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Layout evaluation of building outrigger truss by using material topology optimization

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Abstract. This study presents conceptual information of newly optimized shapes and connectivity of the so-called outrigger truss system for modern tall buildings that resists lateral loads induced by wind and earthquake forces. In practice, the outrigger truss consists of triangular or Vierendeel types to stiffen tall buildings, and the decision of outrigger design has been qualitatively achieved by only engineers' experience and intuition, including information of structural behaviors, although outrigger shapes and the member's connectivity absolutely affect building stiffness, the input of material, construction ability and so on. Therefore the design of outrigger trusses needs to be measured and determined according to scientific proofs like reliable optimal design tools. In this study, at first the shape and connectivity of an outrigger truss system are visually evaluated by using a conceptual design tool of the classical topology optimization method, and then are quantitatively investigated with respect to a structural safety as stiffness, an economical aspect as material quantity, and construction characteristics as the number of member connection. Numerical applications are studied to verify the effectiveness of the proposed design process to generate a new shape and connectivity of the outrigger for both static and dynamic responses.

Keywords: conceptual design information; topology optimization; shape; topology; structural layout; outrigger

1. Introduction

During the last century, numerous researches have been carried out on the analysis and behavior of a whole host of structural systems, running parallel with an increase in building constructions. Currently, various building systems have been introduced (Ali and Armstrong 1995, Taranath 1998, Ali and Moon 2007, Gunel and Ilgin 2007, Lee *et al*. 2014, 2014a) which can be used for the lateral resistance of buildings. According to input construction materials, the building system is classified by steel, reinforced concrete, and composite buildings. With respect to structural systems, buildings are basically classified by frame, braced or shear walled, and tube

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structural systems.

Outrigger systems (Fatima *et al*. 2011, Ali and Armstrong 1995, Stafford Smith *et al*. 1996) are modified from braced and shear walled systems. Differing from other structural systems for buildings, the outrigger system can be used for buildings of over 100 stories due to superior resistant ability against lateral loads.

An outrigger is a stiff girder that connects an interior core such as shear walls to exterior columns in tall buildings. Opposite to low-rise buildings governed by gravity loads like dead or live loads, lateral loads involved by wind and earthquake forces are a critical load condition to tall buildings and they can be resisted by a specific system of the outrigger. The outrigger system consists of steel truss members or reinforced concrete members, which is very similar to typical triangular or Vierendeel truss structures.

Appropriate outrigger layouts improving the shape and connectivity of the truss are significant issues into structural designs for tall buildings, since they are directly linked to three key points of building designs (Ambrose 1993, Underwood and Chiuini 1998), i.e., construction cost, safety, and function. Especially, the layout would be an optimized objective which can simultaneously accomplish three key points, while subjected to given design conditions such as the number of floors occupied by trusses, span length, and input materials of steel or concrete.

In practice, determinations of the outrigger layout design have been qualitatively achieved by only designer' or engineer' experience and intuition, including information of structural building behaviors, although outrigger shapes and truss members' connectivity absolutely affect building stiffness, the input of material, construction ability and so on. There is no definite quantitative ground, for example scientific proofs through reliable optimal design tools, for the decision of some outrigger system.

Jahanshahi and Rahgozar introduced optimum location of outrigger-belt truss in tall buildings by maximizing strain energy of belt truss in 2013, and then not member's connectivity but global location of an outrigger set was treated for tall buildings.

Stromberg *et al*. (2012) concretes on optimizing size and topology of braced frames depending on initial conditions, without the consideration of member constructability for tall buildings in 2012.

In this study, two design steps are newly presented to evaluate the optimized shape and connectivity of outrigger systems. Initially, the shape and connectivity, i.e., topology, of an outrigger truss system are visually evaluated using a conceptual design tool of a classical topology optimization method (Bendsøe and Kikuchi 1988) which produces both the optimal shape and topology. Then the shape and connectivity are quantitatively investigated with respect to stiffness for structural safety, material quantity from an economical aspect, and the number of member connections for construction considerations.

Numerical applications are studied to verify the effectiveness of the proposed design process. Such applications can evaluate a new shape and connectivity of outrigger for both static behaviors driven into mean compliance problems (Sigmund 2001, Lee *et al*. 2010) and dynamic behaviors related to eigenfrequency of free vibration problems (Bogomolny 2010, Du and Olhoff 2007, Huang *et al*. 2010, Pedersen 2000).

The outline of this study is as follows: In Section 2 outrigger systems for tall buildings are described. In Section 3 a given design space of outrigger systems is defined to evaluate optimal shape and topology by using topology optimization. With respect to SIMP formulation, the static and dynamic material topology optimization problems are described, including an updated OC scheme in Section 4. In Section 5, a numerical algorithm for the static and dynamic topology optimization methods is presented. In Section 6 the benefits of the automatic optimal shape and topology extraction are studied with several numerical applications of the present method. Section 7 presents the conclusions of this study.

2. Typical outrigger truss systems for tall buildings in practice

In typical outrigger designs, structural behaviors of a target tall building are initially analyzed. According to the analytical results including practical tests, a specific group of structural designers and engineers measures if maximal displacements exist within limit displacements at the top of the buildings as a critical design condition. And then the decision as to a proper structural system remains and an outrigger truss system of the structural systems is recommended by designers and engineers for the target. Appropriate details of the outrigger truss system is subjected to design conditions such as displacement, stiffness, input material, and the construction conditions are determined to include the size, the number, and material of the outrigger unit.

Fig. 1 describes three practical examples of typical outrigger truss models in tall buildings. Figs. 1(a), (b), and (c) indicate outrigger trusses in the Hyperion Tower (248 m) (Chung 2002) in Seoul, Korea, in the North-East Asia Trade Tower (NEATT, 305 m) (Chung *et al*. 2008) in Incheon, Korea, and in the Petronas Twin Towers (KLCC, 452 m) (Bunnell 1999) in Kuala Lumpur, Malaysia, respectively. As can be seen each outrigger truss is coupled and installed between a mega column and a core. The outrigger is a truss system combining horizontal, vertical, and diagonal members, which resist tension and compression forces in buildings.

Under the assumption of guaranteeing structural safety of all outrigger models, each outrigger

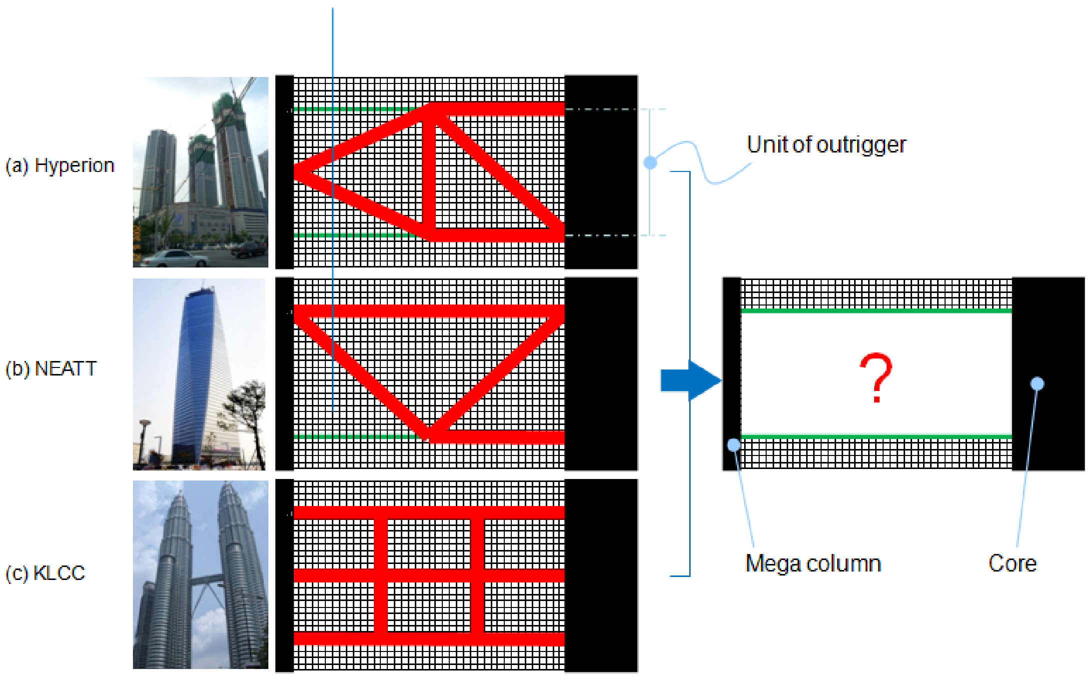

Fig. 1 Typical outrigger truss system for tall buildings

Table 1 Comparisons among outrigger systems: economical and constructive efficiency

Building model of outrigger	Steel material quantity	Number of connection	Economical efficiency	Construction efficiency
Hyperion	132.9	5	●●●	●●
NEATT	116.6	4	●●●	●●●
KLCC	160	12	●	●

model is estimated with respect to an economical aspect involved by steel quantities and construction efficiency by the number of connection points as shown in Table 1. As can be seen, the superiority of the models is KLCC < Hyperion < NEATT. For example, the outrigger truss model of NEATT takes an economical efficiency of 27% and constructive efficiency of 58% more than that of KLCC.

The outrigger truss design and its decision are found to be very significant as can be seen in the estimate of typical outrigger truss models. The question mark in Fig. 1 represents the most optimized outrigger truss which designers and engineers aim to achieve.

3. Descriptions of design space assumed to evaluate outrigger truss systems

As can be seen in Fig. 1 the original design space for evaluating optimized outrigger truss is assumed to be a free boundary condition at the left side of the space which is connected to one mega column and a fixed boundary at the right side of the space linked to one internal core. Input forces are two uniformly distributed loads applied respectively to the vertical (self-weight) and horizontal directions of a given design space.

In here, the rectangular design space where outrigger truss is positioned at the left side of the core is adopted as the design target due to the symmetrical arrangement of outrigger truss.

Fig. 2 sketches the above-mentioned loadings and boundaries as well as the structural mechanism applied to a given design space in which the outrigger system is partially located into a tall building. The building is composed of main three units such as a mega column, a core, and an outrigger.

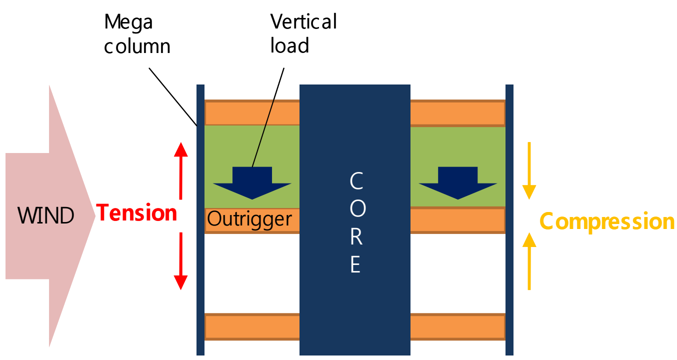

Fig. 2 Structural mechanism and outrigger system of tall buildings

4. Material topology optimization formulations of continuum structures based on mean compliance

In order to provide design information for the decisions of designers and engineers, the so-called structural optimization method, topology optimization is utilized here, and its strongest merit is to produce both optimal shapes and topologies of a target outrigger truss, i.e., connectivity among members. Optimal shapes are directly linked to design information such as appropriate quantities of input material and effective usages of space. Optimal topologies can help designers and engineers with design information like connections among truss members and simplifications of the outrigger construction.

In this study, the homogenization method (Bendsøe and Kikuchi 1988) of the topology optimization methods is treated such that the normalized density ρ is set as the design parameter between almost 0 (due to avoiding numerical singularity) and 1. As to material models, the so-called specific SIMP model offers a simple way to implement zero-one design for the material distribution.

The ratio R_i of the material density to the original material density by using the normalized density is written as

$$R_i = \frac{\rho_i}{\rho_{i0}} \tag{1}$$

where ρ_i and ρ_{i0} denote i-th updated element density and i-th original element density, respectively.

Young's modulus of i-th element indicates E_i, which is assumed to be in a penalty relationship with the original E_{i0} as follows

$$E_i = (R_i)^k E_{i0} \tag{2}$$

where the value of k is the penalty parameter between 2 to 4 (Sigmund 1997).

The problem of optimal topology design for minimum compliance of continuum structures may be described in different ways (Bendsøe 1995, Eschenauer and Olhoff 2001). This minimum mean compliance topology optimization problem can be written by using the material density distribution method as follows.

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$$\begin{aligned} &\underset{R}{Min}: \quad f(R) = U^T KU = \sum_{i=1}^{N} (R_i)^k u_i^T k_0 u_i \\ &\text{subject to} \quad KU = F, \quad \sum_{i=1}^{N} (R_i) v_i \le V, \quad 0 < R_i \le 1 \end{aligned} \tag{3}$$

where U and F are the global displacement and forces, respectively. K is the global stiffness. u_i and k_i are the local displacement and stiffness of the i-th element. R_i denotes the i-th normalized density parameter between almost 0 and 1, and v_i is the volume of i-th element. $KU = F$ in Eq. (3) is the equilibrium equation. N is the number of element discretizing a given design space. V is the input material volume constraint charged in the design space.

In this study, an OC (optimality criteria) method (Sigmund 2001) of gradient-based concepts is used for the optimization method because it can reduce the computational cost associated with having many design variables. The design parameters can be updating using the Lagrangian multiplier λ, which can be solved by a bisection algorithm.

$$\frac{k(\rho_e)^{k-1}(\boldsymbol{u}_e)^T \boldsymbol{K}_e \boldsymbol{u}_e}{\lambda V_e} = \frac{-\dfrac{\partial f}{\partial \rho_e}}{\lambda V_e} \tag{4}$$

and

$$\rho_e{}^{i+1} = \left(\frac{-\dfrac{\partial f}{\partial \rho_e}}{\lambda V_e} \right)^i \rho_e{}^i \tag{5}$$

5. Numerical algorithm generating topologically optimal outrigger trusses for tall buildings

Fig. 3 presents a typical topology optimization procedure consisting of structural analyses, sensitivity analyses, and optimization methods. Concrete descriptions of the procedures are omitted here since this study concentrates on practically evaluating outrigger truss by using typical topology optimization methods. Please see the scripts introduced in the references in this study for further analysis of the mathematical principles of the optimization procedures.

Please note that from the solution of the shape and topology design proper outrigger truss models can be automatically produced in spite of the conceptual design. The developed MATLAB code for optimizing outrigger trusses is extended, and based on an educational version of MATLAB code (Sigmund 2001) for elastostatic problems. Since Sigmund's 99 line code's extension is used, a discrete approach with optimal criteria procedure is focused in this study.

Outrigger truss system is one of mega structures and therefore a lot of finite elements may be

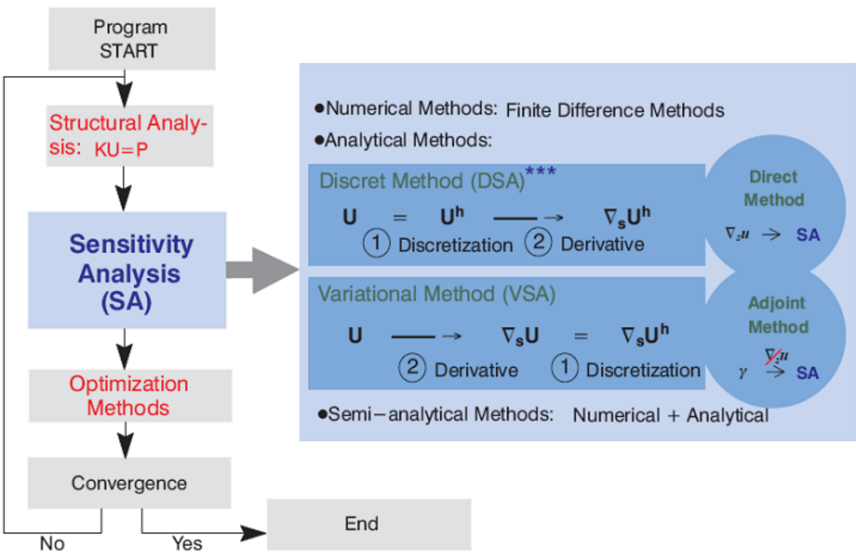


Fig. 3 Typical topology optimization procedures

needed, so that slow convergence of optimization may occur. Moved and regularized Heaviside function (Lee *et al*. 2014b, c) may be adjusted to improve the efficiency and convergent rate, although it is not applied to this study.

6. Numerical applications and discussion

Numerical examples are involved in evaluating appropriate layouts of outrigger trusses for tall buildings by using continuous two-phase (0-1) material SIMP topology optimization methods for the static problem. The objective function is the minimal strain energy (N·m). A plane stress state is assumed.

The given design space of a rectangular form, which outrigger trusses are evaluated by using the present topology optimization method is one half of target outrigger floors as shown in Fig. 2 due to symmetry. The design space is linked to an external column and a R.C. core. Since outrigger trusses are intermediate members connecting an external column and a core, it would be very difficult how to define boundary conditions. Therefore, in this study support conditions are assumed to be a free or a roller support at its left side and a fixed support at the right side.

The span length (L) and the floor height (H) of the given design space are, respectively, 11 m and 7 m as shown in Fig. 4. The thickness of the design space is 1.0 m. As a loading condition the combination of two uniformly distributed loads, i.e., L1 + L2 is considered. The two loads are, respectively, 50 N/m at the horizontal direction assumed to be wind loads and 60 N/m at the vertical direction assumed to be live and dead loads.

One square finite element of approximate 0.273 m × 0.273 m is assumed for discretizing the given design space. Design models (Model 1, 2) depending on defined supports are shown in Table 2. The material of outrigger trusses is steel, and then Young's modulus of steel is 200 GPa and Poisson's ratio is 0.3. The material quantity which is occupied into each design space to generate outrigger trusses is fixed to the volume of 23.1 m^3 during every optimization procedure. The relative volumes are used to input the data of volume fraction for optimization.

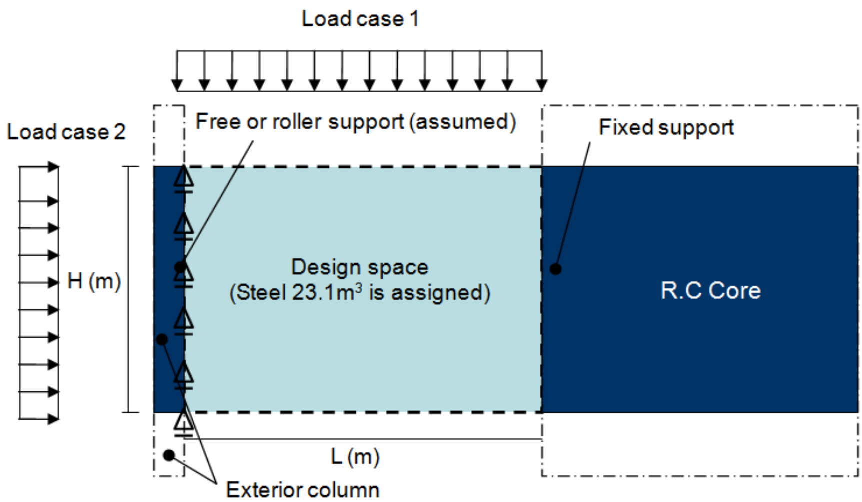


Fig. 4 Design space for outrigger truss design

Table 2 Design input data

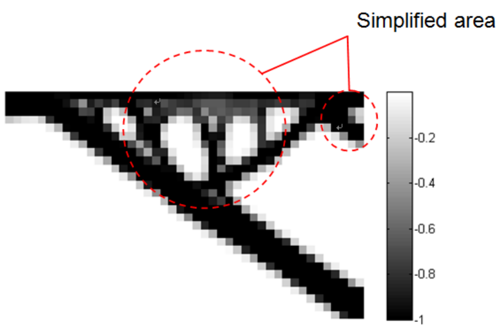
Model number	Design space (L × H × Thickness)	Finite mesh	Relative volume (23.1 m^3 is fixed)	Support	Load case	Material property
Model-1	12 m × 7 m × 1 m	44 × 28	30%	Roller + Fixed	L1 + L2	*Young's modulus: 200 GPa *Poisson's Ratio: 0.3 *Material: steel
Model-2	12 m × 7 m × 1 m	44 × 28	30%	Free + Fixed	L1 + L2	

6.1 Evaluating design information of the outrigger layout using topology optimization

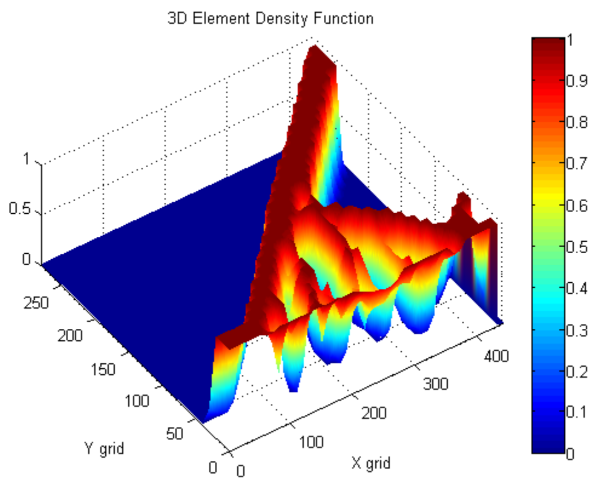
Figs. 5 and 6 show the final optimal outrigger truss layouts of Model-1 and Model-2, respectively. Here Model-1 and Model 2 take different support conditions. The results describe the optimal depositions of steel materials with relative volumes of 30% of the total volume of the design space occupied by steel of 23.1 m^3. The layouts are described by collecting 0 (white), 1 (black), and the intermediate value (gray) of the finite element densities as shown in Figs. 5(a) and 6(a). In addition graphically three dimensionally density contours can be evaluated for the purpose the full understanding of designers and engineers as shown in Figs. 5(b) and 6(b).

As can be seen there are areas reducing material distributions during every optimization procedure, and especially the areas would be removed by designers achieving the simplicity of the design. The result of Model-1 is similar to that of Model-2 in spite of the different supports. Support conditions may have an effect on optimized structures described by connectivity among the members.

The best solution of shape and topology can be achieved through removing blurred material distribution regions in case that the result blurred a given design domain. The blurred regions would be disappeared at the final stage of convergence, i.e., 0-1 material distributions. In addition,

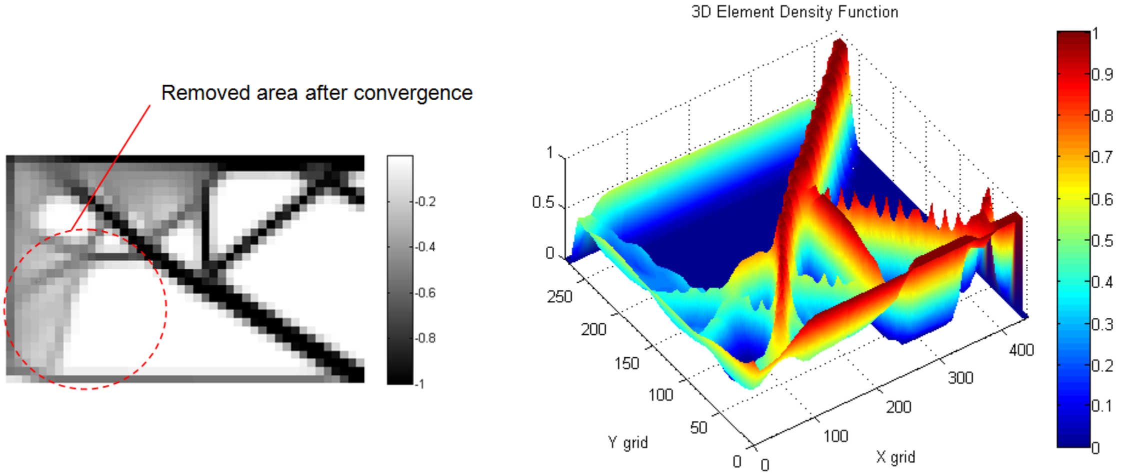


(a) Optimal shape and topology (2D)



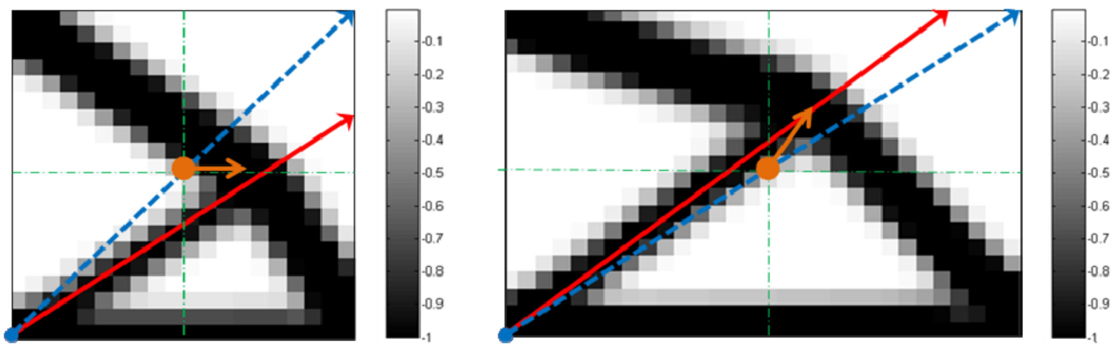
(b) Optimal shape and topology (3D)

Fig. 5 Optimal outrigger truss layout of Model-1 with roller and fixed supports

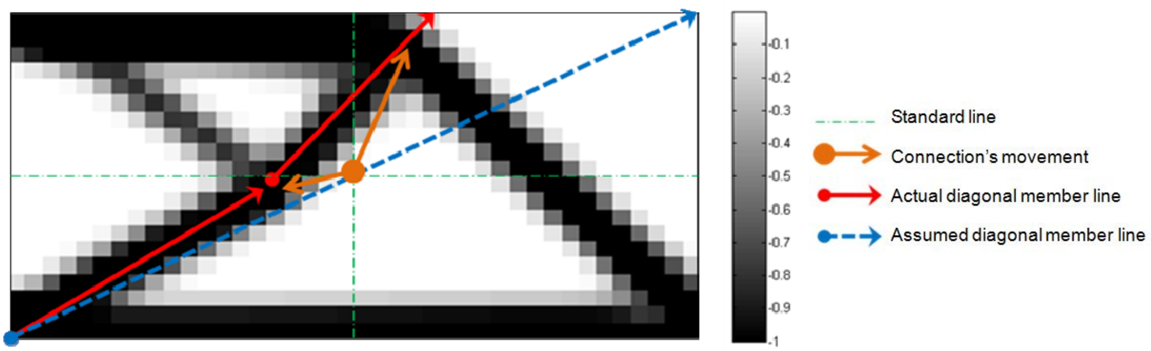


(a) Optimal shape and topology (2D) (b) Optimal shape and topology (3D)

Fig. 6 Optimal outrigger truss layout of Model-2 with free and fixed supports



(a) 1:1 (b) 1.5:1



(c) 2:1

Fig. 7 Topologies of optimal outrigger truss layout depending on the size ratio of width to height of the design space

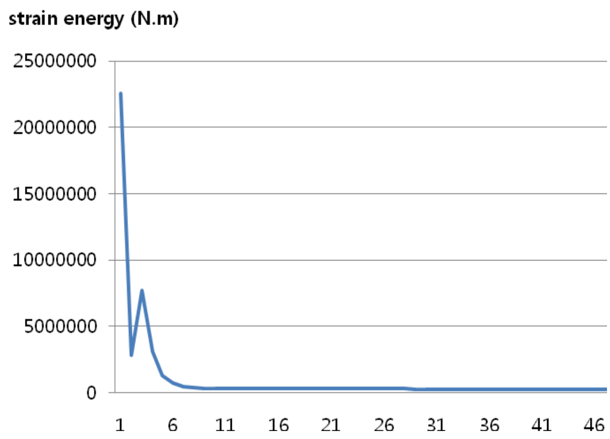


Fig. 8 Convergence histories of strain energy

as outrigger system is one of mega structures, appropriate trends of optimal shape and topology may be helpful to engineers and designers.

6.2 Evaluating design information of outrigger layout depending on the size ratio

In this example, the material quantity of 30% of the total design space is fixed. The three design spaces are considered to take the size ratio of width to height, i.e., 1 to 1, 1.5 to 1, and 2 to 1.

Fig. 7 describes optimal layout results of the member's connectivity depending on size ratio of width to height in design space, in which member's connectivity is assigned after the topology optimization method. As can be seen, in all cases, the actual diagonal member's locations evaluated using topology optimization differ from the assumed diagonal member line, which may be designed by the engineer's intuition and experience.

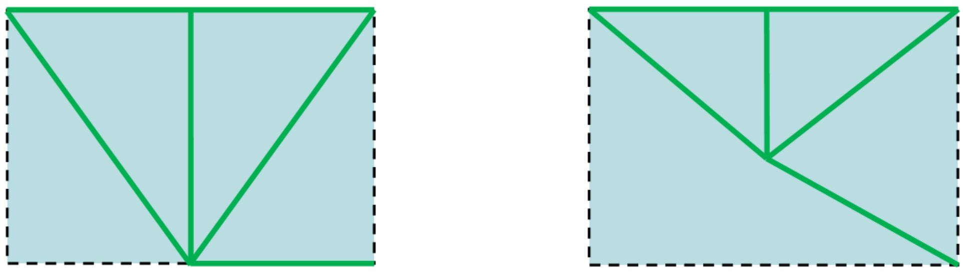
In the ratio of 1 to 1, three members take one connection point, which is located to the right side of the center point in a given design space. In the ratio of 1.5 to 1, one connection point is located to the right and upper side, because the width is larger comparison with the case of 1 to 1. In the size ratio of 2 to 1, two connection points need to the right and upper side as well as the left and lower side due to the extension of width.

It may be almost impossible to make these visual and automative descriptions for outrigger truss design without the proposed topology optimization approach and with only experience and intuition of designers. The design of outrigger trusses needs to be measured and determined according to scientific proofs like as reliable optimal design tools, not engineer's experience and intuition. This study presents alternative to this need of designers and engineers.

Fig. 8 shows convergence curve of mean compliance such as strain energy of Fig. 7(c) during every optimization.

6.3 Performance measurement of simplified evaluated outrigger truss layouts

Fig. 9(a) shows a typical alternative, which is well known as the most generalized outrigger truss detail for tall buildings. In general, the decision of the detail is based on intuition or experience of engineers and designers, and the detail has been chosen for many tall building



(a) Typical Alt (b) Proposed Alt

Fig. 9 Comparison between a typical Alt and a proposed Alt for outrigger truss layouts

Table 3 Alt comparisons with respect to constructional and economical efficiency

Outrigger alternative	The number of connection	Nominal quantity
Typical alternative	5	41.3
Proposed alternative	5	34.1

projects in Korea. Fig. 9(b) describes a simplified alternative directly linked to Figs. 5 and 6. The information evaluation of the conceptual design is generated by using the material topology optimization approach, which has been targeted in the automotive, aeronautic, and naval industries. In order to save construction costs paid by clients, most of all, it is significant to reduce both the number of member connection and material quantities. As can be seen in Table 3, the proposed alternative saves construction cost of outrigger trusses as compared to the typical alternative. Finally the proposed alternative results in reducing material quantities of approximately 10.8%.

Increasing of the number of connections leads to material quantity, and then strength of structure would be improved.

7. Conclusions

This study presents the promising possibility to easily evaluating the optimized outrigger truss to be significant to tall buildings by using typical topology optimization. This study has been made to promote the application of considerations optimizing topology, i.e., connectivity among members, and shape, i.e., global layouts, to the design and detailing outrigger trusses. The “optimized outrigger truss” wholly includes design information to synthetically achieve structural ability such as resisting lateral forces and economical construction such as reducing construction periods for tall buildings.

The practical use of the present design method is adaptable to arbitrary geometrical and loading situations. In addition, there is a considerable potential for applying this method to practical mega structures such as bridges and buildings by easily applying this simple and clear computational program with graphical input and output routines which could replace traditional drawing board methods for developing outrigger truss models.

Although not considered in this study, the appreciation between the practical designs and numerical solutions may be verified by using structural analyses of building behaviors. In other words, several alternatives to numerical solutions are applied to a given building and then displacements or internal forces at specific regions can be estimated by structural analyses such as finite element method.

Finally, the possibility that topology optimization results may be sufficiently applied to practical buildings and civil engineering designs, and not just at the conceptual stages, are very suggestive to engineers and designers. Topology optimization has been widely used in civil and structural engineering. Indeed, structural design is the key area of topology optimization. The present study proposes methods to conceptually model an optimization setup within the existing software to meet building related optimization issues.

Acknowledgments

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References

Ali, M. and Armstrong, P. (1995), *Architecture of Tall Buildings*, Council on Tall Buildings and Urban Habitat Committee, McGraw-Hill, New York, NY, USA.

Ali, M. and Moon, K.S. (2007), "Structural Developments of Tall Buildings: Current Trends and Future Prospects", *Architect. Sci. Rev.*, **50**(3), 205-223.

Ambrose, J.E. (1993), *Building Structures*, John Wiley and Sons.

Bendsøe, M.P. (1995), *Optimization of Structural Topology, Shape, Material*, Berlin, Heidelberg, New York, Springer.

Bendsøe, M.P. and Kikuchi, N. (1988), "Generating optimal topologies in optimal design using a Homogenization Method", *Comput. Methods Appl. Mech. Eng.*, **71**(2), 197-224.

Bogomolny, M. (2010), "Topology optimization for free vibrations using combined approximations", *Int. J. Numer. Method. Eng.*, **82**(5), 617-636.

Bunnell, T. (1999), "View from above and below: The petronas twin towers and/in contesting visions of development in contemporary Malaysia", *Singapore J. Trop. Geo.*, **20**(1), 1-23.

Chung, K.R. (2002), "Application of new structural system in domestic highrise residential buildings", *J. Architect. Inst. Korea*, **46**(6), 64-66.

Chung, K.R., Scott, D., Kim, D.H., Ha, I.H. and Park, K.D. (2008), "Structural system of North-East Asia Trade Tower in Korea", *Proceedings of Council on Tall Buildings and Urban Habitat 8th World Congress*, Dubai, UAE, March, pp. 1-8.

Du, J. and Olhoff, N. (2007), "Topological design of freely vibrating continuum structures for maximum values of simple and multiple eigenfrequencies and frequency gaps", *Struct. Multidiscipl. Optim.*, **34**(2), 91-110.

Eschenauer, H.A. and Olhoff, N. (2001), "Topology optimization of continuum structures – A review", *Appl. Mech. Rev.*, **54**(4), 331-390.

Fatima, T., Fawzia, S. and Nasir, A. (2011), "Study of the effectiveness of outrigger system for high-rise composite buildings for cyclonic region", *Proceedings of ICECECE 2011: International Conference on Electrical, Computer, Electronics and Communication Engineering*, Venice, Italy, November, pp.

937-945.
Gunel, H.M. and Ilgin, E.H. (2007), "A proposal for the classification of structural system of tall buildings", *Build. Environ.*, **42**(7), 2667-2675.
Huang, X., Zuo, Z.H. and Xie, Y.M. (2010), "Evolutionary topological optimization of vibrating continuum structures for natural frequencies", *Comput. Struct.*, **88**(5-6), 357-364.
Jahanshahi, M.R. and Rahgozar, R. (2013), "Optimum location of outrigger-belt truss in tall buildings based on maximization of the belt truss strain energy", *Int. J. Eng.*, **26**(7), 693-700.
Lee, D.K. and Shin, S.M. (2014c), "Advanced high strength steel tube diagrid using TRIZ and nonlinear pushover analysis", *J. Construct. Steel Res.*, **96**, 151-158.
Lee, D.K., Starossek, U. and Shin, S.M. (2010), "Optimized topology extraction of steel-framed diagrid structure for tall buildings", *Int. J. Steel Struct.*, **10**(2), 157-164.
Lee, D.K., Ha, T.H., Jung, M.Y. and Kim, J.H. (2014a-d), "Evaluating high performance steel tube-framed diagrid for high-rise buildings", *Steel Compos. Struct., Int. J.*, **16**(3), 289-303.
Lee, D.K., Lee, J.H. and Ahn, N.S. (2014b), "Generation of structural layout in use for '0-1' material considering n-order eignefrequency dependence", *Mater. Res. Innov.*, **18**(S2), 833-839.
Lee, D.K. Lee, J.H., Lee, K.H. and Ahn, N.S. (2014c-a), "Evaluating Topological Optimized Layout of Building Structures by using Nodal Material Density Based Bilinear Interpolation", *J. Asian Architect. Build. Eng.*, **13**(2), 421-428.
Pedersen, N.L. (2000), "Maximization of eigenvalues using topology optimization", *Struct. Multidisc. Optim.* **20**(1), 2-11.
Sigmund, O. (1997), "On the design of compliant mechanisms using topology optimization", *Mech. Struct. Mach.*, **25**(4), 493-524.
Sigmund, O. (2001), "A 99 topology optimization code written in Matlab", *Struct. Multidisc. Optim.*, **21**(2), 120-127.
Stafford Smith, B., Curvellier, M., Nollet, M-J. and Mahyari, A.T. (1996), "Offset outrigger concept for tall buildings", *Tall Building Structures – A World View, Council on Tall Buildings and Urban Habitat*, pp. 73-80.
Stromberg, L.L., Beghini, A., Baker, W.F. and Paulino, G.H. (2012), "Topology optimization for braced frames: Combining continuum and beam/column elements", *Eng. Struct.*, **37**, 106-124.
Taranath, B. (1998), *Steel, Concrete & Composite Design of Tall Buildings*, McGraw Hill, New York, NY, USA.
Underwood, J.R. and Chiuini, M. (1998), *Structural Design: A Practical Guide for Architects*, John Wiley and Sons.

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