{M}_\varphi(\varphi)\sin\alpha x,\quad V_\varphi(x,\varphi) = \bar{V}_\varphi(\varphi)\sin\alpha x \\
v(x,\varphi) &= \bar{v}(\varphi)\sin\alpha x,\quad u(x,\varphi) = \bar{u}(\varphi)\cos\alpha x \\
N_\varphi(x,\varphi) &= \bar{N}_\varphi(\varphi)\sin\alpha x,\quad N_{\varphi x}(x,\varphi) = \bar{N}_{\varphi x}(\varphi)\cos\alpha x
\end{aligned}
\tag{4}
$$

where, $\alpha = \dfrac{m\pi}{a}$, a: shell panel length, m: vibration mode in axial direction.

Substituting Eq. (4) into Eq. (3), the following ordinary differential equation containing only φ as a variable is obtained:

$$
\begin{pmatrix} \tilde{w} \\ \tilde{\varphi}_\varphi \\ \tilde{M}_\varphi \\ \tilde{V}_\varphi \\ \tilde{v} \\ \tilde{u} \\ \tilde{N}_\varphi \\ \tilde{N}_{\varphi x} \end{pmatrix}^{\bullet}
=
\begin{pmatrix}
0 & -\alpha & 0 & 0 & \frac{1}{R} & 0 & 0 & 0 \\
-\frac{\alpha K_{21}}{K_{22}} & 0 & \frac{\alpha K_0}{K_{22}} & 0 & 0 & -\frac{I_{21}}{RI_{22}} & -\frac{\alpha^2 K_0}{RK_{22}} & 0 \\
0 & -\frac{4\alpha K_{33}}{K_0} & 0 & \alpha & 0 & 0 & 0 & 0 \\
A_{41} & 0 & \frac{\alpha K_{12}}{K_{22}} & 0 & 0 & 0 & \frac{1}{R} & 0 \\
-\frac{1}{R} & 0 & 0 & 0 & 0 & \frac{\alpha I_{21}}{I_{22}} & \frac{\alpha^3 K_0}{I_{22}} & 0 \\
0 & \frac{2\alpha^2 K_{33}}{RI_{33}} & 0 & 0 & -\alpha & 0 & 0 & \frac{\alpha^3 K_0}{I_{33}} \\
0 & 0 & 0 & -\frac{1}{R} & A_{75} & 0 & 0 & \alpha \\
0 & 0 & 0 & 0 & 0 & A_{86} & -\frac{\alpha I_{12}}{I_{22}} & 0
\end{pmatrix}
\begin{pmatrix} \tilde{w} \\ \tilde{\varphi}_\varphi \\ \tilde{M}_\varphi \\ \tilde{V}_\varphi \\ \tilde{v} \\ \tilde{u} \\ \tilde{N}_\varphi \\ \tilde{N}_{\varphi x} \end{pmatrix}
\tag{5}
$$

or

$$
\frac{d}{Rd\varphi}\boldsymbol{Z} = \boldsymbol{Z}^{\bullet} = \boldsymbol{A}\cdot\boldsymbol{Z} \tag{6}
$$

where

$$
A_{41} = \left(\frac{\alpha^3}{K_0}\left(K_{11} - \frac{K_{12}K_{21}}{K_{22}}\right) - \frac{\omega^2\rho t}{\alpha^3 K_0}\right),\quad A_{75} = -\frac{\omega^2\rho t}{\alpha^3 K_0},\quad A_{86} = \frac{1}{\alpha K_0}\left(I_{11} - \frac{I_{12}I_{21}}{I_{22}} - \frac{\omega^2\rho t}{\alpha^2}\right)
$$

$$\tilde{w} = K_0\bar{w},\quad \tilde{\varphi}_\varphi = \frac{K_0}{\alpha}\bar{\varphi}_\varphi,\quad \tilde{M}_\varphi = \frac{1}{\alpha^2}\bar{M}_\varphi,\quad \tilde{V}_\varphi = \frac{1}{\alpha^3}\bar{V}_\varphi$$

$$\tilde{v} = K_0\bar{v},\quad \tilde{u} = K_0\bar{u},\quad \tilde{N}_\varphi = \frac{1}{\alpha^3}\bar{N}_\varphi,\quad \tilde{N}_{\varphi x} = \frac{1}{\alpha^3}\bar{N}_{\varphi x}$$

$K_0 = \dfrac{T_0^3 e}{12(1-\nu)^2}$: standard bending rigidity, T_o: standard shell thickness

Integrating Eq. (6), the transfer matrix $\boldsymbol{F}$ for the shell panel is obtained as follows:

$$\boldsymbol{Z} = \exp(\boldsymbol{A}\varphi) \cdot \boldsymbol{Z}_0 = \boldsymbol{F} \cdot \boldsymbol{Z}_0 \tag{7}$$

where

$\exp(\boldsymbol{A}\varphi) = \boldsymbol{I} + (\boldsymbol{A}\varphi) + \dfrac{1}{2!}(\boldsymbol{A}\varphi)^2 + \dfrac{1}{3!}(\boldsymbol{A}\varphi)^3 +$, $\boldsymbol{I}$: unit matrix

2.4. Point matrix

As the state vectors for each shell panel are referred to the local coordinate system, the relations between the state vectors of consecutive two shell panels are required in order to allow the transfer procedures of the state vectors over the cross section of the member. Considering the relation between the state vectors at the left and right hand sides of the cross section *i* as given in Fig. 3, the point matrix $\boldsymbol{P}$ is obtained as follows:

$$\boldsymbol{Z}_i^L = \boldsymbol{P}_i \cdot \boldsymbol{Z}_i^R \tag{8}$$

where, the superscripts *L* and *R* indicate the left and right hand sides of the cross section *i*.

2.5. Natural frequency and mode shape

Applying the transfer and point matrices described above to the cross section of the thin-walled members composed of cylindrical shell panels as given in Fig. 4, the relation between the state vectors at both ends of the cross section of the member can be obtained as follows:

$$\boldsymbol{Z}_3 = \boldsymbol{F}_3 \cdot \boldsymbol{P}_2 \cdot \boldsymbol{F}_2 \cdot \boldsymbol{P}_1 \cdot \boldsymbol{F}_1 \cdot \boldsymbol{Z}_0 = \boldsymbol{U} \cdot \boldsymbol{Z}_0 \tag{9}$$

Considering the boundary conditions at both lengthwise edges in Fig. 4, the following expression, for

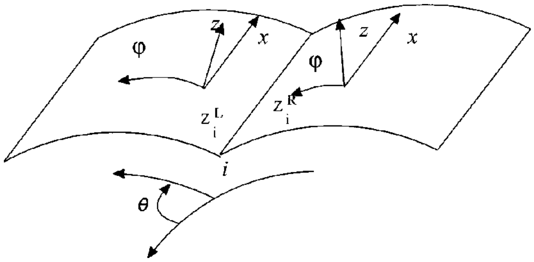


Fig. 3 Relation between consecutive panels

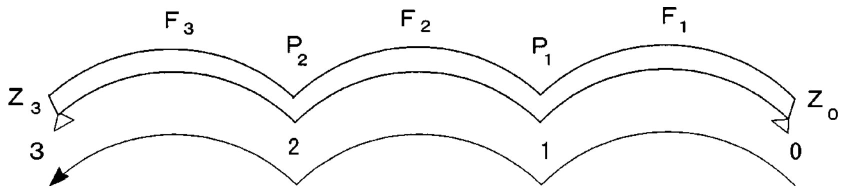


Fig. 4 Cross section with shell panels

example, is obtained for the case of simply supported lengthwise edges:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} \tilde{\varphi}_{\varphi 0} \\ \tilde{V}_{\varphi 0} \\ \tilde{v}_0 \\ \tilde{u}_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{10}$$

or

$$\boldsymbol{U}'\boldsymbol{Z}_0' = \boldsymbol{0} \tag{11}$$

It is required in the case of the natural frequency that the determinant of the matrix of Eq. (11) is zero,

$$|\boldsymbol{U}'| = \boldsymbol{0} \tag{12}$$

Substituting the natural frequency obtained above and setting the first unknown variable of the initial state vector $\tilde{\varphi}_{\varphi 0} \equiv 1$, Eq. (10) can be rewritten as follows:

$$\begin{bmatrix} a_{22} & a_{23} & a_{24} \\ a_{32} & a_{33} & a_{34} \\ a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} \tilde{V}_{\varphi 0} \\ \tilde{v}_0 \\ \tilde{u}_0 \end{bmatrix} = - \begin{bmatrix} a_{21} \\ a_{31} \\ a_{41} \end{bmatrix} \tag{13}$$

or

$$\boldsymbol{U}''\boldsymbol{Z}_0'' = \boldsymbol{a} \tag{14}$$

Solving Eq. (14) for the remaining unknown initial state variables, the relative values of the initial state vector corresponding to $\tilde{\varphi}_{\varphi 0} \equiv 1$ can be obtained. Once the initial state vector is obtained, the state vector at any point of the cross section i.e., mode shape can be obtained by further transfer procedure.

3. Numerical examples

In Table 1, the first five natural frequency parameters $\Omega\,(=R\omega\sqrt{\rho(1-v^2)/E}\,)$ and mode numbers in the axial direction of the cylindrical shell panels (Fig. 2(b): $\theta = 0.14$ rad, $b = 5.4$ cm, $b/t = 90.2$, $a/b = 2.5$) obtained by the proposed method (TMM) are compared with the results obtained by Bardell and Mead (1989). The results are obtained for both simply supported ($v = w = 0$) and fixed ($u = v = w = \varphi_\varphi = 0$) boundary conditions at $\varphi = 0$, θ. As can be seen from that table, good agreements exist between the results. This provides the evidence of accuracy of the proposed transfer matrix method for the shell panels.

Fig. 5 shows the variation of natural frequency parameters $\Omega\,(= b\omega\sqrt{\rho(1-v^2)/E})$ of multi-cylindrical shell panel thin-walled members comprising $N = 2$~5 shell panels (Fig. 1: $b = 2000$ cm, $b/t = 1000$, $a/b = 2.0$, $\theta = 0.0$~1.0). The analytical models made of $N = 3$, 5 shell panels are shown in Fig. 1(b), (c). Both ends ($b = 0$, 2000 cm) are assumed to be simply supported ($v = w = 0$). The vertical axis represents the ratio between natural frequency parameter Ω of the proposed models and that of the simply supported flat plate Ω_0 (Leissa 1969). As can be seen from Fig. 5, every natural frequency parameter decreases and asymptotically approaches to that of simply supported flat plate as the center angle θ of the shell panel decreases. This shows the reliability of the proposed method for the thin-walled members composed of cylindrical shell panels as shown in Fig. 1.

Fig. 6 shows the variation of first five natural frequency parameters $\Omega\,(= b\omega\sqrt{\rho(1-v^2)/E})$ of the thin-walled member composed of only one cylindrical shell panel (Fig. 1(a): $b = 2000$ cm, $\theta = 2.0$, $b/t = 1000$, $a/b = 1.0$~10.0). The mode numbers in the axial direction are also shown in the same figure.

Table 1 Comparison of natural frequencies

Bound	Methods	Ω (m)				
		1st	2nd	3rd	4th	5th
S-S	TMM	0.2736(1)	0.5038(2)	0.7576(3)	0.8635(1)	0.9712(2)
	Bardell	0.2724(1)	0.5039(2)	0.7569(3)	0.8556(1)	0.9636(2)
	Error(%)	0.439	–0.040	–0.040	0.915	0.783
C-C	TMM	0.9542(1)	0.9913(2)	1.0738(3)	1.2180(4)	1.3286(1)
	Bardell	0.9517(1)	0.9887(2)	1.0712(3)	1.2457(4)	1.3164(1)
	Error(%)	0.262	0.262	0.242	0.189	0.918

$\Omega = R\omega\sqrt{\rho(1-v^2)/E}$

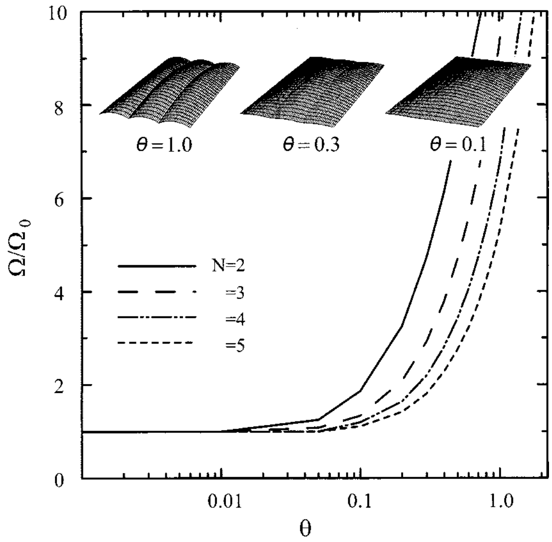


Fig. 5 Relation between natural frequencies and center angles of shell panels

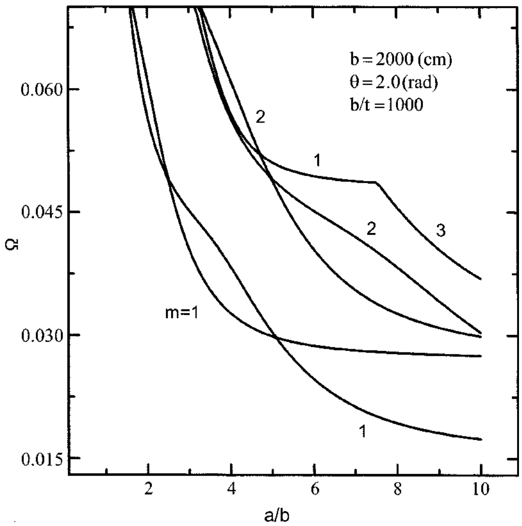


Fig. 6 Natural frequency parameters (N = 1)

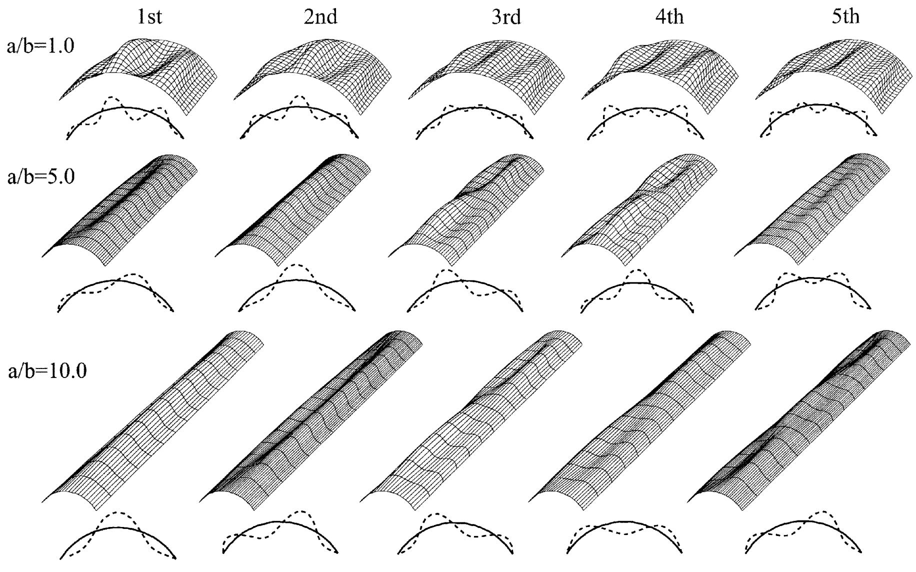


Fig. 7 Vibration mode shapes (N = 1)

As can be in Fig. 6, the first five natural frequency parameters intersect each other and decrease as the aspect ratios *a/b* increase. In Fig. 7, the vibration mode shapes corresponding to the first five natural frequency parameters are shown for the cases of *a/b* = 1.0, 5.0 and 10.0. As shown in Fig. 7, although the mode shapes in the axial direction are relatively simple, those in the circumferential direction are complicated. In addition, the mode number in the circumferential direction increases as the order of vibration mode and the aspect ratio increase. The deformation of middle part of the cross section of shell panels is grater than that of end part as can be seen from Fig. 7.

Fig. 8 shows the first five natural frequency parameters Ω $(= b\omega\sqrt{\rho(1-\nu^2)/E}$ of the thin-walled

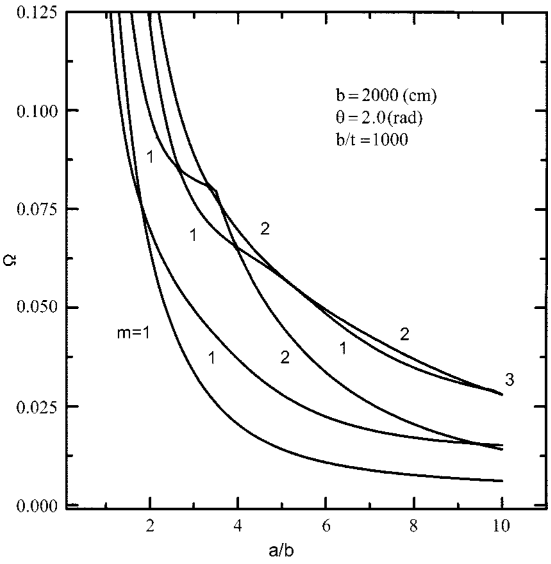


Fig. 8 Natural frequency parameters (N = 3)

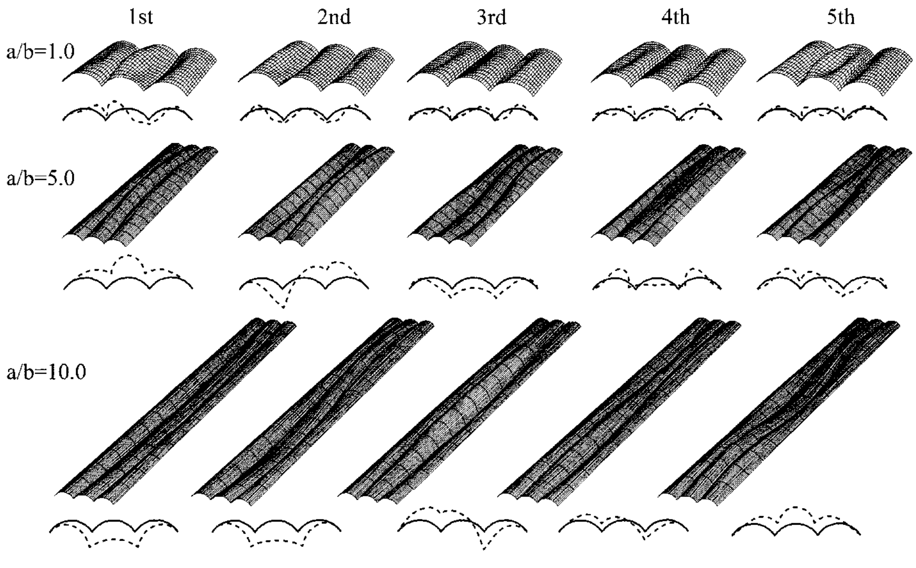


Fig. 9 Vibration mode shapes (N = 3)

member composed of three cylindrical shell panels (Fig. 1(b): b = 2000cm, θ = 2.0, b/t = 1000, a/b = 1.0~10.0). The mode numbers in the axial direction are also shown in Fig. 8. As in the case of the member composed of one cylindrical shell panel, the first five natural frequency parameters intersect each other and decrease as the aspect ratios a/b increase. In Fig. 9, the vibration mode shapes corresponding to the first five natural frequency parameters are shown for the cases of a/b = 1.0, 5.0 and 10.0. As can be seen in Fig. 9, every vibration mode shape for a/b = 1.0 shows local deformation of the shell panels that compose the members. The vibration mode shapes for the 2nd, 3rd and 4th natural frequencies are similar to

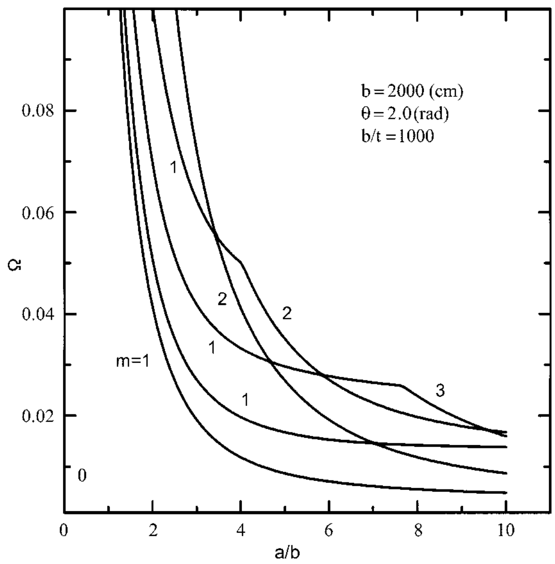


Fig. 10 Natural frequency parameters (N = 5)

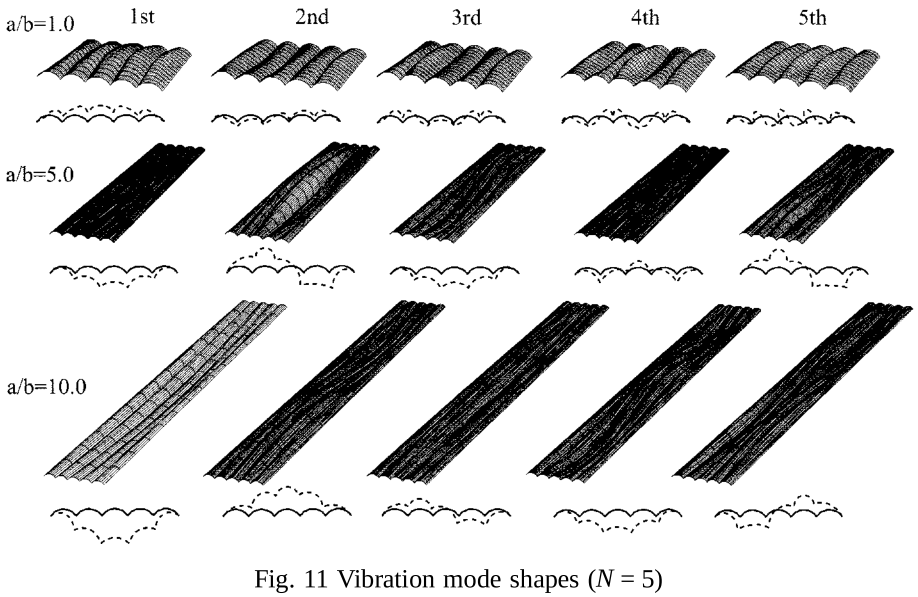


Fig. 11 Vibration mode shapes (N = 5)

each other. The vibration mode shapes for a/b = 5.0 indicate quite distortional combined local and overall deformations. Mode number in the axial direction for 3rd natural frequency is m = 2. However, those for the other natural frequencies are m = 1 as shown in Fig. 9. In the case of a/b = 10.0, the deformation shapes in the circumferential direction indicate simple patterns compared to those of a/b = 1.0 and 5.0. Anyhow, different mode numbers can be seen in the axial direction (the mode number for the 1st and 3rd natural frequencies is m = 1, those for 2nd and 4th is m = 2, and that for 5th is m = 3).

Fig. 10 shows the first five natural frequencies parameters Ω (= $b\omega\sqrt{\rho(1-\nu^2)/E}$ of the thin-walled members composed of five cylindrical shell panels (Fig. 1(c): b = 2000 cm, θ = 2.0, b/t = 1000,

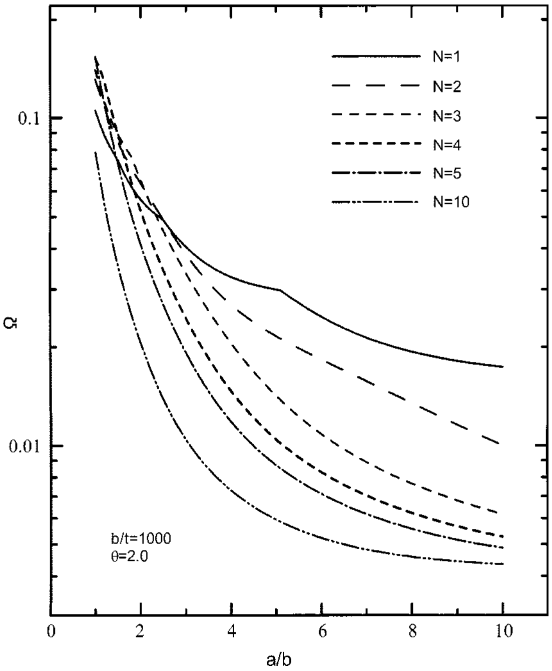


Fig. 12 Natural frequency parameters

a/b = 1.0~10.0). The mode numbers in the axial direction are also shown in Fig. 10. As can be seen in Fig. 10, the first five natural frequency parameters intersect each other and decrease as the aspect ratio a/b increases as in the other cases of one and three cylindrical shell panels. In Fig. 11, the vibration mode shapes corresponding to the first five natural frequency parameters are shown. As can be seen in Fig. 11, the vibration mode shapes show deformation similar to flat plates (overall deformations). However, there are small local deformations in each shell panel.

Fig. 12 shows the variation of natural frequency parameters Ω (= $b\omega\sqrt{\rho(1-v^2)/E}$ of the thin-walled members composed of N = 1~10 cylindrical shell panels (Fig. 1: b = 2000 cm, θ = 2.0, b/t = 1000, a/b = 1.0~10.0). The analytical models of the members made of 1, 3, 5 and 10 shell panels are shown in Fig. 1. The boundary conditions at both ends (b = 0, 2000 cm) are assumed to be $u = v = w = 0$. As shown in Fig. 12, in each case the natural frequency parameter decreases as the aspect ratio a/b increases. In addition, some of them intersect each other in the range of small aspect ratios.

Fig. 13 illustrates the corresponding vibration mode shapes of the natural frequency parameters shown in Fig. 12 (N = 1, 2, 3, 4, 5, 10) for some selected aspect ratios, a/b = 1.0, 2.0, 3.0, 4.0, 5.0 and 10.0. In the case of N = 1, the mode number in the circumferential direction decreases as the aspect ratio increases. Moreover, the deformation of middle part of the cross section is grater than that of end part. In the case of N = 2, the vibration mode shapes for a/b = 1.0, 2.0, 3.0 and 4.0 show only local deformation of the shell panels. However, those for a/b = 5.0 and 10.0 show the distortional deformations or in other words combined local and overall deformations. In the case of N = 3, the vibration mode shape for a/b = 1.0 shows only local deformation. Although, those for a/b = 2.0, 3.0, 4.0, 5.0 and 10.0 show distortional deformations. In the case of N=4, in addition to the local (a/b = 1.0) and distortional deformations (a/b = 2.0, 3.0), the members of a/b = 4.0, 5.0 and 10.0 show overall deformations similar to flat plates. In the cases of N = 5 and 10 only overall deformations can be seen

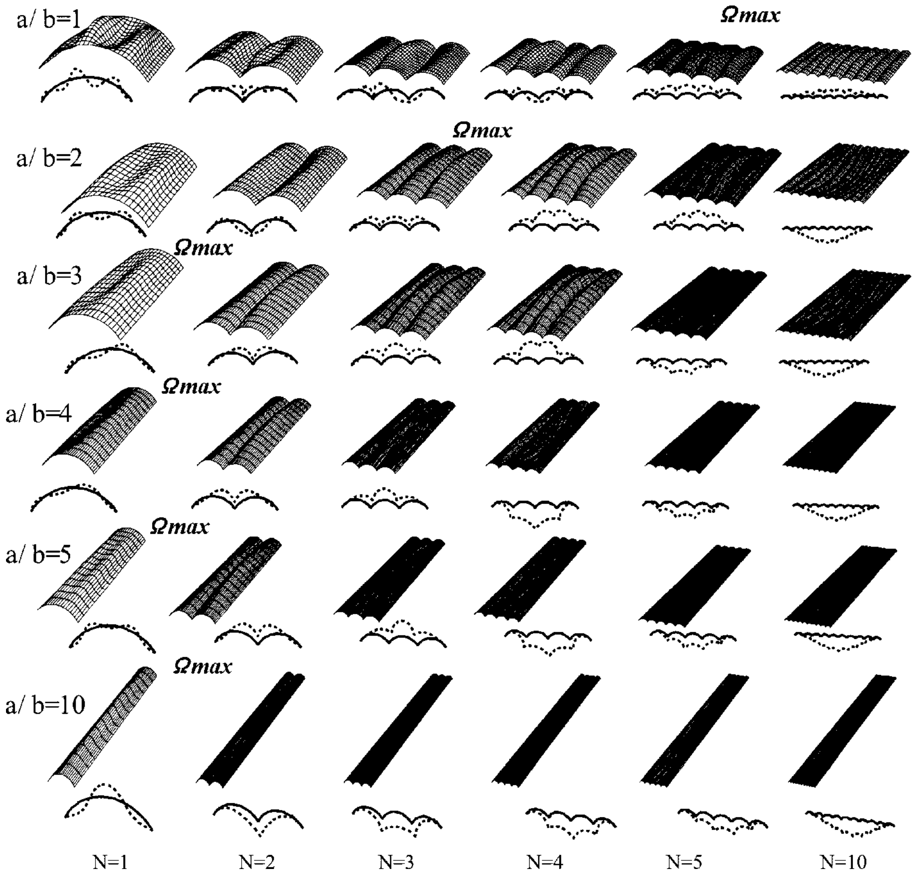


Fig. 13 Vibration mode shapes

in every vibration mode shape.

From the given vibration mode shapes in Fig. 13, although vibration mode shapes induced local deformations can be seen in the cases of small aspect ratios and few number of shell panels, the overall deformations seems to govern the vibration mode shapes of the members as the aspect ratio and number of shell panels increase.

Fig. 14 shows variation of the natural frequencies for different aspect ratios a/b with the number of shell panels composing the thin-walled members (Fig. 1: b = 2000 cm, θ = 2.0, b/t = 1000). In the cases of a/b = 1.0 and 2.0, the natural frequencies become peak at five and three shell panels respectively. Moreover, the vibration mode shapes corresponding to the peak values appear as distortional deformations as can be seen in Fig. 13. In the cases of a/b = 3.0, 4.0, 5.0 and 10.0, the natural frequencies decrease as the number of shell panels increase.

4. Conclusions

The analytical procedure demonstrated in this paper to estimate not only the natural frequencies but also the vibration mode shapes of the thin-walled members composed of cylindrical shell panels is

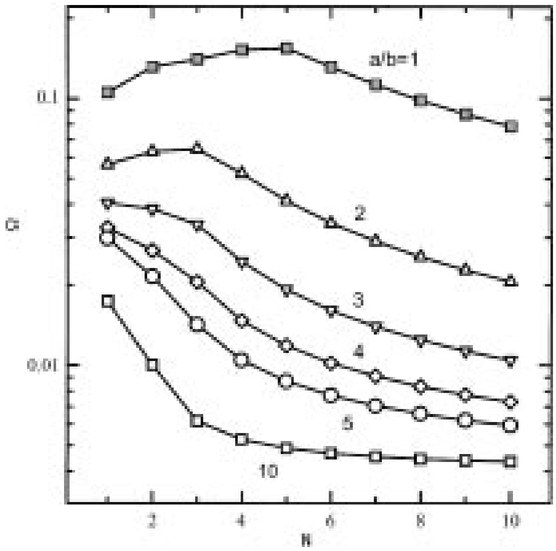


Fig. 14 Relation between natural frequencies and numbers of shells panels

proved to be quite reliable. In addition, it is a good tool to examine the effects of the number of the shell panels on the natural frequencies and vibration mode shapes. From the numerical examples presented in this paper, the following conclusions can be made.

1. The exact natural frequencies and vibration mode shapes of the members are obtained with very small computational efforts.

2. In the cases of small aspect ratios and few number of shell panels, the vibration mode shapes show local deformations. On the other hand, in the cases of large aspect ratios and many shell panels the overall deformations govern the vibration mode shapes of the members.

3. The vibration mode shape corresponding to the peak natural frequency shows distortional deformation i.e. a combination of local and overall deformations.

4. The vibration mode shapes obtained by the transfer matrix method are very effective in clarifying the complicated dynamic phenomenon of the members composed of cylindrical shell panels.

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Notation

The flowing symbols are used in this paper

a	: length of member, and shell panel length
b	: width of member
E	: modulus of elasticity
$\mathbf{F}$	: transfer matrix
K_0	: standard bending rigidity
M_x, M_φ, $M_{x\varphi}$, $M_{\varphi x}$	: bending and twisting moments
m	: vibration mode in axial direction
N	: number of shell panels
N_x, N_φ, $N_{x\varphi}$, $N_{\varphi x}$	: in-plane forces
$\mathbf{P}$	: point matrix
Q_x, Q_φ	: shear forces
R	: radius of shell panel
T_o	: standard shell thickness
t	: shell thickness
u, v, w	: displacements in directions
V_φ	: equivalent shear force
ε_x, ε_φ	: normal strains
$\gamma_{x\varphi}$, $\gamma_{\varphi z}$, γ_{xz}	: shear strains
κ_x, κ_φ, $\kappa_{x\varphi}$, $\kappa_{\varphi x}$	: curvatures of displacements
ν	: Poissonís ratio
θ	: center angle of shell panel
ρ	: mass density
Ω	: frequency parameter; and
ω	: natural frequency

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