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On vibration properties of functionally graded nano-plate using a new nonlocal refined four variable model

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Abstract. In this paper, a new nonlocal hyperbolic refined plate model is presented for free vibration properties of functionally graded (FG) plates. This nonlocal nano-plate model incorporates the length scale parameter which can capture the small scale effect. The displacement field of the present theory is chosen based on a hyperbolic variation in the in-plane displacements through the thickness of the nano-plate. By dividing the transverse displacement into the bending and shear parts, the number of unknowns and equations of motion of the present theory is reduced, significantly facilitating structural analysis. The material properties are assumed to vary only in the thickness direction and the effective properties for the FG nano-plate are computed using Mori–Tanaka homogenization scheme. The governing equations of motion are derived based on the nonlocal differential constitutive relations of Eringen in conjunction with the refined four variable plate theory via Hamilton's principle. Analytical solution for the simply supported FG nano-plates is obtained to verify the theory by comparing its results with other available solutions in the open literature. The effects of nonlocal parameter, the plate thickness, the plate aspect ratio, and various material compositions on the dynamic response of the FG nano-plate are discussed.

Keywords: nonlocal elasticity theory; nano-plates; free vibration; refined plate theory; functionally graded materials

1. Introduction

Nanotechnology is able to create functionally graded materials and engineering structures at a nanoscale, which enables a new generation of materials with revolutionary properties and devices with enhanced functionality. One of these structures is the nano-scale plates, which have attracted attention of scientific community in solid-state physics, materials science, and nano-electronics due to their superior mechanical, chemical and electronic properties. The understanding of

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mechanical behavior of nano-plate is essential in developing of such structures due to their great potential engineering applications.

Both experimental and atomistic simulation results have proved a significant size influence in mechanical properties when the dimensions of these structures become very small. For this reason, the size influence has a considerable role on static and dynamic response of micro, nanostructures and cannot be ignored. It is well-known that classical continuum mechanics does not account for such size influences in micro-, nano-scale structures. In order to overcome this problem, many nonlocal theories that consider additional material constants, such as the strain gradient theory (Aifantis 1999), the micropolar theory (Eringen 1967), and the nonlocal elasticity theory (Eringen 1972) have been developed to characterize the size effect in micro, nano-scale structures by introducing an intrinsic length scale in the constitutive relations. Among these theories, the nonlocal elasticity theory, which was developed by Eringen (1983) to account for scale effect in elasticity, was employed to investigate lattice dispersion of elastic waves, wave propagation in composites, dislocation mechanics, fracture mechanics and surface tension fluids. After this, Peddieson *et al*. (2003) first applied the nonlocal continuum theory to the nanotechnology in which the static deformations of beam structures were obtained by using a simplified nonlocal beam model based on the nonlocal elasticity theory of Eringen (1983). Xu (2006) presented the integral equation approach and the non-local elasticity theory to investigate the free vibration of nano-to-micron scale beams. Reddy (2007) reformulated local beam theory by using the nonlocal differential constitutive relations of Eringen to study bending, vibration, and buckling behaviors of nanobeams. Analytical solutions are obtained to bring out the effect of the nonlocal behavior of nanobeams. Benzair *et al*. (2008) investigated the thermal effect on frequency of single walled carbon nanotubes using nonlocal Timoshenko beam model. Heireche *et al*. (2008a) applied nonlocal Timoshenko beam models to the studies of wave properties of single walled carbon nanotubes. Tounsi *et al*. (2013a) investigated the thermal buckling properties of double-walled carbon nanotubes (DWCNTs) using also nonlocal Timoshenko beam model, including the effects of transverse shear deformation and rotary inertia. Tounsi *et al*. (2013b) proposed an efficient higher-order nonlocal beam theory for the thermal buckling of nanobeams. Their model is capable of capturing both the small-scale effect and transverse shear deformation effects of nanobeams, and it has strong similarities with the nonlocal Euler–Bernoulli beam theory in aspects such as equations of motion, boundary conditions, and stress resultant expressions. A unified nonlocal shear deformation theory is proposed by Berrabah *et al*. (2013) to study bending, buckling, and free vibration of nanobeams based on Eringen model. Recently, Tounsi *et al*. (2013c) developed a new nonlocal thickness-stretching sinusoidal shear deformation beam theory for the static and vibration of nanobeams. Benguediab *et al*. (2014) investigated the chirality and scale effects on mechanical buckling properties of zigzag double-walled carbon nanotubes.

With the rapid development of technology, functionally graded (FG) beams and plates have been started to use in micro/nanoelectromechanical systems (MEMS/NEMS), such as the components in the form of shape memory alloy thin films with a global thickness in micro- or nano-scale (Fu *et al*. 2003, Witvrouw and Mehta 2005, Lü *et al*. 2009), electrically actuated MEMS devices (Hasanyan *et al*. 2008, Mohammadi-Alasti *et al*. 2011, Zhang and Fu 2012), and atomic force microscopes (AFMs) (Rahaeifard *et al*. 2009). Since the dimension of these structural devices typically falls below micron- or nanoscale in at least one direction, an essential feature triggered in these devices is that their mechanical properties such as Young’s modulus and flexural rigidity are size-dependent. So far, only a few works have been reported for FG nanostructures based on the nonlocal elasticity theory. Janghorban and Zare (2011) investigated nonlocal free vibration axially FG nanobeams by using differential quadrature method. Daneshmehr *et al*. (2014) presented a nonlocal higher order plate theory for stability analysis of FG nanoplates subjected to biaxial in plane loadings. Recently, Larbi Chaht *et al*. (2015) studied the bending and buckling response of FG size-dependent nanoscale beams including the thickness stretching effect.

In the current study, free vibration characteristics of FG nano-scale plates are studied using a new nonlocal hyperbolic refined plate theory. The partition of the transverse displacement into the bending and shear parts leads to a reduction of the number of unknowns, and subsequently, makes the new theory simple to use. In addition, the small scale effect is taken into account by using the nonlocal constitutive relations of Eringen. The effects of nonlocal parameter, aspect ratio, various material compositions on the free vibration responses of the FG nano-plate are discussed. Some illustrative examples are also presented to verify the present formulation and solutions. Good agreement is observed.

2. Theoretical formulation

2.1 Functionally graded material

Fig. 1 shows a functionally graded (FG) rectangular nano-plate of length a, width b, and thickness h. The material on the top surface ($z = +h/2$) of the plate is ceramic and is graded to metal at the bottom surface of the plate ($z = -h/2$) by a power law distribution. According to Mori–Tanaka homogenization scheme, the effective Bulk Modulus (K) and the effective shear modulus (G) are given by (Belabed *et al*. 2014, Valizadeh *et al*. 2013, Cheng and Batra 2000, Qian *et al*. 2004)

$$\frac{K-K_m}{K_C-K_m}=\frac{V_C}{1+\left(1-V_C\right)\dfrac{3\left(K_C-K_m\right)}{3K_m+4G_m}} \tag{1a}$$

$$\frac{G-G_m}{G_C-G_m}=\frac{V_C}{1+\left(1-V_C\right)\dfrac{\left(G_C-G_m\right)}{G_m+f_1}} \tag{1b}$$

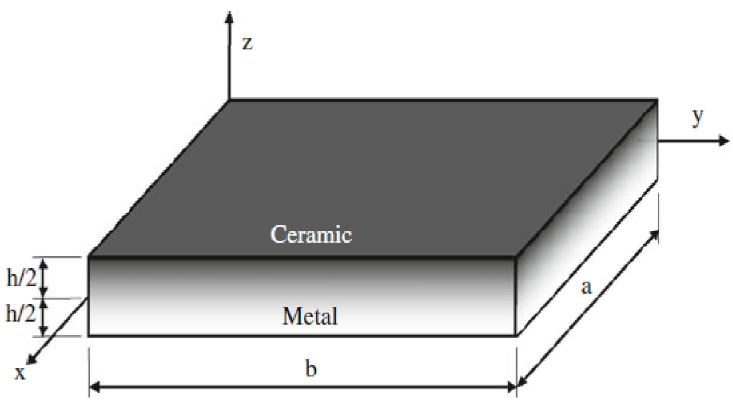


Fig. 1 Schematic representation of a rectangular FG plate

where

$$f_1 = \frac{G_m(9K_m + 8G_m)}{6(K_m + 2G_m)} \tag{2}$$

Here, V_i ($i = c$, m) is the volume fraction of the phase material. The subscripts c and m refer to the ceramic and metal phases, respectively. The volume fractions of the ceramic and metal phases are related by $V_C + V_m = 1$, and V_C is expressed as

$$V_C(z) = \left(\frac{2z + h}{2h}\right)^n, \qquad n \geq 0 \tag{3}$$

where n in Eq. (3) is the volume fraction exponent, also referred to as the gradient index. Fig. 2 shows the variation of the volume fraction of the ceramic phase in the thickness direction z for the FG plate. The effective Young's modulus E and Poisson's ratio ν can be computed from the following expressions

$$E = \frac{9KG}{3K + G} \tag{4a}$$

$$\nu = \frac{3K - 2G}{2(3K + G)} \tag{4b}$$

The effective mass density ρ is given by the rule of mixtures as (Natarajan *et al*. 2011, Benachour *et al*. 2011, Bessaim *et al*. 2013, Yaghoobi and Torabi 2013, Tounsi *et al*. 2013d, Ould Larbi *et al*. 2013, Bouremana *et al*. 2013, Hebali *et al*. 2014)

$$\rho = \rho_C V_C + \rho_m V_m \tag{5}$$

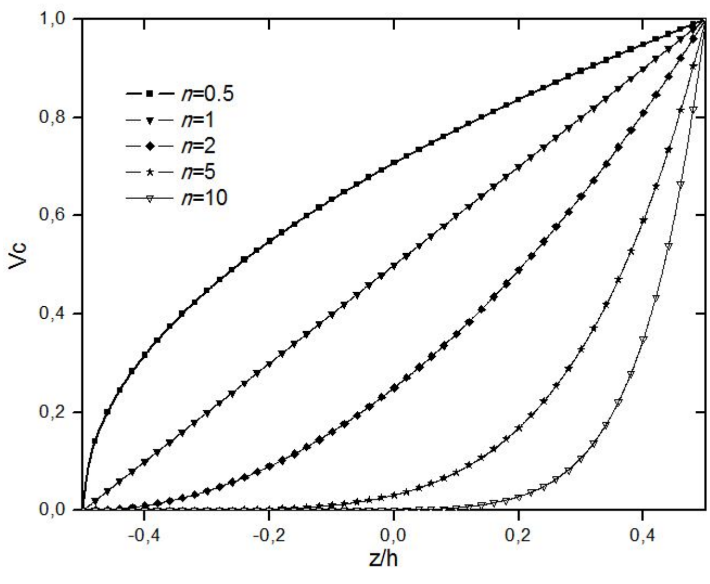


Fig. 2 Variation of ceramic phase through the thickness of the plate

2.2 Review of nonlocal elasticity

According to Eringen (1972, 1983), the stress field at a point x in an elastic continuum not only depends on the strain field at the point (hyperelastic case) but also on strains at all other points of the body. Eringen attributed this fact to the atomic theory of lattice dynamics and experimental observations on phonon dispersion. Thus, the nonlocal stress tensor components σ at point $\boldsymbol{x}$ are expressed as

$$\sigma = \int_V \alpha\left(|x' - x|, \tau\right) t(x') dx' \tag{6}$$

where $t(x)$ are the components of the classical macroscopic stress tensor at point $\boldsymbol{x}$ and the kernel function $\alpha(|x' - x|, \tau)$ represents the nonlocal modulus, $|x' - x|$ being the distance (in Euclidean norm) and τ is a material constant that depends on internal and external characteristic lengths (such as the lattice spacing and wavelength, respectively). Eringen (1972, 1983) numerically determined the functional form of the kernel. By appropriate choice of the kernel function, Eringen (1983) showed that the nonlocal constitutive equation given in integral form (see Eq. (6)) can be represented in an equivalent differential form as

$$(1 - \tau^2 L^2 \nabla^2)\sigma = t, \qquad \tau^2 = \frac{\mu}{L^2} = \left(\frac{e_0 \overline{a}}{L}\right)^2 \tag{7}$$

where $\mu = (e_0 \overline{a})$, e_0 is a material constant and $\overline{a}$ and L are the internal and external characteristic lengths, respectively.

2.3 Four variable plate theory

Recently, using a new four variables refined plate theory against five in case of other shear deformation theories, Tounsi and his co-workers (Ait Yahia *et al.* 2015, Ait Amar Meziane *et al.* 2014, Draiche *et al.* 2014, Klouche Djedid *et al.* 2014, Nedri *et al.* 2014, Zidi *et al.* 2014, Sadoune *et al.* 2014, Tounsi *et al.* 2013d, Bachir Bouiadjra *et al.* 2013, Bouderba *et al.* 2013, Kettaf *et al.* 2013, Bachir Bouiadjra *et al.* 2012, Bourada *et al.* 2012, El Meiche *et al.* 2011) studied a series of buckling, bending and vibration behavior of functionally graded plate and laminated plate. In the present work, a new nonlocal hyperbolic four variables plate theory is presented.

2.3.1 Kinematics

The displacement field of the present theory is chosen based on the following assumptions: (1) the in-plane and transverse displacements are partitioned into bending and shear components; (2) the bending parts of the in-plane displacements are similar to those given by the classical plate theory (CPT); and (3) the shear parts of the in-plane displacements give rise to the hyperbolic variations of shear strains and hence to shear stresses through the thickness of the plate in such a way that the shear stresses vanish on the top and bottom surfaces of the plate. Based on these assumptions, the following displacement field can be obtained

$$u(x, y, z, t) = u_0(x, y, t) - z\frac{\partial w_b}{\partial x} - f(z)\frac{\partial w_s}{\partial x} \tag{8a}$$

$$v(x,y,z,t) = v_0(x,y,t) - z\frac{\partial w_b}{\partial y} - f(z)\frac{\partial w_s}{\partial y} \tag{8b}$$

$$w(x,y,z,t) = w_b(x,y,t) + w_s(x,y,t) \tag{8c}$$

with

$$f(z) = \frac{h\sinh\left(\dfrac{10z}{h}\right)}{10\cosh(5)} - \frac{h}{100} \tag{8d}$$

here u_0 and v_0 denote the displacements along the x and y coordinate directions of a point on the midplane of the plate; w_b and w_s are the bending and shear components of the transverse displacement, respectively; and h is the plate thickness. The nonzero strains associated with the displacement field in Eq. (8) are

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + f(z)\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix}, \qquad \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g(z)\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix}, \tag{9}$$

where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial x} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_b}{\partial x^2} \\ -\dfrac{\partial^2 w_b}{\partial y^2} \\ -2\dfrac{\partial^2 w_b}{\partial x \partial y} \end{Bmatrix}, \quad \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_s}{\partial x^2} \\ -\dfrac{\partial^2 w_s}{\partial y^2} \\ -2\dfrac{\partial^2 w_s}{\partial x \partial y} \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial w_s}{\partial y} \\ \dfrac{\partial w_s}{\partial x} \end{Bmatrix}, \tag{10a}$$

and

$$g(z) = 1 - f'(z) \tag{10b}$$

2.3.2 Constitutive relations

The nonlocal constitutive Eq. (7) has been recently employed for the study of micro and nanostructural elements. However, these works are mainly limited to one-dimensional problems (CNTs, micro/nano beams, etc.) (Peddieson *et al.* 2003, Xu 2006, Reddy and Pang 2008, Heireche 2008a, b, c, Tounsi *et al.* 2008, 2013a, b, c, Berrabah *et al.* 2013). From Eq. (7) the two-dimensional nonlocal constitutive relations for elastic FG nano-plate can be expressed as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} - \mu\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{22} & 0 & 0 & 0 \\ 0 & 0 & C_{66} & 0 & 0 \\ 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & C_{44} \end{bmatrix}\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{11}$$

where (σ_x, σ_y, τ_{xy}, τ_{yz}, τ_{yx}) and (ε_x, ε_y, γ_{xy}, γ_{yz}, γ_{xz}) are the stress and strain components, respectively. Using the material properties defined in Eq. (4), stiffness coefficients, C_{ij}, can be expressed as

$$C_{11} = C_{22} = \frac{E(z)}{1-\nu(z)^2}, \tag{12a}$$

$$C_{12} = \frac{\nu\, E(z)}{1-\nu(z)^2}, \tag{12b}$$

$$C_{44} = C_{55} = C_{66} = \frac{E(z)}{2[1+\nu(z)]}, \tag{12c}$$

2.3.3 Equations of motion

Hamilton's principle is used herein to derive the equations of motion. The principle can be stated in analytical form as (Reddy 2007)

$$0 = \int_0^t (\delta U - \delta K)\, dt \tag{13}$$

where δU is the variation of strain energy; and δK is the variation of kinetic energy.

The variation of strain energy of the plate stated as

$$\begin{aligned}\delta U &= \int_V \left[\sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \tau_{xy}\delta\gamma_{xy} + \tau_{yz}\delta\gamma_{yz} + \tau_{zx}\delta\gamma_{zx}\right] dA dz \\ &= \int_A \begin{bmatrix} N_x\delta\varepsilon_x^0 + N_y\delta\varepsilon_y^0 + N_{xy}\delta\varepsilon_{xy}^0 + M_x^b\delta k_x^b + M_y^b\delta k_y^b + M_{xy}^b\delta k_{xy}^b \\ + M_x^s\delta k_x^s + M_y^s\delta k_y^s + M_{xy}^s\delta k_{xy}^s + S_{yz}^s\delta\gamma_{yz}^s + S_{xz}^s\delta\gamma_{xz}^s \end{bmatrix} dA\end{aligned} \tag{14}$$

where the stress resultants N, M, and S are defined by

$$\left(N_i, M_i^b, M_i^s\right) = \int_{-h/2}^{h/2} (1, z, f)\sigma_i dz, \quad (i = x, y, xy) \quad \text{and} \quad S_i = \int_{-h/2}^{h/2} g\sigma_i dz, \quad (i = xz, yz) \tag{15}$$

The variation of kinetic energy of the plate is expressed as

$$\begin{aligned}\delta K &= \int_{-h/2}^{h/2}\int_A \left[\dot{u}\delta\dot{u} + \dot{v}\delta\dot{v} + \dot{w}\delta\dot{w}\right]\rho(z)\, dA\, dz \\ &= \int_A \{I_0\left[\dot{u}_0\delta\dot{u}_0 + \dot{v}_0\delta\dot{v}_0 + \left(\dot{w}_b + \dot{w}_s\right)\left(\delta\dot{w}_b + \delta\dot{w}_s\right)\right] \\ &\quad - I_1\left(\dot{u}_0\frac{\partial\delta\dot{w}_b}{\partial x} + \frac{\partial\dot{w}_b}{\partial x}\delta\dot{u}_0 + \dot{v}_0\frac{\partial\delta\dot{w}_b}{\partial y} + \frac{\partial\dot{w}_b}{\partial y}\delta\dot{v}_0\right)\end{aligned} \tag{16}$$

↓

$$
\begin{aligned}
&\uparrow \\
&-J_1\left(\dot{u}_0\frac{\partial\delta\dot{w}_s}{\partial x}+\frac{\partial\dot{w}_s}{\partial x}\delta\,\dot{u}_0+\dot{v}_0\frac{\partial\delta\dot{w}_s}{\partial y}+\frac{\partial\dot{w}_s}{\partial y}\delta\,\dot{v}_0\right) \\
&+I_2\left(\frac{\partial\dot{w}_b}{\partial x}\frac{\partial\delta\,\dot{w}_b}{\partial x}+\frac{\partial\dot{w}_b}{\partial y}\frac{\partial\delta\,\dot{w}_b}{\partial y}\right)+K_2\left(\frac{\partial\dot{w}_s}{\partial x}\frac{\partial\delta\,\dot{w}_s}{\partial x}+\frac{\partial\dot{w}_s}{\partial y}\frac{\partial\delta\,\dot{w}_s}{\partial y}\right) \\
&\left.+J_2\left(\frac{\partial\dot{w}_b}{\partial x}\frac{\partial\delta\,\dot{w}_s}{\partial x}+\frac{\partial\dot{w}_s}{\partial x}\frac{\partial\delta\,\dot{w}_b}{\partial x}+\frac{\partial\dot{w}_b}{\partial y}\frac{\partial\delta\,\dot{w}_s}{\partial y}+\frac{\partial\dot{w}_s}{\partial y}\frac{\partial\delta\,\dot{w}_b}{\partial y}\right)\right\}dA
\end{aligned}
\tag{16}
$$

where dot-superscript convention indicates the differentiation with respect to the time variable t; and (I_0, I_1, J_1, I_2, J_2, K_2) are mass inertias defined as

$$
\left(I_0,I_1,J_1,I_2,J_2,K_2\right)=\int_{-h/2}^{h/2}\left(1,z,f,z^2,z\,f,f^2\right)\rho(z)dz \tag{17}
$$

Substituting the expressions for δU and δK from Eqs. (14) and (16) into Eq. (13) and integrating by parts, and collecting the coefficients of δu_0, δv_0, δw_b and δw_s, the following equations of motion of the proposed beam theory are obtained

$$
\begin{aligned}
&\delta\,u_0:\ \frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=I_0\ddot{u}_0-I_1\frac{\partial\ddot{w}_b}{\partial x}-J_1\frac{\partial\ddot{w}_s}{\partial x} \\
&\delta\,v_0:\ \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=I_0\ddot{v}_0-I_1\frac{\partial\ddot{w}_b}{\partial y}-J_1\frac{\partial\ddot{w}_s}{\partial y} \\
&\delta\,w_b:\ \frac{\partial^2M_x^b}{\partial x^2}+2\frac{\partial^2M_{xy}^b}{\partial x\partial y}+\frac{\partial^2M_y^b}{\partial y^2}=I_0\left(\ddot{w}_b+\ddot{w}_s\right)+I_1\left(\frac{\partial\ddot{u}_0}{\partial x}+\frac{\partial\ddot{v}_0}{\partial y}\right)-I_2\nabla^2\ddot{w}_b-J_2\nabla^2\ddot{w}_s \\
&\delta\,w_s:\ \frac{\partial^2M_x^s}{\partial x^2}+2\frac{\partial^2M_{xy}^s}{\partial x\partial y}+\frac{\partial^2M_y^s}{\partial y^2}+\frac{\partial S_{xz}^s}{\partial x}+\frac{\partial S_{yz}^s}{\partial y} \\
&\qquad=I_0\left(\ddot{w}_b+\ddot{w}_s\right)+J_1\left(\frac{\partial\ddot{u}_0}{\partial x}+\frac{\partial\ddot{v}_0}{\partial y}\right)-J_2\nabla^2\ddot{w}_b-K_2\nabla^2\ddot{w}_s
\end{aligned}
\tag{18}
$$

Substituting Eqs. (10) and (11) into Eq. (15) and integrating through the thickness of the plate, the stress resultants are related to the generalized displacements (u_0, v_0, w_b, w_s) by the relations

$$
\begin{Bmatrix}N\\M^b\\M^s\end{Bmatrix}-\mu\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\begin{Bmatrix}N\\M^b\\M^s\end{Bmatrix}=\begin{bmatrix}A&B&B^s\\B&D&D^s\\B^s&D^s&H^s\end{bmatrix}\begin{Bmatrix}\varepsilon\\k^b\\k^s\end{Bmatrix},\qquad S-\mu\left(\frac{\partial^2S}{\partial x^2}+\frac{\partial^2S}{\partial y^2}\right)=A^s\gamma, \tag{19}
$$

where

$$
N=\left\{N_x,N_y,N_{xy}\right\}^t,\quad M^b=\left\{M_x^b,M_y^b,M_{xy}^b\right\}^t,\quad M^b=\left\{M_x^b,M_y^b,M_{xy}^b\right\}^t, \tag{20a}
$$

$$\varepsilon = \left\{\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0\right\}^t, \quad k^b = \left\{k_x^b, k_y^b, k_{xy}^b\right\}^t, \quad k^s = \left\{k_x^s, k_y^s, k_{xy}^s\right\}^t, \tag{20b}$$

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}, \quad B = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix}, \quad D = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix}, \tag{20c}$$

$$B^s = \begin{bmatrix} B_{11}^s & B_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & B_{66}^s \end{bmatrix}, \quad D^s = \begin{bmatrix} D_{11}^s & D_{12}^s & 0 \\ D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & D_{66}^s \end{bmatrix}, \quad H^s = \begin{bmatrix} H_{11}^s & H_{12}^s & 0 \\ H_{12}^s & H_{22}^s & 0 \\ 0 & 0 & H_{66}^s \end{bmatrix}, \tag{20d}$$

$$S = \left\{S_{xz}^s, S_{yz}^s\right\}^t, \quad \gamma = \left\{\gamma_{xz}, \gamma_{yz}\right\}^t, \quad A^s = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix}, \tag{20e}$$

where A_{ij}, B_{ij}, D_{ij}, etc., are the plate stiffness, defined by

$$\begin{Bmatrix} A_{11} & B_{11} & D_{11} & B_{11}^s & D_{11}^s & H_{11}^s \\ A_{12} & B_{12} & D_{12} & B_{12}^s & D_{12}^s & H_{12}^s \\ A_{66} & B_{66} & D_{66} & B_{66}^s & D_{66}^s & H_{66}^s \end{Bmatrix} = \int_{-h/2}^{h/2} C_{11}\left(1, z, z^2, f(z), z\,f(z), f^2(z)\right) \begin{Bmatrix} 1 \\ \nu \\ \dfrac{1-\nu}{2} \end{Bmatrix} dz, \tag{21a}$$

$$\left(A_{22}, B_{22}, D_{22}, B_{22}^s, D_{22}^s, H_{22}^s\right) = \left(A_{11}, B_{11}, D_{11}, B_{11}^s, D_{11}^s, H_{11}^s\right), \tag{21b}$$

$$A_{44}^s = A_{55}^s = \int_{-h/2}^{h/2} C_{44}\left[g(z)\right]^2 dz, \tag{21c}$$

Substituting from Eq. (19) into Eq. (18), we obtain the following equation

$$\begin{aligned} & A_{11}\frac{\partial^2 u_0}{\partial x^2} + A_{66}\frac{\partial^2 u_0}{\partial y^2} + \left(A_{12} + A_{66}\right)\frac{\partial^2 v_0}{\partial x \partial y} \\ & - B_{11}\frac{\partial^3 w_b}{\partial x^3} - \left(B_{12} + 2B_{66}\right)\frac{\partial^3 w_b}{\partial x \partial y^2} - B_{11}^s\frac{\partial^3 w_s}{\partial x^3} - \left(B_{12}^s + 2B_{66}^s\right)\frac{\partial^3 w_s}{\partial x \partial y^2} \\ & = \left(1 - \mu\nabla^2\right)\left[I_0\ddot{u}_0 - I_1\frac{\partial \ddot{w}_b}{\partial x} - J_1\frac{\partial \ddot{w}_s}{\partial x}\right] \end{aligned} \tag{22a}$$

$$\begin{aligned} & \left(A_{12} + A_{66}\right)\frac{\partial^2 u_0}{\partial x \partial y} + A_{66}\frac{\partial^2 v_0}{\partial x^2} + A_{22}\frac{\partial^2 v_0}{\partial y^2} \\ & - \left(B_{12} + 2B_{66}\right)\frac{\partial^3 w_b}{\partial x^2 \partial y} - B_{22}\frac{\partial^3 w_b}{\partial y^3} - B_{22}^s\frac{\partial^3 w_s}{\partial y^3} - \left(B_{12}^s + 2B_{66}^s\right)\frac{\partial^3 w_s}{\partial x^2 \partial y} \\ & = \left(1 - \mu\nabla^2\right)\left[I_0\ddot{v}_0 - I_1\frac{\partial \ddot{w}_b}{\partial y} - J_1\frac{\partial \ddot{w}_s}{\partial y}\right] \end{aligned} \tag{22b}$$

$$
\begin{aligned}
&B_{11}\frac{\partial^3 u_0}{\partial x^3}+\left(B_{12}+2B_{66}\right)\frac{\partial^3 u_0}{\partial x\partial y^2}+\left(B_{12}+2B_{66}\right)\frac{\partial^3 v_0}{\partial x^2\partial y}+B_{22}\frac{\partial^3 v_0}{\partial y^3}\\
&-D_{11}\frac{\partial^4 w_b}{\partial x^4}-2\left(D_{12}+2D_{66}\right)\frac{\partial^4 w_b}{\partial x^2\partial y^2}-D_{22}\frac{\partial^4 w_b}{\partial y^4}\\
&-D_{11}^s\frac{\partial^4 w_s}{\partial x^4}-2\left(D_{12}^s+2D_{66}^s\right)\frac{\partial^4 w_s}{\partial x^2\partial y^2}-D_{22}^s\frac{\partial^4 w_s}{\partial y^4}\\
&=\left(1-\mu\nabla^2\right)\left[I_0\left(\ddot{w}_b+\ddot{w}_s\right)+I_1\left(\frac{\partial\ddot{u}_0}{\partial x}+\frac{\partial\ddot{v}_0}{\partial y}\right)-I_2\nabla^2\ddot{w}_b-J_2\nabla^2\ddot{w}_s\right]
\end{aligned}
\tag{22c}
$$

$$
\begin{aligned}
&B_{11}^s\frac{\partial^3 u_0}{\partial x^3}+\left(B_{12}^s+2B_{66}^s\right)\frac{\partial^3 u_0}{\partial x\partial y^2}+\left(B_{12}^s+2B_{66}^s\right)\frac{\partial^3 v_0}{\partial x^2\partial y}+B_{22}^s\frac{\partial^3 v_0}{\partial y^3}\\
&-D_{11}^s\frac{\partial^4 w_b}{\partial x^4}-2\left(D_{12}^s+2D_{66}^s\right)\frac{\partial^4 w_b}{\partial x^2\partial y^2}-D_{22}^s\frac{\partial^4 w_b}{\partial y^4}\\
&-H_{11}^s\frac{\partial^4 w_s}{\partial x^4}-2\left(H_{12}^s+2H_{66}^s\right)\frac{\partial^4 w_s}{\partial x^2\partial y^2}-H_{22}^s\frac{\partial^4 w_s}{\partial y^4}+A_{44}^s\frac{\partial^2 w_s}{\partial x^2}+A_{55}^s\frac{\partial^2 w_s}{\partial y^2}\\
&=\left(1-\mu\nabla^2\right)\left[I_0\left(\ddot{w}_b+\ddot{w}_s\right)+J_1\left(\frac{\partial\ddot{u}_0}{\partial x}+\frac{\partial\ddot{v}_0}{\partial y}\right)-J_2\nabla^2\ddot{w}_b-K_2\nabla^2\ddot{w}_s\right]
\end{aligned}
\tag{22d}
$$

3. Exact solution for a simply-supported FG plate

Rectangular plates are generally classified according to the type of support used. Here, we are concerned with the exact solutions of Eq. (22) for a simply supported nanoplate. The following boundary conditions are imposed at the side edges

$$v_0=w_b=w_s=\frac{\partial w_s}{\partial y}=N_x=M_x^b=M_x^s=0 \quad \text{at} \quad x=0,\ a \tag{23a}$$

$$u_0=w_b=w_s=\frac{\partial w_s}{\partial x}=N_y=M_y^b=M_y^s=0 \quad \text{at} \quad y=0,\ b \tag{23b}$$

Following the Navier solution procedure, we assume the following solution form for u_0, v_0, w_b and w_s that satisfies the boundary conditions given in Eq. (23)

$$
\begin{Bmatrix} u_0 \\ v_0 \\ w_b \\ w_s \end{Bmatrix}=
\begin{Bmatrix} U_{mn}e^{i\omega t}\cos(\alpha\, x)\sin(\beta\, y) \\ V_{mn}e^{i\omega t}\sin(\alpha\, x)\cos(\beta\, y) \\ W_{bmn}e^{i\omega t}\sin(\alpha\, x)\sin(\beta\, y) \\ W_{smn}e^{i\omega t}\sin(\alpha\, x)\sin(\beta\, y) \end{Bmatrix}
\tag{24}
$$

where U_{mn}, V_{mn}, W_{bmn} and W_{smn} are arbitrary parameters to be determined, ω is the eigenfrequency associated with (m, n)th eigenmode, and $\alpha = m\pi / a$ and $\beta = n\pi / b$.

Substituting Eqs. (24) into Eq. (22), the analytical solutions can be obtained from

$$\left(\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{12} & a_{22} & a_{23} & a_{24} \\ a_{13} & a_{23} & a_{33} & a_{34} \\ a_{14} & a_{24} & a_{34} & a_{44} \end{bmatrix} - \lambda\omega^2 \begin{bmatrix} m_{11} & 0 & 0 & 0 \\ 0 & m_{22} & 0 & 0 \\ 0 & 0 & m_{33} & m_{34} \\ 0 & 0 & m_{34} & m_{44} \end{bmatrix}\right) \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{bmn} \\ W_{smn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{25}$$

where

$$\begin{aligned}
a_{11} &= A_{11}\alpha^2 + A_{66}\beta^2 \\
a_{12} &= \alpha\,\beta\left(A_{12} + A_{66}\right) \\
a_{13} &= -\alpha\,[B_{11}\alpha^2 + (B_{12} + 2B_{66})\,\beta^2] \\
a_{14} &= -\alpha\,[B_{11}^s\alpha^2 + (B_{12}^s + 2B_{66}^s)\,\beta^2] \\
a_{22} &= A_{66}\alpha^2 + A_{22}\beta^2 \\
a_{23} &= -\beta\,[(B_{12} + 2B_{66})\,\alpha^2 + B_{22}\beta^2] \\
a_{24} &= -\beta\,[(B_{12}^s + 2B_{66}^s)\,\alpha^2 + B_{22}^s\beta^2] \\
a_{33} &= D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4 \\
a_{34} &= D_{11}^s\alpha^4 + 2(D_{12}^s + 2D_{66}^s)\alpha^2\,\beta^2 + D_{22}^s\,\beta^4 \\
a_{44} &= H_{11}^s\alpha^4 + 2(H_{12}^s + 2H_{66}^s)\alpha^2\beta^2 + H_{22}^s\beta^4 + A_{55}^s\alpha^2 + A_{44}^s\beta^2 \\
m_{11} &= m_{22} = I_0 \\
m_{33} &= I_0 + I_2\left(\alpha^2 + \beta^2\right) \\
m_{34} &= I_0 + J_2\left(\alpha^2 + \beta^2\right) \\
m_{44} &= I_0 + K_2\left(\alpha^2 + \beta^2\right) \\
\lambda &= 1 + \mu\left(\alpha^2 + \beta^2\right)
\end{aligned} \tag{26}$$

4. Analytical results and discussion

In this part of this work, the size-dependent free vibration response of a simply supported nano-plate made of functionally graded material is investigated. The free vibration analysis is conducted by considering the top surface of the plate is ceramic rich (Si_3N_4) and the bottom surface is metal rich (SUS304). The mass density ρ and the Young's modulus E are: ρ_c = 2,370 kg/m^3, E_c = 348.43e^9 N/m^2 for Si_3N_4 and ρ_m = 8,166 kg/m^3, E_m = 201.04e^9 N/m^2 for SUS304. Poisson's ratio v is considered to be constant and taken as 0.3 for the present work.

In all cases, we present the non-dimensionalized frequency defined as

$$\overline{\omega} = \omega h \sqrt{\frac{\rho_c}{G_c}} \tag{27}$$

where $\overline{\omega}$ is the natural frequency, ρ_c and G_c are the mass density and shear modulus of the ceramic phase, respectively.

In order to validate the present model, some numerical examples are solved to prove the performance in vibrational analysis. For this purpose, we firstly began to investigate simply supported plate for different values of nonlocal parameter, the plate thickness and the plate aspect ratio. The computed results are presented in Table 1 and are compared with those predicted by the third shear deformation theory (TSDT), the first shear deformation theory (FSDT) and the classical plate theory (CPT) developed by Aghababaei and Reddy (2009). The numerical results from the

Table 1 Comparison of fundamental frequency ($\overline{\omega} = \omega h \sqrt{\rho / G}$) of nano-plate ($a$ = 10, E = 30 × 10^6, ρ = 1, ν = 0.3)

a/b	a/h	μ	Present	TSDT[(a)]	FSDT[(a)]	CPT[(a)]
1	10	0	0.0930	0.0935	0.0930	0.0963
		1	0.0850	0.0854	0.0850	0.0880
		2	0.0787	0.0791	0.0788	0.0816
		3	0.0737	0.0741	0.0737	0.0763
		4	0.0695	0.0699	0.0696	0.0720
		5	0.0659	0.0663	0.0660	0.0683
	20	0	0.0238	0.0239	0.0239	0.0241
		1	0.0218	0.0218	0.0218	0.0220
		2	0.0202	0.0202	0.0202	0.0204
		3	0.0189	0.0189	0.0189	0.0191
		4	0.0178	0.0179	0.0178	0.0180
		5	0.0169	0.0170	0.0169	0.0171
2	10	0	0.0588	0.0591	0.0589	0.0602
		1	0.0555	0.0557	0.0556	0.0568
		2	0.0527	0.0529	0.0527	0.0539
		3	0.0503	0.0505	0.0503	0.0514
		4	0.0481	0.0483	0.0482	0.0493
		5	0.0463	0.0464	0.0463	0.0473
	20	0	0.0149	0.0150	0.0150	0.0150
		1	0.0141	0.0141	0.0141	0.0142
		2	0.0134	0.0134	0.0134	0.0135
		3	0.0127	0.0128	0.0128	0.0129
		4	0.0122	0.0123	0.0123	0.0123
		5	0.0117	0.0118	0.0118	0.0118

(a) Aghababaei and Reddy (2009)

Table 2 Comparison of natural frequency of FG nano-plate (a = 10, n = 5)

a/b	a/h	μ	Mode 1		Mode 2		Mode 3	
			Ref [(b)]	Present	Ref [(b)]	Present	Ref [(b)]	Present
1	10	0	0.0441	0.0432	0.1051	0.1029	0.1051	0.1915
		1	0.0403	0.0395	0.0860	0.0842	0.0860	0.1358
		2	0.0374	0.0366	0.0745	0.0730	0.0746	0.1110
		4	0.0330	0.0323	0.0609	0.0596	0.0610	0.0861
	20	0	0.0113	0.0111	0.0278	0.0274	0.0279	0.0536
		1	0.0103	0.0101	0.0228	0.0224	0.0228	0.0380
		2	0.0096	0.0094	0.0197	0.0194	0.0198	0.0310
		4	0.0085	0.0083	0.0161	0.0158	0.0162	0.0241
2	10	0	0.1055	0.1029	0.1615	0.1574	0.2430	0.2397
		1	0.0863	0.0842	0.1208	0.1177	0.1637	0.1587
		2	0.0748	0.0730	0.1006	0.0980	0.1310	0.1269
		4	0.0612	0.0596	0.0793	0.0772	0.0999	0.0968
	20	0	0.0279	0.0274	0.0440	0.0432	0.0701	0.0688
		1	0.0229	0.0224	0.0329	0.0323	0.0464	0.0455
		2	0.0198	0.0194	0.0274	0.0269	0.0371	0.0364
		4	0.0162	0.0158	0.0216	0.0212	0.0283	0.0277

(b) Natarajan *et al*. (2012)

present theory are found to be in very good agreement with the existing shear deformation theories. Noted that the present model has only four unknowns, while the number of unknowns in FSDT (e.g., Thai *et al*. 2012) and TSDT (Aghababaei and Reddy 2009) is five. Also, the present theory does not required shear correction factors as in the case of FSDT. It can be also concluded that the local elasticity theory overestimates the natural frequency than the nonlocal elasticity theory.

The second comparison is carried out for FG plates (n = 5) with different values of nonlocal parameter, the plate thickness and the plate aspect ratio. The natural frequencies computed using the present theory, are compared with those of Natarajan *et al*. (2012) in Table 2. Again, good results are achieved for both formulations. The difference observed in the case of mode 3, is due to the calculation of frequency which is carried out with (1, 3)th eigenmode instead to (1, 2)th eigenmode. It should be noted that the present theory involves four unknowns as against five unknowns in both FSDT and TSDT.

Fig. 3 presents the variation of the frequency ratio ($\overline{\omega}_{NL} / \overline{\omega}_L$) against the nonlocal parameter (μ) for different values of thickness parameter (a / h). Where $\overline{\omega}_{NL}$ and $\overline{\omega}_L$ are the nondimensional frequency based on nonlocal and local elasticity, respectively. These results indicate that the responses vary nonlinearly with the small scale parameter. It can be seen that the increase of the nonlocal parameter leads to a decrease in the frequency ratio. This result proves that the effect of the nonlocal parameter softens the nano-plate. The main finding from the figure is that all responses of FG nano-plate with lower thickness parameter (i.e., a/h = 5) are strongly affected by the small scale parameter than those of the FG nano-plate with relatively higher thickness

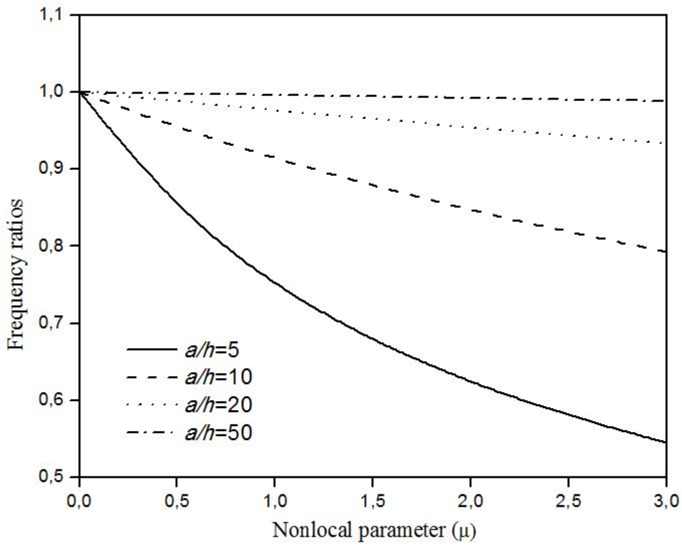


Fig. 3 Effect of the nonlocal parameter (μ) and the thickness parameter (a/h) on dimensionless frequency ratio ($\overline{\omega}_{NL} / \overline{\omega}_L$) for a simply supported square FG plate with volume fraction exponent $n = 5$

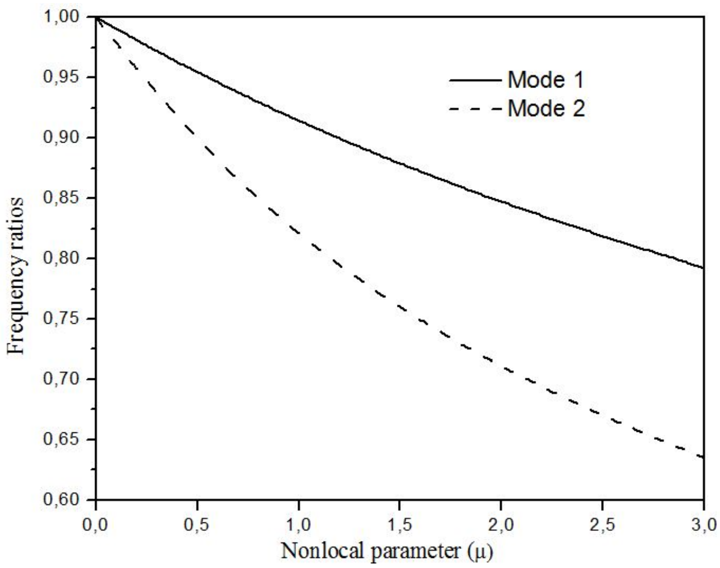
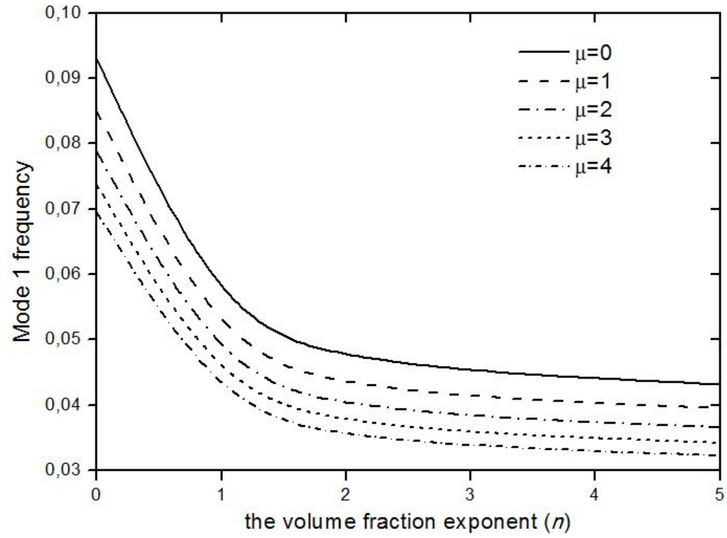


Fig. 4 Effect of the nonlocal parameter (μ) on dimensionless frequency ratio ($\overline{\omega}_{NL} / \overline{\omega}_L$) for a simply supported square FG plate for the first two frequencies with $a/h = 10$ and volume fraction exponent $n = 5$

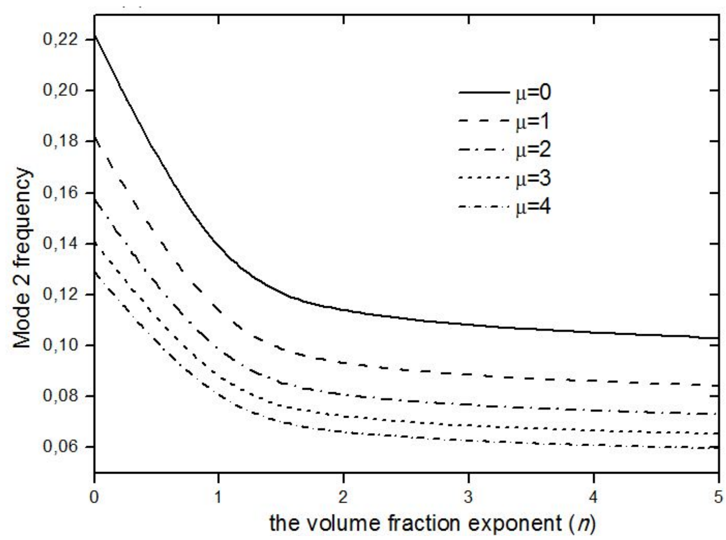
parameter. From this observation, it can be concluded that modeling based on the local (classical) plate models is not suitable, and the nonlocal plate models may provide an adequate approximation for the nano-sized structures.

It can be seen from Fig. 4 that the frequency ratios are smaller than unity for both modes 1 and 2, indicating that the inclusion of the small scale effect leads to a reduction in the vibration frequencies. This underprediction of frequency values is amplified for higher vibration modes.

Fig. 4 illustrated the effect of the volume fraction exponent on the dimensionless two first frequencies of FG nano-plate with a/h = 10 for various values of the small scale parameter. One can observe that the dimensionless frequency decreases as the volume fraction exponent increases. This is due to the fact that an increase in the volume fraction exponent yields a decrease in the stiffness of the FG nano-plate. There is an abrupt change in the responses when the volume fraction exponent changes from 0 to 2, but after passing n = 2 all of the curves become flatter.



(a)



(b)

Fig. 5 Effect of the volume fraction index (n) and the nonlocal parameter (μ) on dimensionless frequency for a simply supported square FG plate with a/h = 10: (a) first frequency; (b) second frequency

5. Conclusions

The size-dependent vibration properties of FG nano-plate are numerically investigated by employing a new hyperbolic refined plate model based on the nonlocal differential constitutive relations of Eringen. By dividing the vertical displacement into bending and shear components, the number of unknowns and governing equations of the present model is reduced to four and is therefore less than alternate theories. The influences of small scale parameter, thickness parameter, and various material compositions on the vibration response of the FG nano-plate are discussed. Numerical results prove that the small scale effects play a considerable role on the vibration properties of the FG nano-plate. The novel nonlocal plate model produces smaller frequencies than the classical (local) plate model. Therefore, the nonlocal effects should be considered in the analysis and modeling of dynamic behavior of nanostructures. Further, it is proved that the volume fraction exponent has a great effect on the behaviors of FG nano-plate, and the responses can be controlled by selecting proper values of the volume fraction exponent. The formulation lends itself particularly well in analysing nanostructures with including the stretching effect (Larbi Chaht *et al*. 2015, Hamidi *et al*. 2015, Belabed *et al*. 2014, Fekrar *et al*. 2014, Hebali *et al*. 2014, Bourada *et al*. 2015, Houari *et al*. 2013, Saidi *et al*. 2013, Bessaim *et al*. 2013) which will be considered in the near future.

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