Steel and Composite Structures, Vol. 2, No. 3 (2002) 195-208
DOI: http://dx.doi.org/10.12989/scs.2002.2.3.195

Elasticity solution of multi-layered shallow cylindrical panels subjected to dynamic loading

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(Received February 14, 2001, Accepted January 30, 2002)

Abstracts. Elasticity solutions to the boundary-value problems of dynamic response under transverse asymmetric load of cross-ply shallow cylindrical panels are presented. The shell panel is simply supported along all four sides and has finite length. The highly coupled partial differential equations are reduced to ordinary differential equations with constant coefficients by means of trigonometric function expansion in the circumferential and axial directions. The resulting ordinary differential equations are solved by Galerkin finite element method. Numerical examples are presented for two (0/90 deg.) and three (0/90/0 deg.) laminations under dynamic loading.

Key words: elastictiy; panel; shallow; dynamic; composite.

1. Introduction

With the development of new manufacturing techniques, more and more advanced materials have been introduced lately. Fiber-reinforced composite is one such material which has been widely used in the aircraft and aerospace industry, since the properties, such as high mechanical strength, high stiffness, and low density, are crucial in the design of aircraft and space structures. Because of their wide usage, the characteristics of these materials under static and dynamic conditions are of considerable importance and interest to designers. Modeling of composite cylindrical panels is usually based on one of the following three types of theories: classic laminated theory (CLT), shear deformation theories, and three-dimensional elasticity theory. Because of the high ratio of in-plane Young’s modulus to transverse shear modulus, ignorance of the shell transverse shear deformation in CLT can lead to serious errors even for thin cylindrical panels. Although the shear deformation theories have met with success in many cases, three-dimensional elasticity solutions are still needed for the purpose of assessment of various approximate theories and better understanding of actual distributions of stresses, and displacements in composite panels, especially for thick ones. There are not many solutions for laminated cylindrical panels on three-dimensional elasticity, because of the considerable mathematical difficulties in solving governing differential equations for the general boundary and loading conditions. Some elasticity solutions used for stress and vibration analysis of shallow shells under static and dynamic loads are reviewed here.

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Free vibration analysis of doubly curved shallow shells on rectangular platform using three-dimensional elasticity theory was studied by Bhimaraddi (1991). In this paper the governing partial differential equations are reduced to ordinary differential equatiuons by assuming the solution, in the axial and circumferential directions, to be composed of trigonometric functions and solving the resulting equation. An exact three-dimensional thermo elasticity solution is employed to derived the elastic response of a doubly-curved cross-ply laminated panel under mechanical loading and temperature variation by Huang and Tauchert (1992) using the power series method.

A three-dimensional elasticity solution for the static response of simply supported orthotropic cylindrical shells was presented by Bhimaraddi and Chandrashekhara (1992). In this paper, the solution is obtained assuming the ratio of the panel thickness to its middle surface radius is negligible as compared to unity. It is shown that the two-dimensional shell theories are quite inaccurate when the thickness to length ratio of the panel is more than 1/20. Three-dimensional elasticity solution for static response of orthotropic doubly curved shallow shells on rectangular platform was studied by Bhimaraddi (1993). He obtained displacements and stresses in X, Y and Z directions by assuming the variables in the form of the trigonometric functions expansion. Bending analysis of thick cross-ply laminated doubly-curved shells was presented by Jing and Tzeng (1993). In this paper and approximate approach based on the assumption suggested by Soong (1970) is used instead of using exact three-dimensional elasticity. Natural vibration of free, laminated composite triangular and trapezoidal shallow shells was investigated by Qatu (1995). This paper deals with algebraic polynomials in a Ritz analysis to determine the natural frequencies.

Recently the authors have studied the response of orthotropic cylindrical panel under dynamic patch load and anisotropic cylindrical panels under dynamic load (Shakeri *et al*. 2000, 921-927, Shakeri *et al*. 2000 1-11). In these papers three-dimensional theory of elasticity with Galerkin finite element method have been used. Review of the published literature shows that elasticity solution to the problem of laminated, cross-ply cylindrical shallow panel of finite length under dynamic load has not yet been investigated.

Thus, the dynamic response of axisymmetric cross-ply laminated shallow panels subjected to asymmetric loading based on three-dimensional elasticity equations are studied.

2. Problem description

Consider a laminated circular cylindrical panel, as shown in Fig. 1, composed of N uniformly thick layers. The layers are so arranged that the panel in general is orthotropic, and the material symmetry axis in radial, circumferential and axial directions are parallel to the Z, Y and X axes, respectively. The panel is simply supported on four edges. The constitutive equations of each layer are stated as;

$$\begin{bmatrix} \sigma_X \\ \sigma_Y \\ \sigma_Z \\ \tau_{ZY} \\ \tau_{XZ} \\ \tau_{XY} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_X \\ \varepsilon_Y \\ \varepsilon_Z \\ \gamma_{ZY} \\ \gamma_{XZ} \\ \gamma_{XY} \end{bmatrix} \tag{1}$$

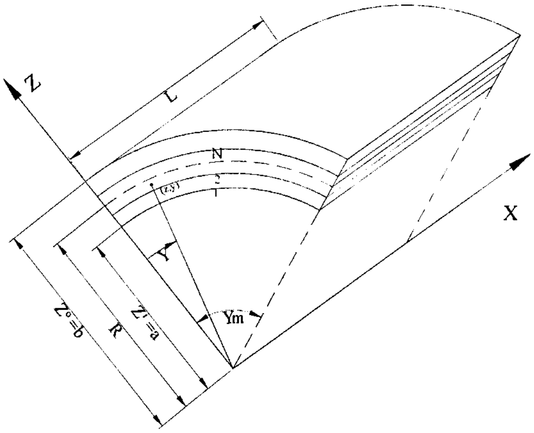

Fig. 1 Geometry and the coordinate system of laminated panel

where:

$\sigma_i (i = Z, Y, X)$ are the normal stresses
$\varepsilon_i (i = Z, Y, X)$ are the normal strains
$\tau_{ZY}, \tau_{XZ}, \tau_{XY}$, are the shear stresses
$\gamma_{ZY}, \gamma_{XZ}, \gamma_{XY}$, are the shear strains

and

$C_{ij}(i, j = 1, 2, 3, 4, 5, 6)$ are the elastic constants

The stratin displacement relations of the 3-D elasticity equations in the cylindrical coordinate system are written as:

$$\varepsilon_X = \frac{\partial U_X}{\partial X}, \quad \varepsilon_Y = \left[\frac{R}{R+Z}\right]\left[\frac{\partial U_Y}{\partial Y} + \frac{U_Z}{R}\right], \quad \varepsilon_Z = \frac{\partial U_Z}{\partial Z}, \quad \gamma_{XY} = \frac{\partial U_Y}{\partial X} + \left[\frac{R}{R+Z}\right]\frac{\partial U_X}{\partial Y}$$

$$\gamma_{XZ} = \frac{\partial U_Z}{\partial X} + \frac{\partial U_X}{\partial Z}, \quad \gamma_{YZ} = \left[\frac{R}{R+Z}\right]\left[\frac{\partial U_Z}{\partial Y} - \frac{U_Y}{R}\right] + \frac{\partial U_Y}{\partial Z} \tag{2}$$

In order to reduce the system of equations with variable coefficients to one with constant coefficient, the only assumption we need to make is:

$$\left[\frac{R}{R+Z}\right] \approx 1 \tag{3}$$

which is a basic assumption of shallow shell theory. Utilizing Eq. (3) in Eqs. (2), we write the strain displacement relations as:

$$\varepsilon_X = \frac{\partial U_X}{\partial X}, \quad \varepsilon_Y = \frac{\partial U_Y}{\partial Y} + \frac{U_Z}{R}, \quad \varepsilon_Z = \frac{\partial U_Z}{\partial Z}, \quad \gamma_{XY} = \frac{\partial U_Y}{\partial X} + \frac{\partial U_X}{\partial Y}$$

$$\gamma_{XZ} = \frac{\partial U_Z}{\partial X} + \frac{\partial U_X}{\partial Z}, \quad \gamma_{YZ} = \left[\frac{\partial U_Z}{\partial Y} - \frac{U_Y}{R}\right] + \frac{\partial U_Y}{\partial Z} \tag{4}$$

Using Eq. (3), 3-D stress equilibrium equations can be written as:

$$\frac{\partial \sigma_X}{\partial X} + \frac{\partial \tau_{XY}}{\partial Y} + \frac{\partial \tau_{XZ}}{\partial Z} + \frac{\tau_{XZ}}{R} = \rho \frac{\partial^2 U_X}{\partial t^2}$$

$$\frac{\partial \tau_{XY}}{\partial X} + \frac{\partial \sigma_Y}{\partial Y} + \frac{\partial \tau_{ZY}}{\partial Z} + \frac{2\,\tau_{ZY}}{R} = \rho \frac{\partial^2 U_Y}{\partial t^2}$$

$$\frac{\partial \tau_{XZ}}{\partial X} + \frac{\partial \tau_{ZY}}{\partial Y} + \frac{\partial \sigma_Z}{\partial Z} + \frac{\sigma_Z - \sigma_Y}{R} = \rho \frac{\partial^2 U_Z}{\partial t^2} \tag{5}$$

Substituting the stress-strain relations (1), via strain-displacement relations (4), the above governing equations can be written in terms of three displacements as:

$$C_{11}^{(k)} \frac{\partial^2 U_X}{\partial X^2} + C_{12}^{(k)}\left(\frac{\partial U_Z}{R\partial X} + \frac{\partial^2 U_Y}{\partial X \partial Y}\right) + C_{13}^{(k)} \frac{\partial^2 U_Z}{\partial Z \partial X} + C_{66}^{(k)}\left(\frac{\partial^2 U_Y}{\partial X \partial Y} + \frac{\partial^2 U_X}{\partial Y^2}\right)$$

$$+C_{55}^{(k)}\left(\frac{\partial^2 U_X}{\partial Z^2} + \frac{\partial^2 U_Z}{\partial Z \partial X}\right) + \frac{C_{55}^{(k)}}{R}\left(\frac{\partial U_X}{\partial Z} + \frac{\partial U_Z}{\partial X}\right) = \rho \frac{\partial^2 U_X}{\partial t^2}$$

$$C_{66}^{(k)}\left(\frac{\partial^2 U_Y}{\partial X^2} + \frac{\partial^2 U_X}{\partial X \partial Y}\right) + C_{12}^{(k)} \frac{\partial^2 U_X}{\partial X \partial Y} + C_{22}^{(k)}\left(\frac{\partial U_Z}{R\partial Y} + \frac{\partial^2 U_Y}{\partial Y^2}\right) + C_{23}^{(k)} \frac{\partial^2 U_Z}{\partial Z \partial Y} +$$

$$C_{44}^{(k)}\left(\frac{-\partial U_Y}{R\partial Z} + \frac{\partial^2 U_Y}{\partial Z^2} + \frac{\partial^2 U_Z}{\partial Z \partial Y}\right) + \frac{2C_{44}^{(k)}}{R}\left(\frac{-U_Y}{R} + \frac{\partial U_Y}{\partial Z} + \frac{\partial U_Z}{\partial Y}\right) = \rho \frac{\partial^2 U_Y}{\partial t^2}$$

$$C_{55}^{(k)}\left(\frac{\partial^2 U_X}{\partial Z \partial X} + \frac{\partial^2 U_Z}{\partial X^2}\right) + C_{44}^{(k)}\left(-\frac{\partial U_Y}{R\partial Y} + \frac{\partial^2 U_Y}{\partial Z \partial Y} + \frac{\partial^2 U_Z}{\partial Y^2}\right) + C_{13}^{(k)} \frac{\partial^2 U_X}{\partial Z \partial X} +$$

$$\frac{1}{R}\left[(C_{13}^{(k)} - C_{12}^{(k)})\frac{\partial U_X}{\partial X} + (C_{23}^{(k)} - C_{22}^{(k)})\left(\frac{U_Z}{R} + \frac{\partial U_Y}{\partial Y}\right) + (C_{33}^{(k)} - C_{23}^{(k)})\frac{\partial U_Z}{\partial Z}\right]$$

$$+C_{33}^{(k)} \frac{\partial^2 U_Z}{\partial Z^2} + C_{33}^{(k)}\left(\frac{\partial U_Z}{R\partial Z} + \frac{\partial^2 U_Y}{\partial Z \partial Y}\right) = \rho \frac{\partial^2 U_Z}{\partial t^2} \tag{6}$$

The simply supported boundary conditions are taken as:

$$U_Z = \sigma_Y = \tau_{YX} = 0 \quad \text{at} \quad Y = 0, Y_m$$

$$U_Z = \sigma_X = \tau_{XY} = 0 \ \text{at} \quad X = 0, L \tag{7}$$

For a laminate consisting of N laminae, the continuity conditions to be enforced at any arbitrary interior (k)th interface can be written as:

$$\sigma_Z)_k = \sigma_Z)_{k+1} \quad \tau_{ZY})_k = \tau_{ZY})_{k+1} \quad \tau_{ZX})_k = \tau_{ZX})_{k+1} \tag{8a}$$

$$U_Z)_k = U_Z)_{k+1} \quad U_Y)_k = U_Y)_{k+1} \quad U_X)_k = U_X)_{k+1} \tag{8b}$$

The boundary conditions on the inner and outer surfaces of the panel are:

$$\sigma_Z = p(Y, t), \quad \tau_{XZ} = \tau_{ZY} = 0 \quad \text{at the outer surface} \tag{9a}$$

$$\sigma_Z = \tau_{XZ} = \tau_{ZY} = 0 \quad \text{at the inner surface} \tag{9b}$$

3. Solution of the governing equations

The solution which satisfies the boundary conditions (7) are:

$$\begin{aligned} U_Z &= u_Z(Z, t)\sin\beta_m Y \cdot \sin p_n X \\ U_Y &= u_Y(Z, t)\cos\beta_m Y \cdot \sin p_n X \\ U_X &= u_X(Z, t)\sin\beta_m Y \cdot \cos p_n X \end{aligned} \tag{10}$$

where:

$$\beta_m = \frac{m\pi}{Ym}, \quad p_n = \frac{n\pi}{L}$$

After substituting Eq. (10) into Eqs. (6), the partial differential equations in terms of the variables, Z, Y, X and t are reduces to ordinary differential equations in terms of the variable Z and t as:

$$-C_{11}^k P_n^2 U_x + C_{12}^k\left(\frac{P_n}{R}U_z - \beta_m P_n U_y\right) + C_{13}^k P_n \frac{\partial U_z}{\partial z} + C_{66}^k(-\beta_m^2 U_x - \beta_m P_n U_y)$$

$$+C_{55}^k\left[\frac{\partial^2 U_x}{\partial z^2} + P_n\frac{\partial U_z}{\partial z} + \frac{1}{R}\left(\frac{\partial U_x}{\partial z} + P_n U_z\right)\right] = \rho^k \frac{\partial^2 U_x}{\partial t^2}$$

$$-C_{66}^k(P_n^2 U_y + \beta_m P_n U_x) - C_{12}^k \beta_m P_n U_x + C_{22}^k\left(\frac{\beta_m}{R}U_z - \beta_m^2 U_y\right) + C_{23}^k \beta_m \frac{\partial U_z}{\partial z}$$

$$+C_{44}^k\left[\frac{-1}{R}\frac{\partial U_y}{\partial z} + \frac{\partial^2 U_y}{\partial z^2} + \beta_m \frac{\partial U_z}{\partial z} + \frac{2}{R}\left(\frac{-1}{R}U_y + \frac{\partial U_y}{\partial z} + \beta_m U_z\right)\right] = \rho^k \frac{\partial^2 U_y}{\partial t^2}$$

$$-\left[C_{55}^k P_n^2 + \beta_m^2 C_{44}^k - \frac{1}{R^2}(C_{23}^k - C_{22}^k)\right]U_z + \frac{\beta_m}{R} \times (C_{44}^k - C_{23}^k + C_{22}^k)U_y + \frac{P_n}{R}(C_{12}^k - C_{13}^k)U_x - P_n$$

$$(C_{55}^k + C_{13}^k)\frac{\partial U_x}{\partial z} - \beta_m(C_{44}^k + C_{23}^k)\frac{\partial U_y}{\partial z} + \frac{C_{33}^k}{R}\frac{\partial U_z}{\partial z} + C_{33}^k\frac{\partial^2 U_z}{\partial z^2} = \rho^k\frac{\partial^2 U_z}{\partial t^2} \tag{11}$$

Eqs. (11) are solved by means of the finite element method. Each layer is devided into an arbitrary number of radial elements. Linear shape functions may be considered for each element (k) as

$$u_s^{(k)}(z,t) = \langle N_i N_j \rangle \begin{Bmatrix} U_{si} \\ U_{sj} \end{Bmatrix} \quad s = z, y, x \tag{12}$$

Substituting Eqs. (12) into the first of Eqs (11). and applying the formal Galerkin method yields

$$\int\Big[(-C_{11}^k P_n^2 - C_{66}^k \beta_m^2)U_x + \frac{1}{R}(C_{12}^k + C_{55}^k)P_n U_z - (C_{12}^k + C_{66}^k)P_n \beta_m U_y + (C_{13}^k + C_{55}^k)P_n\frac{\partial U_z}{\partial z}$$

$$+C_{55}^k\frac{\partial^2 U_x}{\partial z^2} + \frac{C_{55}^k}{R}\frac{\partial U_x}{\partial z} - \rho^k\frac{\partial^2 U_x}{\partial t^2}\Big]N_s dr = 0 \quad s = i, j \tag{13}$$

Similarly, substituting Eqs. (12) into the second and third of Eqs. (11) yield four other equations which combined with Eqs. (13) provide six equations for six unknowns U_{zi}, U_{zj}, U_{yi}, U_{yj}, U_{xi} and U_{xj}. Eq. (13) for $s = i$ reduces to

$$A_4 u_{zi} + B_4 u_{zj} + C_4 u_{yi} + D_4 u_{yj} + E_4 u_{xi} + F_4 u_{xj} + G_4 \ddot{u}_{xi} + H_4 \ddot{u}_{xj} = C_{55}^k \frac{\partial u_z}{\partial z}\bigg|_i \tag{14}$$

where the constants A_4, B_4, ... and H_4 are given in the Appendix. The system of six equations for the six unknowns obtained for the base element (k) may be written as:

$$[M]_k\{\ddot{X}\}_k + [K]_k\{X\}_k = \{F(t)\}_k \tag{15}$$

where:

$[M]$ and $[K]$ are the 6×6 mass and stiffness matrices
$\{F(t)\}$ is the 6×1 force matrix

Expressing Eq. (8a) in term of displacements, by using the Eqs. (1) and (4) and replacing the derivatives by proper finite difference approximation, results into the expressions for displacement components on the inter laminar boundaries in term of the displacement values of neighbouring nodes (Fig. 2) as follows:

$$U_{ZkI}^k = U_{ZkI+1}^{k+1} = A.U_{ZkI-1}^k + B.U_{ZkI+2}^{k+1} + C.U_{YkI-1}^k + D.U_{YkI+2}^{k+1} + E.U_{XkI-1}^k + F.U_{XkI+2}^{k+1}$$

$$U_{XkI}^k = U_{XkI+1}^{k+1} = A'.U_{ZkI-1}^k + B'.U_{ZkI+2}^{k+1} + C'.U_{YkI-1}^k + D'.U_{YkI+2}^{k+1} + E'.U_{XkI-1}^k + F'.U_{XkI+2}^{k+1}$$

$$U_{YkI}^k = U_{YkI+1}^{k+1} = A''.U_{ZkI-1}^k + B''.U_{ZkI+2}^{k+1} + C''.U_{YkI-1}^k + D''.U_{YkI+2}^{k+1} + E''.U_{XkI-1}^k + F''.U_{XkI+2}^{k+1} \tag{16}$$

where:

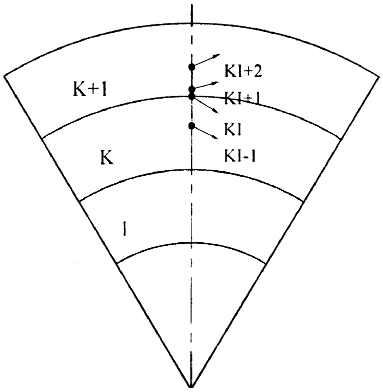

Fig. 2 Configuration fo K and K+1 elements

U^k_{ZkI}, U^k_{YkI}, U^k_{XkI} are the displacements at (kI)th node of (k)th element

A, B, ..., F'' are constant coefficients (see Appendix).

Substituting Eq. (16) into (15), the dynamic finite element equilibrium equations for two neighbouring elements at interior (k)th and (k+1)th interfaces are obtained as:

$$[M]_k\{\ddot{X}\}_k + [K]_k\{X\}_k = \{0\} \tag{17a}$$

$$[M]_{k+1}\{\ddot{X}\}_{k+1} + [K]_{k+1}\{X\}_{k+1} = \{0\} \tag{17b}$$

By applying Eq. (16) for the first and last nodes, displacement values for these nodes can be obtained, and then from Eq. (15), the dynamic equations for the first and last element becomes:

$$[M]_1 \cdot \{\ddot{X}\}_1 + [K]_1 \cdot \{X\}_1 = \{0\} \tag{18a}$$

$$[M]_{MI} \cdot \{\ddot{X}\}_{MI} + [K]_{MI} \cdot \{X\}_{MI} = \{F(t)\}_{MI} \tag{18b}$$

By assembling Eqs. (15), (17a,b), (18a,b), the general dynamic finite element equilibrium equations are obtained as:

$$[M] \cdot \{\ddot{X}\} + [K] \cdot \{X\} = \{F(t)\} \tag{19}$$

where in Eqs. (18b) and (19):

$[M]$ and $[K]$ are matrices of ($3MI$–6–$3N$)×($3MI$–6–$3N$) sizes.
MI and N are the number of elements and layers respectively.

Once the finite element equilibrium is established, the Newmark direct integration method with suitable time step is used and the equations are solved.

4. Numerical results and discussion

Two- and three-layered cross-ply shallow cylindrical panels composed of graphite-epoxy are considered. The forcing function is chosen as:

$$p(Y, X, t) = P_0(1 - e^{-13100t})\sin\beta_m Y \sin p_n X \tag{20}$$

The material properties are

$$E_L = 85 \text{ Mpa},\ E_T = 2.125 \text{ Mpa},\ G_{LT} = 1.0625 \text{ Mpa}$$

$$G_{TT} = 0.425 \text{ Mpa},\ \nu_{LT} = \nu_{TT} = 0.25,\ \rho = 1408(\text{kg/m}^3)$$

Figs. 3 and 4 show the radial stress (σ_z) across the thickness in two and three layered panels at 2.5 msec. At this time the dynamic loading in figure forward (Eq. 20) has the maximum value and is contant from this point. It is seen that the boundary and inter laminar conditions are satisfied. Circumferential normal stress (σ_Y) and it's time history for two and three layers are shown in Figs. 5

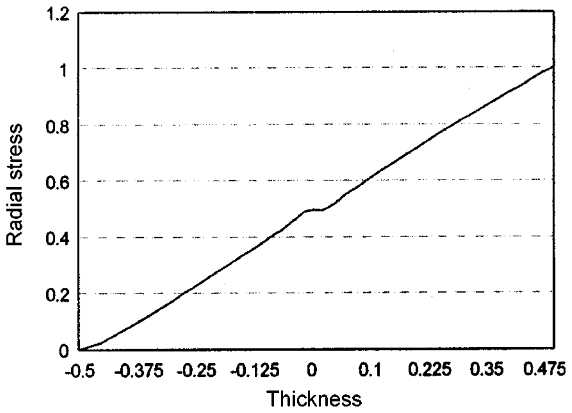

Fig. 3 Distribution of σ_Z across thickness at $Y = \phi/2$, $z = 1/2$ (0/90 deg.)

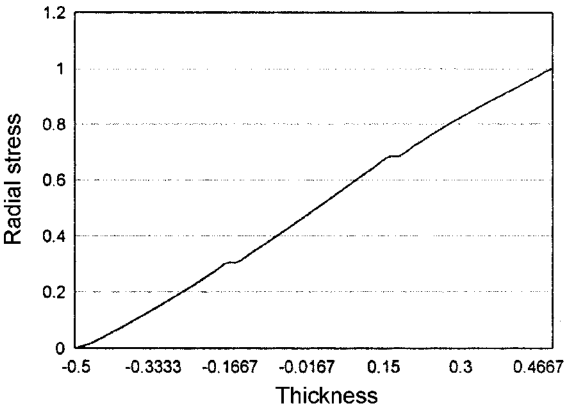

Fig. 4 Distribution of σ_Z across thickness at $Y = \phi/2$, $z = 1/2$ (0/90/0 deg.)

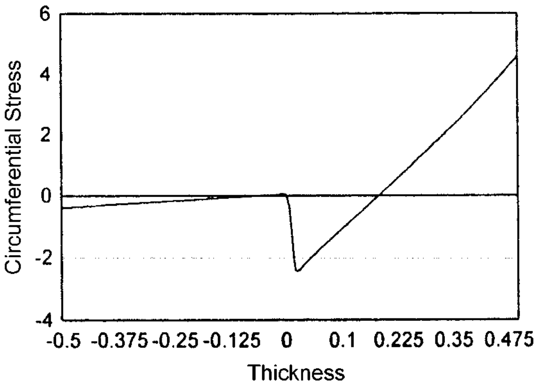

Fig. 5 Distribution of σ_Y across thickness (0/90 deg.)

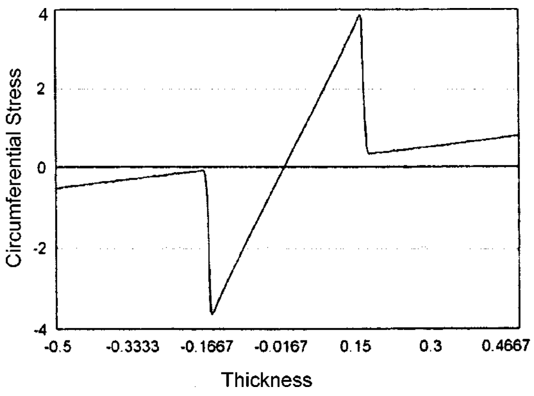

Fig. 6 Distribution of σ_Y across thickness (0/90/0 deg.)

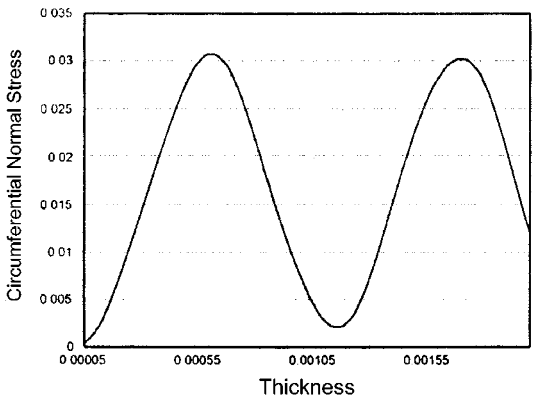

Fig. 7 Variation of σ_Y with time (0/90 deg.)

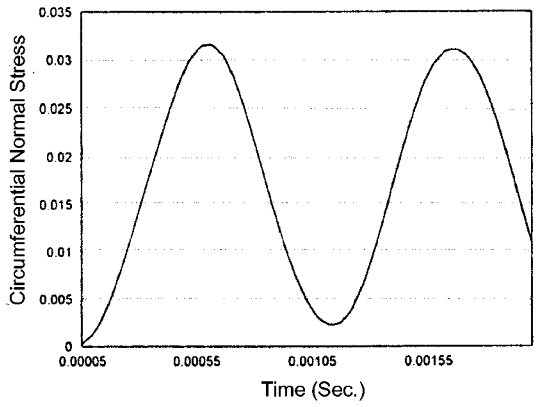

Fig. 8 Variation of σ_Y with time (0/90/0 deg.)

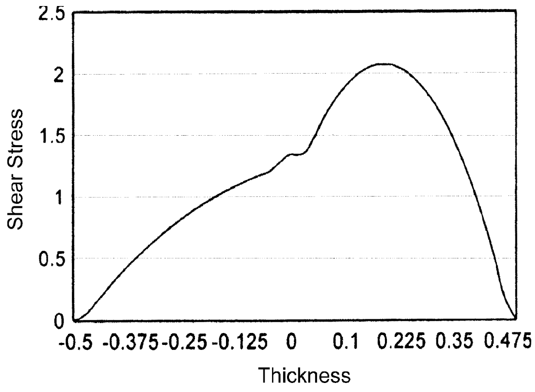

Fig. 9 Distribution of τ_{ZY} across thickness (0/90 deg.)

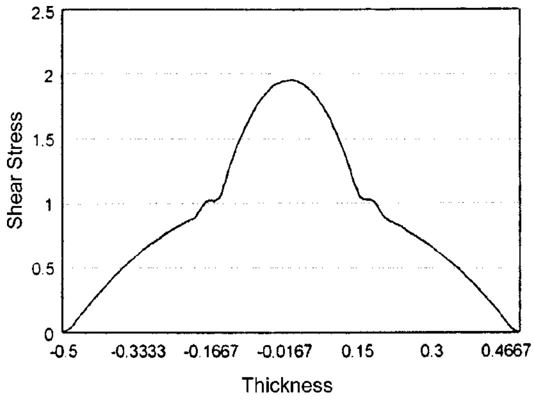

Fig. 10 Distribution of τ_{ZY} across thickness (0/90/0 deg.)

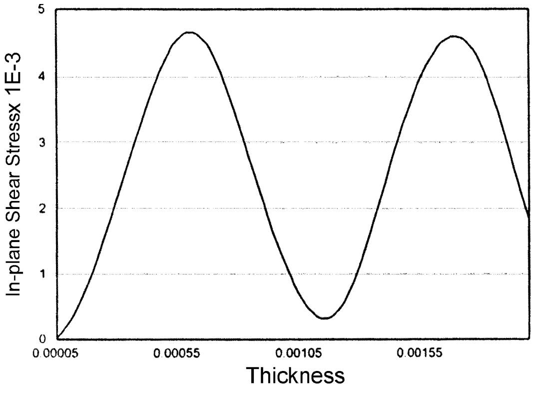

Fig. 11 Variation of τ_{ZY} with time (0/90 deg.)

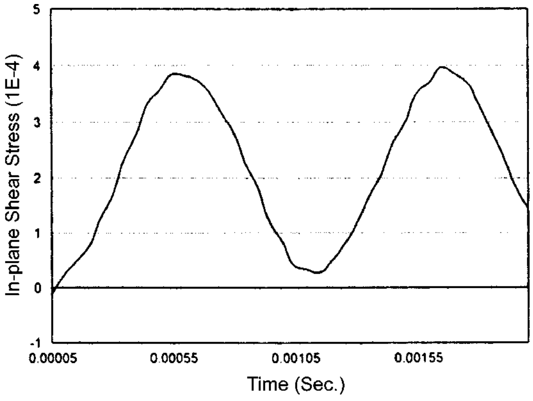

Fig. 12 Variation of τ_{ZY} with time (0/90/0 deg.)

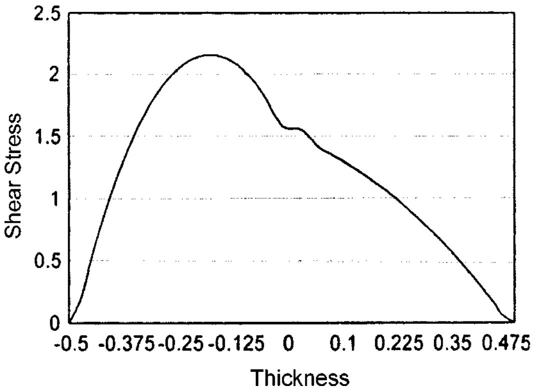

Fig. 13 Distribution of τ_{XZ} across thickness (0/90 deg.)

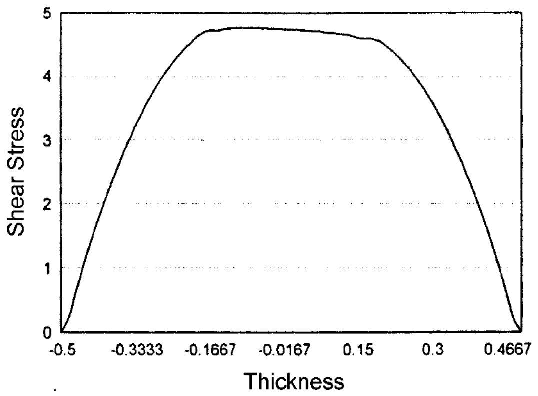

Fig. 14 Distribution of τ_{XZ} across thickness (0/90/0 deg.)

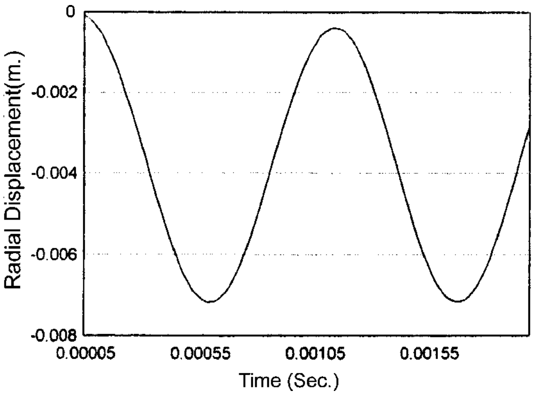

Fig. 15 Variation of U_Z with time (0/90 deg.)

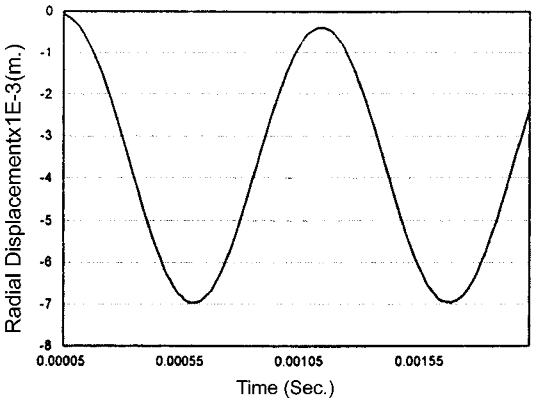

Fig. 16 Variation of U_Z with time (0/90/0 deg.)

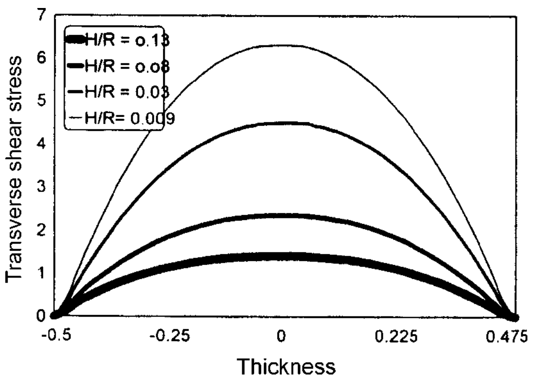

Fig. 17 Variation of τ_{XZ} with H/R (0/90 deg.)

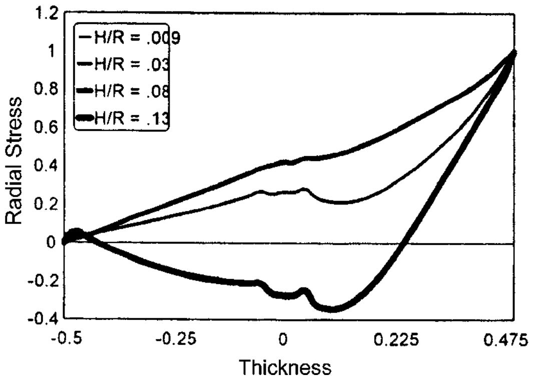

Fig. 18 Variation of σ_z with H/R (0/90 deg.)

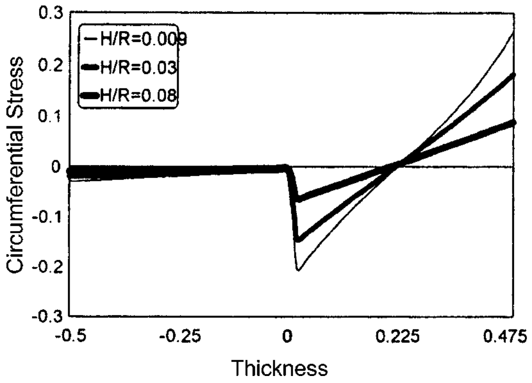

Fig. 19 Variation of σ_y with *H/R* (0/90 deg.)

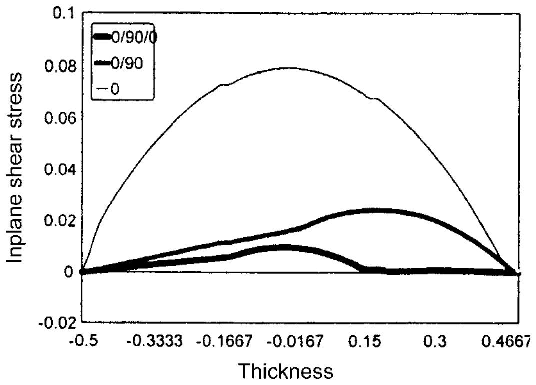

Fig. 20 Variation of τ_{yz} with layer

and 6; and 7 and 8, respectively. As the Figures show, this stress is not continuous at inter laminae. In the case of three-layered panel this stress is symmetric about neutral axis and in mid lamina is greater than the other two laminae.

The in-plane shear stress in the circumferential direction (τ_{ZY}) and it's variation with respect to time are shown in Figs. 9 through 12. This shear stress distribution is parabolic and satisfies the boundary and inter laminae conditions. It is also symmetric about the neutral axis in circumferential direction. The other in-plane shear stress (τ_{XZ}) for two and three layer are shown in Figs. 13 and 14 respectively. This distribution also has an approximate parabolic form. The time history of the radial displacement distribution (U_Z) are shown in Figs. 15 and 16.

Influence of depth to mid radius ratio (*H/R*) of panel in transverse shear stress (τ_{XZ}), normal radial stress (σ_Z) and normal circumferential stress (σ_Y) are shown in Figs. 17, 18 and 19, respectively. in Fig. 18 with increasing the *H/R*, shallow panel changes from thin shell to the thick one and this cause that the radial stress (σ_Z), changes from tension in the outer surface to the compression toward the inner layer. According to the Fig. 19 as *H/R* increases, the distribution of σ_Y across the thickness becomes more linear and close to the bending behavior of plate. Also discontinuity of this stress decreases with

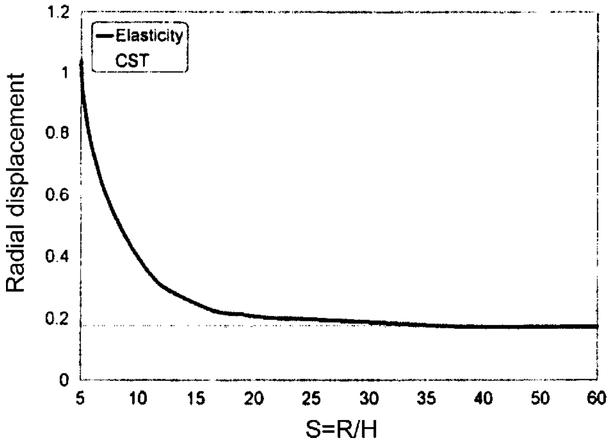

Fig. 21 Variation of radial displacement (U_Z) with *R/H* (0/90)

increasing the *H/R*. This is because that with increasing the thickness of shallow panel with constant radius, depth of panel decreases and consequently it's behavior is close to the behavior of the plate. Variations of in plane shear stress (τ_{ZY}) with layers is shown in Fig. 20. Fig. 21 shown distribution of radial displacement (U_Z) in three-dimensional elasticity and classic shell theory (CST). As expected the CST gives very poor results at relatively high ratio of *H/R*, and the elasticity solution asymptotically approaches to the CST solution.

5. Conclusions

The main aspect of an elasticity solution for any mechanical structure is to be able to solve the problem which can be used as a basic to access the degree of accuracy of other approximate methods. The elasticity solution of multi-layered shells and panels under dynamic loading is quite complicated, but as is shown in the paper, is possible with some restrictions for boundary conditions.

In this paper the response of multi-layered shallow cylindrical panels under dynamics loading is presented. The results achieved could be used for above mentioned purpose, and that, the following conclusions can be made;

1. The through-thickness distribution of transverse shear stresses, (τ_{ZY}) and (τ_{XZ}) are very close to parabola. These distributions in two layered panel are not symmetric and their maximum are not located at mid radius of panels.
2. The sign of σ_Z changes with increasing the thickness to mid radius ratio *H/R*.
3. It is shown that CST underestimates the radial displacement (U_Z) and gives a poor estimate for relatively high value of *H/R*. With increasing the thickness to mid radius ratio, the CST results approach the elasticity results.
4. Amplitude of vibration in radius direction (U_Z) is increased in three layered panel (0/90/0) in comparison with two layered one (0/90).
5. Through thickness shear stress (t_{ZY}) decreased with increasing the number of layers.

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Appendix

$$A_4 = P_n\left[\frac{1}{3R}(C_{12}^k + C_{55}^k)(z_j - z_i) - \frac{1}{2}(C_{13}^k + C_{55}^k)\right]$$

$$B_4 = \frac{P_n}{2}\left[\frac{1}{3R}(C_{12}^k + C_{55}^k)(z_j - z_i) + C_{13}^k + C_{55}^k\right]$$

$$C_4 = \frac{-P_n\beta_m}{3}(C_{12}^k + C_{66}^k)(z_j - z_i) \qquad D_4 = \frac{-P_n\beta_m}{6}(C_{12}^k + C_{66}^k)(z_j - z_i)$$

$$E_4 = -\left[\frac{1}{3}(C_{11}^k P_n^2 + \beta_m^2 C_{66}^k)(z_j - z_i) + C_{55}^k\left(\frac{1}{2R} + \frac{1}{z_j - z_i}\right)\right]$$

$$F_4 = -\frac{1}{6}(C_{11}^k P_n^2 + \beta_m^2 C_{66}^k)(z_j - z_i) + C_{55}^k\left(\frac{1}{2R} + \frac{1}{z_j - z_i}\right)$$

$$G_4 = -\frac{\rho}{3}(z_j - z_i),\ H_4 = \frac{G_4}{2}$$

$$A = D_1 \times K_1,\ B = E_1 \times K_1,\ C = -D_2 \times K_2$$

$$D = -F_2 \times K_2,\ E = -E_3 \times K_3,\ F = -G_3 \times K_3$$

$$A' = -D_1 \times K_4,\ B' = -E_1 \times K_4,\ C' = D_2 \times K_5$$

$$D' = F_2 \times K_5,\ E' = E_3 \times K_6,\ F' = G_3 \times K_6$$

$$A'' = -D_1 \times K_7,\ B'' = -E_1 \times K_7,\ C'' = D_2 \times K_8$$

$$D'' = F_2 \times K_8,\ E'' = E_3 \times K_9,\ F'' = G_3 \times K_9$$

where: Δh is the thickness increment and:

$$K_1 = \frac{B_2 \times C_3}{detA_1^1},\ K_2 = \frac{B_1 \times C_3}{detA_1^1},\ K_3 = \frac{B_2 \times C_1}{detA_1^1},\ K_4 = \frac{A_2 \times B_3}{detA_1^1}$$

$$K_5 = (A_1 \times C_3 - A_3 \times C_1) \times \frac{1}{detA_1^1},\ K_6 = \frac{A_2 \times C_1}{detA_1^1},\ K_7 = \frac{A_3 \times B_2}{detA_1^1}$$

$$K_8 = \frac{A_3 \times B_1}{detA_1^1},\ K_9 = (A_1 \times B_2 - A_2 \times B_1) \times \frac{1}{detA_1^1}$$

$$A_1 = \frac{C_{23}^k - C_{23}^{k+1}}{R} + \frac{C_{33}^k + C_{33}^{k+1}}{\Delta h},\ B_1 = \beta_m \times (C_{23}^{K+1} - C_{23}^K)$$

$$C_1 = \beta_m \times (C_{13}^{K+1} - C_{13}^{K}),\ D_1 = \frac{C_{33}^{K}}{\Delta h},\ E_1 = \frac{C_{33}^{K+1}}{\Delta h},\ \ A_2 = \beta_m \times (C_{44}^{K} - C_{44}^{K+1})$$

$$B_2 = C_{44}^{k}\left(\frac{1}{\Delta h} - \frac{1}{R}\right) + C_{44}^{k+1}\left(\frac{1}{\Delta h} + \frac{1}{R}\right),\ D_2 = \frac{C_{44}^{K}}{\Delta h},\quad F_2 = \frac{C_{44}^{K+1}}{\Delta h}$$

$$A_3 = \beta_m \times (C_{55}^{K} - C_{55}^{K+1}),\ C_3 = \frac{1}{\Delta h} \times (C_{55}^{K} + C_{55}^{K+1}),\ E_3 = \frac{C_{55}^{K}}{\Delta h}$$

$$G_3 = \frac{C_{55}^{K+1}}{\Delta h},\ detA_1^1 = A_1 \times B_2 \times C_3 - A_2 \times B_1 \times C_3 - A_3 \times B_2 \times C_1$$

CC