Steel and Composite Structures, *Vol. 18, No. 3 (2015) 711-737*
DOI: http://dx.doi.org/10.12989/scs.2015.18.3.711

Development of a simplified equivalent braced frame model for steel plate shear wall systems

Arghya Kamal Chatterjee [a], Anjan Bhowmick [b] and Ashutosh Bagchi [*]

Department of Building, Civil and Environmental Engineering Concordia University, Montreal, Canada

(*Received September 13, 2013, Revised July 17, 2014, Accepted September 24, 2014*)

Abstract. Steel Plate Shear Walls (SPSWs) have been accepted widely as an effective lateral load resisting system. For seismic performance evaluation of a multi-story building with SPSWs, detailed finite element models or a strip model can be used to represent the SPSW components. However, such models often require significant effort for tall or medium height buildings. In order to simplify the analysis process, discrete elements for the framing members can be used. This paper presents development of a simplified equivalent braced model to study the behavior of the SPSWs. The proposed model is expected to facilitate a simplification to the structural modeling of large buildings with SPSWs in order to evaluate the seismic performance using regular structural analysis tools. It is observed that the proposed model can capture the global behavior of the structures quite accurately and potentially aid in the performance-based seismic design of SPSW buildings.

Keywords: steel-plate-shear-wall; simplified modeling; strut and tie; non-linearity; finite element method

1. Introduction

The use of ductile steel plate shear wall (SPSW) is gaining grounds in modern construction. This is primarily because of its ability to resist lateral loads and add ductility to the performance of the structure. SPSW has a natural tendency to buckle due to the imperfection of geometry. Once buckles, the load resisting mechanism changes from plane shear to inclined tension field. Capturing the geometric non-linearity involved in buckling has always been challenging. Modeling technique turn out to be more complicated and time consuming. Particularly for industrial design of multi-storey structures, where the local behavior of individual plate is not a major concern but the overall behavior of the system is important, a simplified modeling technique has to come up. With complicated models, repeated performance test with several ground motion records consumes huge amount of time and thus makes work un-economic.

SPSWs have been put to use since 1960s, however consideration of the post-buckling strength in design is relatively new. The "strip model" introduced by Thorburn *et al.* (1983) provided a

∗Corresponding author, Ph.D., E-mail: ashutosh.bagchi@concordia.ca
[a] E-mail: arghyachatt@gmail.com
[b] E-mail: bhowmick@encs.concordia.ca

http://www.techno-press.org/?journal=scs&subpage=8 ISSN: 1229-9367 (Print), 1598-6233 (Online)

simple technique to illustrate the behavior of the thin plate as a SPSW. This model has been later criticized for underestimating the stiffness (Driver *et al*. 1997). From 1990s finite element method gained ground for analysis of SPSW (e.g., Elgaaly *et al*. 1993, Driver *et al*. 1997, Elgaaly 1998). Subsequently, there are many experimental and numerical studies available in steel-plate shear walls (e.g., Lubell *et al*. 2000, Mohammad *et al*. 2003, Kharrazi 2005, Berman and Bruneau 2005, Dastfan and Driver 2008, Topkya and Atasoy 2009, Neilson 2010, Bhowmick *et al*. 2010, Vatansever and Yardimci 2011, Shahi *et al*. 2013, Memarzadeh *et al*. 2010, Choi *et al*. 2013, Bozdogan 2013).

There have been several attempts made to develop a proper method for analysis of SPSW systems. The most widely accepted method of modeling SPSW systems is detailed Finite Element Method (FEM), where the infill-plate is modeled as shell element. Its results are mostly reliable (though accuracy depends on the user's choice on modeling technique) but modeling is quiet complicated and analysis time is significantly high. Strip model is another popular model which was first proposed by Thorburn *et al*. (1983). However, modeling for reversal of loading is very cumbersome in strip model. Later, in 1998, Elgaaly presented a modified strip model for both welded and bolted infill plate connections. Though his model was able to predict the pushover and hysteric responses of the tested specimens, it was not a simple and time efficient model. One efficient way of modeling SPSW system would be with a strut and tie model. There has been counted number of attempts made to develop an equivalent braced frame (Eq.BF) model. Thorburn *et al*. (1983) proposed a truss model where the property of the equivalent braces was derived based on principle of least work done. While calculating the area of braces it was assumed that the boundary members were rigid. Another truss model was recently proposed by Topkaya and Atasoy (2009). Their model assumed beams to be rigid and a combination of both empirical and analytical relations were used to develop the model. In this truss model along with the calculation of area of truss brace, modified stiffness of columns were also needed to be computed. Though truss model by nature is time efficient, but still owing to the assumptions and approximations these models do not always yield reliably accurate results. Three sample specimens (Table 1) have been shown to compare the reliability of detailed Finite Element (FE) model with the existing truss models. These models can estimate the initial stiffness within acceptable limits. A more robust and reliable model is needed that can capture the SPSW behavior within acceptable limits of accuracy without any compromise in time efficiency.

In this paper an attempt has been made to develop a simplified braced model, with varying properties of non-concentric cross bracings, which can successfully replace the complicated non-linear behavior of SPSW systems. The equivalent braced model has been tested under several available experimental and theoretical test results and found to have satisfactory performance. Pushover and hysteretic curves for several SPSW structures obtained from pre-reported experiments or Finite Element models have been compared with results obtained through the

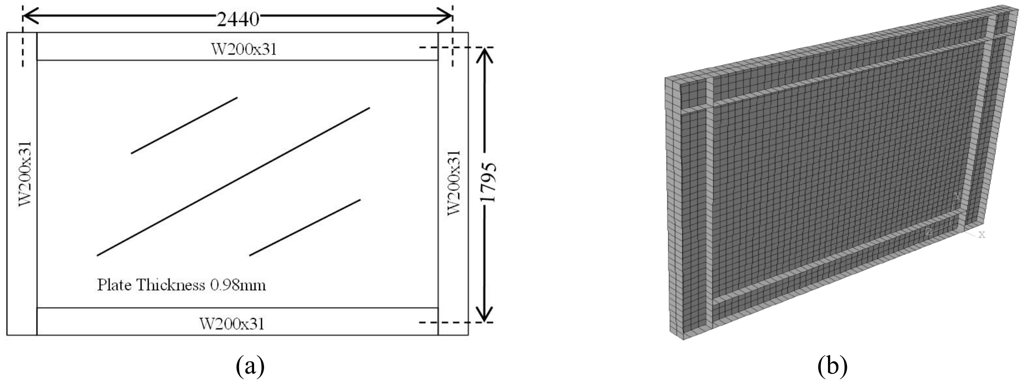
Table 1 Details for the selected single-storey SPSW specimen

S.No	Specimen	Boundary		Plate		
		Beam	Column	Thickness	E(MPa)	σ_y(MPa)
1	CSA-S16-09 design	W530x272	W360x509	3 mm	200,000	385
2	Lubell et al. (2000)	S75x8	S75x8	1.5mm	200,00	320
3	Neilson (2010)	W200x31	W200x31	0.98mm	210,000	275

Equivalent Braced Frame (Eq.BF) model.

2. Methodology

To develop Eq.BF model a detailed parametric study is needed to be carried out numerically. Initially, detailed FE modeling (with infill plates modeled as shell element) of benchmark single-storey SPSW systems are developed. For example, Fig. 1 shows the details of the single-storey SPSW specimen tested by Neilson (2010) and the corresponding finite element model developed for the present study. A concentrated lateral (horizontal) load is applied at a corner at the top (left or right) and to simulate the test condition, nonlinear static pushover analysis is performed by gradually increasing the load from zero to the maximum level until the system fails. The pushover curve (plot of base shear or reaction at the base against the displacement of the top floor) for this specimen and other benchmark problems are validated with available experimental or numerical results. Fig. 2 shows the comparative performance of the existing bracing models for SPSW systems (e.g., Thorburn *et al*. 1983, Topkaya and Atasoy 2009) as compared to results of the detailed finite element models and available experimental results (e.g., Neilson 2010, Lubell *et al*. 2000). It is observed from Fig. 2 that the existing bracing models for SPSW systems do not capture the push curves of the systems adequately. For three dimensional detailed FE modeling, commercial package Abaqus (2011) has been chosen. To test the proposed Eq.BF model, which is essentially restricted to two dimensions for simplicity, Opensees software system (Mazzoni *et al*. 2007) has been used. In detailed FE modeling both material and geometric non-linearity can easily be incorporated. For simplified modeling, the modeling technique itself should take account of these effects. In equivalent braced frame model, beams and columns are modeled as beam-members rigidly connected at beam-column joints and the non-concentric diagonal trusses are made of truss-members. The boundary members in Eq.BF are kept unchanged and adjustment on the truss properties are attempted so as to achieve desired behavior. A detailed parametric study on SPSW systems is required to establish geometric and material property of truss members in Eq.BF. Failure of infill plate in any SPSW system is progressive even though its material property may be assumed perfectly elasto-plastic. It is because of its high degree of redundancy i.e., even though some part of the plate has yielded, the remaining is still capable for maintaining a load path. To achieve similar failure pattern equivalent truss system, its material



(a) (b)

Fig. 1 Sketch and FE-mesh of the specimen tested by Neilson (2010)

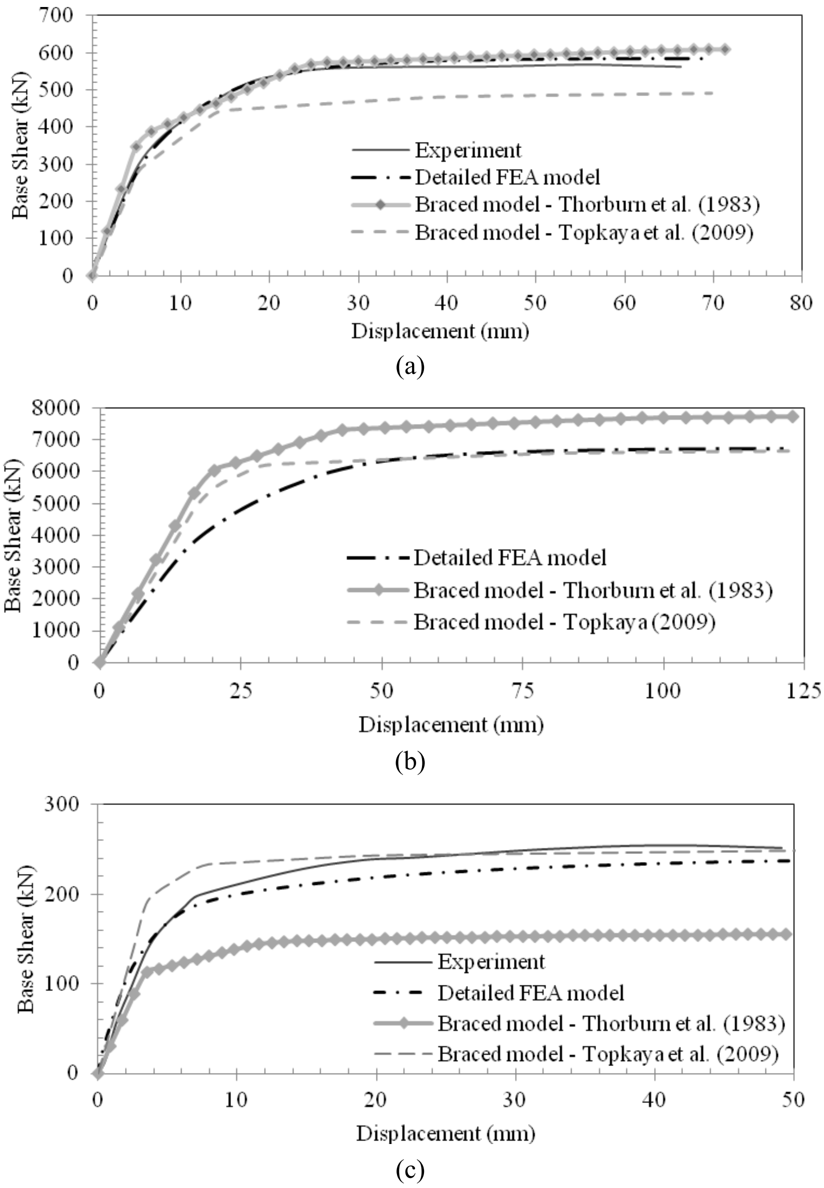


Fig. 2 Performance of existing equivalent bracing models: (a) Neilson (2010) test specimen; (b) a newly designed single-storey specimen; and (c) Lubell *et al*. (2000) test specimen

property needs to be modified from perfectly elasto-plastic to at least a tri-linear curve. Fig. 3 shows the uniaxial hysteretic stress-strain properties of the equivalent brace material, where $\varepsilon_{t\text{-}i}$ and $\sigma_{t\text{-}i}$ represent the strain and stress, respectively, for the *i*th point in the stress-strain curve in tension. Similarly, on the compression segment, $\varepsilon_{c\text{-}i}$ and $\sigma_{c\text{-}i}$ represent the respective strain and stress. Ebrace represents the initial Young's Modulus of brace material. In compression a bi-linear curve for truss is enough as the buckling of plate is almost instantaneous.

Identifying the influencing independent parameters which characterize the behavior and

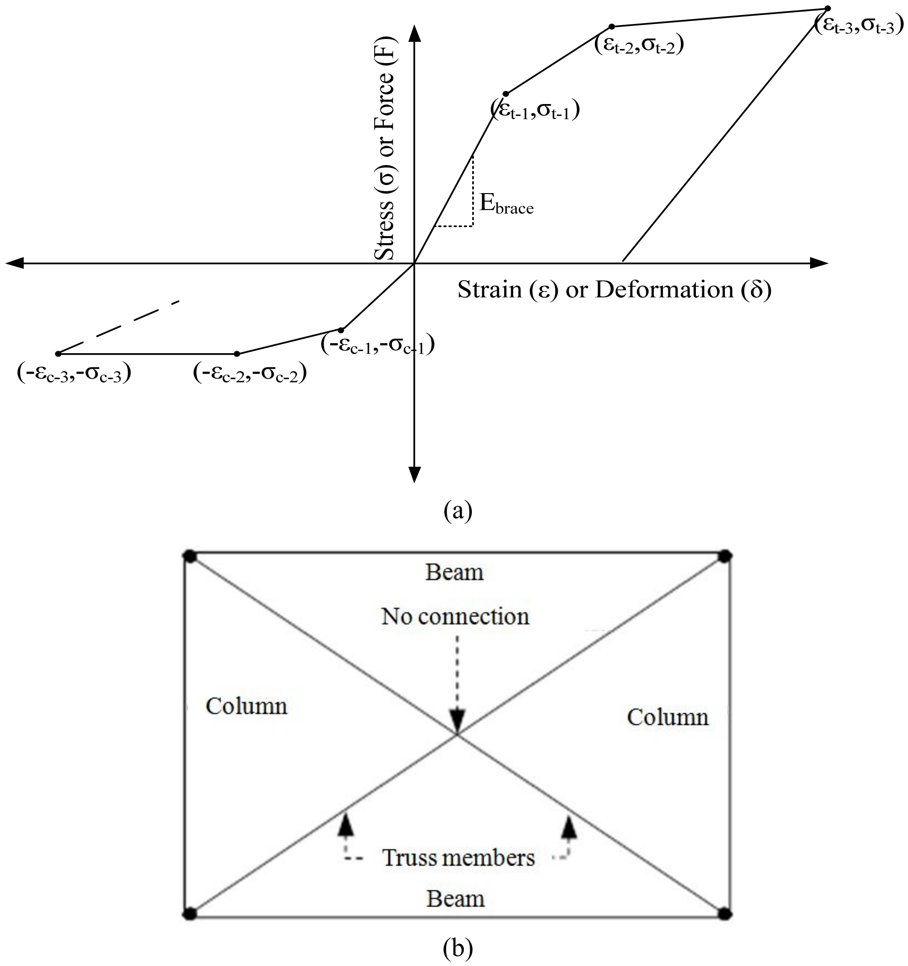


Fig. 3 Uniaxial Hysteretic material for braces

strength of SPSW systems is itself a challenging task. Mohammad *et al*. (2003) conducted a dimensionless parametric study to identify these parameters. Some of those parameters like aspect ratio, column flexibility, etc. are considered for the development of the Eq.BF model. Additional parameters like the ratio of diagonal length to the thickness of plate are also observed to have significant importance in Eq.BF model.

Fig. 4 shows the relationship of various parameters affecting the properties of the equivalent bracing system. To achieve the target behavior of SPSW system using the diagonal braces, only the selected properties of the equivalent truss (replacing the original infill plate) need to be calibrated with SPSW parameters. A change in any of these properties can be considered as the "cause" that will have a significant "effect" in the final performance of the structure. With the mentioned objective, proper methodology is developed through which the "cause" can be calibrated based on "effect". An outline to establish the "cause" properties has been shown in Fig. 5. Statistical approach has been used extensively to obtain empirical relation from SPSW parameters. Only shear relations has been used to develop stiffness of equivalent truss and thus it

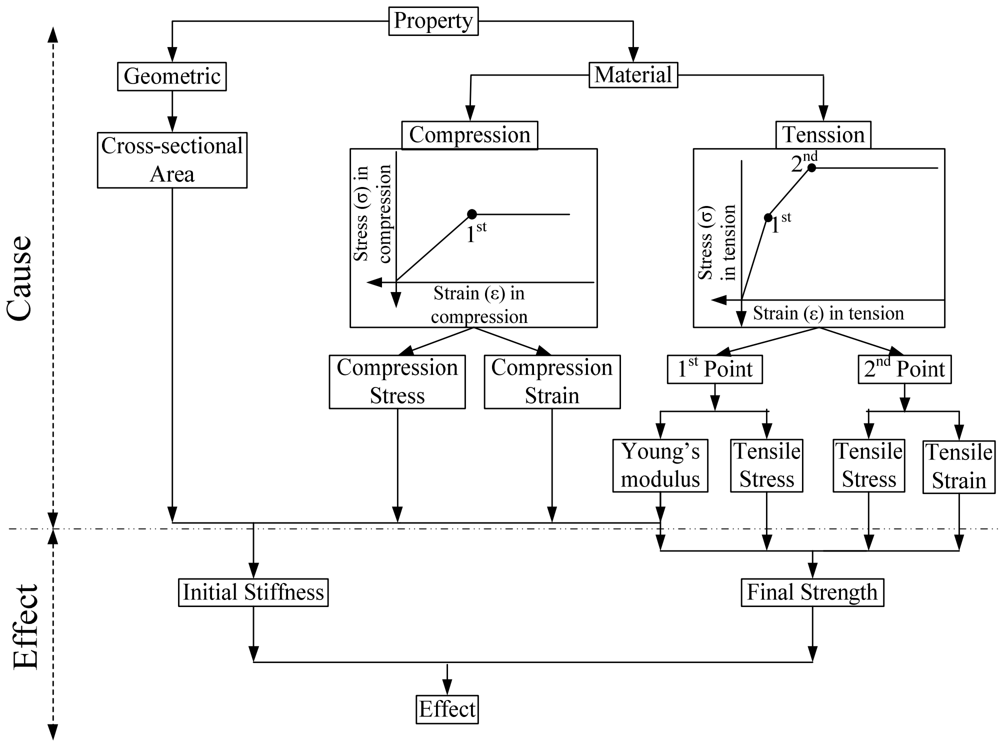


Fig. 4 Property of truss braces to be adjusted to get the desired effect from the Eq.BF model

is assumed that bending of boundary frame has no significant effect in determining plate strength. This assumption can be applied in most cases where aspect ratio is relatively high or the building is not too tall.

3. Evolution of equivalent braced frame (Eq.BF) model

To model the behavior of infill-plate in a steel plate shear wall system by equivalent truss bracing, a linear relation between their stiffness is established. Then the required cross-sectional area of truss bracing is calculated. An equivalent linear model is created by parametric study to establish the geometric non-linear behavior of the steel plate to equivalent bracing. Several parameters have been introduced to simulate the non-linear behavior of the plate in the equivalent behavior in the brace element. Stiffness of the brace element can be adjusted by multiplying the brace area with appropriate parameters obtained by linear equations to simulate the non-linear brace stiffness. Also, to capture the ultimate strength of a SPSW system in Eq.BF model, the material properties of the bracing system need to be parametrically modified.

3.1 Stiffness reduction due to buckling of the plates

As introduced by Topkaya and Atasoy (2009), a parameter αs which is the ratio of the post-buckled stiffness of the plate to the pre-buckled original stiffness is important to represent the

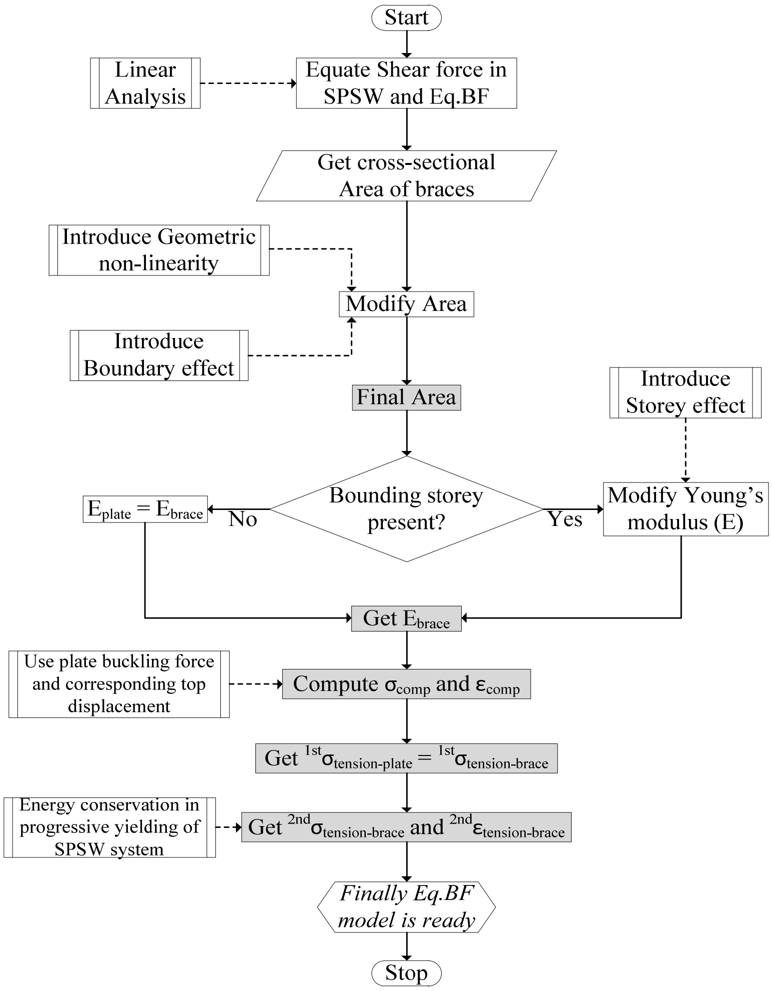


Fig. 5 Flowchart representation of Eq.BF model development methodology

reduction of the stiffness of the plate due to buckling. It is a parameter which can capture geometric non-linearity of plates in SPSW system. Through a trial and error analysis, it has been observed that the two main dimensionless parameters responsible for the geometric non-linear behavior of steel plate are the thickness to panel size (the size of a plate panel expressed as the length of diagonal) ratio of the plate and aspect ratio of infill plate. The parameters can be mathematically expressed as, b / L ($= \beta_1$) and l / h ($= \beta_2$), which are similar to the relevant parameters used by Topkaya and Atasoy (2009) (b, L, h, l being thickness, diagonal length, height, length of infill plate respectively). Their attempt in establishing a proper relation of αs with the primary variable has been extended through this work.

Since, the concern here is the geometric non-linearity only all analysis for parameterizing αs is kept within the elastic limit. Also, no bounding beam column is considered since the study is related only to the panel plate. The parametric study is carried out using detailed FEM models in Abaqus. In that case, the modeling technique is kept close to the one used for the validation study, as far as possible. Shell elements (S4R from Abaqus element directory) have been used to model the plate. All four edges are restrained against lateral rotation and translation out of the plane. Due to the presence of heavy boundary members in steel plate shear wall, the out-of-plane rotation of the plate elements at the edges is practically negligible. Quasi-static load was applied along the plane of the plate along the top edge, to represent the shear from an imaginary axially rigid beam. Fig. 6 shows a sample image for a bucked plate. To compare the stiffness of the perfect geometry (linear behavior) with that of the buckled plate (geometric non-linear behavior), the same model was analyzed once without imperfection and then with imperfection. The model without imperfection shows a much higher stiffness than the one with imperfection. Fig. 7 shows a sample example of the stiffness reduction with and without imperfection for plate with β_2 = 0.7 and β_1 = $3\sqrt{2}$. It should be noted that no attempt was made to reach the plastic limit of the plate and introduce material non-linearity. After a certain limit of the lateral load (sufficiently large for the plate) the perfect geometry suddenly fails and an abrupt change in the load-displacement curve is observed. The stiffness below that limit is constant and is the one considered. As mentioned earlier, with and imperfection, the load-displacement curve is not linear even within the elastic limit. However, an approximate straight line, representing an equivalent linear relation the strength and displacement can be assumed in that case. For all the cases considered here the approximate linearization of the force-deformation curve result in R^2 > 0.8.

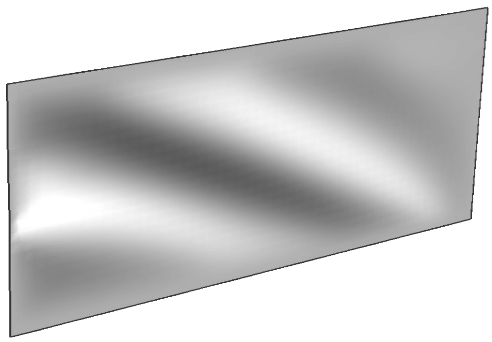


Fig. 6 Sample image of a buckled plate (i.e., geometric non-linearity for buckling)

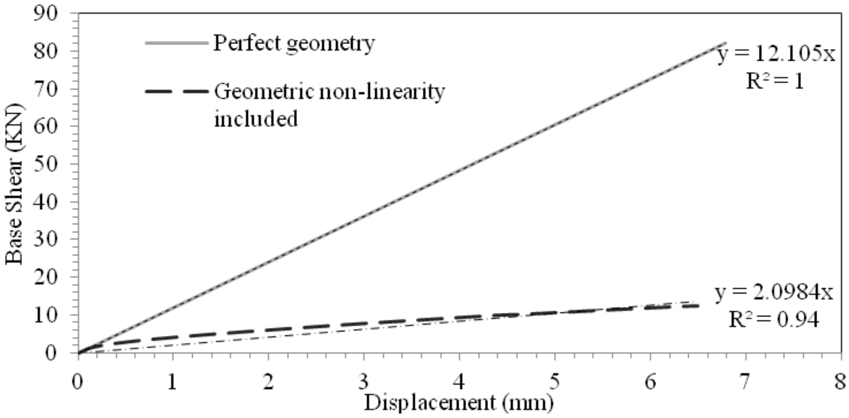


Fig. 7 Sample example of shear stiffness reduction for geometric non-linearity (achieved with imperfection) in plates

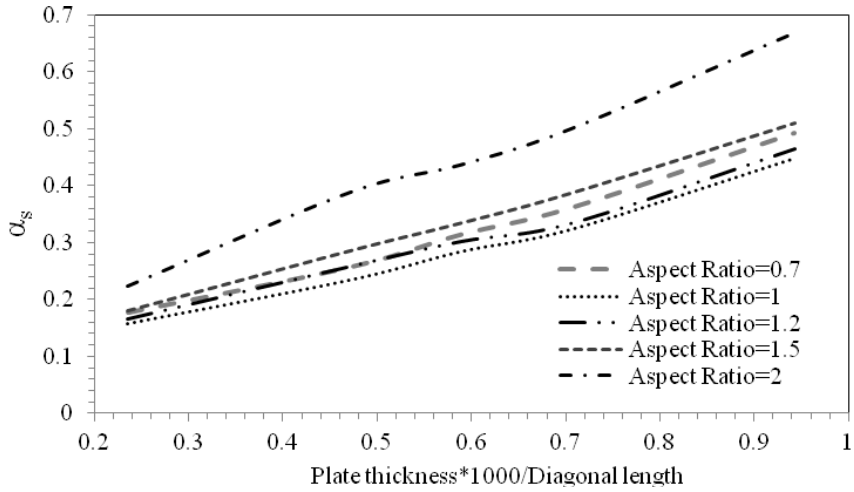


Fig. 8 Variation of α_s in relation to β_1

Fig. 8 shows the sensitivity of αs to the plate thickness to diagonal length ratio, β_1; and they are found to be linearly related. However, but depending on the aspect ratio (β_2), the slope of the line may change. It is also observed that αs is almost constant when β_1 = 0, irrespective of different values of β_2. Thus, fitting a linear Eq. (1) with varying slope (slope given by m) of the line yielded an $R^2 > 0.99$. The variation of m with β_2 is then represented through a polynomial fit as shown in Fig. 9. The relation between m and β_2 is established in Eq. (2) with the coefficient of determination of $R^2 = 0.99$.

$$\alpha_s = m * \beta_1 + 0.06 \tag{1}$$

$$m = -0.2102(\beta_2)^3 + 1.1062(\beta_2)^2 - 1.5923(\beta_2) + 1.0862 \tag{2}$$

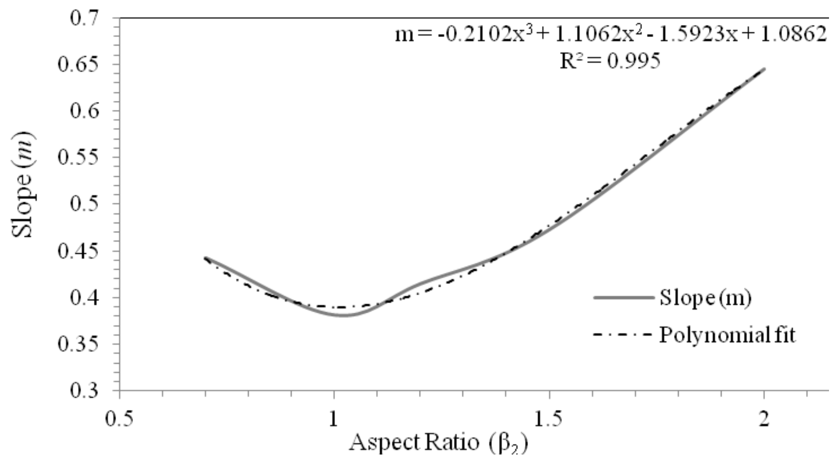


Fig. 9 Variation of 'm' with aspect ratio

3.2 Linear stiffness relation of SPSW system and Eq.BF model

The stiffness of SPSW system and Eq.BF model are equated under linear conditions to obtain the cross sectional area of the cross braces. In SPSW system the significant force is in the form of infill plate shear. Bending effect becomes significant only when the aspect ratio is very low and for taller SPSWs. For this study it is assumed that the bending effect on the plate is negligible. Thus, the stiffness of the plate can be established by shear rigidity alone. The symbols used in the derivation of an expression for a lateral stiffness of the SPSW system are introduced as below, and they are illustrated in Figs. 10 and 11.

Diagonal length of brace = $L = \sqrt{l^2 + h^2}$
b = Thickness of plate μ = Poisson's ratio
I = Moment of inertia of transverse section = $\dfrac{b * l^2}{12}$

$$Q = \bar{y}' + A' = \frac{l}{4} * \frac{l * b}{2} \tag{3}$$

V = Applied shear force on plate
G = Shear modulus = $\dfrac{\tau}{\theta}$
Since
Shear stress = $\dfrac{VQ}{Ib}$; So, $V = \left(\dfrac{Ib}{Q} * G\right) * \theta$ or, $V = \left(\dfrac{Ib}{Q} * \dfrac{G}{l}\right) * \Delta l$

or

$$V = K_{plate-linear} * \Delta l \tag{4}$$

where, α_s is the ratio of stiffness of the buckled plate to that of the plate with perfect geometry (linear), as already discussed;
So

$$\alpha_s = \frac{K_{plate-buckled}}{K_{plate-linear}}$$

For bracing,

$$K_{brace} = \left(\frac{A_d E}{L}\right) * \cos^2 \varphi \tag{5}$$

where, φ is the brace angle, L is the length of the brace, A_d is the equivalent cross-sectional area for bracing and E is Young's modulus.
Thus,

$$\alpha_s * K_{plate-buckled} = K_{brace} \tag{6}$$

$$\alpha_s = \left(\frac{Ib}{Q} * \frac{G}{l}\right) = \left(\frac{A_d E}{L}\right) * \cos^2 \varphi \tag{7}$$

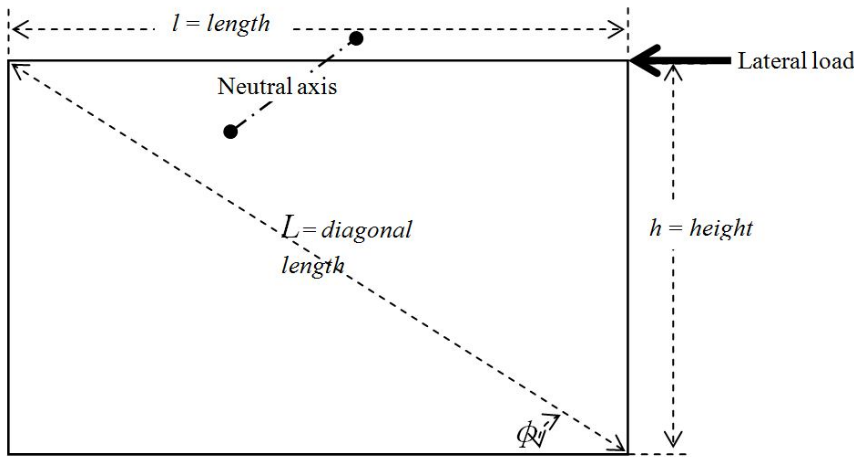


Fig. 10 Shear load on infill plate

Fig. 11 Shear deformation of plate frame under linear conditions

Modulus of elasticity E is related to the shear modulus G and the Poisson's Ratio μ, as $E = 2G(1 + \mu)$

Therefore

$$A_d = \frac{\alpha_s * \left(\frac{Ib}{Q}\right)}{2(1+\mu)\frac{\cos^2 \varphi}{L}} \tag{8}$$

The above expression for A_d is derived by assuming the absence of boundary members. A noticeable strength increase is observed in the presence of strong boundary members. Unless the boundary members are strong enough, tension field in the plate remains incomplete (Mohammad *et al*. 2003). Thus, to estimate the increase in the capacity or strength of the plate due to presence of boundary members another parameter (α_m) is used.

3.3 Stiffness reduction due to buckling of the plates

Stiffness reduction considered for buckling of plate (α_s) has been computed in the absence of boundary frame. With inclusion of the boundary frame, tension fields start to develop in the plate and for optimizing the use of plate i.e., for complete development of tension field boundary members should be strong enough. So, a parameter α_m is introduced which accounts for this increase of stiffness of the plate in presence of boundary frame, and thus the area of equivalent braces is increased. The physical entities responsible for parameter α_m are the overall non-linear strength of the plate and the boundary frame. Size, thickness and aspect ratio of a plate are the primary geometric parameters that determine the strength of steel plate within the elastic limit. Thus, it can be safely assumed that αs is responsible for the variation in α_m. However, standardizing the strength of the boundary members is a formidable task. Wrangler (1931) introduced a flange flexibility parameter w_h to study the behavior of tension fields in W-sections. Owing to the behavioral similarity between SPSW and web girders, standard S16 of the Canadian Standards Association (CSA 2009) and the AISC Specification (AISC 2005) accepted this flexibility parameter as a measure for the strength of the boundary members in SPSW systems. Kuhn *et al*. (1952) simplified this parameter as given in Eq. (9). The same parameter was as found to have an effect on the overall capacity of the plate by Mohammad *et al*. (2003). Dastfan and Driver (2008) modified the parameter as w_L (Eq. (10)) for the end panels (top and bottom). Thus, α_m is studied by varying the flexibility of the boundary elements, β_3 which is equal to w_h for an intermediate storey and w_L for the top storey. All other independent dimensionless parameters responsible for change in strength of SPSW as identified by Mohammad *et al*. (2003) are kept constant.

$$w_h = 0.7 * h * \sqrt[4]{\frac{b}{2LI_c}} \tag{9}$$

where, w_h is column flexibility for intermediate storey

$$w_h = 0.7 * \sqrt[4]{\left(\frac{h^4}{I_c} + \frac{L^4}{I_c}\right)\frac{b}{4L}} \tag{10}$$

where, w_L is column flexibility for intermediate storey.

For this parametric study similar plate model was created as for the study of α_s. The bounding members were considered as beam elements (B31 in Abaqus elements library) for simplicity. The change in column flexibility was brought about by changing the cross sectional area of the column profile. As the study is done for a single storey structure, the effect of beam is indirectly accounted for by considering the change in column flexibility corresponding to the top storey (i.e., by using w_L instead of w_h). The cross-sectional area of the beam was never changed. Also, the ratio of the moment of Inertia to area of a column was kept constant throughout the computation as that ratio is supposed to be an independent parameter affecting the behavior of SPSW system (Mohammad *et al*. 2003). Rigid connection between boundary and plate was been assumed. The boundary members were restrained against lateral rotation and out of plane translation. Hinge support was provided at the column base. Assuming the beam to be axially rigid, a quasi-static shear force was applied on the top beam (as in the case of the analysis of the plate alone while computing αs). Imperfection was introduced in the plate such that the plate buckles with the application of load and the geometric non-linear behavior is taken into account. With the variation of β_3 and α_s, the variation in the stiffness is estimated from the load displacement curves. The slope of the load-displacement curve is obtained by linear fitting of the curve (with co-efficient of determination, R^2 >0.9). As an example, a sample load-deflection curve for a single-storey SPSW system with the aspect ratio of 1.0 and the plate thickness of 2 mm is shown in Fig. 12. An exactly similar analysis was carried out with the bare frame model (where the plate is absent, but all other parameters remain the same) and using the same process as above, the stiffness was estimated (with accuracy of $R^2 > 0.99$). The difference between the stiffness of the full SPSW system and that of the corresponding bare frame gives the portion on the stiffness contributed by the plate in the SPSW system. This stiffness of the plate when analyzed with the frame as above is significantly higher than that of a very similar plate analyzed alone without the boundary members. This is primarily because of the interaction between the structural members (one supporting the other collectively). The ratio of the stiffness of the plate in presence of boundary members to that of without boundary members is expressed as *αm* (Eq. (11)).

$$\alpha_m = \frac{K_{plate|boundary}}{K_{plate|only}} \tag{11}$$

The relation between β_3 and *αm* as shown in Fig. 13 can be best represented by a quadratic function as given by Eq. (12). The values of R^2 for all samples are found to be more than 0.99, indicating a close fit. The coefficients of Eq. (12) can be further used to establish a relation with α_s as illustrated in Fig. 14. The independent co-efficient, 'C_1' is always found to 1.0. The variation of the other two coefficients ('A_1' and 'B_1') with *αs*, is given by Eqs. (13)-(14).

$$\alpha_m = A_1 * \beta_3^2 + B_1 * \beta_3 + C_1 \tag{12}$$

$$A_1 = -4.497 * \ln(\alpha_s) - 0.6184 \tag{13}$$

$$B_1 = -7.5789 * \alpha_s^2 + 2.2279 * \alpha_s - 0.6997 \tag{14}$$

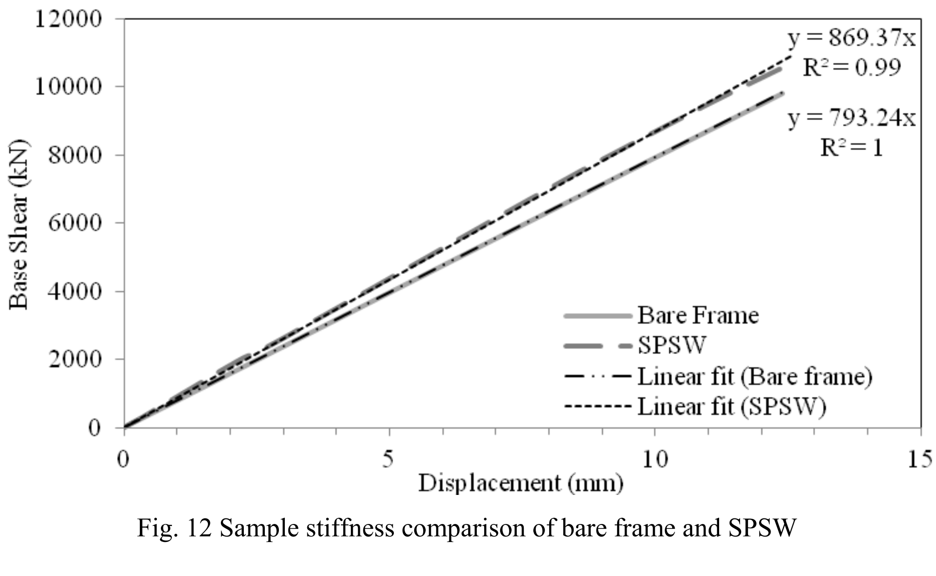


Fig. 12 Sample stiffness comparison of bare frame and SPSW

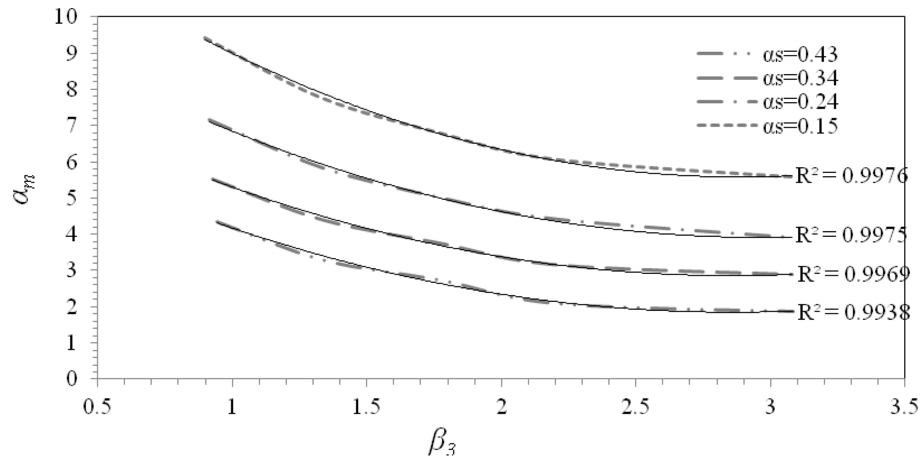
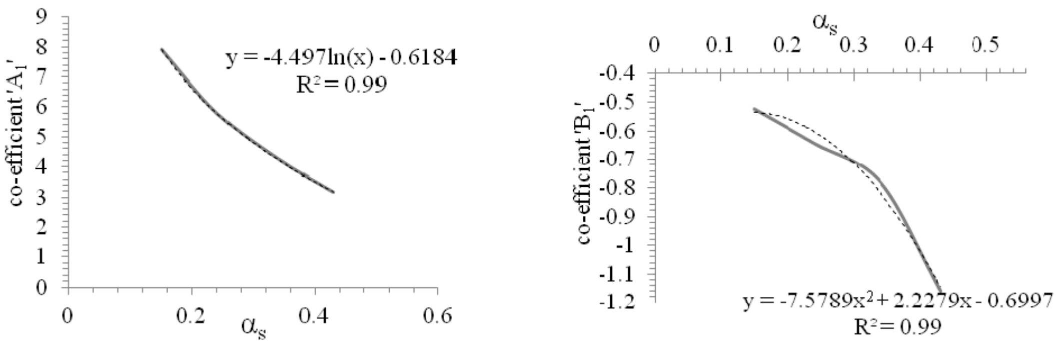


Fig. 13 Relation between β_3 and α_m



(a) (b)

Fig. 14 Variation of co-efficient A_1 and B_1 with α_s

This parameter (α_m), being responsible for strength increase in plate, is used as a multiplier to the area of the equivalent brace. Thus, the new non-concentric brace area can be represented by Eq (15). This area is computed based on parametric study as presented above on the single storey structure within elastic limit of the plate. The parameters to account for the material nonlinearity and multi storey effect are used to develop a suitable material property for the equivalent bracing system as presented later in this chapter.

$$A_d = \frac{\alpha_m \alpha_s * \left(\dfrac{Ib}{Ql} \right)}{2(1+\mu)\dfrac{\cos^2 \varphi}{L}} \tag{15}$$

3.4 Compression struts in Eq.BF model

Once the area of the bracing is determined, the behavior of a compression strut needs to be characterized. Under a cyclic lateral loads, the plate in a SPSW system may alternately develop tension fields and buckling of the plate due to compression along its diagonals. When the plate is modeled using the equivalent diagonal braced, they will also undergo tension or compression, depending on the direction of the lateral load. Though a brace as s compression strut does not have a very significant influence in the overall behavior of a SPSW system, it is an important component of the Eq.BF model. When the plate was studied without the boundary members, it was found that only up to a small magnitude of the lateral force, the stiffness of the plate with and without any imperfection are close to each other. With a higher level of the lateral force, the stiffness of the plate with imperfection reduces significantly because of the buckling of the plate along the compression diagonal. A sample example of pushover curves indicating the limit of the compression force in the plate at which buckling occurs, is given in Fig. 15, where the plate has an aspect ratio of 1.0 and thickness of 2 mm. This limiting force, up to which the behavior of the plate is linear, is observed to depend on the aspect ratio and the thickness of the plate. The buckling force to thickness relation can be established by a quadratic equation (Eq. (16), Fig. 16) with $R^2 >$ 0.98. The co-efficients X_1, X_2 and X_3 can be related to aspect ratio as given by Eqs. (17), (18) and (19), respectively, which are shown in Fig. 17.

$$F_{bucle} = X_1 b^2 + X_2 b + X_3 \tag{16}$$

where, X_1, X_2 and X_3 are constants depending on the aspect ratio of plates

$$X_1 = 7.2945(\beta_2)^2 + 5.6749(\beta_2) \tag{17}$$

$$X_2 = 11.265(\beta_2)^2 + 14.785(\beta_2) \tag{18}$$

$$X_3 = 16.325(\beta_2)^{1.4848} \tag{19}$$

Once F_{buckle} is known, taking its component along the brace and dividing by the area of the brace (A_d), the compression yield strength (i.e., buckling strength) of the brace can be calculated. This parameter does not have a very significant effect on the final behavior of the model except

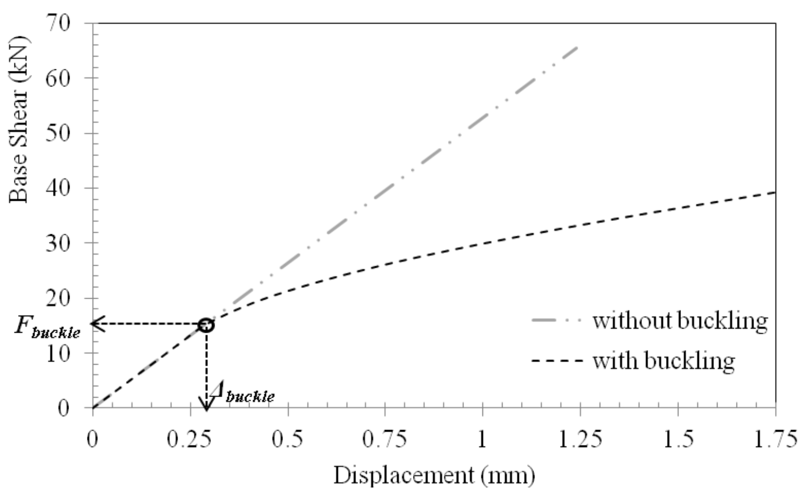


Fig. 15 Sample comparison of plate pushover with and without imperfection

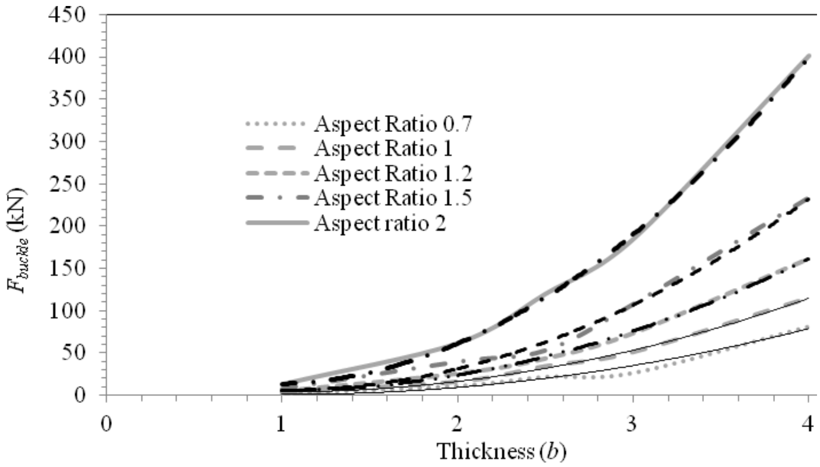
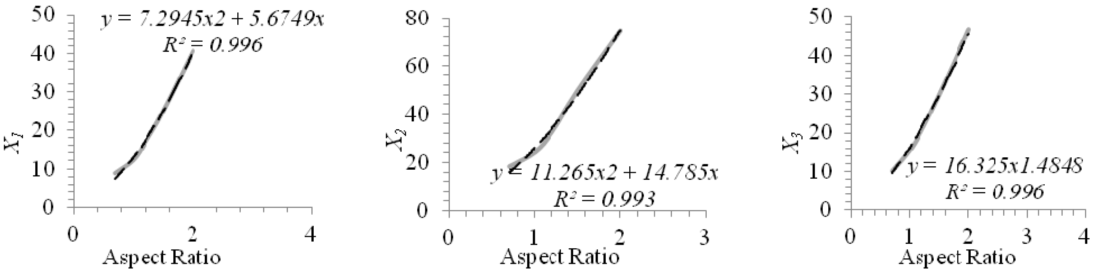


Fig. 16 Relation between buckling force and thickness of plate



(a) (b) (c)

Fig. 17 Relation between coefficients X_1, X_2 and X_3 with aspect ratio

a slight increase in the initial stiffness. At the onset of buckling, the top displacement of the plate without the boundary elements can be approximately related to the thickness of the plate by Eq. (20). On application of the lateral force corresponding to F_{buckle} (which is a very small force), the change in top displacement (Δ_{buckle}) is negligibly small for different aspect ratios. However, Δ_{buckle} will change appreciably if a significant variation of the overall stiffness of the system is observed. But that is not a concern in regard to this parametric study. Therefore, for all aspect ratios under consideration, the average displacement is taken and is related to the plate thickness (Fig. 18) by Eq. (20). Thus, for a SPSW represented using bracing, the modulus of elasticity for the compression strut does not remain the same as that of the brace in tension.

Once F_{buckle} is known, taking its component along the brace and dividing by the area of the brace (A_d), the compression yield strength (i.e., buckling strength) of the brace can be calculated. This parameter does not have a very significant effect on the final behavior of the model except a slight increase in the initial stiffness. At the onset of buckling, the top displacement of the plate without the boundary elements can be approximately related to the thickness of the plate by Eq. (20). On application of the lateral force corresponding to F_{buckle} (which is a very small force), the change in top displacement (Δ_{buckle}) is negligibly small for different aspect ratios. However, Δ_{buckle} will change appreciably if a significant variation of the overall stiffness of the system is observed.

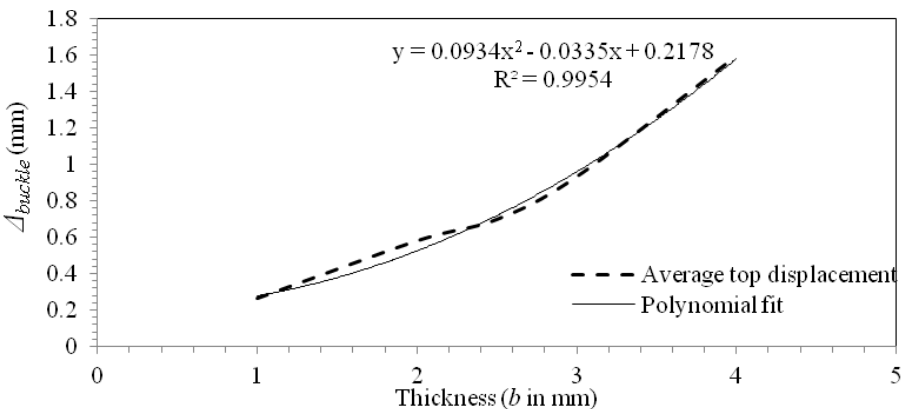


Fig. 18 Variation of average top displacement with thickness for lateral force F_{buckle}

But that is not a concern in regard to this parametric study. Therefore, for all aspect ratios under consideration, the average displacement is taken and is related to the plate thickness (Fig. 18) by Eq. (20). Thus, for a SPSW represented using bracing, the modulus of elasticity for the compression strut does not remain the same as that of the brace in tension.

$$\Delta_{buckle} = 0.0934b^2 - 0.0335b + 0.2178 \tag{20}$$

3.5 Tension strut in Eq.BF model

In a multi-storey steel plate shear wall, the presence of plates above and below causes a neutralizing effect on yield forces in the plates. Since, the plate is distributed throughout the width of the bay, as compared to a bracing system connected only at the corners, the vertical forces during plate yielding is higher than that of the vertical component of force from the brace. The vertical yield force from the plate can be taken as $0.5 * \sigma_y * bl$, where it is assumed that only half the width of the bay is responsible for tension yielding of the plate; in this case, σ_y is the yield stress of the plate. This assumption arises from the fact that if the equivalent area of the brace is divided by the thickness of the plate, the observed length is very close to half the width of the bay. At yielding the vertical component of the force from the equivalent brace in tension is $\sigma_y * A_d * \cos(\gamma)$, where γ is the angle of inclination of the brace with vertical column. The tensile force in the braces of the two consecutive stories also has a neutralizing effect at the corresponding beam column joints. The ratio of these balancing forces on a SPSW and equivalent brace is represented as α_{bal} as given by Eq. (21).

$$\alpha_{bal|storey-i} = \frac{Lower\{(0.5lb\sigma_y)_{storey-i}, (0.5lb\sigma_y)_{storey-i+1}\}}{Lower\{(A_d\sigma_y)_{storey-i}, (A_d\sigma_y \cos\theta)_{storey-i+1}\}} \tag{21}$$

The direct effect of this balancing of the storey forces is observable in the increased stiffness in case of multi-storey structures as compared to the single storey ones. To account for such increase in the stiffness in the corresponding braced model, the modulus of elasticity of a brace in tension is increased by α_{bal} (Eq. (21)). Instead of increasing the modulus of elasticity, increasing the cross sectional area of the brace was considered in other models (like Thorburn *et al*. 1983, Topkaya and

Atasoy 2009). However, such strategy will make the model more complicated, as iterative techniques are needed to be introduced so that the behavior of the brace is represented correctly both in tension and in compression. For single-storey structures there is no need to increase the stiffness of the braces as there is no internal force balance as observed in the muti-storey systems. However, it is observed that if the beam web thickness is more than nearly fifteen times the thickness of the plate, a similar equilibrium of forces between the beam and the plate should be considered. In those cases, even though the external semi-supports from the upper and lower stories are absent, for its high stiffness as compared to plate, the beam acts as a rigid member. For an intermediate storey in multi-storey structure, α_{bal} is the sum of both upper and lower storey ratio as both have an increasing effect on the stiffness of the plate at that storey level.

$$E_{brace} = \alpha_{bal} * E \tag{22}$$

where, E represents the young's modulus of plate in SPSW systems and E_{brace} is the modulus of elasticity of the equivalent brace.

In the Eq.BF model, with perfectly elastic-plastic material property, the tensile yield stress indicates the stress beyond which the tension brace will stop taking further load. However, in case of a plate with the same perfectly elastic-plastic material property, the behavior is more like a bunch of parallel connected strips (with collective area same as that of brace). In that case, even though the yield stress is reached in some areas of the plate, other areas of the plate still continue taking further load (Fig. 19). Elgaaly (1998) reported somewhat similar increase of strength by indicating that the yield strain distribution in diagonal tension field is parabolic. So, a progressive failure curve of the material model needs to be defined in case of Eq.BF model. For a given tensile load, let the elongation of bracing element is Δh_b, and for the same load, the maximum elongation in a parallel strip is Δh_p (Fig. 19). Thus, the volumetric change in the brace (with area A_d) is $A_d * \Delta h_b$ and that of parallel-strips is $\alpha_k * A * \Delta h_p$; where αk is a factor which depends on the shape of the yield area formed by the nodes of the parallel strips (Fig. 19). If energy dissipated by both SPSW and Eq.BF systems are equated, relationship between Δh_b and Δh_p can be established as Eq. (23).

$$\left(\frac{\Delta h_b}{\Delta h_p}\right) = \alpha_k \tag{23}$$

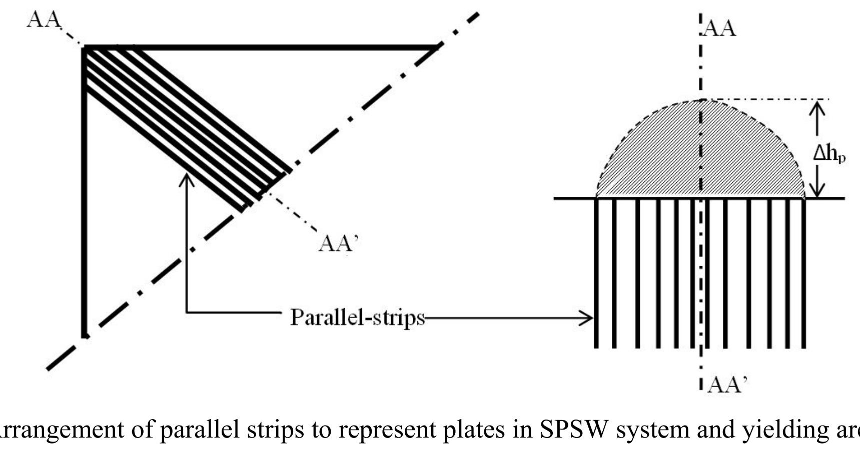


Fig. 19 Arrangement of parallel strips to represent plates in SPSW system and yielding area covered

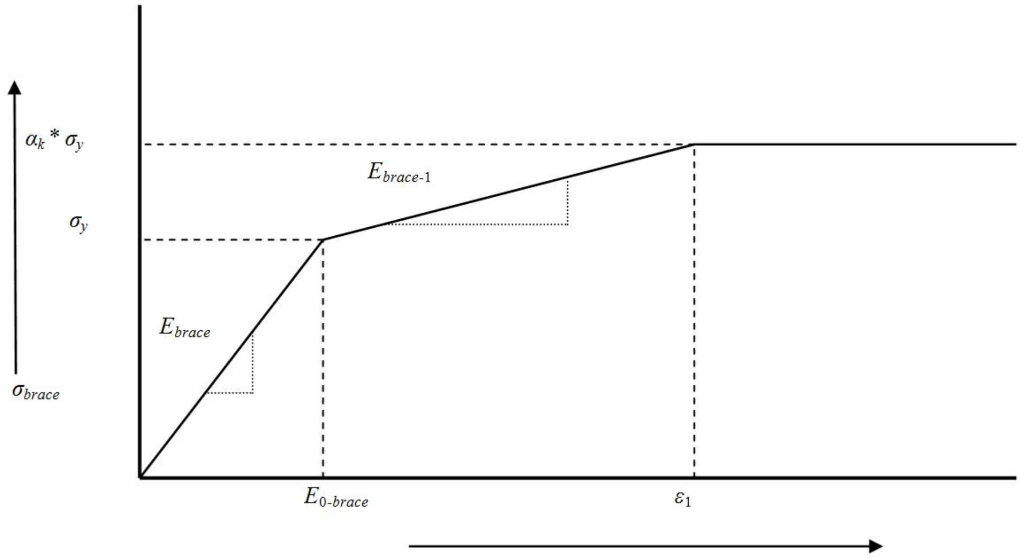


Fig. 20 Material properties in tension of truss braces in Eq.BF model, where σ_y and ε_0 are yield stress and strain of plate in SPSW system

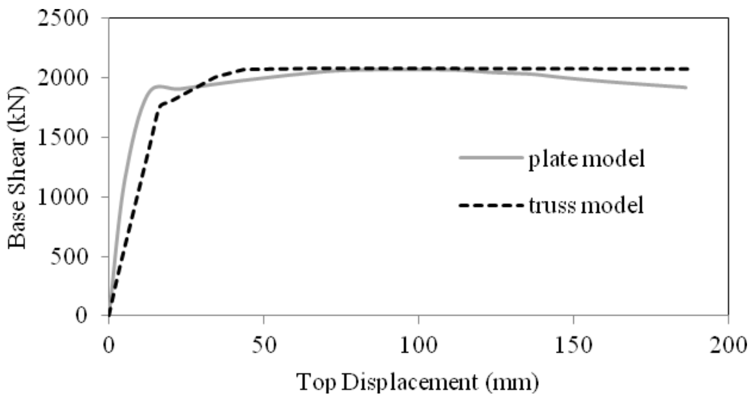


Fig. 21 Sample pushover curve with enhanced material properties in Eq.BF model to match the SPSW system

To achieve the same level of the final deformation in the Eq.BF model, as in the corresponding SPSW system, the original yield stress needs to be multiplied by factor α_k. In this case, the enhanced material properties for the bracing are represented by tri-linear stress-strain curve as shown Fig. 20. Since the stress, $\alpha_k * \sigma_y$ represents the point of final yielding point beyond which the stress-strain curve is perfectly plastic, a larger value of strain ($\varepsilon_1 > \varepsilon_y$) corresponding to that stress is assumed. Through a repeated study with varying parameters, it has been observed that α_k depends upon the ratio of the web thickness of the beam ($t_{web|beam}$) to the sum of the thickness of the connecting plates Eq. (24).

$$\beta_4^i = \frac{t_{web|beam}}{(b_i + b_{i+1})} \tag{24}$$

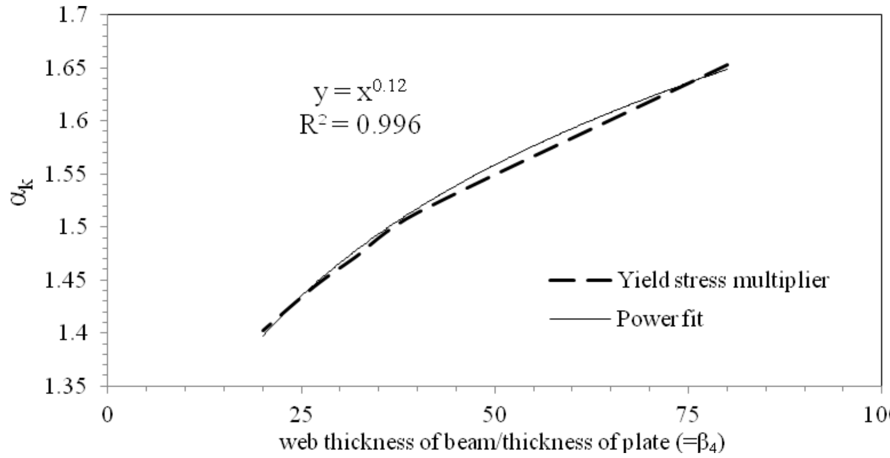


Fig. 22 Relation between α_k and β_4

where, *I* represents the *i*th storey in a multi-storey system. For single storey structures $b_{i+1} = 0$.

The relation between α_k and β_4 is established by carrying out parametric study with both plate frame model and equivalent braced model. In the braced model, the yield strength of braces in tension is enhanced experimentally so that both the plate model and the equivalent braced model have similar pushover curves. A sample pushover curve for a SPSW system with square beams (80 mm × 80 mm), square columns (200 mm × 200 mm) and plate thickness of 3 mm is shown in Fig. 21. Since, thickness of beam to that of plate in the sample pushover study (Fig. 22) is very large, an increased initial stiffness is observed in plate model as compared to equivalent bracing model. However, that can be neglected for this part of the study where the only concern is the representation of the yield strength. In this case, α_k is obtained by dividing the enhanced stress with that of the original yield stress of plate. The relation between α_k and β_4 as shown in Fig. 22 can be expressed by a power relation fitted with Eq. (25), where $R^2 = 0.996$.

$$\alpha_k = (\beta_4)^{0.12} \tag{25}$$

4. Validation of the proposed model

Three single storey samples have been subjected to pushover test for validation of the model. Equivalent areas for the braces are computed as 1115 mm^2 and 642.3 mm^2 for models of Lubell *et al*. (2000) and Neilson (2010), respectively. The third sample is designed as per NBCC 2010 and CSA-S16-09 (Table 1). The details of the model parameter for the designed sample and experimental specimen from Lubell *et al*. (2000) and Neilson (2010) are given in Tables 2, 3 and 4 respectively.

Results obtained from both the models are shown in Figs. 23 and 24. For both the models the initial stiffness is correctly estimated by the Eq.BF model. For Neilson's specimen (Fig. 24) the ultimate strength and the sequence of yielding match almost perfectly with shell-plate model and with Eq.BF model. The amount of error estimated with Eq.BF model for experimental validation is less than 2% in term of ultimate strength. The results from the Eq.BF model are in excellent agreement with those from the detailed three dimensional FE model with shell elements. With the

Table 2 Details of EQ.BF model parameters for designed single storey SPSW validation

α_s	0.31	Compression		Tension			
α_m	3.82	Stress	-16.2	σ_y (MPa)	385	$\alpha_k\sigma_y$	486.5
Area of brace	7982.3	Strain	0.00016	ε_0 (mm/mm)	0.00193	$\alpha_k\varepsilon_0$	0.00243

Table 3 Details of EQ.BF model parameters for validation of Lubell's (2000) specimen

α_s	0.56	Compression		Tension			
α_m	1.25	Stress	-10	σ_y (MPa)	320	$\alpha_k\sigma_y$	363.1
Area of brace	642.3	Strain	0.00032	ε_0 (mm/mm)	0.0016	$\alpha_k\varepsilon_0$	0.00182

Table 4 Details of EQ.BF model parameters for validation of Neilson's (2010) specimen

α_s	0.22	Compression		Tension			
α_m	4.87	Stress	-5.7	σ_y (MPa)	275	$\alpha_k\sigma_y$	344.5
Area of brace	1115	Strain	0.0001	ε_0 (mm/mm)	0.0013	$\alpha_k\varepsilon_0$	0.00164

specimen tested by Lubell *et al*. 2000 (Fig. 23), the agreement of pushover curves for Eq.BF model is satisfactory with approximately 6% error in ultimate strength. The initial stiffness is correctly estimated. The sequence of yielding is predicted correctly (as observed from the push-over curves) by both the models. However, the experimental pushover curve shows a slightly higher degree of strain hardening than those produced by the numerical models (both FEA and Eq.BF models.

Comparing the pushover curves from the designed single-storey sample (Fig. 25), it can be said that with Eq.BF model predicts accurately until the first yielding and the ultimate strength is estimated with less than 2% error. In the zone between first yielding and the yield plateau, the stiffness is over estimated by nearly 8%. Since, the Eq.BF model is an approximate model,

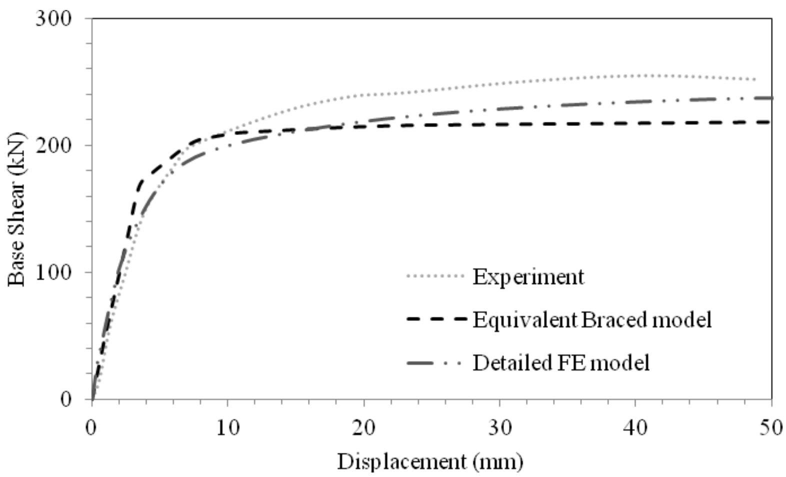


Fig. 23 Pushover curves from different models based on Lubell *et al*. (2000) specimen

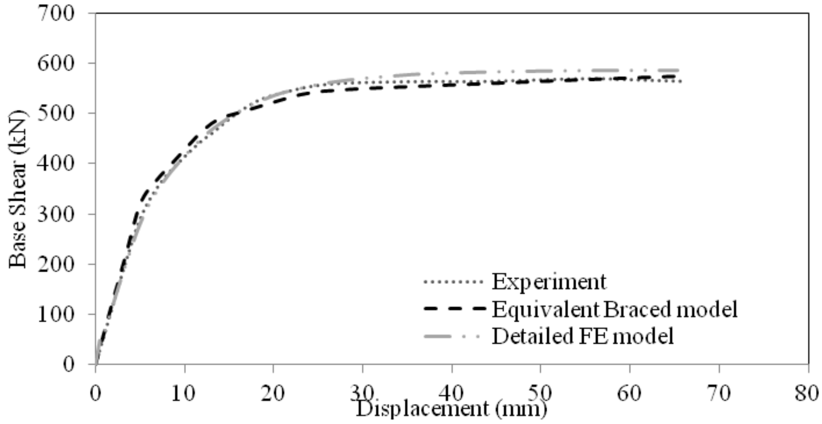


Fig. 24 Pushover curves from different models based on Neilson (2010) specimen

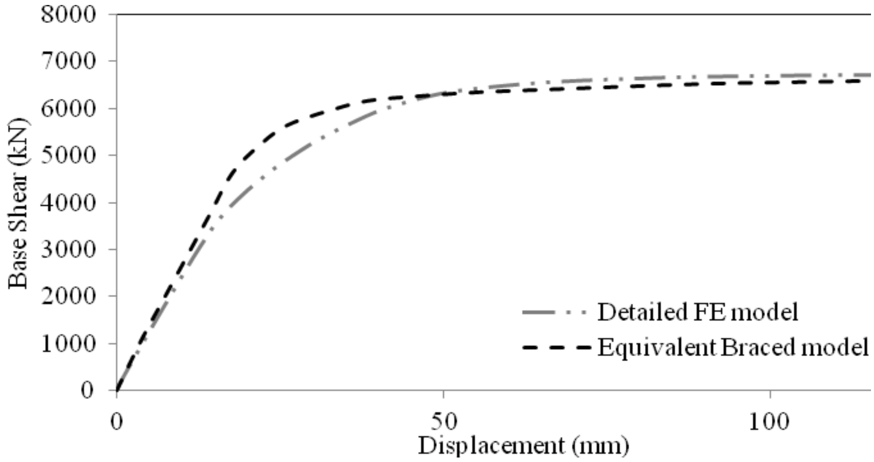


Fig. 25 Pushover curves from different numerical models for single-storey specimen

a perfect agreement with detailed FE model may not entirely achievable. However, the performance of the Eq.BF model in reproducing the pushover curve is adequate for assessing the overall behavior of a SPSW system.

Comparing the pushover curves from the designed single-storey sample (Fig. 25), it can be said that with Eq.BF model predicts accurately until the first yielding and the ultimate strength is estimated with less than 2% error. In the zone between first yielding and the yield plateau, the stiffness is over estimated by nearly 8%. Since, the Eq.BF model is an approximate model, a perfect agreement with detailed FE model may not entirely achievable. However, the performance of the Eq.BF model in reproducing the pushover curve is adequate for assessing the overall behavior of a SPSW system.

Three multistory sample structures (4-storey, 6-storey and 10-storey) were designed based on NBCC 2010 and CSA S16-09. All buildings have identical plan with a total plan area of 2,014 m^2 and represent hypothetical office buildings which are assumed to be located in Vancouver. The building has two identical shear walls to resist lateral forces in each direction and thus each shear wall will resist one half of the design seismic loads. For simplicity, torsion is neglected. Each shear wall panel is 7.6 m wide, measured from centre to centre of columns, and has an aspect ratio of 2.0 (i.e., storey height of 3.8 m).

The boundary members are designed according to CSA S16-09 (CSA 2009) to develop full capacity of infill plates. For 4-storey and 6-storey shear walls, a beam size of W610 × 372 is selected at the base of SPSW walls to anchor the forces developed due to yielding of bottom storey infill plates. For all other storeys, the beam section of W460 × 158 has been selected. For the 10-storey structure the base beam is selected as W690 × 419. For the 10-storey SPSW, similar beam sections of W610 × 372 are selected for first to sixth storey and for the top four storeys similar beam sections of W460 × 286 are selected. CAN/CSA-S16-09 (CSA 2009) also has provisions for the stiffness of the columns to ensure the development of an essentially uniform tension field in the infill plate. Table 5 presents the final columns sections and plate thicknesses for the four, six and ten storey SPSWs.

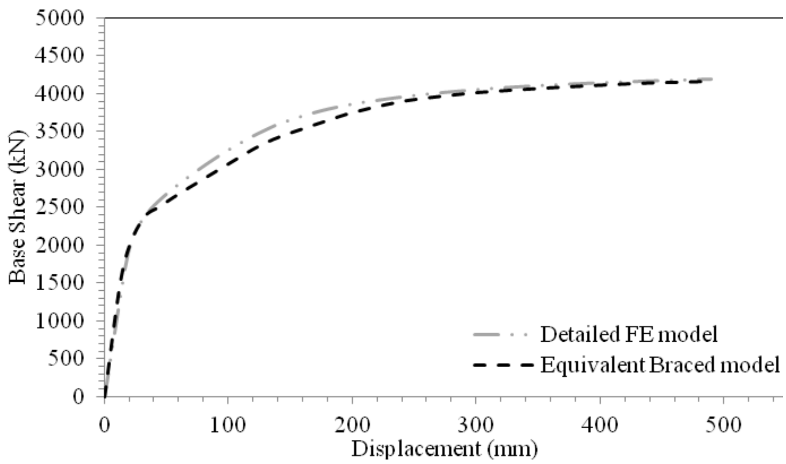


Fig. 26 Pushover curves from different models for 4-storey SPSW system

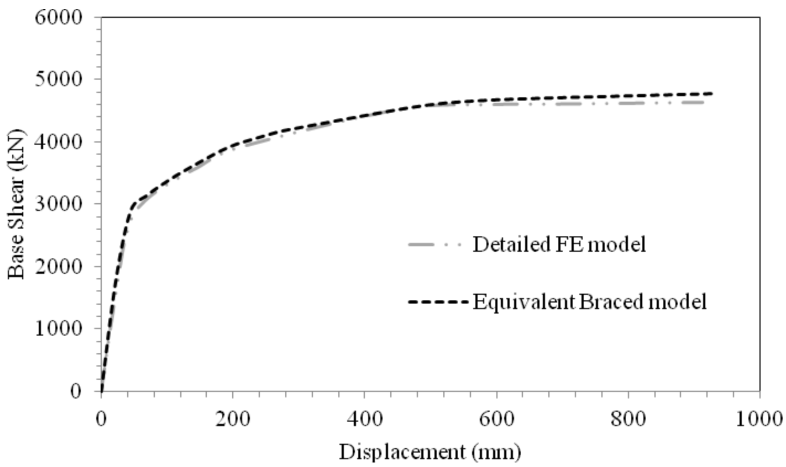


Fig. 27 Pushover curves from different models for 6-storey SPSW system

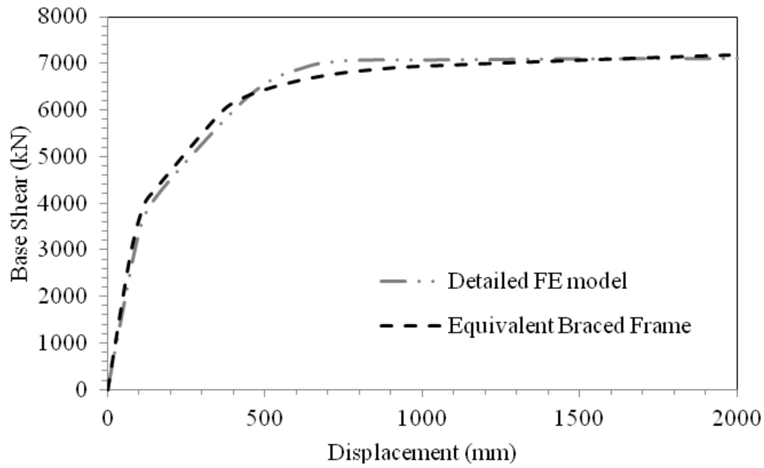


Fig. 28 Pushover curves from different models for 10-storey SPSW system

Table 5 Details of structural elements for 4-storey, 6-storey and 10-storey SPSW systems

Storey	10-storey wall		6-storey wall		4-storey wall	
	Plate thickness (mm)	Column	Plate thickness (mm)	Column	Plate thickness (mm)	Column
1	5	W360 × 900	3	W360 × 744	2.75	W360 × 634
2	5	W360 × 900	3	W360 × 744	2.5	W360 × 634
3	5	W360 × 677	2.75	W360 × 382	2	W360 × 382
4	4.5	W360 × 677	2	W360 × 382	1	W360 × 382
5	4	W360 × 509	1.5	W360 × 262		
6	3.5	W360 × 509	1	W360 × 262		
7	3	W360 × 463				
8	2.5	W360 × 463				
9	1.5	W360 × 463				
10	1	W360 × 463				

These multi-storey structures are modeled both in Abaqus (detailed FE model with shell-plate element) and Opensees (simplified Eq.BF model). Six-storey model parameters are given in Table 6 as sample calculation. Pushover curve corresponding to each storey level displacement and base shear are compared. It's worth mentioning that the time required for developing and analyzing the Eq.BF models is significantly less as compared to the detailed FE models. For all three multi-storey structures, pushover curves (Figs. 26, 27 and 28) obtained by the two types of models are in excellent agreement. The initial stiffness is very accurately estimated by Eq.BF model. Also, for all three cases, the ultimate strengths obtained from the simplified equivalent braced frame models are in excellent agreement with that from the detailed FE models. In addition, the sequence of hinge development in columns and the progress of material non-linearity induced in members also have a reasonably good match. The pushover analysis time using Eq.BF was less than a minute whereas, whereas in detailed FE using Abaqus, it took almost two hours. Thus, it can be

Table 6 Calculated Eq.BF model properties for 6-storey

Storey	1	2	3	4	5	6
α_s	0.42	0.41	0.37	0.32	0.23	0.15
α_m	3.68	3.70	3.72	4.39	5.72	6.08
Area (mm^2)	15206.16	15172.03	11991.05	10667.15	6771.65	2262.47
σ_{comp} (MPa)	30.86	29.34	26.93	20.12	10.04	5.79
ε_{comp}	0.000202	0.000201	0.000157	0.000121	6.6E-05	3.5E-05
$\alpha_k\sigma_y$ (MPa)	186.55	188.01	191.26	197.38	209.86	239.43
$\alpha_k\varepsilon_0$	0.000402	0.000191	0.000185	0.000187	0.000154	0.000262

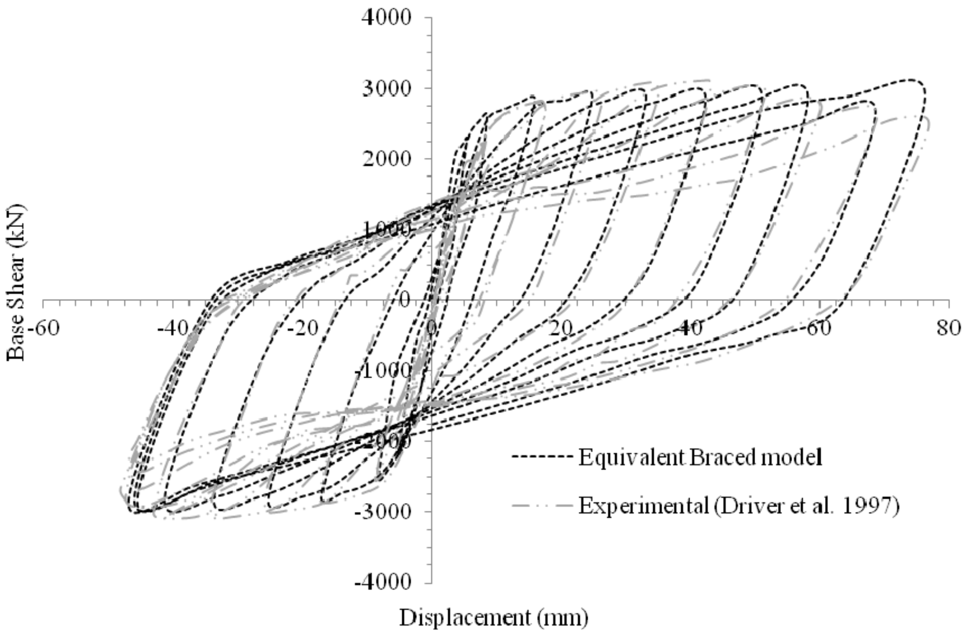


Fig. 29 Validation of hysteretic curve result for Driver *et al.* (1997) specimen

concluded that time efficient Eq.BF model is reasonably accurate and advantageous in terms of the modeling ease and analysis time to study the overall behavior SPSW systems. At every storey level pushover curves are in good agreement for all three multi-stories.

It must be noted that Eq.BF model is significantly more efficient in time and effort than any detailed FE model. A normal pushover analysis of single storey using detailed FE model takes about an hour, while using an Eq.BF model the analysis takes less than a minute in same computer. With less elements and parameters to deal with in Eq.BF model than in a detailed FE analysis, it is not only easier to model the structure, but also saves significant amount of time for repeated analysis. The real efficiency in the Eq.BF model is in its ability to develop response curves in cyclic loadings. For multi-storey cyclic analysis, Eq.BF took less than 5 minutes whereas, detailed the FE model took approximately 45 hours in the same computing hardware. A sample validation for cyclic curve has been shown with Driver *et al.* (1997) specimen of four storey SPSW structure (Fig. 29). The hysteric behavior is very accurately estimated by Eq.BF. Stability of the hysteretic curve shows stability of the model performance. However, in terms of pinching effect there is still scope for further improvement. There is excellent agreement of initial stiffness and ultimate strength both in positive and negative side.

5. Conclusions

An Equivalent Braced Model (Eq.BF) has been proposed in this research to study the behavior of steel plate shear walls. A series of nonlinear validations has been carried out to check the accuracy and efficiency of the proposed model. The proposed model provided excellent estimation of initial stiffness, final strength with an average error of less than 5%. Calculating the model parameters from the configuration of a given SPSW system is easy and not at all cumbersome. The main efficiency of the model lies in its time for analysis. A huge saving of time and effort is possible by using Eq.BF model rather than detailed FE models. Particularly, for industries dealing with performance based design, repeated analysis is required. So, this model can help speeding the design process since analysis is short and reliable. However, it must always be accounted that Eq.BF model is a simplified modeling technique shown to be an excellent approximation only for predicting the overall global performance of structure. Where detailed structural behavior is a concern, it is recommended that analysis with full scale detailed FE model is carried out. There is also scope for improvement in this simplified model like introducing factors that accounts the bending action of the bounding frame, introducing a proper calibration for the pinching effect of infill plates, etc. The parametric study for modeling involved through this study has also introduced a statistical method for modeling equivalent linear models when complex non-linearity is involved in structures. Development of simplified FE models that can estimate the complex behavior of SPSW systems in a global sense has been successfully achieved through this research.

Acknowledgments

The support of the Natural Sciences and Engineering Research Council of Canada (NSERC) is gratefully acknowledged.

References

Abaqus (2011), Abaqus/Standard user's manual, Version 6.11. 2011; Pawtucket (RI): Hibbitt, Karlsson, & Sorensen, Inc., HKS.

AISC (2005), Seismic Provisions for Structural Steel Buildings, ANSI/AISC 341-05; American Institute of Steel Construction, Chicago, IL, USA.

Berman, J.W. and Bruneau, M. (2005), "Experimental investigation of light-gauge steel plate shear", *J. Struct. Eng.*, **131**(2), 259-267.

Bhowmick, A.K., Grondin, G.Y. and Driver, R.G. (2010), "Performance of type D and type LD steel plate", *Can. J. Civil Eng.*, **37**, 88-98.

Bozdogan, K.B. (2013), "Free vibration analysis of asymmetric shear wall-frame buildings using modified finite element-transfer matrix method", *Struct. Eng. Mech.*, *Int. J.*, **46**(1), 1-17.

Choi, B.-J., Kim, W.-K., Kim, W.-B. and Kang, C.-K. (2013), "Compressive performance with variation of yield strength and width-thickness ratio for steel plate-concrete wall structures", *Steel Compos. Struct.*, *Int. J.*, **14**(5), 473-491.

CSA (2009), Limit states design of steel structures, CAN/CSA-S16-09; Canadian Standards Association,

Toronto, ON, Canada.
Dastfan, M. and Driver, R.G. (2008), "Flexural stiffness limits for frame members of steel plate shear wall systems", *Proceeding of Annual Stability Conference*, Structural Stability Research Council, Nashville, TN, USA, April.
Driver, R.G., Kulak, G.L., Kennedy, D.J.L. and Elwi, A.E. (1997), "Seismic behaviour of steel plate shear walls", Structural Engineering Report No. 215; Department of Civil and Environmental Engineering, University of Alberta, Edmonton, AB, Canada.
Elgaaly, M. (1998), "Thin steel plate shear walls behavior and analysis", *Thin-Wall. Struct.*, **32**(1-3), 151-180.
Elgaaly, M., Caccese, V. and Du, C. (1993), "Post-buckling behavior of steel-plate shear walls under cyclic loads", *J. Struct. Eng.*, **119**(2), 588-605.
Kharrazi, M.H.K. (2005), "Rational method for analysis and design of steel plate walls", Ph.D. Thesis, University of British Columbia, Vancouver, BC, Canada.
Kuhn, P., Peterson, J.P. and Levin, L.R. (1952), "A Summary of Diagonal Tension, Part I: Methods of Analysis", NACA technical note 2661; National Advisory Committee for Aeronautics, Washington, D.C., USA.
Lubell, S., Prion, H.G.L., Ventura, C.E. and Rezai, M. (2000), "Unstiffened steel-plate-shear wall performance under cyclic loading", *J. Struct. Eng.*, **126**(4), 453-460.
Mazzoni, S., McKenna, F., Scott, M.H. and Fenves, G.L. (2007), "OpenSees command language manual", *Pacific Eathquake Engineering Research Center*, University of California, Berkeley, CA, USA.
Memarzadeh, P., Azhari, M. and Saadatpour, M.M. (2010), "A parametric study on buckling loads and tension field stress patterns of steel plate shear walls concerning buckling modes", *Steel Compos. Struct., Int. J.*, **10**(1), 87-108.
Mohammad, R.B., Grondin, G.Y. and Elwi, A.E. (2003), "Experimental and numerical investigation of steel plate shear wall", Structural Engineering Report No. 254; Department of Civil and Environmental Engineering, University of Alberta, Edmonton, AB, Canada.
Neilson, D.A.H. (2010), "Welding of light gauge infill panels for steel plate shear walls", M.Sc. Thesis, Department of Civil and Environmental Engineering, University of Alberta, Edmonton, AB, Canada.
NRC (2010), National Building Code of Canada (NBCC), Canadian Commission on Building and Fire Codes; National Research Council of Canada, Ottawa, ON, Canada.
Shahi, R., Lam, N., Gad, E. and Wilson, J. (2013), "Protocol for testing of cold-formed steel wall in regions of low-moderate seismicity", *Earthq. Struct., Int. J.*, **4**(6), 629-647.
Topkaya, C. and Atasoy, M. (2009), "Lateral stiffness of steel plate shear wall systems", *Thin-Wall. Struct.*, **47**(827-835), 827-835.
Thorburn, L.J., Kulak, G.L. and Montgomery, C.J. (1983), "Analysis of steel plate shear walls", Structural Engineering Report No. 107; Department of Civil Engineering, University of Alberta, Edmonton, AB, Canada.
Vatansever, C. and Yardimci, N. (2011), "Experimental investigation of thin steel plate shear walls with different infill-to-boundary frame connections", *Steel Compos. Struct., Int. J.*, **11**(3), 251-271.
Wagner, H. (1931), "Flat sheet metal girders with very thin metal web, Part III", NACA technical note 605; National Advisory Committee for Aeronautics, Washington, D.C., USA.

DL