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Cost minimization of prestressed steel trusses considering shape and size variables

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Abstract. There are many studies on the optimization of steel trusses in literature; and, a large number of them include a shape optimization. However, only a few of these studies are focused on the prestressed steel trusses. Therefore, this paper aims to determine the amounts of the material and cost savings in steel plane trusses in the case of prestressing. A parallel-chord simply supported steel truss is handled as an example to evaluate the used approach. It is considered that prestressing tendon is settled under the bottom bar, between two end supports, using deviators. Cross-sections of the truss members and height of the truss are taken as the design variables. The prestress losses are calculated in two steps as instantaneous losses and time-dependent losses. Tension increment in prestressing tendon due to the external loads is also considered. A computer program based on genetic algorithm is developed to solve the optimization problem. The handled truss is optimized for different span lengths and different tendon eccentricities using the coded program. The effects of span length and eccentricity of tendon on prestressed truss optimization are investigated. The results of different solutions are compared with each other and those of the non-prestressed solution. It is concluded that the amounts of the material and the cost of a steel plane truss can be reduced up to 19.9% and 14.6%, respectively, by applying prestressing.

Keywords: prestressed steel; prestressed truss; plane truss; genetic algorithm; optimization

1. Introduction

Prestressing techniques have been widely used in structural engineering, especially in concrete structures, since 1928 when the initial prestressed concrete application was realized by Eugene Freyssinet. The first application of prestressing to steel structures was carried out by Prof. Dr. Ing. Dischinger in 1935, as applying prestressing to steel frame beams. Prof. Magnel conducted a preliminary study on prestressed steel trusses in 1950. Since then, studies about prestressed steel structures have generally focused on prestressed steel frame beams and prestressed steel columns. Nowadays, long-span beams, high poles and structural strengthening are the main application areas of prestressed steel structures. However, application of prestressing to steel structures has been very limited.

The aim of this study is the investigation of the material and cost savings in prestressed steel

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trusses. For this purpose, a selected truss beam is optimized for different span lengths and tendon eccentricities. Optimum height of the truss and optimum tendon geometry are also examined. A genetic algorithm is adopted to solve the optimum design problem. The trusses are analyzed using finite element method, and designed according to AISC-ASD (1989) manual. A computer program was coded in BASIC for the analysis, design and optimization of the trusses. Cross-sectional and geometrical optimization of several prestressed steel trusses was accomplished using the coded program. Results of the prestressed steel trusses with the same topology are compared with each other and those of the non-prestressed solutions.

As pointed out above, there are not many studies on the prestressed steel structures in the literature. Some of the related studies are mentioned here. Öztürk (1979) carried out one of the preliminary studies on prestressed steel structures, and specified that they are especially useful for long spans. Hanaor (1988) presented an algorithm for the analysis and design of prestressed pin-jointed structures. You (1997) studied on displacement control of prestressed truss network structures. Ronghe and Gupta (1999) compared some prestressed steel beams with the ordinary steel beams. Arda and Yardımcı (2000) gave the information about materials, calculation and application methods of prestressed steel trusses. Han and Park (2005) examined elastic behavior of post-tensioned trusses with straight and draped tendon profiles for truss strengthening. Dong and Yuan (2007) proposed an initial internal force method for pretension process analysis of prestressed space grid structures. Albrecht and Lenwari (2008) used prestressing in order to strengthen of steel truss bridges. Park *et al*. (2010) studied on flexural behavior and strengthening effect of a bridge using an externally prestressed steel I-beam. Belletti and Gasperi (2010) investigated the middle span prestressed steel roof beams, and especially focused on the amount of prestressing force and direction of the prestressing tendons.

None of the studies given above includes an optimization process. There are also a few studies focused on optimization of prestressed steel structures. Kirsch (1972) developed a method for the optimum design of prestressed indeterminate beams with uniform cross-section. Levy and Hanaor (1992) examined the effect of prestress on the minimum weight design of singly loaded trusses. Levy and Hanaor (1992) did not comprise cost optimization, which is different from this study. Because of using two different materials (structural steel and prestressing steel) in prestressed steel structures, cost of the structure should be considered as the optimization criterion. Another difference of this study is the employed genetic algorithm, which has not been used in optimization of prestressed trusses until now. However, genetic algorithm is widely used for the optimization of ordinary trusses in the literature (Rajan 1995, Rajeev and Krishnamoorthy 1997, Kaveh and Kalatjari 2003, Rahami *et al*. 2008, Cheng 2010, Dede *et al*. 2011, Guo and Li 2011, Kociecki and Adeli 2013).

2. Description of handled truss

A parallel-chord (Warren configuration) plane truss is adopted as an example in this study. It is assumed that the selected truss is a simply supported roof girder. The prestressing tendon is settled under the bottom chord, between two ends. Deviators are used to form the geometry of prestressing tendon. Distance between the vertical bars of the truss is determined as 1/12 of the span length. Out-of-plane displacement at the each node of the truss is restricted. Snow, coating, fitment and self-weight loads are considered. The handled prestressed steel truss is presented in Fig. 1.

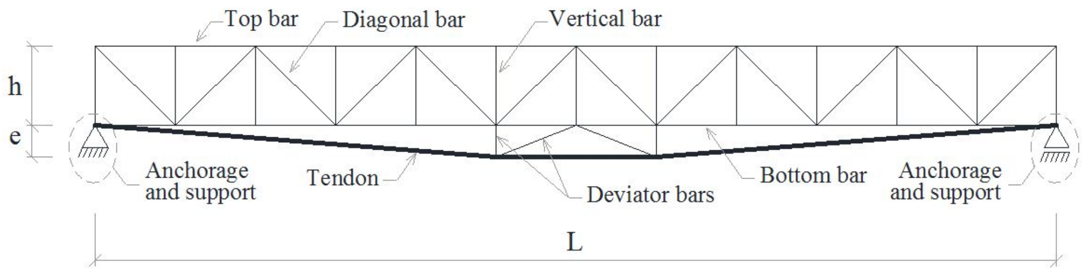

Fig. 1 Topology of the handled prestressed steel truss

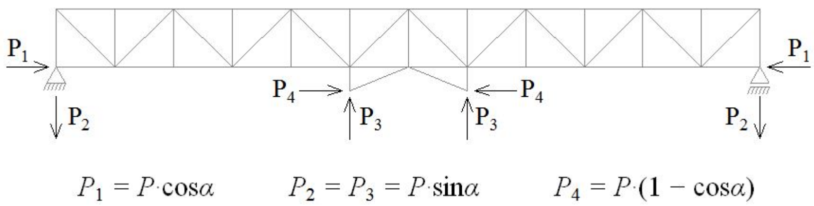

Fig. 2 Application of equivalent prestressing loads to the truss

3. Analysis of prestressed steel trusses

Prestressed trusses are indeterminate structures generated by statically determinate truss and prestressing tendon. In the analysis, prestressing tendon is removed and equivalent prestressing loads are applied to the truss. Thus, a statically determinate truss is solved instead of indeterminate one. An example for application of equivalent prestressing loads to a truss is demonstrated in Fig. 2. In this figure, α is the angle between tendon and bottom bar; P is the prestressing force.

Finite element method is used for static analysis of trusses, as mentioned before. Detailed information about finite element analysis of trusses can be found in the study of Bathe (1996). Prestress losses and stress increment are considered in this study, as explained below.

3.1 Prestress losses

Prestress losses are calculated in two steps as instantaneous losses and time-dependent losses. The instantaneous losses occur during or shortly after the prestressing process, whereas the time-dependent losses occur after the prestressing in time.

Losses due to anchorage set, prestressing method, friction and elastic shortening are the instantaneous losses. The anchorage losses occur due to permanent deformations on the anchorage zones and sliding of the anchorages. The amount of the anchorage losses is dependent on the material properties. The friction between tendons and deviators on the inflection points causes the friction losses. In prestressed steel trusses, prestressing tendon can be consisted of more than one tendon, and these tendons can be tensioned at the same time or separately. If tendons are tensioned separately, prestress losses occur between firstly and secondly tensioned tendons. These prestress losses are named as the losses due to prestressing method. In this study, the instantaneous losses are approximately taken as 5% of prestressing force (Öztürk 1979).

Relaxation of the prestressing steel causes the time-dependent losses. These losses cannot be calculated exactly. Producers can inform designers about relaxation properties of their steel products. In this study, the time-dependent losses are obtained according to TS3233 (1979). Therefore, if tensile stress of tendon does not exceed 70% of the ultimate stress, prestress loss is 8% of the tensile stress; and similarly, if it does not exceed 50% of the ultimate stress then prestress loss is 6% of the tensile stress. It is also specified in TS3233 (1979) that intermediate values can be determined by interpolation.

3.2 Stress increment

The stress increment occurs in prestressing tendon due to external loads. Depending on this stress increment, tendon force increases as

$$\Delta P = \frac{\delta_p}{(\delta_{tr} + \delta_{tn})/1} \tag{1}$$

where, δ_p is the change in the axis length of the tendon due to external loads; δ_{tr} and δ_{tn} are the change in the axis length of the tendon due to unit prestressing force according to rigidity of the truss and the tendon, respectively. δ_p and δ_{tr} are calculated using analysis of statically determinate truss, and δ_{tn} is calculated as

$$\delta_{tn} = \frac{1 \cdot L_{tn}}{E_{tn} \cdot A_{tn}} \tag{2}$$

where, L_{tn} , E_{tn} and A_{tn} are the length, the elasticity modulus and the cross-sectional area of the prestressing tendon, respectively. ΔP has the same unit with the unit force that is applied to calculate the δ_{tr} and δ_{tn}.

3.3 Prestressing force

It can be concluded from the study realized by Cakir (2011) that the optimum prestressing force can be calculated, approximately, as

$$P_0 = \frac{q \cdot L^2}{8} \cdot \frac{(h+e)}{((h+e)^2 + h^2)} \tag{3}$$

where, q is the equivalent uniformly distributed external load; h is the height of the truss; L is the span length of the truss and e is the eccentricity of the tendon. This prestressing force must be revised for the load conditions, considering the prestress losses and the stress increment. Therefore, the instantaneous prestressing force (P_i) and the time-dependent prestressing force (P_s) are calculated as given below.

$$P_i = P_0 - 0.05 \cdot P_0 \tag{4}$$

$$P_s = P_i - (0.06 \sim 0.08) \cdot P_0 + \Delta P \tag{5}$$

4. Statement of optimization problem

An objective function, constraints, design variables and design parameters must be clarified in order to define an engineering optimization problem. These characteristics of the handled optimization problem are explained below.

4.1 Objective function

The total weight of structural steel is generally regarded as the objective function in an ordinary truss optimization. In the case of prestressed trusses, the cost of prestressing steel cannot be ignored. Thus, the objective function is calculated as the total cost of prestressed steel truss and formulated as

$$C_T = C_{ss} + C_{ps} \tag{6}$$

where, C_{ss} and C_{ps} are the cost of the structural steel and the prestressing steel, respectively, and calculated with formulas given below.

$$C_{ss} = UP_{ss} \cdot W_{ss} \tag{7}$$

$$C_{ps} = UP_{ps} \cdot W_{ps} \tag{8}$$

where, W_{ss} and W_{ps} are the total weight of the structural steel and the prestressing steel, respectively. UP_{ss} and UP_{ps} are the unit price of the structural steel and the prestressing steel, respectively. These unit prices include the cost of labor and other necessary equipment.

4.2 Constraints

In this study, stress, slenderness and deflection constraints are considered, that are used in most of the similar optimization studies existing in the literature.

4.2.1 Stress constraints

Three load conditions (LC1, LC2 and LC3) are considered in design according to prestress loss stages, as presented in Table 1.

The stress constraints are calculated according to AISC-ASD (1989) specification, and written in normalized form for the i^{th} truss member under the first load case (LC1) with the equations given below.

Table 1 Considered load conditions

Load condition	Loads	Prestress	Prestress loss	Stress increment
LC1	Self-weight	No	No loss	No
LC2	Self-weight	Yes	Instantaneous losses	No
LC3	Self-weight + external loads	Yes	Instantaneous + time-dependent losses	Yes

$$g_{1,i} = \frac{F_i}{F_t} - 1 \quad \text{if} \quad F_i \geq 0 \tag{9}$$

$$g_{1,i} = \frac{F_i}{F_{a,i}} - 1 \quad \text{if} \quad F_i < 0 \tag{10}$$

where, F_i and $F_{a,i}$ are the calculated stress and the allowable compressive stress for the i^{th} member, respectively; F_t is the allowable tensile stress and calculated as

$$F_t = 0.6 \cdot F_y \tag{11}$$

where, F_y is the yield stress of the structural steel.

The allowable compressive stress of the i^{th} member is calculated with the equations given below, depending on the axial stability of the bar.

$$F_{a,i} = \frac{\left[1 - \frac{\lambda_i^2}{2 \cdot C_c^2}\right] \cdot F_y}{\frac{5}{3} + \frac{3 \cdot \lambda_i}{8 \cdot C_c} - \frac{\lambda_i^3}{8 \cdot C_c^3}} \quad \text{if} \quad \lambda_i < C_c \tag{12}$$

$$F_{a,i} = \frac{12 \cdot \pi^2 \cdot E}{23 \cdot \lambda_i^2} \quad \text{if} \quad \lambda_i > C_c \tag{13}$$

where

$$\lambda_i = \frac{K_i \cdot l_i}{r_i} \tag{14}$$

$$C_c = \sqrt{\frac{2 \cdot \pi^2 \cdot E}{F_y}} \tag{15}$$

In these formulas, l_i , K_i and r_i are the system length, the effective length factor and the radius of gyration for the i^{th} bar, respectively; E is the elasticity modulus of the structural steel.

The stress constraints for the second and third load cases (LC2 and LC3) are also calculated as explained above and represented by $g_{2,i}$ and $g_{3,i}$ for the i^{th} member of the truss.

4.2.2 Slenderness constraints

According to AISC-ASD (1989), slenderness must be smaller than 200 for compression bars and 300 for tension bars. This regulation is also written in normalized form for the i^{th} truss member as given below.

$$g_{4,i} = \frac{\lambda_i}{200} - 1 \quad \text{for compression bars} \tag{16}$$

$$g_{4,i} = \frac{\lambda_i}{300} - 1 \quad \text{for tension bars} \tag{17}$$

4.2.3 Deflection constraints

The deflection constraints are checked in upward and downward directions. The upward deflection constraint is calculated for the first load case (LC1) and expressed as

$$g_{5,1} = \frac{f_1}{L/240} - 1 \tag{18}$$

where, f_1 is the maximum upward deflection at the midpoint of the truss, L is the span length of the truss. The downward deflection constraint is calculated for the third load case (LC3) as

$$g_{5,2} = \frac{f_2}{L/240} - 1 \tag{19}$$

where, f_2 is the maximum downward deflection at the midpoint of the truss.

4.3 Design variables and design parameters

This study includes the cross-sectional and shape optimization, simultaneously. The purpose of the cross-sectional optimization is the determination of the fittest cross-sections for the bars; on the other hand, the purpose of the shape optimization is the determination of the fittest height for the truss beam. The members of the truss are categorized into five groups as top, bottom, vertical, diagonal and deviator bars. Therefore, the cross-sections of the members in these five groups and the height of the truss are considered as design variables.

The main design parameters of the handled truss are the material properties of the structural steel and the prestressing steel, the loads, the unit prices, the span length and the topology of the truss and the geometry of the prestressing tendon.

5. Optimization using genetic algorithm

Genetic algorithm, which is one of the artificial intelligence methods, uses the principles of Darwin's natural selection theory. According to this theory, strong individuals will survive in the next generation while weak individuals will not, in a population. Children of survived strong individuals will constitute the next generation; therefore, the next generation will have stronger individuals than previous generation. At the end of a certain number of iterations, the last generation will be constituted by excellent individuals (Holland 1975, Goldberg 1989).

Genetic algorithm is an evolutionary optimization technique, and, it needs an initial generation that includes randomly determined solutions of the problem. Firstly, the solutions in the generation are graded according to their fitness; after the grading, genetic operators (reproduction, crossover and mutation) are applied in order to constitute the next generation. Genetic algorithms use discrete design variables; therefore, a certain number of probable values for each design variable are determined before the optimization. These probable values are named as the design variable values set (Goldberg 1989).

In genetic algorithms, the solutions in the generation are represented by the strings that are obtained by encoding the row number of the used value of each design variable within the design variable values set (Rajeev and Krishnamoorthy 1992). The binary coding which is one of the

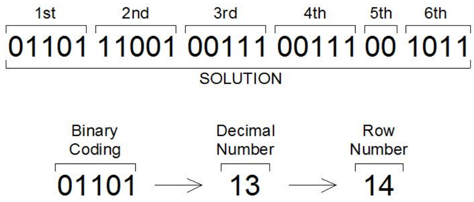

Fig. 3 Encoding of a solution and a design variable

different types of coding techniques in literature is preferred in this study. An example for the binary coding of a solution and a single design variable is demonstrated in Fig. 3. Detailed information about binary coding can be found in the study realized by Aydın and Ayvaz (2010).

After the genetic process, encoded solutions are decoded reversely in order to obtain the solutions of the new generation.

5.1 Penalized objective function

Genetic algorithm is appropriate for unconstrained optimization problems like most of the other optimization techniques. Therefore, the objective function given in Eq. (6) must be transformed into an unconstrained function. A penalty function is added to the objective function for this transformation, and the penalized objective function is constituted. In this study, the penalized objective function is calculated as given below (Rajeev and Krishnamoorthy 1992).

$$\Phi = C_T \cdot \left[1 + K \cdot P_T\right] \tag{20}$$

In this equation, K is the penalty coefficient whose value is determined according to type of the problem. P_T is the penalty function and calculated as

$$P_T = \sum_{i=1}^{nc} p_i \tag{21}$$

where, nc is the number of the constraints and its value is five for this study; p_i is the violation factor of the i^{th} constraint and calculated as given below.

$$p_i = \sum_{j=1}^{n} p_{i,j} \tag{22}$$

$$\text{if} \quad g_{i,j} > 0 \quad \text{then} \quad p_{i,j} = g_{i,j} \quad \text{else} \quad p_{i,j} = 0 \tag{23}$$

In Eq. (22), value of n is equal to the number of truss member for the stress and slenderness constraints, and value of n is two for the deflection constraint.

5.2 Reproduction

There are two essential duties of genetic operators: to make better the generation and to modify the generation. Reproduction is the unique operator to make better the generation. The worst solutions are taken out of the generation and the fittest ones are copied instead of them by reproduction operator. The solutions, which will be affected by reproduction, are decided according to their fitness factor; and, the fitness factor of the i^{th} solution is calculated as

$$f_{c,i} = f_i / f_a \tag{24}$$

where, f_i is the fitness value of the i^{th} solution; f_a is the average of the fitness values of all solutions in the generation. The fitness value of each solution is also calculated with the equation given below.

$$f_i = (\Phi_{\max} + \Phi_{\min}) - \Phi_i \tag{25}$$

In this equation, Φ_i is the value of the penalized objective function for i^{th} solution; $\Phi_{\max}$ and $\Phi_{\min}$ are the maximum and minimum values of the penalized objective functions in the generation, respectively (Rajeev and Krishnamoorthy 1992).

5.3 Crossover

The crossover operator is used to create the new solutions depending on the existing solutions. In crossover operator, some characters of strings are exchanged between the mated solutions in the mating pool that is obtained by reproduction operator. Mated two solutions are generally named as parent solutions. There are different types of crossover operator like uniform crossover, single point crossover and multipoint crossover.

It is demonstrated in the study of Aydın and Ayvaz (2010) that better solution can be reached in less iteration using uniform crossover instead of the one-point crossover or the two-point crossover. Therefore, the uniform crossover is preferred in this study, in which a crossover mask (randomly determined an extra binary string) is used for each crossover operation. An example for the application of the uniform crossover to twelve-character solution strings is given in Table 2.

5.4 Mutation

The mutation operator is also used to modify the solutions in the generation. The values of a predetermined number of the randomly selected characters in the population are changed by the mutation. The mutation rate is generally decided between 1.0%~0.1%. Different types of the mutation operator are used in the literature. The controlled mutation is preferred in this paper because of its speed to reach the optimum solution in less iteration. In the controlled mutation, the initially decided mutation rate is reduced according to providing of convergence (Aydın and Ayvaz 2010). In this study, the mutation rate is reduced by 50% when one-third of the convergence is provided; and, the mutation is ended when two-thirds of the convergence is provided.

Table 2 Uniform crossover

Parent solutions	Crossover mask	New solutions
1 0 0 1 1 0 1 1 1 0 0 1	0 0 1 1 0 0 0 1 0 0 1 1	1 0 **1 0** 1 0 1 **0** 1 0 **0 0**
1 1 1 0 1 0 1 0 1 1 0 0		1 1 **0 1** 1 0 1 **1** 1 1 **0 1**

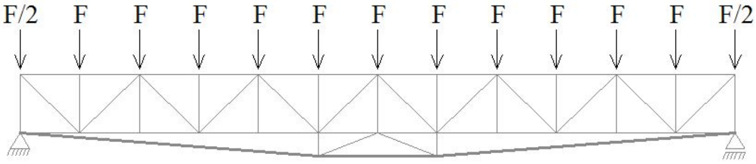

Fig. 4 External load scheme

Detailed information about the genetic operators can be found in the literature (Aydın and Ayvaz 2010, Dede *et al*. 2011).

6. Numerical examples

A parallel-chord prestressed truss, which is described in Section 2, is handled as an example to evaluate the approach used in this study. The handled truss is optimized for three different span lengths (60 m, 80 m and 100 m) and for five different tendon eccentricities (0.5 m, 1.0 m, 1.5 m, 2.0 m and 2.5 m). Non-prestressed optimizations of the trusses are also realized in order to compare the results. In addition to self-weight, the total of the external loads (snow, coating and equipment) on the truss is assumed as 1.0 kN/m^2. The concentrated loads (F) are calculated depending on the external loads for the 6 m spacing between girders, and applied to the top nodes of the truss, as shown in Fig. 4. The self-weight of the truss are also applied to all nodes of the truss, proportionally. The values of the other design parameters of the truss are given in Table 3.

Thirty-two circular tube cross-sections are predetermined as the design variable values set for the top, bottom and web bars; and four tube cross-sections are predetermined for the deviator bars. Circular tube cross-sections are selected from the list of DIN 2448. Eight different values are also predetermined for the height of the truss. The design variable value sets for all of the six design variables are shown in Table 4.

Five-character binary strings must be used to encode the values of design variables for top, bottom, vertical and diagonal bars. Similarly, a two-character string and a four-character string must be used for deviator bar and the height of truss, respectively. Therefore, a twenty-six-character binary string is used to represent each solution. The other optimum design parameters for the genetic algorithm are determined as follows:

- The number of solution in generation : 30
- Penalty coefficient : 0.5~1.0
- Convergence criterion : 60%
- Crossover type : Uniform
- Mutation type : Controlled
- Rate of mutation : 0.4%
- Maximum number of iteration : 300

The values of the optimum design parameters used in this study are decided according to the previous studies in which the effects of different optimization parameters (different types of genetic operator, population size, number of value in design variable values sets, etc.) are investigated. Several GA runs are utilized to determine the appropriate penalty coefficient for each

Table 3 Values of the design parameters

Design parameter	Unit	Value
Yield stress of structural steel	MPa	235
Ultimate stress of structural steel	MPa	340
Ultimate stress of prestressing steel	MPa	1860
Unit cost of structural steel	$/kN	240
Unit cost of prestressing steel	$/kN	950
Elasticity modulus of steel	MPa	207300
Unit weight of steel	kN/m^3	78.5
Diameter of a tendon	mm	15.24

solution. It can be never argued that the solutions obtained by a genetic algorithm are the best. There may be a better solution obtained in the case of using different parameters for the genetic algorithm.

The value of the external concentrated load (F) is calculated as 30 kN, 40 kN and 50 kN, for 60 m, 80 m and 100 m span lengths, respectively. Six optimum solutions are produced for each span length, considering the five tendon eccentricities and non-prestressed truss. The values of the design variables, the material weights and the costs of these optimum solutions are given in Tables 5-7. The values of the penalty functions of all these solutions are calculated to be zero; it means that none of the constraints is violated.

It is seen from Tables 5-7 that the minimum costs are determined as 18,851 $, 35,286 $ and 62,216 $ for the 60 m, 80 m and 100 m span trusses, respectively. Accordingly, the total weights of structural materials are calculated as 74.698 kN, 139.347 kN and 243.247 kN for the 60 m, 80 m and 100 m span trusses, respectively. These minimum costs and minimum weights are achieved at 2.5 m tendon eccentricity for all of three span lengths.

Table 4 Design variable values sets

Design variable	Number of values	Predetermined values							
Top bar Bottom bar Vertical bar Diagonal bar (*Tube cross-section*, *mm*)	32	139.7×6.3 139.7×7.1 152.4×7.1 159.0×7.1 168.3×7.1 177.8×7.1 193.7×7.1 219.1×6.3		219.1×7.1 219.1×8.0 219.1×8.8 244.5×8.0 244.5×8.8 273.0×8.0 244.5×10.0 244.5×11.0		273.0×10.0 323.9×8.8 355.6×8.0 355.6×8.8 323.9×10.0 323.9×11.0 355.6×10.0 406.4×8.8		355.6×11.0 406.4×10.0 355.6×12.5 406.4×11.0 355.6×14.2 406.4×12.5 355.6×16.0 406.4×14.2	
Deviator bar (*Tube cross-section*, *mm*)	4	76.1×5.0		101.6×5.0		114.3×5.0		139.7×5.0	
Height of truss (*m*)	16	3.0 3.5	4.0 4.5	5.0 5.5	6.0 6.5	7.0 7.5	8.0 8.5	9.0 9.5	10.0 10.5

Table 5 Results of 60 m span truss

	No prestress	Tendon eccentricity (m)				
		0.5	1.0	1.5	2.0	2.5
Top bar	244.5/8.0	244.5/8.0	219.1/8.0	219.1/8.0	219.1/7.1	219.1/7.1
Bottom bar	219.1/8.0	168.3/7.1	159.0/7.1	159.0/7.1	159.0/7.1	177.8/7.1
Vertical bar	139.7/6.3	139.7/6.3	139.7/6.3	139.7/6.3	139.7/6.3	139.7/6.3
Diagonal bar	168.3/7.1	168.3/7.1	168.3/7.1	159.0/7.1	159.0/7.1	152.4/7.1
Deviator bar	-	76.1/5.0	76.1/5.0	101.6/5.0	101.6/5.0	101.6/5.0
Truss height (m)	5.0	5.0	5.0	5.0	5.0	4.5
Struc. steel (kN)	88.682	81.766	77.960	77.136	74.625	73.398
Pres. steel (kN)	-	1.295	1.296	1.297	1.298	1.300
Cost ($)	21,284	20,854	19,941	19,745	19,143	18,851

Table 6 Results of 80 m span truss

	No prestress	Tendon eccentricity (m)				
		0.5	1.0	1.5	2.0	2.5
Top bar	323.9/8.8	323.9/8.8	323.9/8.8	273.0/10.0	323.9/8.8	244.5/10.0
Bottom bar	244.5/10.0	219.1/8.8	219.1/8.8	219.1/8.0	244.5/8.0	219.1/7.1
Vertical bar	193.7/7.1	168.3/7.1	159.0/7.1	168.3/7.1	152.4/7.1	168.3/7.1
Diagonal bar	219.1/8.0	219.1/7.1	219.1/7.1	219.1/7.1	219.1/6.3	219.1/6.3
Deviator bar	-	101.6/5.0	114.3/5.0	114.3/5.0	114.3/5.0	114.3/5.0
Truss height (m)	6.5	6.0	5.5	6.0	5.0	6.5
Struc. steel (kN)	171.763	152.016	148.023	146.650	141.362	136.751
Pres. steel (kN)	-	3.453	3.454	3.456	3.458	2.596
Cost ($)	41,223	39,765	38,807	38,479	37,212	35,286

Table 7 Results of 100 m span truss

	No prestress	Tendon eccentricity (m)				
		0.5	1.0	1.5	2.0	2.5
Top bar	355.6/10.0	406.4/8.8	323.9/11.0	406.4/8.8	323.9/11.0	355.6/8.8
Bottom bar	323.9/8.8	244.5/11.0	273.0/8.0	244.5/11.0	273.0/8.0	273.0/8.0
Vertical bar	244.5/8.0	219.1/8.0	219.1/8.0	219.1/7.1	219.1/7.1	219.1/7.1
Diagonal bar	273.0/10.0	273.0/8.0	273.0/8.0	273.0/8.0	273.0/8.0	273.0/8.0
Deviator bar	-	139.7/5.0	139.7/5.0	139.7/5.0	139.7/5.0	139.7/5.0
Truss height (m)	9.5	8.0	8.5	7.5	8.0	8.0
Struc. steel (kN)	303.678	263.359	256.139	254.638	247.075	237.843
Pres. steel (kN)	-	5.396	5.397	5.398	5.401	5.404
Cost ($)	72,883	68,332	66,600	66,242	64,429	62,216

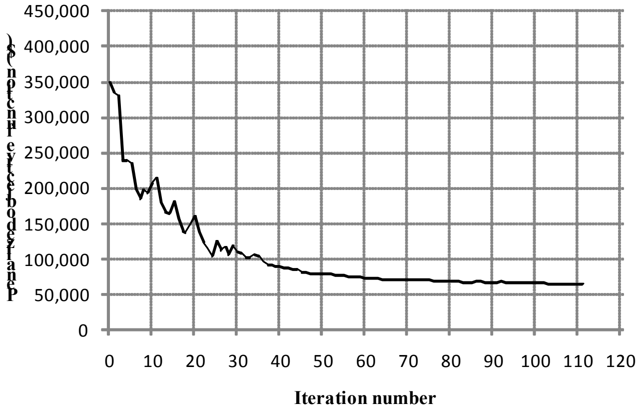

Fig. 5 Convergence history for the average of all solutions in generation (L = 100 m, e = 2.5 m)

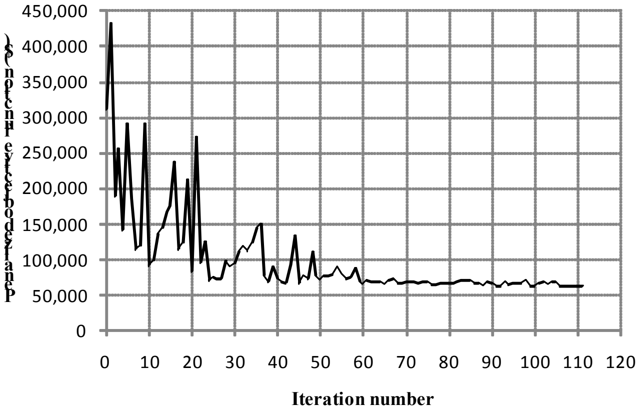

Fig. 6 Convergence history for the first solution in generation (L = 100 m, e = 2.5 m)

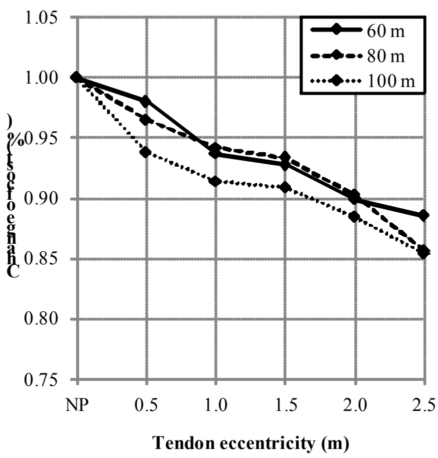

Fig. 7 Proportional change of cost versus tendon eccentricity

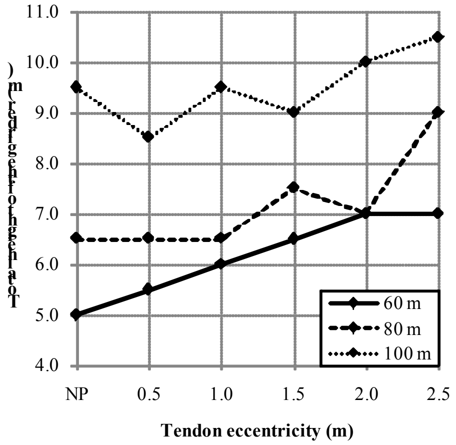

Fig. 8 The total height of the girder versus tendon eccentricity

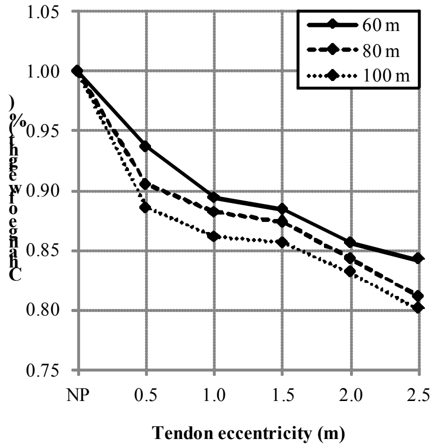

Fig. 9 Proportional decrease in the weight versus tendon eccentricity

Due to lack of space, only one design is selected among eighteen designs to demonstrate the convergence performance of the used algorithm. So, convergence history of penalized objective function (Φ) for the last design (100 m span and 2.5 m eccentricity) are illustrated in Figs. 5 and 6 for the average of all solutions in generation and for the first solution of generation, respectively.

It is seen from these figures that there are reverse jumps in these convergence curves, as expected. The crossover and mutation operators cause these reverse motions. The jumps are bigger especially at the preliminary iterations depending on the initial mutation rate 0.4%. The mutation rate reduces the 0.2% at the 63th iteration, and it ends at the 89th iteration, according to providing of convergence. On the other hand, there is a general convergence supplied by reproduction operator. The optimum solution is obtained at 111th iteration, which is a success of the genetic algorithm. Similarly, the optimum solutions are obtained within less than 150 iterations in most of the other seventeen designs.

Proportional change in the cost of prestressed truss compared to the cost of the non-prestressed (NP) truss is given in Fig. 7 for all of three span lengths. From this figure, the cost of the truss is saved up to 11.4%, 14.4% and 14.6% for the 60 m, 80 m and 100 m span trusses, respectively, by applying prestressing. Furthermore, the amount of the cost saving in prestressed trusses is greater in the case of longer span.

It can also be seen from Fig. 7 that the total cost of truss decreases while the eccentricity of prestressing tendon increases. On the other hand, the total height of the truss (including deviators) mostly increases depending on the eccentricity of the tendon (see Fig. 8), which is not desired.

Change in the total weights of prestressed trusses is given in Fig. 9 as proportionally compared to non-prestressed (NP) one. The total weights of the structural materials decrease up to 15.8%, 18.9% and 19.9% for the 60 m, 80 m and 100 m span trusses, respectively. In considered prestressed steel trusses, the amount of the material saving is greater than those of the cost saving.

In this paper, size and shape optimization of a prestressed steel truss is realized, simultaneously. A parallel-chord simply supported truss girder is selected as example. It is assumed that the prestressing tendon is settled under the bottom bar between two end supports, and it is formed by using deviators. External loads and self-weight of the truss is taken into account in analysis and design. The objective function of optimization problem is determined as the total cost of the truss. Stress, slenderness and deflection constraints are considered. Design variables are cross-sections of the truss members as size variable, and the height of the truss as shape variable. A computer program is coded using the genetic algorithm. Optimum design of the selected truss is performed for three different span lengths and five different tendon eccentricities. Conclusions drawn from this study and some recommendations are given below.

It is difficult to formulate handled optimization problem using classical optimization methods; however, it is solved easily by the genetic algorithm and most of solutions are obtained less than 150 iterations. Genetic algorithm uses discrete design variables like most of the other artificial intelligence based algorithms; thus, the optimum solutions obtained in this study can be constructed without any modification.

The total cost is saved up to 14.6% and the total weight of structural materials reduced up to 19.9% by using prestressing in trusses. These are considerable percentages, and according to this viewpoint, prestressing should be applied more extensively to steel trusses. The amount of decrease in material weight is much more than one in the cost; therefore, it can be predicted that prestressed trusses will be more useful in the case of higher raw material prices. The cost saving increases in the case of longer spans, as expected; and, it is proved one more time that prestressing must be remembered especially for long-span trusses. Tendon eccentricity is considered from 0.5 m to 2.5 m; the total cost of truss decreases while the eccentricity of prestressing tendon increases. On the other hand, the total height of the truss (including deviators) mostly increases depending on the eccentricity of the tendon. Therefore, optimum tendon eccentricity can be investigated as further, considering the total height of the truss as a constraint. This study includes a specific truss topology and fixed tendon geometry. It will be useful to expand the study for different type of trusses and different tendon configurations using the coded program.

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